INSTITUTE FOR NUCLEAR STUDY
UNIVERSITY OF TOKYO
Tanashi, Tokyo 188
Japan

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Hosotani Model in Closed String Theory

Kiyoshi Shiraishi*

Institute for Nuclear Study, University of Tokyo

Midori-cho, Tanashi, Tokyo 188, Japan

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Abstract

Hosotani mechanism in the closed string theory with current algebra symmetry is described by the (old covariant) operator method. We compare the gauge symmetry breaking mechanism in a string theory which has SU(2) symmetry with the one in an equivalent compactified closed string theory. We also investigate the difference between Hosotani mechanism and Higgs mechanism in closed string theories by calculation of a four-point amplitude of "Higgs" bosons at tree level.

* JSPS fellow.

§1. Introduction

String theories[1] have given a great impact on development of unification in forces and matters in nature. The original revolution appeared in higher dimensional space-time[2,3]. A considerable number of interesting possibilities in multidimensional theories follow it by various inspections[4].

Compactification of extra spaces is utilized in such theories to generate gauge symmetries, which may govern the forces in nature[5]. This scheme is the extension of the idea of Kaluza and Klein[6]. The gauge interaction can be derived from gravity theory. A lot of works have been made on the generalized Kaluza-Klein theory, including cosmological considerations[7].

Recently, it has been clarified that string theory demands a large gauge symmetry before the compactification in general to maintain quantum consistency[3]. Thus, if we want to construct unified theory from string theory in higher dimensions, we should consider symmetry breaking mechanisms rather than generating of symmetry around the stage of the compactification. In fact, the compactification onto the manifolds[8] (or orbifolds[9]) which have no continuous or few symmetries to generate gauge symmetry is considered in such string theories.

More recently, the investigation by fermionic construction [10] and other methods[11] on consideration of (super) Kac-Moody algebra reveals the concept of four-dimensional string theory.

Those four-dimensional superstrings have often no counterpart in the compactified string theories. Most recently, however, Gepner[12] showed that some of the four-dimensional models can be interpreted as compactified theories on Calabi-Yau manifolds, which give rise to phenomenologically favored low-energy aspects [13]. Thus it can be said, in some sense, the geometrical interpretation of compactification "strikes back" in string theories.

Now let us go back to the issue of gauge symmetry breaking. The mechanisms for geometrical symmetry breaking are known at the level of field theory with extra dimensions. The so-called Wilson loop mechanism or Hosotani mechanism[14,15] is one of them; the breakdown of symmetry is caused by, roughly speaking, the "vacuum expectation value" of gauge fields on an extra, non-simply connected space. This vacuum gauge fields play a role of the order parameter in the mechanism. Simplest example for a non-simply connected manifold is a circle, S^1. The model with extra S^1 is originally investigated by Hosotani[15] and by virtue of its simplicity, many authors made persuit on the Hosotani model from various points of view[16,17]. Of course, the knowledge for realistic breaking pattern cannot be extracted from the simplest model, but it is believed that we can examine qualitative properties of the mechanism by the study of the model with the extra space S^1. From a cosmological viewpoint, phase transition in the universe is an interesting subject to investigate[18]. The present author studied the one-loop free energy of the Hosotani model with several sorts of matter fields at zero and finite temperature, although no remarkable dependence on temperature was found[17]. Similar analysis was made for a model in open string theory and almost the same conclusion was drawn[19]. In the case of open strings, we anticipate the similarity to the "particle" case, since the interaction with external gauge fields is restricted at the edges of the world sheet, which behave like a trace of a moving particle.

We expect purely "stringy" effect on dynamical determination of the order parameters to discover interesting behaviors in spontaneous symmetry breaking which cannot be obtained in models mentioned above. Thus we hope to look into the symmetry breaking mechanism in closed string theory. Indeed the subject of current research concentrate on the closed string theory, which is often associated even with phenomenological study[8,20]. This is another reason why we investigate the mechanism in closed string theory.

In this paper we will set up a framework for description of the symmetry breaking mechanism in closed string theory. We suppose that the treatment of the Hosotani mechanism is easy in the operator formalism[1]. We adopt a heuristic introduction of vacuum gauge fields. The dynamical determination of the order parameter by quantum effects is left to be examined in future publications.

This paper is organized as follows. In section 2 we review the old (-fashioned) operator method and we describe torus compactification in closed string theory by the method. In section 3, firstly we introduce gauge symmetry into closed string theory by current algebra in a usual way[1]. Secondly using an analogy with the torus compactification, we formulate Higgs mechanism in the theory. The Hosotani mechanism in closed string theory is described in section 4. As an exercise, we give an evaluation of a four-scalar scattering amplitude in the model. We also calculate the *same* amplitudes in Higgs model explained in the section 3. It is shown that the difference can be seen in the amplitude even if the masses of scalar bosons are identical in both models. The last section is devoted to discussion.

In this paper we consider bosonic strings only. We set $\alpha'=1/2$ throughout this paper.

§2. Torus compactification in closed string theory

We begin with a review of the operator approach to string theory[1]. First we introduce the Virasoro generators which is constructed from the "harmonic oscillator" operators associated with excitation of string modes:

$$L_n = \frac{1}{2}\sum_{m=-\infty}^{\infty} :\alpha_{n-m}\cdot\alpha_m: \quad . \tag{2.1}$$

Here the "normal order" should be taken as

$$\begin{aligned} :\alpha_n^\mu\ \alpha_m^\nu: &= \alpha_n^\mu\ \alpha_m^\nu \qquad \text{when}\quad n<m \\ &= \alpha_m^\nu\ \alpha_n^\mu \qquad \text{when}\quad n>m \quad . \end{aligned} \tag{2.2}$$

By the virtue of this treatment, L_0 is simply written as

$$L_0 = \frac{1}{2}P^2 + \sum_{m=1}^{\infty} :\alpha_{-m}\cdot\alpha_m: \quad , \tag{2.3}$$

where we rewrite α_0 as P .

In closed string theory we must prepare a set of copies of Virasoro generators $\bar{L}_n$, which are expressed in right moving oscillator modes $\bar{\alpha}_m$.

Next we adopt a tachyonic state $|0>$ as usual. It satisfies

$$L_0|0> = \bar{L}_0|0> = |0> \quad . \tag{2.4}$$

Now, since we wish to investigate gauge symmetry breaking in subsequent sections, we ought to pay attention to the masses of the light fields.

The "mass operator" is defined as

$$M^2 = 4(L_0 + \bar{L}_0 - 2) \ , \tag{2.5}$$

for closed strings. The quartation marks imply that this operator gives masses for any external string state whose momentum is set equal to zero.

In the closed strings, this mass operator is always accompanied with the following mass-matching condition:

$$(L_0 - \bar{L}_0)|\cdot\rangle = 0 \ , \tag{2.6}$$

where $|\cdot\rangle$ is an external physical state. The physical states are expected to be made from the "creation" operators α_{-n} (n>0) applied on the tachyonic state. To make a complete account for physical string states, we necessarily mention on the existence of spurious states[1]. Nevertheless, we put off the painfulness on them in this paper and we only remark that the construction of spurious states is related with the Virasoro algebra.

Now consider compactification of one dimension to a circle, S^1. We denote the dimension as the I-th direction and the other directions are labelled by the index i. Momenta of strings are discretized in the I-th direction. In closed strings, it is well known that there are windings on the torus; these are also specified by an integer. These quantum numbers are rearranged according to momenta of left and right moving sectors of strings;

$$P_L{}^I = \frac{\ell'}{2R} + mR$$
$$\text{and} \qquad (\ell', m : \text{integers}) \tag{2.7}$$
$$P_R{}^I = \frac{\ell'}{2R} - mR \quad ,$$

where R is the radius of S^1. ℓ' and m represent descreteness of momentum on S^1 and winding number around S^1, respectively. The following two quantities will be important in later analysis.

$$\frac{1}{2}\{(P_L{}^I)^2+(P_R{}^I)^2\} = \left(\frac{\ell'}{2R}\right)^2 + (mR)^2 \ , \tag{2.8}$$

and

$$\frac{1}{2}\{(P_L{}^I)^2-(P_R{}^I)^2\} = \ell' m \ . \tag{2.9}$$

(2.8) plus the contribution from a number of string oscillators is just the mass of a certain external state. (2.9) is closely connected with the mass-matching condition on each string excitation level.

Let us consider a linear transformation in P^I's. For instance, suppose the following mixing:

$$\begin{pmatrix} P_L{}^{I'} \\ P_R{}^{I'} \end{pmatrix} = \begin{pmatrix} \cosh\alpha & \sinh\alpha \\ \sinh\alpha & \cosh\alpha \end{pmatrix} \begin{pmatrix} P_L{}^I \\ P_R{}^I \end{pmatrix}, \tag{2.10}$$

where α is a constant. Then, the quantities (2.8) and (2.9) become

$$\frac{1}{2}\{(P_L{}^{I'})^2+(P_R{}^{I'})^2\} = \left(\frac{\ell'}{2R}\right)^2 e^{2\alpha} + (mR)^2\, e^{-2\alpha}, \tag{2.11}$$

and

$$\frac{1}{2}\{(P_L{}^{I'})^2-(P_R{}^{I'})^2\} = \ell' m \ , \tag{2.12}$$

under the transformation (2.10).

As seen from (2.11), this transformation gives rise to the variation of the radius of S^1 according to

$$R \to R' = R\, e^{-\alpha}. \tag{2.13}$$

Note that the combination (2.12) (or (2.9)) is invariant under the mixing of this type. The exercise above can be regarded as a very primitive example for a general lattice compactification considered by many authors[11]. The information of the momentum lattice is usually assumed to be carried by external states. But we can think another possible formulation to describe this.

We consider a perturbation on the Virasoro operator. In the case mentioned above, we define two operators L_0' and L_0' as follows:

$$\begin{aligned} L_0' &= L_0 - \frac{1}{2}(P_L{}^I)^2 + \frac{1}{2}(P_L{}^{I\prime})^2 \\ \bar{L}_0' &= \bar{L}_0 - \frac{1}{2}(P_R{}^I)^2 + \frac{1}{2}(P_R{}^{I\prime})^2 , \end{aligned} \tag{2.14}$$

where $P^{I\prime}$ is to be considered as an operator and is defined as (2.10). Thus the states don't have to carry all the information about the compactification. In the present case, it is sufficient that states have information of a fixed-radius compactification, say, $R = 1/\sqrt{2}$. The value of the radius R is attributed to $P^{I\prime}$ in L_0' of (2.14).

The momentum in an extra space in string theory can be given in two equivalent ways above. Eventually, replacement $P_{L,R}{}^I \to$

$p_{L,R}{}^{I\prime}$ in L_n's produces sets of Virasoro generators L_n' and $\bar{L}_n'$ which obey the same Virasoro algebra as L_n.

The method described in this section will be extended to the case with current-algebra symmetry in closed strings in the subsequent sections.

§3. Higgs mechanism

First of all, we introduce current algebra into the closed string theory[1]. The current which has an index of the adjoint representation can be analysed by the mode expansion on the world-sheet coordinate:

$$J^a(z) = \sum_{-\infty}^{\infty} J_n{}^a z^{-n-1}, \tag{3.1}$$

The coefficients as the operators satisfy the commutation relations:

$$[J_m{}^a, J_n{}^b] = if^{abc}J_{m+n}{}^c + \frac{k}{2}m\delta^{ab}\delta_{m+n,0}, \tag{3.2}$$

where the level of the Kac-Moody algebra k is a constant. Gauge symmetry arises from this (affine) Kac-Moody algebra. The zero modes $J_0{}^a$ form a Lie algebra G with the relation of the generators

$$[J_0{}^a, J_0{}^b] = if^{abc}J_0{}^c, \tag{3.3}$$

where f^{abc} are the structure constants of G. We equip another current $\bar{J}(\bar{z})$ for the right moving modes. Therefore we can describe the gauge symmetry $G\times\bar{G}$ in low-energy sector of the theory. In this paper we assume only the case with $\bar{G}\sim G$. The massless spectrum consists of not only the graviton, antisymmetric tensor field and dilaton but also non-abelian vector bosons and scalars. The string states $J_{-1}{}^a\tilde{\alpha}_{-1}{}^i|0\rangle$ and

$\alpha_{-1}{}^{i}\bar{J}_{-1}{}^{a}|0\rangle$ correspond to the gauge bosons which transform as the (adj,1) and (1,adj) representation. of $G\times\bar{G}$. $J_{-1}{}^{a}\bar{J}_{-1}{}^{b}|0\rangle$ corresponds to a multiplet of scalars of the (adj,adj) representations[21,22]. Here and in the following we choose SU(2) as symmetry group G for simplicity. Moreover, we take Cartan-Weyl basis, and then the adjoint index a={1,2,3} is reformed to {+,-,3}. Namely, (3.3) yields $[J_0{}^{+},J_0{}^{-}] = 2J_0{}^{3}$, $[J_0{}^{3},J_0{}^{+}] = J_0{}^{+}$, and so on. The (unperturbed) Virasoro operator is expressed as[21,22,23,24]

$$L_0 = \frac{1}{2}(P_L)^2 + \frac{1}{2}\Sigma :\alpha_{-n}\cdot\alpha_n: + \frac{1}{K}\sum_n :J_{-n}{}^{a}J_n{}^{a}:, \qquad (3.4)$$

where $K= c_v+k$. c_v is defined by the relation $f^{acd}f^{bcd}=c_v\delta^{ab}$. $\bar{L}_0$ is written in an expression similar to (3.4).

Now we try to examine gauge symmetry breaking of this model in analogy with the example illustrated in the preceding section. We are going to take following principles. First, we use the Fourier components of original string modes and the Kac-Moody currents as building-blocks of new Virasoro operators L_0' and $\bar{L}_0'$. Second, the perturbed Virasoro operators L_0' and $\bar{L}_0'$ should involve continuous parameters which indicate the deviation from the original L_0 and $\bar{L}_0$. Third, low-lying external states, at least, are found trivially. The other points to be required will be clarified through the construction below.

The concept of perturbed operators was considered by authors of ref.[25] in a different context.

Similarly to the case with momenta on a circle, we consider following transformation among zero modes of currents belonging to Cartan subalgebra:

$$J_0^{3\prime} = \cosh\beta\ J_0^3 + \sinh\beta\ \bar{J}_0^3$$
$$\bar{J}_0^{3\prime} = \cosh\beta\ \bar{J}_0^3 + \sinh\beta\ J_0^3, \tag{3.5}$$

where $J_0^a = \frac{1}{2\pi i}\oint dz\ J^a(z)$, as previously; β is a parameter of perturbation.

Proceeding along this line, we can construct new L_0' and $\bar{L}_0'$;

$$L_0' = L_0 + A\{(J_0^{3\prime})^2-(J_0^3)^2\}$$
$$\bar{L}_0' = \bar{L}_0 + A\{(\bar{J}_0^{3\prime})^2-(\bar{J}_0^3)^2\} \quad , \tag{3.6}$$

where A is a constant to be determined. This choice of the new Virasoro operators makes the left-right level matching unchanged (for any A),

$$L_0' - \bar{L}_0' = L_0 - \bar{L}_0 \quad . \tag{3.7}$$

The constant A is hence determined by consideration of other Virasoro generators. They are naturally defined as

$$L_n' = L_n + B\{J_0^{3\prime}J_n^3 - J_0^3J_n^3\}$$
$$\bar{L}_n' = \bar{L}_n + B\{\bar{J}_0^{3\prime}\bar{J}_n^3 - \bar{J}_0^3\bar{J}_n^3\} \quad , \tag{3.8}$$

where B is a constant.

We assume that the new Virasoro generators (3.6) obey the same form of the Virasoro Algebra

$$[L_n', L_m'] = (n-m)L_{n+m}' + (\text{the central term}) \ . \tag{3.9}$$

Then the followings are required:

$$A = 1/k \quad \text{and} \quad B = 2/k \ . \tag{3.10}$$

Incidentally, the case with torus compactification is regained formally by supposing that the gauge group is abelian and $k = 2$.

Now the mass-shifts for charged states are given by

$$\delta M^2{}_{op} = \frac{4}{k}\,[2(\sinh\beta)^2\{(J_0{}^3)^2+(\bar{J}_0{}^3)^2\}+4(\sinh\beta\cdot\cosh\beta)J_0{}^3\bar{J}_0{}^3], \tag{3.11}$$

where $J_0{}^3$ and $\bar{J}_0{}^3$ act as "charge" operators. Obviously, gauge bosons ($J_{-1}^{\pm}\bar{\alpha}_{-1}{}^i|0\rangle$ and $\bar{J}_{-1}^{\pm}\alpha_{-1}{}^i|0\rangle$) and scalar bosons ($J_{-1}^{\pm}\bar{J}_{-1}^{\pm}|0\rangle$) are found by applying (3.11) to be massive when β is slightly different from zero.

The physical meaning of this symmetry breaking can be read as follows[26]. The vertex operator for a scalar boson at zero momentum is written in the form:

$$\phi^{ab} \sim \int dz d\bar{z}\ J^a(z)\bar{J}^b(\bar{z}) \ . \tag{3.12}$$

Therefore the second term proportional to $\sinh\beta\cosh\beta$ in (3.11) can be taken as the effect of condensation of zero-mode of ϕ^{33}. The "vacuum parameter" β indicates the magnitude of the order parameter $\langle\phi^{33}\rangle$ in some nonlinear manner. In this way, we can deal with a spontaneously broken (SU(2)) gauge theory with massive gauge and Higgs bosons.

It is well known that, on the other hand, a level one (k=1) SU(2)×SU(2) Kac-Moody symmetry can be generated from the left and right moving modes of strings on a circle S^1[27]. To see this, we only examine the left-moving algebra formed by the currents defined as[28]

$$\begin{aligned} J^+(z) &= :\exp(+i\sqrt{2}X_L{}^I) \\ J^-(z) &= :\exp(-i\sqrt{2}X_L{}^I) \\ J^3(z) &= \frac{i}{\sqrt{2}}\,\partial_z X_L{}^I \quad , \end{aligned} \tag{3.13}$$

where $X_L{}^I$ is the left-moving mode of strings on S^1. If the radius of the circle R is set to $1/\sqrt{2}$, they form SU(2) Kac-Moody algebra. If the value of R deviates from $1/\sqrt{2}$, the SU(2) symmetry will become broken and only U(1) symmetry will be left, which trivially exists in a model with a circle.

Alternatively, we can consider the background metric in path integral as

$$G_{\mu\nu} = (-1,1,1,\ldots,R^2) \quad , \tag{3.14}$$

and we take the string coordinate $X^I(R=1/\sqrt{2})$. This results in the same partition functions as before. Thus we can regard R^2 as a zero mode of Kaluza-Klein scalar field; but in the string case, this Kaluza-Klein scalar is interacting with the stringy excitations which form SU(2)-adjoint scalar, and vector fields, and other massive fields. The deviation of R from $1/\sqrt{2}$ can be interpreted as the expectation value of a scalar field.

Thus one can check the mass spectrum in the "Higgs" mechanism by comparison in each other model. The inclusion of background fields is well described in path integral approach[29,30]. For our purpose, we consider zero-mode part of the partition function, particularly. The zero-mode piece of the bosonic string coordinate on S^1 is of the form

$$\bar{X}^I = 2\pi R(m\sigma_1 + \ell\sigma_2) \ , \tag{3.15}$$

where σ_1 and σ_2 are coordinates of the world sheet and m and ℓ are integers[29]. Here we take the notion $G_{\mu\nu} = (-1,1,\ldots,1)$ as in the former case above. Then the integrand of the partition function is proportional to

$$\begin{aligned} &\sum_{\ell m} e^{-S} \\ &= \left(\frac{\tau_2}{2R^2}\right)^{1/2} \sum_{\ell' m} \exp\left[-\pi\tau_2\left(\frac{\ell'^2}{2R^2} + 2m^2R^2\right) + 2\pi i\tau_1 m\ell'\right] \ . \end{aligned} \tag{3.16}$$

We can read the mass spectrum from this expression, i.e.,

$$M^2 = 2\left[\frac{\ell'^2}{2R^2} + 2m^2R^2 - 4\right] + (\text{oscillators}) \ , \tag{3.17}$$

where -4 in the parentheses comes from tachyon in bosonic strings. ℓ' and m are integers. The last term including τ_1 in (3.17) is concerned with the mass-matching constraint such as (2.9). In order to compare with our model, we define

$$\begin{aligned} Q_L &\equiv \frac{1}{\sqrt{2}}(\ell' + m) \\ Q_R &\equiv \frac{1}{\sqrt{2}}(\ell' - m) \end{aligned} \tag{3.18}$$

and further, we set

$$
\begin{aligned}
Q_L' &= \cosh\beta\, Q_L + \sinh\beta\, Q_R \\
Q_R' &= \cosh\beta\, Q_R + \sinh\beta\, Q_L
\end{aligned} \quad , \qquad (3.19)
$$

where $\sqrt{2}R = \exp(-\beta)$. Using this set of variables, (3.17) is rewritten as

$$
M^2 = 4\{(Q_L')^2+(Q_R')^2-2\} + (\text{oscillators}) \quad . \qquad (3.20)
$$

Thus the mass shift is given by

$$
\delta M^2 = 4[(\sinh\beta)^2\{(Q_L)^2+(Q_R)^2\}+2(\sinh\beta\cosh\beta)Q_L Q_R] \quad . \qquad (3.21)
$$

It seems that we may identify Q_L and Q_R with the eigen values of charge operators $\sqrt{2/k}\, J_0{}^3$ and $\sqrt{2/k}\, \bar{J}_0{}^3$ with $k=1$, respectively. Indeed, one can easily check the case for massless external states. Namely, the gauge bosons $J_{-1}^{\pm}\bar{\alpha}_{-1}{}^i|0>$ have $Q_L=\pm\sqrt{2}$ and $Q_R=0$, whilst $\bar{J}_{-1}^{\pm}\alpha_{-1}{}^i|0>$ have $Q_L=0$ and $Q_R=\pm\sqrt{2}$. The scalar Higgs bosons $J_{-1}^{\pm}\bar{J}_{-1}^{\pm}|0>$ also acquire masses. We are safely to say that we can construct a modular invariant partition function with Higgs mechanism in $SU(2)\times SU(2)$ gauge theory, because of the correspondence with torus compactification when $k=1$.

§4. Hosotani model in closed strings

Now let us consider Hosotani model in the closed string theory. The model we consider has SU(2)×SU(2) symmetry at first, in the same fashion as discussed in sec. 3, as well as an compact extra-space S^1. First of all, we give a specific example for a mixed transformation among internal momenta and charge operators. That is,

$$\begin{pmatrix} P_L{}^{I\prime} \\ Q_L{}' \\ P_R{}^{I\prime} \\ Q_R{}' \end{pmatrix} = \begin{pmatrix} \cosh\theta & 0 & 0 & \sinh\theta \\ 0 & \cosh\theta & \sinh\theta & 0 \\ 0 & \sinh\theta & \cosh\theta & 0 \\ \sinh\theta & 0 & 0 & \cosh\theta \end{pmatrix} \begin{pmatrix} P_L{}^{I} \\ Q_L \\ P_R{}^{I} \\ Q_R \end{pmatrix} , \qquad (4.1)$$

where $Q_L = \sqrt{2/k}\, J_0{}^3$ and $Q_R = \sqrt{2/k}\, \bar{J}_0{}^3$, and the notations are same as before, but the number of large dimensions is less than that in the Higgs model treated in sec. 3 by one. Subsequently, we construct the Virasoro operators as follows:

$$\begin{aligned} L_0{}' &= L_0 + \frac{1}{2}\{(Q_L{}')^2 + (P_L{}^{I\prime})^2 - (Q_L)^2 - (P_L{}^{I})^2\} \\ \bar{L}_0{}' &= \bar{L}_0 + \frac{1}{2}\{(Q_R{}')^2 + (P_R{}^{I\prime})^2 - (Q_R)^2 - (P_R{}^{I})^2\} . \end{aligned} \qquad (4.2)$$

This example is clearly found to be a straightforward extension of the previous models in sec. 2 and 3. The Virasoro generators L_n can also be constructed in a similar method.

Generally speaking, a transformation matrix T which belongs to SO(2,2) can be utilized in the model of this tvpe. Namely, if, by such a matrix the relation

$$ {}^{t}T \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} T = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} , \tag{4.3} $$

is satisfied, then the condition $L_0'-\bar{L}_0'=L_0-\bar{L}_0$ holds by use of T as the transformation matrix in (4.1). As is known, the number of parameters which characterize a general T is six. But transformations which belong to a subgroup $SO(2)_L \times SO(2)_R$, for instance, the rotation

$$ \begin{pmatrix} P_L{}^{I\prime} \\ Q_L{}' \\ P_R{}^{I\prime} \\ Q_R{}' \end{pmatrix} = \begin{pmatrix} \cos\phi_L & -\sin\phi_L & 0 & 0 \\ \sin\phi_L & \cos\phi_L & 0 & 0 \\ 0 & 0 & \cos\phi_R & -\sin\phi_R \\ 0 & 0 & \sin\phi_R & \cos\phi_R \end{pmatrix} \begin{pmatrix} P_L{}^{I} \\ Q_L \\ P_R{}^{I} \\ Q_R \end{pmatrix} , \tag{4.4} $$

leaves each Virasoro operator unchanged:

$$ L_0' = L_0 \quad \text{and} \quad \bar{L}_0' = \bar{L}_0 \ . \tag{4.5} $$

In other words, the transformation such as (4.4) makes no change in the mass spectrum in the string model. As long as symmetry breaking is concerned, the number of parameters which govern the whole mass spectrum, is four. Again, in these four parameters, the two correspond to the mechanism considered before, in sec. 2 and 3; one is concerned with the variation of the size of S^1 and one is concerned with the Higgs mechanism. Thus the number of the parameters which "describe" the geometrical symmetry breaking is two. One of two parameters is, say, θ in (4.1). The last one will be discussed later, because we wish to clarify the physical implication about the example, first of all.

To see the physical interpretation, we think about the difference in mass operators:

$$
\begin{aligned}
&(L_0{}'+\bar{L}_0{}')-(L_0+\bar{L}_0)\\
&= (\sinh\theta)^2\{(P_L{}^I)^2+(P_R{}^I)^2+(Q_L)^2+(Q_R)^2\}\\
&\quad +2(\sinh\theta\cosh\theta)(P_LQ_R+Q_LP_R)\,] \quad . \qquad (4.6)
\end{aligned}
$$

In the limit of small θ, this reduces to

$$(2\theta)\left(\frac{2}{k}\right)^{1/2}(P_L{}^I\bar{J}_0{}^3+P_R{}^IJ_0{}^3) \;\equiv\; -V_0 \;. \qquad (4.7)$$

This expression can be regarded as the "zero-mode" of the vertex, of vector bosons:

$$\left(\frac{2}{k}\right)^{1/2}(\bar{J}(\bar{z})\partial X_L{}^I(z)+J(z)\bar{\partial}X_R{}^I(\bar{z}))e^{ikx}. \qquad (4.8)$$

In addition, when we analyze the new propagator with $\theta\neq0$, we can give the following expansion in terms of the original propagator ($\theta=0$), up to the left-right matching constraint:

$$
\begin{aligned}
\frac{1}{L_0{}'+\bar{L}_0{}'-2} &= \frac{1}{L_0+\bar{L}_0-2} + \frac{1}{L_0+\bar{L}_0-2}V_0\frac{1}{L_0+\bar{L}_0-2}\\
&\quad + \frac{1}{L_0+\bar{L}_0-2}V_0\frac{1}{L_0+\bar{L}_0-2}V_0\frac{1}{L_0+\bar{L}_0-2} + \cdots \quad . \qquad (4.9)
\end{aligned}
$$

This shows the insertion of interactions with zero-modes of gauge fields on S^1. Therefore we can say that the mechanism described above is a simplest example of the Hosotani mechanism in closed string theory. It can be also said that in this example the scale of the vacuum gauge field is given by

$$\langle A_I{}^3 \rangle \sim (-2\theta) \quad , \tag{4.10}$$

in the small θ limit. $A_I{}^3$ denotes a certain combination of zero modes of two gauge fields.

Now let us turn to examine scattering amplitudes in this framework. A tree amplitude of four particles contains poles associated with the intermediate states[1]. Thus we can obtain much knowledge about the mass spectrum and we can compare the amplitude with that in other models such as Higgs model. For simplicity we show a scattering amplitude of four doubly-adjoint scalar bosons:

$$\phi^{+3} + \phi^{33} \to \phi^{+3} + \phi^{33} \quad . \tag{4.11}$$

The superscripts indicate the left and right SU(2) "charge" classified as in sec. 3. The external states are represented as

$$\phi^{+3} \sim \frac{\sqrt{2}}{k} \, J_{-1}^{+} \, \bar{J}_{-1}{}^{3} |0, P^I = 0\rangle \quad , \tag{4.12}$$

and the ϕ^{33}-emission vertex is determined to be

$$V'^{33}(z = \bar{z} = 1) = \frac{2}{k} \left(\sum_n J_n{}^3 - J_0{}^3 + J_0{}^{3\prime}\right)\left(\sum_n \bar{J}_n{}^3 - \bar{J}_0{}^3 + \bar{J}_0{}^{3\prime}\right) e^{ikX(1)}, \tag{4.13}$$

by investigation of the ghost-decoupling condition[1].

Using these preparations, we can calculate the four-point amplitude of scalars and this reduces to:

$$A(\theta) = \frac{\kappa^2}{4} \times$$

$$\left\{ \left\{ \frac{\Gamma(-s/8+m^2/8+1)\Gamma(-t/8-1)\Gamma(-u/8+m^2/8+1)}{\Gamma(s/8-m^2/8)\;\Gamma(t/8+2)\;\;\Gamma(u/8-m^2/8)} \right.\right.$$

$$+\frac{\Gamma(-s/8+m^2/8+1)\Gamma(-t/8-1)\Gamma(-u/8+m^2/8-1)}{\Gamma(s/8-m^2/8)\quad\Gamma(t/8)\quad\Gamma(u/8-m^2/8)}$$

$$+\left.\frac{\Gamma(-s/8+m^2/8-1)\Gamma(-t/8-1)\Gamma(-u/8+m^2/8+1)}{\Gamma(s/8-m^2/8)\quad\Gamma(t/3)\quad\Gamma(u/8-m^2/8)}\right\}$$

$$+\frac{2}{k}(\cosh\theta)^2\left\{-\frac{\Gamma(-s/8+m^2/8)\Gamma(-t/8-1)\Gamma(-u/8+m^2/8)}{\Gamma(s/8-m^2/8)\quad\Gamma(t/8)\quad\Gamma(u/8-m^2/8)}\right.$$

$$+\frac{\Gamma(-s/8+m^2/8)\Gamma(-t/8+1)\Gamma(-u/8+m^2/8-1)}{\Gamma(s/8-m^2/8)\quad\Gamma(t/8)\quad\Gamma(u/8-m^2/8+1)}$$

$$+\left.\left.\frac{\Gamma(-s/8+m^2/8-1)\Gamma(-t/8+1)\Gamma(-u/8+m^2/8)}{\Gamma(s/8-m^2/8+1)\quad\Gamma(t/8)\quad\Gamma(u/8-m^2/8)}\right\}\right], \tag{4.14}$$

where κ^2 is the coupling in closed strings and $m^2(\theta)=8/k(\sinh\theta)^2$. In the amplitude (4.14), the exchange of tachyon and graviton appears in the first term in the first parentheses. This term has poles at $t = -8, 0, \cdots$. The gauge interaction and interaction among charged particles are contained in the second parentheses. For instance, the last term describes the contribution of massive gauge bosons to intermediate states. The term contains poles in the s-channel at $s = m^2, m^2+8, \cdots$. Note that the ratio of couplings of gauge boson and graviton exchange at tree level turns out to be

$$\frac{2}{k}(\cosh\theta)^2 . \tag{4.15}$$

The k-dependence has been mentioned by Ginsparg[31]; a new feature is the dependence of effective coupling on the "vacuum parameter" θ.

We can also carry out calculation of the same amplitude in the Higgs model introduced in sec. 3. The result is

$$A_{Higgs}(\beta) = A(\theta\to\beta)$$
$$+\ \frac{2}{k}\ (\sinh\beta)^2\left\{-\ \frac{\Gamma(-s/8+m^2/8)\Gamma(-t/8-1)\Gamma(-u/8+m^2/8)}{\Gamma(s/8-m^2/8)\ \ \Gamma(t/8)\ \ \Gamma(u/8-m^2/8)}\right\}$$
$$+\left(\frac{2}{k}\right)^2(\sinh\beta)^2(\cosh\beta)^2\left\{\frac{\Gamma(-s/8+m^2/8)\Gamma(-t/8+1)\Gamma(-u/8+m^2/8)}{\Gamma(s/8-m^2/8+1)\Gamma(t/8)\ \Gamma(u/8-m^2/8+1)}\right\}, \tag{4.17}$$

where $m^2=8/k(\sinh\beta)^2$. We find that the amplitudes slightly differ between two models, even though the masses of gauge bosons are same. This difference of course is due to the ways of coupling to the background fields, i.e., gauge and scalar bosons. The last term in the additional terms in (4.17) contains two four-scalar couplings and two Higgs background fields $<\phi^{33}>$ in scattering of low-lying states.

So far we study Hosotani model through an example (4.1) which includes only one vacuum parameter. We must return to investigate another freedom in modification of mass spectrum. Needless to say, two parameters originate from (the combinations of) two vacuum gauge fields associated with $SU(2)_L$ and $SU(2)_R$ symmetry. Again we give another example for taking the parameter:

$$\begin{pmatrix} P_L{}^{I\prime} \\ Q_L{}' \\ P_R{}^{I\prime} \\ Q_R{}' \end{pmatrix} = \begin{pmatrix} 1 & B/2 & 0 & -B/2 \\ -B/2 & 1 & B/2 & 0 \\ 0 & B/2 & 1 & -B/2 \\ -B/2 & 0 & B/2 & 1 \end{pmatrix} \begin{pmatrix} P_L{}^I \\ Q_L \\ P_R{}^I \\ Q_R \end{pmatrix}, \tag{4.18}$$

where B is the new vacuum parameter. Though the matrix in (4.18) looks like a bizarre one, this rotation matrix evidently obeys

SO(2,2) symmetry. And multiplication of the matrices of the same form defines a small group. The investigation of the mass operator with this parameter reveals a possibility that eight massless vector bosons emerge when B=1. Probably these bosons take a form of some mixture of SU(3) gauge fields. These enlargement of symmetry can be understood, when k=1, using the equivalent torus compactification[27,32]. In this case, we must employ a two-torus as an internal space. Let us assume background fields, that is to say, metric and antisymmetric tensor fields on the torus, as

$$G_{mn} = \begin{pmatrix} \cosh 2\theta & -\sinh 2\theta \\ -\sinh 2\theta & \cosh 2\theta \end{pmatrix}, \tag{4.19}$$

and

$$B_{mn} = \begin{pmatrix} 0 & B \\ -B & 0 \end{pmatrix}, \tag{4.20}$$

where m and n are the indices of the two-torus; θ and B are constants. Note that $\det G_{mn}=1$. When we consider the partition function of the compactified model, we can read the mass spectrum from the path-integral form. The comparison will be made in a similar manner as in sec. 3, but now we have two compactified dimensions; both radii of two scales are set to R=1. First we can construct the eigenvalue of the "charge" from a set of quantized numbers, of momentum and winding, in the direction of one dimension. Next, we interpret that another dimension will be remained as a one-torus, that is, a circle. There the notation of the left-right momenta is maintained. In consequence, we obtain a model which can be compare with the Hosotani model in the operator formulation.

In this situation mentioned above, one can calculate the partition function by path integral method and after slightly tedious rearrangement using Jacobi's transformation, we find the mass spectrum:

$$M^2 = \frac{1}{2}\{(P_L{}^{I\prime})^2+(P_R{}^{I\prime})^2+(Q_L{}^{\prime})^2+(Q_R{}^{\prime})^2\} + (\text{oscillators}). \quad (4.21)$$

We have only to know how we can construct momenta P′ and charges Q′ at finite θ and B from these at θ=B=0 (denoted as P and Q). The relation turns out to be:

$$\begin{pmatrix} P_L{}^{I\prime} \\ Q_L{}^{\prime} \\ P_R{}^{I\prime} \\ Q_R{}^{\prime} \end{pmatrix} = \begin{pmatrix} \cosh\theta & 0 & 0 & \sinh\theta \\ 0 & \cosh\theta & \sinh\theta & 0 \\ 0 & \sinh\theta & \cosh\theta & 0 \\ \sinh\theta & 0 & 0 & \cosh\theta \end{pmatrix} \begin{pmatrix} 1 & B/2 & 0 & -B/2 \\ -B/2 & 1 & B/2 & 0 \\ 0 & B/2 & 1 & -B/2 \\ -B/2 & 0 & B/2 & 1 \end{pmatrix} \begin{pmatrix} P_L{}^{I} \\ Q_L \\ P_R{}^{I} \\ Q_R \end{pmatrix} \quad (4.22)$$

The correspondence of parameters is apparent. The symmetry enhancement (e.g., when θ=0 and B=1) can be explained, by explicit construction of currents from bosonic coordinates similarly to (3.13) in the compactified model. As is well known the maximal symmetry group has the rank equal to the dimension of the torus. Note, however, that in our model in operator method, the maximal symmetry is a single SU(3), not SU(3)×SU(3): the latter is attained by the previous two-torus compactification. There is not one-to-one correspondence between the spectrum of the strings with currents and that of the naive compactified string model. If we want exactly same partition functions, we need an appropriate projection in compactification of the background. Anyway, the occurrence of symmetry enhancement is a purely "stringy" effect.

This feature clearly originates from condensation of the antisymmetric tensor field, as is manifestly shown in this parametrization; though this is less obvious when we take a general way of parametrization of the vacuum parameters.

§5. Discussion

In this paper we showed the method to describe the gauge symmetry breaking mechanism, especially *Hosotani mechanism*, in bosonic string theory by heuristic introduction of modified Virasoro operators. And also the comparison with associated models of compactified strings was made.

We have left two problems to study further. One is on the derivation of Virasoro generators from "first principles" such as the consideration of stress tensor in two dimensional theory. The study of Wess-Zumino-Witten models will help the investigation of some algebraic structure when background fields exist. We particularly expect the progress in the study of the case with $k>1$. Reconciliation with conformal-field theorists may be in need!

Apart from the trivial extension to general symmetry groups, another problem is the application to the supersymmetric model. Based on such a model, we can examine the dynamical determination of vacuum parameters and finite temperature effect on it because of the disease of tachyon in bosonic strings. Even if the model will not be realistic, we can look into the stringy effect on the mechanism. In particular, the enlargement of the gauge group is anticipated in some models. In the supersymmetric case, projections on to physical states are essential and then the symmetry enhancement mechanism may not have a straightforward generalization. Nevertheless we are convinced of the interesting aspects, particularly, in the thermal property of the model, because of the possible change of light degrees of freedom. In this respect, we hope that the consideration of symmetry breaking (or enhancement) mechanisms in string theory bring a new perspective to string cosmology.

Finally, we wonder how Hosotani model on torus can be extended to Wilson loop mechanism on general nonsimply connected manifolds. Perhaps an analogous construction of Wilson loops can be performed in terms of the description of strings on Calabi-Yau manifolds by Gepner[12]. In such a model, if possible, vacuum parameters will be no longer continuous quantity and only discrete numbers will be permitted.

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