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AN ENLARGED SUPERFLUID MODEL OF ATOMIC NUCLEUS*

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ABSTRACT

The well known superfluid model (or quasiparticle phonon nuclear model (QPNM)) of atomic nucleus is enlarged by including an adequate four-nucleon effective interaction in addition to the pairing and long-range effective residual interactions. New experimental data can be explained without affecting those observables already described by the QPNM and in addition new features can be enumerated: 1) superfluidities of the neutron and proton systems may be generated by one another; 2) the phase structure is enriched by a new superfluid phase dominated by alpha-type correlations (ATC) and 3) superfluid isomers and their bands of elementary excitations are predicted. Unusual large two-nucleon and alpha transfer reactions cross sections as well as some unusual large alpha decay widths can be explained.

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1 Introduction

By including an adequate four-nucleon effective interaction[1,2] in addition to the pairing and multipole-multipole effective interactions,global correlations between proton and neutron fluids other than two-nucleon pairing and long-range correlations in atomic nuclei are induced.New metastable (isomeric) superfluid and normal states are predicted.A new type of elementary excitations may be constructed on these metastable states as those constructed on the BCS superfluid ground states[3,4].The region of superfluid nuclei is enlarged due to the fact that the neutron and proton superfluidity can mutually be induced via the four-nucleon effective interaction and the phase structure is enriched by a new superfluid phase dominated by the so-called alpha-type correlations (ATC).The power of this model - an enlarged superfluid model (ESM) - is the fact that it can explain at the same level of accuracy or even better those physical observables that the nuclear superfluid model (QPNM)[3,4] has explained and in addition new features and predictions could be enumerated:1)the explanation of the odd-even staggering of the mean-square charge radii of isotopes of one element[5-8];2)the explanation of the unusual large alpha decay rates of neutron deficient lead isotopes[9];3)the description of the unusual large cross sections of the two-nucleon-[10,11] and alpha-[12]transfer reactions in Sn-Sm-Ce-Gd region;4)the prediction of the existence of the superfluid isomers[2] and their elementary excitations by analogy with the fission isomers and 5)the prediction of the existence of the first and second order phase transitions studied e.g.by using the experimental evidences[1] concerning the two-nucleon and alpha transfer reactions.

From the confrontation[13] between the QPNM[3,4] and the interaction boson model(IBM)[14-18] one realizes that the last model seems to explain many more low-lying 0^+-states than QPNM.Our ESM gives the possibility to remove this difficulty of the QPNM by predicting the above-mentioned metastable 0^+ states and their elementary 0^+ excitations.

This paper is organized as follows.In section 2 we formulate the (ESM) model.The discussion of the RPA secular equations is presented in section 3.An enlarged quasiparticle phonon nuclear model for the excited states other than the 0^+ ones is discussed in section 4.The explanation of the odd-even staggering of the nuclear charge radii of the isotopes of some elements is presented in section 5.Some experimental evidences in favour of the prediction of the superfluid isomers are discussed in section 6.Section 7 is devoted to the conclusions.

2 Formulation of the Model

Some years ago[19] the theory of the finite Fermi systems of Migdal[20] has been applied to describe the alpha decay process.It was necessary to introduce an effective four-nucleon interaction to cover the lack of momentum transfer in this process.First results[21−23] looked rather promising,since they were for the first time at the right order of magnitude.A four-body interaction of similar strength has been proposed also earlier[24],in order to explain the Coulomb energy difference of mirror nuclei.Another experimental evidence for introducing a four-nucleon interaction in the nuclear structure theory was the odd-even staggering of the charge radii of isotopes of one element extracted from the isotope shifts experiments[5−8].The calculations of these charge radii using a realistic two body interaction either within linear response or HFB treatments[25,26] have shown a systematic discrepancy as compared to the experiment.More interesting was the assumption[1,2,27,28] of the existence of the effective four-nucleon (2p2n) interactions in addition to the two nucleon ones which led to the first positive results[27−30] in the staggering problem.

In this paper we show that within the ESM it is possible to reproduce the normal odd-even staggering of nuclear charge radii and to construct the structure of both the low-lying collective and non-collective states of the pairing superfluid ground state band and alpha-like superfluid 0^+-state band respectively.Collective excitation in doubly even nuclei are commonly considered to be well described as harmonic vibrations of small amplitude around some equilibrium density matrix (ρ_0).Expanding the total energy in terms of $\delta\ \rho = \rho - \rho_0$ to second order,the first functional derivative to be taken at the equilibrium density is the stationary single-particle Hartree-Fock (HF) Hamiltonian and the second functional derivative describes how the single particle Hamiltonian reacts to a small change in the density matrix ($\delta\rho$). The RPA equations may be derived in order to describe small-amplitude harmonic vibrations around the HF equilibrium position.The RPA eigenstates correspond to the boson states (or the so-called phonon[3] collective vibrations of the nucleus).When some symmetries are broken (as e.g. the rotational and the particle number[31] symmetries) the RPA solutions describe harmonic small amplitude vibrations around the appropiate mean field equilibrium position.

Our ESM that includes, besides pairing correlations, the so-colled dynamically induced ATC is presented in detail in Refs.[1,2].The Hamiltonian

$$H = H_0 + H_{coll} \qquad H_0 = H_{av} + H_{pair} + H_4 \tag{1}$$

includes the (average) selfconsistent single particle part[1,3]

$$H_{av} = \sum_{i=p,n} \sum_{s_i\sigma_i} E_{s_i} a^\dagger_{s_i\sigma_i} a_{s_i\sigma_i} \tag{2}$$

the pairing part[1,3]

$$H_{pair} = -\sum_{i=p,n} G_i P_i^\dagger P_i \qquad P_i = \sum_{s_i} a_{s_i-} a_{s_i+} \tag{3}$$

the *four-nucleon interaction part*

$$H_4 = -G_4 P_p^\dagger P_n^\dagger P_n P_p \tag{4}$$

and the collective part

$$H_{coll} = -\frac{1}{2} \sum_{i=p,n} \sum_{\lambda\mu>0} [\kappa_i^{(\lambda)} Q_{\lambda\mu}^\dagger(i) Q_{\lambda\mu}(i) + \kappa_{np}^{(\lambda)} (Q_{\lambda\mu}^\dagger(n) Q_{\lambda\mu}(p) + Q_{\lambda\mu}^\dagger(p) Q_{\lambda\mu}(n))] \tag{5}$$

where[3]

$$Q_{\lambda\mu}(i) =$$

$$\sum_{s_i\sigma_i} \sum_{s_i'\sigma_i'} < s_i\sigma_i \mid (2(1+\delta_{\mu 0}))^{-\frac{1}{2}} r^\lambda (Y_{\lambda\mu} + (-1)^\mu Y_{\lambda-\mu}) \mid s_i'\sigma_i' > a_{s_i\sigma_i}^\dagger a_{s_i'\sigma_i'}$$

$$= \sum_{s_i s_i' \sigma} (f^{\lambda\mu}(s_i s_i') a_{s_i\sigma}^\dagger a_{s_i'\sigma} + \sigma \bar{f}^{\lambda\mu}(s_i s_i') a_{s_i\sigma}^\dagger a_{s_i'-\sigma}) \tag{6}$$

The ground state is determined by minimizing the energy functional[1]

$$W = < BCS \mid H - \sum_{i=p,n} \lambda_i \hat{N}_i \mid BCS > \tag{7}$$

with respect to the variational parameters u_{s_i} and v_{s_i} subject to the constraints

$$< BCS \mid \hat{N}_i \mid BCS > = N_i \tag{8}$$

$N_{p(n)}$ being the number of protons(neutrons) contained in the corresponding A - shell[31]. The new proton and neutron gap energies ($\Delta_{p(n)}$) and constraint equations

$$\frac{1}{2}(G_{p(n)} + G_4 \chi_{n(p)}^2) \sum_{s_{p(n)}} (\epsilon_{s_{p(n)}}^2 + \Delta_{p(n)}^2)^{-\frac{1}{2}} = 1 \tag{9}$$

$$\sum_{s_i} (1 - \epsilon_{s_i} (\epsilon_{s_i}^2 + \Delta_i^2)^{-\frac{1}{2}}) = N_i \tag{10}$$

now are coupled due to the H_4 - interaction (4). Here

$$\epsilon_{s_i} = E_{s_i} - \lambda_i \tag{11}$$

and

$$\chi_i = \frac{1}{2} \Delta_i \sum_{s_i} (\epsilon_{s_i}^2 + \Delta_i^2)^{-\frac{1}{2}} \tag{12}$$

where E_{s_i} are the renormalized[1] single particle energy levels in a selfconsistent potential well (e.g. a deformed Hartree-Fock or Saxon-Woods one).

As pointed out in Refs 1 the equations (9) and (10) tell us that the *superfluidities of the proton and neutron systems may be generated by one another* and by studying in addition the function $W = W(\Delta_p, \Delta_n)$[1,32,33] (see eq.7) the phase structure is enriched by a new superfluid phase dominated by the ATC - the so-colled alpha-type superfluid phase. Among the consequences concerning the nuclear structure and dynamics figures the prediction of new bands of elementary excitations corresponding to the so-called[2] superfluid isomers. When one obtains second (local) minima versus pairing deformations (Δ_i) of W we can construct new (superfluid) bands of elementary excitations in addition to the (superfluid) ground state band. This spectra should be a trace of the above mentioned superfluid isomers analogous to the fission isomers first discovered in 1962[34], which are actually shape isomers corresponding to a second minimum in the potential energy along the elongation degree of shape deformation.

Within the RPA - treatment with new gap (Δ_i) and Fermi level (λ_i) energies, by adding the H_{coll} - term (5) we obtain the structure of the low-lying collective and non-collective states belonging to different (superfluid) bands in the same spirit of the well-known superfluid model[3,4] (or QPNM[3,13]). The influence of H_4 - term on the structure of these states is performed by replacing the old u_{s_i} and v_{s_i} entering the structure of the collective and non-collective states by those obtained by solving the equations (9,10). The only exception to this rule concerns the 0^+ states whose structure becomes slightly more complicated[2]. For such states the phonon operator has the expression:

$$Q^{\dagger}_{(r)} = \frac{1}{4} \sum_{i=p,n} \sum_{s_i s_i'} [w^{(r)}_{s_i s_i'} (A^{\dagger}(s_i s_i') + A(s_i s_i')) - g^{(r)}_{s_i s_i'} (A^{\dagger}(s_i s_i') - A(s_i s_i'))] \quad (13)$$

with

$$A(s_i s_i') = 2^{-\frac{1}{2}} \sum_{\sigma} \sigma \alpha_{s\sigma} \alpha_{s'-\sigma} \quad (14)$$

$$\alpha^{\dagger}_{s\sigma} = u_s a^{\dagger}_{s-\sigma} + \sigma v_s a_{s\sigma} \quad (15)$$

The secular equation is, as in Ref.3, obtained within RPA by using the variational principle:

$$\begin{vmatrix} X_{pp}-1 & X_{pn} & V_p & 0 & W_p & T_{pn} \\ X_{np} & X_{nn}-1 & 0 & V_n & T_{np} & W_n \\ F_{pp} & F_{pn} & A_p-1 & 0 & B_p & C_{pn} \\ F_{np} & F_{nn} & 0 & A_n-1 & C_{np} & B_n \\ S_{pp} & S_{pn} & K_p & 0 & R_p-1 & Q_{pn} \\ S_{np} & S_{nn} & 0 & K_n & Q_{np} & R_n-1 \end{vmatrix} = 0 \quad (16)$$

where

$$X_{ij} = 2\kappa_{ij} \sum_{s_i s_i'} (f^{(2)}(s_i s_i') u_{s_i s_i'})^2 \varepsilon_{s_i s_i'} (\varepsilon^2_{s_i s_i'} - \omega^2)^{-1} \quad (17)$$

$$V_i = \sum_{s_i} f^{(2)}(s_i s_i) u_{s_i s_i} \omega \Delta_i [\chi_i (\varepsilon^2_{s_i s_i} - \omega^2)]^{-1} \quad (18)$$

$$W_i = \sum_{s_i} f^{(2)}(s_i s_i) u_{s_i s_i} \varepsilon_{s_i s_i} \Delta_i (u^2_{s_i} - v^2_{s_i}) [\chi_i (\varepsilon^2_{s_i s_i} - \omega^2)]^{-1} \quad (19)$$

$$T_{ij} = \sum_{s_i} f^{(2)}(s_i s_i) u_{s_i s_i} 2\varepsilon_{s_i s_i} G_4 (u^2_{s_i} - v^2_{s_i}) \chi_i \chi_j (\varepsilon^2_{s_i s_i} - \omega^2)^{-1} \quad (20)$$

$$F_{ij} = \kappa^{(2)}_{ij} \sum_{s_i} 2\omega f^{(2)}(s_i s_i) u_{s_i s_i} (\varepsilon^2_{s_i s_i} - \omega^2)^{-1} \quad (21)$$

$$A_i = \sum_{s_i} \varepsilon_{s_i s_i} \Delta_i [\chi_i (\varepsilon^2_{s_i s_i} - \omega^2)]^{-1} \quad (22)$$

$$B_i = \sum_{s_i} \omega \Delta_i (u^2_{s_i} - v^2_{s_i}) [\chi_i (\varepsilon^2_{s_i s_i} - \omega^2)]^{-1} \quad (23)$$

$$C_{ij} = \sum_{s_i} 2\omega G_4 (u^2_{s_i} - v^2_{s_i}) \chi_i \chi_j (\varepsilon^2_{s_i s_i} - \omega^2)^{-1} \quad (24)$$

$$S_{ij} - \kappa^{(2)}_{ij} \sum_{s_i} f^{(2)}(s_i s_i) u_{s_i s_i} (u^2_{s_i} - v^2_{s_i}) 2\varepsilon_{s_i s_i} (\varepsilon^2_{s_i s_i} - \omega^2)^{-1} \quad (25)$$

$$K_i = \sum_{s_i} (u^2_{s_i} - v^2_{s_i}) \omega \Delta [\chi_i (\varepsilon^2_{s_i s_i} - \omega^2)]^{-1} \quad (26)$$

$$R_i = \sum_{s_i} (u^2_{s_i} - v^2_{s_i})^2 \varepsilon_{s_i s_i} \Delta_i [\chi_i (\varepsilon^2_{s_i s_i} - \omega^2)]^{-1} \quad (27)$$

$$Q_{ij} = \sum_{s_i} (u^2_{s_i} - v^2_{s_i})^2 2\varepsilon_{s_i s_i} G_4 \chi_i \chi_j [\varepsilon^2_{s_i s_i} - \omega^2]^{-1} \quad (28)$$

$$u_{s_i s_i'} = u_{s_i} v_{s_i'} + v_{s_i} u_{s_i'} \quad (29)$$

$$\varepsilon_{ss'} = \varepsilon_s + \varepsilon_{s'} \quad (30)$$

$$\varepsilon_{s_i} = (\epsilon^2_{s_i} + \Delta^2_i)^{\frac{1}{2}} \quad (31)$$

$$u^2_{s_i} = \frac{1}{2}(1 + \epsilon_{s_i} \varepsilon^{-1}_{s_i}) \qquad v^2_{s_i} = \frac{1}{2}(1 - \epsilon_{s_i} \varepsilon^{-1}_{si}) \quad (32)$$

(see eq.11)

The amplitudes $g^{(r)}_{s_i s_i'}$ and $w^{(r)}_{s_i s_i'}$ of the 0^+ - phonon states have the following expression:

$$g^{(r)}_{s_i s_i'} = \delta_{s_i s_i'} (\omega \Delta_i L_i [\chi_i (\varepsilon^2_{s_i s_i} - \omega^2)]^{-1} + \varepsilon_{s_i s_i} \Delta_i (u^2_{s_i} - v^2_{s_i}) M_i [\chi_i (\varepsilon^2_{s_i s_i} - \omega^2)]^{-1} +$$
$$2\varepsilon_{s_i s_i} G_4 \chi_i \chi_j (u^2_{s_i} - v^2_{s_i}) M_j (\varepsilon^2_{s_i s_i} - \omega^2)^{-1})$$

$$+2\epsilon_{s_is_i'}f^{(2)}(s_is_i')u_{s_is_i'}(\epsilon^2_{s_is_i'}-\omega^2)^{-1}(\kappa^{(2)}_{ii}D_i+\kappa^{(2)}_{ij}D_j) \tag{33}$$

$$w_{s_is_i'}=\delta_{s_is_i'}(\epsilon_{s_is_i}L_i[\chi_i(\epsilon^2_{s_is_i}-\omega^2)]^{-1}+$$
$$\omega\Delta_i(u^2_{s_i}-v^2_{s_i})M_i[\chi_i(\epsilon^2_{s_is_i}-\omega^2)]^{-1}+$$
$$2\omega G_4(u^2_{s_i}-v^2_{s_i})\chi_i\chi_jM_j(\epsilon^2_{s_is_i}-\omega^2)^{-1}+$$
$$2\omega f^{(2)}(s_is_i')u_{s_is_i'}(\epsilon^2_{s_is_i'}-\omega^2)^{-1}(\kappa^{(2)}_{ii}D_i+\kappa^{(2)}_{ij}D_j) \tag{34}$$

in which when i=p(n) j=n(p). The quantities D_i,M_i and L_i are defined by

$$D_i=\sum_{s_is_i'}f^{(2)}(s_is_i')u_{s_is_i'}g_{s_is_i'} \tag{35}$$

$$M_i=\sum_{s_i}(u_{s_i}^2-v^2_{s_i})g_{s_is_i'} \tag{36}$$

$$L_i=\sum_{s_i}w_{s_is_i} \tag{37}$$

and are solutions of the following set of equations

$$D_i=V_iL_i+W_iM_i+T_{ij}M_j+X_{ii}D_i+X_{ij}D_j \tag{38}$$

$$M_i=K_iL_i+R_iM_i+Q_{ij}M_j+S_{ii}D_i+S_{ij}D_j \tag{39}$$

$$L_i=A_iL_i+B_iM_i+C_{ij}M_j+F_{ii}D_i+F_{ij}Dj \tag{40}$$

together with the normalization

$$\frac{1}{2}\sum_{i=p,n}\sum_{s_is_i'}g_{s_is_i'}w_{s_is_i'}=1 \tag{41}$$

3 Discussion of the RPA Secular Equations for 0^+ - Excited States

Swiching off the four nucleon interaction ($G_4=0$) the equation (16) becomes identical to the equation (8.167) of Ref.3 where $\kappa^{(2)}_{ij}=\kappa^{(2)}$,which is the the usual RPA equation for the energies of the collective vibrational phonon 0^+ - states.If further assuming a slightly more simplified model in which all the diagonal quadrupole moment matrix elements are,say,equal to f_0,one obtains two types of excited 0^+ - states.The energies of the first type of states are determined by the non-diagonal matrix elements of the quadrupole moment operator and by the quadrupole-quadrupole interaction coupling constant.The other type of excited 0^+ - states are determined by the pairing interaction only.The first type of collective modes are usually called shape quadrupole β - vibrations and the second type - pairing collective proton (neutron) vibrations of the superfluid nucleus.Usually the diagonal matrix elements f(ss) are not equal and consequently the two types of collective 0^+ - modes are coupled.Swiching on the four nucleon interaction ($G_4\neq 0$) and keeping the simplified model with $f(ss)=f_0$ the proton and neutron pairing vibration modes are coupled.In the model we discussed in the previous section,i.e. $f(ss)\neq f_0$ and $G_4\neq 0$,we obtain the coupling of the three shape vibration and pairing proton and neutron vibration modes.If there exists a second set of solutions for the equations (9,10) i.e. new gap ($\Delta_{2p(n)}$) and Fermi ($\lambda_{p(n)}$) energies corresponding to a local minimum of the energy functional (7) another group of excited 0^+ - states should appear.Their structure depends,of course,on these new gap and Fermi energies.These new spectra should be a trace of the recently predicted[2] superfluid isomers.

The introduction in the long range part of the Hamiltonian of the spinmultipole interaction[3] causes an increase in the density of low-lying 0^+ states and can even bring two or more 0^+ states inside the energy gap.

It is useful to point out that in the ESM as well as in the QPNM the spurious state due to the particle number nonconservation is automatically removed.

The introduction of the four nucleon interaction (4) in the QPNM leads to an enlarged superfluid model (ESM),i.e. the structure of the low-lying collective and non-collective excited states is slightly modified in the region of nuclei where the pairing superfluidity is already installed and for some nuclei the ESM predicts a new bands of elementary excitations corresponding to the so-called superfluid isomers which are evidences of a new superfluid phase - the alpha like superfluid phase.

4 Enlarged Quasiparticle Phonon Nucear Model for the Excited States Other than 0^+ - States

The treatment of γ - and octupole vibrational states in the doubly even superfluid nuclei as well as the excited states of odd-mass and odd-odd superfluid nuclei is a straightforward generalization of the approach developed in chapters 9 and 10 of the Ref.3.When the structure of these states involves other phonons than 0^+ - vibrational ones the only modification in the corresponding wave functions is to replace the gap and Fermi energies by those obtained as solutions of equations (9) and (10).Concerning the β - vibrational phonons,they should be obtained within the procedure developed in section 3.Due to the mutual induction of the proton and neutron superfluidity via the four-nucleon interaction (see eqs.9 and 10) the position of the excited energy levels of the superfluid nuclei could be slightly mod-

ified,however for some transition probabilities or density distributions we may obtain large quantitative modifications.An interesting case is the odd-even staggering of the nuclear charge radii of the isotopes of one element.

5 The Origin of the Odd-Even Staggering of Nuclear Charge Radii of Isotopes of one Element

Experimental evidences on isotopes shifts[5-8] show a distinctive "odd-even staggering" concerning the magnitude of the nuclear charge radii of isotopes of one element.This staggering refects the fact that the radii for the odd-neutron isotopes are permanently smaller than the averages of their even-neutron neighbours'radii.With few exceptions,this behaviour is found over the whole table of nuclei and is confirmed with increasing accuracy.In some regions as e.g. the very neutron deficient Hg,Au,Sm and Ce isotopes,this staggering is very large and has been attributed to special deformation effects[37].In some other regions as e.g. the lead and tin isotopes specific shell effects may express themselves[38].But apart from these special cases,the charge radii of odd-neutron isotopes are found, as we already have mentioned,to be smaller than the averages of their even-neutron neighbours'radii - effect usually called "the normal odd-even staggering".

Attempts of theoretical explanations were based on blocking of ground-state quadrupole vibrations by the odd neutron[39,40] and blocking of pairing correlations[41] together with the polarization of the proton distribution by the neutrons[42].However ,such calculations involving phenomenological assumptions or fit parameters yield - at best - a much too small effect.Among other theoretical explanations we mention the application of the theory of finite Fermi systems to the normal staggering problem with a careful adjustment of the single particle energies which failed to improve the agreement with the experiment,as long as it has been postulated that the two particle interaction should be a realistic one[25,26].More interesting was the assumption of the existence of the effective four-nucleon interactions (two protons and two neutrons) in addition to the two-nucleon ones which led to the first positive results in the staggering problem[28] obtained by Zawischa.

Within the Zawischa approach the staggering mechanism is explained by the corrections to the normal proton density[31] induced by the parts of the selfconsistent field, generated by three- or four-body interactions,which are functions of the neutron pairing density[27].

Within the ESM the staggering is generated by the corrections to the proton density due to the mutual induction of the proton and neutron superfluidity via the four-nucleon interaction (see eqs. 9 and 10) and the blocking of pairing and alpha type correlations.The charge radii staggering for the lead isotopes is one of the most important cases for the ESM,because by swiching off the H_4-interaction,for the proton system the critical Belyaev condition[1] is not satisfied,that is the proton system is in a normal phase.By swiching on the H_4-interaction the proton superfluidity is induced by the already installed neutron superfluidity.Thus the staggering of the charge radii is due to the induced proton superfluidity and not to the induced changes[43] in the canonical basis[31].

In Table 1 the isotopic shifts of the nuclear charge radii for lead isotopes are shown.We compare the results of calculations using HFB with density-dependent two nucleon interaction[27],HFB including the four-nucleon interaction and our ESM,together with the experimental data[5-8].In the calculations we have used the Woods-Saxon single particle levels with a very

small deformation ($\beta_{20} = 0.005$ and $\beta_{40} = 0$),the Woods-Saxon parameters taken from Ref.3 page 21 and the ESM parameters $C_p = 36$ MeV,$C_n = 30$ MeV and $C_4 = 12$ MeV.In spite of the quantitative difference between the results of the Zawischa model and ESM ones the trends are qualitatively the same (see fig.1).This fact suggest that at least in the lead region the ESM describes the largest part of the staggering amplitude.

6 Superfluid Isomers

The name "superfluid isomer" we have chosen in analogy to the fission isomer which is actually a shape isomer corresponding to a second minimum in the potential energy along the elongation degree of shape deformation[34].The analogy can be done in the Bohr-Mottelson[44] spirit of the symmetry breaking.The appearance of the pairing rotations[45],for instance,as a mode of elementary excitations,is related to the symmetry breaking of the nucleon number conservation.This type of symmetry breaking generates the superfluid phase of atomic nuclei with non-zero static deformation of the pairing field.This is in complete analogy with the presence of the rotational spectra of the deformed nuclei,i.e. the breaking of the rotational symmetry leads to the appearance of non-zero static deformation β of the selfconsistent field and,hence,to the rotational levels in the excitation spectrum of the atomic nucleus.The deformation β corresponds to the global minimum of the potential energy of the system along the elongation degree of shape deformations that determines the ground state of the system.The fission isomer or other shape isomers correspond to the local minima of the mentioned potential energy. By analogy we may obtain second (local) minima versus pairing field static deformations $\Delta_{p(n)}$,as we shall show in the following.

Our problem is to minimize the energy functional[1] of e.g.doubly even nucleus

$$E(v_{s_i}) = 2\sum_{i=p,n}(\sum_{s_i} E_{s_i}v_{s_i}^2 - G_i\chi_i^2) - G_4\chi_p^2\chi_n^2 \tag{42}$$

$$\chi_i = \sum_{s_i} u_{s_i}v_{s_i} \quad u_{s_i}^2 + v_{s_i}^2 = 1 \tag{43}$$

with respect to the variational parameters v_{s_i} subject to the constraints

$$\sum_{s_{n(p)}} 2v_{s_{n(p)}}^2 = N_{n(p)} \tag{44}$$

where $N_{n(p)}$ is the number of protons(neutrons) contained in their respective Λ - shells defined by the cutoff and E_{s_i} - the renormalized single particle levels energies. This problem is analogous to minimizing the functional

$$F(v_{s_p}, v_{s_n}, \lambda_p, \lambda_n) = E(v_{s_p}, v_{s_n}) - \sum_{i=n,p}\lambda_i(2\sum_{s_i} v_{s_i}^2 - N_i) \tag{45}$$

with respect to $v_{s_{p(n)}}$ and $\lambda_{p(n)}$ from which we obtain the equations(32),(31) and (10) ,thus getting, $\lambda_i = \lambda_i(\Delta_i^2)$ and the gap equations (9),or to minimize the functional (42)

$$E' = E(v_{s_p}(\Delta_p^2), v_{s_n}(\Delta_n^2), \lambda_p(\Delta_p^2), \lambda_n(\Delta_n^2)) \tag{46}$$

with respect to $\Delta_{p(n)}^2$ subject to the constraints (10) The functions $v_{s_{p(n)}}(\Delta_{p(n)}^2)$ should have the parametrization (32).

The role of the potential energy for the fission isomers,in our case is played by the E' as a function of Δ_p^2 and Δ_n^2 - static deformations of the pairing and four-nucleon fields.

In this section we shall analyse the ${}^{152}_{62}Sm_{90}$ - case.This nucleus presents well defined rotational bands[35] corresponding to the following intrinsic K^π

- states: 0^+ - ground state,0^+ - β-vibrational state (E = 0.685 MeV),2^+ - γ-vibrational and 1^--octupole - vibrational states.There are three additional intrinsic 0^+ - states at E_2 = 1.082 MeV,E_3 = 1.656 MeV and E_4 = 1.736 MeV.On the second 0_2^+ - state (E_2 = 1.082 MeV) is built a rotational band with a smaller moment of inertia than for the ground and β-vibrational bands moments of inertia. This is the indication for an existence of the superfluid isomer in this nucleus,because the second minimum corresponds to larger gap parameters (see Table 2 and Fig.2) implying thus a smaller *moment of inertia.A possible interpretation of the second* 0^+ *- excited state* as a two - quasiparticle state or a neutron pairing vibrational state does contradict the experimental evidence concerning the moment of inertia (see Fig.3).The calculations for the theoretical data entering the Table 2 and Fig.2 have been done nsing the Woods-Saxon single particle levels with the deformation (β_2 = 0.3,β_4 = 0.05) and shape parameters taken from Ref.3 page 21 and the ESM parameters C_p = 22.4 MeV ,C_n = 16.2 MeV and C_4 = 28 MeV.For the global minimum of (46) - the ground state - the gap and Fermi energies are found to be Δ_{p_1} = 0.597 MeV, λ_{p_1} = -6.938 MeV; Δ_{n_1} = 0.586 MeV, λ_{n_1} = -6.372 MeV while for the second (local) minimum -the superfluid isomeric state - the gap and Fermi energies are found to be Δ_{p_2} = 4.567 MeV,λ_{p_2} = 7.117 MeV;Δ_{n_2} = 3.920 MeV, λ_{n_2} = -6.508 MeV.

Applying the theory developed in section 2 we calculated the energies of the β-virational states (see Table 2) built on the ground and isomeric states minima.The existence of the first RPA root of eq.(16) smaller than the energy of the first pole confirms the stability of our model in describing the superfluid isomer.The mentioned solutions of eqs.(9) and (10) corresponding to global and local minima define the pairing and alpha type superfluid phases respectively.The ground state pairing superfluid band contains the rotational bands built on the ground-,β-vibrational (0.685 MeV)-, γ-vibrational- and octupole-vibrational states while the isomer superfluid band may contain the rotational band built on the second 0^+-state[35](see Fig.3).

Some other evidences for possible existence of superfluid isomers we may obtain by studying e.g.two-nucleon- and/or alpha transfer reactions.For instance,by studying ^{138}Ba(^{3}He,n)^{140}Ce and ^{142}Ce(p,t)^{140}Ce reactions(10,11), there can be observed five 0^+ - excited states[46] in^{140}Ce - nucleus.Among them it is interesting to consider the 0^+ (E_x = 3.233 MeV) as a possible candidate for a superfluid isomer,because the corresponding spectroscopic factors for the mentioned two-nucleon transfer reactions are ten times larger than those given by considering the 0^+ - state under the discussion as a possible candidate for pairing monopole or quadrupole elementary excitation.The factor ten is in agreement with our rough estimation presented in Ref.30.

7 Conclusions

With admixture of four-nucleon effective forces to an effective pairing interaction,we can reproduce the normal odd-even staggering of nuclear charge radii quite well.In spite of the differences between the Zawisha model and our ESM the staggering trends are qualitatively the same.Bearing in mind that our ESM is qualitatively much simpler than the Zawischa approach we can conclude that our particular four nucleon interaction (4) dominates the staggering mechanism which is generated by the corrections to the proton density due to the mutual induction of the proton and neutron superfluidity via the mentioned interaction (4)(see eqs.9 and 10) and the blocking

of pairing and alpha type correlations.Moreover,the ESM may predict the superfluid isomers (e.g. in Sm and Ce regions) which are evidences for a new superfuid phase of atomic nuclei - the alpha like superfuid phase.The anomalous large two-nuceon transfer reaction cross sections and alpha decay widths may also be explained by admiting the existence of this new superfluid phase.

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Table captions

Table 1. Comparison of the experimental isotopic shifts[5-8] with the results calculated *in HFB approximation*[27],*HFB with four-nucleon interaction included*[27] and within the present ESM theory.

Table 2. The experimental[35] and the calculated within the ESM energies of the first low-lying 0^+-states separated in two superfluid bands.

Figure captions

Fig.1. The odd-even staggering of the mean-square charge radii of the lead isotopes.The experimental values are taken from Ref.8 and the calculated ones from Ref.27 and present work (ESM).The lines are drawn for guidance.

Fig.2. The correlation energy (46) versus proton and neutron gap energies.The two minima have the co-ordinates described in the text.

Fig.3. The experimental rotational bands built on the ground and first two excited 0^+-states of ^{152}Sm nucleus.

Table 1:

$\delta < r^2 >$ (fm^2)

A	HFB(2N)	HFB(4N) Zawischa	ESM	Exp. Heilig	Exp. Rebel
197	0.052	0.034	0.055	-0.037	0.002
198	0.052	0.098	0.088	0.082	0.082
199	0.052	0.033	0.050	0.009	0.005
200	0.043	0.075	0.091	0.085	0.088
201	0.053	0.032	0.048	0.023	-0.0007
202	0.053	0.074	0.088	0.080	0.081
203	0.054	0.031	0.043	0.027	0.014
204	0.053	0.074	0.078	0.080	0.081
205	0.055	0.029	0.034	0.029	0.026
206	0.055	0.077	0.082	0.077	0.079
207	0.057	0.054	0.058	0.045	0.044
208	0.063	0.075	0.078	0.047	0.074
209	0.066	0.081	0.078	0.091	0.093
210	0.057	0.116	0.090	0.112	0.117
211	0.065	0.077	0.068		0.089
212	0.056	0.115	0.117		0.115
213	0.064	0.073	0.065		
214	0.057	0.115	0.112		

Table 2:

Superfluid band	$I^\pi K$	E_{exp}(Mev)	E_{th}
ground state	0^+0	0.	0.
	0^+0	0.685	0.670
isomeric state	0^+0	1.082	0.837
	0^+0	1.737 ?	4.500

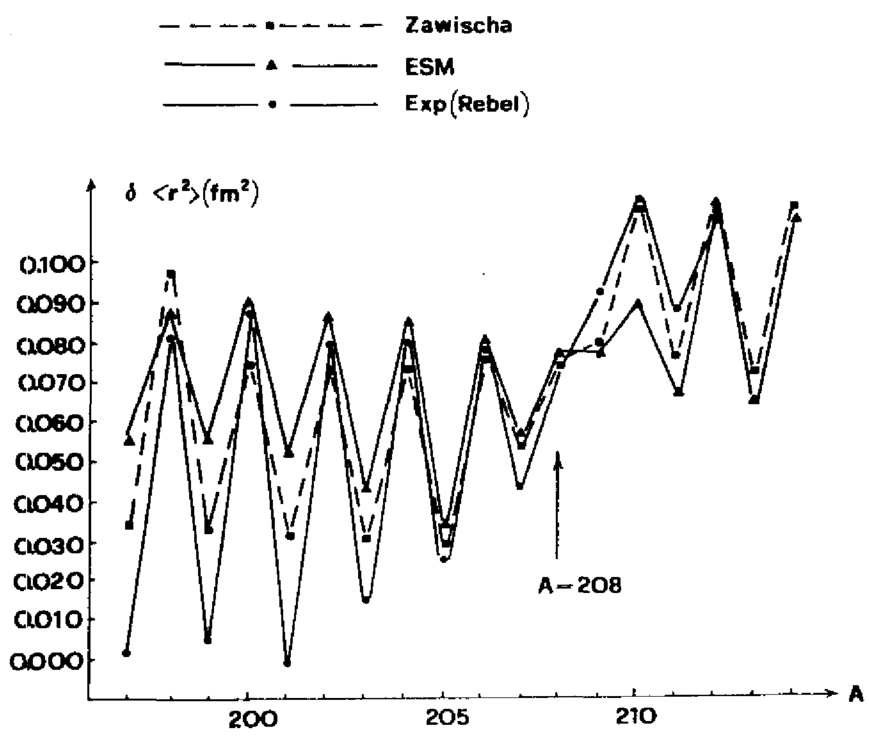
Zawischa
ESM
Exp(Rebel)
δ <r²>(fm²)
0.100
0.090
0.080
0.070
0.060
0.050
0.040
0.030
0.020
0.010
0.000
A-208
200
205
210
A

Fig. 1

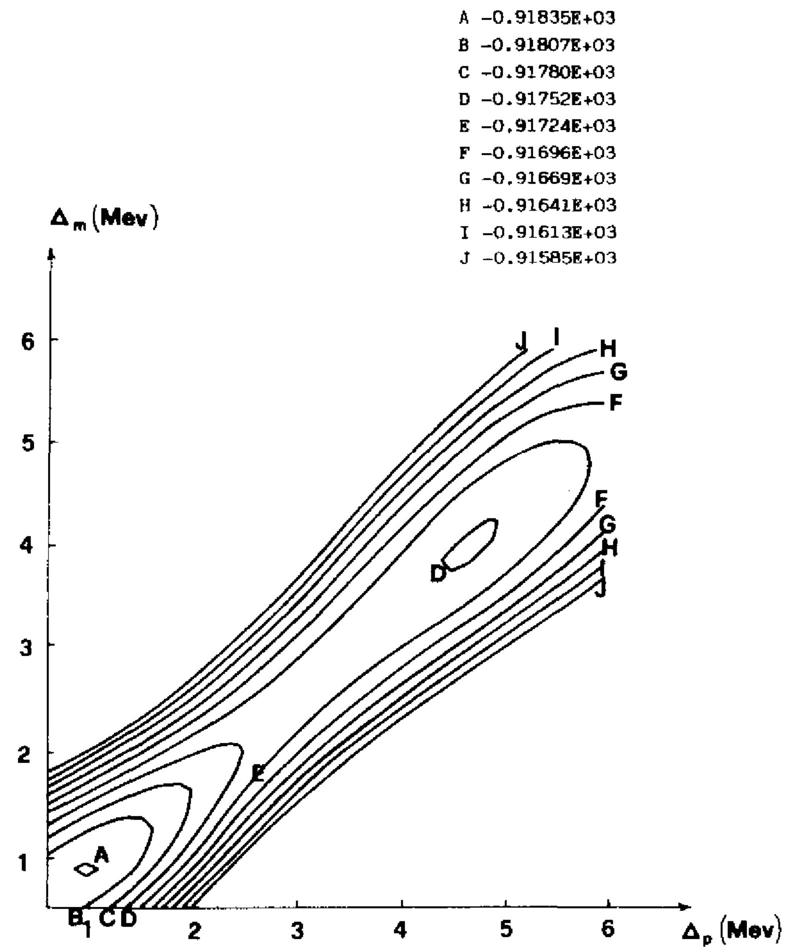
A -0.91835E+03
B -0.91807E+03
C -0.91780E+03
D -0.91752E+03
E -0.91724E+03
F -0.91696E+03
G -0.91669E+03
H -0.91641E+03
I -0.91613E+03
J -0.91585E+03
Δm (Mev)
Δp (Mev)
1
2
3
4
5
6

Fig. 2

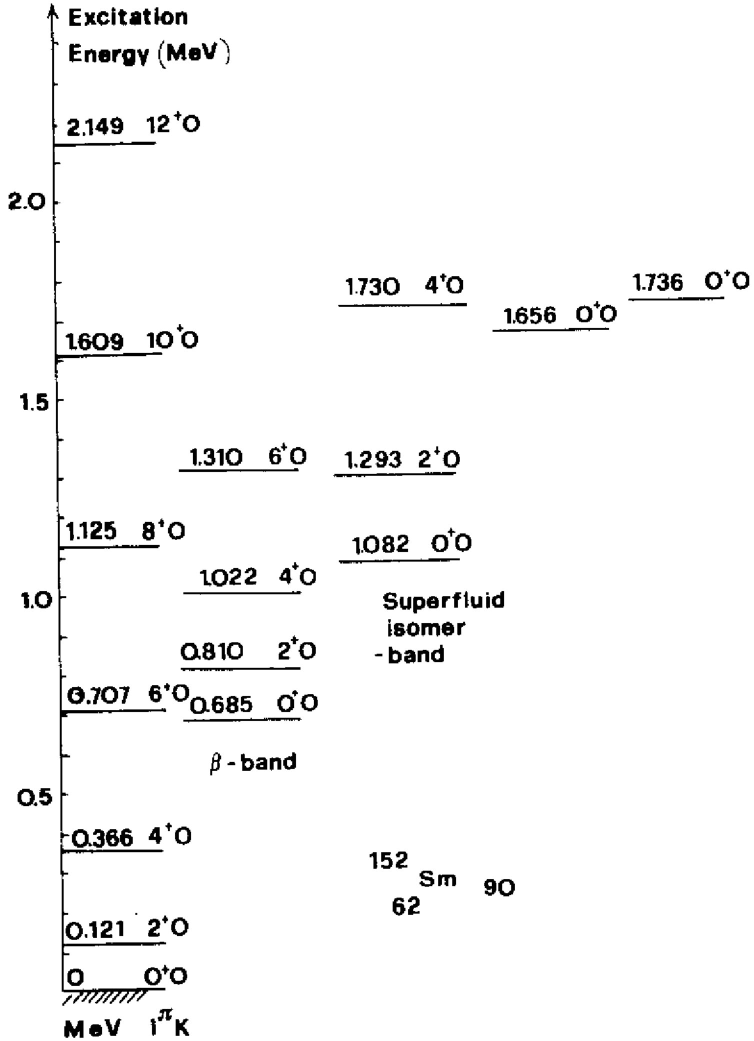
Excitation
Energy (MeV)
2.149 12+0
2.0
1.730 4+0
1.656 0+0
1.736 0+0
1.609 10+0
1.5
1.310 6+0
1.293 2+0
1.125 8+0
1.082 0+0
1.022 4+0
1.0
Superfluid
isomer
-band
0.810 2+0
0.707 6+0
0.685 0+0
β-band
0.5
0.366 4+0
152
Sm
62
90
0.121 2+0
0 0+0
MeV
I^π K

Fig. 3

Stampato in proprio nella tipografia
del Centro Internazionale di Fisica Teorica