PARTICLE-PARTICLE AND HOLE-HOLE RPA CORRELATIONS AT FINITE TEMPERATURE AND THE TEMPERATURE DEPENDENCE OF THE LEVEL DENSITY PARAMETER

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Abstract

The pp-hh RPA equations obtained by summing the infinite series of ladder, upwards and backwards going diagrams in the temperature two particle Green's functions are derived at finite temperature. The contribution to the thermodynamic grand potential due to pp-hh RPA correlations is calculated simultaneously to that of ph RPA correlations. A schematic model is constructed which shows that, as for ph RPA states, the energies of pp and hh RPA states have no temperature dependence at not too high temperature. Within the same model, the temperature dependence of the level density parameter is discussed.

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I. INTRODUCTION

The two body correlations of RPA type have been extensively studied in the past to study the properties of nuclear ground state. Two types of such correlations have been considered : the particle-hole (ph) RPA which corresponds to summing the series of bubble diagrams and the particle-particle (pp) and hole-hole (hh) RPA which corresponds to summing the series of ladder, upwards and backwards going diagrams. The p-h series of diagrams correcting the Hartree-Fock results was first introduced in the derivation of ground state density and energy [1-5]. Somewhat later the series of ladder diagrams was added simultaneously to the previous ph one in the framework of Green's function formalism [6,7]. More recently several developments or applications or extensions have been proposed [8-11] in particular treating the case of the effect of RPA correlations on the Brueckner - Hartree-Fock energy.

Since recently very excited nuclei can be produced in heavy ion collisions and their properties can be studied experimentally as functions of temperature. One is then faced with the same problem of evaluating the effect of correlations on the thermodynamical functions describing a hot nucleus which can all be derived from the knowledge of the thermodynamic potential Ω . The contribution to Ω of particle-hole bubble diagrams has been already studied [12-14]. The correction Ω_{ph} to Ω_{HF}, the Hartree-Fock grand potential, is expressed in terms of the eigenvalues of the p.h RPA equations. The aim of the present paper is to calculate the pp and hh RPA ladder diagrams contribution in a general formalism where pp-hh and ph RPA correlations are introduced simultaneously. A convenient method is to relate Ω to a two body temperature Green's function which, assuming two body correlations only, can be expressed as a function of pp-hh and ph Green's functions. The p-h terms have been already studied and here we concentrate on the terms involving pp and hh Green's functions which are calculated by summing the series of ladder diagrams. This leads to a system of equations describing nuclei with two nucleons added or subtracted to the equilibrium state which are the pp-hh RPA equations at finite temperature. These equations are a generalization of the zero temperature

pairing vibration equations. Knowing the solutions of these equations one can calculate Ω_{pp}, the contribution to Ω of the pp-hh RPA correlations. From Ω_{pp} the energy of the system is derived from which an effective level density parameter is defined by the Fermi gas formula. The temperature dependence of this effective level density parameter is of particular interest. Indeed all mean field calculations give a too small value which stays constant up to $T \simeq 5MeV$ with eventually a very weak temperature dependence at higher temperatures [15-19]. Experimentally the situation is very different. At low excitation energies ($T \simeq 1 - 2\ MeV$) the measured value of $a \approx A/8$ is much larger than the mean field results and drops for $T > 3\ MeV$ to get at $T \simeq 5\ MeV$ a value $a \approx A/13$ which is about the mean field value [20] . This behaviour is not compatible with mean field theories and should come from the contribution of two body correlations. Several papers have been devoted to this subject [21-25]. In refs.[21] to [23], ph RPA correlations are shown to give a contribution which is about 30 % of the mean field value at $T \simeq 1\ MeV$ but which disappears around $T = 4\ MeV$. The trend of the temperature dependence is then in agreement with data with however a too small effect. In ref.[24] second order contributions treated in a semi-classical approximation show an enhancement of a close to a factor 2 at small temperatures then compatible with data but with a too slow decreasing of the effect which disappears at $T \simeq 10\ MeV$only. At last, a very recent paper [25] calculates the same second order contribution but with a g-matrix calculated from realistic nucleon-nucleon interaction. There the effective level density appears too small with also a weak temperature dependence.

In section 2, we study the temperature two particle Green's function when the ladder diagrams are summed up and derive the RPA equations describing $A+2$ and $A-2$ nuclei at finite temperature. In section 3, we calculate the thermodynamic potential with inclusion of both ph and pp-hh RPA correlations. In section 4, we use a schematic model to study the behaviour of pp-hh RPA states as functions of temperature and to derive the analytical expressions of the corrections to energy and level density parameter due to ladder diagrams summation. Then we consider the case of Pb^{208} and study the temperature dependence

of the level density parameter. In section 5, we present our conclusions.

II. LADDER APPROXIMATION OF THE TEMPERATURE TWO PARTICLE GREEN'S FUNCTION

1. *General formulae*

The temperature two particle Green's function is defined as :

$$\mathcal{G}_2(ij,k\ell;\tau-\tau') =< T[a_i(\tau^+)a_j(\tau)(\tilde{a}_\ell(\tau'^+)\tilde{a}_k(\tau')] > \tag{1}$$

where the thermal average of an operator Q is :

$$< Q >= \frac{1}{Z} Tr\ e^{-\beta(H-\mu N)}Q$$

Z is the grand partition function :

$$Z = Tr\ e^{-\beta(H-\mu N)} \tag{2}$$

In eq.(1) the notation τ^+ means that the corresponding operator should be taken at the left of the operator taken at "time τ " when the "time" ordering of operators is performed. The fictitious time τ varies between 0 and β ($\beta = 1/kT$) and the operators $a_i(\tau)$ and $\tilde{a}_i(\tau)$ are written in "Heisenberg" representation and such that :

$$\begin{aligned} a_i(\tau) &= e^{-(H-\mu N)\tau} a_i e^{(H-\mu N)\tau} \\ \tilde{a}_i(\tau) &= e^{-(H-\mu N)\tau} a_i^+ e^{(H-\mu N)\tau} \end{aligned} \tag{3}$$

The operator a_i and a_i^+ are the usual annihilation and creation operators of the Schrödinger representation.

By substituting relations (3) in eq.(1) we get the spectral decomposition of $\mathcal{G}_2$. Defining

$$\begin{aligned} \tilde{E}_n &= E_n - \mu N_n \\ \tilde{E}_{mn} &= \tilde{E}_m - \tilde{E}_n \end{aligned} \tag{4}$$

where $E_n(E_m)$ and $N_n(N_m)$ are respectively the energy and particle number of the system in state $n(m)$, we find $\mathcal{G}_2$ as :

1) $\tau - \tau' > 0$

$$\mathcal{G}_2(ij, k\ell; \tau - \tau') = \frac{1}{Z} \sum_{n,m} e^{-\beta \tilde{E}_n} e^{-\tilde{E}_{mn}(\tau - \tau')} X_{ji}^{mn^*} X_{\ell k}^{mn} \tag{5}$$

the amplitudes X are defined by

$$X_{pq}^{mn} = < m \mid a_p^+ a_q^+ \mid n > \tag{6.a}$$

It is then obvious that we have relations :

$$\begin{aligned} N_m &= N_n + 2 \\ \tilde{E}_{mn} &= E_m - E_n - 2\mu \end{aligned} \tag{6.b}$$

For $\tau - \tau' > 0$, $\mathcal{G}_2$ describes the propagation of a pair of particles and is the temperature two particle propagator.

2) $\tau - \tau' < 0$

$$\mathcal{G}_2(ij, k\ell; \tau - \tau') = \frac{1}{Z} \sum_{mn} e^{-\beta \tilde{E}_n} e^{\tilde{E}_{mn}(\tau - \tau')} Y_{ij}^{mn} Y_{k\ell}^{mn^*} \tag{7}$$

the amplitudes Y are defined as :

$$Y_{pq}^{mn} = < m \mid a_p a_q \mid n > \tag{8}$$

In this case we have relations :

$$\begin{aligned} N_m &= N_n - 2 \\ \tilde{E}_{mn} &= E_m - E_n + 2\mu \end{aligned} \tag{9}$$

$\mathcal{G}_2$ now describes the propagation of two holes.

It is useful for the following to work with the Fourier representation of $\mathcal{G}_2(\tau - \tau')$. Let's define $\mathcal{G}_2(\omega_\nu)$ such that :

$$\mathcal{G}_2(\tau - \tau') = \frac{1}{\beta} \sum_{\nu=0}^{\infty} e^{i\omega_\nu(\tau - \tau')} \mathcal{G}_2(\omega_\nu) \tag{10}$$

or

$$\mathcal{G}_2(\omega_\nu) = \frac{1}{2}\int_{-\beta}^{+\beta} e^{-i\omega_\nu \tau}\mathcal{G}_2(\tau)d\tau \tag{11}$$

where we have used the property of $\mathcal{G}_2$ to depend only on the difference $\tau - \tau'$ which varies between $-\beta$ *to* $+\beta$. Because of the invariance of traces by cyclic permutation of the operators involved, $\mathcal{G}_2$ satisfies the following important relation :

$$\mathcal{G}_2(\tau < 0) = \mathcal{G}_2(\tau + \beta)$$

It follows that eq.(11) takes the form

$$\omega_\nu = \frac{2\pi\nu}{\beta} \tag{12}$$

$$\mathcal{G}_2(\omega_\nu) = \int_o^\beta \mathcal{G}_2(\tau)e^{-i\omega_\nu \tau}d\nu = \int_{-\beta}^{o} \mathcal{G}_2(\tau)e^{-i\omega_\nu \tau}d\tau \tag{13}$$

Several equivalent expressions can be derived for $\mathcal{G}_2(\omega_\nu)$ by substitution of expressions (5) or (7) in eq.(13) but the most symmetrical and convenient to use writes :

$$\mathcal{G}_2(ij,k\ell;\omega_\nu) = \frac{1}{Z}\sum_{n,m} e^{-\beta(E_n-\mu N_n)}\left\{\frac{Y_{ij}^{mn}Y_{k\ell}^{mn^*}}{i\omega_\nu + \tilde{E}_{mn}} - \frac{X_{ji}^{mn^*}X_{lk}^{mn}}{i\omega_\nu - \tilde{E}_{mn}}\right\} \tag{14}$$

where we have used the identity

$$X_{ij}^{mn} = < m \mid a_i^+ a_j^+ \mid n > = < n \mid a_j a_i \mid m >^* = Y_{ji}^{nm^*} \tag{15}$$

In the first term of the bracket $N_m = N_n - 2$ and $\tilde{E}_{mn}$ is given by eq.(9) while in the second one $N_m = N_n + 2$ and $\tilde{E}_{mn}$ given by eq.(6.b).

2. Finite temperature particle-particle and hole-hole RPA equations

At zero temperature the summation of ladder diagrams including backwards going ones in the expansion of two particle Green functions yields to the well known pp and hh RPA or pairing vibration equations which determine amplitudes and energies of two valence particles and holes nuclei simultaneously. In this ladder approximation the zero temperature two particle Green's function satisfies an integral equation which, following Matsubara [26], can be generalized to finite temperature as :

$$\mathcal{G}_2(ij,k\ell;\tau-\tau') = \mathcal{G}_2^L(ij;k\ell;\tau-\tau') - \int_o^{+\beta} \sum_{\substack{mn \\ pq}} \mathcal{G}_1(im;\tau-\tau'') \mathcal{G}_1(j,n;\tau-\tau'') < mn \mid V \mid pq > \mathcal{G}_2(pq,k\ell;\tau''-\tau')d\tau'' \tag{16}$$

$\mathcal{G}_2^L$ is the Hartree-Fock approximation to $\mathcal{G}_2$, therefore is the antisymmetrized product of two Hartree-Fock one particle Green's function $\mathcal{G}_1$. We choose our one particle basis as the complete set of Hartree-Fock states of energy ϵ_i, $\mathcal{G}_1$ is then diagonal in this basis and given by :

$$\begin{aligned} \mathcal{G}_1(im;\tau) = \mathcal{G}_i(\tau)\delta_{im} &= -f_i e^{-\tilde{\epsilon}_i\tau}\delta_{im} \qquad \text{if } \tau < 0 \\ &= \tilde{f}_i e^{-\tilde{\epsilon}_i\tau}\delta_{im} \qquad \text{if } \tau > 0 \end{aligned} \tag{17}$$

with

$$\begin{aligned} \tilde{\epsilon}_i &= \epsilon_i - \mu \\ f_i &= \frac{1}{1+e^{\beta\tilde{\epsilon}_i}} \\ \tilde{f}_i &= 1 - f_i \end{aligned} \tag{18}$$

The series of diagrams corresponding to the integral equation (16) are given in fig.1. Going to Fourier representation eq.(16) leads to :

$$\mathcal{G}_2(ij,k\ell;\omega_\nu) = \mathcal{G}_2^L(ij,k\ell;\omega_\nu) - \sum_{\substack{mn \\ pq}} \mathcal{G}_2^{LD}(ij,mn;\omega_\nu) < mn \mid V \mid pq > \mathcal{G}_2(pq,k\ell;\omega_\nu) \tag{19}$$

where $\mathcal{G}_2^{LD}$ is the Fourier transform of the direct product of two Hartree-Fock one particle Greens'function. With expression (17) of $\mathcal{G}_1$ we can easily calculate :

$$\begin{aligned}\mathcal{G}_2^{LD}(ij,k\ell;\omega_\nu) &= \beta\tilde{f}_i\tilde{f}_j\delta_{ik}\delta_{j\ell}\delta_{\nu o} \qquad \text{if } \tilde{\varepsilon}_i+\tilde{\varepsilon}_j=0 \\ &= -\frac{F_{ij}}{i\omega_\nu-\tilde{\varepsilon}_i-\tilde{\varepsilon}_j}\,\delta_{ik}\delta_{j\ell} \qquad \text{if } \tilde{\varepsilon}_i+\tilde{\varepsilon}_j\neq 0\end{aligned} \tag{20}$$

$$\begin{aligned}\mathcal{G}_2^{L}(ij,k\ell;\omega_\nu) &= \beta\tilde{f}_i\tilde{f}_j(\delta_{ik}\delta_{j\ell}-\delta_{i\ell}\delta_{jk}) \qquad \text{if } \tilde{\varepsilon}_i+\tilde{\varepsilon}_j=0 \\ &= -\frac{F_{ij}}{i\omega_\nu-\tilde{\varepsilon}_i-\tilde{\varepsilon}_j}(\delta_{ik}\delta_{j\ell}-\delta_{i\ell}\delta_{jk}) \qquad \text{if } \tilde{\varepsilon}_i+\tilde{\varepsilon}_j\neq 0\end{aligned} \tag{21}$$

$$F_{ij}=\tilde{f}_i\tilde{f}_j-f_if_j \tag{22}$$

By substituting eqs.(14), (20) and (21) into eq.(19) one gets, for $\tilde{\varepsilon}_i+\tilde{\varepsilon}_j\neq 0$, an equation with poles at $\tilde{E}_{mn}(N_m=N_n+2)$ and $-\tilde{E}_{mn}(N_m=N_n-2)$ on left and right hand sides and a pole at $\tilde{\varepsilon}_i+\tilde{\varepsilon}_j$ on the right hand side. The residue at a given pole should be the same on both sides what leads to three sets of equations determining energies and amplitudes of $A+2$ and $A-2$ systems, namely :

$$\tilde{\varepsilon}_i+\tilde{\varepsilon}_j\neq 0$$

$$(\tilde{E}_\alpha(A+2)-\tilde{\varepsilon}_i-\tilde{\varepsilon}_j)X_{ij}^{(\alpha)}-\frac{1}{2}F_{ij}\sum_{pq}<ij\mid V\mid pq>_A X_{pq}^{(\alpha)}=0 \tag{23}$$

$$(\tilde{E}_\alpha(A-2)+\tilde{\varepsilon}_i+\tilde{\varepsilon}_j)Y_{ij}^{(\alpha)}+\frac{1}{2}F_{ij}\sum_{pq}<ij\mid V\mid pq>_A Y_{pq}^{(\alpha)}=0 \tag{24}$$

$$\sum_{\alpha(A+2)}X_{ij}^{(\alpha)}X_{k\ell}^{(\alpha)}-\sum_{\alpha(A-2)}Y_{ij}^{(\alpha)}Y_{k\ell}^{(\alpha)}=F_{ij}(\delta_{ik}\delta_{j\ell}-\delta_{i\ell}\delta_{jk}) \tag{25}$$

where $<ij\mid V\mid pq>_A$ is the antisymmetrized matrix element.

By changing $\tilde{E}_\alpha(A-2)$ into $-\tilde{E}_\alpha(A-2)$ eq.(24) is the same than eq.(23) : $\tilde{E}_\alpha(A-2)$, $\tilde{E}_\alpha(A+2)$ and the amplitudes X and Y are solutions of one set of equations only :

$$(\omega_\alpha-\tilde{\varepsilon}_i-\tilde{\varepsilon}_j)Z_{ij}^{(\alpha)}-\frac{1}{2}F_{ij}\sum_{pq}<ij\mid V\mid pq>_A Z_{pq}^{(\alpha)}=0 \tag{26}$$

with the condition that $\tilde{\varepsilon}_i + \tilde{\varepsilon}_j \neq 0$.

For the $A+2$ system one has

$$\tilde{E}_\alpha(A+2) = \omega_\alpha \qquad X^{(\alpha)} \equiv Z^{(\alpha)} \tag{27}$$

and for the $A-2$ system :

$$\tilde{E}_\alpha(A-2) = -\omega_\alpha \qquad Y^{(\alpha)} \equiv Z^{(\alpha)} \tag{28}$$

It is easy to separate the two kinds of solutions : $\tilde{E}_{mn}$ defined in eq.(4) is always positive therefore positive eigenvalues of eq.(26) correspond to the $A+2$ system while negative eigenvalues correspond to the $(A-2)$ system.

The closure relation (25) obtained automatically from eq.(14) implies that the amplitudes should satisfy the following orthonormalization conditions :

$$\begin{aligned} \sum_{ij} X_{ij}^{(\alpha)^*} F_{ij}^{-1} X_{ij}^{(\alpha')} &= \delta_{\alpha\alpha'} \\ \sum_{ij} Y_{ij}^{(\alpha)^*} F_{ij}^{-1} Y_{ij}^{(\alpha')} &= -\delta_{\alpha\alpha'} \\ \sum_{ij} X_{ij}^{(\alpha)^*} F_{ij}^{-1} Y_{ij}^{(\alpha')} &= 0 \end{aligned} \tag{29}$$

These relations are not very convenient for numerical calculations. Indeed when the temperature increases, F_{ij} can be very small and can induce lack of precision in the calculation of normalized eigenvectors. It is then convenient to defined renormalized amplitudes x and y such that :

$$\begin{aligned} X_{ij}^{(\alpha)} &= < m \mid a_i^+ a_j^+ \mid n > = F_{ij} x_{ij}^{(a)} \\ Y_{ij}^{(\alpha)} &= < m \mid a_i a_j \mid n > = F_{ij} y_{ij}^{(\alpha)} \end{aligned} \tag{30}$$

The equation (26) and relations (25) and (29) become :

$\tilde{\varepsilon}_i + \tilde{\varepsilon}_j \neq 0$

$$(\omega_\alpha - \tilde{\varepsilon}_i - \tilde{\varepsilon}_j) z_{ij}^{(\alpha)} - \frac{1}{2} \sum_{pq} < ij \mid V \mid pq >_A F_{pq} z_{pq}^{(\alpha)} = 0 \tag{31}$$

$$\begin{cases} \sum_{ij} x_{ij}^{(\alpha)} F_{ij} x_{ij}^{(\alpha')} &= \delta_{\alpha\alpha'} \\ \sum_{ij} y_{ij}^{(\alpha)} F_{ij} y_{ij}^{(\alpha')} &= -\delta_{\alpha\alpha'} \end{cases} \tag{32}$$

$$\sum_{\alpha(A+2)} x_{ij}^{(\alpha)} F_{k\ell} x_{k\ell}^{(\alpha)} - \sum_{\alpha(A-2)} y_{ij}^{(\alpha)} F_{k\ell} x_{k\ell}^{(\alpha)} = \delta_{ik}\delta_{j\ell} - \delta_{i\ell}\delta_{jk} \tag{33}$$

It is easy to show that when T goes to zero, eqs.(31-33) are the so called pp and hh RPA or pairing vibration equations [6,27]. We shall call our equations (31-33) the finite temperature pp-hh RPA equations which describe the properties of the two valence particle or two valence holes nuclei at finite temperature.

In terms of these RPA amplitudes and energies solutions of eqs.(31-33) the two particle Green's function of eq.(14) which we shall need later on simplifies and can be written as :

$$\mathcal{G}_2(ij, k\ell; \omega_\nu) = \sum_{\alpha(A-2)} \frac{F_{ij} y_{ij}^{(\alpha)} F_{k\ell} y_{k\ell}^{(\alpha)}}{i\omega_\nu + \tilde{E}_\alpha(A-2)} - \sum_{\alpha(A+2)} \frac{F_{ij} x_{ij}^{(\alpha)} F_{k\ell} x_{k\ell}^{(\alpha)}}{i\omega_\nu - \tilde{E}_\alpha(A+2)} \tag{34}$$

III. DERIVATION OF THE THERMODYNAMIC GRAND POTENTIAL

In a previous paper [14] the grand potential Ω has been derived taking into account the particle-hole RPA correlations due to the summation of bubble diagrams. In the present paper we want to add the contribution due to the series of ladder diagrams which have been summed in the previous section to calculate the two particle Green's functions. To do so we follow the methods of ref.[7] where the properties of ground state of nuclei were studied.

The grand potential Ω is related to the grand partition function by the relation :

$$\Omega = \frac{1}{\beta} \ell n Z$$

where Z is defined in eq.(2).

The total hamiltonian H can be written as the sum of an independent particle model hamiltonian H_o and a residual interaction W such that :

$$W = V - U \tag{35}$$

V is the two body effective interaction and U the one body potential contained in H_o.

Writing

$$H = H_o + W$$

and replacing W by λw where λ is a perturbative parameter varying between 0 and 1 one has

$$\frac{\partial(\ell n Z)}{\partial \lambda} = -\beta \frac{< W >}{\lambda}$$

what is equivalent to write :

$$\begin{aligned} \Omega &= \Omega_o + \Omega_1 \\ \Omega_1 &= -\int_0^1 \frac{< W >}{\lambda} d\lambda \end{aligned} \tag{36}$$

Ω_o is the grand potential of the independent particle system described by H_o and Ω_1 the contribution due to correlations.

To calculate Ω_1 we then need calculate $< W >$ which can be expressed in terms of the two particle Green's function $\mathcal{G}_2$ of eq.(1). Indeed we can write :

$$\begin{aligned} < W > &= \frac{1}{2} \sum_{ijk\ell} < k\ell \mid V \mid ij >< a_\ell^+ a_k^+ a_i a_j > \\ &= \frac{1}{2} \sum_{ijk\ell} < k\ell \mid V \mid ij > \left\{ \lim_{\tau - \tau' \to -0} \mathcal{G}_2(ij, k\ell; \tau - \tau') \right\} \end{aligned} \tag{37}$$

Substituting directly the two-particle Green's function calculated in the previous section into eq.(37) would sum the ladder diagrams only while we want to include both series

of ladder and bubble diagrams. To do so it is necessary to introduce the three particle Green's function, $\mathcal{G}_3$, by using the dynamical equation relating $\mathcal{G}_2$ to $\mathcal{G}_1$ and $\mathcal{G}_3$. This equation which we write in space-time coordinate representation where the equations are more compact avoiding a multiplicity of indices writes [28,29]

$$\begin{aligned}\mathcal{G}_2(12;1'2') &= \mathcal{G}_1(11')\mathcal{G}_1(22') - \mathcal{G}_1(12')\mathcal{G}_1(21') \\ &- \int d3d4\mathcal{G}_1(1,3)V(3-4)[\mathcal{G}_3(324;1'2'4^+) - \\ &- \mathcal{G}_1(4,4^+)\mathcal{G}_2(32;1'2') - \mathcal{G}_1(3,4)\mathcal{G}_2(42;1'2')]\end{aligned} \tag{38}$$

where the notation $1,2...$ means $r_1^-t_1, r_2^-t_2....$

The next step consists in writing $\mathcal{G}_3$ as an antisymmetrized sum of products $\mathcal{G}_1\mathcal{G}_2$ corrected from second order double counting [29]. Assuming that H_o is the Hartree-Fock one body hamiltonian and approximating $\mathcal{G}_1$ by its Hartree-Fock value of eq.(17), eq.(37) reduces to :

$$\begin{aligned}< W >= -\frac{1}{2}\sum_{ijk\ell} &< k\ell \mid V \mid ij >_A \left\{\lim_{\tau-\tau'\to-0}[\mathcal{G}_i(\tau-\tau')\mathcal{G}_j(\tau-\tau')\right\} \\ \delta_{ik}\delta_{j\ell} - \int d\tau_1 &\mathcal{G}_i(\tau-\tau_1)\mathcal{G}_k(\tau_1-\tau) < iq \mid V \mid kp >_A \mathcal{G}^{II}(pq;\ell j;\tau_1-\tau') \\ + \int d\tau_1 &\mathcal{G}_i(\tau-\tau_1)\mathcal{G}_j(\tau-\tau_1) < ij \mid V \mid pq > \mathcal{G}_2(pq;k\ell;\tau_1-\tau') - 2 < W >_2\end{aligned} \tag{39}$$

where V is the two body interaction of eq.(35), $< k\ell \mid V \mid ij >_A$ the antisymmetrized matrix element of V, $\mathcal{G}^{II}$ the particle hole Green's function of ref.[14] and $\mathcal{G}_2$ the two particle Green's function studied in the section II.

The first term of the braces is the Hartree-Fock self-energy term, the second one gives the contribution of the series of the bubble diagrams when $\mathcal{G}^{II}$ is replaced by its RPA expression and the third one the contribution of the series of ladder diagrams when $\mathcal{G}_2$ is replaced by its RPA expression. The term $2 < W >_2$ cancels double counting in second order. The eq.(39) can be written as :

$$< W >=< W >_{HF} + < W >_{ph} + < W >_{pp} - < W >_2 \tag{40}$$

with

$$< W >_{HF}= -\frac{1}{2}\sum_{ij} < ij \mid V \mid ij >_A f_i f_j \tag{41}$$

$$< W >_{ph} = \frac{1}{2} \lim_{\tau - \tau' \to -0} \sum_{ijk\ell} \int_o^\beta < k\ell \mid V \mid ij >_A \mathcal{G}_i(\tau - \tau_1)\mathcal{G}_k(\tau_1 - \tau) < iq \mid V \mid kp >_A$$
$$\mathcal{G}^{II}(pq; \ell j; \tau_1 - \tau')d\tau_1 - < W >_2 \tag{42}$$

$$< W >_{pp} = -\frac{1}{2} \lim_{\tau - \tau' \to -0} \sum_{ijk\ell} \int_o^\beta < k\ell \mid V \mid ij >_A \mathcal{G}_i(\tau - \tau_1)\mathcal{G}_j(\tau - \tau_1) < ij \mid V \mid pq >$$
$$\mathcal{G}_2(pq, k\ell; \tau_1 - \tau')d\tau_1 \tag{43}$$

To obtain eq.(41) we have used the expression (17) of $\mathcal{G}_i$ in the first term of eq.(39). Substituting eq.(40) into eq.(36) one gets the grand potential as a sum of three contributions

$$\Omega = \Omega_{HF} + \Omega_{ph} + \Omega_{pp} - \Omega_2 \tag{44}$$

$$\Omega_{HF} = \Omega_o - \frac{1}{2} \sum_{ij} < ij \mid V \mid ij >_A f_i f_j$$
$$\Omega_{ph} = -\int_0^1 \frac{< W >_{ph}}{\lambda} d\lambda \tag{45}$$
$$\Omega_{pp} = -\int_0^1 \frac{< W >_{pp}}{\lambda} d\lambda$$

Ω_2 which is the second order contribution to Ω can be found in ref.[14].

Eqs.(40-43) have been written in such a way that $\Omega = \Omega_{HF} + \Omega_{ph}$ when particle-hole correlations only are taken into account and $\Omega = \Omega_{HF} + \Omega_{pp}$ for particle-particle (or hole-hole) correlations alone. Both functions Ω_{ph} and Ω_{pp} contain the second order contribution once, thus when they are summed in eqs.(40,44) this term has to be subtracted once.

$< W >_{ph}$ and Ω_{ph} have been calculated in ref.[14] with the result that :

$$\Omega_{ph} = \frac{1}{2} \sum_{i \neq j} (1 - f_i) f_j < ij \mid V \mid ij >_A$$
$$ln \left(\prod_{i>j} sinh\left[\beta(\varepsilon_i - \varepsilon_j)/2\right] \Big/ \prod_{n>0} sinh\left[\beta\omega_n/2\right] \right) - \Omega_2 \tag{46}$$

The index n characterizes an RPA state of the A nucleus with energy ω_n.

We are left with the calculation of $< W >_{pp}$. Using the integral equation (16) satisfied by $\mathcal{G}_2$ in the ladder approximations and relations (17) we get a simple expression :

$$< W >_{pp} \;=\; \frac{1}{2} \lim_{\tau - \tau' \to -0} \sum_{ijk\ell} < k\ell \mid V \mid ij > \left\{ \mathcal{G}_2(ij, k\ell; \tau - \tau') - \mathcal{G}_2^L(ij, k\ell; \tau - \tau') \right\} \tag{47}$$

where $\mathcal{G}_2^L$ is the free two particle Green's function and $\mathcal{G}_2$ the two particle RPA Green's function. The quantity $< W >'_{pp} = < W >_{HF} + < W >_{pp}$ is identical to what we would have obtained by approximating directly the $\mathcal{G}_2$ of eq.(37) by its RPA approximation. The second term in the braces of eq.(47) just cancels $< W >_{HF}$. However such a direct substitution would not have lead to a coherent inclusion of both types of correlations but to a derivation limited to ladder diagrams summation.

To perform the limit $\tau - \tau' \to -0$, it is convenient to introduce the Fourier transform of $\mathcal{G}_2$ as defined in eqs.(10-13). The equation (47) writes then :

$$\begin{aligned} < W >_{pp} &= < W >'_{pp} - < W >_{HF} \\ < W >'_{pp} &= \frac{1}{4\beta} \lim_{\eta \to -0} \sum_{\nu} e^{-i\omega_\nu \eta} \sum_{\substack{ij \\ k\ell}} < k\ell \mid V \mid ij >_A \mathcal{G}_2(ij, k\ell; \omega_\nu) \\ \omega_\nu &= \frac{2\pi\nu}{\beta} \end{aligned} \tag{48}$$

where $\mathcal{G}_2(\omega_\nu)$ is given by eq.(34) in terms of the solutions of the pp-hh RPA equations.

The calculation of the limit $\eta \to -0$ can be carried out first. It is somewhat lengthy but without difficulty and is performed by replacing $\mathcal{G}_2$ by its expression, eq.(34), and by using two well known relations given in refs.[30] and [31] respectively :

$$\lim_{\eta \to -0} \sum_{\nu} e^{-i\omega_\nu \eta} \frac{1}{i\omega_\nu - x} = -\frac{\beta}{e^{\beta x} - 1} \quad \text{if} \quad \omega_\nu = \frac{2\pi\nu}{\beta} \tag{49}$$

and

$$coth \pi x = \frac{1}{\pi x} - \frac{2x}{\pi} \sum_{k=1}^{\infty} \frac{1}{x^2 + k^2}$$

The result for $< W >'_{pp}$ is :

$$< W >'_{pp} = \frac{1}{4\beta} \sum_{\substack{ij \\ k\ell}} \sum_{\nu=0}^{\infty} < k\ell \mid V \mid ij >_A \mathcal{G}_2(ij, k\ell; \omega_\nu) - \frac{1}{4} \sum_{ij} < ij \mid V \mid ij >_A F_{ij} \tag{50}$$

where F_{ij} has been defined in eq.(22) in terms of occupation number of states i and j.

Substituting eq.(50) into eqs.(48) gives :

$$< W >_{pp} = < W >^c_{pp} - \frac{1}{4} \sum_{ij} < ij \mid V \mid ij >_A (\tilde{f}_i \tilde{f}_j + f_i f_j) \tag{51}$$

$$< W >^c_{pp} = \frac{1}{4\beta} \sum_{\substack{ij \\ k\ell}} \sum_{\nu} < k\ell \mid V \mid ij >_A \mathcal{G}_2(ij; k\ell; \omega_\nu) \tag{52}$$

To calculate Ω_{pp} from W_{pp} we have to integrate on a perturbative parameter λ. Let's define three matrices V, $\mathcal{G}_2$ and $\mathcal{G}_2^L$ in the two particle and two hole subspace by their matrix elements :

$$V_{k\ell,ij} = < k\ell \mid V \mid ij >_A$$
$$\left(\mathcal{G}_2(\omega_\nu)\right)_{ij,k\ell} = \mathcal{G}_2(ij, k\ell; \omega_\nu) \tag{53}$$

and the same relation for $\mathcal{G}_2^L$. The equations (16) and (52) can be written as :

$$\mathcal{G}_2(\omega_\nu) = \mathcal{G}_2^L(\omega_\nu) - \frac{1}{4} \mathcal{G}_2^L(\omega_\nu) V \ \mathcal{G}_2(\omega_\nu)$$
$$< W >^c_{pp} = \frac{1}{4\beta} \sum_{\nu} Tr\left[V \ \mathcal{G}_2(\omega_\nu)\right] = \frac{1}{4\beta} \sum_{\nu} Tr \ \frac{V \ \mathcal{G}_2^L(\omega_\nu)}{1 + \frac{1}{4} \ \mathcal{G}_2^L(\omega_\nu) V}$$

Ω^c_{pp} related to $< W >^c_{pp}$ by eq.(36) is :

$$\Omega^c_{pp} = -\frac{1}{4\beta} \sum_{\nu} Tr \int_0^1 d\lambda \frac{V \ \mathcal{G}_2^L(\omega_\nu)}{1 + \frac{1}{4} \lambda \mathcal{G}_2^L(\omega_\nu) V} = \frac{1}{\beta} \sum_{\nu} \ell n \left[\frac{Det \ \mathcal{G}_2(\omega_\nu)}{Det \ \mathcal{G}_2^L(\omega_\nu)}\right] \tag{54}$$

With eqs.(21) and (34), we find :

$$\Omega^c_{pp} = -\frac{1}{\beta}\sum_{\nu} \ell n \left[\frac{\prod_{\alpha}(i\omega_\nu - \omega_\alpha)}{\prod_{ij}(i\omega_\nu - \tilde{\varepsilon}_i - \tilde{\varepsilon}_j)}\right]$$

ω_α are the eigenvalues of eq.(31) and ω_ν is defined in eq.(12).

By use of the series [31] :

$$\prod_{n=1}^{\infty}\left(1 + \frac{x^2}{\pi^2 n^2}\right) = \frac{sh\ x}{x}$$

we obtain :

$$\Omega_{pp} = \frac{1}{\beta}\ell n\left[\frac{\prod_{ij} sh(\beta\omega_{ij}/2)}{\prod_{\alpha} sh(\beta\omega_a/2)}\right] - \frac{1}{4}\sum_{ij} < ij \mid V \mid ij >_A (f_i f_j + \tilde{f}_i \tilde{f}_j) \qquad (55)$$

with $\omega_{ij} = \tilde{\varepsilon}_i + \tilde{\varepsilon}_j \neq 0$.

We can easily verify that the limit of Ω_{pp} when T goes to zero gives the well known correction to the Hartree-Fock ground state energy due to pp-hh RPA correlations [6,7].

$$\begin{aligned}\Delta E_{pp} = \frac{1}{2}\Bigg[&\sum_{n(A+2)} E_n - \sum_{ij}(\omega_{ij} + \frac{1}{2} < ij \mid V \mid ij >_A) \\ &+ \sum_{m(A-2)} E_m + \sum_{i'j'}(\omega_{i'j'} - \frac{1}{2} < i'j' \mid V \mid i'j' >_A)\Bigg] \qquad (56) \\ = &- \sum_{\substack{n(A+2)\\ i'>j'}} E_n \mid X^{(n)}_{i'j'} \mid^2 - \sum_{\substack{m(A-2)\\ i>j}} E_m \mid Y^{(m)}_{ij} \mid^2\end{aligned}$$

At zero temperature i, j are "pure" particle states ($f_i = f_j = 0$) and $i'j'$ "pure" hole states ($f_{i'} = f_{j'} = 1$).

The correction to the Hartree-Fock grand potential corresponding to the summation of bubble diagrams (ph RPA) and ladder diagrams (pp-hh RPA) has been derived and expressed in terms of the eigenvalues of ph and pp-hh RPA equations. From Ω, all thermodynamic functions can be calculated by taking differentiations of Ω. Therefore with

eqs.(44,46 and 55) we are able to study all properties of nuclei at finite temperature when correlations are taken into account.

In the next section we propose a schematic model where all calculations can be performed analytically. The model will be applied to the calculation of the RPA corrections to the excitation energy and level density parameter as functions of temperature.

IV. SCHEMATIC MODEL

1. Temperature dependence of the pp-hh RPA states

To find the eigenvalues of the RPA eqs.(31), we assume a schematic model similar to that one already proposed in refs.[21,23] to study the RPA giant resonances at finite temperature and the effect of ph RPA correlations on the level density parameter.

The schematic model assumes equidistant shells with identical degeneracy. The particle and hole spectrum is shown in fig.2 where our notations are defined. At zero temperature the unperturbed two particles and two holes configurations with the lowest energies correspond to two particle in states $i_o, j_o...$ with energy $\tilde{\varepsilon}_{io} + \tilde{\varepsilon}_{jo} = \Delta\varepsilon$ and two holes in states $i'_o, j'_o...$ with energies $\tilde{\varepsilon}_{i'o} + \tilde{\varepsilon}_{j'o} = -\Delta\varepsilon$. When the temperature increases new configurations of the same energy $\Delta\varepsilon$ or $-\Delta\varepsilon$ are available : the configurations such as $(i_n i'_{n-1})$ and $(i'_n i_{n-1})$ have energies $\Delta\varepsilon$ and $-\Delta\varepsilon$ respectively and coefficients F different from zero. Assuming that the matrix elements are separable (this is true [27] for a zero range effective interaction if the exchange term is neglected and all radial integrals assumed to be equal) the pp-hh RPA equations (31) reduce for eigenvalues $\omega_\alpha \neq \Delta\varepsilon$ or $-\Delta\varepsilon$ to

$$1 = \frac{\lambda}{2}\left[\frac{F_o \sum_{i_o j_o} v^2_{i_o j_o} + \sum_{n=1}^{N} F_n \sum_{i_n i'_{n-1}} v^2_{i_n i'_{n-1}}}{\omega - \Delta\varepsilon} + \frac{F'_o \sum_{i'_o j'_o} v^2_{i'_o j'_o} + \sum_{n=1}^{N'} F'_n \sum_{i_{n-1} i'_n} v^2_{i_{n-1} i'_n}}{\omega + \Delta\varepsilon}\right] \tag{57}$$

where we have defined :

$$< k\ell \mid V \mid ij >= \lambda v_{k\ell} v_{ij}$$

$$\begin{cases} F_o &= F_{i_o j_o} \\ F'_o &= F_{i'_o j'_o} \\ F_n &= F_{i_n i'_{n-1}} \\ F'_n &= F_{i_{n-1} i'_n} \end{cases} \tag{58}$$

The upper limits N and N' in the summations are determined by $F_{N+1} \simeq 0$, $F'_{N'+1} \simeq 0$.

In our schematic model we can easily show that

$$\begin{aligned} F_o &= -F'_o = th\beta\Delta\varepsilon/4 \\ F_n &= -F'_n = \frac{sh(\beta\Delta\varepsilon/2)}{ch(\beta\Delta\varepsilon/2) + ch(n\beta\Delta\varepsilon)} \end{aligned} \tag{59}$$

Moreover we assume the following relations between matrix elements

$$\sum_{i_o j_o} v^2_{i_o j_o} = \sum_{i'_o j'_o} v^2_{i'_o j'_o} = \frac{1}{2} \sum_{i_n i'_{n-1}} v^2_{i_n i'_{n-1}} = \frac{1}{2} \sum_{i_{n-1} i'_n} v^2_{i_{n-1} i'_n} = \frac{2V}{\lambda} \tag{60}$$

The factor $1/2$ comes from the fact that, if d is the degeneracy of each shell, we can have $d(d-1)/2 \simeq d^2/2$ pairs if the particles (or holes) are on the same shell but d^2 pairs if the particles (or holes) are on different shells.

With assumption (60) the eq.(57) reduces to

$$1 = VF(\beta)\left[\frac{1}{\omega - \Delta\varepsilon} - \frac{1}{\omega + \Delta\varepsilon}\right] \tag{61}$$

with

$$F(\beta) = F_o + 2\sum_{n=1}^{N} F_n(\beta) \tag{62}$$

Eq.(61) has two solutions :

$$\omega = \pm\Delta\varepsilon\sqrt{1+\frac{2V}{\Delta\varepsilon}F(\beta)}$$

Let's represent by α the set of quantum numbers (angular momentum, parity and isospin for light nuclei) characterizing each state of our system. For each set α, the RPA equation (61) has two "collective" solutions with eigenvalues :

$$\begin{cases} \omega_\alpha &= \pm\Delta\varepsilon\sqrt{1+v_\alpha F(\beta)} \\ v_\alpha &= \dfrac{2V_\alpha}{\Delta\varepsilon} \end{cases} \tag{63}$$

The positive eigenvalue gives the energy $\tilde{E}_\alpha(A+2)$ corresponding to the state α in the (A+2) nucleus while the negative eigenvalue gives minus $\tilde{E}_\alpha(A-2)$ which corresponds to the state α in the (A-2) nucleus. Therefore one has :

$$\tilde{E}_\alpha(A+2) = \quad \tilde{E}_\alpha(A-2)$$

Assuming that the effective interaction and the distance between shells are independent of temperature (this is well verified in Hartree-Fock calculations [15,32,33] up to $T \simeq 5\ MeV$), the temperature dependence of the RPA energies is determined by the function $F(\beta)$ only.

We now show that $F(\beta) = 1$ independently of $\beta(\beta \neq 0)$. In eq.(62) we can replace $\sum_{n=1}^{N}$ by $\sum_{n=1}^{\infty}$ since when $n > N$ $F_n(\beta) \simeq 0$. Therefore we have to calculate :

$$I = sh\left(\beta\,\frac{\Delta\varepsilon}{2}\right)\sum_{n=1}^{\infty}\ \frac{1}{ch\left(\beta\dfrac{\Delta\varepsilon}{2}\right)+ch(n\beta\Delta\varepsilon)} \tag{64}$$

The Euler Mac Laurin's formula says that

$$\sum_{n=0}^{\infty} f(n) = \int_o^{\infty} f(x)dx + \frac{1}{2}f(o) - \frac{1}{12}f^{(1)}(o) + \frac{1}{720}f^{(3)}(o) + ...$$

where $f^{(1)}$ and $f^{(3)}$ are the first and third derivatives of $f(x)$ respectively.

All odd derivatives of the function $f(x)$ of eq.(64) are easily shown to be zero for $x = 0$, so that we have after calculation of the integral the exact relation :

$$I = \frac{1}{2}\left(1 - th\beta\frac{\Delta\varepsilon}{4}\right) \tag{65}$$

By substituting (59), (64) and (65) into eq.(62) we get

$$F(\beta) = 1$$

It follows that :

$$\tilde{E}_\alpha(A+2) = \Delta\varepsilon\sqrt{1+v_\alpha} = \tilde{E}_\alpha(A-2) \tag{66}$$

The energies of RPA states in (A+2) and (A-2) nuclei are independent of temperature. We expect this property to hold to a large extent in more realistic calculations. Indeed it has been verified that the temperature independence of RPA giant resonances established in the same schematic model is very closely reproduced in more realistic models [34-37]. However our schematic model may not be extended to too high temperature ($T > 5 - 6\ MeV$). Our assumptions of equal degeneracy of the shells and equality (eq.60) of matrix elements will fail if we go too far away from the Fermi level. Moreover the assumption that $\Delta\varepsilon$ is temperature independent will fail at higher temperature. This was also observed in the calculation of ph RPA states.

2. Thermodynamic functions and level density parameter

Within the schematic model the grand potential has a simple form :

$$\Omega_{pp} = \Omega_{HF} + \frac{\Delta\varepsilon}{2}G(\beta)\sum_\alpha v_\alpha - \frac{2}{\beta}\sum_{\alpha(A+2)}\left[\ell nsh\frac{\beta\tilde{E}_\alpha}{2} - \ell nsh\ \frac{\beta\Delta\varepsilon}{2}\right]$$

$$G(\beta) = \frac{1}{2}\left(1 + th^2\frac{\beta\Delta\varepsilon}{4}\right) + 2\sum_{n=1}^{N} \frac{ch\beta\Delta\varepsilon/2}{ch\beta\Delta\varepsilon/2 + ch\omega\beta\Delta\varepsilon} \qquad (67)$$
$$= coth\beta\Delta\varepsilon/2$$

To derive (67) we have used the fact that $\tilde{E}_\alpha$ is the same for A+2 and A-2 systems. The function $G(\beta)$ is very easily derived from the quantity I of eqs.(64,65).

From eq.(67) we can derive any thermodynamic function. We are here specially interested by calculating the corrections to the level density parameter a due to RPA correlations and their variation with temperature. A recent experiment [20] determines a from the measurement of the excitation energy E^* of the hot nucleus which is assumed to be related to a by the Fermi gas formula

$$E^*(T) = a\ T^2 \qquad (68)$$

We follow the same method and calculating the excitation energy at temperature T we determine a from relation (68). Note that when we go beyond the Fermi gas model the relations

$$\frac{E^*}{T^2} = \frac{S}{2T} = \frac{S^2}{4E^*} = a$$

are not anymore satisfied. One always may define an "effective" a but it will not be unique and to each quantity of the previous relations will correspond a different value of a which will depend on temperature.

The excitation energy E^* is determined from the expression (67) of the grand potential by using the following formulae :

$$E^*(T) = E(T) - E(T=0)$$
$$E(T) = -\frac{\partial}{\partial\beta}(\beta\Omega)$$

It has been shown that when Ω is approximated by the series of RPA diagrams, the derivative with respect to β of this approximated Ω introduces in the energy terms

which correspond to a different type of diagrams. These terms are generated by the β-dependence of the Hartree-Fock mean field. They should not be retained when calculating E if one wants the correction to E due to RPA correlations only. However in our schematic model all Hartree-Fock quantities entering in the expression (67) of Ω are assumed to be independent of β and in this case there is no ambiguity : E given by differentiating Ω of eq.(67) with respect to β is exactly the result of the summation of RPA pp and hh diagrams. Because we have shown that the RPA energies are independent of β, we get simply :

$$
\begin{aligned}
E^*(T) &= E^*_{HF}(T) + \delta E^*_{pp}(T) \\
\delta E^*_{pp}(T) &= \sum_{\alpha(A+2)} \left[-\frac{1}{2} \left(coth\beta\frac{\Delta\varepsilon}{2} - 1\beta\frac{\Delta\varepsilon}{2}\frac{1}{sh^2\beta\frac{\Delta\varepsilon}{2}} \right) v_\alpha \Delta\varepsilon \right. \\
&\left. + \tilde{E}_\alpha \left(coth\beta\frac{\tilde{E}_\alpha}{2} - 1 \right) - \Delta\varepsilon \left(coth\beta\frac{\Delta\varepsilon}{2} - 1 \right) \right]
\end{aligned}
\tag{69}
$$

With eq.(68) we calculate the correction δa_{pp} to the Hartree-Fock level density parameter due to RPA particle-particle and hole-hole correlations as

$$
\delta a_{pp}(T) = \frac{1}{T^2}\delta E^*_{pp}(T) \tag{70}
$$

where δE^*_{pp} is given by eq.(69).

3. Level density parameter in Pb^{208}

We now apply our model to Pb^{208}. We determine our parameters $\Delta\varepsilon$ and μ from the neutron and proton one particle spectra in the lead region given in refs.[38,39]. We assume that the levels $h_{9/2}...p_{1/2}$ for neutrons and $g_{7/2}...s_{1/2}$ for protons are concentrated in one shell, the one labelled $i'_o, j'_o...$ in fig.2 and that the levels $g_{9/2}...d_{3/2}$ for neutrons

and $h_{9/2}...i_{13/2}$ for protons are concentrated in one shell, the one labelled $i_o, j_o...$ in fig.2. Taking an average value for the energies of these two shells gives similar values for neutrons and protons such that :

$$\Delta\varepsilon \simeq 6\ MeV \qquad \mu \simeq -6\ MeV$$

The effective matrix elements v_α are determined from the lowest part of the experimental spectra of nuclei with two nucleons subtracted or added to Pb^{208}. We assume that all parameters are the same for neutron-neutron, proton-proton or neutron-proton correlations so that we calculate the $v'_\alpha s$ from Pb^{206} and Pb^{210} spectra only which are the best known. The contribution to δE^*_{pp} and to δa_{pp} of neutron-neutron correlations so determined are afterwards multiplied by 4. This is a somewhat crude approximation but it will not change drastically our results which anyway are only qualitative.

The energies $\tilde{E}_\alpha$ defined by eqs.(4) are determined from zero temperature experimental spectra as follows :

in Pb^{210} and for a given state α we have

$$\begin{aligned}\tilde{E}_\alpha(Pb^{210}) &= E_\alpha(Pb^{210}) - E_o(Pb^{208}) - 2\mu \\ &= E^*_\alpha(Pb^{210}) + \big(E_o(Pb^{210}) - E_o(Pb^{208})\big) - 2\mu\end{aligned}$$

where E^*_α is the excitation energy of state α in Pb^{210} and E_o the binding energies of the nuclei-Pb^{210} and Pb^{208}.

In Pb^{206} we have similarly

$$\tilde{E}_\alpha(Pb^{206}) = E^*_\alpha(Pb^{206}) + \big(E_o(Pb^{206}) - E_o(Pb^{208})\big) + 2\mu$$

In our schematic model for a given set α, the energies $\tilde{E}_\alpha$ are the same in A-2 and A+2 systems. When the previous energies are calculated from the experimental spectra [40] of $Pb^{206}$and Pb^{210} the energies $\tilde{E}_\alpha(Pb^{210})$ and $\tilde{E}_\alpha(Pb^{206})$ are very close to each other (they

differ by at most 0.8 MeV) and for each state we take the average of the two results. We have included in our calculation the $0^+, 2^+, 4^+$ and 6^+ states of Pb^{206} and Pb^{210}. With the values of $\tilde{E}_\alpha$ and v_α so determined and the result of eq.(66) that the energies $\tilde{E}_\alpha$ are independent of temperature, we calculate easily δa_{pp} of eqs.(69,70). The results are given in fig.3 where δa_{pp} is plotted as a function of temperature. We find a very strong temperature dependence. At small temperatures, $T \simeq 1\ MeV$, δa_{pp} is very large and comparable to the Hartree-Fock value. For $T > 1 - 2\ MeV$, δa_{pp} decreases very sharply with increasing temperature to become negligible at about $T = 5\ MeV$.

On the same fig.3, we have also plotted the contribution δa_{ph} of the p-h RPA correlations calculated in ref.[23] with the same schematic model. The numerical values of δa_{ph} have been recalculated with the same parameters $\Delta\varepsilon = 6\ MeV$ and $\mu = -6\ MeV$ as δa_{pp} and have been multiplied by 2 compared to ref.[23] to take into account neutron-proton hole and proton-neutron hole correlations as well as neutron-neutron hole and proton-proton hole ones which are the only correlations to be included when one uses collective states of Pb^{208}. We see that δa_{ph} is much weaker than δa_{pp} at small temperature but has a smoother temperature dependence. Note that the behaviour of our δa_{ph} is very similar to what was found in refs.[21,22]. The two contributions δa_{pp} and δa_{ph} are not independent since they both contain the second order contribution once (indeed since our schematic model neglects exchange terms there is no double counting in δa_{ph} what can be verified on the formula of ref.[23]). Therefore they should not be added to give the total correction δa. Nevertheless in ref.[21] more realistic calculations of δa_{ph} and of the corresponding δa_2, the second order correction, have shown that δa_2 is much weaker at small temperature than δa_{ph} and has a weak temperature dependence. Therefore one may say that the ph correlations will increase the correction δa_{pp} at small temperature by a significant amount.

Our results should be considered as qualitative only. An interesting and conforting fact is that contributions of different states are coherent so that, within our model, we get a minimum value of the corrections. On the other side, for each set α of quantum numbers the model puts all the collectivity in one state only and so certainly amplifies

the effect. But we believe that the general features will stay in more realistic studies as it has been verified in the case of ph RPA correlations alone [21-23]. The trend of δa_{pp} and δa_{ph} of fig.3 is in good agreement with experimental data which show that a is nearly twice the Hartree-Fock value at $T \simeq 1\ MeV$ but decreases sharply to this last value at $T = 4 - 5\ MeV$. Our results reproduce quite nicely this behaviour.

In refs.[24,25] the second order contribution to the effective mass, therefore to the level density parameter, has been calculated in a semi-classical approach [24] or in a $\mathcal{G}$-matrix approach [25]. In both works the level density parameter has a weak temperature dependence with however differences in the amount of the second order contribution. While in ref.[25], the authors say that their correction is always small, in ref.[24] it is large at small temperature and disappears at $T \approx 10\ MeV$ only. We cannot make a quantitative comparison between our results and theirs since we work with different approaches and approximations. However one can say that the summation of higher order ladder and bubble diagrams will considerably improve the temperature dependence of their level density parameter and will enhance it at small temperature.

CONCLUSIONS

We have derived the pp-hh RPA equations from the two particle Green's function approximated by summing the series of ladder diagrams including backwards going ones. Then we have calculated the corrections to the thermodynamical potential due to the sum of these ladder diagrams added to the ph bubble diagrams. Using a schematic model we were able to study the temperature dependence of pp-hh RPA states and of the level density parameter when pp-hh and ph RPA correlations are taken into account. These corrections to the Hartree-Fock value of the level density parameter are very important at small temperature ($T \simeq 1\ MeV$) but have a strong temperature dependence and become negligible around $T = 4 - 5\ MeV$. This result agrees with the experimental data - but should be confirmed by more realistic calculations

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FIGURE CAPTIONS

Fig.1 Some of the ladder diagrams contributing to the two particle Green's function ($\tau - \tau' > 0$) of eq.(16). For the two hole Green's function ($\tau - \tau' < 0$) equivalent diagrams can be drawn by inverting the arrows.

Fig.2 One particle spectrum of our schematic model. States with $\varepsilon > \mu$ are labelled by $i, j, ...$, states with $\varepsilon < \mu$ by $i', j', ...$ For each shell the energies $\tilde{\varepsilon} = \varepsilon - \mu$ are given.

Fig.3 Contributions to the level density parameter due to pp-hh RPA correlations (δa_{pp}) and ph RPA correlations (δa_{ph}) calculated in the schematic model as functions of temperature. Because each of these two contributions contain the second order term, to the total correction δa is not the sum of δa_{pp} and δa_{ph}.

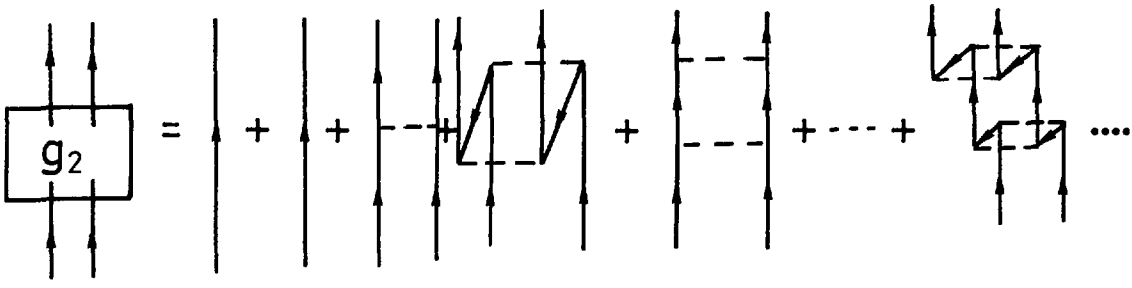
g_2
=
+
+
+
+
+ ---- +
....

Fig.1

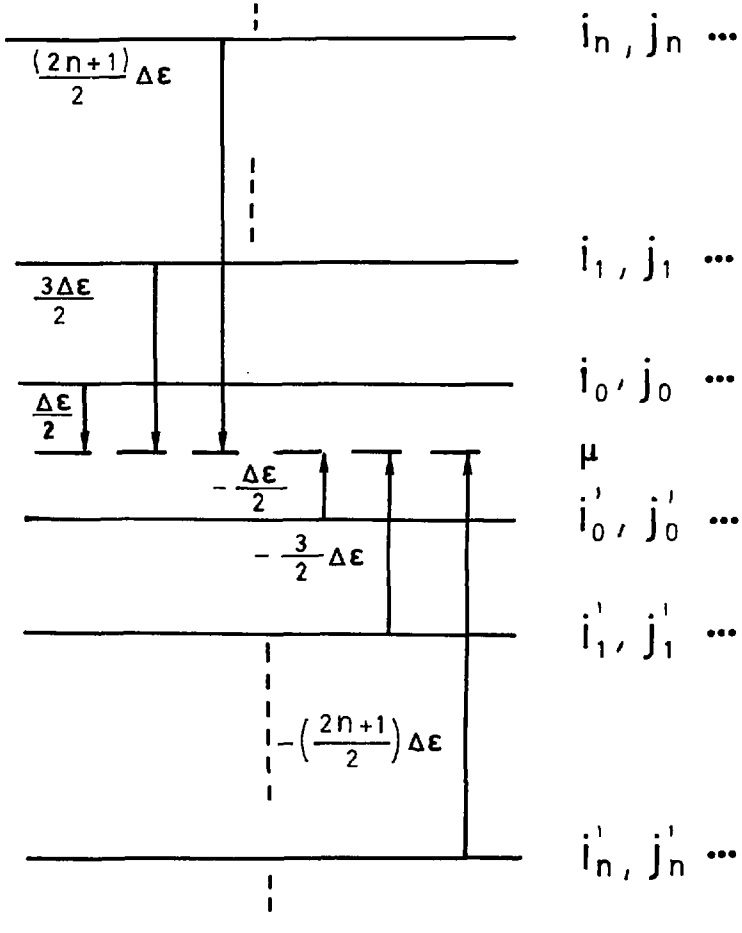
$\frac{(2n+1)}{2}\Delta\varepsilon$
i_n, j_n ...
$\frac{3\Delta\varepsilon}{2}$
i_1, j_1 ...
$\frac{\Delta\varepsilon}{2}$
i_0, j_0 ...
μ
$-\frac{\Delta\varepsilon}{2}$
i_0', j_0' ...
$-\frac{3}{2}\Delta\varepsilon$
i_1', j_1' ...
$-\left(\frac{2n+1}{2}\right)\Delta\varepsilon$
i_n', j_n' ...

Fig.2

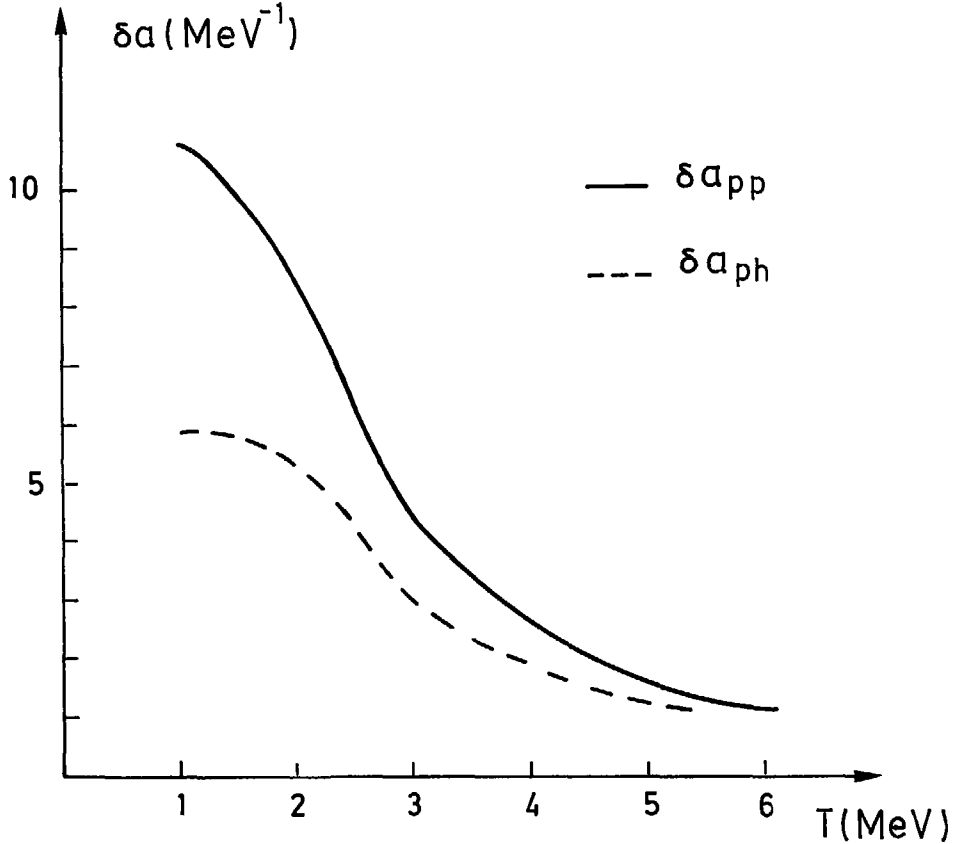

Fig.3