PREPARED FOR THE U.S. DEPARTMENT OF ENERGY,
UNDER CONTRACT DE-AC02-76-CHO-3073

PPPL-2613
UC-420

PPPL-2613

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APRIL 1989

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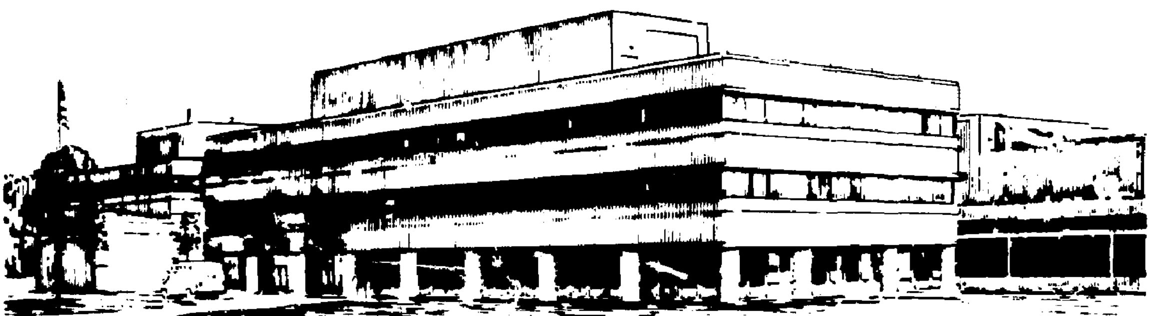

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MAGNETIC ISLAND FORMATION IN TOKAMAKS

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Abstract

The size of a magnetic island created by a perturbing helical field in a tokamak is estimated. A helical equilibrium of a current-carrying plasma is found in a helical coordinate and the helically flowing current in the cylinder that borders the plasma is calculated. From that solution, it is concluded that the helical perturbation of ~ 10^{-4} of the total plasma current is sufficient to cause an island width of approximately 5% of the plasma radius.

A tokamak plasma is ideally axisymmetric. In practice, the axisymmetry is broken either by unsuppressible magnetic field errors caused by the discreteness of toroidal field coils or by suppressible errors caused by the current feeds to the coils or by the misalignment of toroidal or poloidal field coils. The uniform earth magnetic field is usually compensated by the equilibrium coils and/or feedback coils. Of course, accidentally placed magnetic materials cause field distortion. But this accidental source of errors is almost completely eliminated in modern-day tokamak experiments.

The asymmetric field errors cause the reduction of plasma confinement in two ways. One is distortion of the magnetic surfaces that cause particles (chiefly trapped particles) to deviate from the axisymmetric orbits into three-dimensional trajectories. The second is the creation of magnetic islands near the rational surfaces. The island structure effectively reduces the plasma radius, hence the enhanced transport results. Both effects are well known. However, there apparently is considerable misunderstanding with regards to the appearance of magnetic island structures due to field errors. Some maintain that the field errors will be shielded by the presence of conducting plasmas. We shall here show that field errors in general give rise to the appearance of magnetic islands and estimate the magnitude of the island size.

<u>ASSUMPTIONS</u>. We shall assume a tokamak of a large aspect ratio. Then a tokamak can be represented by a straight cylinder with the periodic boundary condition $[f(z + 2\pi R) = f(z)]$ in the z direction.

Now the field error in a tokamak can be expressed as $\Sigma_{n,m} A_{n,m}(r) \exp i(nz/R + m\theta)$ where n and m are integers. The island structure is generated by the field error component, n and m, that matches the rational magnetic surface of rotational transform, $1/q = n/m$. Therefore, we focus attention here to the helical perturbation to a straight cylindrical tokamak. We shall first examine the case $n = 1$ and $m = \ell = q$. Then the problem at hand is exactly the problem of helical equilibrium discussed in ref. 1. For the model, we shall assume that the plasma pressure is sufficiently small so that we can neglect it. Then the force-free equilibrium equation in helical coordinates is given by (ref. 2)

$$\left(k_z^2 + \frac{\ell^2}{r^2}\right) \frac{\partial^2 \psi}{\partial \phi^2} + r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{2}{r} \frac{\ell^2}{(k_z r)^2 + \ell^2} \frac{\partial \psi}{\partial r}$$

$$+ \frac{2\ell k_z B_\phi}{(k_z r)^2 + \ell^2} = - \frac{\partial}{\partial \psi} \left(\frac{1}{2} B_\phi^2\right) , \tag{1}$$

$$B_\phi = B_\phi(\psi) . \tag{2}$$

Here ϕ is the helical coordinate angle and is given as

$$\phi = \ell\theta + k_z z . \tag{3}$$

Other components, such as B_ϕ, ψ, etc., are defined in ref. 1.

A particular solution can be found as in ref. 1. We summarize it here briefly. If we let

$$B_\phi^2 = B_o^2 + k^2\psi^2, \qquad k^2\psi^2 \ll B_o^2 \tag{4}$$

we can obtain instead of Eq. (1),

$$\left(k_z^2 + \frac{\ell^2}{r^2}\right) \frac{\partial^2 \psi}{\partial \phi^2} + r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{2}{r} \frac{\ell^2}{(k_z r)^2 + \ell^2} \frac{\partial \psi}{\partial r}$$

$$+ \frac{2\ell k_z B_o}{(k_z r)^2 + \ell^2} \left(1 + \frac{1}{2} \frac{k^2}{B_o^2} \psi^2 + \ldots\right) = - k^2 \psi \ . \tag{5}$$

For the purpose of understanding the general features of the above equation, it suffices to solve the case where $k_z^2 r^2 \ll \ell^2$. Then the solution of Eq. (5) becomes

$$\psi = \frac{2k_z}{k^2 \ell} B_o \left\{1 + \gamma \left[J_o(kr) + \alpha J_\ell(kr) \cos \phi\right]\right\}, \tag{6}$$

here γ and α are arbitrary constants. The boundary condition requires

$$J_\ell(ka) = 0 \ . \tag{7}$$

Now if the wall at $r = a$ is to carry helical surface currents, J_s^* such that

$$\mu_o J_s^* = \frac{2k_z}{\ell^2 k} B_o \gamma \alpha J_\ell' \ (kr) \cos \phi \tag{8}$$

magnetic islands determined by the flux function (6) will be generated.

If we are to replace this current by ℓ pairs of helical coils, each coil current will be approximately

$$I_h = a \int_{-\frac{\pi}{2\ell}}^{\frac{\pi}{2\ell}} J_s^* \, d\theta = \frac{4k_z a}{\ell^3 k \mu_o} B_o \gamma \alpha \ J_\ell'(ka) \ . \tag{9}$$

This helical current may be compared with the axisymmetric current flowing inside the rational surface whose radius is defined by the condition $J_o'(kr_o) = 0$. We note that this axisymmetric current does not depend on the amplitude of γ in Eq. (6). Then the total current within the radius r_o is,

$$I_t = -\frac{k_z r_o}{\ell u_o} 2\pi r_o B_z = \frac{2\pi k_z r_o^2}{\ell^2} B_o. \tag{10}$$

Hence it follows

$$I_h = \frac{2a}{\ell k r_o^2 \pi} \gamma\alpha J_\ell'(ka) I_t . \tag{11}$$

Experimentally I_t is somewhat lower than the total tokamak current, k is determined by Eq. (7) and r_o is the first zero of $J_\ell(kr)$. The parameter γ (of the order of unity) is to determine the shear of a tokamak and α determines the size of the magnetic island. For example, if $\gamma = 1$, $|q|$ at the center is 1 and $|q|$ at $r = a$ is about 2.3. If $\alpha = 0.25$, the total width of the magnetic island is about a/2 for $\ell = 2$. Assuming $\gamma = 1$, we get for the figure of ref. 1

$$I_h \approx 9.5 \times 10^{-3} I_t . \tag{12}$$

Therefore, a relatively small helical current creates a large magnetic island.

Small helical perturbation. The total width of a magnetic island may be estimated, if α is small. That is, if the helical perturbation is small compared with the shear represented by the term $\gamma J_o(kr)$.

At the rational surface $J_o'(kr_o) = 0$, $J_o(kr)$ is expanded. Then the magnetic island width (the maximum width of the separatrix), Δ, is

$$k^2\Delta^2 = 16 \frac{\alpha J_\ell(kr_o)}{J_o''(kr_o)} = 16\alpha \text{ (for } \ell = 2) . \tag{13}$$

For the example cited above, α will be 2.3×10^{-3} for $\Delta/a = 0.05$. That is, if the magnetic island width is about 4.3 cm for a radius of 85 cm tokamak, the helical perturbing current is approximately

$$I_h = -1.56 \times 10^{-4} I_t . \qquad (14)$$

This helical external current ($n = 1$, $m = 2$) is needed at the plasma surface of radius a to generate the magnetic islands of width 4.3 cm. Suppose that the total current, I_t, is 2×10^6 amperes. Then the helical current is, from Eq. (14), approximately 300 A. The helical magnetic field at the half radius is then of the order of 1.4 gauss. Thus we conclude that a small amount of the helical perturbation gives rise to a relatively large magnetic island.

<u>Generalization</u>. The calculation presented so far requires that the expansion coefficient, k, of Eq. (4) must satisfy the relation (7). If we remove both this restriction and the somewhat artificial condition that an infinitely conducting wall exists at $r = a$, can we still obtain the helical equilibrium solution? If we assume that the non-axisymmetric solution is small compared with the symmetric solution, the answer is affirmative. Since a similar method was used previously to obtain a large aspect ratio tokamak equilibrium[3,4] we shall here sketch the outline of the solving technique.

We choose a radius $b(> a)$ such that $(b - a) \sim a$. On the cylindrical surface at $r = b$, helical coils are located. The objective is to determine the coil current density. At or near $r = a$, the outermost plasma boundary is located. For arbitrary k, the magnetic surface ψ is determined by Eq. (6). The surface at the plasma boundary (noncircular) is determined by the condition that ψ = const. Between $r = a$ and $r = b$, the vacuum solution can be obtained from ($k_z^2 r^2 \ll \ell^2$)

$$\frac{\ell^2}{r^2}\frac{\partial^2\psi}{\partial\phi^2} + \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial\psi}{\partial r} + \frac{2k_z B_\phi}{\ell} = 0 . \quad (15)$$

The non-axisymmetric part of the vacuum solution at $r = a$ is matched with the plasma solution using two arbitrary constants. At $r = b$, the internal solution, ψ_-, is a function of both r and ϕ. This solution is matched to the external vacuum solution, ψ_+, with the condition that $\psi_+ = \psi_-$. The external solution ψ_+ converges as $r \to \infty$. The difference in $\partial\psi/\partial r$ between the two solutions represents the helical current density. The axisymmetric part of the solution is well known.

Conversely, if the calculated helical currents are at the cylinder of the radius b, the equilibrium solution thus obtained determines magnetic surfaces within the plasma. That is, the magnetic islands are produced by the external helical currents at $r = b$.

Higher order islands. In actual tokamaks, the discreteness of toroidal coils introduce the perturbation which is $n' = N$ and $m' = Nm$ for a $q = m$ magnetic surface. Here N is the number of the toroidal field coils. For the same helical coordinate, we obtain instead of Eq. (6),

$$\psi = \frac{2k_z}{k^2\ell} B_o \{1 + \gamma [J_o(kr) + \alpha J_{N\ell}(kr) \cos N\phi]\} . \quad (16)$$

Thus for a fixed r_o/a, $J_{N\ell}(kr_o)/J_{N\ell}(ka) \to 0$ as $N \to \infty$. Hence the higher-order magnetic islands are not expected to cause large effects. We also note that if there is an N-fold symmetry, the number of islands must be Nm or higher. In particular, the m-fold island will not be generated by the $N(\neq 1)$ discrete coils.

Discussions. Experimentally, the non-systematic field error of the order of 10^{-4} to 10^{-5} of the main (toroidal) confining magnetic field is very difficult to avoid. (See refs. 5-6). Since the present large-scale tokamaks have the confining magnetic field of approximately 5×10^4 gauss, the non-systematic field error of 0.5 to 5 gauss is expected. The helical component of the error field which resonates with the magnetic surface rotational transform, $2\pi/q$, will give rise to the creation of the magnetic island, whose width was estimated in this paper. According to this analysis, the magnetic island width could reach 5% of the plasma radius for 1.5×10^{-4} of the tokamak poloidal magnetic field for a $q = 2$ surface. This size of the island should be observable with present-day diagnostic techniques.

Although the magnetic islands due to the error are not expected to destroy the magnetic surfaces entirely, it is conceivable that the effective radius of the tokamak is significantly reduced by the error field so as to increase the transport of particles and energy across the magnetic field as observed in refs. 4 and 5.

Experimentally, it is also possible to increase the radial transport by deliberately applying helical current to a tokamak plasma. The helical currents of less than 1000 amperes should be easily applied. One could conceivably improve the plasma confinement by nullifying the helical error field.

For future toroidal devices, it is probably necessary to increase the accuracy of the magnetic field coil alignment by one order of magnitude and/or to install the error field-correcting coils to reduce the sizes of the magnetic islands.

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