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MOTT TRANSITION:
LOW-ENERGY EXCITATIONS AND SUPERCONDUCTIVITY

L.B. Ioffe

and

A.I. Larkin

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MOTT TRANSITION: LOW-ENERGY EXCITATIONS AND SUPERCONDUCTIVITY *

L.B. Ioffe
Landau Institute for Theoretical Physics, Moscow, USSR

and

A.I. Larkin
International Centre for Theoretical Physics, Trieste, Italy
and
Landau Institute for Theoretical Physics, Moscow, USSR.

ABSTRACT

It is possible that metal-dielectric transition does not result in changes of magnetic or crystallographic symmetry. In this case a fermionic spectrum is not changed at the transition, but additional low-energy excitations appear which can be described as a gauge field that has the same symmetry as an electromagnetic one. In the case of a non half-filled band gapless scalar Bose excitations also appear. Due to the presence of additional gauge field the physical conductivity is determined by the lowest conductivity of the Fermi or Bose subsystems.

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1. INTRODUCTION

Recent discovery of high temperature superconductivity and especially the observation that doping transforms the superconductors into dielectrics revived interest in the Mott-Hubbard transition metal-dielectric. Qualitative properties of metal are well understood. Even at strong interaction metal can be described by the Fermi-liquid theory. The dielectric state is usually studied by variational approach or in the mean field approximation. In the considered problem both methods do not have a small parameter, thus, it is hard (or even impossible) to estimate the accuracy and reliability of their results.

The simplest model describing the system of strongly repulsing electrons is the Hubbard model:

$$H_H=\sum_{i,j} t_{ij} c^+_{i\alpha} c_{j\alpha} + \sum_i U\ (c^+_{i\alpha} c_{i\alpha})^2 \qquad \alpha=1,2 \qquad (1)$$

If the electron band is half-filled and repulsion is strong: U>>t then the Hamiltonian (1) comes to a simpler form:

$$H= \sum_{i,j} J_{ij} c^+_{i\alpha} c_{j\alpha} c^+_{i\beta} c_{j\beta} \qquad J_{ij}=t^2_{ij}/U \qquad (2)$$

which can be represented as Hamiltonian of Heisenberg antiferromagnet:

$$H_A=\sum_{i,j} J_{ij} S_i S_j \qquad (3)$$

Presumably if the interaction J_{ij} is not zero only for the nearest neighbours then the ground state of this Hamiltonian is Neel antiferromagnet. The antiferromagnetic interaction of the next-nearest neighbours (frustration) results in the transition to the spin-liquid state. In this state the mean spin at each site is exactly zero. Two questions arise immediately: can we prove the existence of such a state in a model with a small parameter and what is the spectrum of low energy excitations in this state?

First of all we discuss the possibility of the existence of a spin liquid in the Heisenberg model (3) at large S>>1. At zero temperature and very large $S\to\infty$ thermal and quantum fluctuations are absent and the ground state is magnetic.

The increase of the next-nearest interaction results in the transition into a helicoidal antiferromagnet. This transition can be the second order transition, in this case it can be described by the Landau theory. At large but finite S the long-wave quantum fluctuations change the nature of the transition. The fluctuations are described by the effective action:

$$A(n)=\frac{1}{2}\int dt d^2x\{M\dot{n}^2-\rho(\nabla n)^2-\sigma_1(\Delta n)^2-\sigma_2\partial_x^2 n\ \partial_y^2 n\} \qquad (4)$$

In the vicinity of the transition point $\rho\to 0$, the theory becomes logarithmic and the effective charge g that describes the spin-wave interaction obeys the renormalization group equations [1]:

$$dg/d(\ln R)=g^2 \qquad (5)$$

It is very important that the sign of the r.h.s. of the equation is plus. This point differentiates the present problem from the phase transitions theory and makes it similar to the theory of two-dimensional (classic) magnets. The effective charge that was small at short distances ($g\alpha 1/S$) increases with scale. It means that the system has no magnetic long range order.Finite ρ cuts off the logarithmic divergences. Therefore, at S>>1 the spin liquid state exists in a narrow region $|\rho|\leq\exp(-1/S)$. Probably at S=1/2 this region becomes very large.

The renormalization group approach does not allow to study the

systems with strong interaction. Pomeranchuk [2] and Anderson [3] supposed that the excitations in the spin liquid are neutral fermions (spinons). At least two origins of these fermions are possible.

In the first scenario [4,5] the spinons are topological excitations of the soliton type which appear on the background of the short-range antiferromagnetic order. If in the effective action the term proportional to Hopf invariant (Chern-Simons term) is present with a half integer coefficient then the spin of these excitations is also half-integer. In this scenario the spin has a topological origin and does not interact with a physical magnetic field. The precise form of these excitations in the spin liquid is not clear, but in Neel state they surely exist. However, in the Heisenberg model of antiferromagnet this coefficient is zero [1] in the ordinary Neel state so that skyrmions have an integer spin.

In the alternative scenario the Mott transition metal-dielectric has little effect on the spectrum of fermionic excitations, but the local gauge symmetry is restored. The creation operators of quasiparticles in the dielectric state are creation operators of electrons dressed by a phase factor. In the metal state the local gauge symmetry is broken, the phase factor acquires a mean value and fermionic Green function is not zero at large times and distances.In the dielectric state the gauge symmetry is restored, the mean value of the phase factor is zero ,fermionic Green function decreases rapidly at large times and distances, auxilary gauge field screens out the electromagnetic one so that conductivity is zero. However, the two-particle correlation functions that correspond to a process without

charge transfer (e.g. spin-spin correlation function) are the same as in metal. The collective excitations of the spinon liquid are described by the gauge field which is similar to electromagnetic and by scalar Bose field. In the case of a strong repulsion U>>t the number of holes in the whole system coincide with the number of bosons. We discuss the semiphenomenological theory of this state in the main body of this paper. Specifically we find that auxilary gauge field that appears in that problem can screen out the physical electromagnetic one so that the response to the physical electromagnetic field is determined by the largest of the resistance of Fermi or Bose subsystems. Thus the whole system is superconducting only if both Fermi and Bose subsystems are superconducting.

Quantitative description of this state can be obtained only in a model with a small parameter, for instance in a model introduced in [6,7]. This model is also described by Hamiltonian (1) (or by sum of Hamiltonians (1) and (2) with J_{ij} being regarded as an independent parameter), but α runs over N values: α=1..N (N>>1). The interaction of spinons with collective excitations is small in this model since it is proportional to 1/N so that quantitative results can be obtained. These results reproduce [6,11] the qualitative results of the semiphenomenological theory discussed in this paper.

2. SEMIPHENOMENOLOGICAL THEORY AND ELECTROMAGNETIC PROPERTIES

We start with the discussion of the form of the Hamiltonian that we choose to describe all important low-energy excitations of the system in the vicinity of the metal-dielectric transition and in dielectric state. Besides the real ("bare") electrons these modes comprise the phase fluctuations of the auxiliary field Δ_{ij}. The physical meaning of this auxiliary field becomes transparent in the mean field approximation in which $\Delta_{ij}=J_{ij}<c^{+}_{i\alpha}c_{j\alpha}>$. The fluctuations of the amplitude of Δ_{ij} has a gap and, as we believe, has no impact on the qualitative properties of the low-energy excitations. In the model with large N (see the end of Introduction) these fluctuations are really small and can be neglected so that quantitative results can be obtained. The phase fluctuations of Δ_{ij} are always important, here we shall discuss their effects.

To justify our form of semiphenomenological Hamiltonian of low-energy excitations we start from the bare electronic Hamiltonian for which we choose a slightly more general form than the Hubbard one:

$$H=\sum_{i,j} t_{ij} c^{+}_{i\alpha} c_{j\alpha} + \sum_{i,j} J_{ij} c^{+}_{i\alpha} c_{j\alpha} c^{+}_{j\beta} c_{i\beta} + \frac{1}{2}\sum_{i} U(c^{+}_{i\alpha} c_{i\alpha})^{2} \qquad (6)$$

in which, besides Hubbard terms, we include also additional superexchange interaction governed by independent parameter J_{ij} which has the same form as exchange interaction (2) describing the Hubbard model at U/t>>1 and a half-filled band.

To treat the superexchange interaction we introduce auxiliary field Δ_{ij} and employ Hubbard-Stratanovich transformation. We get the effective action:

$$S=\int_0^\beta L d\tau$$

$$L=\sum_i \bar{c}_{i\alpha}\partial_\tau c_{i\alpha} - \sum_{i,j}\bar{c}_{i\alpha}c_{j\alpha}\exp(iAe_{ij})\ (t_{ij}-\Delta^*_{ij}) - \sum_i [\tfrac{1}{2}U(\bar{c}_{i\alpha}c_{i\alpha})^2 - \mu\bar{c}_{i\alpha}c_{i\alpha}] \qquad (7)$$

$$\Delta_{ij}=\exp[i(\Phi_i-\Phi_j+a_{ij})]$$

where A is external electromagnetic field, μ - chemical potential. Below we shall not take into account the fluctuations of the amplitude $|\Delta_{ij}|$, thus we omit the terms which depend only on $|\Delta_{ij}|^2$ in (7). The separation of the phase of Δ_{ij} into $(\Phi_i-\Phi_j)$ and a_{ij} is ambiguous, it should be determined from some auxiliary condition imposed on a_{ij} (choice of the gauge) which we discuss below. To treat the fermionic interaction remaining in (7) we introduce one more auxiliary scalar field ϕ_i:

$$L=\sum_i \bar{c}_{i\alpha}\partial_\tau c_{i\alpha} - \sum_{i,j}\bar{c}_{i\alpha}c_{j\alpha}\exp(iAe_{ij})\ (t_{ij}-\Delta^*_{ij}) - \sum_i (i\phi_i-\mu)\bar{c}_{i\alpha}c_{i\alpha} + \tfrac{1}{2}U^{-1}\phi_i^2] \qquad (8)$$

Then we perform the gauge transformation of variables:

$$c_i \to c_i \exp(-i\Phi_i)$$
$$\phi_i \to \phi_i - \dot{\Phi}_i \qquad (9)$$

and get:

$$L=\sum_i \bar{c}_{i\alpha}\partial_\tau c_{i\alpha} - \sum_{i,j}\bar{c}_{i\alpha}c_{j\alpha}\exp(iAe_{ij})\ [t_{ij}\exp(i\Phi_i-i\Phi_j) - \exp(-ia_{ij})|\Delta_{ij}|] -$$
$$-\sum_i [(i\phi_i-\mu)\bar{c}_{i\alpha}c_{i\alpha} + \tfrac{1}{2}U^{-1}(\phi-\dot{\Phi})^2] \qquad (10)$$

The transformation properties of the fields a_{ij}, ϕ_i are equivalent to transformation properties of vector and scalar fields of lattice QED.

It is convenient to choose the gauge in which scalar potential does not fluctuate and equals some constant ϕ_0 which should be determined from the condition of the free energy minimum:

$$i\sum_i \langle c^+_{i\alpha}c_{i\alpha}\rangle = \frac{1}{U}\sum_i \langle \dot{\Phi}_i - \phi_0\rangle \qquad (11)$$

where $\langle..\rangle$ means average with weight $\exp[\int L d\tau]$ with L being defined by (10). The last term in the effective action (10) describes a system of non-interacting rotators governed by Lagrangian L_0:

$$L_0=\frac{1}{2U}\sum_i (\dot{\Phi}_i - i\phi_0)^2 \qquad (12)$$

where we use real-time representation. The presence of the term $\dot{\Phi}_i\phi_0$ in Lagrangian L_0 distinguishes these rotators from the ordinary ones. To obtain their energy spectrum we employ the Schrödinger representation. The canonical momentum M conjugated to the variable Φ_i is

$$M=\frac{1}{U}(\dot{\Phi}_i - i\phi_0) \qquad (13)$$

Inserting the expression (13) for a canonical momentum into Hamiltonian $H=M\dot{\Phi}_i - L$ we get $H=\frac{1}{2}U(M+i\phi_0/U)^2$. The wave function $Y(\Phi)$ should be periodic over Φ so that eigenvalues of the operator N are integers m and the corresponding energy levels are

$$\epsilon_m=\frac{1}{2}U(m+i\phi_0/U)^2 \qquad (14)$$

In the rotator ground state $m=m_0$ where m_0 is the integer which is closest to $-i\phi_0/U$ (it can be shown that ϕ_0 obeying the condition (11) is purely imaginary so that $i\phi_0$ is real).

Now the equation (11) acquires a simple meaning: it ensures that the mean number of electrons in the system equals the mean value of operator M.

The first term in the effective action (10) describes the interaction of rotators with each other and with fermions. For the qualitative analysis it is sufficient to replace operator $c^+_{i\alpha}c_{j\alpha}$ in it by its mean $\langle c^+_{i\alpha}c_{j\alpha}\rangle$. The resulting term in the effective action describes interaction between neighbouring rotators.

We consider first the case of a strong repulsion U>>t. In this

case the interaction between rotators is small. For the half-filled band we can choose $i\phi_0/U=-1$. In this case the level spacing of each rotator is of the order of U, therefore in the ground state all rotators are in the same state m=1. If holes are present then the mean value of m is fractional, which implies that rotator wave function is superposition of wave functions with m=0 and with m=1. Thus in this state $i\phi_0/U$ is close to -1/2 so that the level spacing between levels m=0 and m=1 is of the order of t. The level spacing between other levels remains U>>t. Therefore in this state each rotator can be described by a two-level system. The excitations to the higher level are Bose particles, they can be described by the operators S_i which are equal to the operator $\exp(i\Phi_i)$ projected on the remaining two levels. If the density of holes is small then the number of Bose excitations is also small and it is convenient to represent operators S_i, S_i^+ as a series over creation-anihilation operators of bose field. We make use of the Holstein-Primakoff transformation and get

$$S_i^+ \approx b_i^+(1-b_i^+b_i) \tag{15}$$

where we retain only the leading and the next term of the expansion over boson density. In the bose representation Hamiltonian of rotators becomes

$$\begin{aligned} H\{b\}&=H_0\{b\}+H_{int}\{b\} \\ H_0\{b\}&=\sum_{i,j} t_{ij} b_i^+ b_j + \epsilon \sum_i b_i^+ b_i \\ H_{int}\{b\}&=-\sum_{i,j}{}' t_{ij}^* [b_i^+ b_j^+ b_i b_i + b_j^+ b_j^+ b_i b_j] - \epsilon \sum_i b_i^+ b_i^+ b_i b_i \end{aligned} \tag{16}$$

where $t_{ij}^*=t_{ij}\langle c_{i\alpha}^+ c_{j\alpha}\rangle$, ϵ is the level spacing between m=0 and m=1 levels:

$$\epsilon=\frac{1}{2}U+i\phi_0 \tag{17}$$

If the hole density is small then the gas approximation can be employed to study Bose system (16). In this case only long-wave bosons are important, their bare spectrum follows from Hamiltonian $H_0\{b\}$:

$$\epsilon_0(k)=t^*(k)+\epsilon=\epsilon-t_0+t_1k^2 \tag{18}$$

At $\epsilon>t_0$ there are no bosons in the ground state. At $\epsilon<t_0$ the bare spectrum becomes unstable, but Bose interaction described by $H_{int}\{b\}$ corresponds to repulsion of long-wave bosons, therefore in this case bose condensate is formed $\langle b\rangle\neq 0$. In the gas approximation the density of this condensate is governed by the scattering amplitude Γ of two bosons:

$$n_c=|\langle b\rangle|^2=\frac{t_0-\epsilon}{2\Gamma(\omega=0,k=0)} \tag{19}$$

The scattering amplitude Γ at zero frequency and zero momentum can be obtained with logarithmic accuracy:

$$\Gamma=\frac{4\pi t_1}{\ln[t_0/(t_0-\epsilon)]} \tag{20}$$

In the leading order over $\{\ln[t_0/(t_0-\epsilon)]\}^{-1}$ the total density of bosons n coincide with n_c, the difference between n and n_c has the next order of the small parameter $\{\ln[t_0/(t_0-\epsilon)]\}^{-1}$:

$$n-n_c\approx(t_0-\epsilon)/4\pi t_1\approx n_c\{\ln[t_0/(t_0-\epsilon)]\}^{-1} \tag{21}$$

Thus in the considered system the total number of bosons (and, thus the hole density) is zero at $n_c=0$ that means that at $n_c=0$ the band is half-filled. However another scenario is possible if there are a few types of Bose fields in the system. In this case a pair of Bose condensates can be formed: $\langle b^\alpha b^\beta\rangle\neq 0, \langle b^\alpha\rangle=0$. For instance, this is the case for a system consisting of a number of planes in which a weak tunneling between planes result in an effective attraction between bosons on adjacent planes, whereas bosons on the same plane still repulse each

other (this mechanism of boson attraction was proposed in [10]).

The appearance of these Bose condensates results in the breakdown of the local gauge symmetry related with the fields a_{ij}, ϕ.

The averaging over Bose and Fermi fields leads to the effective action of electromagnetic field **A** and gauge field a_{ij}. Generally, in the long-wave limit this action has a form

$$S(A,a)=\frac{T}{2}\int d^2k \sum_{\omega} \{[A_\alpha(\omega,k)-a_\alpha(\omega,k)]\Pi_{\alpha\beta}(\omega,k)[A_\beta(\omega,k)-a_\beta(\omega,k)]+ \\ a_\alpha(\omega,k)\pi_{\alpha\beta}(\omega,k)a_\beta(\omega,k)\} \qquad (22)$$

where the first term is generated by fermions that interact with both fields $\mathbf{A}, a_{ij}$ and the second is generated by bosons which interact only with field a_{ij}. It is convenient to single out from $\Pi_{\alpha\beta}$ and $\pi_{\alpha\beta}$ their longtitudinal and transverse parts:

$$\Pi_{\alpha\beta}=(\delta_{\alpha\beta}-\frac{k_\alpha k_\beta}{k^2})\Pi_1+\frac{k_\alpha k_\beta}{k^2}\Pi_2$$
$$\pi_{\alpha\beta}=(\delta_{\alpha\beta}-\frac{k_\alpha k_\beta}{k^2})\pi_1+\frac{k_\alpha k_\beta}{k^2}\pi_2 \qquad (23)$$

The appearance of Bose condensate means that π_1 remains non-zero at $\omega,k\to 0$: $\pi_1(\omega=0,k=0)=\rho_b$. At temperatures above the critical point of Bose condensation $\pi_1(\omega\to 0,k=0)=\sigma_b\omega$. If the repulsion U is strong then the number of bosons equals the number of holes that implies, in particular, that in the case of a half-filled band σ_b becomes zero.At finite U we should take into account that rotator is described by two Bose fields: one field corresponds to creation of excitations with $m>m_0$ and the other to excitations with $m<m_0$. The number of holes equals the difference between the numbers bosons of these two types. At finite temperature and half-filled band the conductivity becomes finite but exponentially small: $\sigma_b \propto \exp(-U/2T)$,since the minimal energy of

excitation energy in this state is U/2. If the band is half-filled the gap in bosonic spectrum which is nearly U/2 at U>>t decreases with U decrease and becomes zero at $U=U_c\approx t$. At $U<U_c$ bose condensate is formed that breaks the gauge symmetry.

Now we turn to fermionic excitations. The Green function $G_{ij}(t)$ of real electrons differ from the Green function of spinons by a phase factor:

$$G_{ij}(t)=<\bar{c}_{i\alpha}(t)c_{j\alpha}(0)\exp(i\Phi_i-i\Phi_j)> \qquad (24)$$

In a state with a one-particle Bose-condensate the mean value of the phase factor is non zero at $t,r_{ij}\to\infty$ so that the residual of the electron Green function is also non-zero in this state, whereas in a state without Bose condensate the mean value of the phase factor tends to zero at $t,r_{ij}\to\infty$ thus in this state the residual of the electron Green function is zero on the Fermi surface. However gapless fermionic excitations (spinons) are present in this state

The exchange by virtual quantums of the field a_{ij} leads to the repulsion between fermions. The exchange by virtual quantums of the Bose field can lead to attraction but its strength is proportional to the density of bosons (i.e. density of holes) and is weak. Thus in the framework of the simplest one-plane model with equivalent sites the effective interaction of fermions is repulsive. In this case the system of fermions is normal and $\Pi_1(k=0)=\sigma_f\omega+O(\omega^2)$. In more complicated models the interaction between fermions can become attractive, in this case at low temperature the system of fermions becomes superconductive and $\Pi_1(k=0)=\rho_f+O(\omega)$.

To get the effective action of the electromagnetic field which

describes its interaction with the whole electronic system we should average the effective action (22) over long-wave fluctuations of the gauge field a_{ij}. Performing the averaging we get:

$$S(A,)=\frac{T}{2}\int d^2k \sum_{\omega} A_{\alpha}(\omega,k)P_{\alpha\beta}(\omega,k)A_{\beta}(\omega,k)$$
$$P_{\alpha\beta}=(\delta_{\alpha\beta}-\frac{k_{\alpha}k_{\beta}}{k^2})P_1+\frac{k_{\alpha}k_{\beta}}{k^2}P_2 \qquad (25)$$

where

$$P_1=\frac{\Pi_1\pi_1}{\Pi_1+\pi_1} \qquad (26)$$

The current j appearing as a reaction to the external electromagnetic field $\mathbf{A}$ is given by $\mathbf{j}=P\mathbf{A}$, thus formulae (26) means that the physical conductivity of the whole system is determined by the lowest conductivity of the Fermi or Bose subsystems.

This means that if Bose subsystem is superconductive and Fermi subsystem is not then the physical conductivity is finite and equals σ_f. In the opposite case if Fermi subsystem is superconductive and Bose subsystem is not, then the conductivity is also finite and equals σ_b. If both subsystems are superconductive then the superconductive density of the whole system is $\rho=\rho_f\rho_b/(\rho_f+\rho_b)$. If both subsystems have a finite conductivity then the resistivity of the whole system is sum of the resistivities of the subsystems: $\sigma^{-1}=\sigma_b^{-1}+\sigma_f^{-1}$. If the band is half-filled and $U>U_c$ then σ_b tends to zero at $T\to 0$ therefore the conductivity of the whole system is zero in this state at T=0 independently of the state of the fermion subsystem.

Now we discuss the quantization of the flux in the superconductive state ($\rho_f,\rho_b\neq 0$). Generally, the free energy of the superconductive state can be expanded over variations of the phases of the order

parameters of Fermi and Bose subsystems:

$$F=\frac{1}{2}|\nabla\psi_f-2(\mathbf{A}-\mathbf{a})|^2\rho_f+\frac{1}{2}|\nabla\psi_b-e^*\mathbf{a}|^2\rho_b \qquad (27)$$

where $e^*=1$ if Bose condensate is one-particle condensate ($\langle b\rangle\neq 0$) and $e^*=2$ if it is two-particle Bose condensate ($\langle b^{\alpha}\rangle=0, \langle b^{\alpha}b^{\beta}\rangle\neq 0$) (we note that in our units the charge of electron is unity).

If $\rho_b>\rho_f$ then the vortices formed by Fermi order parameter ψ_f (so that $\mathbf{a}=0$ and $\nabla\psi_b=0$) are more energetically favourable. In this case the charge which determines the flux quantization condition is 2, as usual. In the opposite case, if $\rho_b<\rho_f$ then the vortices formed by bose order parameter (so that $\mathbf{A}=\mathbf{a}$ and $\nabla\psi_f=0$) are more favourable. In this case the charge which determines the flux quantization condition is e^*.

3. CONCLUSIONS

Usually metal-dielectric transition is accompanied by a change in crystallographic symmetry: e.g. lattice period doubling which happens if the antiferromagnetic order parameter is formed or if the Pierls transition happens. In these transitions the fermionic spectrum is changed. In any case the metal-dielectric transition results in the restoration of the gauge symmetry in the dielectric state. In this paper we have discussed the semiphenomenological theory of the metal-dielectric transition which is not accompanied by changes in fermionic spectrum. This transition can be related with the restoration of gauge symmetry in the dielectric state. As a consequence of the transition the gapless gauge excitations appear in the dielectric state which have the same transformation properties as the electromagnetic field.

In the framework of the semiphenomenological approach it is impossible to establish which state has the lowest energy. Even if phenomenological Hamiltonian (7) is justified for the description of some material the possibility that a state with a period doubling is the genuine ground state is still open. This doubling can result from either the wave of the amplitude of Δ_{ij} (which results in molecular crystal state) or from oscillation of signs of Δ_{ij} such that the product of Δ_{ij} over the smallest lattice plaquette is -1 (this state can also be described as a state with a flux of a-field which is equal to π per each plaquette). The density of states on the Fermi-surface is zero (at the half-filled band) in both states so that the occurrence of superconductivity in both states seems unlikely. This is the reason why we regard the uniform phase without the period doubling as the most interesting one.

It is possible that high-temperature superconductors based on copper oxides which easily become dielectrics at a small doping can be described by the semiphenomenological theory considered in this paper. In this case a question arises: how is the superconductivity destroyed at T_c? Three cases are possible. If the transition temperature of the bose subsystem is above the transition temperature of the Fermi one then the superconductive phase transition is driven by the transition of the Fermi subsystem and its properties differ slightly from the properties of the usual BCS transition. If the transition temperature of Bose subsystem is below the transition temperature of the Fermi one then the superconductive phase transition is driven by the transition of the Bose system, this scenario of the phase transition was proposed

by Anderson and co-workers [3,8-10]. It is possible, finally, that phase transitions help each other and their critical temperatures are close.

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REFERENCES

1.L.B.Ioffe, A.I.Larkin Intern.Journ.Mod.Phys.B 2, 203 (1988).

2.I.Ya.Pomeranchuk ZhETP 11, 226 (1941).

3.P.W.Anderson Science 235, 1196 (1987).

4.F.Wilczek, A.Zee Phys.Rev.Lett. 57,2250 (1983).

5.I.E.Dzyaloshinskii, A.M.Polyakov. P.B.Wiegmann Phys.Lett.A 127, 112 (1988).

6.L.B.Ioffe, A.I.Larkin Phys.Rev.B to be published.

7.I.Affleck, J.B.Martson Phys.Rev.B 37, 3774 (1988).

8.G.Bascaran, P.W.Anderson Phys.Rev. B37, 580 (1988).

9.P.W.Anderson, G.Bascaran, Z.Zou, T.Hsu Phys Rev.Lett. 58, 2790 (1987).

10.J.M.Wheatly, T.C.Hsu, P.W.Anderson Phys.Rev. B37, 5897 (1988).

11.L.B.Ioffe, A.I.Larkin ZhETP to be published.

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