IC/88/353

INTERNATIONAL CENTRE FOR
THEORETICAL PHYSICS

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INTERNATIONAL
ATOMIC ENERGY
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EDUCATIONAL,
SCIENTIFIC
AND CULTURAL
ORGANIZATION

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and
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GENERATION OF HIGHER-ORDER SQUEEZING OF QUANTUM ELECTROMAGNETIC FIELDS BY DEGENERATE FOUR-WAVE MIXING AND OTHER PROCESSES*

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ABSTRACT

It is found that the field of the combined mode of the probe wave and the phase-conjugate wave in the process of degenerate four-wave mixing exhibits higher-order squeezing to all even order. The degree of squeezing increases with the order N, and the higher-order squeeze parameter q_N may approach -1.

MIRAMARE – TRIESTE
November 1988

* To be submitted for publication.

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The new concept of higher-order squeezing has been introduced by Hong and Mandel [1],[2] as a natural generalization of the usual second-order squeezing.

In the usual approach to squeezing in the context of quantum optics, the real field $\hat{E}$ decomposed into two quadrature components $\hat{E}_1$ and $\hat{E}_2$ which are canonical conjugates. Then the state is squeezed to the Nth order in $\hat{E}_1 (N = 1, 2, 3\ldots)$ if there exists a phase angle ϕ such that $\langle(\Delta\hat{E}_1)^N\rangle$ is smaller than its value in a completely coherent state of the field.

By using the Campbell-Backer-Hausdorff (C-B-H) identity Hong and Mandel readily obtained the relation

$$\langle(\Delta\hat{E}_1)^N\rangle = \langle:(\Delta\hat{E}_1)^N:\rangle + \frac{N^{(2)}}{1!}\left(\frac{1}{2}C_0\right)\langle:(\Delta\hat{E}_1)^{N-2}:\rangle$$

$$+\frac{N^{(4)}}{2!}\left(\frac{1}{2}C_0\right)^2\langle:(\Delta\hat{E}_1)^4:\rangle + \ldots + (N-1)!!C_0^{N/2}, \quad \text{if N is even} . \tag{1}$$

Here $N^{(r)}$ stands for $N(N-1)\ldots(N-r+1)$, the commutator $C_0 = \frac{1}{2i}[\hat{E}_1, \hat{E}_2]$. Now the normally ordered moments $\langle:(\Delta\hat{E})^N:\rangle$ all vanish for a coherent state. It follows that the state is squeezed to any even order N if

$$\langle(\Delta\hat{E}_1)^N\rangle < (N-1)!!C_0^{N/2} . \tag{2}$$

Hong and Mandel have applied this concept of higher-order squeezing to several physical situations, such as Degenerate Parametric Down Conversion, Second Harmonic Generation, etc. In this paper, we consider the processes of Degenerate Four-Wave Mixing (DFWM) and Non-degenerate Parametric Down Conversion (NPDC).

In the process of DFWM, we shall study the standard geometry shown in Fig. 1. Two strong, classical pump waves of complex amplitude ($v_1 = |v_1|e^{i\theta_1}$, $v_2 = |v_2|e^{i\theta_2}$) are incident on a nonlinear crystal, possessing a third-order ($\chi^{(3)}$) nonlinearity. The length of the medium is L. The frequencies of the pump waves, transmitted-probe wave and phase-conjugate wave are the same (ω). The simplest Hamiltonian has the form

$$\hat{H} = \hbar\omega\hat{a}_3^+\hat{a}_3 + \hbar\omega\hat{a}_4^+\hat{a}_4 + \hbar g(v_1 v_2 \hat{a}_3^+\hat{a}_4^+ e^{-2i\omega t} + H.C.) . \tag{3}$$

Here g is a coupling constant, $\hat{a}_j$ and $\hat{a}_j^+$ (j = 3,4) are annihilation and creation operators respectively, of phase-conjugate wave (j = 3) and probe wave (j = 4) (see Fig. 1).

It is convenient to use the more slowly varying variables

$$\hat{A}_j = \hat{a}_j e^{i\frac{\omega}{c}z}, \quad (j = 3, 4) \tag{4}$$

where z is the path travelled in the medium, and c the speed of light in the medium. Then we get the Heisenberg equations of motion for $\hat{A}_3$ and $\hat{A}_4$,

$$\frac{d\hat{A}_3}{dz} = i|K|\hat{A}_4^+, \quad (z = L - ct \quad for \quad \hat{A}_3) \tag{5}$$

$$\frac{d\hat{A}_4}{dz} = -i|K|\hat{A}_3^+, \quad (z = ct \quad for \quad \hat{A}_4) \tag{6}$$

where

$$|K| = g\frac{|v_1||v_2|}{c} . \tag{7}$$

The solutions of the output modes are

$$\hat{A}_3(0) = \xi\hat{a}_3(L) - \eta\hat{a}_4^+(0) \tag{8}$$

$$\hat{A}_4(L) = \xi\hat{a}_4(0) - \eta\hat{a}_3^+(L) , \tag{9}$$

where

$$\xi = sec|K|L \tag{10}$$

$$\eta = ie^{i(\theta_1+\theta_2)}tan|K|L . \tag{11}$$

It can be verified that the fields of $\hat{A}_3$ mode and $\hat{A}_4$ mode do not exhibit higher-order squeezing, so we consider the combined mode

$$\hat{c} = \frac{\hat{A}_3(0) - i\hat{A}_4(L)}{\sqrt{2}} . \tag{12}$$

We then define the quadrature component $\hat{E}_1$ by

$$\hat{E}_1 = \hat{c}e^{-i\phi} + \hat{c}^+e^{i\phi} \tag{13}$$

where ϕ is some phase angle that may be chosen at will.

Let

$$\hat{c}_{in} = \frac{\hat{a}_3(L) - i\hat{a}_4^+(0)}{\sqrt{2}} \tag{14}$$

$$\hat{c}_{in}^+ = \frac{\hat{a}_3^+(L) + i\hat{a}_4(0)}{\sqrt{2}} \tag{15}$$

They are linear combinations of the input modes to the four-wave mixer. Then from Eqs.(8),(9),(12),(13),(14) and (15), we get

$$\hat{E}_1 = f\hat{c}_{in} + f^*\hat{c}_{in}^+ , \tag{16}$$

where

$$f = \xi e^{-i\phi} - i\eta^* e^{i\phi} . \tag{17}$$

We now apply the C-B-H identity in the form

$$\langle e^{(\Delta\hat{E}_1)x}\rangle = \langle :: e^{(\Delta\hat{E}_1)x} ::\rangle e^{\frac{1}{2}x^2c_0} , \tag{18}$$

but with the normal ordering applicable to the $\hat{c}_{in}$, $\hat{c}_{in}^+$ operators, and with the commutator c_0 therefore identified with $|f|^2$, we then obtain, after equating coefficients of $\frac{x^N}{N!}$,

$$\langle(\Delta|\hat{E}_1)^N\rangle = \langle :: (\Delta\hat{E}_1)^N ::\rangle + \frac{N^{(2)}}{1!}(\frac{1}{2}|f|^2)\langle :: (\Delta\hat{E}_1)^{N-2} ::\rangle +$$
$$\frac{N^{(4)}}{2!}(\frac{1}{2}|f|^2)^2\langle :: (\Delta\hat{E}_1)^{N-4} ::\rangle + \ldots + (N-1)!!|f|^N \qquad \text{(N even)} \tag{19}$$

where :: :: denotes normal ordering with respect to $\hat{c}_{in}, \hat{c}_{in}^+$.

Now we take the initial quantum state to be $|\epsilon\rangle_4|0\rangle_3$ which is a product of the coherent state $|\epsilon\rangle_4$ for $\hat{a}_4(0)$ mode and the vacuum state for $\hat{a}_3(L)$ mode, then

$$\begin{aligned}
&{}_3\langle 0|_4\langle\epsilon| :: (\Delta\hat{E}_1)^N :: |\epsilon\rangle_4|0\rangle_3 = {}_3\langle 0|_4\langle\epsilon :: (h\Delta\hat{c}_{in} + h^*\Delta\hat{c}_{in}^+)^N :: |\epsilon\rangle_4|0\rangle_3 \\
&= \sum_{r=0}^{N}[{N \atop r}]_3\langle 0|_4\langle\epsilon|(\Delta\hat{c}_{in}^+)(\Delta\hat{c}_{in})^{N-r}|\epsilon\rangle_4|0\rangle_3 \; f^{*r}f^{N-r} \\
&= \frac{1}{(\sqrt{2})^N}\sum_{r=0}^{N}[{N \atop r}]_3\langle 0|_4\langle\epsilon||\Delta\hat{a}_4^+(0) + \Delta\hat{a}_3^+(L)]^r[\Delta\hat{a}_4(0) + \Delta\hat{a}_3(L)]^{N-r}|\epsilon\rangle_4|0\rangle_3 f^{*r}f^{N-r} \\
&= 0 .
\end{aligned} \tag{20}$$

Hence from Eq.(19), the higher-order moments

$$\begin{aligned}
\langle(\Delta\hat{E}_1)^N\rangle &= (N-1)!!|f|^N \\
&= (N-1)!!|\xi e^{-i\phi} - i\eta^* e^{i\phi}|^N \\
&= (N-1)!![sec^2|K|L + tan^2|K|L - 2sec|K|Ltan|K|L\cos(2\phi - \theta_1 - \theta_2)]^{N/2}
\end{aligned} \tag{21}$$

If ϕ is chosen to satisfy

$$2\phi - \theta_1 - \theta_2 = 0 \quad , \quad \cos(2\phi - \theta_1 - \theta_2) = 1 . \tag{22}$$

Then Eq.(21) leads to the result

$$\langle(\Delta\hat{E}_1)^N\rangle = (N-1)!![sec|K|L - tan|K|L]^N , \tag{23}$$

when $0 < |K|L < \pi$, the right-hand side is less than $(N-1)!!$, which is the corresponding Nth order dispersion for a coherent state. Then the field of the combined mode of the probe wave and the phase-conjugate wave exhibits higher-order squeezing to all even order.

Moreover, in order to determine whether the squeezing is intrinsically of higher-order in the sense defined by Hong and Mandel [2], we calculate the normally ordered moments, for any even N,

$$\langle : (\Delta \hat{E}_1)^N : \rangle = (N-1)!!(-1)^{N/2}[1-(sec|K|L - tan|K|L)^2]^{N/2} \, .$$

It follows that when $\frac{N}{2}$ is odd, $\langle : (\Delta \hat{E}_1) : \rangle < 0$, therefore there is intrinsic Nth order squeezing for all odd values of $\frac{N}{2}$, viz. for $N = 2, 6, 10, 14, \ldots$.

Finally we calculate the squeeze parameter q_N for measuring the degree of Nth order squeezing

$$\begin{aligned} q_N &= \frac{(N-1)!![sec|K|L - tan|K|L]^N - (N-1)!!}{(N-1)!!} \\ &= [sec|K|L - tan|K|L]^N - 1 \end{aligned} \, .$$

We find that $|q_N|$ increases with N, and the higher-order squeeze parameter may approach -1 (Fig. 2).

Besides, we calculate the higher order squeezing of the quantum electromagnetic field in the process of non-degenerate parametric down conversion. It is found that the field of the combined mode of the probe wave and the idler wave in NPDC also exibits the higher-order squeezing with strong, classical pump wave [3]. The degree of squeezing increases with N as well. So the higher-order squeezing of the quantum field has potential applications in optical communication, interferometry, spectroscopy and gravitation-wave detection.

Acknowledgments

One of the authors (L.X.) would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste. This research was also supported by the National Science Foundation of China.

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Figure Captions

Fig. 1. Schematic for generation of squeezing light via DFWM. M_1, M_2, M_3 = Mirrors, $BS = 50\% - -50\%$ beam splitter.

Fig. 2 The higher-order squeeze parameter q_N is plotted as a function of $|K|L$.

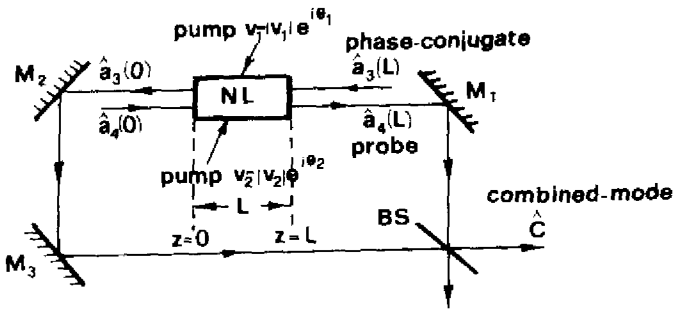

Fig.1

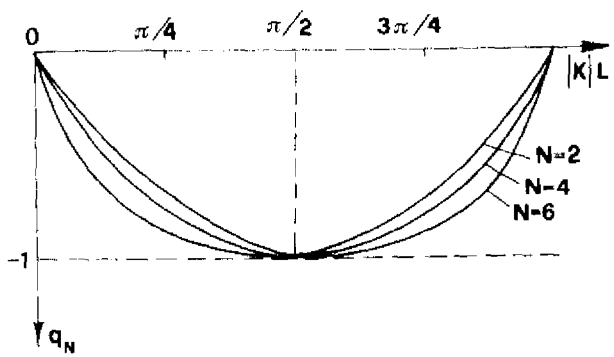

Fig.2

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