AN OVERVIEW ON THE EQUATORIAL ELECTROJET THEORETICAL GROUNDS

C.J. ZAMLUTTI, J.H.A. SOBRAL and M.A. ABDU
Instituto de Pesquisas Espaciais INPE
C.P.515 - 12201 São José dos Campos, S.P., Brazil

ABSTRACT

The grounds on which the equatorial electrojet theory is based are reexamined in a way as to suggest specific additional implementations in the existing electrodynamical modeling of this phenomena, making use of now existing improved computer processing speeds.

1. INTRODUCTION

The equatorial electrojet is part of a whole gama of phenomena produced by the action of electric and magnetic forces. All such phenomena are grounded on the steady state Maxwell equations, together with the steady state equations of motion for all ionized particles (see review by Forbes, 1981). Collecting together all these equations the basic relations for the dynamo action are established. The equations are:

$$\nabla \cdot \underline{j} = 0 , \tag{1}$$

$$\underline{E} = -\nabla\phi , \tag{2}$$

$$\underline{j} = \underline{\underline{\sigma}} \cdot (\underline{E} + \underline{u} \times \underline{B}) ; \tag{3}$$

where $\underline{j}$ stands for the electric current, $\underline{E}$ for the electric field, $\underline{u}$ for the neutral wind, B for the magnetic field and $\underline{\underline{\sigma}}$ for the electrical conductivity. Underlined symbols represent vector entities and symbols underlined twice denote tensor entities.

Even though considerable progress has been achieved with the modelling by Takeda and Maeda (1980), who solved the problem of a globally self consistent equatorial electrojet structure, there are still some aspects to be explored as far as the global electrodynamics is concerned. Some problems have been raised before. Cole (1969) calculated the magnetic disturbances produced by E-W currents in terms of the time dependent continuity equation; K. Maeda (1977) included the effect of the Coriolis force and ion neutral collision frequency to compute the wind velocity. Still questioning the accuracy on the computational procedure is the work by Maeda and Murata(1968) concerning the shape of the diurnal variation of the wind velocity. An actual questioning of the basic equations to explain mutually interacting phenomena, arised with the theory for the two-stream ion wave instability associated with the equatorial electrojet (see Farley, 1963). In the last theory Equations 1 and 2 are no long true.

Regardless the hierarchy of the phenomena, as far as completeness of the system of equations is concerned, it seems necessary a reexamination of the grounds of what would be a fairly general theory, to suggest a consistent system of equations compatible with nowadays capabilities of simulations using macro systems of equations.

2. THE BASIC EQUATIONS

The governing equations to describe the phenomena involving the mechanical and electromagnetic aspects of the upper atmospherere are:

$$\partial \underline{h}/\partial t = \delta \underline{h} - \nabla \cdot \underline{\phi} , \tag{4}$$

$$\frac{\partial}{\partial t}\begin{pmatrix}\varepsilon_0 \underline{E}\\ -\underline{B}\end{pmatrix} = \nabla \times \begin{pmatrix}\mu_0^{-1}\underline{B}\\ -\underline{E}\end{pmatrix} - \begin{pmatrix}\underline{j}\\ \underline{\Omega}\end{pmatrix} , \tag{5}$$

where Equation 4 is a vector equation representing the hydrodynamics equations and Equation 5, also of vector type, is the representation of the electromagnetic equations. Here $\underline{h}$ is the vector whose components are the hydrodynamics parameters: density, velocity and temperature. The symbol δ stands for the local rate of change of the parameter which follows it. The matrix $\underline{\phi}$ is the matrix of the fluxes of the respective parameters.

The symbols ε_0 and μ_0 are respectively the permitivity and permeability of the vacuum, and j represents the total macroscopic current.

The hydrodynamics contribute with five equations for each particle species. The electromagnetics contributes with six equations no matter the number of ions present. Therefore, if only one type of ion is present we must consider 15 hydrodynamics equations and 6 electromagnetic equations in a total of 21 equations to properly simulate the desired phenomena. If two ion species are present 5 more equations are necessary and the total is increased to 26 equations. The situation becomes more complex because we are dealing with nonlinear coupled equations. In practice, physical criteria are invoked to eliminate either nonlinearities or coupling.

We will not digress on the ways to reduce the number of transport equations to the equations for density, velocity and temperature. This is a lengthy task and has been considered in detail by Burgers (1969), Schunk and walker (1971) and Schunk (1975). As for the electromagnetic equations they are the well known Maxwell equations and need no further comment. Moreover, assumption is made that the lighter species already come into equilibrium with themselves. This means that the effect of photoelectrons can be modelled as sources for energy and charge.

With the above considerations it is possible to identify:

$$\delta h_1 = Q_1 - L_1 \quad ,$$

$$\delta \underline{h}_2 = -\nabla p_2 + \rho_2 \underline{g} + q_2 \, (\underline{E} + \underline{v}_2 \times \underline{B}) - \sum_c \rho_2 \, \nu_2 \, (\underline{v}_2 - \underline{c}_2) - 2\rho_2 \, \underline{\Omega} \times \underline{v}_2$$

$$\delta h_3 = (Q_3 - L_3) + \underline{v}_2 \cdot [\delta \underline{h}_2 + \nabla(\gamma p_2)] \quad ,$$

where Q_1 and L_1 stand for production and loss of mass density respectively. Q_3 and L_3 represent production and loss of energy density, P is the pressure, ρ the density of mass, q the charge density, g the acceleration of gravity, $\underline{v}_2$ the average velocity of the considered specie, Ω is the earth's angular velocity, γ is the ratio between specific heats, ν_2 the collision frequency and c_2 the velocity of the colliding specie. Here, the numbered symbols have to be considered for each particle specie present For the neutrals $q_2 = 0$.

The corresponding fluxes are:

$$\underline{\phi}_1 = \rho_2 \underline{v}_2 \quad ,$$

$$\underline{\underline{\phi}}_2 = \underline{\underline{\phi}}_{2\,momentum} + \underline{\underline{\phi}}_{2\,viscosity},$$

$$\underline{\underline{\phi}}_{2\,momentum} = \rho_2 \underline{v}_2 \underline{v}_2 \quad ,$$

$$\underline{\underline{\phi}}_{2\,viscosity} = - \eta_2 [\nabla \underline{v}_2 + (\nabla \underline{v}_2)^t - (2/3)(\nabla \cdot \underline{v}_2) \underline{\underline{I}}] \quad ,$$

$$\underline{\phi}_3 = (\rho_2 \varepsilon + \gamma p_2) \underline{v}_2 + \lambda \nabla T_2 \quad ,$$

where ε stands for internal energy; λ is the thermal conductivity and T_2 is the temperature of the considered particle. Usually $\rho\varepsilon$ is replaced by $3p/2$. The current $\underline{j}$ may be identified as:

$$\underline{j} = (\sum_{ions} q_2 \underline{v}_2) - (q_2 \underline{v}_2)_{electrons}$$

The above system of hydrodynamic equations is by no means complete but it contains all the terms that have been consagrated as significant in the literature.

As for the electromagnetic equations they provide the necessary complementary relations such that the number of equations and unknowns be the same.

3. THE MAGNETOHYDRODYNAMIC APPROACH TO ELECTROJETS

The modeling of any phenomena involves essentially three aspects:

a) The selection of the appropriate governing equations;

b) limitation of the solution by either initial conditions or boundary conditions or both;

c) the efficiency to describe the known parameters (e.g. sources and sinks, the geomagnetic field).

As far as part b is concerned considerable discussion can be founded in the literature(see Forkes, 1981, Takeda and Maeda, 1980 and references therein) and we will not pursue this matter further on. Part c also received considerable attention (Torr et al 1980; Vickrey et al 1982). We will therefore restrict ourselves to the considerations on part a.

The governing equations were presented in Section 2 and constitute a magnetohydrodynamic formulation for the phenomena. From them, simplified sets were chosen before all of which comprise severe limitations to the proposed models. We will comment on next about their significance.

The steady state approach to both Equations 4 and 5 for the ionized particles has the intrinsic constraint of the electric field being electrostatic, from Equation 5. Moreover, from Equation 4 this approach implies that rearrangement of ionization is much more rapid than the diurnal variation of the source mechanisms for h_1 and h_3. While this approximation is not so severe at equatorial latitudes it is certainly not valid at auroral latitude, where the particles precipitation mechanism presents time scales as short as 1 hour. Additionally, the steady state simulation is only adequate to flow type phenomena since it excludes all time dependence. Therefore, instabilities connected with electrojets connot be obtained with this type of approach.

Another approach usually adopted, in connection with the steady state for ionized particles, is to assume a wavelike behavior for the neutral particles. Again this may be tolerated at non-auroral latitudes but is questionable at auroral latitudes, for the reason that particles precipitations are non periodic phenomena. The separation of the coupled system of equations for ionized and neutral particles is also questionable, mainly at auroral latitudes where Joule heating is a very effective energy interaction mechanism (Strauss 1978, Straus and Schulz, 1976).

Probably the only acceptable simplifications, in current use, are those which concern orders of magnitude like, for instance, the neglect of gravity, collision and Coriolis terms in the equation of motion for the electrons.

In order to study the strongly related phenomena which occurs at both equatorial and auroral latitude, we are then left with the only alternative of solving the whole system of magnetohydrodynamic equations. This used not to be possible until very recently as ten years ago. The now existing improved computer processing speeds are very encouraging for the solution of large system of equations. For this reason we suggest that attempts be made to study the time evolution of the system of Equations 4 and 5. Time stepping procedures are thus a possible fate for the theoretical study of the equatorial electrojet. As for the boundary conditions the present situation as presented by Takeda and Maeda (1980) seems not to be very critical.

4. DISCUSSION AND CONCLUSION

In this work we raised some questions as for the efficiency of the now existing theoretical models to describe what is actually going on in electrojet phenomena (equatorial and auroral). In particular, the strong connection between the electrojet phenomena and instabilities can only be studied with a model which reproduces both phenomena at once. Questions concerning the aplicability of now existing theoretical models for instabilities were raised before by Fejer (1984), regarding auroral E region irregularities, and by Fejer and Kelley (1980), as for the nonlinear evolution of equatorial E region irregularities. The prsent theoretical proposal is an answer to these questions.

REFERENCES

Burgers, J.M. "Flow equations for composite gases", Academic Press, New York, N.Y. 1969.

Cole, K.D. "Theory of electric currents in ionospheric E-layers", Planet. Space Sci. 17, 1977-1992, 1969.

Farley, D.T. "A plasma instability resulting in field-aligned irregularities in the ionosphere", J. Geophys. Res. 68, 6083-6097, 1963.

Fejer, B.G. and M.C. Kelley. "Ionospheric rregularities", Rev. of Geophys. Space Phys. 18, 401-454, 1980.

Fejer, B.G. "Small scale plasma irregularities in the auroral lower ionosphere", Proceedings of the MIT Symposia on the physics of space plasmas SPI Conference Proceedings and Reprint Series, 5, 73-97, 1984.

Forbes, J.M. "The equatorial electrojet", Rev. Geophys. Space Phys, 19, 469-504, 1981.

Maeda, H. and Murata, H. "Electric currents induced by nonperiodic winds in the ionosphere, 1". J. Geophys. Res., Space Phys. 73, 1077-1092, 1968.

Maeda, K. "Conductivity and drifts in the ionosphere". J. Atmos. Ter. Phys. 39, 1041-1053, 1977.

Schunk, R.W. and Walker, J.C.G. "Transport Processes in the E region of the ionosphere", J. Geophys. Res. 76, 6159-6171, 1971.

Schunk, R.W. "Transport equations for Aeronomy", Planet Space Sci, 23, 437-485, 1975.

Straus, J.M and Schulz, M. "Magnetospheric convection and upper atmospheric dynamics J. Geophys. Res. 81, 5822,1976.

Straus, J.M. "Dynamics of the thermosphere at high latitudes", Reviews of Geophys. Space Phys. 16, 183-194, 1978.

Takeda, M. and Maeda, H. "Three dimensional structure of ionospheric currents 1, Currents caused by diurnal tidal winds", J. Geophys. Res. 85, 6895-6899, 1980.

Torr, M.R., Richards, P.G. and Torr, D.G. "A new determination of the ultraviolet heating efficiency of the thermosphere, J. Geophys. Res. 85, 6819-6826, 1980.

Vickrey, J.F., Vondrak, R.R. and Matthew, S.J. "Energy deposition by precipitating particles and joule dissipation in the auroral ionosphere", J. Geophys. Res. 87, 5184-5196, 1982.