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ABSTRACT

An interpretation of three-dimensional simple supergravity as a pure Chern-Simons gauge action is shown to be valid up to the one loop level. Canonical quantization of this system does not lead to an explicit definition of the physical Hilbert space. Hence another formulation of the $N = 1$ three-dimensional supergravity is introduced. In this formalism an explicit definition of the physical Hilbert space is possible, but still one has to solve the problems of showing that there exists a global set of coordinates and of defining the inner product.

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1 Introduction

Quantization of gravity is one of the most important problems in physics. Even though this problem can be solved in the context of string theories, one still hopes that standard gravitation can be quantized without running into problems of non-renormalizability by a canonical formalism. The canonical formalism of standard gravity does not lead directly to a physical Hilbert space definition[1]. A new step in this direction is a formalism due to Ashtekar[2], in which there are some additional degrees of freedom which become eliminated dynamically by some new constraints. In this formalism the coordinates of the phase space are connection one forms and in [3], even though the problems of normal ordering and inner product are unsolved, an attempt to construct a physical Hilbert space is given.

In three space-time dimensions gravity is a completely integrable theory. Witten has shown that there exists an interpretation of this theory as Chern-Simons action[4], thus due to the unitarity of the latter theory it solves also the problem of physical interpretation of the Wheeler-de Witt equation. He has also shown that there exists an explicit definition of the physical Hilbert space of three-dimensional gravity, when the space-time manifold is taken as $\mathbf{R} \times \Sigma$, where Σ represents the Riemann surface.

In fact it was already known that some supergravity actions can be interpreted as pure Chern-Simons gauge action[5].

In this work we intend to generalize this procedure to the $N = 1$ supergravity in three dimensions and to give a procedure for finding an explicit definition of the physical Hilbert space. In the following section we will examine the pure Chern-Simons gauge action and show that the one loop quantum effective action is the same as the classical one (see also [7]). In section 3 we will show that three-dimensional supergravity can be interpreted as a pure Chern-Simons gauge action. But a real polarization is lacking so that an explicit definition of the physical Hilbert space is not obvious. Hence in the last section we will give another formalism of $N = 1$ supergravity, which has the desired properties for defining explicitly a physical Hilbert space. But still one has to show that there exists some global coordinates and a consistent definition of the inner product.

2 Chern-Simons Action

In $d=3$ there exists a gauge action which does not depend on the space-time metric.In terms of the 1-form $A = A^A_\mu T_A dx^\mu$ it is given as

$$I_{CS} = \frac{k}{2}\int_M Tr(A\wedge dA + \frac{2}{3}A\wedge A\wedge A), \qquad (1)$$

where the integral is taken over the three-manifold M and the trace is over the gauge (Lie or super Lie) group G which is generated by T_A. This action is invariant under the infinitisimal gauge transformations

$$\delta A_\mu = \partial_\mu \Lambda + [A_\mu, \Lambda], \qquad (2)$$

where $\Lambda = \Lambda^A T_A$ is an infinitisimal gauge parameter[1]. But generally it is not invariant under finite gauge transformations [8]. The physically important quantity is the partition function and its invariance under the finite transformations leads to the quantization of k. We deal only with the infinitisimal gauge transformations so that k is arbitrary. In fact in the following sections we will set k=1 for convenience. The above action can be written in terms of the Killing metric g_{AB}, structure constants f_{ABC} of the gauge algebra and the totally antisymmetric tensor $\epsilon^{\mu\nu\rho}$ as

$$I_{CS} = \frac{k}{2}\int_M d^3x\epsilon^{\mu\nu\rho}(g_{AB}A^A_\mu\partial_\nu A^B_\rho + \frac{2}{3}f_{ABC}A^A_\mu A^B_\nu A^C_\rho). \qquad (3)$$

Thus for a given group one must first show that there exists a nonvanishing Killing form.

The partition function

$$Z = \int DA^A_\mu exp[iI_{CS}(A)] \qquad (4)$$

gets the largest contribution from the solution of the classical equations of motion in the large k limit. Hence in this limit we may expand A in the neighbourhood of the classical solution A_{cl} :

$$A = A_{cl} + A_{qu},$$

[1] In the case of super-Lie groups for which a nonvanishing Killing form exists the invariance follows from the generalized Jacobi identities and from the property of Killing metric: $g_{AB} = (-1)^{\epsilon(A)\epsilon(B)}g_{DA}$, where $\epsilon(A) = \epsilon(B)$.

where A_{qu} represents the quantum fluctuations. Using this in the partition function yields

$$Z = e^{iI_{CS}(A_{cl})}\int DA_{qu}exp[\int_M Tr(A_{qu}\wedge DA_{qu})], \qquad (5)$$

where $\frac{2}{ik}D$ is the covariant derivative with the connection 1-form A_{cl}. By using the Faddeev-Popov procedure for fixing the gauge as

$$D_\mu A^\mu_{qu} = 0,$$

leads to [6]

$$Z = e^{iI_{CS}(A_{cl})}\int DA_{qu}DBD\eta D\bar\eta exp\{\int_M[Tr(A_{qu}\wedge DA_{qu}) + Tr2BD_\mu A^\mu_{qu} + Tr\bar\eta D_\mu D^\mu\eta]\}, \qquad (6)$$

where η ($\bar\eta$) is ghost(antighost) and B is an auxiliary field. The Grassmann parities, which we indicate by ϵ, of these fields are given as

$$\epsilon(\eta^A) = \epsilon(\bar\eta^A) = \epsilon(A^A_\mu) + 1$$
$$\epsilon(B^A) = \epsilon(A^A_\mu).$$

Now by introducing a compact notation:

$$\phi = (A^\mu_{qu}, B),$$

the partition function (6) can be written as

$$Z = e^{iI_{CS}(A_{cl})}\int D\phi D\eta D\bar\eta exp\{\int_M[Tr(\phi L\phi) + Tr(\bar\eta D^2\eta)]\}, \qquad (7)$$

where $D^2 = D_\mu D^\mu$ and the matrix L is given as

$$L = \begin{pmatrix} -\epsilon^{\mu\nu\rho}D_\rho & -D^\mu \\ D^\nu & 0 \end{pmatrix}.$$

One can formally perform the integral in (7) to find

$$Z = e^{iI_{CS}(A_{cl})}\frac{det D^2}{det^{1/2}L}. \qquad (8)$$

Now one may write

$$det L = det^{1/2}(\tilde{L}L),$$

where $\tilde{L}$ indicates the transpose of the matrix L. This can be easily calculated to yield

$$det^{1/2} L = det^{1/4}(\tilde{L}L) = det D^2, \tag{9}$$

by using the fact that the classical equations of motion are

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] = 0. \tag{10}$$

Thus in the large k limit the partition function yields

$$Z = exp[i I_{CS}(A_{cl})], \tag{11}$$

which means that the quantum effective action at the one loop level is still given by the classical action.

3 Simple Supergravity

In this section we would like to give an interpretation of simple supergravity as Chern-Simons action by following the procedure used by Witten for the standard gravity[4]. $N = 1$ super-Poincaré algebra in three dimensions ($ISO(2,1|2)$)is given in terms of real generators as

$$\begin{aligned} [J^a, J^b] &= \epsilon^{abc} J_c, \\ [J^a, P^b] &= \epsilon^{abc} P_c, \\ [J^a, Q_\alpha] &= -\tfrac{1}{2}\gamma_\alpha^{a\beta} Q_\beta, \\ \{Q_\alpha, Q_\beta\} &= -\tfrac{1}{4}(\gamma^a C^{-1})_{\alpha\beta} P_a, \\ [P^a, P^b] &= [P^a, Q_\alpha] = 0, \end{aligned} \tag{12}$$

where $a, b, c = 0, 1, 2$; $\alpha, \beta = 1, 2$. Three dimensional gamma matricies are defined to satisfy:

$$\begin{aligned} \{\gamma_a, \gamma_b\} &= 2\eta_{ab}, \\ [\gamma_a, \gamma_b] &= 2\epsilon_{abc}\gamma^c, \end{aligned}$$

and we use a real representation so that non-zero elements of the charge conjugation matrix $C^{\alpha\beta}$ are $C^{12} = -C^{21} = 1$. We also used the definition $J^a = \frac{1}{2}\epsilon^{abc} J_{bc}$.

It is easy to generalize the non-degenerate, invariant, quadratic expression given by Witten for the $ISO(2,1)$, to the above algebra:

$$K = P_a J^a + \bar{Q}^\alpha Q_\alpha, \tag{13}$$

where $\bar{Q}^\alpha = Q_\beta C^{\beta\alpha}$. The invariance of it under the group generators can easily be shown.

If we normalize the trace of this expression as

$$Tr\ K = Tr\ v_A d^{AB} v_B = 1,$$

where

$$v_A = (P_a, J_a, Q_\alpha),$$

the Killing metric will be

$$g_{AB} = d^{-1}_{AB} = \begin{pmatrix} 0 & 2\eta_{ab} & 0 \\ 2\eta_{ab} & 0 & 0 \\ 0 & 0 & C_{\alpha\beta} \end{pmatrix}, \tag{14}$$

by using the definition:

$$C^{\alpha\beta} C_{\beta\gamma} = \delta^\alpha_\gamma.$$

Now by making use of this metric and the gauge field

$$A_\mu = e^a_\mu P_a + \omega^a_\mu J_a + \bar{\Psi}^\alpha_\mu Q_\alpha,$$

(1) can easily be shown to read

$$\begin{aligned} \mathcal{L}_{CS} &= \tfrac{1}{2}\int_M d^3x Tr \epsilon^{\mu\nu\rho}\{A_\mu \partial_\nu A_\rho + \tfrac{1}{3} A_\mu [A_\nu, A_\rho]\} \\ &= \int_M d^3x \epsilon^{\mu\nu\rho}\{e^a_\mu(\partial_\nu \omega_{a\rho} - \partial_\rho \omega_{a\nu} + \epsilon_{abc}\omega^b_\nu \omega^c_\rho) + \\ &\quad + \tfrac{1}{2}\bar{\Psi}_\mu(\partial_\nu + \tfrac{1}{2}\omega^a_\nu \gamma_a)\Psi_\rho\}. \end{aligned} \tag{15}$$

This is the simple supergravity action given in[9]. Due to its construction this action is invariant under the infinitesimal gauge transformations (2) with a gauge parameter:

$$\lambda = \rho^a P_a + \tau^a J_a + \bar{\kappa}^\alpha Q_\alpha.$$

Use of this parameter in (2) leads to the following transformations

$$\begin{aligned} \delta e^a_\mu &= \partial_\mu \rho^a - \epsilon^{abc}\omega_{b\mu}\rho_c + \epsilon^{abc} e_{b\mu}\tau_c - \tfrac{1}{4}\bar{\kappa}\gamma^a\Psi_\mu, \\ \delta\omega^a_\mu &= \partial_\mu \tau^a + \epsilon^{abc}\omega_{b\mu}, \\ \delta\bar{\Psi}_\mu &= \partial_\mu\bar{\kappa} - \tfrac{1}{2}\bar{\kappa}\omega^a_\mu\gamma_a + \bar{\Psi}_\mu\gamma_a\tau^a. \end{aligned} \tag{16}$$

The κ and τ transformations coincide, respectively, with the usual local supersymmetry and Lorentz transformations of the simple supergravity. The unique symmetry which is not explicitly seen to be satisfied is the diffeomorphisms.

The supergravity action is invariant under the diffeomorphisms generated by a vector field ℓ as

$$\delta_D \phi^A_\mu = \partial_\mu(\ell^\nu \phi^A_\nu) + \ell^\nu(\partial_\nu\phi^A_\mu - \partial_\mu\phi^A_\nu), \tag{17}$$

where $\phi^A_\mu = (e^a_\mu, \omega^a_\mu, \Psi^\alpha_\mu)$. One can show that by using the equations of motion and setting

$$\rho^a = \ell^\nu e^a_\nu \; ; \; \tau^a = \ell^\nu \omega^a_\nu \; ; \; \kappa = 0,$$

in (16) the infinitesimal local transformations coincide with the diffeomorphisms (17) of e^a_μ and ω^a_μ. Local Lorentz transformation with the above value of the parameter accompanied by an infinitisimal supersymmetry transformation with the parameter

$$\kappa = \ell^\nu \Psi_\nu,$$

leads to the desired diffeomorphisms of the gravitino field, when one uses the equations of motion.

Because of using equations of motion for showing that gauge transformations of Chern-Simons action are equivalent to the standard local transformations of the simple supergravity action, interpretation of the three dimensional simple supergravity as Chern-Simons action is correct classically.But we may enlarge this to the one loop level after quantization, on the grounds of the results of section 1: in the large k limit, the quantum effective action at one loop is equal to the classical one.

For quantizing this system canonically we take the space-time manifold as $M = \mathbf{R} \times M_2$, where M_2 indicates the two dimensional space. In the

case of taking the Wilson lines as observables M_2 can be chosen as Riemann surface[4]. Because of having a first order Lagrangian, the Poisson brackets and the classical constraints can easily be read. In fact from the following form of the action:

$$\begin{aligned} \mathcal{L}_{CS} = & \textstyle\int_M d^3x \epsilon^{ij} \{ -2e^a_i \partial_0 \omega_{aj} + e^a_0(\partial_i\omega_{aj} - \partial_j\omega_{ai} + \epsilon_{abc}\omega^b_i\omega^c_j) + \\ & + \omega_{a0}(\partial_i e^a_j - \partial_j e^a_i + \epsilon^{abc} e_{bi} e_{cj} - \tfrac{1}{4}\bar{\Psi}_i\gamma^a\Psi_j) - \\ & - \tfrac{1}{2}\bar{\Psi}_i\partial_0\Psi_j + \bar{\Psi}_0(\partial_i + \tfrac{1}{2}\omega^a_i\gamma_a)\Psi_j \}. \end{aligned} \tag{18}$$

Now one can easily see that the canonical momenta which correspond to the time components of the fields vanish. These fields play the role of the Lagrange multipliers and yield the constraints:

$$\begin{aligned} &\partial_i\omega_{aj} - \partial_j\omega_{ai} + \epsilon_{abc}\omega^b_i\omega^c_j = 0, \\ &\partial_i e_{aj} - \partial_j e_{ai} + 2\epsilon_{abc} e^b_i \omega^c_j + \tfrac{1}{4}\epsilon^{ij}\bar{\Psi}_i\gamma_a\Psi_j = 0, \\ &\epsilon^{ij}(\partial_i + \tfrac{1}{2}\omega^a_i\gamma_a)\Psi_j = 0. \end{aligned} \tag{19}$$

Non-zero generalized Poisson brackets[2] can easily be read as

$$\begin{aligned} \{\omega^a_i(x), e^b_j(y)] &= \tfrac{1}{2}\epsilon_{ij}\eta^{ab}\delta^2(x-y), \\ \{\Psi_{i\alpha}(x), \Psi_{j\beta}(y)] &= 2\epsilon_{ij}\delta^\alpha_\beta\delta^2(x-y). \end{aligned} \tag{20}$$

From the constraints, (19), one can easily see that the procedure of separating the phase space variables into coordinates and momenta and eliminating the latter to reach a physical Hilbert space defined in terms of the coordinates which obey some definite constraint equations, is not possible directly, or in other words, a real polarization does not exist.

In the next section we will give another formalism of $N = 1$ supergravity, which is useful for an explicit definition of the physical Hilbert space.

4 Complex Gravitino

Now let us change the fermionic part of the algebra (12) by letting the fermionic charge to be complex. Thus by defining $\bar{Q}^\alpha = Q^*_\beta C^{\beta\alpha}$ we enlarge

[2]Generalized Poisson brackets are defined as

$$\{F, G] = \frac{\partial_r F}{\partial p_A}\frac{\partial_l G}{\partial q^A} - (-1)^{\epsilon(F)\epsilon(G)}\frac{\partial_r G}{\partial p_A}\frac{\partial_l F}{\partial q^A}.$$

the algebra as:

$$\begin{array}{rcl} [J^a, J^b] & = & \epsilon^{abc} J_c, \\ [J^a, P^b] & = & \epsilon^{abc} P_c, \\ [J^a, Q_\alpha] & = & -\frac{1}{2}\gamma^{a\beta}_{\alpha} Q_\beta, \\ [J^a, \bar{Q}^\alpha] & = & \frac{1}{2}\bar{Q}^\beta \gamma^{a\alpha}_{\beta}, \\ \{Q_\alpha, \bar{Q}^\beta\} & = & -\frac{1}{4}\gamma^{a\beta}_{\alpha} P_a, \\ \{Q_\alpha, Q_\beta\} & = & \{\bar{Q}^\alpha, \bar{Q}^\beta\} = 0, \end{array}$$

where the other commutators vanish.

When one wants to have an action which is invariant under a rigid group which has the generators satisfying the above algebra ($N = 2$ supergravity) he has to couple to $(2,3/2)$ multiplet a $(3/2,1)$ multiplet which has the following supersymmetric action[10]

$$S = \int_M d^3x(\epsilon^{\mu\nu\rho}\bar{\chi}_\mu \partial_\nu \chi_\rho - B^2_\mu),$$

when he would like to have only first derivative equations of motion for the real spin-3/2 field χ. Our aim is to give a quantization scheme for the $N = 1$ supergravity, so that we do not need to couple a new multiplet. From the above action it is also clear that $d = 3$, $N = 2$ supergravity cannot have a Chern-Simons interpretation.

We will use the generators of the above algebra for defining a gauge field:

$$A_\mu = e^a_\mu P_a + \omega^a_\mu J_a + \bar{\psi}^\alpha_\mu Q_\alpha + \bar{Q}^\alpha \psi_{\mu\alpha}, \tag{21}$$

where $\psi_{\alpha\mu}$ is complex and $\bar{\psi}^\alpha_\mu = \psi^*_{\beta\mu} C^{\beta\alpha}$.

The invariant , nondegenerate metric on this super Lie algebra is defined by

$$< J^a, P^b >= 2\eta^{ab} \; ; \; < Q_\alpha, \bar{Q}^\beta >= \delta^\beta_\alpha, \tag{22}$$

which follows from the invariant quadratic form

$$W = J^a P_a + 2\bar{Q}Q. \tag{23}$$

Making use of (21) and (22) in (1) leads to

$$\mathcal{L} = \int_M d^3x \epsilon^{\mu\nu\rho}\{\frac{1}{2}e^a_\mu(\partial_\nu\omega_{a\rho} - \partial_\rho\omega_{a\nu} + \epsilon_{abc}\omega^b_\nu\omega^c_\rho) + \bar{\psi}_\mu(\partial_\nu + \frac{1}{2}\omega^a_\nu\gamma_a)\psi_\rho\}. \tag{24}$$

As one can easily see, by using the definiton of gauge transformations given in (2) with

$$\lambda = \rho^a P_a + \tau^a J_a + \bar{\Lambda}Q + \bar{Q}\Lambda,$$

this action has the following local supersymmetry invariance

$$\begin{array}{rcl} \delta e^a_\mu & = & -\frac{1}{4}\bar{\psi}_\mu\gamma^a\Lambda - \frac{1}{4}\bar{\Lambda}\gamma^a\psi_\mu, \\ \delta\omega^a_\mu & = & 0, \\ \delta\bar{\psi}_\mu & = & \partial_\mu\bar{\Lambda} - \frac{1}{2}\bar{\Lambda}\omega^a_\mu\gamma_a, \\ \delta\psi_\mu & = & \partial_\mu\Lambda - \frac{1}{2}\omega^a_\mu\gamma_a\Lambda. \end{array} \tag{25}$$

In fact this leads to a global N=1 supersymmetry invariance. Thus adding another real gravitino to the simple supergravity action as in (24) does not change the properties of it. This is due to the fact that there is no local excitation, which follows from the equations of motion:

$$\begin{array}{l} \epsilon^{\mu\nu\rho}(\partial_\nu + \frac{1}{2}\omega^a_\nu\gamma_a)\psi_\rho = 0, \\ \epsilon^{\mu\nu\rho}(\partial_\nu + \frac{1}{2}\omega^a_\nu\gamma_a)\bar{\psi}_\rho = 0. \end{array} \tag{26}$$

Now as we have done in the previous section we take $M = \mathbf{R} \times M_2$ for canonical quantization. Thus the generalized Poisson brackets will be

$$\begin{array}{rcl} \{\omega^a_i(x), e^b_j(y)] & = & \frac{1}{2}\epsilon_{ij}\eta^{ab}\delta^2(x-y), \\ \{\bar{\psi}^\alpha_i(x), \psi_{j\beta}(y)] & = & \epsilon_{ij}\delta^\alpha_\beta\delta^2(x-y), \end{array} \tag{27}$$

and the constraints are

$$\partial_i\omega_{aj} - \partial_j\omega_{ai} + \epsilon_{abc}\omega^b_i\omega^c_j = 0, \tag{28}$$

$$\partial_i e_{aj} - \partial_j e_{ai} + 2\epsilon_{abc}e^b_i\omega^c_j - \frac{1}{2}\epsilon^{ij}\bar{\psi}_i\gamma_a\psi_j = 0, \tag{29}$$

$$\epsilon^{ij}(\partial_i + \frac{1}{2}\omega^a_i\gamma_a)\psi_j = 0, \tag{30}$$

$$\epsilon^{ij}(\partial_i + \frac{1}{2}\omega^a_i\gamma_a)\bar{\psi}_j = 0. \tag{31}$$

In this case we can define the physical Hilbert space to be that which is constructed by ω^a_i and ψ^α_i which satisfy (28) and (30) and also invariant under the gauge transformations

$$\begin{array}{rcl} \delta\omega^a_i & = & \partial_i\tau^a + \epsilon^{abc}\omega_{bi}\tau_c, \\ \delta\psi_i & = & \partial_i\Lambda - \frac{1}{2}\omega^a_i\gamma_a\Lambda + \gamma_a\tau^a\psi_i. \end{array} \tag{32}$$

Of course these conditions are not sufficent for defining the physical Hilbert space, because in the set of all solutions of (30)there are also $\bar{\psi}$. Thus in addition to the above conditions, we must still eliminate the complex conjugate of the solutions of (28) for achieving a correct definition of the physical Hilbert space. Of course for having an explicit definition of the physical Hilbert space we must find the global parameters which can be used to parametrize the above-cited solutions. The bosonic part is already known, when M_2 is taken as Riemann surface, and it is given as the moduli space of flat $SO(2,1)$-connection modulo gauge transformations [4], but for the time being the fermionic part is not available yet and we don't intend to find it in this work. Of course after finding the explicit definition of the physical Hilbert space one still needs to define the inner product and show that it is positive definite. Either to demonstrate that there exists a desired global set of parameters or to define an inner product for defining the physical Hilbert space are important for showing that the above procedure for quantizing the $N = 1$ supergravity in three dimensions works.

As it can easily be seen, the above procedure can be generalized to M complex gravitini ψ_M . In this case the bosonic part remains as before but the fermionic part of the physical Hilbert space is defined as to be constructed by the complex fields which satisfy

$$\epsilon^{ij}(\partial_i + \frac{1}{2}\omega_i^a\gamma_a)\psi_{Mj} = 0. \qquad (33)$$

Now the set of all the solutions of (33) is the same as the set of all solutions of (30). Thus also the fermionic part of the physical Hilbert space will be as above if we define it by eliminating the complex conjugates of the solutions of (33).

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