ROTATIONAL MOTION OF AN ARTIFICIAL SATELLITE PERTURBED BY SOLAR RADIATION PRESSURE

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ABSTRACT

The motion of a satellite about its center of mass is studied using a semi-analytical method. Torques produced by conservativa and non conservative forces are considered. An analytical model is proposed for solar radiation torques. Andoyer variables are used to describe the rotational motion. Analytical equations are used to transform osculating to a mean set of differential equations. Since the mean equations are more slowly varying, a numerical integration using large step size can be perfomed to obtain the mean state at a later time.

INTRODUCTION

The use of analytical methods to analyse the rotational motion of artificial satellites has been of prime importance in connection with many satellite missions. However an analytical solution become very cumbersome when we use realistic models for non gravitational perturbative torques.

In this paper we present a semi-analytical method to propagate the elements that describe the attitude of an artificial satellite. The solution is obtained by first using an averaging method to eliminate short period terms. These mean set of differential equations are then integrated numerically (or analytically when it is possible). Since the mean equations are more slowly varying than the osculating ones the numerical integration can be perfomed using larger step size than usual.

Furthermore it will be considered here an artificial Earth satellite of cilindrical shape, traveling in a fixed elliptical orbit, so that all the torques, with the exception of gravitational and solar radiation pressure, can be ignored. The solar radiation pressure torque will be considered of second order with respect to the gravitational.

SOLAR RADIATION PRESSURE TORQUES

The general expression for the solar radiation pressure torques can be given by (Georgevick (1973)):

$$\vec{N} = -\frac{k}{R^2}\iint \{ B(\theta)\vec{r} \times \hat{n} + b\cos\theta\, \vec{r} \times \hat{u} \} dA \tag{1}$$

where: a) $k = 1{,}01 \times 10^{17}$ kg m/s; b) R is the Sun-satellite distance; c) $\vec{r}$ gives the position of the surface element dA with respect to the center of mass of the satellite; d) $\hat{n}$ is a unit vector along the outer normal; e) $\hat{u}$ is a unit vector along the direction of the flux ; f) $\cos\theta = \hat{n} \cdot \hat{u}$; g) $B(\theta) = a\cos\theta + c\cos^2\theta$; h) a, b and c are parameters related with the coefficients of specular and diffuse reflexions.

Let Oxyz be a system with its origin at the center of mass of the satellite with its axes parallel to the principal central axes of inertia of the satellite. This system can be related to the absolute equatorial system by the matrix L whose elements are functions of the Andoyer variables (Kinoshita (1972)). Thus the vectors $\vec{r}$, $\hat{n}$ and $\hat{u}$ can be expressed in the system Oxyz and then:

$$\vec{M} = N_1\,\hat{e}_x + N_2\,\hat{e}_y + N_3\,\hat{e}_z \tag{2}$$

where the N_i $(i = 1,2,3)$ are functions of the Andoyer variables (Kinoshita (1972)), of the right ascension α and declination δ of the Sun, of the spherical coordinates of the surface element dA and its outer normal.

EQUATIONS OF MOTION

The equations of the rotational motion of a rigid cylindrical artificial satellite of mass M, taking into account the gravitational potencial between the bodies and the direct solar radiation pressure can be put in the following form:

$$\frac{d\ell_i}{dt} = \frac{\partial H}{\partial L_i} + P_i \tag{3a}$$

$$(i = 1, 2, 3)$$

$$\frac{dL_i}{dt} = -\frac{\partial H}{\partial \ell_i} + Q_i \tag{3b}$$

with

$$H = H_0(L_1, L_2) + H_1(L_i, \ell_i) \qquad i = 1,\ldots,6 \tag{4}$$

and

$$H_o = \frac{1}{2}\{(\frac{1}{C} - \frac{1}{A})L_1^2 + \frac{1}{A} L_2^2 \} \qquad (5a)$$

$$H_1 = \frac{\mu^4 M^6}{L_4^6} (C - A)f \qquad (5b)$$

Here, L_i and ℓ_i, for $i = 1,2,3$, are the Andoyer variables, and, for $i = 4,5,6$, are the Delaunay variables. Also: a) $A(B = A)$ and C are the principal moments of inertia of the satellite; b) f is a function of the ℓ_i and L_i, where the ℓ_i variables $(i = 1,\ldots,6)$ appear in the arguments of the cosines. This function is shown in Zanardi (1986) taking $B = A$; c) P_i and Q_i $(i = 1,2,3)$ come from the solar radiation pressure (shadow was not considered).

When expressed in terms of Andoyer variables the functions P_i and Q_i take the following form

$$P_i = P_i(\ell_i, L_i, \alpha, \delta, a, b, c) = P_{is} + P_{ip} \qquad (6a)$$

$$Q_j = Q_{jp}(\ell_i, L_i, \alpha, \delta, a, b, c) \qquad (6b)$$

$$Q_3 = Q_3(\ell_i, L_i, \alpha, \delta, a, b, c) = Q_{3s} + Q_{3p} \qquad (6c)$$

$$i = 1,2,3 \quad ; \quad j = 1,2.$$

where P_{is} and Q_{3s} are terms independent of ℓ_2 and ℓ_3

APPLICATIONS OF THE HORI METHOD

The solution of system (3) when $P_i = Q_i = 0$, can be obtained by the Hori method (Hori (1966)). The first order solution will be (Zanardi (1986)):

$$L_i = L_i^* + \delta L_i^* \qquad (7a)$$

$$(i = 1, 2, 3)$$

$$\ell_i = \ell_{io} + \delta \ell_i^* \qquad (7b)$$

where L_i^* are constants; $\ell_{io} = \ell_i^* + n_i t$; ℓ_i^* are constants; δL_i^* and $\delta \ell_i^*$ are periodic functions; $n_i = n_i(L_1^*, L_2^*)$ and, $n_3 = 0$.

FIRST ORDER PERTURBATION IN P_i AND Q_i

Applying Lagrange's method of variation of parameters in the system (3) we obtain the following system (Vilhena de Moraes (1981)):

$$\dot{\ell}_i^* = (n_i + \delta n_i) + \bar{P}_i + \sum_{k=1}^{3} \{ \bar{P}_k \frac{\partial(\delta\bar{\ell}_i)}{\partial \ell_k^*} - \bar{Q}_k \frac{\partial(\delta\bar{\ell}_i)}{\partial L_k^*} \} \quad (8a)$$

$$(i = 1, 2, 3) \quad (8b)$$

$$\dot{L}_i^* = \bar{Q}_i - \sum_{k=1}^{3} \{ \bar{P}_k \frac{\partial(\delta\bar{L}_i)}{\partial \ell_k^*} - \bar{Q}_k \frac{\partial(\delta\bar{L}_i)}{\partial L_k^*} \}$$

where the barred variables are mean functions of L_i^* and ℓ_i^* and $\delta\bar{L}_i$ and $\delta\bar{\ell}_i$ are parts of the solution without solar radiations pressure given by equations (7a) and (7b). Also

$$n_i + \delta n_i = \frac{\partial H^*}{\partial L_i^*} \quad (9)$$

where H^* is the Hamiltonian of the unperturbed problem ($P_i = Q_i = 0$) after the application of the Hori's method.

It is worthy to note that a transformation was introduced to avoid spurious mixed terms.

The system (8) is, generally, difficult to be integrated analitically when we consider perturbations that are not deterministically known. Meanwhile, neglecting coupling terms we get:

$$\dot{\ell}_i^* = n_i + \delta n_i + \bar{P}_i \quad (10a)$$

$$(i = 1, 2, 3)$$

$$\dot{L}_i^* = \bar{Q}_i \quad (10b)$$

where, by equations (6a),(6b),(6c), $\bar{P}_i = P_{i_s}, \bar{Q}_3 = Q_{3_s}, \bar{Q}_j = 0$ for $j = 1,2$.

These simplified averaged variational equations of motion, where the right-hand sides are free from the fast varying variables ℓ_1 and ℓ_2, can be integrated numerically with large step size.

Given the initial conditions for the osculating elements the associated averaged elements can be evaluated from the transformation (7) with reasonable accuracy by iteration.

Therefore the analytical expression of the solution of the system (10) can be obtained by the method of sucessive approximations and then

$$\ell_i^* = \ell_{i_o}^* + \ell_{i_s}^* t \; ; \; \ell_{i_o}^* \text{ and } \ell_{i_s}^* \text{ are constants}; \; i = 1, 2, 3 \quad (11a)$$

$$L_j^* = L_{j_o}^* = \text{const.} \; ; \; j = 1, 2 \quad (11b)$$

$$L_3^* = L_{3_0}^* + L_{3_s}^* t \; ; \; L_{3_0}^* \text{ and } L_{3_s}^* \text{ are constants} \quad (11c)$$

Thus, the solar radiation pressure torque gives rise to secular perturbations in all the angular variables t_i and to a secular variation in the inclination of the plane which is perpendicular to the angular momentum.

CONCLUSIONS

It has been showed a semi-analytical method to study the influence of the gravity-gradient and solar radiation pressure torques on the rotational motion of an artificial Earth's satellite. An analytical expression was given for the solar radiation pressure torque so that analytical solution to averaged equations could be obtained by the method of the successive approximation. For a rather sophisticated models and /or other perturbations, a numerical evaluation may be the most feasible approach to estimate the mean state.

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