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IFUSP/P-742

COMPTON'S KINEMATICS AND EINSTEIN-EHRENFEST'S RADIATION THEORY

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Setembro/1988

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PACS : 03.50-z , 12.20.Ds , 41.70

ABSTRACT

The Compton kinematic relations are obtained from entirely classical arguments, that is, without the corpuscular concept of the photon. The calculations are nonrelativistic and result from Einstein and Ehrenfest's radiation theory modified in order to introduce the effects of the classical zero-point fields characteristic of Stochastic Electrodynamics.

I - INTRODUCTION

The discovery of light quanta (later called *photons*) by Einstein (Einstein 1905) was one of the most important in the Old Quantum Theory. Einstein's ideas about the corpuscular structure of the radiation influenced tremendously the further development of Quantum Theory, in spite of the resistance demonstrated by most of the physicists and the difficulties of incorporation within a wide class of phenomena. Einstein himself tried to become unavoidable the concept of the photon by studying the processes of absorption and emisssion of radiation by atoms in his classical paper of 1917 (Einstein 1917). Even though the main purpose of Einstein was to demonstrate the *necessity of photons*, the paper is famous nowadays not only because it contains one of the the first probabilistic treatements within Quantum Theory, but also the notion of spontaneous (as well as stimulated) emission. Due to the simplycity of the theory, part of the ideas of the work (associated with the A and B coefficients) have became popular and are well known by students of Modern Physics. Obviously the text-books try to harmonize Einstein's model with Quantum Mechanics, and some features from the original work are often overlooked (Lewis 1973), e.g., the fact that it also contains a deduction of Planck's radiation law and that Bohr's condition is a result, not an assumption. Less known is the fact that were made efforts by Einstein and Ehrenfest (Einstein 1923) in order to "improve" the achievements of the early paper. For them this improvement meant to *reinforce the concept of the photon* and remove the hypothesis that the "molecules"can only occupy discrete states of energy i.e., the model is also valid for free particles (Lewis 1973). Coincidently, at the same year of 1923, Compton issued his paper (Compton 1923) which explained the misterious features of the X and γ-ray scattering based on the corpuscular model for the radiation.

Our aim is to explore further Einstein and Einstein-Ehrenfest's work in the realm of a completely classical theory, the so-called Classical Stochastic

Electrodynamics (SED) (Peña 1982)-(Boyer 1975). According to SED the Universe is fulfilled by zero-point (and random) electromagnetic radiation characterized by the spectral distribution $\rho_0(\omega) = \hbar\,\omega^3/2\pi^2c^3$ (a Lorentz-invariant pattern). The Planck constant h appears naturally within the theory as a multiplicative constant ("not as a quantum of action") that simply fixes the scale of the spectrum. These different boundary conditions permit us to extract new and interesting informations from Einstein-Ehrenfest's work. We will show that in a system of particles immersed in both thermal and zero-point fields, the exchange of energy and momentum through the radiation must obey some relations in order to maintain the equilibrium. Curiously these relations are exactly the same that characterizes the conservation of the momentum and energy in a process like the Compton effect, for example. Without using the corpuscular concept of the photon we conclude that it's possible for such a system to exchange definite quantities of electromagnetic energy of the order of $\hbar\,\omega$, with an associated electromagnetic momentum $\hbar\,\vec{k}$, where $\omega = c|\vec{k}|$ is the frequency of the radiation. The energy and momentum are *extracted* from or *added* to radiation beams *"made up" of electromagnetic classical waves,* as is expected in Maxwell theory.

II - EINSTEIN'S ORIGINAL MODEL FOR CAVITY RADIATION

In this chapter we will present a synopsis of Einstein's 1917 paper, in order to show some interesting features and to improve the understanding of our further analysis about it.

In the early days of the Quantum Theory it was not very clear the relationship between Planck's theory of blackbody radiation and Bohr's theory of line spectra. Only in 1916 Einstein conceived an elegant and illuminating approach to this problem; he derived both Planck's radiation formula and Bohr's condition from general (probabilistic) hypotheses about the interaction between radiation and matter. After this he issued a paper (Einstein 1917) where he proves that his model (related to emission and absorption of radiation by "molecules") is consistent only if it's correct the hypothesis that the electromagnetic radiation is made up by quanta (Einstein 1905).

Einstein first considered that the "molecules" (actually polarizable objects, as we shall see later) were immersed in thermal radiation characterized by the spectral density $\rho_T(\omega)$. Secondly he assumed the following simple hypotheses:

1- The "molecules" only have discrete energy states.

2- The Boltzmann distribution is valid for the "molecules" in these states.

3- Wien's law is valid for the spectral distribution at temperature T, that is, $\rho_T(\omega) = \omega^3 F(\omega/T)$ where F is an arbitrary function.

Only the first hypothesis is a quantum one, due to the discrete character of the energy states. The other two are completely classical, based on thermodynamics and electromagnetism.

Let us see in which way Einstein treated the energy exchange between radiation and matter. For the sake of simplycity he studied only transitions between two states of energy, namely, E_i and E_j, with $E_i > E_j$. He also supposed that the transition between the states is done through the absorption or emission of a quantity of energy $E_i - E_j$ under the form of electromagnetic radiation. After this he formulated some hypotheses about the laws governing such transitions. He considered that the "molecules" behave as harmonic oscillators when they emit and absorb electromagnetic radiation. If the oscillator (frequency ω) is immersed in thermal radiation [spectral density $\rho_T(\omega)$], the absorved (or emitted) power is proportional to $\rho_T(\omega)$ (Max Born *Atomic Physics*). So Einstein assumed that the transition probability (per unit of time), from the state E_j to the state E_i, *induced* by the random thermal radiation was:

$$\frac{dW_{ji}}{dt} = \mathrm{B}_{ji}\, \rho_T(\omega) \tag{1.1}$$

where B_{ji} is a constant (temperature independent). In a similar way he wrote the transition probability from the state E_i to the state E_j, *also induced* by the random thermal radiation, as:

$$\frac{dW_{ij}}{dt} = \mathrm{B}_{ij}\, \rho_T(\omega) \tag{1.2}$$

where B_{ij} is another constant.

However it's also possible the emission of radiation (by such a oscillator) without an external stimulation. Einstein took this fact into account simply adding a term [in eq. (1.2)] which represents the probability of *spontaneous emission* of radiation with energy $E_i - E_j$. This term cannot be proportional to $\rho_T(\omega)$ (it's not a emission induced by thermal radiation), but it eventually can depend on the frequency ω. He wrote it as A_{ij}, and then equation (1.2) becomes:

$$\frac{dW_{ij}}{dt} = \mathrm{B}_{ij}\, \rho_T(\omega) + \mathrm{A}_{ij} \tag{1.3}$$

Following he imposed that the system is in equilibrium and proceeded in order to find the spectral density of the exchanged radiation which is in accordance to the assumed Boltzmann distribution for the "molecules". If n_i is the number of "molecules" with energy E_i and n_j the number of "molecules" with energy E_j, the condition of equilibrium (detailed balance) can be written as:

$$n_i \frac{dW_{ij}}{dt} = n_j \frac{dW_{ji}}{dt} \tag{1.4}$$

According to the Boltzmann statistics, we must have $n_i/n_j = e^{-(E_i-E_j)/kT}$, and since we expect an increasing $\rho_T(\omega)$ with $T \to \infty$, formula (1.4) will become, in this limit,

$$B_{ji}\,\rho_T(\omega) \approx B_{ij}\,\rho_T(\omega). \tag{1.5}$$

The equation above is valid at any high temperature and is equivalent to $B_{ij} = B_{ji}$. Inserting equation (1.5) within (1.4), one easily obtains:

$$\rho_T(\omega) = \frac{A_{ij}/B_{ij}}{e^{(E_i-E_j)/kT} - 1} \tag{1.6}$$

Now Einstein used his third hypothesis [Wien's displacement law to $\rho_T(\omega)$] and thus obtained:

$$\frac{A_{ij}}{B_{ij}} = \alpha\,\omega^3 \tag{1.7}$$

and

$$E_i - E_j = \hbar\,\omega \tag{1.8}$$

where α and $\hbar$ are constants. Therefore the expression for $\rho_T(\omega)$ will be:

$$\rho_T(\omega) = \frac{\alpha\,\omega^3}{e^{\hbar\omega/kT} - 1} \tag{1.9}$$

In this first part, Einstein deduced two important results in a simple way: the expression (1.8), that represents the Bohr relation for the energy emitted or absorved in an atomic transition, and Planck's formula for the spectral density of thermal radiation [expression (1.9)].

However, if it's not well known the fact that the work presents a deduction of both expressions, less known is the fact that the deductions are based on *classical grounds*, as we shall see in what follows. From the three initial hypotheses, only the first, referred to the discreteness of the energy states, has a quantum character. Curiously, in a posterior work by Einstein and Ehrenfest (Einstein 1923) it is shown that this hypothesis is unnecessary. In this almost unknown article they generalize the earlier result in such a way that the "molecules" are allowed to occupy a *continuous range of energies* (Lewis 1973). They also assume that the events of emission an absorption are *statistically independent*, i.e., there is *no interference* among the elementary processes. According to Einstein and Ehrenfest, then, this no interference assumption is plausible if the electromagnetic radiation "is made up of something (photons) which is emitted or absorved *instantaneously* (corpuscules)", and this is the only "quantum hypothesis" of the model.

In the second part of Einstein's 1917 work, he studied the momentum exchange between the "molecules" and the thermal radiation in which they are immersed. This analisys (Milonni 1976) was considered by Einstein himself the *most important* of the work, because it suggests the unavoidability of a quantum theory of the radiation. The reasoning of Einstein is based on the fact that the spontaneous emission, a process which is not stimulated by radiation beams, is not necessarilly a *directed* process, according to the *classical theory of the radiation.* Thus he proved that *all* the processes (induced and spontaneous)*must* be directed, otherwise the situation of thermodynamic equilibrium reached by the "molecules" would be incompatible with the spectral distribution given by (1.9).

The detailed calculations concerning the second part of Einstein's 1917 paper will be presented in the next section, but with a different interpretation, that is, without the corpuscular concept of the photon.

III - CLASSICAL STOCHASTIC ELECTRODYNAMICS AND THE EINSTEIN-EHRENFEST (1923) MODEL

a- Energy Exchange

The reasons why an exited atom radiates have remained somewhat misterious even after the phenomenological introduction of the concept of spontaneous emission by Einstein. Only in 1927, with the efforts of Dirac, (Dirac 1927) it was posible "to derive the A coefficient directly from the first principles"(Milonni 1984). According to Dirac, spontaneous emission is due to the so-called *radiation reaction* (J.D. Jackson *Classical Electrodynamics*), a force caused by the self fields of a radiating charge. However this is not the only interpretation, and in 1948, Welton (Welton 1948) pointed out that spontaneous emission may be considered a forced (induced) emission due to the zero point (quantum) electromagnetic field. In a work by Park and Epstein (Park 1949), Welton's idea is applied directly to the Einstein's 1917 model (exposed briefly in the last section), in such a way that the "molecules" interact with both thermal and zero-point radiation fields. In this modified model there is no "spontaneous emission", only emissions (and absorptions) *induced* by these fields.

Now we would like to draw attention to the fact that the notions of *radiation reaction* and *zero-point fluctuating electromagnetic fields* also appear in a consistent way within *Classical Electrodynamics*. If one postulates the existence of *classical* electromagnetic (fluctuating) fields persistent even at zero temperature, Classical Electrodynamics is provided with new boundary conditions. In this way it can predict some of the "quantum behaviour" of the microscopic matter *in entirely classical grounds.* Some examples may be found in the reviews by de la Peña (Peña 1982) and by Boyer (Boyer 1980), where are discussed the microscopic properties of the harmonic oscillator, the blackbody radiation, the diamagnetic behaviour of free and harmonically bound charges, the Casimir forces between macroscopic objects and other few phenomena. This new version of Classical Electrodynamics is often called Classical Stochastic Electrodynamics or simply Stochastic Electrodynamics (SED).

The fundamental hypothesis of SED is that the zero point fluctuations of the electromagnetic field (which pervade all space) are ***real*** and ***classical***, with a spectral distribution (density of energy per frequency) given by

$$\rho_0(\omega) = \frac{\hbar\,\omega^3}{2\pi^2 c^3} \quad . \tag{2.1}$$

Here $\hbar$ is simply a multiplicative constant which later will be identified with $h/2\pi$=Planck constant/2π. This is the way in which the Planck constant is introduced within classical SED, i.e., without any quantisation procedure. The constant gives the intensity of the zero point fields, whose spectral distibution has the remarkable property of being the *only* one which is ***Lorentz invariant*** (Marshall 1965)-(Boyer 1969).

A convenient way to describe the zero-point fields and to study their interaction with matter is to consider them as *random fields*, that can be written formally as a superposition of plane waves:

$$\vec{E}(\vec{r},t) = \sum_{\lambda=1}^{2} \int d^3\vec{k}\;\; \hat{\epsilon}(\vec{k},\lambda)\; g(\vec{k},T)\; \cos\left[\omega_k t - \vec{k}\cdot\vec{r} + \eta(\vec{k},\lambda)\right] \quad . \tag{2.2}$$

The polarization vectors $\hat{\epsilon}(\vec{k},\lambda)$ obey:

$$\begin{gathered} \hat{\epsilon}(\vec{k},\lambda)\cdot\hat{\epsilon}(\vec{k},\lambda') = \delta_{\lambda\lambda'} \qquad \vec{k}\cdot\hat{\epsilon}(\vec{k},\lambda) = 0 \\ \sum_{\lambda=1}^{2} \epsilon_i(\vec{k},\lambda)\epsilon_j(\vec{k},\lambda) = \delta_{ij} - \frac{k_i k_j}{k^2} \end{gathered} \tag{2.3}$$

and $\eta(\vec{k},\lambda)$ are statistically independent random phases, uniformly distributed in the interval $[0,2\pi]$ (Boyer 1975), that contains all the statistical character of the fields. The function $g(\vec{k},T)$ is the amplitude, and is related to the spectral density as $g^2(\omega,T) = c^3\rho(\omega,T)\omega^2$. Obviously in such an approach the relevant quantities will be the *averages* [performed over the random phases $\eta(\vec{k},\lambda)$] of the physical entities.

The detailed study of the interaction of the random fields with matter is not an easy task, mainly when one deals with nonlinear systems, e.g., the hydrogen atom (Peña 1982). Thus we will try to overcome this kind of difficulty joining the simplicity of Einstein's model ("molecules" in equilibrium with radiation) with the hypotheses of SED, as we shall see in what follows.

We will assume, firstly, that not only the thermal, but also the zero point radiation fields [with spectral distribution given by (2.1)] stimulate *emissions and absorptions* of radiation in a polarizable particle with internal harmonic oscillations. For the sake of simplicity we will initially study, as well as Einstein, only transitions between the energies E_2 and E_1 (with $E_2 > E_1$). Later on we shall consider the continuous case, that's based on Einstein-Ehrenfest's work (Einstein 1923)-(Lewis 1973).

If such a system absorbs energy and suffers a transition from the state with energy E_1 to the state with energy E_2, then, according to our modification of Einstein's model, the probability of transition from 1 to 2 will be:

$$\frac{dW_{12}}{dt} = \mathrm{A}_{12}\,\rho_0(\omega) + \mathrm{B}_{12}\,\rho_T(\omega) \tag{2.2}$$

where A_{12} and B_{12} are constants, i.e., temperature and frequency independent. We would like to stress again that the expression above can be justified on classical grounds, because it is well known (Max Born *Atomic Physics*) that a harmonic oscillator, with frequency ω, absorbs energy from the radiation at a rate proportional to the spectral density at the same frequency. Both terms in the r.h.s. represent a *classical transition probability*, because they are connected with the spectral densities $\rho_0(\omega)$ and $\rho_T(\omega)$ of the fluctuating classical electromagnetic fields.

In analogy with (2.2) we are going to write the transition probability per unit time from the state E_2 to the state E_1 as:

$$\frac{dW_{21}}{dt} = \mathrm{A}_{21}\,\rho_o(\omega) + \mathrm{B}_{21}\,\rho_T(\omega) \tag{2.3}$$

Here the term $\mathrm{A}_{21}\rho_o(\omega)$ is replacing the term corresponding to spontaneous emission in Einstein and Ehrenfest's original calculation. This means that we are assuming that the ***spontaneous*** emission is in fact ***induced*** by the zero point radiation (França 1988).

The second initial assumption by Einstein (Boltzmann statistics for the particles) will be maintained, that is, if we have $n(E_1)$ particles in the state E_1 and $n(E_2)$ particles in the state E_2 the relation

$$\frac{n(E_2)}{n(E_1)} = e^{-(E_2-E_1)/kT} \tag{2.4}$$

is valid on the average.

If we assume the detailed balance condition:

$$n(E_2)\,\frac{dW_{21}}{dt} = n(E_1)\,\frac{dW_{12}}{dt} \tag{2.5}$$

the particles and the radiation will be in equilibrium.

Analysing the expression above in the limits $T \to \infty$ [when $n(E_2) \approx n(E_1)$ and $\rho_T(\omega) \gg \rho_o(\omega)$] and $T \to 0$ [when $n(E_2) \ll n(E_1)$ and $\rho_o(\omega) \gg \rho_T(\omega)$], we find, respectively, the relations $\mathrm{B}_{12} = \mathrm{B}_{21} = \mathrm{B}$ and $\mathrm{A}_{12} = 0$ with $\mathrm{A}_{21} = \mathrm{A} \neq 0$. We note here that the fact $A_{12} = 0$ appears naturally, i.e., the zero point radiation does not stimulate absorptions in the equilibrium situation (França 1988).

It's easy to show from (2.5) that:

$$\rho_T(\omega) = \frac{\frac{A}{B}\,\rho_0(\omega)}{e^{(E_2-E_1)/kT} - 1} \tag{2.6}$$

and Wien's law (the third classical hypothesis by Einstein) demands that $E_2 - E_1 = \hbar\omega$, where $\hbar$ is a universal constant.

The value of the constant A/B can be fixed by using the Rayleigh-Jeans law for the blackbody radiation $[\rho_{RJ}(\omega)]$. The expresion is valid for low frequencies ($\hbar\omega \ll kT$) and must coincide with (2.6) in this limit, when $\rho_T(\omega) \approx \frac{A}{B}\,\rho_0(\omega)\,kT/\hbar\omega$. Since $\rho_{RJ}(\omega) = kT\pi^2 c^3/\omega^2$, we must have A = 2B. The constant $2\pi\hbar$ which appears in (2.1) can be identified again with Planck's constant and, as we have already quoted, it's strictly connected with the intensity of the zero-point radiation fields. With this we conclude that Einstein's derivation of Planck's formula is compatible with the existence of zero-point classical electromagnetic fluctuations.

At first sight the relation A = 2B means that the zero point electromagnetic fluctuations are *twice* more effective than the thermal ones in order to induce the emission of radiation. In the realm of classical SED, however, both the zero-point fluctuations and the so-called radiation reaction force play an important role. Then we prefer to understand the result A = 2B in the same way as was suggested by (Milonni 1984) and (França 1988). The main idea is that the self-fields of the charge also induces emissions at the same rate that the zero point fields with spectral density $\rho_0(\omega) \propto \omega^3$.

In what follows, based on the work of Einstein and Ehrenfest we are going to remove the hypothesis of discrete energy levels for the particles (Lewis 1973).

Let us assume that one particle suffers N *absorptions* in the frequencies $\omega_1, \omega_2, \ldots, \omega_N$ and M *emissions* in the frequencies $\omega'_1, \omega'_2, \ldots, \omega'_M$ in such a way that the particle goes from an initial state with energy E_I to a final state with energy E_F (where E_I and E_F are arbitrary). In the didactical diagram showed in fig. 1 we can have an intuitive feeling of the Einstein-Ehrenfest proposition.

It is necessary a generalization of the expressions (2.2) and (2.3), in order to have a mathematical description to the processes indicated above. Therefore, we can write (similarly to Einstein and Ehrenfest) the following expression for the transition probability (per unit time) dW_{IF}/dt, representing the change from the state with energy E_I to the state with energy E_F:

$$\frac{dW_{IF}}{dt} = \overbrace{\prod_{i=1}^{N} [\mathrm{B}\,\rho_T(\omega_i)]}^{\text{absorptions}} \overbrace{\prod_{j=1}^{M} [\mathrm{A}\,\rho_0(\omega'_j) + \mathrm{B}\rho_T(\omega'_j)]}^{\text{emissions}} \tag{2.7}$$

For the inverse process we have:

$$\frac{dW_{FI}}{dt} = \overbrace{\prod_{i=1}^{N} [\mathrm{A}\,\rho_0(\omega_i) + \mathrm{B}\rho_T(\omega_i)]}^{\text{emissions}} \overbrace{\prod_{j=1}^{M} [\mathrm{B}\,\rho_T(\omega'_j)]}^{\text{absorptions}} \tag{2.8}$$

It's important to mention at this point that the above expressions are valid only if the "elementary" processes (emission and absorption) are *statistically independent*. Such a situation could be verified if these processes occur in very short times, so that there is no interference among them (Lewis 1973).

By the other hand, we expect that under the *influence* of thermal and zero point radiation, the particles are *induced* to add or subtract energy and momentum to the radiation field. This field is represented by a superposition of plane waves with all frequencies. For this reason it is reasonable to expect that each absorption (in a frequency $\omega_i = c|\vec{k}_i|$) is accompained by a transference of momentum (from the wave to the particle) which is *directed* according to the corresponding wave vector $\vec{k}_i$. If we consider ***induced emission*** as the reverse of ***induced absorption*** then it is natural to assume that also these processes involve the superposition of plane waves each one with a definite direction for the momentum. With these considerations, it is reasonable to accept that the energy removed (or added) from the radiation inside the cavity will be converted into translational kinetic energy added (or removed) to the particle. All these considerations are consistent with the Einstein-Ehrenfest model and with SED.

If we take into account these observations, the final energy (E_F) and the initial energy (E_I) of the particle are expected to be related by:

$$E_F - E_I = \sum_{i=1}^{N} \Phi(\omega_i) - \sum_{j=1}^{M} \Phi'(\omega'_j) \tag{2.9}$$

where $\Phi(\omega)$ and $\Phi'(\omega')$ are positive unknown quantities to be fixed below. The first sum in (2.9) represents the energy extracted from the radiation field after N absorptions, and the second sum is the energy added to the radiation field after M emissions.

From now on our discussion departs from the original one by Einstein and Ehrenfest. This happens because our intention is not to derive again Planck's expression for $\rho_T(\omega)$. This formula has been derived many times in the classical context of SED (Boyer 1969)-(Peña 1982). Then we shall assume that $\rho_0(\omega)$ and $\rho_T(\omega)$ are well known and we change our goal, that is, we want to obtain the unknown quantities $\Phi(\omega)$ and $\Phi'(\omega')$.

As before, we shall assume the Boltzmann distribution, that is:

$$\frac{n(E_I)}{n(E_F)} = e^{(E_F - E_I)/kT} \tag{2.10}$$

The detailed balance condition:

$$n(E_I)\frac{dW_{IF}}{dt} = n(E_F)\frac{dW_{FI}}{dt} \tag{2.11}$$

is also assumed in order to keep the equilibrium between radiation and matter.

Introducing (2.7), (2.8), (2.9) and (2.10) into (2.11) we get:

$$\prod_{i=1}^{N}\left[\frac{\mathrm{B}\,\rho_T(\omega_i)\,\mathrm{e}^{\Phi(\omega_i)/kT}}{\mathrm{A}\,\rho_o(\omega_i)+\mathrm{B}\,\rho_T(\omega_i)}\right] = \prod_{j=1}^{M}\left[\frac{\mathrm{B}\,\rho_T(\omega'_j)\,\mathrm{e}^{\Phi'(\omega'_j)/kT}}{\mathrm{A}\,\rho_o(\omega'_j)+\mathrm{B}\,\rho_T(\omega'_j)}\right] \tag{2.12}$$

The expression above must be valid for any N and M and also for arbitrary sets of ω_i and ω'_j This means that each term in the square brakets must be equal to 1. Since A/B = 2 and $\rho_o(\omega)$ and $\rho_T(\omega)$ are well known (from previous (and different) analyses based on SED), the only unknown quantities are $\Phi(\omega)$ and $\Phi'(\omega')$. From these considerations it's simple to show that

$$\begin{aligned} \Phi(\omega) &= \hbar\,\omega \\ \Phi'(\omega') &= \hbar\,\omega' \end{aligned} \tag{2.13}$$

If we use these results and write (2.9) for $N = M = 1$, we get

$$E_I + \hbar\,\omega = E_F + \hbar\,\omega' \tag{2.14}$$

as a relation to be valid on the average.

We have concluded that the energy $\hbar\omega$ must be extracted (or added) from the *classical radiation beams* in order to maintain the thermodynamical equilibrium. It doesn't mean that we have considered the radiation as being made up of corpuscules. The relation (2.14) is a direct consequence of the hypothesis that it is possible "to count" the number of elementary processes (statistically independent) of emission and absorption of radiation.

b- Momentum Exchange:

The detailed study of the particles motion under the influence of radiation is found in the second part of Einstein's 1917 article (Einstein 1917) -(Milonni 1976), and aimed to prove the necessity that all elementary processes (emission or absorption of radiation) are directional. This study, which is performed imposing the equilibrium condition again, is somewhat more sophisticated than the method employed in the last section, when we calculated the energy exchange. In our work, the calculations will be done in the nonrelativistic regime (as well as presented originally by Einstein), because in this way important simplifications are introduced.

Now we shall proceed with the calculation of the momentum exchange, based on the work of Einstein, and in agreement with the hypotheses of Classical Stochastic Electrodynamics.

Consider that the polarizable particle is in movement inside a recipient with temperature T, so that we can divide the radiation influence in two parts; the first one will be identified with a fluctuating interaction and the second one will have a dissipative character, in analogy with the phenomenological theory of the brownian motion. If initially the particle has a linear momentum with projection $m\,v$ in the x direction, then after a short interval of time τ it will have a momentum (Einstein 1917)-(Milonni 1976):

$$m\,v' = m\,v + \Delta - \mathrm{R}\,v\,\tau \qquad (2.15)$$

Here Δ is referred to the fluctuating part of the momentum given to the particle by the random action of the thermal and zero-point radiation fields. The term $-\mathrm{R}\,v\,\tau$ is just the dissipative part (we shall see that it only acts when $T \neq 0$) that slows down the particle. In what follows the dissipative force $-\mathrm{R}\,v$ will be calculated explicitly.

Since the hypotheses about the emission and absorption were formulated in a coordinate system in which the particle is at rest, we shall investigate how the radiation looks in such a inertial system. In the laboratory system (attached to the recipient where the particles are), the electromagnetic radiation is isotropic, that is, the energy density between ω e $\omega + d\omega$ within the solid angle $d\Omega$, around some (arbitrary) direction of propagation, will be written as:

$$\frac{d\Omega}{4\pi}\,\rho(\omega,T)\,d\omega \tag{2.16}$$

We would like to stress that the spectral density $\rho(\omega,T)$ above also include the effects of the zero-point radiation fields, and must be written as:

$$\rho(\omega,T) = \rho_o(\omega) + \rho_T(\omega) \tag{2.17}$$

where $\rho_T(\omega)$ is the well known Planck's formula, and $\rho_o(\omega)$ the zero point spectral density given by (2.1).

However for a particle that is moving with velocity v (constant), along the x axis (in the laboratory system), the density of energy is not isotropic in the coordinate system attached to it, and consequently will be denoted by:

$$\frac{d\Omega'}{4\pi}\,\rho'(\omega',\theta')\,d\omega' \tag{2.18}$$

Here θ' is the angle between the x axis and the wave vector $\vec{k}'$ associated to a wave with frequency $\omega' = c|\vec{k}'|$.

In the Appendix we present the detailed calculation of the Lorentz tranformation for the spectral density. In the case that $v/c \ll 1$ the result is:

$$\rho'(\omega',\theta') = \left[\rho(\omega') + \frac{v}{c}\frac{\partial\rho(\omega')}{\partial\omega'}\,\omega'\,\cos\theta'\right]\left[1 - \frac{3\,v}{c}\,\cos\theta'\right] \tag{2.19}$$

where $\rho(\omega')$ is the spectral density in the laboratory system. The anisotropy of $\rho'(\omega',\theta')$, naturally, will cause a dissipative force on the particle.

For the sake of simplicity we shall consider that the particle will suffer transitions between two states of energy E_1 and E_2, with $E_2 > E_1$. This is not a quantum assumption, because E_2 can be arbitrarily close to E_1 and the generalization to the continuous case is straighforward, as we have seen in the last section. Following the earlier model for the interaction between radiation and the "molecules", a beam of radiation (thermal and zero-point) associated with the solid angle $d\Omega'$ induces (Einstein 1917)-(Milonni 1976):

$$N_2 = n(E_2)\,[\mathrm{B}\,\rho_T'(\omega',\theta') + \mathrm{A}\,\rho_0'(\omega')]\,\frac{d\Omega'}{4\pi} \tag{2.20}$$

emission processes and

$$N_1 = n(E_1)\,\mathrm{B}\rho_T'(\omega',\theta')\frac{d\Omega'}{4\pi} \tag{2.21}$$

absorption processes per unit of time. We have selected those which involve the same frequency ω', because we intend to calculate the linear momentum associated to each elementary process as a function of the frequency. In the expressions above $n(E_1)$ is the number of particles with energy E_1 and $n(E_2)$ the number of particles with energy E_2, and it is valid the classical relation $n(E_1)/n(E_2) = e^{(E_2-E_1)/kT}$. The coefficients A and B are the same as in the expressions (2.7) and (2.8), with A/B= 2.

At this point there is a fundamental difference between Einstein's approach and ours, based on Stochastic Electrodynamics. In Einstein's view, the fact that the absorptions and emissions *induced* by radiation beams are *directional*, is classicaly plausible, and the necessity of directioning of the *spontaneous emission* lead him to the hypothesis of radiation quanta. For us, however, *all* the emission and absorption processes are induced (by thermal and zero-point fields), and we therefore expect that in all of them will occur a directional transfer of linear momentum to the particle. We have already supposed that each process involves a frequency, for instance, ω', and a wave vector $\vec{k}'$ well defined. Such a wave carries momentum in the $\vec{k}'$ direction, and this will be the direction of the exchanged momentum. We intend to calculate its modulus, that we shall denote as $Q(\omega')$, since we expect it will be a function of the frequency ω'.

In this way the fraction of linear momentum (direction x) added to the particle per unit of time will be:

$$d\dot{p} = (N_1 - N_2)\, Q(\omega')\, \cos\theta' \tag{2.22}$$

The whole variation of momentum, considering all the propagation directions, will be:

$$\int d\dot{p} = \frac{dp}{dt} =$$
$$= Q(\omega')\, B\,[n(E_1) - n(E_2)] \int \frac{d\Omega'}{4\pi} \cos\theta' \left[\rho(\omega') + \frac{v}{c}\frac{\partial \rho\omega'}{\partial\omega}\, \omega' \cos\theta'\right] \left[1 - \frac{3\,v}{c}\right]$$
$$- n(E_2)\, A\, Q(\omega')\, \rho_o'(\omega') \int \frac{d\Omega'}{4\pi} \cos\theta' \tag{2.23}$$

where are already included (2.20) and (2.21).

The second integral is zero, and the first one is easily calculated. We obtain, then, after retaining only terms of the order of v/c:

$$\frac{dp}{dt} = -\frac{Q(\omega)}{c}\,[n(E_1) - n(E_2)]\, B \left[\rho(\omega) - \frac{\omega}{3}\frac{\partial \rho(\omega)}{\partial\omega}\right] v \equiv -R(\omega)\, v \tag{2.24}$$

where we have simplified the notation by putting $\omega' \Rightarrow \omega$ (since ω' is arbitrary).

The expresssion (2.24) represents the dissipative force applied to the particle [the last term in (2.15)].

We would like to draw attention to the fact that when $T = 0$, we have $\rho(\omega) = \rho_o(\omega) = \hbar\omega^3/2\pi^2 c^3$, and the expression above is zero, i.e., the zero point fields (classical vacuum) do not contributes to slow down the motion of the particle. This result is also valid in the relativistic regimen (Rueda 1981).

In what follows we shall calculate the fluctuating action of the radiation in order to find the transfered momentum $Q(\omega)$.

We shall assume that each process of emission or absorption (in the frequency ω) contributes with a linear momentum (projection in x direction) λ_i to the particle, and that in a short time τ, it's possible to *count* a number $\aleph$ of such processes which we shall suppose do not interfere among themselves. In this interval of time, then, the total contribution (to the x component) of the fluctuating part to the momentum will be:

$$\Delta = \sum_{i=1}^{\aleph} \lambda_i \tag{2.25}$$

Since we are considering the λ_i as random variables, we have that $\langle \lambda_i \rangle = 0$, and consequently $\langle \Delta \rangle = 0$ on the average. However $\langle \Delta^2 \rangle \neq 0$, so that:

$$\langle \Delta^2 \rangle = \sum_{i=1}^{\aleph} \langle \lambda_i^2 \rangle \tag{2.26}$$

We have already quoted that each elementary process involves a momentum transfer $\lambda_i = Q(\omega)\cos(\theta_i)$, where θ_i is the angle between the x axis and the wave vector of the emitted (or absorved) radiation. Squaring λ_i and averaging over all directions θ_i of the wave vector, we obtain:

$$\langle \lambda_i^2 \rangle = \frac{Q(\omega)^2}{4\pi}\, 2\pi \int_0^{\pi} d\theta_i \,\mathrm{sen}\theta_i \cos^2\theta_i = \frac{1}{3} Q(\omega)^2 \tag{2.27}$$

Thus the expression (2.26) is simplified to:

$$\langle \Delta^2 \rangle = \sum_{i=1}^{\aleph} \langle \lambda_i^2 \rangle = \frac{1}{3} \aleph\, Q(\omega)^2 \tag{2.28}$$

The next step is just to express $\aleph$, the total number of absorption and emission processes that occur in the time interval τ, as a function of known quantities. We know that $A\rho_0(\omega) + B\rho_T(\omega)$ is the rate of stimulated emissions, and $B\rho_T(\omega)$ the rate of absorption processes. Thus, the total number of emissions and absorptions $\aleph$ during τ will be:

$$\aleph = n(E_2)\,A\,\rho_0(\omega)\,\tau + [n(E_2) + n(E_1)]\,B\,\rho_T(\omega)\,\tau \tag{2.29}$$

By putting (2.29) into (2.28), we obtain:

$$\langle\Delta^2\rangle = \frac{1}{3}\,Q(\omega)^2[n(E_2)\,A\,\rho_0(\omega) + [n(E_2) + n(E_1)]\,B\,\rho_T(\omega)]\tau \tag{2.30}$$

Now we shall return to the equation (2.15) $m\,v' = m\,v + \Delta - R\,v\,\tau$. If we square this expression and take an "ensemble" average, we obtain:

$$m^2\langle v'^2\rangle = m^2\langle v^2\rangle + \langle\Delta^2\rangle + \\ + R^2\,\tau^2\,\langle v^2\rangle + 2\,m\langle v\,\Delta\rangle - 2R\langle v\,\Delta\rangle\,\tau - 2\,m\,R\langle v^2\rangle\,\tau \tag{2.31}$$

We shall assume now that there is no correlation between Δ and v, i.e., the fluctuating part of the momentum does not depend on the velocity of the particle. Then we can write $\langle\Delta\,v\rangle = \langle\Delta\rangle\langle v\rangle = 0$. If we neglect the term of the order τ^2, in comparison with $\langle\Delta^2\rangle$ and $2\,m\,R\langle v^2\rangle\tau$ that have the order τ, we obtain:

$$m^2\langle v'^2\rangle = m^2\langle v^2\rangle + \langle\Delta^2\rangle - 2\,m\,R\,\tau\langle v^2\rangle \tag{2.32}$$

Since we are considering that the equilibrium situation is reached through the interaction with the radiation and matter only, we can use the equipartition principle, and write:

$$\frac{1}{2}m\,\langle v'^2\rangle = \frac{1}{2}m\,\langle v^2\rangle = \frac{1}{2}\,k\,T \tag{2.33}$$

according to our assumption of Boltzmann distribution for the particles.

Thus equation (2.32) becomes:

$$\frac{\langle\Delta^2\rangle}{\tau} = 2\,\mathrm{R}\,k\,T \tag{2.34}$$

The expressions (2.24), for R, and (2.30), for $\langle\Delta^2\rangle$, have already been found, so that (2.34) becomes:

$$\mathrm{B}\left[n(E_1) - n(E_2)\right]\left[\rho_T(\omega) - \frac{\omega}{3}\frac{\partial\rho_T(\omega)}{\partial\omega}\right] =$$
$$= \frac{\mathrm{Q}(\omega)}{6\,k\,T}\left\{n(E_2)\,\mathrm{A}\,\rho_0(\omega) + \left[n(E_1) + n(E_2)\right]\mathrm{B}\,\rho_T(\omega)\right\} \tag{2.35}$$

In order to simplify this expression we can invoke the hypothesis of detailed balance. i.e.:

$$n(E_2)\,\mathrm{A}\,\rho_0(\omega) + \left[n(E_1) + n(E_2)\right]\mathrm{B}\,\rho_T(\omega) = 2\,n(E_1)\,\mathrm{B}\,\rho_T(\omega) \tag{2.36}$$

If we put (2.36) into (2.35) and use $n(E_2)/n(E_1) = \mathrm{e}^{(E_2-E_1)/kT}$, we reach finally:

$$\left[1 - \mathrm{e}^{(E_1-E_2)/kT}\right]\left[\rho_T(\omega) - \frac{\omega}{3}\frac{\partial\rho_T(\omega)}{\partial\omega}\right] = \frac{\mathrm{Q}(\omega)\,c}{3\,k\,T}\,\rho_T(\omega) \tag{2.38}$$

The expression for $\rho_T(\omega)$ has been already deduced in the classical realm of SED, (Boyer 1969)-(Peña 1982) without any quantisation hypothesis either of the energy or the radiation field. Therefore we can introduce it into equation (2.38), as well as Bohr's relation $E_2 - E_1 = \hbar\,\omega$, which is a result of the first part (concerning the energy exchange) of Einstein's model itself (see, for instance, expression (2.6) and the subsequent discussion). Then equation (2.38) lead us to:

$$\mathrm{Q}(\omega) = \frac{\hbar\,\omega}{c} \tag{2.39}$$

The quantitie $\mathrm{Q}(\omega)$ is just the modulus of the linear momentum transferred to the particle in each process of emission (or absorption) of radiation, with frequency ω, necessary to maintain the thermodynamical equilibrium of the system. The result can be generalized if we consider processes which involves N absorptions in the frequencies ω_i and M emissions in the frequencies ω'_j, as we have done in the case of the energy. If the vector $\vec{p}_F(E_F)$ is the final momentum(energy) and $\vec{p}_I(E_I)$ is the initial momentum(energy), the relationship among the momenta, energy and frequency will be such that:

$$\vec{p}_I + \sum_{i=1}^{N} \hbar \vec{k}_i = \vec{p}_F + \sum_{j=1}^{M} \hbar \vec{k}'_j \tag{2.40}$$

$$E_I + \sum_{i=1}^{N} \hbar \omega_i = E_F + \sum_{j=1}^{M} \hbar \omega'_j \tag{2.41}$$

where $\hbar$ is the Planck constant and $\omega = c|\vec{k}|$.

We have joined the *classical hypotheses* contained within Einstein's model with the Classical Stochastic Electrodynamics, and we have obtained the value of $\hbar\omega$ for the energy exchanged in each process of emission (or absorption) of radiation with frequency ω. Each process is accompanied of a momentum transfer $Q(\omega) = \hbar\omega/c$ which has the direction of the wave vector $\vec{k}$. The relations above are commonly associated with the conservation of energy-momentum in processes of scattering of electromagnetic radiation by massive particles. In quantum language, $\hbar\omega/c$ is just the momentum of the *photon of frequency* ω, and $\hbar\omega$ its energy. Such scattering (when the frequency ω is high enough, i.e., X-rays, for example) is called Compton scattering, a phenomenon that contributed greately to the affirmation of the Quantum Theory of the radiation.

During the calculations we have done some simplifying assumptions that are the same contained in the original work by Einstein. We must take some care, however, if we want to keep these assumptions compatible with the hypotheses of classical SED.

We will begin the discussion with the aplication of the equipartition principle to the translational motion of free particles. This principle is utilized when, in the calculation of the momentum exchange, we attribute to the translational energy the value $\frac{1}{2}\langle m v^2 \rangle = \frac{1}{2} k T$ as being the average of the kinetic energy acquired by the particle immersed in the Planck radiation with temperature T. This approximation, however, implies that the zero point fluctuations, important to maintain the equilibrium between radiation and matter, are supposed to have a very little contribution in its kinetic (translational) energy when $T \neq 0$. As we have verified before, the emissions and absorptions change the kinetic energy of the particle from E_1 to E_2, with $E_2 - E_1 = \hbar \omega$, in the case which is involved only one process with frequency ω. We have also calculated the momentum exchange between radiation and matter, and we have found $\langle \Delta \rangle^2$, that is, the momentum square average acquired after $\aleph$ absorptions and emissions (in a short time τ). We have seen that [expression (2.30)]:

$$\frac{\langle \Delta^2 \rangle}{Q^2(\omega)} = \frac{\langle \Delta^2 \rangle}{\hbar^2 \omega^2 / c^2} =$$
$$= \frac{\tau}{3} n(E_1) \left[e^{-(E_2 - E_1)/kT} A \rho_o(\omega) + (e^{-(E_2 - E_1)/kT} - 1) B \rho_T(\omega) \right] \quad (2.42)$$

In the limit $T \to 0$, we shall have $\rho_T(\omega) \to 0$ and $e^{-(E_2 - E_1)/kT} \to 0$, and so obtaining $\langle \Delta^2 \rangle / \frac{\hbar^2 \omega^2}{c^2} \to 0$. T. is approximated result for the momentum fluctuations is obtained due to the Boltzmann factor $e^{-(E_2 - E_1)/kT}$, which is compatible with the equipartition principle. If we want a fully incorporation of the zero-point fluctuations, we must modify this point. However we do not want to loose the simplicity of Einstein's model and so we decided not to change either the expression (2.30) or the equipartition hypothesis. We prefer to justify the hypotheses already done. For this we shall remind the result obtained by Einstein and Hopf (Einstein 1910) in 1910. In this work they have obtained the momentum square average of a polarizable particle under the influence of random radiation. The calculation is performed only with the utilization of Classical Electrodynamics, and the result is:

$$\langle\Delta^2\rangle = \frac{8\pi^4 e^2 c}{15m} \frac{\rho^2(\omega,T)}{\omega^2} \tau \tag{2.43}$$

where e and m are the charge and the mass of the oscillating part (frequency ω) of the polarizable particle. This is a general result which does not depend on the particular form of the spectral distribution $\rho(\omega,T)$. The only hypothesis is that the particles are immersed in *random electromagnetic fields*. According to our view based on Stochastic Electrodynamics, matter is *always* immersed in the zero point radiation fields and therefore expression (2.43) must include them. It's usual to consider the thermal and the zero-point radiation statistically independent, and so the momentum fluctuations can be written as (Jiménez 1980):

$$\langle\Delta^2\rangle = \langle\Delta^2\rangle_{thermal} + \langle\Delta^2\rangle_0 = \langle\Delta^2\rangle_{thermal} + \frac{8\pi^4 e^2 c}{15m} \rho_0(\omega)\tau \tag{2.44}$$

In the limit $T \to 0$, the thermal part will vanish, and so formula (2.43) will become:

$$\langle\Delta^2\rangle = \frac{8\pi^4 e^2 c}{15m} \frac{\tau}{\omega^2} \left(\frac{\hbar\omega^3}{2\pi^2 c^3}\right)^2 = \frac{2}{15} \frac{\tau}{\Gamma} \left(\frac{e^2}{\hbar c}\right)^2 \left(\frac{\hbar\omega}{mc^2}\right)^4 m^2 c^2 \tag{2.45}$$

where $\Gamma = e^2/mc^3$ is the time that the light spends to cross the radius r_o of the *classical electron* ($r_o \approx e^2/mc^2$). We can rewrite equation (2.45) in order to obtain a ratio between energies, i.e.:

$$\frac{\langle\Delta^2\rangle/2M}{\hbar\omega} = \frac{1}{15} \frac{\tau}{\Gamma} \left(\frac{e^2}{\hbar c}\right)^2 \left(\frac{\hbar\omega}{mc^2}\right)^2 \left(\frac{\hbar\omega}{Mc^2}\right) \tag{2.46}$$

where $\Delta^2/2M$ is the kinetic (translational) energy of the particle (total mass M) and $\hbar\omega$ is the exchanged energy.

Now it's possible to verify the importance of each factor in the r.h.s. of (2.46). Within the time τ there are $\aleph$ emissions and absorptions, and its reasonable to assume that $\tau \geq \Gamma$. Consequently this term does not contribute to decrease the r.h.s. of (2.46). However we also know that $(e^2/\hbar c)^2 \approx (1/137)^2$, but the vanishing observed in (2.42) when $T \rightarrow 0$ is guaranteed in (2.46) only when $mc^2 \gg \hbar\omega$ (obviously this also implies that $\dot{M}c^2 \gg \hbar\omega$). In other words; at zero temperature the average kinetic (translational) energy of the particle is much smaller than the characteristic exchanged energy $\hbar\omega$. Our analisys, then, lead us to the limit of massive particles, in which is also valid the nonrelativistic approximation that we have used during the calculations. Different arguments (Boyer 1969a), based on the interaction of the particles with the recipient also lead to the same conclusion.

IV - CONCLUSIONS

As in the original spirit of Einstein's model, we have studied the interaction of polarizable particles (an "ensemble" that obeys Boltzmann statistics) with electromagnetic radiation. However, our treatement is somewhat different, since we have considered explicitly the action of the *classical fluctuating zero-point radiation* that *induces*, together with thermal radiation, emissions and absorptions of electromagnetic energy under the form of *waves*. The particle energy is changed at a rate $dW/dt \propto \rho(\omega)$, which also represents the transition probability among "states" of energy. Here $\rho(\omega)$ represents the spectral distribution of the fields (zero-point and thermal) that stimulates the processes.

Since all the processes are induced, a well defined direction may be associated to each one. Thus we have concluded that each *energy transfer* is accompanied by a *momentum transfer*, in such a way that, if there are N processes of absorption and M processes of emission, the following relations:

$$\begin{aligned} E_I + \sum_{i=1}^{N} \hbar\,\omega_i &= E_F + \sum_{j=1}^{M} \hbar\,\omega_j' \\ \vec{p}_I + \sum_{i=1}^{N} \hbar\,\vec{k}_i &= \vec{p}_F + \sum_{j=1}^{M} \hbar\,\vec{k}_j' \end{aligned} \tag{4.1}$$

must be valid in order to maintain the equilibrium. We are considering the expressions above as simply the classical energy and momentum conservation laws which always hold when free charges are in interaction with electromagnetic radiation. Nowadays the definite quantities $\hbar\,\omega$ and $\hbar\omega/c$ are interpreted as being the energy and momentum of the *massless particles* (called "photons" of frequency ω), which represent the quanta of the electromagnetic field within Quantum Electrodynamics. These relations are well known since they were of decisive importance for the elucidation of the problems involving the X-ray scattering (Compton 1923). He deduced, based on the relativistic generalizations of the kinematic relations (4.1) we have derived, the famous expression

$$\frac{\Delta\lambda}{\lambda_0} = \frac{h\,\nu_0}{m\,c^2}(1-\cos\theta) \quad , \quad \lambda_0\nu_0 = c \tag{4.2}$$

for the wavelenght-shift observed in X-ray scattering. This was done almost twenty years after the introduction of the concept of photon by Einstein (Einstein 1905). The interesting history concerning the attempts to understanding the X-ray scattering may be appreciated in the book *The Compton Effect - Turning Point in Physics* (Stuewer 1975).

Here, revisiting Einstein's 1917 and Einstein and Ehrenfest's 1923 papers on the new light of SED, we have concluded that it is possible to obtain the kinematic relations (4.1) by using only Classical Electrodynamics with classical zero-point radiation. There was no need to introduce the "photon" as a particle-like entitie. In other words; our analysis suggests that we can consider the "photon" as an almost monochromatic (frequency ω) *pulse of electromagnetic waves* with average energy $\hbar\,\omega$. This is interesting because we know that Compton himself was able to derive the wavelenght-shift (4.2) by using a reasoning based on the *classical Doppler effect* (Compton 1923)-(Dodd 1983)-(Kidd 1985)-(Schroedinger 1927)-(Strnad 1986). This classical reasoning also helped him in the calculation of the X-ray scattering cross section as we can see in Compton's original paper (Compton 1923), as well as in a work by Woo (Woo 1925). More recently we have discussed this point with detail (Barranco 1988).

It's grateful to see that seventy years after Einstein's work one still can find new features in it, as we have perceived in joining it with Classical Stochastic Electrodynamics. We could see the range of Einstein's 1917 theory by appreciating its connections with Compton scattering, a phenomenon that constitutes itself a landmarck in the study of the interaction of radiation with matter. We think that Classical Physics can still give us many interesting explanations concerning the microscopic world phenomena.

EPILOGUE

"For the rest of my life I will think what are light quanta".

Einstein - 1917

"All these 50 years of conscious brooding have brought me no near to the answer to the question: what are light quanta? Nowadays every Tom, Dick and Harry thinks he knows it, but he is mistaken".

Einstein - 1951

APPENDIX

LORENTZ TRANSFORMATION FOR THE SPECTRAL DISTRIBUTION

We are going to calculate how the spectral density $\rho(\omega,T)$ changes when it's observed from an inertial system which is in movement with velocity v (x direction) in relation to the laboratory system (recipient with temperarure T). For this we shall write the fluctuating fields as a superposition of plane waves, and the electric filed (at the point $\vec{r}$ and time t) will be:

$$\vec{E}(\vec{r},t)=\sum_{\lambda=1}^{2}\int d^3\vec{k}\;\hat{\epsilon}(\vec{k},\lambda)\;g(\vec{k},T)\cos[\omega_k t-\vec{k}\cdot\vec{r}+\eta(\vec{k},\lambda)] \qquad (A.1)$$

where $g(\omega_{\vec{k}})\equiv g(\vec{k},T)$ characterizes the amplitude of the wave with frequency $\omega_{\vec{k}}=c|\vec{k}|$. The polarization vectors $\hat{\epsilon}$ satisfy:

$$\hat{\epsilon}(\vec{k},\lambda)\cdot\hat{\epsilon}(\vec{k},\lambda')=\delta_{\lambda\lambda'} \qquad \vec{k}\cdot\hat{\epsilon}(\vec{k},\lambda)=0$$
$$\sum_{\lambda=1}^{2}\epsilon_i(\vec{k},\lambda)\epsilon_j(\vec{k},\lambda)=\delta_{ij}-\frac{k_ik_j}{k^2} \qquad (A.2)$$

In (A.1) $\eta(\vec{k},\lambda)$ are random phases uniformly distributed between 0 and 2π. The average (over η) of the square of $\vec{E}$ will be:

$$\langle\vec{E}^2(\vec{r},t)\rangle=\sum_{\lambda_1=1}^{2}\sum_{\lambda_2=1}^{2}\int d^3\vec{k}_1\int d^3\vec{k}_2\;\hat{\epsilon}(\vec{k}_1,\lambda_1)\;\hat{\epsilon}(\vec{k}_2,\lambda_2)\;g(\omega_1)\;g(\omega_2)\times$$
$$\times\langle\cos[\omega_1 t-\vec{k}_1\cdot\vec{r}+\eta(\vec{k}_1,\lambda_1)]\cos[\omega_2 t-\vec{k}_2\cdot\vec{r}+\eta(\vec{k}_2,\lambda_2)]\rangle \qquad (A.3)$$

and since (Boyer 1975):

$$\langle\cos[\omega_1 t-\vec{k}_1\cdot\vec{r}+\eta(\vec{k}_1,\lambda_1)]\cos[\omega_2 t-\vec{k}_2\cdot\vec{r}+\eta(\vec{k}_2,\lambda_2)]\rangle=\cos^2(\omega_1 t-\vec{k}_1\cdot\vec{r})\frac{1}{2}$$
$$\times\delta^3(\vec{k}_2-\vec{k}_1)\,\delta_{\lambda_1\lambda_2}+\mathrm{sen}^2(\omega_1 t-\vec{k}_1\cdot\vec{r})\frac{1}{2}\delta^3(\vec{k}_2-\vec{k}_1)\,\delta_{\lambda_1\lambda_2}=$$
$$=\frac{1}{2}\delta^3(\vec{k}_2-\vec{k}_1)\,\delta_{\lambda_1\lambda_2}. \qquad (A.4)$$

we obtain:

$$\langle \vec{E}^2(\vec{r},t)\rangle = \sum_{\lambda_1=1}^{2}\sum_{\lambda_2=1}^{2}\int d^3\vec{k}_1 \int d^3\vec{k}_2\ \hat{\epsilon}(\vec{k}_1,\lambda_1)\ \hat{\epsilon}(\vec{k}_2,\lambda_2)\ g(\omega_1)\ g(\omega_2)\times$$

$$\times\frac{1}{2}\delta^3(\vec{k}_2-\vec{k}_1)\,\delta_{\lambda_1,\lambda_2} = \int d^3\vec{k}\, g(\omega_{\vec{k}})^2 = \frac{4\pi}{c^3}\int_0^\infty d\omega\,\omega^2\, g(\omega)^2 \quad (A.5)$$

It's easy to see that if $\vec{B}$ is the magnetic random field, then $\langle\vec{E}^2\rangle=\langle\vec{B}^2\rangle$. According to the definition of spectral density (density of energy per frequency) we can write:

$$\frac{\langle\vec{E}^2+\vec{B}^2\rangle}{8\pi} \equiv \int_0^\infty d\omega\,\rho(\omega,T) = \frac{1}{c^3}\int_0^\infty d\omega\,\omega^2\, g^2(\omega,T), \qquad (A.6)$$

and from the last relations:

$$\rho(\omega) = \frac{g(\omega)^2\omega^2}{c^3} \qquad (A.7)$$

In the inertial system attached to the particle, which is in movement with velocity $\vec{v} = v\,\hat{\imath}$, the polarization vectors $\epsilon'_x,\epsilon'_y,\epsilon'_z$ may be written as functions of $\epsilon_x,\epsilon_y,\epsilon_z$ (laboratory system):

$$\begin{aligned}\epsilon'_x &= \epsilon_x\\ \epsilon'_y &= \gamma\left[\epsilon_y - \frac{v}{c}\frac{(\vec{k}\times\hat{\epsilon})_z}{k}\right]\\ \epsilon'_z &= \gamma\left[\epsilon_z + \frac{v}{c}\frac{(\vec{k}\times\hat{\epsilon})_y}{k}\right]\end{aligned} \qquad (A.8)$$

where $\gamma \equiv 1/\sqrt{(1-v^2/c^2)}$.

In the particle system, the electric field is given by:

$$\vec{E}'(\vec{r},t) = \sum_{\lambda=1}^{2}\int d^3\vec{k}\ g(\omega)\ (\epsilon'_x\hat{\imath}+\epsilon'_y\hat{\jmath}+\epsilon'_z\hat{k})\cos(\omega_k t - \vec{k}\cdot\vec{r} - \eta) \qquad (A.9)$$

Note that $g(\omega_{\vec{k}})$ is the amplitude defined in the laboratory system. Squaring $\vec{E}'$ and taking the average over η, we obtain:

$$\langle \vec{E}'^2(\vec{r},t)\rangle = \int d^3\vec{k}\; g(\omega)^2 \sum_{\lambda=1}^{2}(\epsilon_x'^2 + \epsilon_y'^2 + \epsilon_z'^2) \tag{A.10}$$

Using (A.8) and summing over λ [see (A.2)], we arrive at:

$$\langle \vec{E}'^2(\vec{r},t)\rangle = \int d^3\vec{k}\; g(\omega)^2\; \gamma^2(1 - \frac{\vec{v}\cdot\vec{k}}{\omega})^2 \tag{A.11}$$

According to the transformation properties of the four-vector $k_\mu \equiv (\vec{k}, \omega/c)$, we have:

$$\begin{aligned} d^3\vec{k} &= d^3\vec{k}'\gamma(1 + \frac{\vec{v}\cdot\vec{k}'}{\omega_k'}) \\ k_x' &= \gamma(k_x - \frac{v}{c^2}\omega_k) \\ \omega_k' &= \gamma(\omega_k - v\, k_x) \end{aligned} \tag{A.12}$$

With this (A.11) becomes:

$$\langle \vec{E}'^2(\vec{r},t)\rangle = \int \frac{d^3\vec{k}'\; g(\omega)^2}{\gamma(1 + \frac{\vec{v}\cdot\vec{k}'}{\omega_k'})} \tag{A.13}$$

In the system attached to the particle, the amplitude will be $g'(\vec{k}')$, which we desire to relate with $g(\omega)$. The definition of $g'(\vec{k}')$ is obviously:

$$\langle \vec{E}'^2(\vec{r},t)\rangle \equiv \int d^3\vec{k}'\; g'(\vec{k}')^2 \tag{A.14}$$

Comparing (A.14) with (A.13), we obtain:

$$g'(\vec{k}')^2 = \frac{g(\omega)^2}{\gamma(1 + \frac{\vec{v}\cdot\vec{k}'}{\omega_k'})} \tag{A.15}$$

Now we can use $(A.7)$ in order to obtain:

$$\rho'(\vec{k}') \equiv \rho'(\omega',\theta') = \rho(\omega)\gamma(1 - \frac{\vec{v}\cdot\vec{k}}{\omega})^2 \Big/ (1 + \frac{\vec{v}\cdot\vec{k}'}{\omega'_k}) \qquad (A.16)$$

We can simplify the expression above by retaining only terms of the order of v/c (nonrelativistic approximation), so that $\gamma \approx 1$ and $\cos\theta' \approx \cos\theta - \frac{v}{c}\mathrm{sen}^2\theta$. Thus $(A.16)$ becomes:

$$\rho'(\omega',\theta') \approx \rho(\omega)(1 - \frac{3v}{c}\cos\theta') \qquad (A.17)$$

Now we can expand $\rho(\omega)$ around ω', $\rho(\omega) = \rho(\omega' + \frac{v}{c}\omega'\cos\theta') \approx \rho(\omega') + \frac{\partial\rho(\omega')}{\partial\omega'}\omega'\,\frac{v}{c}\cos\theta' + \ldots$, and finally get:

$$\rho'(\omega',\theta') = \left[\rho(\omega') + \frac{v}{c}\frac{\partial}{\partial\omega'}\rho(\omega')\,\omega'\cos\theta'\right]\left[1 - \frac{3v}{c}\cos\theta'\right] \qquad (A.18)$$

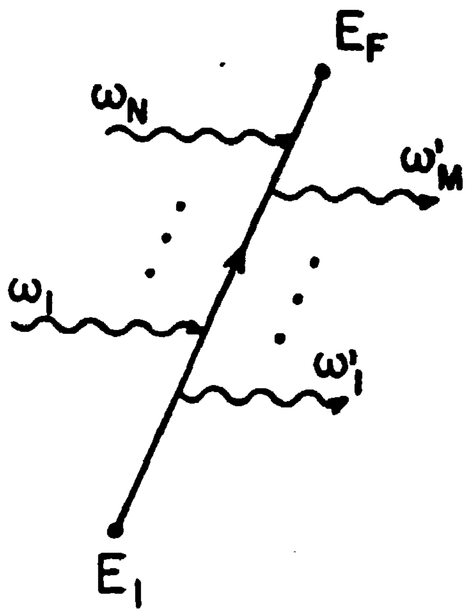
which is what we need in order to calculate the momentum exchange in section III-b.

ACKNOWLEDGMENTS

We would like to thank the partial support by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) and Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq). One of us (H.M.F.) acknowledges the hospitality received at Depertamento de Fisica Moderna, Facultad de Ciencias, Universidad de Cantabria, Santander (Spain) where part of this work was done.

Figure caption:

Schematic picture representing the processes of N absorptions in the frequencies $\omega_1, \omega_2, \ldots, \omega_N$ and M *emissions* in the frequencies $\omega'_1, \omega'_2, \ldots, \omega'_M$. E_F and E_I are respectively the (arbitrary) final and the initial particle energies.



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