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Baby Universes, Fine Tuning Problems — A Theory of Everything Robbing the Throne by Killing the Rivals —*

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*Lecture Note based on the Talk presented at the XXII International Symposium on the Theory of Elementary Particles, Ahrenshoop, DDR, October 17-21, 1988

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Abstract

We review the recently popular "theory of baby universes" put forward by Banks, Coleman and Hawking. We then derive the strong CP breaking coefficient $\bar{\theta}$ to be very small, in a similar manner to the derivation of the cosmological constant being zero. A solution for an old controversy concerning the entropy creation in black holes is also discussed. We finally confront the baby universe theory with random dynamics. We conclude that the theory of baby universes is so successful that the essential features are likely true and might have to go into a right theory even if with some troubles at first.

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Section 1. Introduction

Recently a new look on quantum gravity has attracted a lot of interest: baby universe theory[1)2)]. The baby universe is normally a small closed universe which liberates itself from a bigger universe. As a theory it is a rather nasty baby because it kills all its rivals in order to take the throne as the theory of everything (TOE): In fact he lets the competitors go down the wormhole.

The baby universe "theory" is not much of a theory in the sense of a full model, but rather some procedures and/or some remarks. The following two ideas are the crucial ingradients:

1) It is argued that the effect of wormholes with a cross section of topology of an S^3 sphere is to make extra contributions to the action of the types that can be added in agreement with the gauge symmetries of the original field theory. This effect is a kind of renormalization of the coupling constants. Here included under this notion is also masses and coefficients to kinetic terms in the Lagrangian. The extra terms are "dynamical" third quantization variables, they are quantum mechanical operators on the same footing as a field or the position of a particle. The only difference is that these "dynamical" coupling constants are constants as functions of space and time. This is due to the fact that Einstein field equation prevents a wormhole from carrying energy or momentum.

2) Whatever the original statistical distribution of these "dynamical" and therefore quantum mechanical random coupling constants are, it gets modified by an extremely rapidly varying

factor in the partition function in such a way that one obtains at the end almost a definite prediction for these couplings or at least some of them.

This prediction is for the full coupling constants – the sum of the original couplings and the contribution from the wormholes. So it becomes irrelevant what the original values might have been. Therefore the theory – the TOE – predicting the original values turns out non-testable. At least one cannot test the predictions for the coupling constant, because it is overwritten by the wormholes. This is how the baby universe theory robs the throne as TOE: it make the complete theories untestable via their coupling constants (herein included masses).

A most impressive victory of this baby universe theory is that the couplings make with very high probability the cosmological constant extremely close to zero[2)-5)]. However we think that there are even more important reasons for believing in this theory which is hard to avoid:

1) It is based on quantizing general relativity only in a very primitive manner. So it really does not need for implementation any full quantization of gravity. It suffices with such features of an eventual quantum gravity as can hardly be missing from any reasonable quantum gravity. Practically any sensible quantum gravity will in some region of scales be described (formally, or with sense made by the help of a cutoff) by means of a Euclideanized functional integral over all Riemann surfaces of four dimensions. So whatever quantum gravity should finally turn out to be the baby universe like theory would have to be

true. One may say this in the following way: The whole argumentation has a phenomenological support in the sense that no new assumptions are introduced without a phenmenological basis or a logical argument. No purely estetic guess is used.

2) Precisely because it makes the TOE go down the wormhole as untestable, there is no place for any competing theory. So all other TOE's are made into metaphysics, only the baby universe theory itself remain as candidate for TOE.*)

In the following section, 2 we review the baby universe theory. In section 3 we then go through our own contribution[6]: the argumentation that the strong CP problem – why there is no $F\tilde{F}$ term in the QCD Lagrangian within the accuracy of measurement – can be solved in a manner quite similar to the solution of the cosmological constant problem by means of baby universes. In section 4 we collect several remarks on the baby universe theory: We stress the robustness that the theory is not sensitive to details. We argue for the solution of the conflict between the argument by 't Hooft that black holes are just particles and the entropy increase due to them. We also discuss the relation with Random Dynamics Project. In the last section 5 we give a short conclusion.

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*)See, however, the remarks below on random dynamics and baby universe theory and the remark that the number and types of degrees of freedom may still be tested.

Section 2. Review of Baby Universe Theory*)

The main starting point for the baby universe theory is to take into account some special topological structures in space time foam as predicted by Wheeler many years ago. Almost whatever quantum gravity might finally turn out to be, it must be in some relatively long distance scale approximation described by a functional integral formulation in which one functionally integrates over all possible Riemann surfaces of dimension four. This is so because we expect the gravitational field to be identical to the Riemann surface metric tensor field from the convincing arguments by Einstein and from the basically successful experimental tests of classical Einstein gravity. We also believe that we can quantize it by means of the Feynman path way integral method. Of course this type of quantum gravity – like most quantum gravities – leads to divergencies when the evaluation of quantum corrections is attempted. That fact strongly suggests that cutoffs or modifications are needed if not at lower energies then at least the Planck energy. Superstring theory presumably gives finite results and thus manages to regularize quantum gravity. However, even for instance in superstring theory there is some low energy approximation in which quantum gravity can be treated as a functional integral over the Riemann surfaces of dimension four, also functionally

*)There exist several nice review articles, e.g. refs. 7) and 8).

integrating at the same time over the matter fields. So such a functional integral formulation is indeed rather general.

Now, since Riemann surfaces of dimension four can have quite complicated topologies, the functional integral of quantum gravity supposedly includes summation over all these various topologies. That is to say that there are quantum fluctuations in the topology of space time. In other words we have space time foam.

The special topological element of interest for the "baby universe fashion" is the wormhole. It leaves or meats a more flat space time manifold within a limited region in both space and time as is shown in Fig.1. Such a wormhole would typically

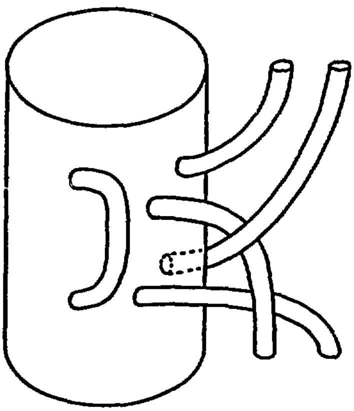

Fig.1

have a cross section with topology of an S^3 sphere. The tube thus has the topology of $S^3 \times I$ where I is an interval of the real axis. Such wormholes are attached to more smooth space time regions after removal of a (little) four ball (the set of points closer to a Euclidean space time point than some radius).

Presumably it is not really important whether such wormholes can be obtained as classical solutions of the equations of motion, the Einstein field equations, since according to the rules of the Feynman path way integral we have to integrate over them anyway. However there are arguments[10)] that a so called axion – an antisymmetric tensor gauge field – can make the wormhole a classical solution. Anyway it seems difficult to avoid wormholes (B. De Witt seems though to be of another opinion). Now what is their *effect?* This is where there has been the progress in understanding: They make the coupling constants (herein included masses and the coefficients to the kinetic terms in the Lagrangean) "dynamical".

In fact suppose that we have a Lagrangean

$$\mathcal{L}_0(x) = \sum_i g_i \mathcal{L}_i(x) \tag{2.1}$$

where $\mathcal{L}_i(x)$ is some local operator and g_i is a coupling constant. By the effect of baby universes dynamical terms $\sum_i \alpha_i \mathcal{L}_i(x)$ are added to give an effective action

$$S_{eff} = \int d^4x \sqrt{g(x)}\, \mathcal{L}_{eff}(x) \ ,$$

$$\mathcal{L}_{eff}(x) = \sum_i (g_i + \alpha_i) \mathcal{L}_i(x) \quad . \tag{2.2}$$

Now the coupling constants

$$\tilde{\alpha}_i = g_i + \alpha_i \qquad (2.3)$$

are dynamical in the sense that they lead to a formalism in which these couplings are to be integrated over on an equal footing with fields of the theory: Gravitational field $g_{\mu\nu}$ and all other fields (matter field) φ. The difference from ordinary path integral is that there is only one variable of integration for each coupling, i.e. it does not depend on space time. Thus the partion function is given by

$$Z = \int \prod_i d\alpha_i f(\alpha) Z(\alpha) \qquad (2.4)$$

where

$$\log Z(\alpha) = \int \mathcal{D} g_{\mu\nu} \mathcal{D}\varphi \exp\{-S(g_{\mu\nu}, \varphi ; g_i + \alpha_i) \qquad (2.5)$$

and $f(\alpha)$ is a function of α determined by the boundary condition. For the Hartle-Hawking boundary condition[9] we obtain

$$f(\alpha) = N^{-1} \exp(-D_{ij}\alpha_i\alpha_j) \log Z. \qquad (2.6)$$

Arguments for introducing the extra term in (2.2) is given by a third quantized formalism[1)10)] in which there are operators a_i and a_i^+ for creating or annihilating whole universes, at least small ones, the baby universes. In eq.(2.2) i denotes a state of a baby universe i.e. its contents, geometry and field configurations etc.. Let the amplitude for coupling at a point x be a function $K_i(\varphi(x))$ of the fields at the point x. We denote by $2S_i$ the action of a wormhole representing the passage of a baby universe in the state i. The third quantization is performed in the "path integral" formalism by

$$\sum e^{-S} = \langle n_1', n_2', \cdots | e^{-S_{eff}} | n_1, n_2, \cdots \rangle \tag{2.7}$$

where the initial and final states of $(n_1, n_2, \cdots)$ and $(n_1', n_2', \cdots)$ baby universes are given by

$$|n_1, n_2 \cdots\rangle = (\sqrt{n_1! \cdots})^{-1} (a_1^+)^{n_1} (a_2^+)^{n_2} \cdots |0\rangle \tag{2.8}$$

and

$$|n_1', n_2' \cdots\rangle = (\sqrt{n_1'! \cdots})^{-1} (a_1^+)^{n_1'} (a_2^+)^{n_2'} \cdots |0\rangle \tag{2.9}$$

respectively. The sum in (2.7) stands for summing up all fluctuations for n_i and n_i' fixed. In the semiclassical approximation we obtain

$$S_{eff} = \int d^4x \sqrt{g}\, \mathcal{L}_{eff}(x) \quad , \tag{2.10}$$

$$\mathcal{L}_{eff} = \mathcal{L}_0 + \sum_i (a_i^+ + a_{i*}) \mathcal{L}_i \tag{2.11}$$

with $\mathcal{L}_i = e^{-S_i} K_i$. We have set in (2.2)

$$\alpha_i = a_i^+ + a_{i*} \; . \tag{2.12}$$

where i* denotes TCP conjugate of the state i. The operators a_i and a_i^+ obey

$$[a_i^+, a_j] = \delta_{ij} \; . \tag{2.13}$$

They are third quantized because they create or annihilate baby universes in second quantized states.

Even without fully presenting the third quantized formalism it should be possible to understand the main features of the effective theory. One important feature is that one approximates the effect of a small wormhole entering a more large scale space time at a point x^{μ} by a contribuion to the effective action S_{eff} given by a polynomical $\mathcal{L}_i(x)$ in $\varphi(x)$ and $\partial_{\mu}\varphi(x)$. The effective Lagrangean density $\mathcal{L}_i(x)$ depends on the state i of the baby universe the propagation of which is constituted by the wormhole in question, so we have a different polynomial $\mathcal{L}_i(x)$ corresponding to each state i of the baby universe. The coefficients on such contributions will depend on the "third quantized" state of the system of all the baby universes which could possibly bump into the bigger universe space time.

It is, however, a further important point that the Einstein field equations guarantee that a baby universe leaving or being absorbed by a bigger universe cannot possibly take away four momentum from or deliver it to the bigger universe. One rough argument for this fact is that the baby universe which has left the bigger universe exists nowhere in the geometrical sense since a position is something only making sense inside the (bigger) universe. Since a momentum takng up should correspond to a taking up wave number in space, it is impossible to carry momentum without having (a superposition of) positions. A better argument for this is that the four momentum conservation in Einstein gravity theory is guaranteed by combining the Bianchi identities and the Einstein field equations. Now the fact that the flux of four momentum into a little region of space time is zero is guaranteed by the mere existence of the metric and the Christoffel symbol along the surface of this region. No reference to the interior is needed. Therefore it does not really matter whether there is a wormhole meeting the space time inside the region or not. In any case no energy and momentum can escape. In turn no taking up four momentum implies that the baby universe must enter at any space time point with the same amplitude. That is a form of Heisenberg uncertainty principle which guarantees this fact. It follows that the contributions to the effective action from the baby universes are no longer x dependent but are given by integrals of the form

$$\int \mathcal{L}_i(x)\sqrt{g(x)}d^4x \tag{2.14}$$

which are just so as to mean an effective renormalization of a "coupling constant" in the original action.

We can say that the extra terms are "coupling constants" because of the impossibility that energy momentum is taken up by the baby universes. Therefore they are constant as functions of space and time.

We have now reviewed or reargued for the effective action (2.2) which take into account the effect of the (small) wormholes. It is this effective action S_{eff}, eq.(2.2) which shows that the competing theories go down the wormhole, because it is the extra terms with coefficients α_i that come from the baby universes in the third quantization which swamps any predictions about coupling constants coming from the fundamental theory.

It turns out that in general — for instance using the Hartle-Hawking initial state condition[9)] — the coefficients α_i are like values of measurements of *quantum mechanical* variables for a system initially being in an eigenstate of an other variable complementary to the α_i's. So they are a priori as unpredictable as the position of a particle prepared in a momentum eigenstate. However, the following is one of the greatest features of interest in baby universe theory: <u>although these variables α_i are thus random they get so narrow distributions (or at least some of them do) that it is really rather like a perfect prediction!</u> However, in order to get this result a very suspicious technique is used: one plays with Euclideanized space time. That is to say one uses Euclidean metric or essentially equivalently imaginary time. In this way the cosmological constant Λ_{eff} as effectively observed at long distances contributes, not as would be the case in Minkowski space time, a phase which is proportional to Λ_{eff} times the space time volume V_4 but rather it gives an exponentially raising or lowering path way integrand. It is this ability of the partition function to blow up, to become huge, which is the main point of calculating the cosmological coupling constant. In fact this exponential blow up, gets exponentiated *once more* before showing up as the weight factor. But exponentiation once more is not even so significant for getting the cosmological constant zero. Just getting $e^{V_4\Lambda_{eff}}$ would be enough to see that a small positive cosmological constant is favored strongly. Exponentiating even more would just make the Λ_{eff} be even more strongly peaked at 0^+.

Coleman[2)] uses that the solution to the *Einstein field* equation without matter is an S^4 sphere with radius proportional to the inverse square root of the cosmological constant

$$r^2 = \frac{3}{8\pi G\Lambda} \qquad (2.15)$$

It is then easily seen that *the behavior of the partition* function in (2.4) which is the extra weight factor becomes

$$Z(\alpha) = \exp\{\exp(\frac{3}{8G^2\Lambda})\} \ . \qquad (2.16)$$

The distributions of $f(\alpha)Z(\alpha)$ in (2.4) of the "third quan*tization* variables" $\alpha = \{\alpha_i\}$ becomes

$$f(\alpha)Z(\alpha) = N^{-1}e^{-D_{ij}\alpha_i\alpha_j}\cdot Z(\alpha)\cdot \log Z(\alpha) \ , \qquad (2.17)$$

where

$$Z(\alpha) = \exp\{\exp(\frac{3}{8G^2\Lambda} - \frac{8}{3}\pi^2 A_1(\alpha) - \frac{64}{9}\pi^3 A_2(\alpha) + C)\} \ . \qquad (2.18)$$

In the following we explain the meaning of *each* factors in (2.17). The first factor N is a normalization. The second one, $e^{-D_{ij}\alpha_i\alpha_j}$ is the *a priori* distribution of the third quantized variables α. It strictly speaking depends on the third quantization of the "super world". However, provided this a priori distribution is smooth it does not really matter what it is in detail because the factor $Z(\alpha)$ completely dominates it for small values of the cosmological constant. The functions A_1 and A_2 in (2.18) are derived from the coefficients in the *ansatz* action (to be used in the classical approximation by definitions) to the R^2 terms. In fact we approximate (2.5) in the low energy regime by

$$\log Z(\alpha) = \int \mathcal{D} g_{\mu\nu} \exp\{-S_{eff}(g_{\mu\nu};\alpha)\} , \qquad (2.19)$$

and then use in the Euclideanized Riemann spaces a phenomenological expansion of the gravitational action

$$S_{eff}(g_{\mu\nu},\alpha) = \int d^4x\sqrt{g}\{\Lambda(\alpha)-[16\pi G(\alpha)]^{-1}R$$
$$+ a(\alpha)R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + b(\alpha)R_{\mu\nu}R^{\mu\nu} + c(\alpha)R^2$$
$$+ O(R^4)\} . \qquad (2.20)$$

The effective coupling constants are determined by a maximization of the α-distribution $Z(\alpha)$ by using an S_4 ansatz with the radius (2.15) for space time. Eq.(2.20) becomes

$$S_{eff}(\alpha) = -3[8G^2(\alpha)\Lambda(\alpha)]^{-1} + \frac{8}{3}\pi^2A_1(\alpha) + \frac{64}{9}\pi^3A_2(\alpha) , \qquad (2.21)$$

which gives (2.18) in the semiclassical approximation. In (2.18) $A_i(\alpha)$ are some functions of $\{\tilde{\alpha}\}$ which can be expressed in terms of the coefficients in the expansion (2.20), e.g.[7)]

$$A_1(\alpha) = 24a(\alpha) + 36b(\alpha) + 144c(\alpha) . \qquad (2.22)$$

From the distribution (2.18) of the α's we see the tremendous dominance of small and positive Λ-values. This is the explanation why in "Baby universe theory" the effective cosmological constant Λ_{eff} is zero. This is the presumably greatest achievement of this "theory".

Another similar prediction is the solution of the problem of why the weak interaction mass scale is so terribly small compared to the Planck scale, which is the presumably fundamental one. This problem is closely related to the so called hierarchy problem. Grinstein and Wise[11)] showed that baby universes may solve the problem of this big scale ratio also. In fact they argue that a light scalar can naturally appear. The point is that the mass of the scalar, after the adjustment of the α's to maximize the partition function $Z(\alpha)$, turns out to depend exponentially on the various parameters. Thus it can "easily" take on an exponentially small value relative to the "fundamental" scale, the Planck scale. The reason for this is that there comes into the derivative of the quantity to be maximized a logarithm of the physical mass of the scalar particle in question. When it is determined it means that the mass comes out as the exponential of the value obtained for it. Usually the problem with scalars is that their masses due to quadratic divergencies with a fundamental cut off becomes most likely of the order of the fundamental mass scale. However, having the bare parameters such as the bare mass square replaced by α's, which can adjust themselves, can help to cancel the quadratic divergencies. Then the way is opened for the exponentially small say Higgs mass relative to the fundamental scale.

Section 3. Strong CP (Theta) Problem*)

In analogy to the cosmological constant problem — why the effective cosmological constant is so exactly zero — it is natural to ask if the baby universe theory can solve other fine tuning problems such as the problem of why the theta term in QCD is not present, which violates CP symmetry in strong interactions.[12)]

The theta term is of the form

$$\mathcal{L}_{\theta}(x) = \theta\cdot\frac{g_s^2}{32\pi^2} F^a_{\mu\nu}F^a_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma} \tag{3.1}$$

in the QCD Lagrangian, where g_s denotes the gauge coupling constant. This theta term is topological in the sense that it is actually a total divergence. So one might first believe that one could simply throw it away with the remark that it is only a boundary term. This is, however, not quite true since there can be such field configuration — the instantons — that they cannot be deformed continuously to the vanishing field configuration $A^a_\mu(x) = 0$. Even if the boundary terms vanish there can for such topologically nontrivial configurations result a nonzero contributions to the action from the theta term. Really this theta term could be removed and reformulated in terms of the theta vacuum. Originally the effect equivalent to the theta term was formulated as a question about which superposition of various

*)This section is based on our work, ref.6).

vacuum n-states to take as the physical vacuum. If a Belavin-Polyakov-Schwartz-Tyupkin[13] instanton goes on the vacuum shifts from one large gauge to another one.[14] The various "large gauges" for the vacuum are denoted by integers n. An instanton shifts n up by one unit, while an anti-instanton shifts it down by one unit. In order to be an eigenstate of energy in the presence of instanton activity the initial true vacuum must be of the form

$$|\theta\rangle = \sum_n e^{i\theta n}|n\rangle \ , \tag{3.2}$$

where $|n\rangle$ denotes the "vacuum" with the "large" gauge given by n. It is not difficult to see that the effect of taking such a vacuum $|\theta\rangle$, is equivalent to taking as vacuum $|\theta=0\rangle$ but then to include the theta term (3.1) in the Lagrangean[15].

Now it is easily seen that either formulation — the theta vacuum or the total divergence term in the Lagrangian — breaks CP conservation. For instance parity P is broken simply because the theta term (3.1) has $F^a_{\mu\nu}(x)$ factors with an odd number — in fact three — spacelike indices. Under parity each of these changes sign. Since because of the instantons the term has indeed an effect in spite of total divergence it will lead to a true breaking of the CP symmetry. Actually that is only true by the presence of the quark mass terms. The instantons can only proceed provided the quarks have masses, because there would otherwise be zero modes which lead to a factor zero in the amplitude for the instanton to proceed. In fact the coefficient θ in (3.1) is replaced by

$$\bar{\theta} = \theta + \arg\det M \qquad (3.3)$$

where M denotes a quark mass matrix. Even though quark masses are small calculations show that the suppression of the CP-breaking effect is not sufficient to make it small enough not to be observed. Actually the value of the coefficient theta bar is found from the experimental knowledge to be numerically bounded by 10^{-9}. This is a too small value to be acceptable as an accident and there is need for an explanation[12]. Peccei and Quinn[16] proposed an explanation by introducing two Higgs fields and predicting a new particle, the axion[17]. One has, however, not found any axion in nature yet, and the development has rather been to invent models with invisible axions. Using experimental knowledge the bounds on the mass of the axion has become rather tight. The basic idea is that the two Higgs fields of the vacuum can adjust themselves so as to minimize the energy of it. This minimization of the vacuum energy makes the effective theta value zero so as to guarantee CP-conservation. In fact the vacuum is adjusted by minimization of the energy due to the instantons.

It is now the question whether the maximization of the baby universe third quantization distribution — given by $Z(\alpha)$ — can take over the role of the minimization of the vacuum energy in the Peccei-Quinn model, and the adjustable α can take over the role of the adjustable Higgs fields in P-Q model. The point is that the effective terms in the Lagrangian caused by the baby universes can also have the form of an extra contribution to the theta term. In fact one can easily imagine a wormhole into which goes a couple of gluons in a pseudo-scalar state denoted by i_θ.

The interaction of such a baby universe with our universe could be simulated by an effective term which could be just the theta term

$$\alpha_{i_\theta} \frac{g_s^2}{32\pi^2} F^a_{\mu\nu} F^a_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma} \ . \tag{3.4}$$

The final fixing of this extra term would be determined by the effect of the full theta term — the sum of the bare one and the baby universe one — on the $Z(\alpha)$. The coefficient of the full $F\tilde{F}$ term is given by

$$\tilde{\alpha}_{i_\theta} = \bar{\theta}_{eff} = \bar{\theta} + \alpha_{i_\theta} \ . \tag{3.5}$$

The effective coupling constants $\tilde{\alpha}_i = g_i + \alpha_i$ in the baby universe theory are determined by a maximization of the α-distribution $Z(\tilde{\alpha})$, (2.18). As was explained already there is an enormous peak in $Z(\tilde{\alpha})$ of (2.17) at α_i's such that

$$G^2(\tilde{\alpha})\Lambda(\tilde{\alpha}) \to 0^+ \ , \tag{3.6}$$

which explains why the effective cosmological constant is close to zero with very high precision. Eq.(3.6) would fix some of α_i's. Further maximization of $Z(\tilde{\alpha})$ given by (2.18) under the condition (3.6) would fix most of the rest α_i's by the equation for the maximization condition

$$\frac{\partial}{\partial \tilde{\alpha}_i}\left[-\frac{8}{3}\pi^2 A_1(\tilde{\alpha}) - BG^2(\tilde{\alpha})\Lambda(\tilde{\alpha})\right] = 0 \ , \tag{3.7}$$

where B is a Lagrange multiplier introduced to guarantee the condition (3.6). So it is the sum (this full term) which will become CP-invariant (and thereby zero) if it happens that the maximization of $Z(\tilde{\alpha})$ through eq.(3.7) leads to a CP-invariant solutions. So the contribution from the baby universe effective coupling α_{i_θ} will even be able to cancel a CP-breaking in the fundamental Lagrangian (3.1). However, whether it now really does that or not depends upon the CP-properties of the physics relevant for the quantity to be maximized in order to fix the value of the coefficient α_{i_θ} of the term (3.4) caused by the baby universe which influences the CP-breaking in QCD.

Now let us discuss the symmetry of the partition function $Z(\tilde{\alpha})$ under CP transformations. To this end we introduce a definition of a kind of extended CP transformation for the dynamical variables $\{\alpha_i\}$. Consider the CP invariant Lagrangian of the form

$$\mathcal{L}_{eff}(x) = \sum_i \tilde{\alpha}_i \mathcal{L}_i(x) \tag{3.8}$$

where $\tilde{\alpha}_i = g_i + \alpha_i$ and $\mathcal{L}_i(x)$ is a local operator. If $\mathcal{L}_i$ is CP even i.e.

$$(CP)\mathcal{L}_i(CP)^{-1} = \mathcal{L}_i \;, \tag{3.9}$$

we define the CP transformed value α_i^{CP} by

$$\alpha_i^{CP} = \alpha_i \;. \tag{3.10}$$

On the other hand if $_i$ is CP odd

$$(CP)\mathcal{L}_i(CP)^{-1} = -\mathcal{L}_i \;, \tag{3.11}$$

we define

$$\tilde{\alpha}_i^{CP} = -\tilde{\alpha}_i \tag{3.12}$$

so that

$$\alpha_i^{CP} = -2g_i - \alpha_i \ . \tag{3.13}$$

Really it is a hard calculation and it requires knowledge of the relevant laws of nature to calculate if the quantity $Z(\tilde{\alpha})$ actually takes its maximum for a CP-invariant or a CP-breaking value of the effective theta given by (3.5). If all the coupling constants are considered adjustable parameters the whole problem of maximizing a physical quantity is really a CP-invariant one. If we do not consider the couplings as given there is in fact no CP-breaking possible in the following sense: To a given set of couplings we can construct a CP-conjugate set. Thus the partition function $Z(\tilde{\alpha})$ must be invariant under the CP transformations (3.10) and (3.12)

$$Z(\tilde{\alpha}^{CP}) = Z(\tilde{\alpha}) \quad . \tag{3.14}$$

Maximizing $Z(\tilde{\alpha})$ can lead with high likelyhood to either of two possibilities : symmetrical or nonsymmetrical argument as illustrated in Figures 2 a) and 2 b). It is difficult to know without calculating the function. However, once some of the variables α_i take nonsymmetric values the generic expectation would be that they all would do so.

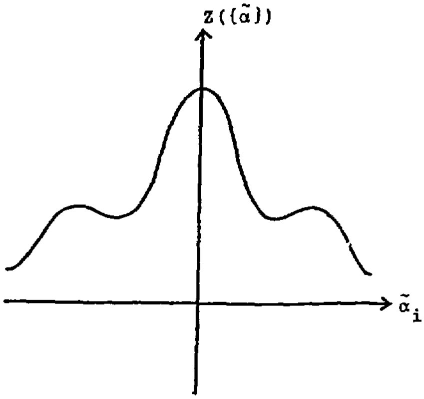

Fig. 2a Symmetric maximum

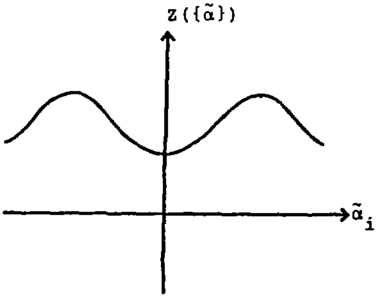

Fig. 2b Asymmetric maximum

In this sense any CP-breaking in the baby universe theory is a kind of spontaneous one: It happens that the maximum of the partition function $Z(\tilde{\alpha})$ occurs for a CP-violating set of couplings in spite of the fact that the problem was set in a CP-symmetric manner. It means that there would be another set of couplings giving precisely the same value for the partition function $Z(\tilde{\alpha})$ but giving just the opposite sign for the CP-violation parameter ε of the K^0 decay.

A priori it would not in the baby universe theory be so strange if CP was not broken at all because $Z(\tilde{\alpha})$ to be maximized would be CP-invariant and thus there could easily be a CP-invariant maximum. However, we know from the phenomenology that the total maximum must be a CP-breaking one, because we know that CP is broken in nature. We know the experiments of K^0 decay, that K_L^0 decays into two π's. We may therefore at first think that also the theta bar term (3.5) will be driven to CP-violating value under the maximization.

We would in fact think that there could be many terms in Z to be maximized which involve a combination of the theta coupling (3.5) and of the other CP-breaking couplings. Such combination would have values depending upon whether the theta and other CP-breaking couplings break CP in the same or the opposite direction. If such terms are relevant there will not be any maximum for theta equal to zero because these terms will give a nonzero slope in general for log Z when the effectve theta is equals to zero. Only if the theta and other CP-breaking couplings are effectively decoupled in the quantity to be maximized it will be likely that the effective theta be zero in spite of the phenomenological fact that CP is broken.

We believe that such a separation is possible or likely. A significant point of our method is to invoke an approximate separation between $\bar{\theta}_{eff}$ governing CP symmetry for strong interactions and other potentially CP breaking couplings in Nature at higher energy scales than the strong interaction scale. The idea is that this separation occurs because the "dynamical" theta i.e. α_{i_θ} term (3.4) which comes from a baby universe with two gluons in a pseudoscalar state only interacts via instantons. Now instantons of QCD are only active at an energy scale of the order of Λ_{QCD}. This fact has two reasons: 1) for small size (or equivalently high energy) instantons the quarks are effectively massless and zero modes kills the instanton activity[18], 2) due to the renormalization group effects the contribution of the instantons to the amplitudes goes as $\exp(-8\pi^2/g_{eff}^2)$ where g_{eff} is the effective gauge coupling constant, which is small at high energy. Of course the separation is not 100 % accurate and we therefore should be able to calculate in an order of magnitude manner the small but nonzero $\bar{\theta}$ predicted by the baby universe thory. Work in this direction is hoped to be reported elsewhere by the present authors. Since other CP breaking physics is not present at the Λ_{QCD} scale physics, there seems little place for $Z(\tilde{\alpha})$ to depend on the combination of α_{i_θ} and other CP breaking couplings. At least it should not be possible to transfer the information on the direction of CP breaking from one to the other.

Now the most important point for our analysis of predicting $\bar{\theta}_{eff}$ zero is the above argued separation with respect to CP properties of the instanton scale physics and the higher energy physics. Corresponding to this separation we define two CP operations on the set of α_i's : One is for strong interactions denoted by $(CP)_S$ and the for the rest of interactions at higher energies denoted by $(CP)_R$. They are defined such that

$$CP = (CP)_S \cdot (CP)_R \tag{3.15}$$

The $\bar{\theta}_{eff}$ term is the only CP odd operator in strong interactions and we define the CP transformations of $\tilde{\alpha}_i$ from (3.12)

$$\tilde{\alpha}_i{}^a = \begin{cases} \tilde{\alpha}_i & \text{for } i \neq i_\theta \\ -\tilde{\alpha}_i & \text{for } i = i_\theta \end{cases} , \tag{3.16}$$

for $a = (CP)_S$. On the other hand for $a = (CP)_R$, we have, in view of (3.10), (3.12) and (3.15),

$$\alpha_i{}^a = \begin{cases} -2g_i - \alpha_i & \text{for } {}_i \text{ CP odd but } i \neq i_\theta \\ \alpha_i & \text{otherwise} \end{cases} \tag{3.17}$$

According to the approximate separation we have

$$Z(\tilde{\alpha}^a) \simeq Z(\tilde{\alpha}) , \tag{3.18}$$

with $a = (CP)_S$ or $(CP)_R$ where $\simeq$ indicates that it holds approximately, while for the total CP trnsformation $Z(\tilde{\alpha})$ is exactly invariant as (3.14). Therefore for $i = i_\theta$, that is for $\tilde{\alpha}_{i_\theta} = \bar{\theta}_{eff} = \bar{\theta} + \alpha_{i_\theta}$ we obtain from (3.16) and (3.18)

$$Z(-\bar{\theta}_{eff}, \tilde{\alpha}') \simeq Z(\bar{\theta}_{eff}, \tilde{\alpha}') , \tag{3.19}$$

where $\tilde{\alpha}'$ denotes $\tilde{\alpha}_i$'s excluding $\bar{\theta}_{eff}$. Thus $Z(\bar{\theta}_{eff})$ could easily take its extremum at the symmetric point $\bar{\theta}_{eff} = 0$. A typical example of this decoupling is depicted in Fig. 3 where Z is drawn as a function of an weak CP odd $\tilde{\alpha}_i$, and $\tilde{\alpha}_{i_\theta}$. It has two peaks over $\tilde{\alpha}_i = \pm$ (a nonzero number), but with $\tilde{\alpha}_{i_\theta}$.

Fig.4 shows symmetries for Z as a function of the variables $\tilde{\alpha}$. It would no longer be a mystery if $\bar{\theta}_{eff} = 0$, since eq.(6) could easily have this solution. We note that the symmetry property (3.19) is supported by the QCD dilute instanton gas calculation[15] for $Z(\bar{\theta}_{eff})$ which gives

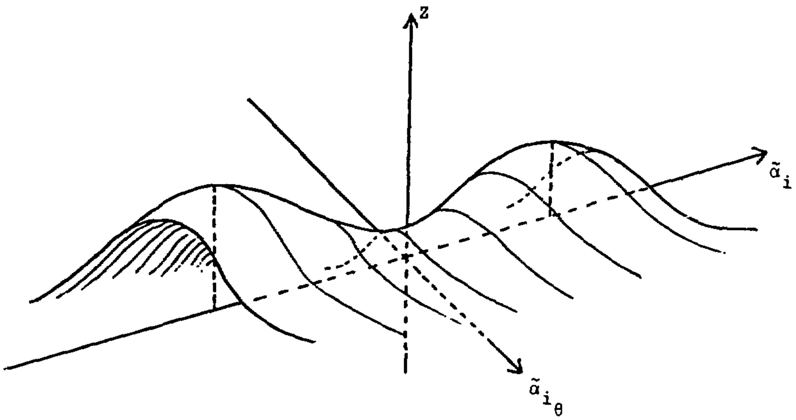

Fig. 3

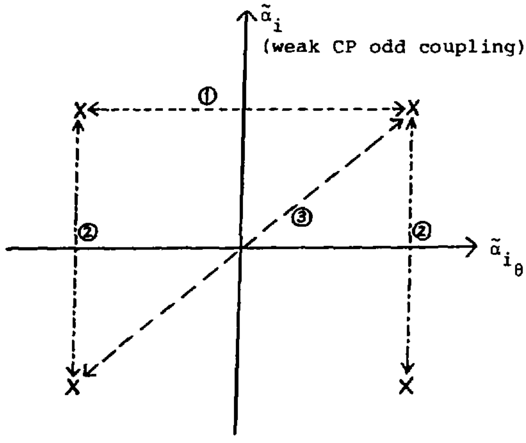

Fig. 4

① : approximate $(CP)_s$ symmetry

② : approximate $(CP)_w$ symmetry

③ : exact CP symmetry

$$Z(\bar{\theta}_{eff},\bar{\alpha}') = \exp[\exp(\text{const.}\cos\bar{\theta}_{eff}+\gamma)] \ , \qquad (3.20)$$

where γ is some terms independent of $\bar{\theta}_{eff}$. This shows that in the dilute gas approximation the maximum occurs for either $\bar{\theta}_{eff}$ = 0 or π.

We have solved the strong CP problem in baby universe theory in the sense that we have argued from this theory that $\bar{\theta}_{eff}$ = 0 or π with high precision can occur very likely. This conclusion follows from very general arguments. If we make the dilute gas approximation for QCD instantons it follows that $\bar{\theta}_{eff}$ must be either 0 or π. Thus the CP conservation for strong interaction is explained.

We do not predict that $\bar{\theta}_{eff}$ takes a CP invariant value exactly because it only does so in the approximation that there is a decoupling between CP transformation $(CP)_S$ for strong interactions and $(CP)_R$ for all the rest of interactions, at higher energies. This decoupling means that the partition function is roughly symmetric under both of these partial CP operations on the α_i's. It results from the separation in energy scale of the CP breaking of the instanton scale and of all the other interactions. We seek to perform more accurate estimate of this decoupling than those in the present article which will be a subject for our forthcoming paper.

There should thus be no need for Peccei-Quinn axions since baby universe theory solves the strong CP problem without need for Peccei-Quinn axions. Therefore it is our prediction that axions should not be observed (They could of course exist for other reasons though). Another prediction is that there is a small strong CP breaking, $\bar{\theta}$ being not exactly zero.

Section 4. Some further remarks and points

We would like to come with several remarks about baby universe theory.

First as stressed by Adler[19], the theory is remarkably robust in the sense that it is not sensitive to the details. In fact one does not need any detailed knowledge about the properties of the wormholes in order to obtain the main effect of making the coupling constants dynamical and next obtaining the values of the cosmological constant and then the $\bar{\theta}$ to be zero. The main point is the nonlocality so that even the existence of wormholes would not be needed if some other similar effect existed. Moreover the details of what to be maximized seems not to be very important. For the cosmological constant it is presumably important that the quantity to be maximized depends on the volume – or some other expression for the size – of the space time Riemann surface. For the strong CP problem we used very little information about the quantity to be maximized.

One of the points of robustness is that it does not really matter what model for quantum gravity will finally turn out to be the truth. At small distances it could be something totally different from the model of the type of Riemann space that we know gravity must be at large distances. The main use of baby universes is to make the couplings "dynamical". This can be done by having a "dynamical" term which stems from physics at any scale of energy. We may then argue that there will come some "dynamical" contribution from the scale just below the one at which physics begins to be very different from the Riemannian manifold gravity. At such a scale we can still trust that wormholes must be present. It would be very strange if the new physics above that scale would precisely work for cancelling the effect of the baby universes coming from lower scale physics. As an expression for robustness you can also take the statement of Adler that a rather general nonlocal form of the action suffice to give the main results of the baby universe theory. Indeed if you take for the action containing the nonlocal term

$$F_M\left(\int d^4x \sqrt{g}\right)$$

i.e. a function of the space time four volume $\int d^4x \sqrt{g}$, one can still show the Coleman's argument for the cosmological constant being zero.

We have already stressed that the baby universe theory let all theories at the deeper level (the TOE's) go down the wormhole. This is also a sign of robustness since it makes irrelevant the coupling constants of the theory at the more fundamental level. Still – for this remark we thank Paulo Di Vecchia –the number of the fields and their spins and gauge quantum numbers seem to survive when going down the wormhole. This is indeed true for the attack simply from the dilute gas approximation of wormholes of the type of $S_3 \times I$. However, it could sensibly be speculated that more general types of space time foam with a high density could mean that there are so many wormholes all over in the large scale space time and that their foot points were close to each other or other local type topological structures. This would mean that there was at the fundamental level several layers of space time (Fig. 5) which from a macroscopic point of view correspond to a single piece. If, so to speak, there are several layers of "fundamental" space time which covers the same bit of "phenomenological" space time we can have a quantum (= particle) running on one or the other of these layers — or rather in one or the other superposition of

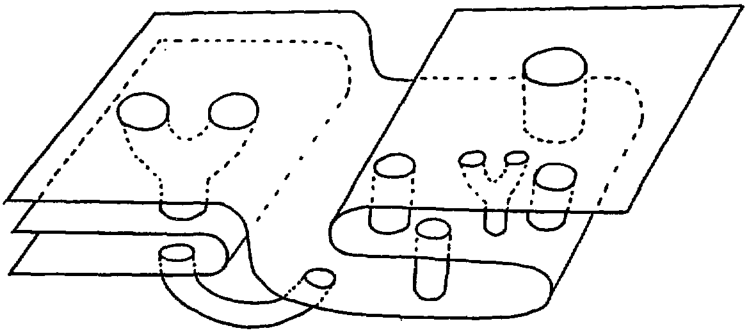

Fig.5

running in them. Such various different superpositions will be conceived of as different types of particles — different flavors say. In this way one could achieve an increase of the number of fields effectively present in the theory. This implies that one can have the number of degrees of freedom also changed by the space time foam. If this works even more predictions of the TOE would go down the drain. Anyway if the baby universes just control the coupling constants (and herein again included the masses) this is enough for controlling also Higgs fields if there just are scalar fields of the right quantum numbers present. This can influence strongly the number of degrees of freedom that shows up at low energies, and that is what matters for real experiments.

A further question on which we want to comment is what we can call the controversy between Hawking[20)] and 't Hooft[21)]. We may represent the point of views ascribed to these authors by the following slogans:

't Hooft: Black hole states are just particles,

Hawking: By the formation and evaporation of a black hole entropy is increased.

There is seemingly a contradiction between these slogans because according to the 't Hooft statement such a formation and subsequent decay of a black hole is just certain scattering processes which should not cause any increase of entropy by principle.

This contraversy gets a nice resolution in baby universe theory. The proposed resolution is the following: If one includes arbitrarily high order polynomials of fields there are infinitely many states of a baby universe provided one allows it to be arbitrarily large. So there are indeed infinitely many "dynamical" coupling constants, i.e. the coefficients of the infinitely many possible field products applicable as Lagrangian terms, and they are dynamical because of the effect of the corresponding infinitely many baby universe states. Now formation of a big – macroscopic – black hole must involve the emission of rather big "baby universes" because we know from the classical description of a black hole that a rather big world falls down below the horizon and really becomes a liberated (baby) universe. That in turn must mean that "dynamical" – and therefore random – couplings get involved in a process of formation and decay of a big black hole. These couplings cannot possibly all have been collected in coupling constant data tables. They must therefore be in practice really random – unless they are fixed in a calculable manner by the maximization principle. If they are not fixed in a practical manner from the maximization there are effectively random couplings in large numbers involved in the decay of the black holes just being particles (in the 't Hooft slogan). That makes the decay become chaotic and really to present entropy increase. So the randomness of entropy increase due to black holes is really just the revelation of the huge number of coupling constants relevant for the production and decay of big black hole "particles". Once one would have measured and tabulated the huge number of couplings needed the black hole could be formed and decay without entropy increase. But as long as we do not have such tables at our disposal there is entropy increase!

Finally we would like to discuss the relation between baby universe theory and Random Dynamics[22]. The strong robustness of the baby universe theory leads the thought towards the idea of "Random Dynamics" that fundamental physics seems random. Taken seriously the Random Dynamics model is an attempt of a TOE; so does it also go down the wormhole? Yes, at first it might seem so, but really this is not necessarily true. There are two ways of looking at the relation between Random Dynamics and baby universe theory depending on which of the two is thought as the more fundamental:

1) If we think of the baby universe theory as the more fundamental we may remark that it predicts the effective coupling constants $\tilde{\alpha}_i = g_i + \alpha_i$ to be (quenched) random. This is indeed one of the main assumptions of random dynamics. So in this sense random dynamics is a consequence of the baby universe theory. We have indeed seen above that the α_i's are random in the sense of being distributed, integrated over. This randomness is quenched in the sense that the α_i's take the same value at all times (and at all space time points too). Strictly formally they are quantum variables and not quenched. But in practice they must appear as quenched. However, the tremendously sharp distribution which is obtained for them – at least for the cosmological constant – in practice make them just predicted numbers, and thereby the argument for random dynamics is working only in principle, but not really in practice.

2) Also when we consider the question of which is the more fundamental oppositely, random dynamics being the more fundamental, we find that baby universe theory deduce to some extend random dynamics. The point is precisely that baby universe theory send all the T.O.E.'s down the wormhole. This in fact means that we have a fundamental problem in seeing through the space time foam to see what theory is really beneath the baby universes. But to the degree we cannot "see" anything it also means that it cannot matter much what goes on at the more fundamental level beneeth the wormhole scale. That is, however, just a basic assumption of the random dynamics that the fundamental physics does not matter. The baby universe act to a large extend as a veil covering for present day physicists and for physicists in many years to come the theory behind. If it covers sufficiently strongly there is no way to test which is the truth behind and it could as well be random and chaotic: it is as if it was chaotic because we cannot see the difference anyway. In this way we can say that Random Dynamics is suggested by the baby universe theory. But it is not quite working because as we discussed above some more spacetime foam effects are needed to make also the number of degrees of freedom and their spins etc. changed by the quantum gravity.

You might protest that in baby universe theory one puts in gravity from the start and thus really use quantum gravity. That is of course true but we already stressed that robustness that what goes on at very small distances is likely not to be relevant.

Actually the principle of locality — that field degrees of freedom at points away from each other in space time do not interact directly — gets violated due to the wormholes in the approximation that the lengths of these wormholes are neglected. It is, however, restored effectively at the end because nonlocality interactions cannot carry neither energy nor momentum. This is so because these quantum numbers cannot pass through the wormholes. Because of that one cannot observe the effect of locality breaking as causng signals to pass faster than light e.g.. One only sees the very mild nonlocality effect of the coupling constants "knowing" about physics at far remote places.

Fig.6

This mild breaking of locality suffice for solving the cosmological constant problem, but is not otherwise easy to observe. Really, however, the original locality put into the general relativity theory is not truly used to explain the phenomenologically observed locality principle. In this way a "random dynamics type" of locality explanation is really what the baby universe theory has. Such an appearance of an only effective locality seems to be very much called for to resolve the cosmological constant problem.[23]

3) A third relation between random dynamics and the baby universe theory is provided by using the baby universe theory to explain why the inequality is nearly saturated. This subject was presented by one of us (H.B.N.)[24] in the other lecture at this Symposium. In fact it is rather easy, by means of a theory predicting coupling constants, to be determined by maximizing some quantity to get an inequality for resulting coupling become just satisfied as an equality. Since the situation is quite different on the forbidded side of the inequality we could expect a very different – and let us assume much smaller – values of $Z(\alpha)$ on this side. Then all that is further needed is that $Z(\alpha)$ increases with the gauge couplings. Then the maximal $Z(\alpha)$ will be pushed up just where the inequality is saturated. Thus the gauge couplings will be as strong as they can without violating the inequality.

Section 5. Conclusion

The resolution of the 't Hooft-Hawking controversy is just one of the achievements of the baby universe theory. The fine tuning problems — the cosmological constant being zero, the Grinstein-Wise understanding of the smallness of the Weinberg-Salam-Glashow scale compared to the fundamental scale, our understanding of the smallness of the $\bar{\theta}$-parameter — constitute very significant solutions of old problems and strongly signals a significant truth in the baby universe way of argumentation. We found that the $\bar{\theta}$ parameter is not exactly zero but only so to the approximation in which there is decoupling between the $\bar{\theta}$ parameter and the other CP violating parameters. Therefore $\bar{\theta}$ is just small, not exactly zero. But that may turn out to be enough to agree with experiment. If no problem of this type remain we will not need the axion of the Peccei-Quinn type and it is thus our prediction: it should presumably not exist.

What is really needed is the dynamical couplings or some dynamical couplings may be enough, and the maximization of some physical quantity which is growing sufficiently for growing universe size. The "dynamical" couplings are needed in order that there be some parameters to adjust to solve the fine tuning problems. The maximization of something growing large for large universes is needed to make the cosmological constant zero by having the largest universes become favored ones. This is really rather general properties for which other models than the baby universes could possibly be contemplated, but why not believe that baby universes do the job when they seem to impose themselves to do it.

If it should turn out that Euclideanized quantum gravity is not the correct treatment - as we think might easily be the case - one might have to replace the maximization principle coming from this treatment by something else. That something else could be that the Hamiltonian for the world is not hermitean but rather has an antihermitean part. Such an antihermitean part implies that the world either decays or proliferates. If we e.g. imagine that empty space - vacuum - proliferated faster than a world with matter in it we might understand that the dominant type of universe today - long time after the creation - would be one that got empty rather fast. So one with an appreciable Hubble expansion would be favored, in good agreement with experiment. In order that the universe should not recollapse the sign of the cosmological constant causing such a recollapse would be disfavored. If spatially infinite universes were for some reason disfavored there could be a way of regaining the zero cosmological constant prediction by using an antihermitean part in the Hamiltonian. In the light of the great achievements of the baby universe theory it might not be a too heavy price for making it work even to take an antihermitean Hamiltonian!?

We would like to close this lecture note by presenting a figure: The nasty baby prince is throwing all the TOE's into the drain (wormhole).

Fig.7

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