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IS HIGHER-DERIVATIVE GRAVITY A GOOD THERAPY TO THE CAUSAL PATHOLOGIES OF GÖDEL-TYPE UNIVERSES? (*) (**)

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ABSTRACT

The possibility of considering higher-derivative gravity as a therapy to the causal pathologies of Gödel-type universes is investigated. As a consequence an unusual cosmological solution is obtained.

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The recent surge of interest in the so-called Gödel-type universes constitutes an apparent puzzle concerning gravity research since, as it is well known, Gödel's original work[1] dates back to 1949. What is it about these models that attracts so much attention? Perhaps the answer lies on the fact that, in general, the aforementioned space-times admit closed timelike curves. In truth, it is difficult to resist to the appealing idea of traveling into one's own past and thus conceivably influencing one's own history!

In the last few years a respectable effort has been devoted to the investigation of Gödel-type models that are homogeneous in space and time (hereafter called ST homogeneous)[2)-10)]. Recently, Rebouças and Tiomno[3)] have showed that the causality features of these ST homogeneous Riemannian manifolds depend on two independent parameters: m and Ω. Only in case $m^2 \geq 4\Omega^2$ there is no breakdown of causality of Gödel-type. They have also found the first exact Gödel-type solution of Einstein's equations describing a completely causal ST homogeneous rotating universe, corresponding to the limiting case $m^2 = 4\Omega^2$. Their solution is generated by a massless scalar field. More recently, Rebouças et al.[4)] have shown that the limiting case $m^2 = 4\Omega^2$ is the sole Gödel-type solution of the algebraic tachyon fluid type.

On the other hand, the well-known difficulties concerning the construction of a correct quantum version for General Relativity have led to the study of alternative theories of gravitation. In particular the $R + R^2$ theory of gravity has been suggested as a possible solution to the infinities plagueing the quantization of General Relativity[11)-15)]. This higher-

-derivative gravity theory is defined by the action

$$I = \int d^4x\sqrt{-g}\,\left[\frac{R}{2\kappa} - \frac{\Lambda}{\kappa} + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu} + L_m\right], \qquad (1)$$

where α and β are dimensionless coupling constants (in natural units), κ and Λ are the Einstein and cosmological constants, respectively, and L_m is the matter Lagrangian density. The correspondent field equations are given by

$$G_{\mu\nu} = - T_{\mu\nu}, \qquad (2)$$

$$G_{\mu\nu} = \frac{1}{\kappa}\,(R_{\mu\nu} - \frac{1}{2}\,Rg_{\mu\nu}) + \frac{\Lambda}{\kappa}\,g_{\mu\nu}$$

$$+ \alpha(-R^2 g_{\mu\nu} + 4RR_{\mu\nu} - 4g_{\mu\nu}\Box R + 4\nabla_\nu\nabla_\mu R) \qquad (3)$$

$$+ \beta(-2\Box R_{\mu\nu} - R_{\rho\theta}R^{\rho\theta}g_{\mu\nu} + 4R_{\mu\rho\theta\nu}R^{\rho\theta} - g_{\mu\nu}\Box R + 2\nabla_\nu\nabla_\mu R).$$

The following interesting question can now be posed:

Is higher-derivative gravity a good therapy to the causal pathologies of ST homogeneous Gödel-type universes?

To answer this question we must find, first of all, a class of exact solutions of the higher-derivative field equations concerning the ST homogeneous Gödel-type models. To accomplish this, we make use of the fact that the necessary and sufficient conditions to a Gödel-type metric, i.e.,

$$ds^2 = [dt + H(r)d\Phi]^2 - D^2(r)d\Phi^2 - dr^2 - dz^2, \qquad (4)$$

be space-time homogeneous are[3]

$$\frac{H'}{D} = \text{const} = 2\Omega, \quad \frac{D''}{D} = \text{const} \equiv m^2 \quad . \tag{5}$$

The source of our geometry is supposed to be a perfect fluid with energy density ρ and pressure p. An observer comoving with the fluid is assumed to have the four-velocity

$$v^{\mu} = \delta^{\mu}{}_{0} \quad . \tag{6}$$

We can show now that the higher-derivative field equations concerning the model under consideration reduce to the following set of equations

$$\rho = \Omega^2/\kappa + 4\Omega^4(\alpha+3\beta) - 2m^4(2\alpha+\beta) - \Lambda/\kappa \ , \tag{7}$$

$$p = \Omega^2/\kappa + 12\Omega^4(\alpha+3\beta) - 16\Omega^2 m^2(\alpha+\beta) + 2m^4(2\alpha+\beta) + \frac{\Lambda}{\kappa} \ , \tag{8}$$

$$m^2/\kappa = 2\Omega^2/\kappa + 16\Omega^4(\alpha+3\beta) - 24m^2\Omega^2(\alpha+\beta) + 4m^4(2\alpha+\beta). \tag{9}$$

In order to have physically significant solutions we must guarantee the positivity of energy and pressure. Consequently Λ must be bounded within the interval

$$- 12\Omega^4(\alpha+3\beta) - 2m^4(2\alpha+\beta) + 16m^2\Omega^2(\alpha+\beta) - \Omega^2/\kappa \leq \Lambda/\kappa \leq$$

$$4\Omega^4(\alpha+3\beta) - 2m^4(2\alpha+\beta) + \Omega^2/\kappa \ , \tag{10}$$

which implies that

$$8\Omega^4(\alpha+3\beta) + \Omega^2/\kappa - 8m^2\Omega^2(\alpha+\beta) \geq 0 \ . \tag{11}$$

In the integration of Eqs. (7)-(9) three cases arise, according to whether m^2 is >, < or = 0. Taking into account that we are interested in completely causal solutions only, we restrict our discussion to the $m^2 > 0$ case.

The metric is then given by

$$ds^2 = dt^2 + \frac{8\Omega}{m^2}\sinh^2\left(\frac{mr}{2}\right)d\phi dt - dr^2 - dz^2 +$$

$$\frac{4}{m^2}\sinh^2\left(\frac{mr}{2}\right)\left[\left(\frac{4\Omega^2}{m^2} - 1\right)\sinh^2\left(\frac{mr}{2}\right) - 1\right]d\phi^2, \qquad (12)$$

and the following inequality holds

$$4\Omega^2/\kappa \geqq m^2/\kappa - 32\Omega^4(\alpha+3\beta) + 40\, m^2\Omega^2(\alpha+\beta) - 4m^4(2\alpha+\beta). \quad (13)$$

Now the completely causal ST homogeneous models of the Gödel-type are characterized by $m^2 \geqq 4\Omega^2$, as we have already mentioned. Undoubtedly, the solution $m^2 = 4\Omega^2$ is compatible with (13). It follows then from (9) and (11) that

$$\frac{m^2}{4} = \Omega^2 = \frac{1}{8(3\alpha+\beta)\kappa}\ . \qquad (14)$$

From Eqs. (7) and (8) we get

$$\Lambda = -\frac{3\Omega^2}{2},\ \rho = p = 0. \qquad (15)$$

We have thus succeeded in finding a completely causal universe of the Gödel-type in the framework of higher-derivative gravity . An interesting feature of this solution is that it has both the energy density and pressure equal to zero, and it is thus a vacuum solution with the symmetries of the Gödel universe (counting the cosmological constant as "vacuum"). As such, it is a rather unusual vacuum solution, having no analogue in the context of the standard General Relativity. It is also interesting because it relates Newton's constant, α, β, and the value of the cosmological constant. Consequently it stablishes a connection between the microphysics and macrophysics.

Let us now look over carefully the metric corresponding to our model. From Eqs. (12) and (14) we get the following line element

$$ds^2 = [dt + \frac{1}{\Omega}\sinh^2(\Omega r)d\Phi]^2 - dr^2 - dz^2 - \frac{1}{4\Omega^2}\sinh^2(2\Omega r)d\Phi^2. \qquad (16)$$

We introduce new coordinates Φ and R defined by

$$\varphi = \Phi - \Omega t, \quad \Omega R = \sinh(\Omega r), \qquad (17)$$

and rewrite the metric for the hyperbolic family of space-times (16) in the form

$$ds^2 = (1+\Omega^2R^2)dt^2 - (1+\Omega^2R^2)^{-1}dR^2 - R^2d\varphi^2 - dz^2. \qquad (18)$$

This metric is nothing but a trivial embedding of the 3-dimensional anti-de Sitter metric in the 4-dimensional space-time. Therefore the existence of such solution is not so surprising in terms of the higher-derivative theory.

The problem concerning the existence of another causal solutions is still an open question, unless we introduce artificial constraints between the parameters α and β.

Just to conclude, we wish to point out that if higher-derivative gravity has not entirely succeeded in healing the causal pathologies of Gödel-type universes, it has at least bequeathed to us an unusual result: a completely causal vacuum solution of the Gödel-type. Moreover, this space-time has a seven-parameter maximal group of motions (G_7) while the remaining Gödel-type metrics have a G_5 [6]. Solutions with a G_7 of motions are known to be very rare.[16]

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