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A. CHATTERJEE, G.S. FOOTE AND S. OGAZA

Department of Nuclear Physics
Research School of Physical Sciences
Australian National University
GPO Box 4, Canberra, ACT 2601, Australia
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J.O. NEWTON, D.J. HINDE†, R.J. CHARITY‡, J.R. LEIGH, J.J.M. BOKHORST+++,
A. CHATTERJEE++++, G.S. FOOTE AND S. OGAZA////

Department of Nuclear Physics
Research School of Physical Sciences
Australian National University
GPO Box 4, Canberra, ACT 2601, Australia

† Present address: Hahn-Meitner Institute, D-1000 Berlin 39, Federal Republic of Germany.

‡ Present address: Lawrence Berkeley Laboratory, University of California, Berkeley, CA 94720, USA.


++++ Present address: Van de Graaff Laboratory, Bhabha Atomic Research Centre, Bombay, 400 085, India.

//// Present address: Institute of Nuclear Physics, Ul. Radzikowskiego 152, PL-31342 Krakow, Poland.
Abstract: The average number of neutrons preceding fission ($u_{\text{pre}}$) was measured for the compound systems $^{188}\text{Yb}$, $^{178}\text{W}$, $^{182}\text{Pt}$, $^{192}\text{Ir}$, $^{200}\text{Pb}$, $^{210}\text{Po}$, $^{213}\text{At}$ and $^{231}\text{Es}$ formed by reactions induced by $^{16}\text{O}$, $^{18}\text{O}$, $^{19}\text{F}$, $^{28}\text{Si}$ or $^{30}\text{Si}$ projectiles with energies (E) between 4.9 and 7.2 MeV/A. In some cases $u_{\text{pre}}$ is seen to increase with increasing E above a threshold energy ($E_{\text{th}}$) whereas the statistical model indicates that it should decrease. For a given projectile, this threshold decreases with increasing fissility, becoming equal to the Coulomb barrier around $A_{\text{cm}}=213$ for $^{18}\text{O}$ projectiles. Below $E_{\text{th}}$ the variation of $u_{\text{pre}}$ with E is consistent with statistical model predictions. The deviations above $E_{\text{th}}$ have been attributed to dissipative effects not included in the model. Extensive statistical model and $\chi^2$ analyses of the pre-fission data below $E_{\text{th}}$ and of fission and fusion excitation function data, previously measured, were made. The diffuseness parameters of the fusion spin-distributions agreed reasonably well with those suggested by the zero-point motion model. The ratios of level densities at saddle and equilibrium deformations ($a_+/a_-$) were found to be consistent with a value of unity, and the fission barriers ($E_+$) consistent with the predictions of the finite-range rotating liquid-drop model. However these values for $a_+/a_-$ and $E_+$ may not represent the true values. Inclusion of dissipation requires higher values, whilst inclusion of the temperature dependence of $E_+$ in statistical model calculations is shown to result in a reduction in the value of $a_+/a_-$.

Since reliable theoretical calculations are unavailable for either effect the consistency of the data with the finite range fission barriers can only be demonstrated to within 10-15% and values for $a_+/a_-$ have an uncertainty of at least 5%.

NUCLEAR REACTIONS $^{150}\text{Sm}(^{18}\text{O},F)$, E=108-122 MeV; $^{159}\text{Tb}(^{18}\text{F},F)$, E=110-124 MeV; $^{160}\text{Ta}(^{18}\text{F},F)$, E=105-135 MeV; $^{164}\text{Er}(^{28}\text{Si},F)$, E=170 MeV; $^{170}\text{Er}(^{28}\text{Si},F)$, E=135-165 MeV; $^{181}\text{Ta}(^{18}\text{F},F)$, E=95-135 MeV; $^{176}\text{Er}(^{28}\text{Si},F)$, E=130 MeV; $^{182}\text{Os}(^{18}\text{O},F)$, $^{197}\text{Au}(^{18}\text{O},F)$, E=95-124 MeV; $^{232}\text{Th}(^{18}\text{F},F)$, E=105-138 MeV; measured (fragment) n-coinc, $\sigma$(fragment $\theta,E_n$); deduced pre-, post-fission neutron multiplicities, $^{188}\text{Yb}$, $^{178}\text{W}$, $^{188}\text{Pt}$, $^{192,198,200}\text{Pb}$, $^{210}\text{Po}$, $^{233}\text{Fr}$, $^{231}\text{Es}$, deduced statistical model parameters, dynamical and temperature effects.
1. Introduction

Substantial progress has been made in recent years in the field of fusion-fission at high angular momentum and excitation energy. Here shell effects are expected to have a minor influence and for a long time data were conventionally analysed in the framework of the transition-state statistical model and the rotating liquid drop model (RLDM). Fission excitation functions were fitted by varying only two parameters of the model, namely the ratio of level density parameters $a_a/a_u$ at the saddle and equilibrium points, and a scaling factor $k'_a$ for the RLDM fission barrier. However it has been shown\(^1\) that a large correlated range of these two parameters can give equally good fits to the experimental data and hence that no precise physical meaning can be attributed to them when derived in this way. Ward et al.\(^2\) showed that the average number of pre-fission neutrons (\(v_{\text{pre}}\)) was sensitive to $a_a/a_u$ but not very sensitive to the value of $k'_a$. Therefore measurement of $v_{\text{pre}}$ in conjunction with fission excitation function data provided much more precise values of the fission barrier (\(E_f\)) and $a_a/a_u$.

Recently Sierk\(^3\) has made calculations of fission barriers which incorporate finite range effects on the nuclear surface energy, Coulomb energy and rotational moment of inertia, and also make use of an improved shape specification over that used in the RLDM. The Sierk barriers are generally lower than those of the RLDM, particularly for lighter nuclei, and appear to account for the fact that many analyses of fission excitation functions suggest values for $k'_a$ which are less than unity. It has also been realised\(^4\) that the angular momentum distribution for fusion has a significant effect on the results of a statistical model analysis. In some work\(^5\) the fission barriers were fixed at the Sierk values, $a_a/a_u$ taken as unity or allowed slight variation from it, and the diffuseness of the angular momentum distribution varied to get fits to fission excitation
functions. Although good fits were claimed, it should be appreciated that varying the diffuseness is in some ways analogous to varying $k'$. Hence good fits resulting from arbitrary variation of diffuseness do not prove the correctness or otherwise of the Sierk barriers. Of course the shape of the fusion angular momentum distribution does reflect the physics of the fusion process and, if it could be determined, the validity of the Sierk barriers might perhaps be established. Attempts to deduce these distributions from experiment in our\textsuperscript{8} and other\textsuperscript{9} cases have been made. This paper reports measurements of $v_{\text{pre}}$ for a wide range of systems, for which excitation functions for fusion, fission, $\gamma$-ray multiplicities\textsuperscript{8} and elastic and transfer reactions\textsuperscript{8} have also been measured.

The measurement of $v_{\text{pre}}$ takes advantage of the fact that the intensity of neutrons emitted from the fast-moving fission fragments is strongly correlated with the fragment direction, whereas those emitted before fission from the relatively slowly moving compound system are only weakly correlated with the beam direction. Thus measurement of the angular correlation of the neutrons with respect to the fragment direction leads to a value of $v_{\text{pre}}$, if it is assumed that the neutrons are only emitted by the compound system or by the fully accelerated fragments. This assumption is often, but not always, a good one; sometimes account has to be taken of neutrons emitted during fragment acceleration\textsuperscript{10}.

The measurements of Ward et al.\textsuperscript{2} taken together with the fission and fusion excitation function measurements, indicated a value for $s_f/s_v$ of 1.02±0.02 for $^{209}$Pb. This appears to be a "reasonable" value since theoretical predictions\textsuperscript{11,12}, though not totally consistent, suggest a value near to unity. In this sense the measurement of $v_{\text{pre}}$ could be said to be in good agreement with expectations from the statistical transition-state model. However, measurements of this type for a variety of reactions
leading to the systems $^{170}\text{Yb}$, $^{190}\text{Ir}$, and $^{181,183,187}\text{Ir}$, mostly at much higher excitation energies ($E_x$) than for the $^{208}\text{Pb}$ measurements, and to the heavier systems $^{251}\text{Es}$, $^{218}\text{Po}$ and $^{213}\text{Fr}$, at similar $E_x$, have indicated higher values of $u_p$ than would have been expected from "reasonable" statistical model calculations, even when the effect of neutron emission during fragment acceleration is taken into account.

It has been suggested that the discrepancy may be explained by dynamical effects not taken into account in the statistical model, namely the high "viscosity" involved in nuclear shape change and the related interaction between the thermal and collective motions which results in a long relaxation time for build up of the fission degree of freedom. These effects allow extra neutrons to be emitted during the times for relaxation and between saddle and scission, both of which are assumed to be negligible in the conventional statistical model compared to the lifetime for neutron emission. Currently there is considerable uncertainty regarding the nature of the "viscosity" and its variation with nuclear shape. In view of these uncertainties it still appears worthwhile to apply a conventional statistical model analysis to our data and to see, for example, whether any useful information can be obtained regarding $a_x/a_y$.

2. Experimental Method

Neutron emission from the compound systems $^{168}\text{Yb}$, $^{178}\text{W}$, $^{188}\text{Fr}$, $^{192,196,200}\text{Pb}$, $^{210}\text{Po}$, $^{213}\text{Fr}$ and $^{251}\text{Es}$ was studied. They were formed by bombardment of $\sim 1$ mg cm$^{-2}$ targets by beams of $^{16}\text{O}$, $^{18}\text{O}$, $^{19}\text{F}$, and $^{28,30}\text{Si}$ from the ANU 14UD pelletron. Details of the reactions are shown in table 1.

The experimental apparatus and methods were similar to those described by Ward et al. and will therefore only be outlined here apart from the improvements that were made. The aluminium scattering chamber was of 10 cm
diameter with its axis along the beam direction. Its walls were thin (0.8 mm) to minimize scattering of neutrons. Three fission detectors were located in the plane perpendicular to the beam at angles of 0°, 45° and 90° to the direction of the neutron detector. The latter was an NE213 liquid scintillator (7.5 cm dia x 10 cm) placed 34 cm vertically above the target. The fission detectors consisted of thin (~20 µm) silicon surface-barrier detectors backed by ~100 µm detectors located 3.5 cm from the target, and subtending ±4.0°. This system enabled beam-like particles passing through the thin detector, in which the fission fragments stopped, to be vetoed by the rear detector; it was essential for the Si-induced reactions, where the beam-like particles deposited similar amounts of energy to the fission fragments in the thin detector. Examples of fission spectra in singles and in coincidence with neutrons are shown in fig.1. Simultaneous measurement of the three neutron-fragment correlation angles without moving the neutron detector is very advantageous; apparent changes of detection efficiency due to change of neutron scattering from surrounding materials is entirely avoided.

The neutron energies (velocities) were determined from the times between the detection of fission fragments and neutrons. The correlation between fragment energy and time was corrected by software in the off-line analysis of the data, using the position of the γ-ray peak to deduce the true origin.

It is easy to distinguish low-velocity neutrons from the γ-rays by time of flight (TOF) but this is not the case for the highest energy neutrons, due to the finite resolution of the timing system and the relatively short flight path. However high-energy neutron-induced events can be readily distinguished from γ-ray-induced events by neutron-γ pulse-shape discrimination (PSD). Hence a combination of these two techniques enables
good discrimination between neutron- and γ-induced events for all neutron energies. Pulse shape discrimination was achieved by timing between pulses derived from constant-fraction discriminators, acting on fast pulses from the scintillator, and pulses from a crossover circuit fed via a linear amplifier. Because of the large dynamic range (~1000:1 range of pulse heights) two linear amplifiers with gains differing by a factor of ten were used.

Data were collected event by event on magnetic tape. Appropriate off-line sorting produced spectra corresponding to PSD signals from the low gain system (high neutron energy), and from the high gain system, excluding those signals which also appeared in the low gain system. Examples of such PSD and TOF spectra together with two-dimensional PSD/TOF spectra are shown in fig.2. Also indicated is a two-dimensional window for obtaining the TOF spectra from the neutrons alone. The weak tail of γ-ray events along the TOF axis, which has a peak at ~10 ns after the main γ-ray peak, may be due to (n,γ) reactions and scattering from the floor of the experimental area.

The efficiency of the neutron detector was determined for a neutron velocity range of 0.6 cm ns\(^{-1}\) to 4.0 cm ns\(^{-1}\). A new method giving a rapid and precise efficiency measurement was developed. A 2.5 cm diameter multi-wire proportional counter (MWPC)\(^{23}\), in which a \(^{252}\)Cf source formed one of the cathode planes, was located so that the \(^{252}\)Cf was in the target position. Two torr of isobutane gas was introduced to the target chamber. For each fission decay one of the resulting fission fragments was emitted into the gas volume between the electrodes, hence ideally all fission events were detected. In practice the efficiency was found to be greater than 97%. With 450 V on the central wire plane, good timing signals were obtained, and in the energy spectrum, α-particles and fission fragments were well-separated. By measuring the neutron velocity spectrum (taking the start
signal from the MWPC) and dividing by the well-known $^{252}$Cf singles neutron spectrum, the neutron-detector efficiency was simply and rapidly determined. The efficiency was measured in exactly the same configuration used for the beam experiments. Thus insofar as the neutron spectrum from $^{252}$Cf is similar to those from the reactions studied, neutron scattering from material around the target is taken into account. Neutron-$\gamma$-ray discrimination was used in conjunction with the time-of-flight information to reject $\gamma$-rays detected in the NE213 scintillator.

Repeat measurements for the system $^{19}$F + $^{181}$Ta were made, in the two month period of investigation, to check for systematic deviations, and the neutron-detector efficiency was measured both before and after the series of experiments.

3. Analysis

3.1 Transformation to neutron velocity spectrum

The neutron TOF-spectra were corrected to take into account the system time resolution and were fitted with cubic splines. The deconvolution was carried out separately on the low-gain and high-gain exclusive TOF spectra. This had the advantage (see fig.2) that the full width at half maximum (FWHM) of the $\gamma$-ray peak in the low gain (high neutron energy) spectrum was $\sim 1.3$ ns rather than $\sim 2$ ns if the TOF spectra were combined. The correspondingly poorer resolution ($\sim 4.5$ ns) of the high gain system was no disadvantage in view of the longer flight times of the low energy neutrons. Examples of experimental, fitted and deconvoluted spectra for the case of 105 MeV $^{19}$F on $^{181}$Ta ($\theta_{yz} = 0^\circ$) are shown in fig.3. After deconvolution the two spectra were added together, giving a total corrected TOF spectrum, and then converted to a velocity spectrum.
3.2 Neutron detector efficiency

The efficiency of the same neutron detector in an almost identical configuration as used by Ward et al.\textsuperscript{21} was measured with the \textit{2n} NNFC. In fig.4 the efficiencies, measured with the source plans at 90° and at 45° to the direction of the neutron detector, are shown. These agree within errors, again indicating that the NNFC is detecting essentially all of the fission events. The full line shows the efficiency curve adopted for the present analysis, whilst the dotted line is that of Ward et al. Considering the higher electronic threshold of the latter measurement, the agreement is good. This agreement is very gratifying since the result of Ward et al. was based on the fission-neutron angular-correlation data of Bowman et al.\textsuperscript{7} rather than on the \textit{\textsuperscript{252}Cf} singles spectrum. The present efficiency curve also agrees quite well with one derived from a simple calculation\textsuperscript{20} shown by the dashed curve. This suggests that contributions to the efficiency from scattering of neutrons by surrounding materials are small.

3.3 Extraction of pre- and post-fission neutron multiplicities

Typical neutron velocity spectra at 0° and 90° to the fission-fragment direction are shown in fig.5. From such spectra, components assigned to "pre-fission" emission and "post-fission" emission are extracted. It is conventionally assumed that emission of pre-fission neutrons is from a source moving with the centre-of-mass velocity of the compound system, and emission of post-fission neutrons is from the fully accelerated fission fragments. In both cases isotropic emission in the rest frames is assumed. With these assumptions, the iterative method described by Bishop et al.\textsuperscript{11} and Ward et al.\textsuperscript{21} (the "free fit" method) was used to deduce the pre- and post-fission neutron velocity spectra and thus multiplicities. The fits to the data in fig.5 show the pre-fission component and the components from
both the detected and complementary fission fragments. Note that the latter are nearly equal at 90° but that the detected fragment component dominates at 0°. A pronounced dip is apparent at the fragment velocity $v_f$ in the component spectrum $H^d_{\text{pre}}$ from the detected fragment and in the total fitted 0° curve. This is due to the assumption of symmetric fission with fixed total kinetic energy (TKE) and to the small probability of neutrons of close to zero velocity being emitted from the detected fragment. The component below the fragment velocity $v_f$ in the $H^d_{\text{post}}$ spectrum arises from neutrons emitted in the frame of the detected fragment with velocities less than $v_f$ and in the opposite direction to that of the neutron detector. From the deduced velocity spectra in the centre-of-mass frame of each neutron source, the pre-fission multiplicity $v_{\text{pre}}$, and the post-fission multiplicity $v_{\text{post}}$ were determined. The latter is defined as the multiplicity per fragment, hence the total multiplicity ($v_{\text{tot}}$) is given by $v_{\text{tot}} = v_{\text{pre}} + 2v_{\text{post}}$.

3.4 Uncertainties in the determination of $v_{\text{pre}}$

In order to determine the magnitude of statistical errors, two methods were used. For those energies where repeat points had been measured, the scatter in the values of $v_{\text{pre}}$ was analysed to determine the statistical uncertainty. This was found to be ±7% and includes errors due to the statistical uncertainty in the velocity spectra, the decomposition into pre- and post-fission components and movements of the beam spot from run to run. In order to determine the errors from the first two causes by a second method, a $\chi^2$ search was performed, using the pre- and post-fission neutron spectra determined by the iterative method. The multiplicities were both varied independently, and envelopes of $\chi^2$ per point ($\chi^2/n$) were generated as a function of $v_{\text{pre}}$. In the zero degree spectra at $v_n = v_f$ there was a
deviation between the data and the calculation (see §3.3). In order to obtain lower values of \( \chi^2 \), the effects of variations in the mass-split and TKE were taken into account, as were the solid angles of the fission and neutron detectors. Nevertheless it was found that the discrepancy noted above was still present, though much reduced. It is suggested that with sufficient statistics, the feature would be apparent in the raw time spectra, and could be deconvoluted with the time resolution appropriate for such neutrons (~0.5 MeV). However, with the current data, the feature is not distinguishable in the time spectra, so remains as a discrepancy in the velocity spectra. To test the effect of this feature on the deduced values of \( v_{\text{neu}} \), the \( \chi^2 \) search was performed with the full data set, and with data in the velocity region \( 0.85 \, v_{\text{g}} < v_{\text{g}} < 1.15 \, v_{\text{g}} \) omitted. Typically, the value of \( v_{\text{neu}} \) was ~50 lower than that obtained for the full data set, and close to the value obtained by the iterative method. In the iterative method\(^2\), only the 0° data for \( v_{\text{g}} > 2v_{\text{g}} \) is used, so the discrepancy does not influence the results. The uncertainties were obtained from the properties of the \( \chi^2 \) distribution, and for a 70% confidence level agreed well with those obtained from an analysis of the repeat points. It was found that the uncertainty was inversely proportional to the square root of the total number of neutrons, as would be expected. Errors for those systems not studied by \( \chi^2 \) analysis were determined by interpolation. The errors were ±10%.

Systematic errors in the extracted values of \( v_{\text{neu}} \) could result from deviation from the assumption of isotropic emission. Any anisotropy would be expected to be small, and has been ignored in studies similar to this\(^1,11,13,14,18,39\). The rather detailed measurements of Zank et al.\(^14\) give some support for this assumption, nevertheless, it remains a source of uncertainty.
4. Results

The results for $v_{pre}$, $v_{post}$ and $v_{tot}$ are given in table 2. Earlier results\textsuperscript{21} for the $^{19}$F + $^{181}$Ta system obtained by the free fit method are also included in the table and agree very well with the present results. In addition, estimates of the average number $v_{m}$ of neutrons produced in HI,xn reactions are also given. They were derived from a simple empirical formula for determining the peaks of (HI,xn) cross sections, $v_{m} = 0.074 E_{cm} + M_{p} + M - M_{m}$, where $E_{cm}$ is the centre-of-mass bombarding energy and $M_{p}$, $M$, and $M_{m}$ are the mass excesses of the target, projectile and evaporation residue (after $x$ neutrons are emitted) respectively in MeV. The quantity $M_{m}$ was taken as the liquid drop value\textsuperscript{20}. It is clear that $v_{m}$ exceeds $v_{tot}$ for the light systems and is less than $v_{tot}$ for the heavy compound nuclei. This is due mainly to the Q-value for fission of a given compound nucleus varying from about -21 MeV for $^{160}$Yb to +46 MeV for $^{251}$Es.

The results for $v_{pre}$ are shown plotted against compound system excitation energy $E_{x}$ in fig. 6. Here $E_{x}$ is measured from the liquid drop ground state. The full lines are the results of statistical model calculations\textsuperscript{17}, with $a_{x}/a_{v} = 1.00$ the Sierk barriers and diffuse fusion angular momentum distributions adjusted to be consistent with the fission and evaporation residue excitation function data\textsuperscript{6}. The agreement between calculation and data is reasonably good for the lighter systems and lower $E_{x}$ but is not good for the heavier systems. In general the experimental values of $v_{pre}$ continue to rise with increasing $E_{x}$ above an apparent "threshold" bombarding energy ($E_{th}$), in conflict with the statistical model results. It is easy to understand why these fall at higher $E_{x}$. As the bombarding energy and $E_{x}$ increases the angular momentum does also. The fission barrier falls with increasing angular momentum and when its value reaches the vicinity of the neutron binding energy $E_{v}$, the ratio of fission to neutron
widths $\Gamma_{f}/\Gamma_{\nu}$ becomes large, both for first and higher chance fission. However if a substantial fraction of the initial compound nuclei decay by fission rather than by neutron emission there will be a much reduced number which can decay by second and higher chance fission. Thus $v_{pre}$ falls even if $\Gamma_{f}/\Gamma_{\nu}$ increases for higher chance fission.

It follows from these results that to determine values for $a_{x}/a_{\nu}$ with the statistical model it is necessary, though it may not be sufficient, to restrict the $v_{pre}$ data to the region which follows the trends predicted by the statistical model, i.e. that of "low" $E_{x}$. However, as fig. 6 shows, "low" differs considerably for different systems, varying from $-90$ MeV for $^{188}$Yb to $-40$ MeV for $^{213}$Fr and $-20$ MeV for $^{251}$Es. In the latter two cases and in the reactions induced by $^{28,30}$Si projectiles $E_{x}$ is sufficiently low that the bombarding energies corresponding to $E_{th}$ are in the vicinity of or well below the Coulomb barrier. Thus cross sections are small and $v_{pre}$ measurements are difficult if not impossible. Consequently the systems $^{192}$Pb, $^{213}$Fr and $^{251}$Es and the $^{28,30}$Si induced reactions are not considered in the statistical model analyses.

5. Statistical Model Analysis

The analysis was carried out with the code ALERT1, modified to use the Sierk fission barriers and to take account of neutron emission during fragment acceleration. Fission barriers were allowed to vary from the Sierk values by multiplying them by a parameter $k_{x}$. The fusion cross sections were obtained from least squares fits to experimental excitation functions. Associated angular momentum distributions were parameterised with the Fermi function

$$
\sigma_{fus}(L) = \frac{2\pi^{2} (2L + 1)}{1 + \exp \left( \frac{L - L_{0}}{\delta L} \right)}
$$

(1)
where $\delta L$ determines the diffuseness. The principal aim of the analysis was to determine the best values of $a_x/a_y$, with secondary aims of seeing whether the best simultaneous fits to both excitation function data\cite{1,3} and the $v_{\text{pre}}$ data were consistent with expected values for $k$ and $\delta L$\cite{2}. Whilst it is difficult to obtain definitive values for $\delta L$, information can be obtained by measuring elastic scattering, $\gamma$-ray multiplicities, and calculating values from theory\cite{7,8,32} which, although not exact, often gives reasonable agreement with sub-Coulomb fusion data.

The excitation energy for the compound system was taken relative to the liquid drop ground state and defined as

$$E_x = E_{\text{CM}} + E_{\text{p}}^{\text{exp}} + E_{\text{t}}^{\text{exp}} - M_{\text{LD}}^{\text{CM}} - 3\delta$$

where $E_{\text{CM}}$ is the centre-of-mass bombarding energy, $E_{\text{p}}^{\text{exp}}$ and $E_{\text{t}}^{\text{exp}}$ are the experimental ground state masses of the projectile and target, $M_{\text{LD}}^{\text{CM}}$ is the liquid drop mass\cite{30} of the compound nucleus and $\delta$ is the pairing energy, taken as $11/N_C^{1/2}$ MeV. The last term was included to reproduce approximately the yrast line of real nuclei in the rare earth region at spins where their moments of inertia approach those of the liquid drop. It should be noted that the liquid drop mass of ref.30 refers to odd-mass nuclei so that in any event an amount $\delta$ should be added to it. The real question for the statistical model with the Fermi-gas level density is: where should the origin of the thermal excitation energy be taken? This may or may not be the yrast line and is something of an open question. In general the effect of taking 3$\delta$ rather than $\delta$ is not large in our calculations.

It is common practice to regard fits to fission excitation functions which differ by factors up to ~two from the data as reasonable fits, and certainly they appear so on logarithmic plots covering several decades. This approach is in many respects reasonable since there are uncertainties
in some features of the statistical model, such as level densities. However in this analysis we have attempted to get better fits. The parameters \( a_x/a_y, k_x \) and \( \delta L \) were allowed to vary freely and a \( \chi^2 \) analysis performed for both fission and \( u_{pre} \) excitation functions. Four to six bombarding energies were chosen for the analyses in view of the lengthy computer time involved.

The values of \( u_{pre} \) which clearly deviated from statistical model predictions were not included for the systems heavier than \( ^{198}\text{Pt} \). Most calculations were done for \( a_y=\text{A}/10 \) but the effect of varying \( a_y \) from \( \text{A}/7.5 \) to \( \text{A}/12 \) was also investigated and included in the errors. We prefer to do this rather than choosing a particular value of \( a_y \) because of present uncertainty in its value. Although the value of \( \text{A}/a_y \) is commonly taken to be \( \sim 8 \) MeV, which is an average value deduced from level densities near neutron binding energies, little data exist for higher excitation energies, different masses and angular momenta. Recent measurements for systems with masses of \( \sim 160 \) and \( \sim 180 \) suggest that \( \text{A}/a_y \) varies from \( \sim 8-9.5 \) MeV at low excitation energies, to \( 10-13 \) MeV at excitation energies of \( \sim 400 \) MeV. A value of \( \text{A}/a_y=7.5 \) MeV is given for \( ^{154}\text{Er} \), but may be underestimated because of the effect of incomplete fusion reactions. Determinations of \( a_y \) can be made from accurate measurements of neutron evaporation spectra. The present measurements, involving a combination of pre- and post-fission neutrons are not suitable for this purpose. However, other measurements to accomplish this are in progress.

In addition to the errors on the fission and \( u_{pre} \) data it was also very important to take into account the errors on the fusion cross sections for the \( \chi^2 \) analysis. For \( \pm 5\% \) errors on \( \sigma_{\text{fus}} \) the average percentage errors on the calculated values of the fission cross section \( \sigma_{\text{fis}} \) vary from \( \pm 40\% \) for \( ^{168}\text{Yb} \) to \( \pm 10\% \) for \( ^{208}\text{Pb} \). They emphasise that accurate values of \( \sigma_{\text{fus}} \) are required, particularly for light systems; typical errors for our
measurements were ±3%. For light systems $\sigma_{\text{fiss}}$ is extremely sensitive, not only to the diffuseness of the angular momentum distribution but also to its shape. This is because at low angular momentum $B_f$ is much higher than $B_v$, resulting in very low fission probability; however $B_f$ decreases with increasing $L$. Hence most of the fission cross section originates from the extreme tail of the distribution. For example in the case of 94 MeV $^{18}O + ^{150}Sm$ with $\delta L = 3.5$ the average fissioning angular momentum is 22% higher than $L_0$ (eq. 1). Unfortunately at the present time there is no theoretical guidance on the shape of the tail and this should be born in mind when considering derived values of $\delta L$, and the somewhat closely related values of $k_f$, for light systems. The sensitivity to the form of the tail is much less for heavier systems because of much lower $B_f$.

An example showing calculated and experimental values of $u_{\text{pre}}$, for good fits to the fission cross sections of the $^{181}Ta + ^{19}F$ system and $\delta L = 6$, is shown in fig. 7. The effect on the calculations of taking $a_v/A/7.5$ and $A/12$ is also shown for $a_f/a_v = 0.98$. For lighter systems the effect is even smaller within the range of interest. Good fits with the statistical model cannot be obtained for the two higher energy points if it is required that the lower energy points be well fitted. Contours of $\chi^2$ per degree of freedom in the $a_f/a_v$, $\delta L$ plane, defined by the $\sigma_{\text{fiss}}$ and $u_{\text{pre}}$ data for the $^{18}F + ^{180}Ta$ system are shown in fig. 8. The $\sigma_{\text{fiss}}$ data define a very long acceptable region, covering a wide range of $a_f/a_v$, but the $u_{\text{pre}}$ data restrict this to a small region. The ratio of calculated to experimental fission cross sections for this case are also shown in fig. 8 (upper) plotted against bombarding energy. The agreement is very good considering that the data cover a cross-section range of three decades.

From such analyses it is possible to derive values for $\delta L$, $a_f/a_v$ and $k_f$. The results are shown in fig. 9; all are for $a_v = A/10$. Only for the
light systems is $\sigma_{\text{fiss}}$ very sensitive to the value of $a_{\nu}$ and, for example, the value for $k_\tau$ changes from 1.09 to 1.02 for $a_{\nu} = A/7.5$ in the case of $^{188}$Yb. The difficulties in fitting the fission excitation function for this case have been discussed. However the value of $a_\tau/a_{\nu}$ is rather insensitive to these fits. The $\sigma_{\text{fiss}}$ fits for $^{210}$Po were relatively poor and somewhat poor for $^{200}$Pb. There is surprisingly good agreement between the derived values of $\delta L$ and average values derived from the zero-point-fluctuation fusion model (ZFM) of Esbensen, which often fits sub-barrier fusion data rather well, and from our elastic scattering data for the three lighter systems. However it is notable that the agreement is less good for $^{200}$Pb and $^{210}$Po. The difficulty with the $^{200}$Pb case may partly arise from taking a constant value of $\delta L$ for all bombarding energies. Both the Esbensen model and the optical model analysis, with which it agrees rather well, suggest that $\delta L$ rises considerably, from a nearly constant value at higher bombarding energies, as the Coulomb barrier is approached; for example $\delta L_{\text{ZFM}}$ is 6.7 for 90 MeV, 4.5 at 100 MeV and 4.2 at 110 MeV. Since the effect of $\delta L$ variation is much greater at lower bombarding energies, where the fission probability is lower, the assumption of a constant $\delta L$ would result in its value being too large. The system $^{19}$F + $^{181}$Ta is the most sensitive of all our cases to this effect. Another reason for discrepancies in the $^{200}$Pb case and, much more so, $^{210}$Po may be shell effects. In the statistical model calculations, liquid drop masses and a Fermi-gas type of level-density function are assumed. These may be adequate at high excitation energies and for nuclei away from closed shells. However it should be appreciated that for both $^{200}$Pb and $^{210}$Po the mean fissioning excitation energy is only about 30 MeV for much of the excitation-function energy-ranges, whilst the shell effects (difference between liquid drop and true ground state masses) are ~5 MeV and ~9 MeV.
respectively. Neglect of the possibility of collective level enhancement may also affect the results of calculations in the framework of the statistical model.

With these reservations, the results in fig. 9 suggest that the values of $a_x/a_y$ are consistent with, but on average slightly less than, unity to within about 2%, and the values of $k_x$ are consistent with unity to within about 10%. Hence the results are consistent with the Sierk barriers being correct, though the precision is not as good as one would like. The values of $\delta L$ are in fair agreement with those expected from the ZPM and from optical model analysis. In addition, if the values of $a_x/a_y$ and $k_x$ derived from analysis of the $^{19}$F + $^{181}$Ta reaction are used in the analysis of the fusion/fission data for the $^{30}$Si + $^{170}$Er system, the deduced ratio $\delta L(30\text{Si})/\delta L(19\text{F}) = 1.54 \pm 0.15$, which is in good agreement with that of ~1.5 predicted by the ZPM model.

It is interesting to compare these conclusions with other recent statistical model analyses. Van der Plicht et al. studied the $^{158}$Er, $^{188}$Os, and $^{204,208,208,210}$Po systems for reactions induced by a variety of projectiles from $^9$Be to $^{64}$Ni. Fission but not fusion cross sections were measured; the latter were deduced from the Bass model. In the calculations the Sierk barriers were assumed, $a_y = A/7.5$, $a_x/a_y$ taken as very close to unity, and $\delta L$ allowed to vary in order to fit the fission cross sections. In most cases their values for $\delta L$, ranging from 0.5 to 6.0 with an average of 1.9, were much smaller than expected from the ZPM. They also made calculations using microscopic level densities. Lesko et al. have measured fission and fusion cross sections for reactions induced by $^{58,64}$Ni ions producing the systems $^{170,188}$Pt. They used the Sierk barriers, $a_y = A/8$, $a_x/a_y = 1.0$ and found that $\delta L = 7.5$ fitted all of the fission data, agreeing rather well with the ZPM value of about $7^2,28$. However this value was too
small to fit the sub-barrier fusion data. Kondo et al.\textsuperscript{36} extended the fission data of ref.7 for $^{58}\text{Ni} + ^{126}\text{Sn}$ to lower energies. From a statistical model analysis of this and the $^{64}\text{Ni} + ^{116}\text{Sn}$ data\textsuperscript{7} using the Sierk barriers, $a_\psi/A/10$ and $a_\zeta/a_\psi=1.00$ they found that much more diffuse distributions than predicted by the ZFM were required, but good fits could be obtained with the elastic fusion model of Udagawa et al.\textsuperscript{37}. Note that the ZFM works well in cases where at least one of the projectile or target is deformed. However for cases where both are spherical, such as Ni + Sn, the effect of transfer reactions is relatively much greater and the model is less successful. Our analysis is probably more detailed and less restrictive than those performed so far. It does appear to give "reasonable" values for $\delta L$, perhaps supporting the procedure used. However it is clear that there are still considerable uncertainties and much more work needs to be done.

6. Deviations of $v_{\text{pre}}$ from statistical model predictions

6.1 Interpretation

So far we have not taken account of the deviations of the measured values of $v_{\text{pre}}$ from the statistical model predictions. We have implicitly assumed that the statistical model does give meaningful results for the regions where it does appear to fit the trend of the data. We now examine possible explanations of the deviations and how likely it is that this assumption is correct.

The experimental analysis which we and others have used assumes that neutrons are emitted from the fully accelerated fragments or from the slowly moving compound system. We have already referred to those emitted during fragment acceleration. However it seems unlikely that these could account entirely for the deviations, even for the favourable cases of very heavy
systems such as $^{231}\text{Es}$ where the Q-value for fission is very large; an unreasonably large value of $a_v$ would be required (fig. 6). It should be noted that the values of $v_{pre}$ for $^{231}\text{Es}$ shown in table 2 and fig. 6 are slightly underestimated since the effect of ~10% transfer induced fission is not taken into account$^{38}$. The extra neutrons could be emitted from the nucleus before the saddle point is reached, and/or between saddle and scission. Neither of these possibilities is included in the statistical model, though saddle to scission emission would not in itself be in conflict with the model; it must occur. However, to account entirely for the deviations a very long saddle to scission time $t_{ss}$ of $\approx 30 \times 10^{-21}$ s, for $a_v = A/10$, appears to be required$^{17}$ implying overdamped highly viscous motion.

If the saddle to scission motion is highly viscous, it is likely that the motion towards and over the saddle is too; A number of authors have begun to develop the relevant theory$^{18}$. There are roughly two implications if the pre-saddle motion is viscous.

(i) A finite delay or relaxation time ($\tau_\phi$) is required before the equilibrium Boltzmann distribution is built up at the saddle point. During this period the fission probability is below its maximum value, but the neutron emission probability is normal.

(ii) Even when equilibrium is attained, the flow over the quasi-stationary saddle point is less than that given by the statistical model. The motion is analogous to Brownian motion; some nuclei can return to equilibrium deformation even after passing the saddle point. Kramers$^{39}$ showed that the maximum flow (width) is given by

$$\Gamma_f = \Gamma_{BW}(1 + \gamma^2)^{1/2} \cdot \gamma$$

(3)

where $\gamma$ is related to the friction (viscosity), $\gamma = 1$ corresponding to critical damping, and $\Gamma_{BW}$ is the Bohr-Wheeler statistical model.
width. Substantial reductions may occur since for critical damping,
\[ \Gamma_f = 0.414 \Gamma_{\text{SM}}. \]

6.2 **Effect of viscosity on values of statistical model parameters**

These effects may be approximately incorporated into statistical model codes\(^{14,17}\). Let us consider how inclusion of them may affect values for the statistical model parameters.

6.2.1 **Saddle to scission emission**

The extra values of \(\sigma_{\text{pre}}\) required to fit the experimental values, calculated assuming that they all originate from saddle to scission emission, are shown in fig. 10, plotted against \(E_x\). There is a significant contribution even for the lower values of \(E_x\). Hence it is not valid to fit the statistical model calculations to the observed values of \(\sigma_{\text{pre}}\); they have to be reduced by the saddle-to-scission contribution. To reduce the calculated \(\sigma_{\text{pre}}\) requires \(a_f/a_V\) to be increased by \(-0.02\) (see fig. 7) and correspondingly \(k_f\) has to be increased by \(-0.04\) to bring the fission cross sections back to their correct values. It is still possible to get excellent fits to the fission excitation functions with this change because of the very large correlated range of \(a_f/a_V\) and \(k_f\) which gives nearly identical fits\(^1\). Examples of fits to the \(\sigma_{\text{pre}}\) data for \(^{180}O + ^{150}Sm, ^{19}F + ^{181}Ta\) and \(^{30}Si + ^{170}Er\), all giving excellent fits to the \(\sigma_{\text{pre}}\) data, are shown in fig. 11. For these the method of Hinde et.al.\(^{17}\) and a fixed \(\tau_{\text{ss}} = 30 \times 10^{-21}\) sec was adopted. Statistical model fits with values of \(a_f/a_V\) reduced by 0.02 are also shown. No attempt was made to get best fits to the \(\sigma_{\text{pre}}\) data in view of the neglect of delay time etc.
6.2.2 Pre-saddle emission and reduction of saddle flow.

The effect of both the finite delay time and the reduced saddle flow, when incorporated into a statistical model code, is to reduce the calculated value of the fission cross section. In order to recover the correct experimental value of $\sigma_{\text{fiss}}$, it is necessary to increase the value of $a_{s}/a_{u}$. If $a_{s} = a_{u}$ in the normal statistical model analysis then, using the equation in ref.18, typical variations might be to values for $a_{s}/a_{u}$ of 1.03, 1.06, 1.08, 1.11 for values of $\gamma$ of 1, 2, 4, 8 respectively. The effect on fits to fission excitation functions over a wide range of energies remains to be demonstrated.

The friction is related to the interaction between the thermal and collective modes as in Brownian motion. Hence if $\gamma = 0$ the relaxation time $\tau_{D}$ is infinite. As $\gamma$ increases $\tau_{D}$ falls but eventually reaches a minimum and then increases again due to high viscosity; the effect on $a_{s}/a_{u}$ is least when $\tau_{D}$ has its minimum value. Generally the effect of $\tau_{D}$ appears to be less than that due to reduction of equilibrium flow.

6.2.3 Relative contributions of saddle-to-scission and pre-scission emission.

It seems likely that the effect of saddle-to-scission emission will increase strongly with increasing $A$ because $\tau_{ss}$ is likely to depend on the difference between the saddle and scission shapes, which increases with increasing fissility. This is illustrated in fig.12 which shows $\tau_{ss}$ plotted against fissility $x=(Z^{2}/A)[50.88(1-1.783t^{2})]^{-1}$, where $t=(N-Z)/A$. Values of $\tau_{ss}$ for zero viscosity, two-body viscosity (0.02TP to fit fission kinetic-energies) and full one-body viscosity, are taken from the L=0 calculations of Carjan, Sierk and Nix. Those for one-body viscosity, which superficially seems most appropriate for nuclei, are not very precise for
x535 as they were obtained by extrapolation from the curves of ref.40. The curves for L=40 and L=60 are also approximate and deduced from the L=0 case by Blann and Komoto's\textsuperscript{41} method of effective fissility. The number of saddle-to-scission neutrons emitted also depends on the weighted excitation energy during this time. This depends on the Q-value for fission (excluding the total fragment kinetic energy), which for example varies from \(-21\) MeV for $^{158}$Yb to \(+46\) MeV for $^{231}$Es, and also on the nature of the viscosity, which determines the division between thermal and deformation energy in the descent to scission.

On the other hand $\tau_0$ seems unlikely to depend strongly on $A^{18,42,43}$ and to have values of a few times $10^{-21}\text{sec}$. Hence we conclude that for lighter systems the excess neutrons may originate in comparable amounts both from pre- and post-saddle emission whereas saddle-to-scission emission is likely to dominate for heavier systems.

6.3 Basic questions in the interpretation

Theoretical study of the effects of friction is at an early stage and a number of features must be considered in more detail than hitherto.

Current calculations of the large scale collective motion leading to fission, before and up to the time that the saddle point is reached, assume that the system begins in thermal equilibrium at equilibrium deformation. This approach is probably too simplistic and it may be necessary to consider the dynamical evolution from the point of contact. For fission induced by heavy projectiles this may be particularly important. It is probably essential to take into account the detailed nature of the viscosity and its variation with nuclear shape. Theoretical and experimental\textsuperscript{44,45} evidence suggests that one-body viscosity\textsuperscript{46} is reduced when the nucleus is not highly deformed and its shape is described by only a few multipoles. Also
in spite of its simple physical appeal and apparent success in describing
dissipative effects in nuclei, some recent theoretical papers\textsuperscript{23,46,47} have
cast doubts on the one-body picture. It may be that the one-body wall
formula\textsuperscript{43} results are mimicked by a combination of a relatively weak one-
body viscosity together with a strong two-body viscosity peaked in the
nuclear surface\textsuperscript{42}. Hence there are many open questions regarding the
correct treatment of the dynamics of fission.

7. The value of \( \alpha_x/\alpha_y \)

Much of this paper is concerned with the value of \( \alpha_x/\alpha_y \), which in
principle is a quantity of physical significance. Although it often is
assumed that it has a value close to unity, remarkably little theoretical
attention has been given to the question of what its value might be. The
most commonly used prescription is that due to Bishop et al.\textsuperscript{11}). They
calculated the level density at the Fermi surface for a Fermi gas contained
in a potential box with rectangular sides, the potential having the same
trapezoidal shape in each of the three mutually perpendicular directions.
They concluded that \( \alpha_x/\alpha_y \) should lie between 1.00 and 1.04, depending on the
def ormation at the saddle point. Gottschalk and Ledergerber\textsuperscript{12)} have carried
out Hartree-Fock calculations and indicate that \( \alpha_x/\alpha_y \) should be slightly
less than unity and that the derivation of Bishop et al. is incorrect.
Carjan et al.\textsuperscript{48)}, who carried out microscopic calculations based on a
realistic set of single-particle levels, conclude that \( \alpha_x/\alpha_y \) should have a
value of about 1.065 for \(^{194}\text{Hg} \) at higher \( E_x \). The question of collective
enhancement of level densities may also be relevant to this matter\textsuperscript{33)} and is
not normally taken into account in statistical model calculations. The
theoretical situation regarding the value of \( \alpha_x/\alpha_y \) is therefore quite
unclear.
Another important question which must be addressed is whether the values of \( \alpha / a_p \) obtained from statistical model analyses, modified for viscous effects as indicated, reflect the values of the true physical quantity. As we shall show, they do not because the temperature dependence of the fission barriers is not taken into account in current statistical codes. The principal effect of ignoring this feature is to increase the derived value of \( \alpha / a_p \) relative to the true value.

As might be expected by analogy with classical liquids, increase of temperature \( T \) produces a reduction in nuclear surface-energy and surface tension; in addition an increase in nuclear diffuseness occurs. Hence the fission barrier is reduced. The probability of fission relative to, say, neutron decay is determined mainly by the level density at the saddle point. This is approximately proportional to \( \exp(2(\alpha U)^{1/2}) \), where \( U = a_T^2 \) is the thermal excitation energy at the saddle point. The excitation energy \( U_0 \), above the zero temperature fission barrier, is used currently in statistical model codes, so that \( U_0 = U - \Delta E_T \), where \( \Delta E_T \) is the difference between the energies of the fission barriers for temperatures zero and \( T \). Hence if \( \Delta E_T = 0 \), the effect of using the zero temperature barrier will be to require a larger value for \( \alpha_T \); it should be remembered that one chooses the parameters of the statistical model to fit observed fission probabilities.

In the liquid-drop model parametrization of Nix, the energy of the fission barrier at zero temperature can be written:

\[
E_c(y_0, 0) = (B_s(y_0) - 1)E_s(0) + (B_c(y_0) - 1)E_c(0),
\]

where \( y_0 = 1 - x(0) \) is the deformation coordinate of Nix, and \( x(0) \), \( E_s(0) \) and \( E_c(0) \) are the fissility, surface and Coulomb energies of the spherical nucleus at zero temperature. The functions \( B_s(y) \) and \( B_c(y) \), which are tabulated by Nix, give the shape dependence of the surface and Coulomb energies respectively. At finite temperature the appropriate energy for the
fission barrier is the free energy of deformation at the saddle point, which is given by:

$$F_f(y_f, T) = (B_s(y_f) - 1)F_s(T) + (B_c(y_f) - 1)E_c(T),$$  \hspace{1cm} (5)

where $F_s(T)$ is the free surface energy at temperature $T$, $y_f = 1 - x(T)$ and $x(T) = E_c(T)/2F_s(T)$. Some calculations of the temperature dependence of fission barriers have been made. They indicate that the quantities $F_s(T)$ and $E_c(T)$ have the forms:

$$F_s(T) = F_s(0)(1 - \alpha T^2), \hspace{1cm} E_c(T) = E_c(0)(1 - \alpha T^2),$$ \hspace{1cm} (6)

Hence in the approximation that $y_0 = y_f$, it follows from (5) and (6) that $\Delta E_f \propto T^2$ and that the true value of $a_f$ is given by:

$$a_f = a_f^{(SM)} \left[ 1 + \frac{\alpha[B_c(y_0) - 1]E_c(0) + \beta[B_s(y_0) - 1]E_s(0)}{a_f} \right]^{-1}$$ \hspace{1cm} (7)

where $a_f^{(SM)}$ is the value from the statistical model calculation. A somewhat more refined analysis is required if the variation of deformation with $T$ is to be taken into account, but for our cases gives little difference to the result. The values of the quantities $\alpha$ and $\beta$ are probably not certain at the moment but appear to be about $1.0 \times 10^{-3}$ and $1.0 \times 10^{-2}$ respectively. With these values and $a_f = A/10$, we find values for $a_f/a_f^{(SM)} = 0.949$ and 0.925 for $^{200}$Pb and $^{188}$Yb respectively.

The above refers to zero angular momentum. However, an estimate of the effect of angular momentum can be made by again using the method of effective fissionility. For angular momentum $60\hbar$ the predictions for $a_f/a_f^{(SM)}$ are changed to 0.963 for $^{200}$Pb and to 0.930 for $^{188}$Yb. Hence the corrections are angular momentum dependent to some degree, especially for the heavier systems.
8. Conclusions

We have measured excitation functions for pre-fission neutrons for a number of systems ranging from $^{168}$Yb to $^{251}$Es, originally with the intention of determining the values of the parameter $a_f/a_0$ and the fission barriers. Excitation functions for evaporation residues, fission fragments and elastic scattering had previously been measured for many of these systems. Some deviations from statistical model predictions at the higher bombarding energies had previously been noted\textsuperscript{17}. However, an extensive statistical model analysis was carried out on all of the fission, fusion and $\nu_{pre}$ data, excluding those points in the latter which were clearly in conflict with the model predictions. Values for $a_f/a_0$, $k_f$ and fusion angular-momentum diffuseness parameters were determined. The results indicated that $a_f/a_0$ was slightly lower than but consistent with unity, that within errors of about 10% the results were consistent with the fission barriers predicted by the Sierk model, and that the deduced values of the diffuseness agreed surprisingly well with those predicted by the zero point model and inferred from optical-model analysis of elastic scattering data.

The effect of viscosity on the large scale collective motion resulting in fission was considered. Such viscosity or friction is currently thought necessary to explain the deviations of $\nu_{pre}$ from the statistical model predictions which we and others have observed. It was shown that inclusion of viscous effects in a statistical model code did have an effect on the deduced values of the parameters $a_f/a_0$ and $k_f$, requiring both to be increased by a few percent. However there is considerable uncertainty in present theory, so that precise values cannot be obtained.

The variation of fission barrier with temperature is not taken into account in current statistical model codes. The effect of this neglect on the deduced parameters was considered. The main one appears to be that the
value of $a_x/a_y$ is increased over the true value by about 5%, the exact value being uncertain because of the present state of theory.

Hence, contrary to what was thought quite recently, there are a number of effects, which to some degree cancel themselves and need to be taken into account in determining $a_x/a_y$ and the other parameters. There are clearly many open and challenging questions, both in theory and experiment, in this area of nuclear dynamics.
References


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28) M. Drosg, Nucl. Inst. and Meth. 105 (1972) 573.


39) W.A. Kramers, Physika 2 (1940) 284.


TABLE 1
Systems Studied

<table>
<thead>
<tr>
<th>Compound system</th>
<th>Fissility</th>
<th>Target</th>
<th>Projectile</th>
<th>E_{lab} (MeV) (mean)</th>
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<tr>
<td>¹⁶⁶Yb</td>
<td>0.60</td>
<td>¹⁵⁰Sm</td>
<td>¹⁸O</td>
<td>107-121</td>
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<td>0.64</td>
<td>¹⁵⁰Tb</td>
<td>¹⁹F</td>
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<td>¹⁶⁰Tm</td>
<td>¹⁹F</td>
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<td>¹⁶⁴Er</td>
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<td>²⁸Si</td>
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¹) The gold target had a thickness of 2.5 mg. cm⁻².
### TABLE 2

Measured pre- and post-fission neutron multiplicities

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<th>Reaction</th>
<th>$E_{lab}$ (MeV) (mean)</th>
<th>$\nu_{pre}$</th>
<th>$\nu_{post}$</th>
<th>$\nu_{tot}$</th>
<th>$\nu_m$ (calc)</th>
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\(^a\) Values from Ward et al.\(^{2}\).

Errors in \(v_{\text{pre}}\) are shown in brackets.
Figure Captions

Fig. 1  Energy spectra of 0° fission fragments for 105 MeV 19F on 181Ta. Scaled-down singles spectra, and spectra in coincidence with neutrons are shown.

Fig. 2  Two dimensional spectra of neutron TOF versus PSD signal for high and low gain systems from the 105 MeV 19F + 181Ta reaction; a typical window for selecting the neutron events is indicated. Projections on to the PSD and TOF axes are also shown.

Fig. 3  High gain (exclusive) and low gain TOF-spectra for 105 MeV 19F on 181Ta. The cubic spline fits to these spectra and deconvolution of the fitted functions are indicated.

Fig. 4  Intrinsic efficiency of neutron detector as a function of neutron velocity measured by the 2π MWPC. Efficiencies adopted (full line), calculated (dashed line) and measured by Ward et al (dotted line) are shown.

Fig. 5  Neutron velocity spectra at 0° and 90° for 115 MeV 19F on 159Tb. The deduced spectra for total, pre-fission, post-fission detected fragment (N^t and complementary fragment (N^c) are indicated.

Fig. 6  Measured values of v^pre as functions of compound nucleus excitation energy. The full lines are statistical model calculations with a_x/a_y = k_x = 1.00 and δL adjusted to fit the fission and fusion excitation function data.

Fig. 7  Experimental and calculated values of v^pre for the 181Ta + 19F system. The full lines show calculations for δL=6.0 and a_y=A/10; the dashed (dotted) lines are for a_y=A/12 (A/7.5) and a_x/a_y=0.98.
Fig. 8  (Lower) Contours of \( \chi^2 \) per degree of freedom for the \(^{19}\)F + \(^{189}\)Ta system. The full and dashed lines are derived from analyses of the fission/fusion excitation functions and \( \nu_{\text{pre}} \) data respectively. The arrow indicates the value of \( \delta L_{\text{MIN}} \).

(Upper) Ratio of calculated and experimental fission cross sections with minimum \( \chi^2 \) versus bombarding energy for the \(^{19}\)F + \(^{189}\)Ta system.

Fig. 9  Parameters \( a_x/a_u \), \( k_f \) and \( \delta L/\delta L_{\text{MIN}} \) derived from statistical model calculations plotted against fissility for our studied cases. Results for \( k_f \) and \( \delta L/\delta L_{\text{MIN}} \) are not given for \(^{210}\)Po because of problems in fitting the fission excitation functions (see text). The absolute values for \( \delta L \), beginning with \(^{152}\)Yb, are 2.7, 4.4, 4.9 and 6.0 respectively.

Fig. 10  Neutrons emitted during the saddle to scission transition versus excitation energy. The curves were calculated for \( a_u = A/10 \), \( \tau_{\text{ss}} = 3 \times 10^{-21} \) s and for good fits to all data.

Fig. 11  Statistical model calculations, including saddle-to-scission emission, of \( \nu_{\text{pre}} \) for three systems versus bombarding energy. Values of \( \tau_{\text{ss}} \), \( a_x/a_u \) and \( \delta L \) are indicated; the values of \( k_f \) were chosen to give minimum \( \chi^2 \) for the fission/fusion excitation functions.

Fig. 12  Calculated saddle to scission times for one body and two body (\( \eta = 0.02 \) T.P and zero) viscosity versus fissility. The curves for \( L = 40 \) and 60 were estimated from those for \( L = 0 \).
Fig. 2
HIGH GAIN (EXC) SPECTRUM

$105 \text{ MeV } ^{19}\text{F} + ^{181}\text{Ta}$

$\theta_{\text{ef}} = 0^\circ$

LOW GAIN (INC) SPECTRUM

Fitted Curve

Deconvolution of Fitted Curve

Fig. 3
CALCULATED

ADOPTED

ANGLE BETWEEN SOURCE AND NEUTRON DETECTOR

\[ \phi = 45^\circ \quad \phi = 90^\circ \]

NEUTRON VELOCITY (cm/ns)
Fig. 5

$\Theta_{vf} = 0^\circ$

$\Theta_{vf} = 90^\circ$

$\bar{U}$ / sr cm/nsec

N$_{total}$

N$_{pre}$

N$_{d}^{post}$

N$_{c}^{post}$

NEUTRON VELOCITY (cm/nsec)

Fig. 5
Fig. 8
Fig. 10

The graph shows the relationship between $E_x$ (MeV) and $\nu_{S-SC}$ for different isotopes:
- $^{210}$Po
- $^{200}$Pd
- $^{186}$Pt
- $^{178}$W
- $^{168}$Yb

The x-axis represents $E_x$ (MeV) ranging from 40 to 100, and the y-axis represents $\nu_{S-SC}$ ranging from 0.5 to 1.5.
Fig. 11
Fig. 12