Proceedings of a Symposium held in Puerto de la Cruz, Tenerife, Spain 26 – 30 September 1988
SEISMOLOGY OF THE SUN & SUN-LIKE STARS

Proceedings of a Symposium held in
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26 – 30 September 1988

Jointly organised by:
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- Instituto de Astrofísica de Canarias (IAC)

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Foreword

The Symposium on the Seismology of the Sun and Sun-like Stars, held in Puerto de la Cruz, from 26 – 30 September 1988, was jointly organised by the European Space Agency (ESA) and the Instituto de Astrofísica de Canarias (IAC).

In line with earlier meetings in Aarhus (1986), Catania (1983) and the Crimea (1981), the purpose was to bring together the experimenters who are preparing the ground-based networks and future space programmes and the helio- and astero-seismology community to exchange ideas and results.

The meeting was timed to coincide with the definition phase of several second generation experiments, with the aim of creating an atmosphere in which experience and future plans could be shared. The combination of ground and space observations that will be available in the next decade, as well as the theoretical work that is being planned, is bound to produce a turning point in the understanding of the Sun’s interior. Similarly, with the extension of helioseismology techniques to other stars and growing experimental effort, major breakthroughs in the understanding of stellar (and solar) interiors can be foreseen in the near future.

The Symposium was extremely well-attended by those currently working in this field, and these Proceedings therefore constitute an up-to-date collection of information on helio- and asteroseismology. The Proceedings follow the structure of the Symposium, which was organised into eight sessions, six devoted primarily to helioseismology and two to asteroseismology, under the following headings:

1. Frequency stability: Excitation and damping of oscillations
2. Internal rotation: Splitting
3. Ground-based networks and techniques
4. Solar cycle, magnetic fields and oscillations
5. Space projects and low-frequency oscillations
6. Solar modelling techniques
7. Observations and techniques for asteroseismology
8. Models of stellar evolution from seismology.

The meeting would not have been possible without the hard work of a number of people, in particular Mrs L. Gonzalez and Miss A. van den Eijkel, J.A. Belmonte and Drs P.L. Pallé and A. Jones who provided enthusiastic and dedicated help with setting up the symposium. The capable handling of all editorial matters by Erica Rolfe has resulted in the timely publication of these Proceedings.

Our thanks are also due to the session chairmen: Drs Ph. Delache, T. Duvall, P. Scherrer, J. Leibacher, R. Bonnet, M. Gabriel, J. Harvey and H. Shibahashi; to the Scientific Organising Committee; to the speakers for the high quality of their papers and to all the participants for the enthusiastic manner in which they participated in this meeting.

Vicente Domingo Teodoro Roca Cortés
Space Science Department of ESA
Instituto de Astrofísica de Canarias

WELCOMING ADDRESS

Prof. F. Sanchez
Director of the Instituto de Astrofisica de Canarias

It is my pleasure to extend to you all a very warm welcome to Spain, to the Canary Islands and to Puerto de la Cruz. We are honoured to receive you here in Tenerife. All of us at the Instituto de Astrofisica de Canarias are at your service and we will endeavour to make your stay as fruitful and happy as possible.

I would like to congratulate in advance the organisers of this Symposium, especially for the excellent idea of dedicating it to 'The Sun and Sun-like Stars'. I would like to emphasize something that I believe to be important; that this is an opportunity which young astronomers from all over the world will enjoy, an opportunity to live and interact with the best specialists in the subject. It is an old trick of the 'vinateros' (wine makers), to mix a good young wine with the finest 'Solera' (the distinctive flavour of mature wine); this mixture always produces excellent results both in wine making and in scientific research. A few years ago, there was no modern astronomy in Spain, but through this 'vinateros' trick we have been able to obtain a batch of excellent young astrophysicists, some of whom are here today.

As you all know, on the heights of these islands, one of the world's most complete groups of telescopes is being installed at the Teide and Roque de los Muchachos observatories. For solar physicists the most attractive telescopes are at the Teide observatory which you will visit this week; particularly the German VTT and Gregory telescopes as well as the Spanish Solar Laboratory, and in the near future the French Themis. However one must not forget the Swedish Solar Tower on La Palma which many of you will visit on Saturday. Some of you may also have an opportunity to see the modern 4.2 m Anglo-Dutch William Herschel Telescope on the Roque de los Muchachos.

The 'Canarian International Astrophysics Club' is being enlarged almost daily; last April we had the pleasure of welcoming France. The door is always open to those who need the best site to install their experiments.

Back in 1975 a group from Birmingham University visited us at the Teide Observatory wishing to set up an experiment to measure accurately the gravitational redshift using a new spectrometric technique. We agreed; the instruments were installed and one of our postgraduate students, Teo Roca Cortés - now well known to you all - joined the group and in 1979 they demonstrated the existence of discrete acoustic modes of low degree in the solar five-minute oscillation. Since then helioseismology has grown and progressed spectacularly thanks to the work of all of you; in particular our own observatory has provided a continuous flow of very valuable data. The need for uninterrupted observations has led, amongst other observational techniques, to international cooperation based on worldwide networks of helioseismology stations. Projects such as GONG and IRIS have expressed interest in setting up one of their stations at our observatory. Moreover the urgent need to obtain observations has led ESA to dedicate a substantial part of the payload of SOHO to helioseismology. International networks and satellites are a fine basis for international scientific cooperation.

The natural extension of helioseismological techniques to the study of the interior of stars now presents a real technological challenge. Very large flux collectors used over extended periods in time in conjunction with important innovations in the photometric and spectrometric techniques will be required to achieve success. In this regard a Spanish project, in collaboration with Birmingham University, on the 4.2 WHT has carried out unique observations of Arcturus over a three-week period. In the future international cooperation will have to be further expanded in order to achieve an adequate observational strategy. Let us hope that ESA and NASA will become so interested in these projects that they incorporated them in the payload of some of their satellites in the near future. The IAC is keen to participate and collaborate in any projects of this type.

We believe that the IAC, which is becoming increasingly involved in projects to design and construct new instruments, is able to make a valuable contribution to research in the fields of helio- and asteroseismology. Such contributions will result not only from our collaboration with other institutes, with whom we will always be happy to work, but increasingly from the new generation of astrophysicists now working at the IAC's own excellent facilities. You will have an opportunity to evaluate some of them through your visit to the IAC and through the communications presented at this meeting.

In conclusion I would like to offer you all the opportunity to work at the IAC for extended periods. We would like to be able to make even more fruitful the experience of those astrophysicists who have been working in this field for many years. They can spend time here among us as research scientists and visiting professors helping us to train the new generation of astrophysicists who will produce the astronomical successes of the future.

In this spirit I am very happy to welcome scientists from so many different countries in the belief that this meeting will provide an important review of the present status of helio- and asteroseismological research and that it will also lead to exciting new ideas and international collaborative efforts.
Session 1

Frequency stability: Excitation and damping of oscillations

Chairman: Ph. Delache
The Excitation and Damping of Solar Oscillations (Invited Review)

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ABSTRACT. I review the present status of our understanding of the excitation and damping of solar p-mode oscillations. While a few simple statements can be made about the expected properties of p-modes, there is currently no complete theory which explains the observed mode amplitudes and linewidths. I discuss below the growing evidence that the p-modes cannot be overstated and therefore self-excited, but are instead probably stochastically excited by turbulent convection. I discuss the present status of the latter model for exciting p-modes, and some possible future directions.

The frequencies of the observed p-mode oscillations in the sun have been theoretically reproduced with roughly one percent accuracy by assuming adiabatic oscillations of a standard solar model. (For a review of solar oscillations see Libbrecht 1989a.) While it is satisfying that we can calculate the p-mode frequencies to this precision, at the same time we do not at present possess even a sound qualitative understanding of what is driving the modes to their observed amplitudes. This is an extremely challenging physical problem, and a solution will require a fairly detailed understanding of the interaction of acoustic modes with radiation and convection in the solar atmosphere. Its solution will also be welcomed by stellar seismologists, since an understanding of mode excitation in the sun should allow the prediction of mode amplitudes on other stars.

Before discussing specific excitation and damping mechanisms that have been proposed, it is instructive to ask what can be said with some certainty about the physics of solar p-modes. First of all, we have the observations, which were recently extended and improved (Libbrecht 1988a), and are shown in Figure 1. The figure shows the measured mode linewidths and velocity amplitudes for low \( \ell \) modes, together with mode energies which were determined from the amplitudes and model adiabatic eigenfunctions (Figure 2). Our observations show that the properties in Figure 1 are roughly independent of \( \ell \) for \( \ell \leq 100 \), and for practical purposes depend only on p-mode frequency.

These data, the mode linewidths and energies, are the most basic pieces of data we would like to explain theoretically. For modes with \( \ell \leq 100 \) the data are quite good, as can be seen in Figure 1. For higher \( \ell \) we have almost no quantitative information on the mode linewidths, while a fair estimate of the energy per mode has now been made by Kaufman (these proceedings). Other relevant data include the amplitudes of p-mode oscillations in surface brightness (two conflicting measurements can be found in Libbrecht 1989b and Jimenez et al. 1988), measurements of the phase difference between brightness and velocity oscillations, measurements of the velocity amplitudes as a function of height in the solar atmosphere, etc. With all of these additional observations either the quality of the measurements is relatively poor, or the measurements are difficult to interpret theoretically. Therefore we will confine our attention here to a desire to explain the mode linewidths and energies.

Second on our list of fairly certain statements, we are confident that whatever mechanisms are responsible for exciting and damping the p-modes, the energy flow from radiation and convection into and out of the p-modes is taking place very near the solar surface. To see this consider some specific processes.
Figure 1. FWHM linewidths $\Gamma$, power per mode, and total energy (kinetic plus potential) per mode as a function of frequency, for low $\ell$ $p$-modes. The squares show fits from individual mode features with $\ell = 10-24$, while the circles are the result of a more indirect high frequency analysis of $\ell \approx 60$ modes (Libbrecht 1988a). Power per mode is normalized to represent the mean square surface (radial) velocity per mode in (cm/sec)$^2$, averaged over time and over the solar surface. Because of an uncertain normalization of the Doppler images from which these results were derived, the overall normalization of the power could be in error by 10-20 percent. The energy per oscillation mode assumes the observations were made at $r_{2000} = 0.05$. 
Driving from the radiation field, the kappa mechanism is occurring primarily in the hydrogen ionization zone, which sits a few scale heights below the photosphere. The competing process of radiative damping is significant near the photosphere, but is negligible in the solar interior since the time for photons to diffuse an oscillation wavelength in the solar interior is much greater than the 5-minute period of the modes. Energy exchange with turbulent convection depends strongly on the Mach number of the convection, and therefore it too is negligible in the solar interior where the convective velocities are smaller and the sound speed is larger than at the photosphere.

This fact, that the excitation and damping are taking place near the photosphere, combined with the fact that the adiabatic eigenfunctions provide a reasonably accurate approximation to the true eigenfunctions, can explain why the mode properties in Figure 1 are seen to be nearly independent of $\ell$ for $\ell \lesssim 100$. The eigenfunctions show that at the solar surface for these low $\ell$ the horizontal wavelength of the p-modes is much greater than the vertical wavelength. Further, for the low $\ell$ modes the vertical component of the eigenfunctions in the photosphere is nearly independent of $\ell$, and depends only on the frequency of the modes. Thus we would expect the mode linewidths and amplitudes to be independent of $\ell$ for low $\ell$. However note that for higher $\ell$ it is observed that the mode energy does decrease (Libbrecht et al. 1986, and Kaufman, these proceedings).

At some point in this discussion we must ask how our understanding of the oscillations of stars other than the sun might provide insight into the excitation of solar p-modes. Classical Cepheid and RR Lyrae pulsating stars, for example, are known to be driven by a radiative overstability, the $\kappa$ mechanism. These stars have extremely large oscillation amplitudes in comparison to the sun, asymmetric light curves, and tend to oscillate in usually one and at most a few different modes. It is certainly possible and probably
likely that the details of the physical mechanisms responsible for the oscillations in these stars are quite different from those operating in the sun.

The rapidly oscillating Ap stars are an interesting class of oscillating stars that exhibit fairly low amplitude oscillations, typically a few millimagnitudes in intensity and 10 m/sec in velocity. It is believed that the observed oscillations are nonradial ($\ell = 1$ and/or 2) p-modes, and 5 out of the 12 known Ap oscillators exhibit oscillations in more than one mode (Kurtz 1986 and Libbrecht 1988b). The mechanism driving the oscillations in these stars is not known, but a best guess is that it is some sort of overstability mechanism related to the large magnetic fields observed in these stars (Shibahashi 1987).

No other stars are known to pulsate with velocity and brightness amplitudes as small as are observed on the sun; however this means very little since such low amplitudes would make a detection very difficult on any star other than the sun. Of the known different classes of oscillating stars, the majority are thought to be driven by some sort of overstability mechanism, such as the $\kappa$ mechanism. Thus we should first ask whether a similar mechanism is working in the sun.

Are Solar P-modes Overstable? A number of linear stability calculations have tried to answer this question, with inconclusive results (Ando and Osaki 1975, Goldreich and Keeley 1977a, Christensen-Dalsgaard and Fransden 1982, Antia, Chitre and Narashima 1986, Kidman and Cox 1984). Although a number of mechanisms could lead to overstable stellar oscillations (see Brown, Mihalas, and Rhodes 1986), it appears that the $\kappa$ mechanism, responsible for the oscillations of Cepheids and other stars, would be the dominant one for solar p-modes. One consensus reached by the workers in this field is that the correct answer to the mode overstability question is exceedingly difficult to find. It is easy to see why the calculation is so difficult. First of all, most of the driving for the oscillations comes from the hydrogen ionization zone, which lies below the photosphere, where most of the sun's energy is being transported by convection. Thus to do the job right one must investigate how an acoustic oscillation affects the energy transport by convection as well as radiation in this region, which is difficult without a complete model of the convection process. Second, the radiative damping which competes with the driving takes place in the photosphere, where the bulk of the sun's energy reaches the surface in convective cells and is given up into radiation. Here again one must deal with how the various processes of energy transport are changed by the presence of a p-mode oscillation. And, on top of this, one must deal with the direct coupling of the oscillation with convection. Clearly this is a difficult problem, and it is not surprising that concrete results have not come easily in this area.

A fair amount of insight and progress in understanding mode excitation by the $\kappa$ mechanism has been made recently by Kumar and Goldreich (1989), who considered not whether the modes are linearly overstable, but rather what sort of non-linear mechanism might arrest the growth of linearly overstable modes. Non-linear mode coupling is one mechanism which could perhaps halt the growth of overstable modes, and to my knowledge it is the only mechanism that has been proposed. Since the mode amplitudes are very small, it is likely that only the lowest order coupling, namely 3-mode coupling, is important for solar oscillations, and these couplings were calculated by Kumar and Goldreich.

A number of interesting results come out of these calculations. First of all, the energy drain from a typical 5-minute p-mode via 3-mode coupling is of the same magnitude as the measured $E = E\Gamma_0$, where $\Gamma_0$ is equal to $2\pi$ times the FWHM linewidth measured in Figure 1. Thus 3-mode coupling may indeed make a significant contribution to p-mode energetics whether or not the modes are overstable. Detailed calculations show that the strongest couplings to a typical p-mode are between the p-mode, an f-mode, and a third propagating wave which tends to remove energy from the first two. The strongest couplings to f-modes are with other f-modes and propagating waves, which drain energy from the f-modes. The result of this is that 3-mode coupling will drain energy from all the bound p- and f-modes.

Many of the linear stability calculations have found that while some p-modes are overstable, the f-modes are stably damped. This can be understood from the fact that the f-modes are nearly compressionless surface waves, and with no compression they cannot be overstable via the $\kappa$ mechanism. The observations, on the other hand, show that the amplitudes of the f-modes are comparable to p-modes with similar $\ell$ and $\nu$. From the above we see that while 3-mode coupling does transfer some energy from p-modes into f-modes, its net result is still a net loss of energy from the f-modes. Thus we reach the conclusion that if the f-modes can be theoretically proven to be linearly
stable as many calculations indicate, and if 3-mode coupling is our only mechanism for transferring energy among the modes, then another mechanism is needed to drive the f-modes.

Now it is possible that f-modes are themselves overstable, via some overstability mechanism other than the \( \kappa \) mechanism. With this, perhaps both the p- and f-modes are overstable, and the above argument is no longer valid. If this is the case, the situation becomes more complex. Consider a p-mode, which we will assume is overstable via some mechanism which we need not specify. Then to first order we can write \( \dot{E}_p = \alpha E_p \), where \( \alpha > 0 \) gives a linear instability to the mode. A desirable nonlinear damping would then introduce a higher order term. \( \dot{E}_p = \alpha E_p - \beta E_p^2 \), where \( \beta > 0 \), which then gives a steady-state energy \( E_p = \alpha/\beta \).

The calculations of Kumar and Goldreich (1989) show that the real 3-mode coupling does not have this form, since a typical 5-minute p-mode couples most strongly to f-modes. There is some coupling between p-modes alone, but this is all but negligible compared to the dominant couplings. Thus the energy equation for a p-mode can be written approximately as \( \dot{E}_p = \alpha E_p - \beta E_p E_f \), where \( E_f \) is the energy of the relevant f-modes. The equation for the f-modes, on the other hand, is approximately \( \dot{E}_f = \alpha_f E_f - \beta_f E_p E_f' \), reflecting the fact that f-modes couple most strongly to other f-modes. Thus the f-mode energies are independent of the energy in the p-modes. The end result of this is that 3-mode coupling will drain energy from p-modes at a rate that is linear in \( E_p \). If the assumed overstability pumps energy into the p-mode at a greater rate than the energy loss from 3-mode coupling, then we are still left with no way to stop the exponential growth of the p-mode energy. Thus the conclusion is reached that in all cases the overstability model does not fit the observations.

In my opinion the case is not iron-clad, primarily because mode coupling and possibly other non-linear damping mechanisms are complex, and could probably use additional study. Nevertheless the investigations of mode coupling to date do indicate that it would be difficult to halt the growth of overstable p-modes.

**Stochastic Excitation by Turbulent Convection.** This is an alternative mechanism to excite the p-modes, originally proposed by Goldreich and Keeley a decade ago (1977b), and recently worked on by Goldreich and Kumar (1988). The basic idea here is that turbulence in the sun's convection zone generates acoustic noise, and acoustic noise in the sun's resonant cavity results in the excitation of the cavity's normal modes, the p-modes.

The processes which generate acoustic waves from turbulence are usually referred to in terms of a multipole expansion, with the dominant processes coming from monopole, dipole, and quadrupole acoustic radiation. Each of these is easily visualized from the movement of fluid blobs or eddies during the process of convection. When a fluid blob loses its heat at the solar surface, it shrinks in size and sinks. The shrinking changes the volume of the blob, resulting in monopole radiation, while the action of gravity on the blob results in dipole radiation as the blob sinks. If two fluid blobs push against one another, then the force on one is equal and opposite to the reaction force on the other, and the two blobs together emit quadrupole radiation.

Lighthill made the first real progress in the emission of acoustic radiation by free turbulence, and found that quadrupole emission was the dominant process. The power radiated depended strongly on the Mach number of the turbulence, varying as \( M^4 \) (Lighthill 1952, 1954). The noise you hear at the airport, however, may not be due to the quadrupole process, since the cause is not pure free turbulence. This is because the walls of a jet engine are available to interact with the fluid, which provides an external force, which can result in dipole emission.

The _emission_ of acoustic radiation by turbulence cannot be the whole story in the sun, however. Since the radiation is trapped inside the solar cavity, it can also be re-absorbed by the turbulence, and this re-absorption of acoustic radiation by the turbulence will tend to limit the amplitudes of p-modes. The combined picture of emission and absorption of acoustic radiation by turbulence was first discussed by Goldreich and Keeley (1977b), who found that the dominant processes were quadrupole emission and quadrupole absorption. This leads, in the case of turbulence in a box, to an energy \( E \) per acoustic mode given by \( E \approx m v^2 \), where \( m \) is the mass of a resonant turbulent convective eddy, and \( v \) is the characteristic velocity of the turbulence.

Recently Goldreich and Kumar (1988) found that this may be incorrect in the sun, however, because it does not deal properly with the role of gravity in the solar convection. Gravity provides an external force which results in dipole emission of acoustic radiation.
but still only quadrupole absorption. Their calculations were done for acoustic radiation in a box rather than in the real solar atmosphere, and real gravity-driven turbulence was replaced by what the authors describe as "pseudo-convection" (see Goldreich and Kumar 1988). Nevertheless the calculations indicate that the energy per mode increases by a factor of $M^2$ to $E \approx mc^2$, where $c$ is the sound speed in the fluid. In the photosphere a convective eddy is essentially a granulation cell, and we can write $E \approx \rho H L^2 c^2$, where $\rho \approx 10^{-7}$ gm/cm$^3$ is the density at $\tau_{5000} = 1$, $H \approx 100$ km is a scale height, $L \approx 1000$ km is the horizontal size of a granule, and $c \approx 10$ km/sec, giving $E \sim 10^{28}$ ergs, in rough agreement with the observations. Note that the turbulent excitation model not only predicts the right order of magnitude for the p-mode energies, but it also naturally explains the observation that millions of modes are simultaneously excited.

Before we can accept stochastic excitation by turbulent convection as the correct driving mechanism for the sun's p-modes, however, we need to make some detailed comparisons between the expectations of the theory and the observed p-mode properties. Unfortunately our timing is a bit premature for this; although the determination of the correct interaction of turbulence with acoustic modes in a box is a significant achievement, the solar atmosphere is not accurately described as a box of turbulent fluid, and the application of these theoretical ideas to a realistic atmosphere is still in an embryonic state.

Nevertheless we can still make some statements about this excitation mechanism. First of all, the strongest interaction between the oscillation modes and convection must be taking place in the highest layer of convection, the solar granulation. This is the case not only because of the Mach number argument above, but also by considering a frequency argument. The strongest interaction between a convection cell and an acoustic oscillation will occur if the two have the same frequency; i.e. if the turnover time of the convective eddies is equal to the period of the oscillation. The short turnover times of the granulation cells (of order five minutes) are therefore about right for driving the five-minute oscillations. It is likely that there is some sort of high-frequency cutoff in the

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**Figure 3.** Log-log plot of $E\Gamma$, where $E$ and $\Gamma$ are separately plotted in Figure 1. This is equal to the power being pumped into the p-mode oscillators, presumably by convection. Note the high frequency cutoff, and the $\sim \nu^8$ falloff at low frequencies. This simple low frequency behavior is presently unexplained.
mode excitation by convection; above some frequency there will no longer be any resonant convective cells to interact with the $p$-modes. This is mentioned further below.

From mixing length theory alone we expect the driving of the modes to be taking place in the evanescent region where the frequency of the $p$-modes is less than the acoustic cutoff frequency. The frequency derived from the turnover time of an eddy is roughly $v/H$, where $H$ is a scale height and $v$ is the convective velocity, while the acoustic cutoff frequency is of order $c/H$. Since $v/c < 1$ we expect a resonance with the turbulence primarily in the evanescent region.

Note that the modes that will be described by the simplest physics are the low $\ell$ low $\nu$ modes. Low $\ell$ is simpler because the oscillation displacements are almost perfectly vertical, which probably leads to simpler interactions than with displacements that are both vertical and horizontal. Three-mode coupling is also less important at lower $\ell$ (Kumar and Goldreich 1989). Low $\nu$ is simpler for a number of reasons. The primary reason is that the eigenfunctions are simpler in the photosphere, since at lower frequencies the modes are more evanescent in the atmosphere. With these low $\nu$ modes the whole atmosphere is moving up and down uniformly, which tends to minimize radiative damping. Thus the adiabatic eigenfunctions better approximate the real eigenfunctions at low $\nu$. The 3-mode coupling is also weaker at low $\nu$, and at low $\nu$ one will be avoiding any difficulties associated with whatever high-$\nu$ convective cutoff is in effect. Since we would like to keep the physics simple when comparing the mode properties with theory, the best place to look is at the low $\ell$ low $\nu$ modes.

If our desire is to achieve an accurate agreement between theory and the observed linewidths in Figure 1, we have a difficult task ahead. Calculations of the damping from nonadiabatic effects using a variety of models (Christensen-Dalsgaard, Gough, and Libbrecht 1989) give results that are in rough agreement with the observations, as does a calculation of the damping from the interaction with convection (turbulent viscosity) (Goldreich and Keeley 1977a). It is probably safe to ignore the damping from 3-mode coupling, since in Kumar and Goldreich (1989) they overestimated the energies on the $f$-mode ridge (Kaufman, these proceedings), and thus overestimated the damping due to 3-mode coupling. Note that the damping calculated from nonadiabatic effects is comparable to that calculated from the turbulent viscosity, and the two even show the same frequency dependence. This is probably a fundamental result, whose explanation lies in the relationship between convective and radiative energy transport (Goldreich, personal communication).

The agreement found between the calculated and observed linewidths is not bad for this business, but unfortunately we cannot at present even think about accurately reproducing the fine structure observed in the linewidth as a function of frequency, seen in Figure 1. In order to do so we would have to very accurately calculate the nonadiabatic effects, and as was mentioned above this is an extremely difficult problem. I believe the most we can hope for in the near future is see the the $\Gamma(\nu)$ curve reproduced for $\nu < 2.4$ mHz, where the curve is simple, and with luck so is the necessary physics. Clearly, additional measurements at much lower frequencies would be very useful; the observations will be quite difficult, however, because the linewidths are narrow and the mode amplitudes are small at low frequencies.

Attempts to theoretically reproduce the energy per mode as a function of frequency have so far not met with a lot of success. The simple $E = mc^2$ estimate would predict that the energy/mode would increase with decreasing $\nu$, because the mass of a resonant eddy is larger at lower frequencies. As is seen in Figure 1, this is not observed to be the case. Perhaps the enhanced power in convection at the granulation timescale may have something to do with this, but unfortunately at present this cannot be said with any certainty.

An interesting observation can be made by plotting $\dot{E}T = \dot{E}$ as a function of frequency (Figure 3), which for a stochastically excited $p$-mode is a measure of the power being pumped into the mode, presumably by convection. If one were to leave this input power fixed and arbitrarily increase, say, the radiative damping, then the energy/mode would increase, but the linewidth would increase, leaving $\dot{E}T$ unchanged. Note in the figure the turnover in $\dot{E}T$ at frequencies above 3 mHz. This may be reflecting the high-$\nu$ cutoff in convection mentioned above, giving an indirect measure of the power spectrum of turbulent convection in the solar atmosphere.

As is also clearly seen in Figure 3, $\dot{E}T(\nu)$ at low frequencies is very accurately described by a power law in $\nu$, $\dot{E}T \propto \nu^8$. It is certainly tempting to suggest that the exponent of almost precisely 8 is not an accident, and may be telling us something significant and
simple about the interaction of p-modes with convection. This story is not finished yet.

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ON THE $\kappa$-MECHANISM AND
THE MULTI-DIMENSIONAL EDDINGTON APPROXIMATION

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ABSTRACT/RESUME

To consider the possibility of a $\kappa$-like overstability mechanism as the driving force of solar modes of oscillation requires a theoretical description of the interaction between the hydrodynamics of the motion and the radiative energy transport. The aim of this paper is to answer the question of whether or not the frequently-used multi-dimensional Eddington approximation is suitable for such purposes. It is found that Eddington's approximation is generally adequate for giving account of the radiative damping effects of Planck function fluctuations, while it is in gross disagreement with the exact results with respect to the radiative transfer effects of opacity fluctuations. Therefore, it is concluded that its use should be avoided when investigating $\kappa$-like excitation mechanisms for nonradial stellar modes of oscillation.

Keywords: Radiative Transfer, Eddington Approximation, Radiative Instabilities: $\kappa$-mechanism, Nonradial Oscillations

1. INTRODUCTION

The problem of the excitation mechanism that pumps energy into the solar modes of oscillation is still an unresolved issue. The modes may be driven stochastically by turbulent convection (cf. Ref. 1,2). The modes may also be able to extract energy from the radiation field via an overstability mechanism similar to the $\kappa$-mechanism (cf. Ref. 3,4).

As discussed recently by Trujillo-Bueno and Kneer (Ref. 5), the radiative transfer effects of Planck function (B) fluctuations generally play a stabilizing role. On the contrary, opacity ($\chi$) fluctuations may in principle originate radiative instabilities, the actual appearance of these requiring large radiative fluxes and/or large values of the opacity's thermal sensitivity. Given this potential importance of the radiative transfer effects of $\chi$ fluctuations as a destabilizing agent, a proper theoretical description of the interaction between the hydrodynamics of the motion and the radiative energy exchange could well be vital for conclusively deciding whether or not a $\kappa$-like mechanism is actually responsible of the excitation of some of the solar modes of oscillation. Since the usual procedure in current theoretical investigations of nonradial stellar oscillations is to use the multi-dimensional Eddington approximation for describing such radiation-matter interaction, an investigation of the ability of Eddington's description to reproduce the actual radiative transfer effects of opacity fluctuations is particularly important.

Christensen-Dalsgaard and Frandsen (Ref. 6) demonstrated that, for investigating radial oscillations in the solar atmosphere, the Eddington approximation is generally adequate. These authors also performed preliminary estimates for nonradial oscillations which indicate, in accord with previous calculations of Kneer and Heasley (Ref. 7), that the radiation field increasingly deviates from the correct value as the horizontal wavelength of the perturbation decreases. This latter behaviour has to be expected, since Eddington's approximation is only exact for those situations in which the angular dependence of the specific intensity of the radiation field is representable as an expansion in spherical harmonics with the coefficient of the second spherical harmonic equal to zero (cf. Ref. 8,9). As the horizontal wavelength of the oscillation modes decreases, the anisotropy of the specific intensity is expected to increase, and therefore so too the inability of the above-mentioned expansion to reproduce the correct angular dependence of the radiation field. However, in radiation hydrodynamic calculations, the relevant quantity is the net energy which the plasma gains or loses per unit volume and time, which is not only determined by the precise value of the radiation field.

Thus, for example, Unno and Spiegel (Ref. 9) demonstrated that, for small temperature perturbations in an infinite homogeneous medium, the multi-dimensional Eddington approximation is an adequate description of energy transfer by radiation. As in an infinite homogeneous medium only fluctuations in the Planck function may have an effect (cf. Ref. 10; see also Ref. 5), one may consider the results of these authors as indicating that the Eddington approximation should also work fairly well for reproducing the radiative...
transfer effects of \( B \) perturbations in stellar atmospheres, i.e. in the presence of a surface and stratification. However, besides the necessity of checking whether or not this expectation is confirmed, there remains the question of the ability of Eddington's description to reproduce the actual radiative transfer effects of opacity disturbances.

In order to answer the above questions we will investigate the radiative response of a schematic grey solar model atmosphere in radiative equilibrium (RE) and in LTE to small horizontal sinusoidal temperature fluctuations. The time behaviour of the imposed temperature perturbations will be assumed to be governed by the following radiative heat equation (cf. Ref. 9):

\[
\frac{\partial T}{\partial t} = -\nabla F = 4\pi \chi (J - B), \tag{1}
\]

where \( F \) is the radiation flux vector, \( J \) the frequency-integrated mean intensity of the radiation field, \( B = (\sigma / \pi) T^4 \) the frequency-integrated Planck function, with \( \sigma \) the Stefan-Boltzmann constant, \( \rho \) the density, and \( c_p \) the specific heat at constant pressure for which the value \( 1.6 \times 10^7 \text{erg K}^{-1} \text{g}^{-1} \) is adopted throughout the atmosphere.

Considering separately the influence of \( B \) and \( \chi \) fluctuations on the radiative energy transfer, a useful answer concerning the applicability of the multi-dimensional Eddington approximation will be given by first calculating Eddington's growth rates for the above-mentioned perturbations, and then comparing them with the results from more sophisticated calculations.

2. FORMULATION OF THE PROBLEM

Consider a grey solar RE model with gravitational stratification prescribed by the following expressions for the unperturbed opacity \( \chi \) and density \( \rho \):

\[
\chi = \chi_0 (\exp(-z/H) - 1), \quad \rho = \rho_0 (\exp(-z/2H)).
\]

Choosing \( \chi_0 \) such that the optical depth \( \tau \) at \( z = 0 \) is unity, and measuring geometrical distances in units of the opacity scale height \( H \) (i.e. 60 km for \( H \) - absorption in the solar photosphere), one has \( \tau = \exp(-z) \), with \( z \) the height in the atmosphere. Assume that small temperature perturbations of the form \( T = T(z) + \Delta T \cos kx \) with wavenumber \( k = 2\pi / \Lambda \) (\( \Lambda \) = horizontal wavelength) are applied to this plane-parallel model atmosphere. As a result of these temperature perturbations both \( B \) and \( \chi \) sinusoidal fluctuations are generated, with amplitudes respectively given by

\[
\Delta B = 4(\sigma / \pi) J^2(z) \Delta T, \tag{2}
\]

\[
\alpha = \Delta \chi = \frac{\partial \ln \chi}{\partial \ln T} \frac{\Delta B}{\Delta B(z)}. \tag{3}
\]

The equation which governs the time evolution of \( \Delta T \) follows from the linearization of the radiative heat equation (1):

\[
\frac{\partial \Delta T}{\partial t} = -[n_{\Delta B}(z,k) + n_{\Delta \chi}(z,k)] \Delta T. \tag{4}
\]

The local growth rates \( n_{\Delta B} \) and \( n_{\Delta \chi} \) are due to the \( B \) and \( \chi \) fluctuations, respectively. The expressions for these growth rates are given by

\[
n_{\Delta B} = \frac{16\sigma J^3}{c_p \rho} \frac{\chi_0}{\Delta B} (1 - \frac{\Delta J}{\Delta B}), \tag{5}
\]

\[
n_{\Delta \chi} = \frac{4\pi \chi J}{c_p \rho T} \frac{\partial \ln \chi}{\partial \ln T} (-\frac{\Delta J}{\Delta B}). \tag{6}
\]

In order to obtain these growth rates the linear response of the radiation field to \( B \) and \( \chi \) perturbations has to be calculated separately. On grounds of symmetry the mean intensity fluctuation responds either in phase or \( \pi \) out of phase to horizontal fluctuations. Therefore both local growth rates are real, and the system can be either locally stable or unstable, stability requiring \( n_{\Delta B} + n_{\Delta \chi} > 0 \).

The mean intensity of the radiation field responds in phase to \( B \) perturbations. This is because fluctuations in the Planck function only give rise to an alteration of the emissivity of the stellar gas. For reasonable spatial variations of \( \Delta B, \Delta J < \Delta B \), thus \( n_{\Delta B} > 0 \), which implies that the radiative transfer effects of \( B \) fluctuations generally play a stabilizing role.

On the contrary, the radiative transfer effects of \( \chi \) fluctuations can originate radiative instabilities. The possible types of such instabilities in grey RE atmospheres have recently been established by Trujillo-Bueno and Kneer (Ref. 5). One of these types of instability mechanisms takes place when the amplitude \( \alpha \) (cf. Eqn (3)) of the opacity perturbation increases with height in the atmosphere. Under such circumstances the mean intensity responds in phase to the opacity perturbation below a height in the atmosphere close to \( \tau = 1 \), but \( \pi \) out of phase above such a critical height. Consequently, if \( \partial \ln \chi / \partial \ln T > 0 \), the atmosphere can in principle be unstable below such a critical height (cf. Eqn (6)). As pointed out in the above-mentioned work stellar modes of oscillation could well be extracting energy from the radiation field through this very mechanism. For this reason the numerical calculations below were performed with an outwardly increasing \( \alpha (\tau) \) obtained by choosing \( \partial \ln \chi / \partial \ln T = 1 \) and \( \Delta B = 1 \) in Eqn (3).

3. APPROXIMATE VERSUS EXACT DESCRIPTIONS

In the Eddington approximation the radiation flux vector is given by the following diffusion-like equation (see e.g. Ref. 9, 11):
Compared with an exact description of energy transfer by radiation Eddington's approximation is computationally-advantageous because one does not have to deal with the angular dependence of the radiation field. The differential equation which one has to solve for obtaining $\Delta J$ follows from the linearization of Eqn (1), with $F$ as given by (7):

$$\frac{1}{3} \frac{\partial^2}{\partial r^2} \Delta J = (1 + \frac{k^2}{3r^2}) \Delta J + \frac{1}{3} \frac{\partial J}{\partial r} \frac{\partial \alpha}{\partial r} - \Delta B. \quad (8)$$

This second-order differential equation, together with an upper and a lower boundary condition, has been numerically solved by using Feautrier's method (cf. Ref. 12). (For details see Ref. 13.) Exact (i.e. multi-ray) solutions of the linearized radiative transfer equation were performed applying the numerical strategy of Vincen and Heasley (Ref. 7).

In the following subsections Eddington's growth rates are compared with the exact growth rates, separately for $B$ and $\chi$ fluctuations.

### 3.1 $B$ - Fluctuations

As seen in Fig. 1 Eddington's description of energy transfer by radiation adequately reproduces the general behaviour of the radiative damping effects of $B$ fluctuations. Both, the optically thick ($k/\chi \rightarrow 0$) and thin ($k/\chi \rightarrow \infty$) limits are recovered for all horizontal wavenumbers $k$. The main disagreement occurs at the atmospheric height where the transition between these two limits takes place. Here, Eddington's approximation gives shorter relaxation times $t_r = 1/n_{\Delta B}$ than the exact values. In particular, for $k \approx 0.2$ (horizontal wavelength $\Lambda \approx 30$ scale heights $= 1800$ Km on the Sun), a factor two difference between the two calculations is found. For $k > 1$, although the anisotropy of the radiation field is still greater, and the Eddington and exact mean intensity amplitudes differ considerably, the efficient horizontal transfer of photons has damped out the fluctuations of the radiative quantities to such small levels that the local emission losses almost play the only role. Consequently, for $k > 1$, the agreement between the two calculations is still better than the factor two found for smaller values of the horizontal wavenumber $k$. This is indeed the reason why, although Eddington's $\Delta J$ tends to zero with increasing $k$ much more rapidly than the real $\Delta J$ does, Eddington's approximation still gives the correct value of the growth rate $n_{\Delta B}$ in the optically thin limit.

Finally, note that the radiative relaxation time $t_r = 1/n_{\Delta B}$ is smallest near $r = 1$. In these atmospheric layers the transfer of grey continuum radiation efficiently dissipates thermal perturbations within a few seconds. That horizontal transfer of radiation is important can be recognized at once by noting that, near $r = 1$, $t_r$ varies approximately between 15 and 1 seconds as the wavenumber $k$ increases from zero (plane-parallel geometry) to infinity (see also Ref. 14).

### 3.2 $\chi$ - Fluctuations

The exact growth rate $n_{\Delta \chi}$ is shown in Fig. 2 (see Ref. 5 for a more detailed discussion on the influence of opacity fluctuations on the radiative relaxation time). For the purposes of this paper it is first important to note that the critical atmospheric height where $n_{\Delta \chi}$ changes sign diminishes as the wavenumber $k$ increases. Note also that the larger $k$ (i.e. the smaller the horizontal wavelength $\Lambda$), the smaller the importance of the radiative transfer effects of opacity fluctuations, and therefore the less likely the possibility of a $\chi$-like overstability mechanism. However, this decreasing-influence (with increasing $k$) of the radiative transfer effects of $\chi$ fluctuations becomes really important for horizontal wavelengths smaller than about 1800 Km (i.e. for $k > 0.2$). For the horizontal wavelengths corresponding to the most extensively studied p-modes in helioseismology (i.e. for $k < 0.2$) one would not require unrealistic large values of $\partial m_1/\partial nT$ for having the possibility of local radiative instabilities (i.e. $n_{\Delta B} + n_{\Delta \chi} < 0$; compare Figs. 1 and 2). In the solar atmosphere, where the $H^-$ ion makes the
primary contribution to the opacity, \( \partial \ln \beta / \partial \ln T \) strongly increases with height just below optical depth unity. It then starts to decrease with height, down from a value slightly larger than 10, once the photosphere has been reached (see e.g. Fig. 2 of Ref. 3). Consequently, if instabilities were to occur, they would most likely be originated in a narrow sub-photospheric region, where energy transport by convection is relevant in the Sun.

Figure 2. Exact growth rate \( n_{\Delta \lambda} \) for horizontal opacity fluctuations with an outwardly increasing \( \alpha \) obtained by choosing \( \Delta B = 1 \) and \( \partial \ln \beta / \partial \ln T = 1 \) in Eqn (3). Solid lines: positive values. Dotted lines: negative values. The parameter is the wavenumber \( k \). The signs would have to be reversed if \( \partial \ln \beta / \partial \ln T \) were -1 instead of 1.

Fig. 3 shows the growth rate \( n_{\Delta \lambda} \) calculated with Eddington's approximation. The figure also includes curves which correspond to horizontal wavelengths \( \lambda \) of 200, 100, and 50 opacity scale heights. These imply wavelengths of about \( 12 \times 10^3 \), \( 6 \times 10^3 \), and \( 3 \times 10^3 \) Km on the solar photosphere (recall that in this atmospheric region the opacity scale height is \( \sim 60 \)Km). It is seen that, whereas the exact \( n_{\Delta \lambda} \) depicted in Fig. 2 essentially predicts radiative relaxation for \( \tau < 1 \) but radiative amplification for \( \tau > 1 \), Eddington's growth rates imply that the greater the wavenumber \( k \), the smaller the critical atmospheric height where this transition takes place. For example, for \( k \leq 0.2 \), the exact height in the atmosphere below which these opacity fluctuations would produce radiative instabilities, if they were the only operating cause of radiative energy exchange, lies close to \( z = 1 \). On the contrary, Eddington's approximation would place such a critical height between \( z \approx 1 \) and \( z \approx 4 \) as \( \lambda \) varies from infinity to \( \sim 1800 \)Km on the Sun.

On the other hand, for wavenumbers \( k \leq 0.2 \), Eddington's approximation predicts that the larger the wavenumber, the more likely the possibility of a \( \kappa \)-like mechanism. This wrong qualitative behaviour can actually be observed in the growth rates obtained by Ando and Osaki (Ref. 3) as the degree \( l \) of the p-modes is increased.

Figure 3. Growth rate \( n_{\Delta \lambda} \) calculated with Eddington's approximation. The horizontal opacity fluctuations are as in Fig. 2. The parameter is the wavenumber \( k \). Note that already for \( k = 0.031 \) (\( \lambda = 200 \) opacity scale heights) there exist significant deviations from the exact growth rate.

4. CONCLUSIONS

The multi-dimensional Eddington approximation is generally adequate for reproducing the radiative transfer effects of \( \beta \) fluctuations in stellar atmospheres, i.e., in the presence of a surface and stratification. However, with the exception of the plane-parallel case, it is in gross disagreement with the exact results concerning the radiative transfer effects of \( \chi \) fluctuations for \( \tau \lesssim 1 \).
Eddington's approximation becomes most unreliable near to optical depth unity. As these are precisely the atmospheric regions which require a proper treatment of energy transfer by radiation in order to correctly investigate plasma atmospheric processes such as granular convection, magnetic flux concentrations, or oscillatory phenomena, it is concluded here that the applicability of Eddington's approximation should be restricted to situations where the temperature sensitivity of the opacity is small enough to be sure that the radiative damping effects of B fluctuations play the dominant role. Accordingly, Eddington's approximation should be avoided when investigating the possibility of a χ-like overstability mechanism for nonradial stellar modes of oscillation.

ACKNOWLEDGMENTS: This work was carried out while I was a Ph. D. student at the Universitäts-Sternwarte in Göttingen. I would like to thank Prof. Franz Kneer for kindly having suggested that I should investigate this problem, and for the many valuable things I have learned from him during these last few years. Support by the Deutsche Forschungsgemeinschaft through grant Kni 152/3 is gratefully acknowledged.

5. REFERENCES

ON THE DIAGNOSTIC OF 5m OSCILLATIONS THROUGH PHOTOSPHERIC LINE PROFILES

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ABSTRACT

Recent measurements show that the 5m oscillations have different amplitudes in the opposite flanks of several photospheric lines. This effect can be suitably described by giving the (blue wing)/(red wing) ratio of the r.m.s. wavelength shift at any given level of residual intensity.

We suggest that the main cause is radiative damping, which produces a depth dependent phase difference between the velocity and the thermodynamical (temperature and pressure) perturbations within the line forming region. Synthetic profiles of the Fe I λ6301.5 Å line, obtained by numerical solution of the time independent radiative transfer equation in the oscillating atmosphere (quasi static radiation field approximation), show that agreement between observed and computed profiles can be achieved.

The consequences of this effect (which is also dependent on the magnetic flux concentration) on the diagnostic properties of line profiles are briefly discussed.

Keywords: solar oscillations, radiative damping, line asymmetries, line profiles

1. INTRODUCTION

During the past decade considerable efforts have been made to observe very carefully the effects of the 5m oscillations on the profiles of several Fraunhofer lines and temporal variations of the observed profiles have clearly been detected.

In particular Roca Cortes et al. (Ref. 1) and Bonet et al. (Ref. 2) have shown that the asymmetry of the K I λ7699.0 Å line, at the disk center, undergoes periodic variations anticorrelated with the oscillations of the whole line. This anticorrelation has been straightforwardly explained by Marmolino et al. (Ref. 3) and by Severino et al. (Ref. 4) as a direct consequence of the (outward) increasing amplitude of the velocity wave within the solar photosphere.

Recent measurements (Cavallini et al., Refs. 5 and 6) show that the 5m oscillations have different amplitudes in the opposite flanks of Fraunhofer lines. This effect can be suitably described by giving the (blue wing)/(red wing) ratio of the r.m.s. wavelength shift (henceforth B/R) at any given level of residual intensity. We suggested (Ref. 7) that the main cause of this effect is the existence of a depth dependent phase lag (different from 90°) between the velocity and the thermodynamical perturbations within the line forming region. The suggestion has been followed, with partial success, by Gomez et al. (Ref. 8), who tried also to include granulation effects to fit the observations.

To support our assertion we obtained (Ref. 9), from a first order perturbative solution of the fluid dynamics equations, the polarization relations for monochromatic, two-dimensional waves (including the effect of radiative damping to get the right values of the phase differences between the perturbations) and computed the effects of such waves on typical line profiles. In the present paper we will outline our main results, giving some hints on the way in which the fit can be improved and evaluating some of the consequences that might concern the diagnostic properties of the line profiles.

2. WAVE MODEL

The solar atmosphere, at least in the photospheric and low chromospheric layers, can be described as a simple one component fluid; hence the fundamental equations can be written in the standard form

\[
\frac{Dp}{Dt} + \rho \nabla \cdot \mathbf{v} = 0
\]

\[
\frac{D\mathbf{v}}{Dt} = -\nabla P + \rho g
\]

\[
\frac{DP}{Dt} - \Gamma_1 \frac{D\rho}{Dt} = (\Gamma_2 - 1) \frac{Dq}{Dt}
\]

where \(Dp/Dt\) is the Stokes derivative and \(Dq/Dt\) is the net rate of radiative energy input, per unit volume, to the gas from external sources; all other notations are standard.

The net energy input is due to radiative transport and can be roughly estimated by Spiegel formula (Ref. 10):

\[
\frac{Dq}{Dt} = -\rho c_v \frac{\delta T}{t_{\text{RB}}}
\]

where \(t_{\text{RB}}\) is the radiative relaxation time as given by Ulrich (Ref. 11) to include the effect of the upper boundary in the
case of a vertically stratified atmosphere.

Within the formation region of the photospheric lines used in the quoted works, a first order perturbation theory can be safely applied. In addition we restricted ourselves to bidimensional perturbations of a plane-parallel static atmosphere having the following form:

\[
v_z = v_z(x, z, t) = X \exp[i(\omega t - k_x x - K_z z)] \\
v_x = v_x(x, z, t) = W \exp[i(\omega t - k_x x - K_z z)]
\]

\[
\frac{\delta T(x, z, t)}{T(z)} = \Theta \exp[i(\omega t - k_x x - K_z z)] \\
\frac{\delta P(x, z, t)}{P(z)} = \Pi \exp[i(\omega t - k_x x - K_z z)]
\]

where \(W, X, \Theta, \Pi\) and \(K_z\) are the complex amplitudes and vertical wavenumber; \(k_x\) is the real horizontal wavenumber and \(z\) is the geometrical height in the atmosphere.

To get an analytical solution, the temperature and the radiative relaxation time were kept constant with \(z\), therefore all our waveforms should have constant coefficients. On the contrary, within the final expressions for \(v_z, v_x, \frac{\delta T}{T}\) and \(\frac{\delta P}{P}\), the temperature and the radiative relaxation time were allowed to vary with depth, simulating the stratification of the solar atmosphere. The final results are plotted in Figure 1, in which the HSRA is used as unperturbed model. Solid lines correspond to vertical waves, dashed lines to the indicated values of the horizontal wavenumber \(\Lambda = 2\pi/k_x\) expressed in Mm.

For our purpose the main change, with respect to the adiabatic case, is in the trend of the \(T - v\) phase difference. Note that, while all phase differences are not very sensitive to \(\Lambda\) variations within the considered range, the amplitudes of the temperature and pressure waves are instead substantially affected by the same variations. Notice also the opposite trend of the perturbation amplitudes for temperature (increasing with \(\Lambda\)) and pressure (decreasing with \(\Lambda\)).

For purely vertical waves (\(\Lambda \to \infty\)) the waveforms reduce to those proposed by Noyes et al. (Ref. 12) and used by Gomez et al. (Ref. 8).

3. OBSERVABLE EFFECTS

Synthetic profiles of the Fe I \(\lambda 6301.5\) Å line, have subsequently been obtained -at any instant- by a numerical solution of the \(LTE\), time independent, radiative transfer equation in the oscillating atmosphere (quasi static radiation field approximation).

The most prominent effects observable in the experimental data are all satisfactorily reproduced by our calculations.

The whole bisector of the computed profiles (i.e. the locus of the points midway between the equal intensity points on either flanks of the line) oscillates with an almost constant phase: the phase variations between different intensity levels (shown in Figure 2) stay within 4°. Our values of \(\phi_x\) are also in substantial agreement with the results obtained by Lites and Chipman (Ref. 13) using the phase differences between different lines. The amplitude of the oscillation increases from the wings to the core of the line, in agreement with all previous results.

The role played by the presence of the radiative damping term and by the choice of different values for \(\Lambda\) in deter-

![Figure 1. Computed waveforms of the \(5^m\) oscillations in the solar photosphere (\(h = 0\) corresponds to \(\tau_{5000} = 0\)).](image-url)
maining the $B/R$ ratio is clearly shown by our numerical experiments (see Figure 3).

For purely vertical waves the oscillation amplitude is always larger in the blue than in the red, as found by Gomez et al. (Ref. 8), but cannot be fitted to the observations ($v_0$ is the only free parameter); qualitative agreement is reached within a limited range of residual intensity (from the line core to about 60% for $\lambda 6305.1$, because the observed $B/R$ becomes smaller than unity in the wings according to Arcetri measurements).

When $A$ is decreased we see that, for any residual intensity level, $B/R$ decreases reaching values less than unity in the line wings when $A$ is about 5 Mm. However the shape of the curve $B/R$ vs. $I$ becomes specular with respect to that obtained for large values of $A$ and does not resemble the observed one.

Even the superposition of modes with different $A$ does not help to fit the data on the whole line profile, because we cannot carry the upper part of the curve beyond $B/R = 1$ without compromising the agreement in the line core; moreover the weight that ought to be given to the modes of small $A$ seems unrealistically high.

A way out could easily be found if we might decrease the amplitude of the temperature wave in the deepest layers (below 200 Km), as we essentially did in our ad hoc model varying $A$ with height (see Figure 3), but then we should derive consistently the new waveforms from the equations of fluid dynamics.

This might suggest that our computed temperature waveforms are not sufficiently accurate throughout the whole atmosphere (and this is certainly possible because of the approximations we used), but another explanation may exist.

4. DISCUSSION

To make our argument clear, we can resort to a 1-st order perturbative solution of the line transfer equation, which for the present waveforms gives essentially the same results of the exact solution as far as fully resolved oscillations are considered (see Ref. 9 for all details) and is much simpler to handle.

At a fixed wavelength distance from the unperturbed line center, the effect on the emergent intensity of any of the considered perturbations can in fact be written as the integral of the perturbation, times the proper response function, along the whole line of sight:

$$
\delta I_A^{(v)} = \int_{-\infty}^{+\infty} R F V_A(h) v(h) dh \\
\delta I_A^{(T)} = \int_{-\infty}^{+\infty} R F T_A(h) \frac{\delta T}{T}(h) dh
$$
\[ \delta I_{\lambda}^{(P)} = \int_{-\infty}^{+\infty} RFP_{\lambda}(h) \frac{\delta P}{P}(h) dh \]

Finally, to derive the corresponding shift of the line flank at a fixed intensity level, we can simply divide each \( \delta f_{\lambda} \) by the wavelength derivative of the unperturbed profile at the same point. The separate effects of the different perturbations on the line profile can then be added to get the combined result.

We immediately see that:

i) the velocity has the same effect on both flanks of the line at a given intensity level, causing a shift of the line bisector that is a function of \( I \);

ii) the temperature and pressure, affecting only the width of the line without inducing a net shift, must cause, at a given intensity level, equal but opposite shifts on the two flanks;

iii) the three intensity perturbations are sinusoidal in time and we must carefully take into account their phase differences to get the total amplitude and phase (they add like two-dimensional vectors).

If the phase difference of both temperature and pressure with respect to the velocity were equal to \(-90^\circ\) throughout the whole atmosphere, as in the adiabatic case, we would obtain the same shift on both flanks (\( B/R = 1 \)). On the contrary we get \( B/R > 1 \) whenever the temperature effects dominate those of the pressure in the line forming layers (e.g. in the core of \( \lambda 6305.1 \), formed around 300 Km). The difference is rapidly decreasing in the wings, because the height of the line forming layers is rapidly decreasing and pressure effects begin to cancel those of the temperature.

As we said, we can never reach a good fit of the \( B/R \) on the whole line profile; but now we gained sufficient insight into the mechanisms of line formation to understand in details how the results are controlled by the different features of the waveforms and this should obviously be of great help for future work, providing useful guidelines for their possible improvement.

An important result is that now we can also estimate the cumulative effects of all unresolved motions that are not linearly averaged in the available experimental data because of lacking resolution, and are not accounted by our calculations, in the following way.

If in our linearized approach we assume that the line profile is symmetrical, to the 0-th order, with respect to the line center, we obtain the same results of the non linear calculations shown in Figure 3. However this is an oversimplification, because the unresolved motions that we have not considered are instead responsible of the average line asymmetry (the contribution of the 5-th oscillation is negligible). To the 1-st order their effect can be evaluated, on the two flanks of the line, dividing each \( \delta I_{\lambda} \), at any intensity level, by the proper (either blue or red) derivative of the average experimental profile (Ref. 9).

The correction we get is approximately the right one, showing that the oscillations are not strongly modified by the unresolved motions (first of all by the granulation), although they are non linearly coupled in determining the detailed solution of the radiative transfer equation. The signature of the unresolved motions is present in the \( B/R \) ratio, but -at least in our case- it is not the main cause of the r.m.s. asymmetry.

The same argument can also be used to explain part of the correlation between the observed variations of the line bisector and of the \( B/R \) in the different regions of the solar disk (e.g. in magnetic and non magnetic regions) and should be useful in disentangling the causes of several observed effects.

Concluding, we might say that the agreement with the most recent experimental results seem to imply that our simple picture of the 5-th oscillations is fairly realistic, provided that non adiabatic effects are taken in due account, and contain valuable information about the behaviour of these waves within the solar photosphere. A promising basis is offered to improve the modelling of the oscillations and test our understanding of line formation processes in a moving atmosphere.

REFERENCES

NONLINEAR STUDIES OF SOLAR GRAavity MODES driven by Nuclear burning of $^3$He IN THE CORE

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ABSTRACT

The finite-amplitude behavior of gravity-mode oscillations driven within the deep interior of the sun has been studied by means of a simple idealized model. Such g modes may be self-excited by their ability to extract energy from the nuclear burning of $^3$He in the core. Both a nonlinear bifurcation analysis and numerical simulations of the behavior of this instability suggest that the growth of $^3$He-driven oscillations is likely to be limited to an amplitude which is insufficient to induce convective instability in the core, a process which has been proposed as a mechanism for core mixing. The numerical results also indicate that if the oscillations are linearly unstable then the degeneracy in linear theory between standing and travelling $g$ modes is broken by nonlinear effects. The oscillations thus develop into a left- or right-travelling wave rather than a standing wave or other superposition of horizontally propagating waves.

Keywords: $g$ modes, mode excitation, core mixing

1. INTRODUCTION

Although solar gravity mode oscillations are almost certainly excited at some finite amplitude, it is not at present universally agreed upon that such modes have been detected in helioseismic data. It would thus seem likely that all solar g modes are linearly stable in the present epoch. This may not always have been the case, however, as numerous linear stability calculations suggest that one or more low-order, low-degree g modes (e.g. $g_{l=1}$) became unstable during the first $5 \times 10^9$ years or so of the sun’s main-sequence lifetime (Refs. 1-3, 14, 16).

These linear calculations indicate that the instability of solar g modes, if achieved, is a consequence of the so-called e-mechanism, i.e. the tendency for a portion of the sun’s luminosity to be channeled into the kinetic energy of an oscillation mode as a result of the steep increase with temperature of the rates of the energy-producing nuclear reactions (e.g. Ref. 5). In this instance, the species primarily responsible for driving the instability is $^3$He. Until the present era, the abundance of $^3$He has been governed principally by the PPI reaction chain, which creates $^3$He through the reaction of hydrogen with itself and then with deuterium, a reaction sequence whose rate is proportional to $T^4$, where $T$ is the temperature (see Table 1). $^3$He is destroyed primarily by reacting with itself, a process whose rate is proportional to $T^{14}$. As a result, the equilibrium abundance of $^3$He is a rapidly decreasing function of temperature, scaling as $T^{-6}$.

Thus, once sufficient time has elapsed for the PPI chain to attain equilibrium, the abundance of $^3$He increases steeply with radius (see Figure 1a). This has two important consequences. The first is that nonradial oscillations tend to be driven. This is because a material element, when displaced downward, tends to be both hotter than its surroundings (since the core is stably stratified) and also richer in $^3$He, and thus produces energy at a rate which exceeds that in the matter surrounding it. This in turn results in a comparative increase in the element’s entropy, which enhances the buoyancy restoring force and thus tends to drive over-stability.

A second consequence of the outward-increasing $^3$He abundance is made apparent by the following argument, first put forth by Dike and Gough (Ref. 7). Suppose that some means were available to transport some of the $^3$He-rich material at larger radius inward. Such a process would increase the abundance of $^3$He in the innermost core and would disrupt the equilibrium of the PPI chain. This surplus $^3$He would be rapidly consumed (see Figure 1b), releasing a substantial amount of thermal energy and steepening the thermal gradient in the core. If this effect were sufficiently large, the Schwarzschild criterion for instability to convection would be met and overturning motions would commence in the core. Dike and Gough (Ref. 7) demonstrated the theoretical possibility of such an occurrence by evolving a solar model in such a way as to simulate previous such mixing episodes, and then imposing an instantaneous mixing of the entropy and $^3$He abundance of the core. In this instance, the consequent rapid burning of $^3$He was indeed found to release sufficient energy to drive overturning motions. It would thus appear that the solar core, although convectively stable in response to infinitesimal perturbations, may be convectively unstable in the presence of large-amplitude disturbances.

Table 1. Reactions of the PPI portion of the proton-proton reaction chain. The quantity $Q$ describes the energy released per reaction. The reaction rates are expressed in terms of their dependences on the hydrogen mass fraction $X_1$, the $^3$He mass fraction $X_3$, and the temperature $T$.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$Q$(MeV)</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^3$He + $^3$He $\rightarrow$ $^4$He + $^4$He $\rightarrow$ $^4$He + $^4$He</td>
<td>12.86</td>
<td>$\propto X_3^2 T^{16}$</td>
</tr>
<tr>
<td>$^3$He + $^3$He $\rightarrow$ $^4$He + $^4$He</td>
<td>6.94</td>
<td>$\propto X_3^2 T^4$</td>
</tr>
</tbody>
</table>

Table 1. Reactions of the PPI portion of the proton-proton reaction chain. The quantity $Q$ describes the energy released per reaction. The reaction rates are expressed in terms of their dependences on the hydrogen mass fraction $X_1$, the $^3$He mass fraction $X_3$, and the temperature $T$. 
The principal matter addressed here is whether $g$-mode oscillations driven by $^3$He might themselves provide a sufficiently large disturbance to trigger overturning motions. Exactly this sort of behavior has been observed in numerical and laboratory experiments involving thermobaline convection (Refs. 6, 15). That such a sequence of events might actually have occurred during the last several million years is suggested by the deficit in the observed flux of solar neutrinos, which could be accounted for if the solar core had recently been wholly or partially mixed (Ref. 7).

In investigating this matter, it is necessary to account in some way for the feedback of the unstable oscillation on the structure of the core. Such a phenomenon is not described by linear theory, and some sort of nonlinear analysis must thus be undertaken. Since no general method at present exists for studying nonlinear nonradial oscillations in stars, the problem must be approached indirectly. Previous such efforts include that of Ulrich (Ref. 17), who imposed a $g$-mode oscillation of a particular amplitude on a solar model and estimated its feedback on the stratification of the core resulting from the adjustment of the time-averaged nuclear reaction rates. He concluded that for oscillations with surface amplitudes of up to 80 km s$^{-1}$, the expected adjustment to the structure of the core is much smaller than that required to induce convective motions. In addition, he found that the velocity of a circulation current that is driven by a pulsation-induced horizontal entropy gradient would be insufficient to mix $^3$He. Roxburgh (Ref. 13) assumed from the outset that the unstable oscillation attains a limit cycle, and then estimated its amplitude, assuming further that the depletion of $^3$He by means of nonlinearities in the nuclear reaction rates serves as the principal nonlinear feedback. He deduced that the oscillation would saturate at an amplitude such that the maximum Lagrangian temperature perturbation is about 0.25. A different approach was taken by Dziembowski (Ref. 8), who argued that a parametric resonance instability would arise as a result of the nonlinear coupling of the unstable mode to very high-degree $g$ modes, limiting the amplitude of the unstable mode to a few cm s$^{-1}$.

To summarize, previous work on this problem presents us with several plausible alternatives:

1) instability to $^3$He-driven $g$ modes has never occurred;
2) such instability has occurred, with the result that the oscillations equilibrated at some finite amplitude; subsequently, a) the oscillations are still unstable in the present epoch, or b) stability has been regained, and the oscillations have shut off;
3) such instability has occurred, and has led to repeated episodes of core mixing: at present, the Sun is in the aftermath of such a mixing event, accounting for the observed deficit of solar neutrinos.

The problem of whether or not $g$-mode instability actually has occurred is a difficult problem in linear theory, and will not be considered further here. Instead, the linear instability of the modes will be assumed, and the possible nonlinear consequences of the instability investigated in an effort to determine which of alternatives 2 and 3 is most probable.

**2. POsing THE PROBLEM**

The present investigation differs from previous studies in that the nonlinear dynamical problem is here solved self-consistently, albeit for a highly idealized theoretical model. This model consists of a layer of fluid confined between impermeable horizontal plates at $z = 0$ and $z = d$, in which the motion is constrained to be two-dimensional. In describing its behavior, we adopt the Boussinesq approximation, which asserts that the motions are nearly those of an essentially uniform, incompressible fluid in which temperature perturbations influence the motion only through buoyancy. For a gas, this approximation is valid so long as the Mach number remains small and the layer depth is shallow compared to its pressure scale height.

Embedded in the fluid is a substance, similar to $^3$He, which serves as a source of thermal energy. Its concentration per unit mass of fluid will here be represented by the variable $X$.

In the solar core, the abundance $X$ adjusts to changes in the reaction rates. The equations describing this system are:

\[
\frac{\partial}{\partial t}(\rho u) + \rho \mathbf{u} \cdot \nabla \rho = \frac{1}{\rho_0} \nabla P + \frac{\rho \mathbf{g}}{\rho_0} + \nu \nabla^2 \mathbf{u}
\]  

(1)

\[
\nabla \cdot \mathbf{u} = 0
\]  

(2)

\[
\frac{\partial}{\partial t} T + \mathbf{u} \cdot \nabla T = -\kappa \nabla^2 T + \frac{Q(T, X)}{\rho_0 c_0}
\]  

(3)
where $u, v, ho$ and $T$ are the fluid velocity, pressure, density, and temperature, and the subscript zero signifies a constant characteristic value. In addition, $\nu$ is the kinematic viscosity, $\kappa$ is the thermal diffusivity, $g$ is the diffusivity of $^3$He, $g$ is the gravitational acceleration, $w$ is the vertical component of velocity, $(DT)_\rho$ is the adiabatic temperature gradient, $c_p$ is the specific heat at constant pressure, and $A$ is the coefficient of isotropic thermal expansion.

The quantities simulating nuclear burning are $S$, the net rate at which nuclear reactions deplete the $^3$He fuel, and $Q$, the rate at which nuclear reactions produce energy. In order to make the analysis of these equations tractable, the following simplified expressions have been chosen for $S$ and $Q$:

$$S(T,X) = -S_1 + S_2 T^a X^b$$
$$Q(T,X) = Q_1 + Q_2 S_2 T^a X^b$$

where $S_1$ and $S_2$ are constants, and $Q_1$ and $Q_2$ are the energies released per hydrogen-burning and $^3$He-burning reaction. In each of these expressions, the first term represents the reactions which produce $^3$He, whose rates are independent of the $^3$He abundance and are relatively insensitive to temperature. The second term in each expression represents the temperature-sensitive reaction which destroys $^3$He, the rate of which is sensitive as well to the $^3$He abundance (see Table 1).

The boundary conditions for this model are chosen for mathematical convenience. They describe horizontal boundaries which are thermally conducting and motionless. The boundaries, represented by solid lines, are permitted across them. We thus have

$$\partial_t u = 0 \quad w = 0 \quad T = T_0 \quad X = X_0 \quad \text{at } z = 0$$
$$\partial_t u = 0 \quad w = 0 \quad T = T_d \quad X = X_d \quad \text{at } z = d$$

The vertical boundaries are also frictionless, and no fluxes are permitted across them. We thus have

$$u = 0 \quad \partial_z w = 0 \quad \partial_z T = 0 \quad \partial_z X = 0 \quad \text{at } r = 0, 2\pi/r_a$$

The latter conditions disallow solutions describing horizontally travelling waves, but permit standing wave solutions.

3. NONLINEAR BIFURCATION ANALYSIS

Equations (1)-(10) have static equilibrium solutions for which $T$ decreases with height and $X$ increases with height, much as in the solar core. A linear stability analysis of this solution reveals that instability to gravity modes driven by $^3$He can occur provided the rate of the reaction which destroys $^3$He is sufficiently sensitive to temperature.

The nonlinear behavior of the $^3$He instability has been investigated by analysing the nonlinear bifurcation structure of the governing equations and also by solving these equations numerically. The object of the bifurcation analysis is to reduce the partial differential equations (1-10) to a low-order system of ordinary differential equations, known as amplitude equations, which describe the essential dynamics of the instability and whose solutions are simpler to compute. That such a reduction can be accomplished (as long as the system is close to marginal stability and the amplitude of the instability does not become too large) is predicted by the center manifold theorem, which is discussed, for example, in Ref. 9.

The method of analysis used here is that of Couillet and Spiegel (Ref. 4). The analysis indicates that near the bifurcation point where $^3$He-driven oscillations become unstable, the temporal evolution of the instability can be described by a single ordinary differential equation, the Hopf or complex Landau equation,

$$\dot{A} = (\alpha - i\omega)A + h|A|^2 A$$

where $A$ represents the amplitude of the oscillation, $\alpha$ is the linear growth rate (which vanishes at marginal stability), $\omega$ is the frequency of the oscillation at marginal stability, and $h$ represents differentiation with respect to time. Note that all terms in (11) that are quadratic in $A$ vanish, as the result of a symmetry characteristic of the bifurcation.

Equation (11) can be derived from the structure of the linear problem, the principal difficulty resting in the determination of the value of the coefficient $h$ of the nonlinear term. The sign of $h$ is particularly important because it determines the qualitative behavior of the instability in the weakly nonlinear regime. The nature of the solutions of (11) can be illustrated by plotting the limit set (i.e. those values of $A$ for which $A = 0$, as a function of the linear stability parameter $\alpha$). Such a plot for the case $h > 0$ is presented in Figure 2a. Here stable fixed points, which attract the solutions, are represented by solid lines, and unstable fixed points, which repel the solutions, are indicated by the dashed lines. It is seen that the growth of the instability in this instance is not limited by the lowest-order nonlinear feedbacks, but continues until higher-order nonlinearities, not accounted for in this analysis, become important. Thus, the oscillation in such an instance might grow to very large amplitudes even when it is only marginally unstable in linear theory, perhaps allowing the $^3$He instability to feed back so strongly on the mean state that a transition to overturning motions is induced. In addition, a finite-amplitude perturbation can here lead to instability even when $\alpha < 0$, i.e. when the static equilibrium state at $A = 0$ is linearly stable.

Upon evaluating $h$ for model parameters representative of the solar core, we find in fact that $h < 0$. In this instance, represented by Figure 2b, the growth of the instability when $\alpha > 0$ saturates at the stable stationary solution $|A| = \sqrt{-\alpha/h}$. This is because nonlinear effects here counteract the mechanism which drives the oscillations, thus slowing their growth as their amplitude increases. Thus, the analysis suggests that near marginal stability, at least, the oscillations might not attain amplitudes sufficiently large to induce overturning and mixing.

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He-driven oscillations in the sun is accurately difficult to say with a great deal of confidence whether the represented by the above results. However, an additional conditions might affect this result considerably.

The simplicity of the model considered here makes it dif­

ficulty idealized, and that the inclusion of spherical ge­

amplitude sufficiently large to trigger mixing in the core by

3

He-driven g-mode

The tentative conclusion that is drawn from this result is

that an unstable 3He-driven g-mode is unlikely to attain an amplitude sufficiently large to trigger mixing in the core by the means considered above. However, it is strongly empha­

ized that the model upon which this conclusion is based is highly idealized, and that the inclusion of spherical ge­

ometry, deeper stratification, and more realistic boundary conditions might affect this result considerably.

4. NONLINEAR FEEDBACK MECHANISMS

The simplicity of the model considered here makes it dif­

ty difficult to say with a great deal of confidence whether the evolution of 3He-driven oscillations in the sun is accurately represented by the above results. However, an additional result of this study which is somewhat more certain is the identification of several specific feedback mechanisms which are likely to be important for determining the nonlinear behavior of such oscillations. Among these are three feedbacks arising from the nonlinear nuclear source terms in equations (1-5) (for instance \(\frac{\partial T}{\partial z} \frac{\partial \Phi}{\partial z}\)), and one feedback which arises from the advective nonlinear terms, such as \(u \cdot \nabla T\). Three of these mechanisms, including that associated with the advective terms, tend to counteract the growth of the instability, and thus contribute negatively to the coefficient \(h\) in equation (11). The fourth causes the oscillation to tend toward runaway growth, and thus contributes positively to \(h\). The sum of these effects, of course, is such that \(h < 0\).

In order to provide an example of these feedback mechanisms, the nonlinear feedback arising from advection is here described in some detail. A description of the other feedback mechanisms arising from nuclear nonlinearities, is given in Ref. 11. Consider once again the static equilibrium state of our plane-layer model. The temperature in the layer decreases with height subadiabatically, and the abundance of 3He increases with height. Since the g-mode oscillation will, on average, tend to transport material initially near the top of the layer downward and material initially near the bottom of the layer upward, the oscillation results in a downward temporally- and horizontally-averaged transport of heat. This tends to steepen the temperature gradient closer to the adiabatic gradient, as indicated in Figure 3. In addition, the same kinematic effect results in a net downward transport of 3He, and thus tends flatten the 3He gradient, as shown also in Figure 3b. Since it is the presence of the 3He and subadiabatic temperature gradients which drives the oscillations, the erosion of these gradients by the advective nonlinear feedbacks counteracts the driving mechanism and thus tends to saturate the growth of the instability.

The steepening of the temperature gradient, since it weak­

ens the buoyancy restoring force, also has the effect of re­

ducing the buoyancy frequency, and hence the frequency of the g-mode oscillations. For the limiting amplitude consid­

ered in the previous section, this is a rather small effect, the frequency decrease amounting to a small fraction of one per­

cent.

The operation of this nonlinear feedback mechanism is sum­

marized in Figure 4.

5. NUMERICAL RESULTS

The nonlinear bifurcation analysis described above is, strictly speaking, valid only in the weakly nonlinear regime close to marginal stability. It is therefore desirable to solve
SOLAR GRAVITY MODES DRIVEN BY NUCLEAR BURNING OF $^3\text{He}$ IN THE CORE

Equations (1)-(10) were solved numerically as well, so that the behavior of the instability can be studied at larger amplitudes still.

The two-dimensional governing equations are solved according to a numerical procedure in which variations in the direction of the horizontal coordinate $x$ are decomposed into $m$ Fourier modes. This procedure yields for the Fourier components of the solution a set of partial differential equations in the vertical coordinate $z$ and time $t$. These are coupled by their nonlinear terms, and are integrated using finite-difference methods. Derivatives with respect to $z$ are represented by second-order-accurate centered differences, and the solution is advanced explicitly in time using a two-step Adams-Bashforth scheme, also of second-order accuracy. The advective nonlinear terms are represented spectrally (e.g. Ref. 10), and the nonlinear nuclear source terms are treated according to a pseudospectral method (e.g. Ref. 12). Typically 25 vertical mesh points are employed, and the horizontal variations are represented by $m = 4$ modes, which is sufficient to represent the lowest-order nonlinear feedbacks. Except for the diffusion coefficients, the parameter values were chosen to represent conditions in the core as closely as possible, while still conforming to the conditions of the approximations employed in the analysis.

Close to marginal stability, the results of the numerical calculations were found to be in good agreement with the analytical predictions. The growth of the $^3\text{He}$-driven oscillations indeed saturated at a limiting amplitude, which agreed with that predicted by the bifurcation analysis to within about 10%.

At larger amplitudes, however, the growth of the oscillations led to the layer becoming thermally unstable. This behavior is a consequence of the presence of the nuclear thermal energy sources, which heat the layer ever more strongly as its temperature increases. The resulting tendency toward thermal runaway is counteracted in the sun because it can expand, and thus cool, in response to heating of its core. The model, however, is not self-gravitating and is confined between impermeable plates, and so cannot respond in this manner. In an attempt to imitate this property of the sun, a procedure was introduced by which the mean temperature of the layer was artificially reduced when the nuclear heating rate increased. This had the effect of slightly reducing the amplitude of the oscillation, and allowed stronger driving of the oscillations to be applied before thermal runaway set in.

This tendency toward thermal instability has thus prevented us from computing numerical solutions at oscillation amplitudes as large as had been hoped. However, the results do verify that the oscillations in the model do not grow to large amplitude near marginal stability, and that, at the moderate amplitudes which we estimate are attained in the sun, the oscillations appear to be incapable of triggering overturning motions which could lead to mixing.

In order to investigate the effect on the solutions of the impermeable side walls represented by boundary conditions (10), some of the above computations were repeated with the side walls removed, so that the horizontal structure of the solution is constrained only by the requirement that it be periodic in $x$ with period $2\pi/a$. An effect of this new boundary condition is to permit solutions that are horizontally travelling waves rather than the standing waves which had been imposed previously. Interestingly, the unstable oscillation was observed to prefer the travelling-wave mode to the standing-wave mode, even though these two types of solutions are degenerate in linear stability theory.

Figure 5 Numerical solutions for $^3\text{He}$-driven $g$ modes. Represented is the velocity field at three successive times $t_1 < t_2 < t_3$ spanning about half an oscillation cycle. (a) Standing-wave solution obtained when boundary conditions (10), representing rigid side walls, are employed. (b) Travelling-wave solution, which is preferred when the side walls are absent.
Nonlinear effects thus appear to break this symmetry, and cause oscillations to develop into a single left- or right-travelling wave rather than a standing wave or other superposition of horizontally-propagating waves.

Examples both of standing-wave and travelling-wave numerical solutions, followed over approximately half an oscillation cycle, are presented in Figure 5.

6. CONCLUSIONS

We have studied the nonlinear behavior of g modes driven by burning of $^3$He in the sun's core by solving for the dynamical behavior of a highly idealized model in which an analogous instability occurs. Both a nonlinear bifurcation analysis and numerical solutions of the model equations suggest that near marginal stability the lowest-order nonlinear feedbacks restrict the growth of the oscillations to some limiting amplitude. Hence, runaway growth does not occur near marginal stability. The results also suggest that at this limiting amplitude, the feedback of the oscillations on the mean thermal stratification is probably insufficient to induce convection.

Suppose, then, that $^3$He-driven g modes have not resulted in mixing of the sun's core. Might they still be present, and perhaps observable, in the present era? According to the model results, together with estimates from linear stability analysis and numerical solutions of the model equations in the sun (e.g. Ref. 2), we estimate that they are likely to saturate at amplitudes such that vertical motions of about 10 km s$^{-1}$ occur in the core. Since the eigenfunction of the unstable $g_1(\ell = 1)$ mode has a sizeable amplitude in the envelope as well as the core, it would seem that such oscillations, if present, should be readily observable at the sun's surface. Thus, presuming the spatial structure of the oscillations is not radically different from the eigenfunctions of linear theory, it would appear that $^3$He-driven solar g modes are not unstable in the present epoch.

We stress that because of the numerous approximations involved in our theoretical model, the application of these results to the sun is quite uncertain. In particular, core mixing driven by the fluid turbulence which might be generated by the oscillations has not been studied. Also, the assertion by Dziembowski (Ref. 8) that the amplitude of the unstable $g$ mode is limited to a few cm s$^{-1}$ by resonant coupling to stable, high-degree g modes has not been addressed. Nonetheless, this work has clarified the possibilities for the nonlinear behavior of $^3$He-driven g modes, and has identified several of the competing nonlinear feedback mechanisms by which the finite-amplitude behavior of the oscillations is determined.

7. REFERENCES

LINEWIDTH OF LOW DEGREE ACOUSTIC MODES OF THE SUN

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ABSTRACT

Estimates of the spectral linewidths of low degree (C=0 and 6=1), "5 minute" p-modes obtained from Doppler shift observations in 1984, 1986 and 1987 are reported. The observed linewidths increase from 0.5 μHz at 2000 μHz to 3.8 μHz at 4300 μHz for δ = 0. Comparison with other data suggest that for a given frequency the linewidth increases with increasing δ value. On the assumption that the linewidth is substantially due to damping processes the line widths are consistent with e-folding times between 3.7 and 0.5 days.

Keywords: Solar oscillations, acoustic modes line widths.

1. INTRODUCTION

The measurements reported in this paper are the linewidths of low degree p-modes. These low degree modes (0<C<3), which are observed in unimaged sunlight (Brookes et al (Ref. 7)), are of special interest as they are the modes which propagate into the core of the sun and they are also the modes which will be measured in observations of oscillations of other stars for which no spatial resolution is at present possible. It is useful to compare the low C-value linewidths with the widths obtained for high C-value because of the weighting of different modes throughout the solar interior; the high C-value modes being confined to the outer regions of the Sun.

There have been several measurements of the linewidths of intermediate and high C modes. In 1986, Libbrecht and Zirin (Ref. 5) provided linewidth estimates for intermediate degree modes based on two-dimensional imaging Doppler measurements. Also in 1986 Duvall et al (Ref. 6) reported linewidths for intermediate degree modes with 20<¿<96. These linewidths were obtained from an analysis of a 50 hour stretch of two-dimensional Call K line intensity fluctuation data taken at the South Pole in 1981. The observational resolution of their data set (5 μHz), precluded the possibility of seeing any variation in linewidth below ~ 3.2 μHz. Above this frequency their data showed an increase in mode linewidth with frequency. A recent paper by Libbrecht (Ref. 10) however, which is based on the analysis of a 100 day stretch of two dimensional imaging Doppler measurements, shows that for r=δ with 6=19-24, linewidth increases with increasing frequency from 0.22 μHz at 1.6 mHz to 10 μHz at 4.0 mHz except over the range 2.4 - 2.8 mHz where it appears to be independent of frequency.

2. RESULTS OF WORK

Runs of full disk Doppler shift measurements obtained in Hawaii and Tenerife during 1984, 1986 and with the addition of the Australian station in 1987 were used to form velocity, power spectra with an observational resolution of approximately 0.2 μHz. Elsworth et al (Ref. 1), Aindow et al (Ref. 2). The measurements were made with a resonant scattering spectrometer working with the potassium Fraunhofer line (Ref. 7) at 769.9 nm.

Two methods are used here to derive the line widths. In both methods it is assumed that in the vicinity of any spectral line the noise background does not vary. The same approximation is used by Duvall et al (Ref. 6) and we find that the slow variations in the background lead to errors in centroid positions and widths at the nHz level. Initial frequency estimates are taken from Jimenez (Ref. 9).

If it can be assumed that a solar oscillation can be represented as a harmonic oscillator with damping, driven by a random forcing function, then the frequency response of the system can be represented by a Lorentzian profile. The full width at half power (Δr) of this profile is then related to the e-folding time (τ) for the decay of modal power by r = (2Δn)−1.

Below the 3.8 mHz the total power and centroid of the spectrum in a frequency region symmetrically placed about the estimated line centre is computed for a range of integration widths up to ±10μHz.

Between 3.5 and 4.3 mHz the data are smoothed and a Lorentzian curve is fitted. Figure 1 shows a typical fit.

Assuming that the lineshape is well represented by a Lorentzian then the line width Δr (FWHM) can be given in terms of r, the e-folding time for the decay of the modal power, by r = (2Δn)−1. The results are presented in table 1 and figures 2a and 2b.
The 1984 data have also been analyzed with background subtraction allowing the sensitivity of the method to the precise details of the background to be tested. As in the results presented by Woodard (Ref. 8) the background was assumed to vary quadratically with frequency. The widths obtained for the \( \ell = 0 \) and \( \ell = 1 \) modes in the range 1.9 - 4.3 mHz are shown in Fig. 2a and 2b. As is to be expected the best results are obtained for isolated peaks with good signal to noise. The errors on the line widths in Fig. 2 can be estimated by comparing the deduced widths of the different data sets. We do not feel that we know the distribution of the errors and therefore we do not mark the error bars. The results obtained when the background is subtracted are shown also on the same graph. They do not differ systematically from those derived with the assumption of a constant background.

It should be noted that the \( \ell = 0 \) modes are slightly contaminated by the side lobe structure of the nearby \( \ell = 2 \) modes. This has the effect of increasing the \( \ell = 0 \) linewidth values slightly. Note also that the intrinsic minimum width associated with the finite frequency resolution of 0.2 mHz has not been removed. This means that the linewidth values obtained using these measured widths will be upper limit values. The widths of the \( \ell = 1 \) modes are greater than the widths of the \( \ell = 0 \) modes in the same frequency region due to the rotational splitting of the degenerate \( \ell = 1 \) mode.

The linewidths presented in this paper agree with low degree linewidth measurements by Isaak (Ref. 5) and with the simple average value of Grec et al. (Ref. 6).

![Fig. 2](image.png)

**Fig. 2**

a) \( \ell = 0 \) line widths as a function of frequency

b) \( \ell = 1 \) line widths as a function of frequency

in a) and b) the solid lines represent eye-guides and should not be taken as precise fits. The dotted lines are the eye-guides for the other \( \ell \)-value.
modes). They extend the suggestion by Duvall et al (Ref. 6) that the linewidth increases with increasing θ-value all the way to the radial mode (θ=0).

Furthermore, the rather steep increase in linewidth above 3000 μHz suggests that there may be two different damping mechanisms at work, probably in the upper levels of the Sun. We speculate that the high frequency modes may show significant leakage into the corona with a commensurate increase in the damping experienced.

5. ACKNOWLEDGEMENTS

The assistance of all members past and present of the Solar Oscillation Groups at Birmingham and I.A.C. in obtaining the Tenerife and Hawaii data is gratefully acknowledged. We acknowledge funding from the SERC for the network, and the Spanish CAICYT for funding the PR84-0005 grant. Some travel support was provided by a British/Spanish Integrated action.

6. REFERENCES


<table>
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**Table 1** Linewidths and excitation times see text for explanation.
MEASUREMENTS OF HIGH DEGREE
SOLAR OSCILLATION AMPLITUDES

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ABSTRACT
We present measurements of high degree solar oscillation velocity amplitudes that have been corrected for the effects of atmospheric seeing. Observations of high degree oscillations, which have very small spatial features, suffer from the effects of atmospheric image blurring and image motion, thereby reducing the amplitudes of their spatial frequency components. In an attempt to correct the velocity amplitudes for these effects, we have simultaneously measured the atmospheric modulation transfer function (MTF) by looking at the effects of seeing on the solar limb. We find that the peak velocity power decreases by ~2.6 and the energy per mode decreases by a factor of ~5 as the degree number increases from 200 to 700. These results allow us to conclude that the modes are not in energy equipartition with convective turbulent eddies in the top of the convection zone.

Keywords: Solar Oscillations, Amplitudes, Atmospheric Seeing, Modulation Transfer Function

1. INTRODUCTION
Much work has been done in calculating the frequencies of the sun’s normal modes of vibration, and the frequencies of observed p-modes have been theoretically reproduced to within one percent. However, very little progress has been made in predicting the amplitudes and linewidths of the oscillations. This is a direct consequence of our lack of understanding of the excitation and damping mechanisms. There are two main candidates for excitation mechanisms. One is that the modes are overstable due to the \( K \)-mechanism (see Ref. 6 for a review and references therein). The other more likely theory (due to Goldreich, Keeley, and Kumar) is that the modes are being stochastically excited by turbulent convection in the convection zone (Refs. 2-5). Possible damping mechanisms include radiative damping, and turbulent viscous damping. (See Libbrecht Ref. 6 and these proceedings for a review of this subject). One prediction of the simplified Goldreich/Keeley/Kumar theory is that the modes are in energy equipartition with the eddies of the turbulent convection, i.e., that mode energy is independent of \( \ell \).

Some work has been done in measuring mode amplitudes and linewidths, and in particular Libbrecht (Ref. 7) has measured mode amplitudes and linewidths for \( \ell \leq 100 \). Almost no accurate measurements exist for p-modes with \( \ell > 100 \). Libbrecht et al. (Ref. 9) have attempted to measure high \( \ell \) mode amplitudes, but their results were inconclusive due to uncertainties produced by atmospheric seeing. This is unfortunate since high \( \ell \) modes propagate only in the top of the convection zone, where it is thought that most of the excitation and damping occurs.

Atmospheric seeing smears out the small spatial features associated with high \( \ell \) modes. Put more quantitatively, the seeing reduces the amplitudes of the high spatial frequency components seen in any solar image. In this work, we have simultaneously made velocity observations at the center of the solar disk, and measured the atmospheric modulation transfer function (MTF). We have measured the MTF by looking at the solar limb, which, before being distorted by seeing, has a known spatial profile. Of course, the profile of the undistorted limb is not truly known, but, because our pixel size is larger than the scale height.
of the solar atmosphere, we can say that the profile is known to the accuracy of our measurements.

2. OBSERVATIONS

All observations were done on the 10 inch refractor at the Big Bear Solar Observatory on 23 July, 1987 from 15:00 to 01:00 UT. This was a day of moderate quality seeing. The data were acquired using the BBSO videomagnetograph system (Ref. 11) configured for Doppler measurements. The Ca line at 6393 Å was used for all measurements.

2.1 Seeing Measurements

Throughout the course of the day, 1148 individual limb profiles were acquired at the east limb of the sun. Each limb profile covered a region about 60 arc sec wide. At approximately 15 minute intervals, the telescope was moved from the center disk to the limb, and short exposure (at video rates) limb profiles were taken continuously for a time equal to the time to make one Dopplergram, about 6 sec. Under this scheme, 41 groups of 28 profiles each were acquired throughout the day.

Each observed limb profile, \( L(x) \), was fit using a non-linear least squares method to a theoretical limb profile, \( L(x - x_0) \), which has been smeared, or blurred, by a model point spread function (PSF), \( S(x) \):

\[
L(x) = \int_{-\infty}^{+\infty} L(x' - x_0)S(x - x')dx'
\]

(1)

where \( x \) is the horizontal image coordinate, and \( x_0 \) is the limb offset. A two-Gaussian PSF was used:

\[
S(x) = \frac{1}{1 + \alpha}[S_1(x) + \alpha S_2(x)]
\]

(2)

where \( S_j(x) \) is a standard normalized Gaussian function of width \( \sigma_j \). We have assumed that \( \sigma_2 \gg \sigma_1 \), and \( \alpha < 1 \). The theoretical limb profile, \( L(x) \), is that of Pierce and Waddell (ref. 10) with an arbitrary multiplicative intensity parameter, \( I_0 \).

Five parameters were fit for: \( I_0, \sigma_1, \sigma_2, \alpha, \) and \( x_0 \). The second through fourth parameters are a function of image blurring ("soft seeing"), and the last parameter is related to image motion ("hard seeing"). Note that image motion can be produced by both the atmosphere and by telescope shake. We do not differentiate between the two, since their effects on the image quality are the same. Image motion is included as time integrated image blurring by allowing

\[
\sigma^2_t = \sigma^2_i + \sigma^2_{x_0}
\]

(3)

where \( \sigma_{x_0} \) is the RMS deviation from the mean of \( x_0 \). Typical values for these parameters were found to be \( \sigma_1 \sim 2 \text{ arc sec}, \sigma_2 \sim 20 \text{ arc sec}, \sigma_{x_0} \sim 0.8 \text{ arc sec}, \) and \( \alpha \sim 0.3 \). These combine to give the PSF a FWHM of \( \sim 4 \text{ arc sec} \). Note that atmospheric blurring and motion, telescope shake, telescope optics, and image focus contribute to this PSF.

The MTF is the Fourier transform of the PSF. Therefore, the function MTF (as a function of spherical harmonic degree, \( \ell \)) is

\[
MTF(\ell) = \frac{1}{1 + \alpha}[e^{-(\sigma^2_i + \sigma^2_{x_0})\ell^2/2R^2_{\ell}}
\]

\[
+ \alpha e^{-(\sigma^2_{x_0})\ell^2/2R^2_{\ell}}]
\]

(4)

The MTF as a function of time is shown in Figure 1a. The MTF averaged over the entire day is shown in Figure 1b. The MTF with a 15 minute sample interval was interpolated using a cubic spline to get a MTF with a regular one minute sample interval.

2.2 Velocity Measurements

The velocity data consists of 633 Dopplergrams, taken at a rate of one per minute, consisting of 498 x 460 pixels, and with an image size of approximately 267 x 197 arc sec centered on the solar disk. As mentioned above, at approximately 15 minute intervals, the telescope was moved to the limb and limb profiles were collected. This was accomplished without interrupting the collection of Dopplergrams.

After interpolating over bad or missing images, and subtracting a best fit planar background from each image, a two-dimensional spatial Fourier transform was performed on each image, giving a \( k_x - k_y - \ell \) diagram. We then assumed that the MTF, which was measured in the horizontal, or \( x \), direction, is also valid in the vertical direction because the atmospheric seeing qualities are assumed to locally isotropic. For each point at \( k_h = \sqrt{k_x^2 + k_y^2} = \ell/R_0 \) in each image we divided by the appropriate value of MTF(\( \ell, \ell \)). This is the seeing correction. For each time series of spatial frequency components we subtracted a best fit linear background. Temporal Fourier transforms were then performed, resulting in a \( k_x - k_y - \omega \) diagram. Finally, we removed temporal frequency shifts produced by solar rotation, and integrated the data along circles of constant \( k_h = \sqrt{k_x^2 + k_y^2} \), to produce a \( k_h - \omega \) (or \( \ell - \nu \)) diagram.

Ridges can be seen out to \( \ell \sim 1320 \). We also produced an uncorrected \( \ell - \nu \) diagram in which ridges can be seen out to \( \ell \sim 1700 \). We believe that the seeing correction breaks down completely at \( \ell \sim 1320 \) because, at this point, the temporal variations in the
MEASUREMENTS OF HIGH DEGREE SOLAR OSCILLATION AMPLITUDES

3. VELOCITY POWER MEASUREMENTS

The velocity power in each ridge and the frequency of each ridge at a given $\ell$ was measured by fitting a series of Gaussians of the form

$$\sum_{i=1}^{N} A_i e^{-(\nu - \nu_i)^2 / \sigma_i^2} + A_B e^{-(\nu - \nu_B)^2 / \sigma_B^2} + C$$

where $\nu_i$ is the frequency of the $i$th ridge, the subscript $B$ refers to a wide background Gaussian, and $C$ is a constant background offset. Typically, $\sigma_B$ was found to be 700–800 $\mu$Hz. The velocity power in the $i$th ridge is the integral of the appropriate Gaussian, or $P_{\nu,i} = A_i \sqrt{\pi} \sigma_i^2$. The $\ell - \nu$ diagram was calibrated by comparing the ridge frequencies with those in Libbrecht and Kaufman (Ref. 8). These frequencies can be ultimately boot-strapped back to low $\ell$ frequency measurements which were done using a spherical harmonic decomposition (Ref. 1). Using this method we found a scale factor of $d\ell/dk = 21.09$. This agreed adequately with a less accurate scale factor determined directly from the images.

The velocity power along each ridge, as a function of $\ell$, was fit to a polynomial for the purposes of smoothing. A cubic spline interpolation in the frequency direction was then done to these fits to create a regularly spaced grid in $\nu$, in addition to $\ell$. A contour plot of these numbers is shown in Figure 2. The numbers demarcating the ridge positions are from the polynomial fits described above. It should be noted that the inter-ridge velocity power depicted by the contours is not well defined. We only show the contours in these regions for clarity in viewing the velocity power in the entire $\ell - \nu$ plane.

4. DISCUSSION

We will first consider a very difficult topic, the accuracy of the seeing correction. In short, we have no easy way at present to quantitatively assess the true accuracy of the seeing correction. We can, how-
Fig 2. Contour plot of velocity power in the $\ell - \nu$ plane corrected for the effects of atmospheric seeing. The numbers demarcating the ridge positions are from polynomial fits along the length of a ridge. Inter-ridge contour data was derived from a cubic spline interpolation in the $\nu$ direction.

However, address certain aspects of the correction shown in Figure 1a. For example, we believe the kink in the MTF at $\ell \sim 150$ is real. We have analyzed the limb profiles using a PSF consisting of five Gaussians of fixed widths and varying amplitudes, and we still obtain a kink at $\ell \sim 150$ in the MTF. We do not see any diurnal variation in the MTF, but this may be a real feature of this one day of data. We plan to further investigate the types of MTFs that arise under different seeing conditions. Given these provisos, we might conservatively say that the MTF is valid out to the e-folding point, which corresponds to $\ell \sim 700$, or a horizontal wavelength of $\sim 8$ arc sec.

In Figure 3 we show the maximum velocity power as a function of $\ell$, as derived from the contour data. From $\ell \sim 200$ to $\ell \sim 700$ we see that the velocity power drops by a factor of about 2.6. The energy of a mode is $E = P_v M$, where $M$ is the mass of the mode. Since $M$ is approximately proportional to $\ell^{-1/2}$ (Ref. 6), the energy per mode drops by a factor of $\sim 5$ as $\ell$ increases from 200 to 700. If energy equipartition between the modes and the convective turbulence eddies were to hold, mode energy would be independent of $\ell$. Even with the somewhat uncertain nature of our seeing correction, this, clearly, cannot be the case. This confirms the rather tentative conclusions raised in Libbrecht et al. (Ref. 9).

The author would like to thank Ken Libbrecht for many useful discussions and guidance. Many thanks are also due to the observing staff at BBSO (Alan Patterson, Bill Marquette, Randy Fear, and Curtis Odell) for their help in obtaining the data. This work...
Fig 3. Maximum velocity power as a function of $\ell$, as derived from the contour data in Fig. 2. From $\ell \sim 200$–700 the power drops by a factor of 2.6. The drop off indicates that the modes are not in energy equipartition with convective turbulence.

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5. REFERENCES

DYNAMICS OF THE OVERSHOOT LAYERS AND BOUNDARY CONDITIONS IN HELIOSEISMOLOGY

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Abstract

The variation of both the horizontal and the vertical small scale rms velocity with height in the photosphere shows a minimum in photospheric layers about 150-200 km above the continuum. In the context of the non-equilibrium thermodynamics we suppose that the small scale rms velocity variation reflects the variation of the entropy production in these layers. We propose that the upper boundary of theoretical helioseismology models should be placed at that height where the entropy production shows its minimum.

Keywords: Convection, Overshoot, Boundary Conditions

The investigation of the dynamics of the overshoot layers includes the calculation of the rms velocity at different levels in the photosphere, and enables the calculation of the small scale velocity variation with height in the photosphere. In this way we can describe quantitatively the dynamical behaviour of the velocity fluctuations which are associated with the granulation, and with the small scale structures (< 3\(^\circ\)), which are product of the fragmentation of the granulation. The significance of the overshoot layers for the upper boundary conditions in theoretical helioseismology models has been described elsewhere (Ref. 4). The form of the small scale velocity variation with height bears some implications for the dynamics of the overshoot layers and the location of the upper boundary. Here, we try a new thermodynamical approach to interpret the form of the small scale velocity variation and also to justify the location of the upper boundary in the overshoot layers.

The variation of the vertical and the horizontal small scale rms velocities \( V_z \) and \( V_y \) respectively, shows the following characteristics: (i) a steep gradient, (ii) about the same rms value, and (iii) qualitatively the same variation with height, with a minimum at a level of about 150 to 200 km above the continuum layers (Ref. 3,4). Besides, the steep gradient of the horizontal velocity \( V_y \) reveals the existence of a strong shear stress tensor in the overshoot layers, and thus the existence of a strong vorticity induced by shear stress. We expect, therefore, the initiation of turbulence and oscillations, and the enhancement of turbulent viscosity.

The variation of the small scale rms velocity with height, in addition to the existence of a velocity minimum, provokes the following questions of a qualitative nature:

1) Which thermodynamical property of the overshoot layers is responsible for the existence of a minimum in the variation of the small scale rms velocity fluctuations?

2) Which velocity variation, an exponential function or one of the form of a catenary, would be favoured on the basis of the thermodynamics of the overshoot layers?

3) How can the increase of the rms velocity with height beyond the minimum be interpreted?

Before we go any further, first we have to realize the physical (thermodynamical) space of the overshoot layers which is shown in Fig. 1.

Here, the overshoot layers can be also considered as a hydrodynamical (thermodynamical) system, located between layers which are strongly structured and convectively unstable (granulation layers and CO instability zone, Ref. 2). Besides, the overshoot layers show steep gradients of the average temperature and density (pressure), and large fluctuations of these quantities. Accordingly, the variation of the rms velocities with height in the overshoot layers can be treated in the context of nonequilibrium thermodynamics.

The fragmentation of the granulation with height in the deep photospheric layers and the variation of the small scale rms velocity reveals the existence of a momentum and an energy flow associated with the velocity fluctuations in the overshoot layers. This flow begins in the convectively unstable layers near \( \tau_{500} = 1 \) (granulation layers), passes through the layers which are in radiative equilibrium and runs up to the convectively unstable layers of the higher photosphere, with CO instability (Ref. 2). (see Fig. 1). However, the thermodynamical properties of the the granulation layers and those layers where the CO instability takes place are completely different in relation to the layers which are in radiative equilibrium, and therefore convectively stable. Namely, their entropy production depends on whether they include dissipative structures or not: The granulation layers and those layers which are convectively unstable (CO instability) include dissipative structures and show, therefore, a higher entropy production compared to the convectively stable photospheric layers which are in radiative equilibrium.

The local entropy production \( \dot{\sigma} \) is defined as the source term in the balance equation of the local entropy \( \dot{s} \):

\[
\frac{d\dot{s}}{dt} + \text{div} (\dot{s} \mathbf{v}) = \dot{q} = \dot{\sigma}
\]
where $S$ is the entropy flow and $q(f)$ the entropy source (entropy production).

By means of the Gibbs Equation and the equation of the internal energy $\dot{u}$, the entropy production $\dot{\sigma}$ can be expressed as follows:

$$\dot{\sigma} = (T_{sz} + p\delta_{sz}) : (\frac{\nabla V}{T}) + \cdots$$

with the shear stress tensor $T_{sz}$:

$$T_{sz} = \mu(\frac{\partial V_x}{\partial z} + \frac{\partial V_z}{\partial x}) + \cdots$$

and the deformation tensor $V_{sz}$:

$$V_{sz} = \frac{1}{2}(\frac{\partial V_x}{\partial z} + \frac{\partial V_z}{\partial x})$$

$p$ is the pressure, and $T$ the temperature.

Here, we realize that the local entropy production $\dot{\sigma}$ is a function of the velocity gradients, especially of the horizontal velocity $V_x$.

Considering now the total variation of $\dot{\sigma}$ in the overshoot layers, beginning at the granulation layers near $z_{5000} = 1$ through the deep photosphere up to the upper convectively unstable photospheric layers, we realize the following: The entropy production $\dot{\sigma}$ first decreases because the structures of the granulation disappear with height and increase again when the photospheric layers become convectively unstable (CO instability, Ref. 2). Thus we realize immediately the existence of a minimum of $\dot{\sigma}$, which is expected in the deep photosphere where a radiative equilibrium exists. On the other hand, we find in the variation of $V_x$ and $V_y$ a minimum in the photospheric layers 150–200 km above the continuum $z_{5000} = 1$.

The fact that $\dot{\sigma}$ is a function of the horizontal velocity gradient allows us to see in the variation of the rms velocities the variation of the entropy production of the system under consideration.

The variation of the vertical and horizontal velocities $V_x$ and $V_y$ reflects the variation of the entropy production $\dot{\sigma}$ in the overshoot layers. Thus, neither the minimum nor the rise of the vertical and horizontal velocities, $V_x$ and $V_y$, with height conflicts with the expected thermodynamics of these layers. We also expect minima of the other quantities, $\Delta p$ and $\Delta \rho$, in these layers.

So far, the overshoot layers have been considered as the atmospheric region where the convective flow gradually disappears; hence, they take the role as proper boundary conditions for theoretical models.

But in the context of nonequilibrium thermodynamics, the overshoot layers can not be considered as the upper limit of the convective zone any longer. These layers possess their own characteristics and have to be considered as an interface between two different thermodynamical regimes, the chromosphere and the convection zone. Here, in the overshoot layers nonlinear processes invoke a hydrodynamical and thermodynamical transformation of energy, according to the existence of entropy sources in these layers. The minimum of the entropy production $\dot{\sigma}$ or the horizontal velocity $V_x$ signifies the minimum of the activity of the nonlinearities in the overshoot layers. Any theoretical model which uses the overshoot layers as boundary conditions has to realize the existence of entropy sources in the overshoot layers.

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MEASUREMENTS OF OSCILLATION PARAMETERS FROM SYNTHETIC TIME SERIES.

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ABSTRACT
The Fourier transforms of time series from a stochastically excited oscillator are fitted by different methods with Lorentz profiles in order to obtain the frequency, amplitude and the damping constant of the oscillator. Many simulations are performed to get estimates of standard errors of the parameters caused by the discrete sampling, the random force and the finite observation time. The effects of varying the damping time relative to the observation time are investigated. Finally, amplitude variations as a function of time are calculated by different methods and are compared with measurements of the real Sun.

Keywords: Oscillation measurements, simulations, stochastically excited oscillator.

1. INTRODUCTION.
What are the fundamental limits of the determination of oscillation parameters of solar global modes? Which method works best? Are the observations of the linewidth of low degree (Ref. 1) and intermediate degree modes (Ref. 2) comparable? Can the observations of amplitude variations as a function of time (Ref. 3) be used for lifetime determination? Computer generated data exposed to different methods of analysis can help us answer these questions.

2. THE SIMULATIONS.
Two times 2500 time series of a stochastically excited oscillator are generated as numerical solutions to the equation
\[ \frac{d^2A}{dt^2} - 2 \pi \frac{dA}{dt} + \omega^2 A = f(t) \] (1)
where \( A \) is the amplitude of the oscillation, \( \omega \) is the damping constant, \( \omega \) is the frequency and \( f(t) \) is a random forcing. This stochastic forcing is modelled by perturbing the oscillator between every time step. Given the \( n \)’th (complex) amplitude \( A_n \), the amplitude in the \( n+1 \)’th time step is given by
\[ A_{n+1} = A_n \exp(-\pi \cdot i \cdot \omega \cdot \Delta t) \cdot X, \] where \( \omega = (\omega - \eta) \eta \) differs at most 0.1 nHz from \( \omega \), which is negligible compared to the precision of the analysis, and \( \Delta t = 60s \) is the time step. \( \Delta t \) is a complex random variable, Gaussian with unit variance in both real and imaginary part. This shall mimic small and frequent 'kicks' with random phase and amplitude. This way of kicking is not the same as Kumar et al (Ref. 4). They had fixed sized kicks distributed randomly in time, and only in the imaginary direction corresponding to a velocity change. The start value of the amplitude is given by a complex random variable, Gaussian in both real and imaginary part with variance \( 1/\pi \). Each series has \( N = 2^{16} = 65536 \) measurements, obtained as the real part of \( A_n \), that is a total duration of 45.5 days, if \( \Delta t = 60s \). The period of the oscillator is arbitrarily set to 5.2 min, so that \( \omega = 2.0138 \times 10^4 \) s\(^{-1} \), corresponding to a cyclic frequency \( \nu_o = 3205 \mu \text{Hz} \). The first 2500 series have an inverse damping constant of 2 days, the last 2500 have 10 days. Each sequence is then Fourier transformed (examples of power spectra are shown in Figure 1). It should be noted, that the simulated data are unrealistic in the sense, that they have no noise and the 'observation period' is uninterrupted.

3. THE NON-LINEAR FIT.
In order to make good estimates of the damping constant \( \eta \) several power spectra are averaged like Libbrecht and Zirin (Ref. 2) have done with the 21 \( \pm 1 \) different power spectra corresponding to the different \( m \)-values for a fixed \( l \). A non-linear least-squares fit (Levenberg-Marquardt method (Ref. 6)) to a Lorentz profile
\[ \text{Power}(\nu) = \frac{P}{4\pi^2(\nu - \nu_0)^2 + \eta^2} \] (2)
over a frequency range of \( \pm 5 \mu \text{Hz} \) (cyclic frequency) around the peak gives now estimates of the three parameters \( \nu_0, \eta \) and \( P \). (See Fig. 1c,f) The typical errors of the parameters are displayed in Figs. 2-4. If \( N_{\text{spec}} \) is the number of spectra averaged before the fit, the standard deviation of the frequency determination is here given by rough fits to Figure 2:
\[ \frac{s(\nu)}{\nu} = 1.4 \times 10^{-4} N_{\text{spec}}^{-0.65} \] (3)
for a damping constant \( \eta^1 = 2 \) days, and
\[ \frac{s(\nu)}{\nu} = 0.8 \times 10^{-4} N_{\text{spec}}^{-0.75} \] (4)
for \( \eta^1 = 10 \) days. For \( N_{\text{spec}} < 10 \) the above estimates
of the standard deviation are too small. The standard
deviations of the damping constant is roughly
\[
\frac{\sigma(\eta)}{\eta} = 0.59 N_{\text{spec}}^{0.7} \quad \eta^{-1} = 2 \text{ days.} \tag{5}
\]
\[
\frac{\sigma(\eta)}{\eta} = 1.2 N_{\text{spec}}^{0.7} \quad \eta^{-1} = 10 \text{ days.} \tag{6}
\]
Note, that the exponents to \( N_{\text{spec}} \) in the interval
5 < \( N_{\text{spec}} \) < 50 is numerically larger than the value \( \frac{1}{2} \) which might have been expected. The measured
 linewidth is slightly greater than the given value (the
 dashed line in Fig. 3a,b). The difference is exactly
 equal to the inverse observation time (45.5
 days)\(^{-1} = 0.25 \times 10^4\) s That is because the expected line
 profiles are not exactly Lorentzian, but are broadened
 by the finite observation time (see Ref. 7).

The reason for the low values of the measurement
 of the power especially for low \( N_{\text{spec}} \) as seen in Fig-
 ure 4 is that low power points in the spectra are
 weighted most (see also fits in Figure 1c and f). A

Figure 1. Power spectra of solutions to Eq. 1. Figure a, b and c have \( \tau^{-1} = 2 \) days, and d, e and
f have \( \tau^{-1} = 10 \) days. Fig. c and f are averages of 10 power spectra with the least-squares
 Lorentz fit superimposed. The structure of the spectra are in accordance with Scherrer's simula-
 tions (Ref. 5).

Figure 2. Standard deviation (dashed line) and mean (thick solid line) of the frequency measured
 by a Lorentz fit as a function of \( N_{\text{spec}} \), i.e. the number of power spectra averaged before the fit.
The thin solid line is \( \nu_0/\nu = \omega_0/2\pi \) from Eq. (1).
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4. FITS OF UNAVERAGED SPECTRA.

The fewer spectra averaged, the more humpy are the curves to be fit with Lorentz profiles, and the more complicated is the \( \chi^2 \)-landscape, where we have to find the minimum. Therefore, if \( N_{\text{spec}} \) is small, the danger of falling into non-global minima is greater, and we get wrong parameters. If I try to use the fit to one single spectrum like Fig. 1, the method breaks down and gives meaningless parameters. It is worth emphasizing, that the fine structure of the spectra in Figure 1 is not due to noise in the data, but to discrete sampling.

Two other methods are therefore applied to the 5000 unaveraged power spectra. The first one uses moments of the power spectrum and compares these to moments of an exact Lorentz profile. It is worth emphasizing, that the fine structure of the spectra in Figure 1 is not due to noise in the data, but to discrete sampling.

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The ratio of the estimates from Eq. (9) and (12) \( M_a/M_R \) is set equal to the ratio of Eq. (7) and (11). This equation has now only one unknown, namely \( \eta \), which is then solved by iteration. \( P \) is then easily found from any of the 3 moments.

A theoretical calculation of the standard deviation of \( \nu \) gives

\[
\frac{\sigma(\nu)}{\nu} = \frac{(\eta/N\Delta t)^2}{2\pi\nu} \tag{13}
\]

where \( N \) is the number of measurements (here 2^16) and \( \Delta t \) is the time between two measurements (here 60s). The derivation of this formula follows the variances in the white noise from \( f(t) \) in Eq. (1) through Eqs. (9) and (10) and finally evaluates the variance of the ratio \( M_a/M_R \). Details of the derivation will be given elsewhere; I have not yet calculated the same thing for \( \eta \) or \( P \). The resulting standard deviations of the oscillation parameters obtained from the analysis of the 5000 simulations are presented in Table 1.

The agreement between theory (Eq. (13)) and computer simulations is satisfactory. The mean of the measured \( \eta \)'s is a few parts of 10^7 from the given value. The mean of the calculated \( \eta \)'s is a few percents over the given value (see fig. 6a,b). It is, however, a small shift compared to the standard deviation.

The second method measures the HWHM directly by solving the equation

\[
\int \frac{f(\omega)d\omega}{\omega} = \int f(\omega)d\omega \tag{14}
\]

for \( x \), where \( f(\omega) \) is the power spectrum, and \( x \) is the width of integration (on the figure in angular frequency). In the case of a Lorentz profile, the HWHM is equal to the damping constant \( \eta \).

![Figure 6. Plot of the function \( \int f(\omega)d\omega \), where \( f(\omega) \) is the power spectrum, and \( x \) is the width of integration (on the figure in angular frequency). In the case of a Lorentz profile, the HWHM is equal to the damping constant \( \eta \).](image)

<table>
<thead>
<tr>
<th>( \eta^{-1} )</th>
<th>( \sigma(\nu)/\nu )</th>
<th>( \sigma(\eta)/\eta )</th>
<th>( \sigma(P)/P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 days</td>
<td>6.2×10^{-5}</td>
<td>0.30</td>
<td>0.25</td>
</tr>
<tr>
<td>theory</td>
<td>6.02×10^{-5}</td>
<td>0.31</td>
<td>0.26</td>
</tr>
<tr>
<td>10 days</td>
<td>3.1×10^{-5}</td>
<td>0.47</td>
<td>0.31</td>
</tr>
<tr>
<td>theory</td>
<td>2.69×10^{-5}</td>
<td>0.32</td>
<td>0.27</td>
</tr>
</tbody>
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for \( x \), where \( f(\omega) \) is the power spectrum (Figure 6); \( \omega_0 \) is found by the first method. I have not taken into account the finite width of the window, hence the \( \eta \)'s from this analysis are slightly underestimated. This does not, however, affect the estimates of the standard deviations. The standard deviation of \( \eta \) is greater than the one found by the moment method: \( \sigma(\eta)/\eta = 0.4 \) for \( \eta^{-1} = 2 \) days, and \( \sigma(\eta)/\eta = 0.85 \) for \( \eta^{-1} = 10 \) days. The reason for the poor estimates of \( \eta \) in the case of \( \eta^{-1} = 10 \) days is obvious from Fig. 6. The determination of the HWHM depends only on the two or three nearest frequency bins to \( \omega_0 \) (and on the...
MEASUREMENTS OF OSCILLATION PARAMETERS FROM SYNTHETIC TIME SERIES

5. AMPLITUDE VARIATIONS.

Finally, the amplitude as a function of time is calculated from the simulations. First, time series with different damping constants, \( \eta \), are simulated. The length of the time series is set to 250 days to be able to compare directly with real solar data on the figures in Ref. 3. In agreement with the same article a window length, \( T \), is chosen to be 25 days. The window is now shifted over the 250 days in steps of 1 day. Each day the 25 days of observation are fit with a sine-wave with the same frequency as the oscillator. The sine-wave amplitude and phase are then plotted as a function of time. Furthermore, the quantity \( \sum_{t=1}^{T} A(t)^2 \)

which is proportional to the mean energy in the window, is plotted, see Figure 8. The abrupt phase jumps of \( 2\pi \) are obviously not real. The curves with an inverse damping time of 10 days are superficially very similar to the curves displayed in Jefferies et al., Ref. 3, who nevertheless estimate the lifetimes to be approximately 50 days. Hence their lifetime certainly cannot be defined as \( \tau' \), as it is done here and in other works (Refs. 2,7).

6. DISCUSSION.

The non-linear least squares Lorentz fit cannot be used for the determination of the oscillation parameters from unaveraged spectra. The method of the moments seems to be superior to the HWHM method, at least in this simplified case with no noise. Thus it would be interesting to apply this method to the analysis of real unaveraged noisy data, such as full disk observations from Ref. 1.

In the analysis of averaged spectra the non-linear fit is better than the method of the moments, but only when averaging more than 30 spectra. (To get this number compare Eqs. (3)-(6) with the numbers in Table 1 times \( N_{obs} \).)

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ON RADIATIVE AND CONVECTIVE INFLUENCES ON STELLAR
PULSATIONAL STABILITY

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ABSTRACT

Recent theoretical investigations have shown that there is a delicate balance between instability and stability mechanisms in certain stars when physical influences, such as convection and radiative transfer, are considered in a finer detail than in a typical pulsation computation. Baker and Gough (1979) and Xiong (1980) demonstrated the susceptibility of pulsations to stabilization towards the red edge of the RR Lyrae and Cepheid instability strips by a convection-pulsation coupling. Moreover, Christensen-Dalsgaard and Frandsen (1983) have illustrated the stabilizing effect of radiative transfer when it is included consistently in the calculation of a solar model. Here we extend the calculations of Baker and Gough to Cepheids and the sun.

Keywords: Convection, stars: pulsation, stars: Cepheids, stars: the sun.

1. INTRODUCTION

The interaction of convection and pulsation in stellar envelopes can be modelled in various ways. The problem has been approached numerically via fully nonlinear hydrodynamic computations (Deupree, 1977). Alternatively, the development of a model from the linear, pulsation equations, utilizing a mixing-length concept, is also possible (Unno, 1976, Gough, 1977). Dimensional analysis and the spectral theory of turbulence may also be applied to provide a basis for a similar, quasi-linear computational method (Xiong, 1978).

The application of such models has produced mixed results: Gonczi and Osaki (1980), using Unno's theory, failed to predict a return to stability at the cool edge of the Cepheid instability strip, unlike Xiong (1980), using his own theory. Baker and Gough (1979), using Gough's (1977) theory, and Deupree's (1977) nonlinear computations were successful in producing a red edge for the RR Lyrae instability strip.

At present there appear to be divided opinions as to whether solar acoustic modes are self-excited (Antia, Chitre and Gough, 1980), or excited stochastically (Goldreich and Keeley, 1977, Christensen-Dalsgaard, Gough and Libbrecht, 1988).
The solid line indicates the evolutionary track of Boker and Kippenhohn (1965). Also indicated by the dotted line are the rough borders of the instability strip according to our calculations.

Figure 2 illustrates the variation of the stability coefficient, $\eta$, across the strip. The cases shown are those for mixing lengths of 1.0 and 1.5 pressure scale heights in both the constant- (labelled Vitense) and variable- (labelled Gough) theories. The variation of this factor appears to make little difference to the results. It could be argued that the actual width of the strip is narrower for the latter theory and so force it to compare better with observations, but the inconsistencies and calibration problems inherent in the mixing-length theory lend no credibility to this conclusion. Furthermore, the assumption that the convective cell can best be approximated by the most rapidly growing eddy is certainly suspect when this eddy is actually predicted to be, in some instances, long and thin, and consequently subject to shear instabilities which destroy the eddy long before it can reach its limiting amplitude. This is actually a failing of the mixing-length formulation in the form given by Gough (1977) ; the mixing length should obviously be determined by a quantity dependent on the shear experienced by the eddy in question (see Gough, 1978).
Much as in the same way as the solar radius can be used to calibrate time independent mixing-length theory, calibration of time-dependent theories is to some extent possible by fitting the theoretical instability strip to the observed one (Xiong, 1982). The physical relevance of this calibration is as questionable as in the former case.

In common with mixing-length theory, Xiong's turbulence theory is also dependent on an undetermined length scale; that theory requires the specification of the scale associated with the wave number describing the local variation of the velocity correlation integral, \( < u_i u_j > \), or what is essentially the mean size of large energy-containing eddies (presumably the dominating cell in a convective envelope). In some sense, then, this is precisely equivalent to a local mixing length, though in Xiong's formulation it is derived from a study of the quasi-linear fluctuation equations and spectral theory of turbulence, and is not a basic physical concept used to describe the terms involving the fluctuations in the mean equations, as in Unno's and Gough's theories.

How much of an improvement Xiong's analysis is above a mixing-length approach is difficult to appreciate.

Our results here are comparable to Xiong's calculations (1980): the double-peaked structure to figure 2 results from the varying abilities of the hydrogen and second helium ionization zones to drive the pulsation. Xiong's models appear to be too sparsely populated in the strip to resolve this feature. It would also appear that his envelope models contained too few mesh points to resolve the spatial oscillations of the local convection theory, though his illustrations do hint to their existence.

There are two main differences between our (and indeed Baker and Gough's, 1979) and Xiong's results:

1. When the convection interaction is included our results indicate that the overall effect is that in hotter models the driving is enhanced whereas in the cooler models it is weakened. Xiong's results, however, indicate that in most cases convection serves only to reduce instability.

2. At the lowest temperatures Xiong's calculations suggest that the growth rate plummets to negative values, whereas here it reaches a minimum and then increases slowly towards neutral stability.

These points may both result from the neglect of various turbulent dissipation mechanisms in our calculations (c.f. Gonzalez, 1981, 1982).

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The stability coefficient for the case where the convective flux perturbation is included can be either greater or smaller than that for the case where it is neglected. In addition to the convective flux producing a contribution to the work integral, the radiative flux contribution is also modified as a result of the alteration to the eigenfunctions. Indeed, it is the interplay of these effects that determines whether the stability coefficient is enhanced or reduced with respect to its value when convective perturbations are ignored. However, the radiative and convective components of the heat flux are inextricably intertwined and their contributions to the stability integral cannot be unambiguously separated. Therefore, any discussion of the balance between the radiative and convective contributions is not a simple matter.

A problem of a different kind appeared in the course of our calculations: in attempting to incorporate the turbulent pressure in the manner of Baker and Gough, we found that the predicted ratio of turbulent to total pressure approached unity towards the middle of the instability strip. Here the convective efficacy is quite large in the envelope and even the moderate value of 1.5 for \( \alpha \) (the mixing length to pressure scale height ratio) predicts an unbelievably large turbulent pressure. Of course the turbulent pressure may be quite successfully constrained to reasonable values by the inclusion of a term in the dynamical equations which models effects such as acoustic wave generation (D.W. Moore and E.A. Speigel, personal communication). However, as noted by Gough (1976), it is possible to choose the mixing length so that it gives an unphysical solution for the turbulent pressure at the lower boundary of the convection zone where higher order terms such as that modelling wave generation presumably do not matter, and possibly this may well be what is happening here.

It would appear, then, that the value of the mixing length is limited, and is not so free a parameter than is commonly assumed. It is also interesting to note that this occurs for the moderate values of \( \alpha \) typically quoted in model calculations.

In the formulation of the local convection theory, the turbulent pressure gradient raises the order of the equations in the convection zone of the equilibrium model. Despite the singularities that then occur at the edges of the convection zone, the equations may still be successfully integrated to give a consistent model. We find that the gradient of the temperature in the superadiabatic region is then reduced, as predicted by, for example, Gough (1977).

Alternatively, neglecting the turbulent pressure gradient in the superadiabatic temperature gradient artificially reduces the order of the equations and allows a lower-order system to be solved (Henney et al., 1986, Baker and Gough, 1979).

The turbulent pressure perturbation is more difficult to incorporate, though it may be included by an iteration procedure of the form used by Antia et al. (1981). It may be included consistently by actually solving the mixing-length equation determining it, imposing a regularity condition at the boundaries. Here, however, we have neglected the gradient of the turbulent pressure perturbation in the pulsation calculations, with the consequence that, again, the order of the system is artificially reduced.

The results of our calculations are illustrated in figure 3. When the Reynolds stresses are introduced, the effect upon the vibrational stability is dominated by the isotropic portions of the stresses. Specifically, the major contribution to the work integral (c.f. Baker and Gough) arises from the part of the fluctuation of the turbulent pressure that \( \phi \) oscillates out of phase with the temperature or density fluctuation, in exactly the manner that the gas and its perturbation performs work. The anisotropic component of the stress tensor is always negligible.

Comparisons with the case where the turbulent pressure is accounted for only through its influence upon the
mean structure of the envelope, suggests that its inclusion may both increase or decrease the vibrational stability of models of different stars. In this manner it is similar to the convective flux and its perturbation.

Xiong (1978) and Gough (1977) have both suggested that the turbulent pressure increases in importance as lower effective temperatures are approached, but it is interesting to note that beyond the red edge the neglect of its perturbation is actually less important than at the higher temperatures inside the strip. Also, the increasing efficacy of convection towards the higher temperatures creates a very sensitive superadiabatic zone. Consequently the approximate solution that artificially reduces the order of the equations begins to deviate quite significantly from that obtained by consistently including the turbulent pressure, and this is reflected in the resulting growth rates of pulsations. Here, then, it would appear that consistency is crucial in determining the stability coefficients, and both the static model and pulsations ought to be computed by consistently including the turbulent pressure and its gradient.

3. THE STABILITY OF SOLAR RADIAL MODES

We have attempted to include the Eddington approximation to radiative transfer into the calculations using the convection theory of Gough (1977), with the aim of applying it to the problem of the stability of solar, radial p modes.

The calculations were performed on a fairly simple set of solar models: The temperature-optical depth relation of the Harvard-Smithsonian Reference Atmosphere (Gingerich et al., 1971) together with the requirement of hydrostatic balance was used to determine the structure of the atmosphere. The outer boundary was located at an optical depth of $10^{-4}$, where the pulsation was matched onto an adiabatic, outwardly propagating wave in an isothermal atmosphere at a temperature of $1.6 \times 10^{6} K$. The envelope was then integrated down to a sufficient depth (which was comparable with that of the model used by Ando and Osaki (1975)) in a manner similar to that for the delta Cephei models discussed above. This gave a solar model with approximately sixty atmospheric mesh points, and four hundred points in the envelope.

The results for the usual Eddington calculation (i.e. calculations such as those of Ando and Osaki (1975,1977) which ignore the convective flux perturbation) are shown in figure 4. The case which includes the Eddington factor was computed by treating the atmosphere in the manner suggested by Christensen-Dalsgaard and Frandsen (1983). The results do not agree exactly with those of, for example, Ando and Osaki, but by adjusting certain features of the equilibrium model, the location of where the modes become stabilized due to the inclusion of increased radiative damping in the atmosphere (when radiative transfer is treated more thoroughly) can be shifted with respect the frequency. The mixing length and the composition are obvious parameters. Indeed, decreasing the mixing length, broadly speaking, lowers the convective efficacy, and consequently a greater proportion of the total energy flux must be carried by convection in order to reduce the temperature gradient by the same degree. For modes that are unaffected by the alteration to the deeper parts of the convection zone resulting from the decrease in mixing length, the associated reduction in the radiative flux in the driving zone could increase the stability of the modes. Decreasing the mixing length may also act to reduce the depth of the convection zone. Consequently, lower-order modes, pene-
trating the more deeply, may experience increased radiative damping in the interior. The results obtained numerically for different static models appear to confirm these simple expectations.

There is also numerical evidence that our envelope is not deep enough to account completely for radiative damping in the interior. This may indeed have led to an overestimate of the growth rates, but we estimate that it does so insubstantially.

Note that the case computed in the diffusion approximation shown in our graphs is inconsistent, because it was computed using a model which was calculated with the Eddington approximation. This, however, does not affect the results appreciably: using the diffusion approximation explicitly gives a form of the equations for the pulsations whose coefficients depend upon $J/B$, rather than the explicit factor $J - B$ that arises in any formulation using the Eddington approximation or a more complicated approximation to radiative transfer theory. This constitutes a major difference: the neglect of $J - B$ in the first case introduces merely a slight inaccuracy, though in the second it involves the neglect of what is possibly an important term. Thus the results for the diffusion approximation should be fairly insensitive to the static model, where the inclusion of radiative transfer only slightly alters the mean structure. Additional computations showed that this was indeed the case. This is not to say that the diffusion approximation is better than expected; it becomes very inaccurate in the atmosphere, and this is particularly serious for high-order modes. Christensen-Dalsgaard and Frandsen's results are a testament to the other case.

The results are in complete disagreement with those of Kidman and Cox (1984) as indeed are most other calculations.

The reasons for this are not obvious. Neither do our calculations completely agree with those of Christensen-Dalsgaard and Frandsen (1982). Those calculations were performed upon a very detailed solar model, and the difference between this and our very crude equilibrium model could account for the lack of agreement. As appears to be the case, the proximity of the lower-order modes to marginal stability causes the frustrating fragility of the sign of the stability coefficient. Consequently every conceivable hydrodynamic effect could be important in determining the stability of these modes, especially stabilizing effects such as viscosity: the calculations reported here are thus incomplete.

We have made a preliminary investigation of the effect of incorporating the perturbation to the convective heat flux. Once again the diffusion approximation appears to underestimate atmospheric radiative damping severely. These results are illustrated in figure 5. The jagged appearance of these plots illustrates a resonance feature between the pulsation period and timescales of the local convection theory. It would appear, then, that the convection theory, at this level of approximation, serves to increase the excitation of at least some solar radial modes. However, turbulent pressure has a damping influence: as can be seen from figure 6 all but two of the modes are stabilised.

4. CONCLUSION

We have applied Gough's (1977) convection theory to pulsating stars and have demonstrated that its inclusion may exert a stabilizing influence on some stars, particularly those towards the cool side of the Cepheid instability strip. We have compared our results briefly with those of Xiong.

We included to some extent the Reynolds stress in the
calculations to find that its role in stellar structure and stability is not easily understood. More detailed computations are planned for the future.

In the case of solar acoustic modes, we have applied a modified version of the theory in order to compute pulsation growth rates within the framework of the Eddington approximation to radiative transfer. It would appear from our calculations that increased radiative damping in the atmosphere occurs as a result of improving the treatment of radiative transfer. These results appear to agree with certain previous calculations. The results of the calculations of Kidman and Cox (1984) do not agree with ours, but there is some similarity between our results and those of Ando and Osaki (1975, 1977), Goldreich and Keeley (1977) and with Antia et al. (1987).

We have found that when we include mixing-length representations of the fluctuations of the convective heat flux and Reynolds stress, almost all of the modes are stabilized. As we have emphasised, the stability of the modes results from a delicate balance of several different mechanisms and we have not yet had time to determine whether this conclusion is robust.

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Session 2

Internal rotation: Splitting

Chairman: T. Duvall
SOLAR INTERNAL ROTATION FROM HELIOSEISMOLOGY

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ABSTRACT

Observations of solar oscillations allow one to determine the properties of internal solar rotation. This branch of helioseismology, like most others, is enjoying rapid growth. Many diverse methods are used to determine the internal solar rotation. Measurements of p-mode frequencies made by several groups are in essential agreement that rotation within the convection zone differs in depth and latitude only slightly from the observed rotation of the surface. Deeper layers seem to rotate with less latitudinal differential rotation. The rotation of the energy-generating core is not well determined by p-mode oscillations and it may be rotating somewhat faster or at about the same rate as the surface. Observations of g-modes support the notion of a rapidly rotating core. Great improvements in knowledge of solar internal rotation are certain as higher resolution is obtained in images, frequency, and spherical harmonic degree and order. Signal-to-noise ratio will also improve greatly with the availability of continuous data from networks and from spacecraft.

Keywords: Helioseismology, Sun, Solar Interior, Solar Rotation

1. INTRODUCTION

The rotation of stars plays an important role in their evolution and the production of stellar magnetic activity. Therefore one of the primary goals of helioseismology is to define the nature of motions inside the sun - especially rotation. The importance of the subject (or perhaps the level of contention) is indicated by the presence of two reviews of the subject in these proceedings. This paper reviews current knowledge of internal rotation as derived from helioseismology with a strong emphasis on observations. Progress is so rapid, like many aspects of helioseismology, that this review will be obsolete by the end of this meeting. Figure 1 emphasizes this point by showing that the size of the helioseismology literature is roughly doubling every three years. Specific works on solar internal rotation show a similar trend. This can be compared with a doubling time for all astronomical papers of 18.3 years and 26 years for solar papers (Ref. 1).

There is a rich theoretical and modeling literature dealing with rotation inside the sun (e.g. Refs. 2-7). Indeed, a cynic is able to find a prediction of almost any type of rotation. Observational helioseismology gives us the means to test these predictions. While this review concentrates on observational matters, the importance of a close interplay with theory must be emphasized.

Several recent reviews discuss observational helioseismology results concerning internal solar rotation (Refs. 8-17). There are also non-helioseismology observations bearing on the nature of internal solar rotation. Recent measurements of the oblateness of the sun’s figure yield a slightly larger value than would be expected if the entire sun rotates like the surface (Ref. 18). In particular, while rapid rotation of much of the interior is not consistent with these results, a core rotating twice as fast as the surface cannot be excluded. Measurements of the rotation of visible features provide only hints about internal rotation. The main difficulty is to assign depths to various phenomena such as sunspots, magnetic field patterns and supergranulation. The bulk of the current evidence suggests that features likely to be anchored below the surface rotate 1-3% more rapidly than the surface however the details are murky (but see Ref. 19). Some recent reviews of surface rotation are found in Refs. 20-22.
As a working hypothesis, suppose that the rotation rate of the sun is constant beneath the convection zone and within the convection zone is constant with radius and possesses the rate seen at the surface. Figure 2 is a pictorial representation of this model. Do observations support or refute such a model?

![Figure 2](image)

Figure 2. A meridional cut of a hypothetical model of internal solar rotation. Rotation rate is indicated by intensity. The rate is constant below the convection zone and constant with radius in the convection zone at the surface rate. The poles are at the top and bottom.

2. BASIC METHODS AND CAUTIONS

The basis for measuring internal rotation by helioseismology is conceptually simple: Frequencies of oscillation modes are altered (perturbed) in a rotating sun compared with a non-rotating sun by an amount which depends on a suitable average of the rotation rate of the cavity within which the mode is confined. The observer's job is to measure the frequency perturbations of modes which sample a wide range of cavities in radius and latitude. This allows the internal rotation to be deduced either by various inversion schemes or by comparison with forward modeling.

This simple picture has a number of complications. Frequencies can be altered by other factors such as magnetic fields, large-scale circulation patterns, large-scale temperature fluctuations, and the slightly non-spherical shape of the sun. Helioseismological probing of these phenomena will become increasingly important in the future but for our present purposes they are an unwelcome complication in a straightforward determination of internal rotation. Rotation is the major cause of frequency changes and we can neglect the other sources for the time being.

There are still complications in the simple methodology. One of the most basic is the issue of global modes and local standing wave patterns. Wave disturbances which retain coherency for a time longer than required for propagation once around the sun's circumference are global. That is, their frequencies depend upon conditions in a cavity which extends completely around the sun. Observationally, this means that one sees only a continuous structure in a spatial-temporal power spectrum, i.e., the oscillation amplitude is not discrete with respect to global quantum numbers $l$ and $m$. Gravity waves are likely to have lifetimes so long that any g modes which may be observed can be considered as global. In the case of f and p modes, both global and local modes exist. The transition from global to local is not yet well defined. Observations from SOHO will provide a definitive answer and in the meantime a South Pole experiment is planned this November to address the question. The data discussed in Ref. 23 show that global p modes can be recognized to $l$ values of at least 130 (at $v=3.4$ mHz) and to frequencies of at least 5.0 mHz (at $l=40$). Until global and local modes can clearly be distinguished, it is important to use care in interpreting observational results.

With these cautions in mind, we follow Brown (Ref. 8) to develop a few results useful in planning and reducing observations. The frequency shift of adiabatic, nonradial oscillations produced by slow differential rotation is a complicated function (Refs. 24-26). If one neglects Coriolis and centrifugal forces and assumes that the variation of $\Omega$, the solar rotation rate, is negligible with latitude compared with the variation of $\Omega^m$, then

$$v(a,l,m) - v(a,l,0) = -m \frac{\int \bar{\Omega}(\theta)(P^m_l(\cos \theta))^2 \cos \theta}{\int (P^m_l(\cos \theta))^2 \cos \theta}$$

where $\bar{\Omega}$ is a suitable average of the rotation rate over depth and latitude and $\theta$ is the colatitude. Coriolis forces can be neglected at high degree but produce significant effects at low degree and frequency (Refs. 4,27). The $m$ symmetry of the integrals in Eq. 1 means that the frequency splitting due to slow rotation is antisymmetric in $m$. It is thus convenient to express observational measurements of rotational frequency splitting as coefficients of series of antisymmetric functions of $m$. For global modes the observer's job is simply to measure the frequencies of as many different modes as high a precision as possible.

At degrees and frequencies where truly global modes do not exist, the observer's job is more complicated. Here we need to measure the frequency shift of advected oscillation patterns which are not discrete with respect to the $l,m$ quantum numbers. In practice this means measuring the frequency difference of the envelope of spectral power associated with a given value of $m$, the radial order number, and the now continuous values of $l$ and $m$ (or equivalently, horizontal wave number vector).

It is useful to visualize the cavities in which modes are confined. Such visualizations remind us of the large amount of volume averaging which attends helioseismological measurements. A few examples are illustrated in Figure 3. These figures are not based on precise calculations.

3. OBSERVATIONAL REQUIREMENTS

3.1 Need for spectral discrimination

We are confronted with an incredibly rich solar oscillation spectrum. Thus the first task of the observer is to plan observations in a way to reduce the complexity of the observed spectrum to a manageable level. Neglecting this point is perilous and can lead to hopelessly confused spectra. The strategy is simple: maximize resolution in $(l,m,v)$ while minimizing the number of spurious spectral features. Many of the points below have been discussed elsewhere (e.g. Refs. 13,28).
Angular range determines resolution in \( l \) and \( m \); while the number of angular elements determines the range covered in \( l_m \). Roughly speaking, the largest value of \( l_m \) observable is the reciprocal of the angular element size in units of the solar diameter. It is worth emphasizing the advantage of angular resolution even for observations of low-degree modes because of the opportunity to simplify otherwise complex and crowded spectra. As more advanced detectors have become available, the number of angular elements which may be observed simultaneously has increased. Several groups have equipment which allows more than \( 10^6 \) elements to be measured at the same time. This offers excellent scientific possibilities but also huge data management problems. If one is interested primarily in low-degree modes then large detector arrays are not required. Indeed, single-element detectors, sometimes combined with clever masking of the solar image, are used to obtain some of the best helioseismology observations.

Angular distortion of the solar image is a serious problem for solar rotation measurements. Any distortion which alters the scale or position of an oscillation pattern is a potential source of trouble. For example, if the diameter of the sun is 1% different than assumed for reduction purposes, then an \( l=100 \) mode will appear to be either 101 or 99 depending on the sign of the error. This type of error is difficult to detect in observations which do not cover the entire disk. Even if one has the diameter correct, one needs to guard against a variable magnification across the field of the focal plane. Another problem is atmospheric seeing which mixes and attenuates the appearance of high-degree patterns (Ref. 29).

Errors in angular pointing need to be monitored and corrected in solar rotation measurements, particularly if one is observing high-degree modes with a detector which records less than the entire solar rotation. A drift in pointing can alter the apparent rotation rate of an oscillation pattern at the rate of about 10% for a drift of 1 arc s per hour. Even if the whole disk is recorded, such subtle effects as differential refraction during a day’s observations can alter the apparent rotation rate of oscillation patterns. The effect is that the true center of the solar disk moves across the sky from east to west more rapidly than the apparent disk center.

### 3.3 Temporal considerations

Time sampling of solar oscillation data should be frequent enough to ensure that the Nyquist frequency of the sampling is greater than any frequency of interest. Global oscillation modes have not been detected at a frequency in excess of 5 mHz so a sampling frequency of 10 mHz would seem to be adequate. Another factor, however, is to suppress aliasing of high-frequency signals into frequencies of interest. Antialiasing can be achieved by rapid subsampling within the basic sample period combined with a suitable weighted averaging of the sub-samples. This technique has not, to my knowledge, been applied to solar oscillation data. The usual approach is to set the sampling frequency high enough to avoid aliasing of any oscillation features. Recent observations have shown that trapped oscillations can be detected at least 7.3 mHz (Ref. 23) so a sampling rate of once per minute should be adequate. Even if one avoids aliasing of oscillation patterns, it is important to avoid aliasing of high frequency noise. This problem can be mitigated by integrating the solar image continuously during the sample period rather than simply taking one ‘snapshot’ during the interval.

The duration of observations governs the frequency resolution. Since our goal is to resolve the frequency splitting of oscillation multiplets and to measure mode frequencies accurately, it follows that we should observe for at least one rotation period. In practice,
one should observe for one and a half times the rotation period (Ref. 30). A number of good helioseismology results on rotation have been obtained from observations as short as half a day. How can this be? Since the frequency shift produced by rotation is a product of the rotation rate and the $m$ value, large frequency shifts are produced at high degrees. Thus, if one takes care to isolate a specific range of $m/l$ values, for example $m=±l$, then rotational frequency splitting can be measured, in this case for $l$ values $>50$ using a half-day set of observations.

The linewidth of individual modes is another important factor in designing observations of rotational line splitting. If a line is intrinsically narrow then the measurements of its frequency can be made more accurately. A number of studies (e.g. Refs. 31,32) have shown that the variance of the most likely center frequency of a wide range of reasonable spectral features,

$$\sigma_f^2 = \frac{\Delta f \Delta v}{snr^2} \tag{2}$$

where $\Delta f$ is the frequency full width at half maximum of the spectral feature, $\Delta v$ is the frequency resolution of the measurement and $snr$ is the signal-to-noise ratio of the measurement (Ref. 33).

Therefore, confirming intuition, high signal-to-noise ratio observations with good frequency resolution lead to good frequency measurements. It is now well established that intermediate and low-degree $p$ modes have spectral line widths of about 1 $\mu$Hz at a frequency around 3 mHz and that width is a strong function of frequency (e.g. Ref. 34). Thus, other factors equal, low-frequency modes offer the best possibilities for accurate measurements of rotational splitting.

Signal-to-noise ratio is an important factor in Eq. 2. Interruptions in a series of oscillation observations cause noise in the form of spectral sidebands around the true spectral features. Any kind of observational temporal modulation may cause trouble. Essentially, the modes act as carriers and power is shifted from them into sidebands depending on the type and strength of temporal modulation. Improving signal-to-noise ratio by eliminating or reducing observational temporal modulation is the main motivation for networks of ground-based oscillation instruments and also for observing from space.

3.4 Signal and noise considerations

In preparing to observe rotational frequency splitting, we seek to produce the largest possible signal-to-noise ratio. As mentioned above, long, uninterrupted sequences are highly desirable. It is also important to maximize the ratio of the oscillation signal to the solar background non-oscillatory fluctuations. Recent measurements have made it possible to be fairly precise about this issue, at least for the case of $p$ modes. Mode line width and velocity amplitude measurements (Ref. 35) can be combined with the measured background velocity amplitude (Ref. 36) to produce the ratio between the rms velocity amplitude of a mode and the rms background velocity amplitude within a frequency band equal to a mode line width.

This function is shown in Figure 4 along with a somewhat less certain similar calculation based on intensity measurements (Ref. 37). Since mode line width increases slowly and background noise decreases slowly with increasing degree, these functions are reasonable estimates for a range of degrees up to about 50.

This illustration shows that Doppler shift observations promise better results than intensity measurements. Naturally, instrumentation should be designed to have intrinsic noise levels well below the level of the solar background noise to take maximum advantage of the signals carried by sunlight. Atmospheric transparency fluctuations are a serious source of noise at low degree and low frequency. Other than raising the background noise level, it seems unlikely that this broadband source has any systematic effect on the measurement of rotational frequency splitting. However, caution is required in the low-frequency range of $g$ modes against interpreting signals produced by atmospheric fluctuations at harmonics of once per day as real solar signals. The only demonstrated remedies for these problems are to use the best observing sites or to observe from multiple sites to distinguish between atmospheric and real solar signals. The best sites can be very good indeed. For example, recent observations from the South Pole have clearly shown $p$ modes of degree zero in intensity observations (Ref. 23).

Noise is a serious problem for the observation of $g$ modes. The problem originates in the strong background of solar fluctuations at low degree and frequency where one hopes to be able to observe $g$ modes. This background results from active regions and supergranulation (Ref. 38). It is also not helpful that the spectrum of $g$ modes is expected to be very crowded and accordingly, long observing sequences are required. This reviewer is not aware of convincing observations of $g$ modes and, based on his own unsuccessful efforts to observe these elusive features, cagily awaits convincing results.

4. OBSERVATIONAL METHODS AND REDUCTIONS

The previous section dealt with requirements and limits. This section considers some of the practical methods in use to obtain rotational splitting measurements.

4.1 Integrated disk

Some of the first results on rotational splitting came from observations of Doppler shifts and intensity fluctuations of the full solar disk obtained without angular resolution. These results are restricted to low degree $(0-3)$ and values of $m$ such that $l-m$ is even. Further, without angular resolution, all the various $l$ and $m$ states are blended in a single spectrum. This technology has matured to the point that at least three ground-based networks are in operation (the Birmingham, IRIS and SLOT groups) and a space-borne instrument (ACRIM on SMM) will soon be joined by new instruments on SOHO (VIRGO and GOLF).

The Birmingham and IRIS groups use resonance scattering spectrometers to detect integrated light Doppler shifts of the $7699\AA$ potassium line (Birmingham, Ref. 39) and $5890.6\AA$ sodium lines (IRIS, Ref. 40). The basic principle is the same: alkali metal vapor is produced within a small volume pervaded by a magnetic field.
Sunlight is selectively scattered by the vapor according to wavelength, vapor pressure, magnetic field strength and polarization state. Switching the state of polarization of scattered light fed to photomultiplier detectors allows one to alternately sample the red and blue wings of the solar spectrum lines and thereby deduce its Doppler shift. The principal advantages of these detectors are relative simplicity, high sensitivity and excellent stability. The main disadvantage, at least in the simplest form of the instruments, is an inability to sample more than two points of the solar line profile. This causes a nonlinear response to Doppler shift and limits dynamic range.

The active cavity radiometer irradiance monitor (ACRIM) on board the Solar Maximum Mission satellite (SMM) measures the flux of sunlight over a very wide spectral region (Ref. 41). Its principle is to use a shutter to alternately block and admit sunlight to an absorbing cavity. When sunlight is blocked, electrical power is used to maintain the cavity at a temperature equal to that when sunlight is admitted. This provides a measure of light flux. The main advantage of this instrument is its location away from atmospheric disturbances and the long duration of available data. The main disadvantage is low sensitivity to oscillations resulting from the relatively high background solar noise level (see Figure 4) and analog-to-digital converter quantization noise.

Full-disk sun photometers operate by measuring the brightness of the sun at a number of different wavelengths (e.g. Ref. 42). To date, the noise associated with terrestrial atmospheric fluctuations has prevented any results about rotational splitting. This problem is avoided by going to space and we hope to soon hear exciting results from the IPHIR instrument on the Phobos Mars probes.

Analysis of integrated-disk observations for p-mode rotational splitting effects involves identifying and measuring the individual l,m states of oscillation multiplets. What is simple in principle is not so simple in practice. Problems arise from overlapping spectral features from different l states and frequency fine structure and sidelobes in temporally interrupted or modulated data series. The finite lifetime of the modes is also a serious difficulty which broadens and introduces fine structure in the spectral profile of a mode. Thus a clear result on the rotational splitting of low-degree p modes is still not in hand.

Turning to low-degree g modes, analysis is typically less direct because of the crowded nature of the expected g-mode spectrum. Identifications of individual g modes and their splitting properties have been made but the agreement among the observers is not as close as one would like (e.g. Ref. 43). Most recent analyses involve generating a family of synthetic g-mode spectra with two parameters: the asymptotic period separation between high-n multiplets and the rotational frequency splitting. These models are cross-correlated with observational data to find the best match.

4.2 Resolved disk

As mentioned earlier, resolving the disk into even a few elements offers the possibility of resolving the oscillation spectrum into separate, simpler spectra with responses maximized for specific l,m values. Almost every kind of photometer and spectrometer has been used for solar oscillation observations. Early work was done with grating spectrometers but these suffer a limitation of the number of spatial elements which may be passed simultaneously. As a result, image scanning of one sort or another is often employed to increase the angular range covered in grating observations. This is a noisy process which limits the ultimate quality of data from spectrographs. The principle of the observations is simple: a suitable line profile is sampled in wavelength in such a way as to allow its Doppler shift to be determined while a map of the solar surface is built up. It is necessary to provide a wavelength reference of some sort. The solar spectrum line itself can be used if one is willing to give up the low-degree modes. There is a class of differential observations used in which one divides the solar image into two or more parts and differentially measures the Doppler shifts from the various sections. This method is robust and effective for observations of low-degree modes and is still used. Other methods have superseded the use of grating spectrographs for observations of higher-degree modes.

Jnarrowband filters of various kinds are now the most widely used spectrometers for imaging observations. This is because an entire image may be observed simultaneously. Among the instruments which have produced results on rotational splitting are birefringent filters, Fabry-Perot etalons and the magneto-optical filter. Each has advantages and disadvantages but the essence of their operation is the same: make two or more images of the sun at different wavelengths across a suitable spectrum line and process the images to obtain measures of Doppler shift across the solar disk. Linearity and dynamic range are increased by making more samples across a line profile but some of the best work to date has been done with just two samples.

A Michelson interferometer can be used to measure Doppler shifts in an instrument called a Fourier Tachometer (Ref. 44). The principle is to measure the phase of one Fourier component (selected by the interferometer) of a small part of the solar spectrum which contains one spectrum line. The phase of the signal is a measure of the Doppler shift of the line. Advantages of this approach include excellent linearity and dynamic range. The main disadvantage is low sensitivity; essentially one trades sensitivity for linearity and dynamic range. This limits the number of spatial elements which may be observed simultaneously given the capabilities of current two-dimensional detector arrays. It is necessary to provide a stable wavelength reference for work at low degrees.

Observing intensity oscillations is very simple in principle: one forms an image of the sun, filtered to some wavelength range, on a two-dimensional detector array (typically a charge-coupled device). A time series of such observations reveals the oscillations. This technique has been used for rotation splitting observations at the South Pole. The signal can be enhanced by observing at a level in the solar atmosphere at which oscillations are relatively strong. The temperature minimum region is an ideal level. The main advantages of this technique are simplicity and an ability to see oscillations without attenuation over more of the solar disk than Doppler measurements. The disadvantage is a lower solar mode-to-background ratio (see Figure 4) and a strong sensitivity to atmospheric transparency fluctuations. The latter problem is particularly serious for low degrees.

4.3 Diameter/limb-darkening measurements

An extensive set of helioseismology results have come from measurements of fluctuations of continuum intensity near the solar limb. Such measurements have been made in Arizona, New Mexico, France and the South Pole and a two-station network is being established. The essence of the observational technique is to make differential measurements of the limb-darkening function at one or more azimuths around the solar limb. Because relatively small portions of the solar disk are measured, the resolution in l,m is rather coarse and one expects to obtain spectra containing large numbers of overlapping modes. This problem is somewhat reduced by combining spectra from different positions around the limb in such a way as to enhance and suppress sensitivity to various combinations of degree and azimuthal order. Also, most work has been done at low frequencies where the problem of overlap in degree is minimized due to a relatively low density of modes. Sensitivity to atmospheric fluctuations is reduced by differentially comparing the limb darkening function measured over different ranges in radius. This observational technique is hard to assess in comparison with other intensity and Doppler shift measurements because quite different results are obtained. Evidently the limb-darkening fluctuations respond very differently to p, f and g modes than do other measures such as disk intensity and Doppler shift fluctuations.
result is a rather strong discrepancy in measurements of modes in regions where limb-darkening and other techniques overlap and an inability of the other techniques to even detect oscillations detected by limb darkening measurements. There has been a tendency to dismiss the limb darkening results in favor of other methods which exhibit large signal-to-noise ratios, at least in the five-minute region. However, the history of science teaches us that prevailing views are often incorrect and one should not judge hastily until the source of the discrepancies is clearly identified.

4.4 Reductions

The reduction of observations is a crucial part of helioseismological determination of internal solar rotation. The goal is to reduce the vast amount of information in a typical oscillation spectrum to a form useful for deducing the internal rotation. The first way this was done was with imaging observations of p modes which was to collect or analyze data only concerning sectoral modes. Here the splitting is a maximum and one deals only with two spectra at a given degree instead of 2l+1 spectra. One measures the frequency difference between the +m and -m spectra and divides by 2l to obtain a measure of rotational frequency splitting. This measure is contaminated by leakage from nonsectoral modes and a correction is required (Ref. 45). Noisy individual results are then averaged. Of course, such modes are confined near the equator and do not allow one to deduce the latitudinal variation of internal rotation.

To gain latitudinal information an entire multiplet must be observed and analyzed. In the case of p modes, the first step is to represent the frequency splitting of a multiplet or a carefully selected set of multiplets as coefficients in a series expansion of frequency splitting. For very low degrees, there are so few members in a multiplet that a series expansion is hardly justified and one simply states the splitting frequency of the various modes. This procedure quickly reaches practical limits and the series expansion becomes attractive.

The first series used to express the frequency splitting was powers of m. Odd powers represented rotational effects and even powers reflected possible effects due to nonspherical distortions of the sun. However, this series is not orthogonal and so the values of coefficients depends on the number of terms used in the expansion. A much more nearly orthogonal series was introduced in Ref. 46 in an effort to reduce the dependence of the coefficients on each other and on the order of the expansion. Here frequency splitting is represented as

$$\nu(n, l, m) = \sum_{i=0}^{N} a_i P_i (-m/l)$$

where $$\nu$$ is the m-averaged frequency of the multiplet, $$P_i$$ are the Legendre polynomials of degree i and $$L=\ell(|l|+1)$$. In practice, current data quality justifies setting $$N=5$$. The argument m/l rather than m/L was chosen to minimize the variation of the coefficients with degree. That is, if one assumes that the entire sun rotates differentially like the surface then the values of $$a_i$$ one computes are nearly independent of degree. Unfortunately, both arguments are in use which means that at low degree there is a slight difference in the values of the coefficients for otherwise identical data. The difference is negligible for degrees above 20 or so, given currently available observational precision, but should be kept in mind as observational quality improves. Note that the coefficients can be added to approximate the sectoral frequency splitting as

$$\Delta \nu_{\text{sectoral}} = a_1 + a_2 + a_3 = a_{135}.$$  

The fit of Eq. 3 to a single multiplet is typically noisy and the data are combined in various ways to reduce the noise. A simple method of doing this is to produce an m-averaged spectrum at a given degree after removing splitting effects and then cross correlating this spectrum with the individual spectra as a function of m (Ref. 47). The set of frequency offsets of peak correlation as a function of m is then used to solve for the coefficients of Eq. 3 and the process is iterated until convergence is achieved. This method of averaging data combines all the radial orders for a given degree which blurs the potential resolution in radius and latitude of any deductions about internal rotation. More importantly, the cross-correlation method is sensitive to modes having different degrees which leak into the spectrum being analyzed. At high degrees, this leakage is not expected to bias the analysis but that is not the case at degrees of about 15 and lower (Ref. 10). The tendency will be for rotational splitting to be overestimated at low degree.

An improvement to averaging by cross correlation when one has good enough signal-to-noise ratio spectra is to fit the modes of each multiplet individually, solve for the coefficients of Eq. 3 and then average the coefficients from different multiplets with suitable weighting. This process has been used in Ref. 34. Here it was found that there is a significant frequency variation of the coefficients as well as a degree dependence. This led to the introduction of modified, n-averaged coefficients, viz.

$$a_i(n) = a_i + \frac{\nu}{2\pi}$$

where $$\nu$$ is in mHz. The $$b_i$$ coefficients are small but should not be forgotten when comparing values of $$a_i$$ and $$a_i(n)$$. A further improvement suggests itself: average the individual multiplet coefficients by combining results from multiplets having similar values of L/n since such modes lie within more or less the same cavity and should have more or less the same coefficients. Figure 5 compares the unaveraged splitting measurements of Ref. 47 plotted against l and against L/n. There appears to be some reduction of scatter in the second plot.

An alternative expansion of rotational frequency splitting has been proposed in Ref. 48:

$$\nu(n, l, m) = \sum_{i=0}^{N} b_i P_i (m/l)$$

where the symbols are the same as for Eq. 3. The advantage of this expansion is a close relation of the $$b_i$$'s to the coefficients of a Legendre series expansion of solar rotation rate as a function of latitude. Error propagation properties are unknown and this expansion has not yet been adopted by observers.

At some point in the development of helioseismology, the utility of using averaged series expansions of frequency splitting will diminish. This is because such expansions are based on concepts of smoothness which may not be justified for the sun. To obtain the maximum resolution inside the sun will require preserving all the frequency splitting information.

5. OBSERVATIONAL RESULTS

This section will quickly become obsolete since it deals with the status of observational results in a field which is rapidly improving. It is thus fairly brief.

5.1 Intermediate degree p modes

Most of the results about rotational splitting have come from observations of p modes in the intermediate-degree range, say 10-100. This region allows one to probe most of the volume of the sun and is readily observable with current technology. The observational results are in basic agreement. Table 1 summarizes the results currently available. There are at least two forthcoming studies which are not yet available for inclusion here.

For purposes of comparison, only the most recent available or most extensive results from an observing group are considered here. This means that the 1982.0 South Pole observations, 1984.4 HAO observations and the 1985.6 Big Bear observations will not be dis-
rotational frequency splitting measurements derived from observations of sectoral $p$ modes (Ref. 47). The results are shown, without enumeration, plotted against different variables. The abscissa of the upper plot is spherical harmonic degree. The abscissa of the lower plot is $L/v$. Reduced scatter in the lower plot suggests that observational results should be averaged in ranges of $L/v$ rather than degree.

| Table 1. Intermediate-$l$ splitting observations (full disk). |
|---|---|---|---|---|
| epoch | span $^a$ | method $^b$ | $l$ range | $m$ range | analysis $^c$ | Ref. |
| 1982.0 | 2 | K | 20-98 | all | C | 46 |
| 1983.5 | 17 | G | 1-100 | ±1 | F | 45 |
| 1984.4 | 5 | F | 8-50 | all | C | 47 |
| 1984.5 | 35 | M | 5-120 | all | C | 49 |
| 1984.6 | 16 | M | 3-89 | all | C | 50 |
| 1984.8 | 15 | F | 15-99 | all | C | 51 |
| 1985.6 | 12 | B | 5-20 | all | F | 52 |
| 1986.4 | 129 | B | 10-60 | all | F | 34 |
| 1987.9 | 2 | K | 20-100 | all | C | 23 |

$^a$days.

$^b$K=CalT K-line intensity; G=grating spectrograph Doppler; F=Fourier tachometer Doppler; M=magneto-optic filter Doppler; B=birefringent filter Doppler.

$^c$C=cross-correlation; F=fit.

It is clear in Figure 6 that $a_1$ shows systematic differences in the results from different groups. In particular, the Mt. Wilson measurements are about 5 nHz larger than the others. The origin of these differences is not clear. One possibility is slight errors in the solar diameter used for the reductions. Another possibility is image distortion or drift in some or all of the data. Finally, it may just be that these differences originate in the sun and indicate changes in internal rotation as a function of time. Further observations are required (as usual). A clear trend in all the results is a reduction of the value of $a_1$ with decreasing degree. This suggests a decrease in average rotation rate with increasing depth.

Values of $a_3$ shown in Figure 7 from various studies are in substantial agreement except for Mt. Wilson results which are systematically smaller at $l>30$ and the South Pole 87 results which tend to be larger at $l<40$. The values cluster around the surface result except at lower degrees where a decline is seen. This suggests that as depth increases, the differential rotation with latitude becomes weaker than seen at the surface.

The values of $a_4$ are fairly noisy but, within this noise, the results are in general agreement with each other. At $l>40$ somewhat larger values than the surface seem to be indicated by the data while at $l<40$ values tend toward zero. This again suggests a decrease in latitudinal differential rotation with depth.
Figure 7. The coefficient \( a_3 \) of Eq. 3 as a function of degree determined from several independent investigations. Symbols are the same as in Figure 6.

Figure 8. The coefficient \( a_5 \) of Eq. 3 as a function of degree determined from several independent investigations. Symbols are the same as in Figure 6.

Figure 9. Rotational frequency splitting of sectoral modes as a function of degree determined from several independent investigations. Symbols are the same as in Figure 6 with a diamond added to indicate the results of Ref. 45.

It is useful to compare the various studies of rotational splitting in terms of sectoral splitting. This is an approximate measure of rotation along the equator. Here we add the direct observations in Ref. 45 and note that Ref. 50 presents measurements of sectoral splitting between degrees 90 and 170. The latter are not included in Figure 9 as no tabular values were presented.

The sectoral splitting measurements show only some slight systematic differences among observers. Some of this can be traced to differences in \( a_i \) as discussed earlier. It is thus not clear whether the differences originate in the observations, reductions, analyses or in the sun itself. We note that the differences between various results are smaller for the \( a_3 \) than the \( a_1 \) coefficients. This would seem to imply that the equatorial rotation rate is either more stable in time or is more accurately measured than the average rotation rate over latitude. The general decline in \( a_{135} \) with decreasing degree indicates that the internal rotation rate sampled by sectoral modes decreases as depth increases. Note that this is not the same as implying that the equatorial rotation rate decreases inward since lower-degree sectoral modes average over a wider range of latitude than do shallow, high-degree sectoral modes (see Figure 3).

An unexpected result from helioseismology is the finding of significant non-zero values of \( a_{24} \). These coefficients represent symmetric variations with respect to the solar equator and should therefore be zero unless the structure of the sun varies in some way with latitude. As the coefficients are not relevant to the subject of this review, they will not be discussed further here.

5.2 High-degree \( p \) modes

The first rotational splitting measurements were made using high-degree \( p \) modes. Only a few results have been published since then. As mentioned earlier, the basic problem is one of determining scale and distortions with high accuracy. Observations to date are listed in Table 2 and, unlike the case for the intermediate degree \( p \) modes, the observations have in most cases been reduced to estimates of equatorial rotation rate as a function of depth. The results are shown in Figure 10. Here the results of Ref. 54 have been shifted upward so that the most shallow depth agrees with the measured equatorial rotation rate of surface magnetic features. Included in the figure are the results of an inversion of the direct sectoral mode observations of Ref. 45 (Refs. 55, 56) and an estimate of equatorial rotation rate with depth from Ref. 50.

<p>| Table 2. High-( l ) sectoral splitting observations. |</p>
<table>
<thead>
<tr>
<th>epoch</th>
<th>method</th>
<th>( l ) range</th>
<th>analysis</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1977</td>
<td>G</td>
<td>125-900</td>
<td>C</td>
<td>57</td>
</tr>
<tr>
<td>1980</td>
<td>G</td>
<td>150-700</td>
<td>C</td>
<td>58</td>
</tr>
<tr>
<td>1981</td>
<td>G</td>
<td>80-995</td>
<td>S</td>
<td>54</td>
</tr>
<tr>
<td>1984</td>
<td>M</td>
<td>90-170</td>
<td>C</td>
<td>50</td>
</tr>
<tr>
<td>1986</td>
<td>E</td>
<td>80-150</td>
<td>S</td>
<td>59</td>
</tr>
</tbody>
</table>

\( G = \) grating spectograph; \( M = \) magneto-optical filter; \( E = \) Fabry-Perot etalon.

\( C = \) cross correlation; \( S = \) spline fits.
It is evident in Figure 10 that the various results are not in excellent agreement. One suspects that the inversions are the basic source of the disagreements and that, as a result, the depth scales may not match well. It is also quite possible that the non-global modes observed at high degrees are responding to local, large-scale velocity flows in addition to rotation. One may conclude from these results that the sectoral rotation rate at first may increase slightly with depth and that it almost certainly decreases at greater depths.

5.3 Low-degree p modes

From an astrophysical viewpoint, the most interesting question about solar rotation may well be its nature in the core. Unfortunately, this region is only probed by low-degree p modes which are hard to measure accurately. Currently available measurements from no- and low-resolution instruments are collected in Table 3.

Table 3. Low-l p-mode splitting observations.

<table>
<thead>
<tr>
<th>epoch</th>
<th>method</th>
<th>l value</th>
<th>analysis</th>
<th>$\Delta f$</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>R</td>
<td>1-2</td>
<td>S</td>
<td>0.75</td>
<td>60</td>
</tr>
<tr>
<td>1980</td>
<td>A</td>
<td>1-2</td>
<td>F</td>
<td>&lt;0.5</td>
<td>61</td>
</tr>
<tr>
<td>1980</td>
<td>R</td>
<td>1-2</td>
<td>L</td>
<td>&lt;0.75</td>
<td>62</td>
</tr>
<tr>
<td>1981</td>
<td>R</td>
<td>1</td>
<td>S</td>
<td>0.75</td>
<td>63</td>
</tr>
<tr>
<td>1981-1984</td>
<td>R</td>
<td>1</td>
<td>M</td>
<td>0.68</td>
<td>64</td>
</tr>
<tr>
<td>1984</td>
<td>R</td>
<td>1</td>
<td>M</td>
<td>0.75</td>
<td>65</td>
</tr>
<tr>
<td>1984</td>
<td>G</td>
<td>2-5</td>
<td>A</td>
<td>0.4</td>
<td>66</td>
</tr>
</tbody>
</table>

$A$=ACRIM radiometer; $R$=resonance scattering spectrometer; $G$=grating spectrometer.
$F$=model fit; $S$=splitting; $L$=line broadening; $M$=modulation period; $A$=autocorrelation.
$\mu$Hz.

The available measurements fall into two classes: values of about 0.75 $\mu$Hz and values less than 0.5 $\mu$Hz. Although this difference is small, it is astrophysically crucial since the former values imply a rapidly rotating core while the latter do not. One of the main problems in making a good measurement of low degree p-mode splitting is the small number of modes which can be observed. Each mode exhibits frequency fine structure because of its finite lifetime and also because of modulation associated with observing window functions. Therefore it is important to be able to average results from a number of modes. As the number of low-degree modes is small, we must probably resort to developing an average by observing for many mode lifetimes. Additionally, efforts could be concentrated on long-lived, low-frequency p modes with intrinsically long lifetimes and therefore frequency fine structure with a scale different from rotational splitting. Unfortunately, these modes are weak and suffer from background noise. We may simply have to wait until very long observing runs can be accumulated and reduced before getting a good value of rotational splitting of low-degree p modes.

5.4 g modes

Low-degree g modes offer the best way of probing the rotation of the deep solar interior. However, there is less confidence in the available g-mode observations compared with p-mode observations because different observers do not agree on the identification and frequencies of various g modes. Nevertheless, there is remarkable agreement about the value of rotational splitting as listed in Table 4.

Table 4. g-mode splitting observations.

<table>
<thead>
<tr>
<th>epoch</th>
<th>method</th>
<th>l value</th>
<th>analysis</th>
<th>$\Delta f$</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1979</td>
<td>G</td>
<td>F</td>
<td></td>
<td>1.2</td>
<td>67</td>
</tr>
<tr>
<td>1980</td>
<td>A</td>
<td>C</td>
<td></td>
<td>1.05</td>
<td>43</td>
</tr>
<tr>
<td>1980</td>
<td>A</td>
<td>C</td>
<td></td>
<td>1.03</td>
<td>58</td>
</tr>
<tr>
<td>1981</td>
<td>R</td>
<td>C</td>
<td></td>
<td>1-1.3</td>
<td>69</td>
</tr>
<tr>
<td>1984-6</td>
<td>R</td>
<td>C</td>
<td></td>
<td>&gt;1</td>
<td>70</td>
</tr>
<tr>
<td>1987</td>
<td>G</td>
<td>P</td>
<td></td>
<td>1.6</td>
<td>71</td>
</tr>
</tbody>
</table>

$A$=ACRIM radiometer; $R$=resonance scattering spectrometer; $G$=grating spectrometer.
$F$=peak extraction; $C$=cross correlation with model.
$\mu$Hz.

All the values in Table 4 indicate a rapidly rotating core. Nonetheless, this reviewer remains cautious about accepting these results until better agreement between the frequencies found in independent observations is obtained. It is such agreement about the frequencies of p modes that gives us confidence to accept these results.

5.5 SCLEERA results

Many oscillation multiplets have been identified in spectra of limb-darkening fluctuations measured obtained by the SCLEERA group. These include g, f and p modes. Rotational frequency splitting of about 1.8 $\mu$Hz is reported for p modes with periods around 5 minutes and l values of 1 to 6 (Ref. 72). These results have high internal consistency but differ by a factor of 4 from measurements of the same modes made using Doppler and intensity methods. At frequencies less than 1 $\mu$Hz, radial-order 1 and 2 p modes of degrees 5-22 have been identified and yield frequency splittings between 1.6 and 0.6 $\mu$Hz with a strong l dependence (Ref. 73). These measurements were extended and used to infer that the latitudinal differential rotation rapidly decreases beneath the surface and has the opposite sign as the surface between r = 0.7-0.9 R (Ref. 74). Again, high internal consistency is reported but no other group has been able to observe p modes in this low-frequency range so it has not been possible to independently verify these rotational frequency splittings. Measurements of intensity fluctuations in a 200 x 200 arc sec aperture centered on the disk at 48 wavelengths between 0.5 and 1.7 $\mu$m have been made at SCLEERA and confirm the low-degree p-mode measurements (Ref. 75).

Low-frequency f modes of degrees 19-36 have been detected in SCLEERA observations (Ref. 76). The analysis has been extended (Ref. 77) and yields rotational splitting frequencies between 467 and 435 $\mu$Hz with a clear l dependence. Significant third- and fifth-order splitting dependencies with m were also measured and second and fourth order dependencies were found to be consistent.
with zero. Again, no independent confirmation of these results has been reported since no other group has been able to observe these modes.

At frequencies around 100 µHz, the SCLERA group has also detected g modes ($l=1-5$) in their observations (Ref. 78). A recent analysis (Ref. 79) indicates rotational frequency splitting of about 2.8 to 2.9 µHz with $l$ and $n$ dependencies. A comparison with independent estimates of g-mode rotational splittings (Table 4) indicates that the SCLERA values are nearly three times as large.

The SCLERA group has combined their frequency splitting results to obtain an estimate of internal solar rotation (e.g. Refs. 80,81). According to the SCLERA findings, rotation rate gradually increases inward to about triple the surface rate near the base of the convection zone and then rises rapidly to about 8 times the surface rate followed by a slow decline all the way to the center. This estimate of internal solar rotation differs greatly from other results as shown in [Figure 11].

In a number of papers (e.g. Refs. 81,82) the SCLERA group has addressed the question of the discrepancies between their results and those obtained by other groups. It has been suggested that multiplet splitting observed using Doppler and spectral line intensity fluctuations reflects the surface rotation rate of active regions rather than an average over the depth sampled by the mode. However, it is not clear why multiplet splitting should not be the same regardless of the detection technique once the multiplet is detected. It is as if the magnitude of a measured atomic Zeeman effect were to depend on the type of spectrometer used to measure the line splitting. In any event, observations using Doppler and intensity fluctuations have now been made under a wide range of solar activity with rotation results that are consistently different from those of the SCLERA group. In my opinion, the resolution of the problem is more likely to lie in the realm of reduction, analysis and interpretation than with the sun or the observations.

6. FUTURE PROSPECTS

It is traditional to close a review with a look to the future. It is a very promising future indeed. The vigor of this field is so great that it is a sure bet we will know much more about the internal rotation of the sun than we do now in just a matter of a few years. If what has gone before has been exciting and interesting, what is to come will be even more so. My enthusiasm is based on expected improvements in three areas as follows.

6.1 Better data

Major improvements in the quality of helioseismological data are about to happen. On the ground we have a number of networks of no- or low-angular resolution instruments already started or about to be started. These will greatly improve the signal-to-noise ratio of oscillation spectra by suppressing spurious spectral sidebands. These networks include the Birmingham stations, IRIS and SLOT. A full-disk imaging network is under development (GONG). A second limb-darkening/diameter instrument will be installed by SCLERA in China in 1989.

The relatively neglected area of high-degree rotational splitting will receive new attention as new instruments are brought into regular service by the Institute for Astronomy in Hawaii and by NSO on Kitt Peak. A new, high-resolution instrument for South Pole operation should produce interesting results this November.

Low-degree observations from the ground remains a difficult observational challenge in the face of terrestrial atmospheric noise and noise from activity on the sun. This is an area where fresh instrumental ideas are needed. Perhaps some highly differential scheme will be able to disentangle the oscillation signals we seek from all the noise.

Of course, the best way of removing the terrestrial noise is to go above it. A decade from now we will have superb data from the MDI, GOLF and VIRGO instruments on board SOHO and we will look back to the present time as quaint prehistory. Other space missions will provide helioseismology data by carrying various irradiance monitors (Ref. 83).

6.2 Better reduction

As data quality improves, it will actually be easier to extract good results from the observations. Some of the strategies that are used today will not be required with good data. In particular, it will be a pleasure to not have to worry so much about temporal sidelobes produced by observing window modulation. On the other hand, we will always be working on the low signal-to-noise fringes of the oscillation spectrum no matter how good the results are in the high signal-to-noise regions of the spectrum. Therefore there will be continued need for advanced reduction techniques to be applied to helioseismology observations. One of the pleasant problems coming soon is a large quantity of data. Automated techniques will be required to deal with large quantities. In this regard, some interesting proposals have been made for such automation (Refs. 84,85).

6.3 Better analysis

Much of the advanced theory needed to deduce the internal solar rotation from observations is already in place. It is simply awaiting high quality data. In this regard, preliminary analysis of the structure-sensitive even coefficients of Eq. 3 suggests fascinating possibilities (Refs. 86-89). A fruitful dialog with the geophysical and oceanographic communities has already developed. This should lead to a mutually beneficial sharing of knowledge and techniques and a rapid development of optimal techniques for analyzing solar data. The field of helioseismology has been characterized by an unusually open interchange between its participants, especially between observers and theoreticians. This openness is a great strength but it must be guarded carefully and continually nourished.

Acknowledgements.

I thank my colleagues in Tucson, Tom Duvall, Frank Hill, Stuart Jefferies and John Leibacher for innumerable discussions on helioseismology. Thanks to Henry Hill, Ken Libbrecht, Ed Rhodes and Roger Ulrich for providing preprints of recent work. My apologies are extended to authors whose work I have overlooked, misunderstood or misrepresented.
8. REFERENCES

ON THE MEASUREMENT OF SOLAR ROTATION USING HIGH-DEGREE P-MODE OSCILLATIONS

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ABSTRACT

We describe the progress made and some of the difficulties encountered in measuring the solar rotation rate with p-modes of degree $100 \leq \ell \leq 400$, using a set of high-resolution solar images taken at Big Bear Solar Observatory. The main conclusion drawn from an analysis of one day of data is that the equatorial angular velocity is essentially equal to the observed surface rate over the radius range $0.87 \leq r/R \leq 0.99$ to within a few percent. Because of likely systematic errors at the 1% level these data do not allow us to distinguish between a surface rotation rate equal to that measured using magnetic tracers and that based on the Doppler shift of photospheric spectral lines.

Keywords: Solar Oscillations, Rotation, P-Modes, Internal Dynamics

1. INTRODUCTION

Measurements of p-mode oscillations of degree $10 \leq \ell \leq 100$ generally show that the latitude variation of the angular velocity seen at the Sun's surface persists throughout much of the convection zone, but changes substantially near its base. In particular, the equatorial angular velocity undergoes a relatively rapid decrease with depth near the convective/radiative boundary. The random error in the splitting measurements is a few tenths of a percent per $\ell$ value at $\ell \approx 60$ in the best measurements.

The rotation rate in the upper convection zone, probed by p-modes of degree $\ell \geq 100$, is considerably less certain and, in particular, observations of high-$\ell$ modes (Refs. 1-3) have not yet shed adequate light on the well-known discrepancy between the surface rate measured by magnetic tracers and the rate inferred from the Doppler shifts of photospheric spectrum lines. This difference, if real, has been cited as evidence that sunspots are somehow 'tied' to deeper, faster-moving layers.

Motivated by this surface rate 'paradox' and the general need for improved rotation measurements in the upper convection zone, we undertook a p-mode rotational frequency shift analysis using full-disk, high-resolution solar images.

2. OBSERVATIONS

Full-disk filtergrams of the Sun over an $\sim 8$ Å passband centered on the Ca II K line were obtained at Big Bear Solar Observatory from 25-27 August 1987 and from 24 June - 1 July 1988, using a Datacopy Model 610 Electronic Digitizing Camera (a scanning CCD camera). Byte-deep images were produced in a 1408 scan line by 1408 pixel format at the rate of one every 90 s. The time required to scan 1408 lines in this mode is 46.5 s. A solar image of diameter approximately 1320 pixels was formed using a 5-inch objective in combination with a small diverging lens. The optics and camera were mounted to the side of the 8-inch Singer telescope, which was itself locked onto the main 26-inch telescope for guiding.

3. ANALYSIS AND RESULTS

Substantial analysis has thus far been performed only on the CaK images from 25 August 1987, during which 346 useful recordings were made. Preliminary analysis follows closely the procedure described by Libbrecht and Zirin (Ref. 4). The raw images were first transformed into a set of 'perfect' images, each having a standard position, size, orientation, and...
center brightness. A multipole decomposition of each perfect image, followed by a temporal Fourier analysis of the resulting complex moments yields the power of brightness fluctuations as a function of degree $\ell$, order $m$, and frequency $\nu$. A great deal of spurious spectral power was present in the first spectrum we computed, including a 'high-$\ell$' component due to the spatial analogue of Nyquist aliasing. Spatial detrending of each image prior to spherical-harmonic and Fourier analysis removes essentially all the unwanted power. A sample $\ell - \nu$ spectrum is shown in Figure 1.

In the first analysis we computed spectra for even order over the range $2 \leq |m| \leq 100$, and for degree $2 \leq \ell \leq 1536$. The signal-to-noise ratio in the $\ell - \nu$ spectrum formed by averaging the initial spectrum over $m$ turned out to be substantially less than we had hoped for in such high spatial resolution a data set. The main barriers to obtaining high-$\ell$ sensitivity in these data appear to be a combination of image motion and inadequate image focus. Evidence of the former inadequacy can be found in the centering signal, while evidence of the latter can be seen in the images themselves, in the form of limb blurring. Both of these problems apparently have their origin in the long time required to scan the solar disk.

Following this, we recomputed power spectra for the range $2 \leq \ell, |m| \leq 400$ (even $m$) of good signal quality. Our first use for this spectrum has been to obtain improved rotational splitting measurements up to $\ell = 400$. In this task, the cross correlation procedure of Brown (Ref. 5), as refined by Duvall et al. (Ref. 6), was employed to obtain the rotational frequency shift $\Delta \nu_{\ell,m}$ for each $m$ and $\ell$. To preempt spurious frequency shifts, we varied the usual procedure by averaging spectra for three consecutive (even) $m$ values before cross-correlating with the appropriate mean spectrum. The coefficients $a_i$ obtained from the fitting

$$\Delta \nu_{\ell,m} = \ell \sum_{i=0}^{5} a_i P_i(m/\ell)$$

to Legendre polynomials, $P_i$, were plotted as functions of $\ell$ and examined.

Two problems complicate the interpretation of these curves. The first is that, because the images are obtained by scanning, the shape of a moving feature - e.g. a travelling wave - is distorted, as noted by previous workers (Ref. 1).
In our data, the correction to the rotation rate due to this 'wavenumber modulation' is approximately 10% at $\ell = 100$ and roughly $\frac{3}{5}$% at $\ell = 400$ and is not negligible in relation to the random error of the observations. (The scan direction was inclined $\sim 25^\circ$ with respect to the solar equator for these observations.) We believe we can calculate this error to $\sim 10\%$. The second, non-instrumental, difficulty occurs because the blending of spatial sidelobes (the inevitable by-product of observing only one solar hemisphere) in our power spectra prevents us from isolating spectral peaks of individual modes. Blending is a consequence of the shortness of the data set and possibly of short mode lifetimes. Thus the frequencies we can measure are not mode frequencies as such. What we are measuring can perhaps be best understood by regarding the power spectrum as an incoherent sum of contributions from waves on different portions of the visible disk. The frequency shift, owing to rotation, of a wave of azimuthal order $m$ and total wavenumber $k = \ell/R$ ($R$ = solar radius), propagating at colatitude $\theta$, is just

$$\Delta \omega(\theta) = m\Omega(\theta)$$

$\langle \Omega(\theta) \rangle$ is a depth average of the angular velocity at colatitude $\theta$.) The measured frequency shift, $\Delta \omega$, will therefore be a latitude average of $\Delta \omega(\theta)$. In spatially decomposing the images one would ideally compute a given multipole moment by integrating the product of a spherical harmonic function and the measured brightness (corrected for limb darkening) over heliographic solid angle. The $\Delta \omega$ implied by this procedure turns out to be just the mode frequency shift, for which a useful, approximate expression was given by Brown (Ref. 5). In practice one always (implicitly or explicitly) decomposes solar images in such a way that most of the contribution to the oscillation signal comes from near the equator. Therefore, to be applicable to the case of incoherent or unresolved modes, Brown's expression must be generalized:

$$\Delta \omega = \frac{\int_{-1}^{+1} \Omega(z) [P_m^l(z)]^2 f(z) \, dz}{\int_{-1}^{+1} [P_m^l(z)]^2 f(z) \, dz}, \quad (1)$$

where the weighting function $f$, which peaks at the equator, is the only modification ($x \equiv \cos \theta$, $P_m^l$ is an associated Legendre function).

The effect of unequal weighting is to suppress the signature of latitudinal differential rotation in the data, as can be seen clearly in the plot of the $a_3$ coefficient (Fig. 2).

\begin{figure}[h]
\centering
\includegraphics[width=\linewidth]{figure2}
\caption{The measured Legendre coefficient $a_3$ as a function of spherical harmonic degree. The horizontal line is the $a_3$ value that one would expect to obtain with high-frequency resolution observations of coherent modes.}
\end{figure}

Since the $P_m^l$ function in eq. (1) is appreciable only near the equator, for $m \approx \ell$, the combination $a_1 + a_3 + a_5$ is still expected to be a reliable measure of the angular velocity in the equatorial plane, as we readily confirm. In principle, the contribution to the oscillation signal from different latitudes could be made equal by appropriately apodizing the solar images, but in practice this degrades the signal-to-noise ratio of the spectra to an unacceptable level. We are currently exploring analysis techniques to circumvent this problem.

An alternative rotational splitting analysis was performed to remove the effect of scanning in a more satisfactory way. The middle 512 latitude lines of each perfect image were interpolated onto a grid of 32 sine-latitude points by 1024 equally-spaced longitude points, after removing gross trends in the image. The longitude range covers only the visible half of the Sun. The multipole and Fourier decompositions were carried out on the collapsed images in a manner similar to that of Ref. 5, except that the temporal transforms were computed before the spatial decompositions so that the time delay introduced by scanning could be trivially eliminated by shifting the phase of the complex Fourier coefficients. In this later analysis we only computed spectra for even $m$ in the range $2 \leq |m| \leq 512$ and all $\ell$ such that $|m| \leq \ell \leq |m| + 19$. 
Figure 3. The solar equatorial rotation rate inferred from p-mode observations. The low-\(\ell\) values (small circles) are from Libbrecht (ref. 9), intermediate-\(\ell\) values (diamonds) are unpublished results based on the data used in Ref. 9, and the high-\(\ell\) values (large, dotted circles) are the result of the present analysis. The high-\(\ell\) values were arbitrarily shifted upward by 10-\(\mu\)Hz before plotting, to make the new results roughly match the old. Although we expect this correction is the right one, this is not certain. Because of this we strongly caution against any over-interpretation of these data.

In this way we examined only modes propagating near the solar equator. A frequency shift \(\Delta \nu_{\ell, m}\) was obtained by cross correlating the appropriate mean spectra of the previously mentioned analysis with the spectra from the present one. For each \(m\) we fit the quantity

\[
\frac{\Delta \nu_{\ell, m} - \Delta \nu_{\ell, -m}}{2m}
\]

to a linear function of \(\ell\) and took the fit value at \(m = \ell\) to the data as the equatorial rotation rate. The results of the fitting appear in Figure 3 together with results at low-\(\ell\) plotted for comparison. A constant, ad hoc correction of 10 nHz has been added to the new rotation curve which forces it to join smoothly to the curve at lower \(\ell\).

4. DISCUSSION AND CONCLUSIONS

We conclude from Figure 3 that over the range of depths probed by p-modes of degree \(100 \leq \ell \leq 400\) there are no large-scale variations in the equatorial angular velocity of amplitude greater than a few percent. In fact, the new data appear to be consistent with a constant rotation rate differing from the surface value by at most 2%. Figure 3 would suggest that the rotation rate of the photosphere is closer to the value based on sunspot motions (Ref. 7) than to the value derived from Doppler measurements (Ref. 8). However, we feel this conclusion is not very firm in view of the ad hoc nature of the correction used to obtain the final curve.

The most likely source of the \(\sim 10 \mu\)Hz discontinuity (removed prior to plotting) between the old and new rotation curves is an error in the method of determining the position of the solar disk in the \(\text{Ca K}\) images. An equatorial component of drift between the actual and measured disk positions of only 2 arc-sec per day would produce a systematic error of about
1% in the rotation rate. The radius of the solar disk in our images is observed to vary by roughly 1 arcsec over the observation day – indirect evidence of low-level image distortion. Lacking a real understanding of these distortions, we are reluctant to believe that a simple constant correction to the rotation rate is applicable.

Finally, we note that our rotational splittings are consistent with those of previous investigators (Refs. 1-3), and have significantly less error. The sensitivity of the p-modes seen in the present analysis formally extends to within 1% of the surface, into the layers where significant (and also contradictory) deviations from the surface rotation rate have been inferred (Refs. 1 & 2). Although we have performed no formal inversion of our data, we are skeptical that the claimed variations are consistent with our data.

5. REFERENCES
RADIAL AND LATITUDINAL GRADIENTS IN THE SOLAR INTERNAL ANGULAR VELOCITY

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ABSTRACT

We recently presented the results of an analysis of the frequency splittings of intermediate-degree (3 < degree < 170) p-mode oscillations which were obtained from a 16-day subset of our 1984 Mt. Wilson 60-foot tower observations (Ref. 1). These results showed evidence for both radial and latitudinal gradients in the solar internal angular velocity. In particular, our results indicated that, from 0.6 R\(\odot\) to 0.95 R\(\odot\), the solar internal angular velocity increases systematically from 440 to 463 nHz, corresponding to a positive radial gradient of \(\pm 66\) nHz/R\(\odot\) for that portion of the solar interior. Our previous analysis also indicated that the latitudinal differential rotation gradient which is seen at the solar surface persists throughout the convection zone, although there was some indication that the differential rotation might disappear entirely below the base of the convection zone. Here we extend our previous analysis to include comparisons with additional observational studies and we also present comparisons between our earlier results and the results of additional inversions of several of the observational datasets. All of these comparisons reinforce our previous conclusions regarding the existence of radial and latitudinal gradients in the internal angular velocity.

1. INTRODUCTION

The first attempt at determining the internal rotation of the sun from the frequency splittings of acoustic oscillations employed high-degree sectoral harmonic modes which probed only the outer few percent of the equatorial zone of the sun (Ref. 2). Since that study several other studies (Refs. 3-6) have also employed high-degree p-modes but have yielded somewhat inconsistent results, due largely to the relatively limited duration of the datasets which have been available thus far. The discrepancies in these high-\(\ell\) studies have been summarized recently by F. Hill (Ref. 7) and by J. Harvey (Ref. 8).

Similar discrepancies have been seen for the lowest degree (1\(\leq\) \(\ell\) \(\leq\) 3) p-modes. The first study to use these modes (Ref. 9) employed whole-disk velocity measurements and found evidence that the rotation of the solar core is substantially higher than that of the photosphere. One year later these results were questioned on the grounds that more peaks were seen in the observed power spectra than simple estimates of the

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which provided estimates of the solar internal angular velocity along the solar equatorial plane, at middle heliographic latitudes, and along the solar polar axis (Ref. 2). In this latter study we employed a simple forward analysis of these three linear combinations to obtain estimates of the radial profiles of the angular velocity in these three latitude zones. We also included a comparison of our radial profiles with those obtained from a formal inversion of our Mt. Wilson 16-day frequency splittings.

In this paper we will extend our previous analysis to include comparisons with additional observational studies and we will also present comparisons between our earlier results and the results obtained by ourselves and by two other groups from the direct inversions of some of the frequency splitting datasets. We will show how all of the studies which have employed full-disk solar images and the inversions give evidence for a positive radial gradient in the internal angular velocity along the equatorial plane which extends over most of the convection zone. We will also show evidence that the latitudinal gradient in the angular velocity observed at the surface persists over most of the convection zone.

In addition, we will discuss the evidence that the latitudinal differential rotation disappears beneath the base of the convection zone. We will indicate which of the observational studies give the most support to this latter idea, as well as indicating which of these studies show possibly contradictory evidence. Finally, we will describe which additional observations will be most crucial in enabling us to determine the true rotational behavior below the convection zone.

2. THE 16-DAY MOUNT WILSON RESULTS AND THE COMPARISONS WITH OTHER STUDIES

The details of our 1984 Mount Wilson 60-foot tower observations are contained in Ref. 1. The observations described there were obtained on 14 out of 16 consecutive days of July and August of 1984. Here we will also compare these results with additional frequency splittings obtained independently from a total of 35 days of 1984 60-foot tower observations by Tomczyk (Ref. 33).

Before comparing either set of Mt. Wilson frequency splittings with those available from other published and submitted full-disk imaging studies, we first analyzed our splittings in ways which allowed us to study the radial dependence of the internal angular velocity along the equator, at middle latitudes, and at polar latitudes. We did this by employing the frequency splittings observed for prograde and retrograde sectoral harmonics for degrees between 90 and 170, while for \( \ell > 90 \), we formed three different linear combinations of the Legendre polynomial expansion coefficients which we had previously obtained from complete \( \ell \) power spectra.

The three linear combinations which we created for \( \ell > 90 \) were: 1) \( a_\ell \), 2) \( a_\ell \), and 3) \( a_\ell \). The first of these combinations was created to provide angular velocity estimates along the equatorial plane which would be directly comparable with the sectoral harmonic frequency splittings for \( \ell > 90 \). The second linear combination is simply equal to \( \frac{1}{2} \) angular degrees evaluated at \( m = 0 \). Hence, this linear combination includes information from these sectoral harmonics which are near the zone (\( m = 0 \)) harmonics. These sectoral harmonics effectively sample the angular velocities between the equator and a limiting heliographic latitude which is dependent upon the degree, \( \ell \), of the modes. Hence, the frequency splittings computed from this second linear combination correspond to averages obtained for ranges of latitudes extending between the equatorial plane and a slightly different limiting high latitude for each degree. The third linear combination given above is the so-called "polar" combination, which would literally yield the angular velocities at a latitude of 90 degrees if the angular velocities were constant with depth.

As soon as we had computed these three linear combinations of the odd \( a_\ell \) coefficients, we binned both our results and those from the other studies into 10-degree wide bins in \( \ell \). This binning analysis was performed in order to allow us to search for large scale \( \ell \) dependencies in the various linear combinations. In essence this binning procedure was a smoothing operation which we employed to minimize the effects of these variations which were only one or two degrees wide.

Figure 1. Binned values of the equatorial frequency splittings versus the degree corresponding to the center of each bin. The triangles and the open squares were obtained from studies of sectoral harmonics alone. All other curves were obtained from Legendre polynomial expansion fits to the frequency shifts as functions of azimuthal order, \( m \). The legend relates the symbols to the various locations where the observations were obtained; it also gives the epoch of each observation; and it lists the authors of each study.
The comparison of the binned values of $a_1 + a_2 + a_3$ for the various studies is shown as a function of degree in Figure 1. Here the binned values of the different studies are identified with different symbols. The legend in the figure identifies the symbols which are used for the different studies. The two most recent studies which were included here are those of Jefferys et al. (Ref. 27) and Libbrecht (Ref. 29). In the case of Libbrecht's most recent results they were averaged about 3 mHz. With the sole exception of the 1982 South Pole results (Ref. 18), all of the other studies show a clear decrease in the equatorial frequency splitting as a function decreasing degree below $l$ 90.

At higher degrees the filled squares of Tomczyk (Ref. 33) extend up to $l$ 120 and are consistent with a lack of variation between $l$ 90 and $l$ 120. On the other hand, the open squares are from our earlier 16-day Mt. Wilson study (Ref. 1) and were obtained from the cross correlation of portions of prograde and retrograde sectoral harmonic power spectra. These points appear to suggest a decrease in the equatorial angular velocity as a function of increasing degree between $l$ 90 and $l$ 120. It is this trend which we initially tried to verify with our new 1984 higher-degree observations which are described in our companion paper in these proceedings (Ref. 31). Unfortunately, any confirmation of this trend will have to wait for the reduction of additional days of higher degree observations. One additional point to be mentioned with regard to Figure 1 is that it does not show any evidence for consistent temporal change in the internal equatorial angular velocity between the beginning of 1982 and the end of 1987, or over roughly one half of one 11 year solar cycle. The systematic differences in the curves of the different observers which were derived from nearly simultaneous observations - are as large as are any of the differences where non-simultaneous observations are shown.

The corresponding comparison of the binned values of the so-called "polar" linear comparison is shown as a function of degree in Figure 2. Here again the results of the different studies have been given different symbols and the legend identifies the various symbols with the authors of the various studies. We note first that the 1983 Mt. Wilson observations of Duvall and Harvey (Ref. 11) are not included here as they were in Figure 1, since those results were obtained from sectoral harmonic $i = m$ power spectra and not from Legendre polynomial expansions of the frequency splitting of different torsional $i = 1$ 01 harmonics as were all of the other points shown here. Consequently, the second linear combination could not even be formed from the Duvall and Harvey results.

Secondly, we note that the values of this mid-latitude linear combination fall systematically below the corresponding equatorial values given in Figure 1 for all of the studies shown. Thirdly, we note that all of the studies whose results are shown in Figure 2 agree to within 5 to 7 mHz of each other except for the 1984 Mt. Wilson studies (Refs. 1 and 33) which are systematically high above $l$ 50 by up to 22 mHz. A fourth point to be made concerning Figure 2 is that, below $l$ 50, the Brown and Morrow (Ref. 22), the Libbrecht (Ref. 29), and the Mt. Wilson (Refs. 1 and 33) studies all show evidence for an increase in the mid-latitude frequency splittings with decreasing degree, with the exception of the Mt. Wilson points for $l$ 5 9 which are comparable to their values for $40^\circ \leq l \leq 100^\circ$.

The corresponding comparison of the binned values of the so-called "polar" linear comparison is shown as a function of degree in Figure 3. Again, the Duvall and Harvey (Ref. 11) study is not represented here because of the nature of their original observations. All of the "polar" points fall systematically below those for the mid latitudes which are shown in Figure 2. Next, as with the mid-latitude expansion, the polar values agree well with each other above $l$ 50 with the exception of our Mt. Wilson results, which are again higher than those of the other studies. Thirdly, as was the case in Figure 2, below $l$ 50 the polar values from Mt. Wilson (Refs. 1 and 33), from Big Bear (Libbrecht, Ref. 29) and from Sacramento Peak (Brown and Morrow, Ref. 22) all show evidence for a decrease with increasing degree except for the Mt. Wilson results for $3^\circ \leq l \leq 9^\circ$ which show a decrease back to values approximating those at $l$ 50. Lastly, Figure 3 shows no evidence for any credible temporal variations in the polar frequency splittings.

![Figure 2](image1.png)

Figure 2. Binned values of the so-called "zonal" or mid-latitude linear combination in $\zeta$ for combination is shown as a function of degree in Figure 2. Here again the results of the different studies have been given different symbols and the legend identifies the various symbols with the authors of the various studies. We note first that the 1983 Mt. Wilson observations of Duvall and Harvey (Ref. 11) are not included here as they were in Figure 1, since those results were obtained from sectoral harmonic $i = m$ power spectra and not from Legendre polynomial expansions of the frequency splitting of different torsional $i = 1$ 01 harmonics as were all of the other points shown here. Consequently, the second linear combination could not even be formed from the Duvall and Harvey results.

![Figure 3](image2.png)

Figure 3. Same as Figure 2 for the polar (1 = 90°) linear combination. Note different vertical scales for Figures 2 and 3. The same symbols are employed in both Figures 2 and 3.

### 3. CREATION OF COMPOSITE COMBINATIONS

Because the comparisons we made between the various studies which were shown in Figure 1, 2, and 3 did not show any evidence for systematic temporal variations in any of the three linear combinations, we next attempted to minimize the effects of the systematic differences between the different analysis techniques employed by the different authors. We did this by combining the binned values from all of the studies shown in Figures 1-3 into three composite linear combinations. (For those $l$-values where there were values from Mt. Wilson from
three linear combinations are described in the text. We performed this combination in an unweighted fashion because, with the exception of the two South Polar runs (Refs. 18 and 27) and the 1986 Big Bear campaign (Ref. 29), all of the other values came from studies which ranged in duration from 14 to 35 days. Hence, we believed that, with the exception of these three cases, any differences in the sizes of the internal error bars from one study to the next were most likely due to systematic differences in the analyses rather than true differences in the inherent accuracy of the different sets of points. Given the agreement between the 16-day long 1984 Mt. Wilson and the 100-day long 1986 Big Bear results, we believe that in all but a few cases the use of a weighting algorithm in the computation of the composite mean values and standard deviations would have altered the results by no more than one to three mHz at the most.

The composite, binned linear combinations which resulted from the unweighted combination of the various studies shown in Figures 1 through 3 in the fashion described above are given in Figure 4. In this figure the composite equatorial, mid-latitude, and polar linear combinations are shown as functions of degree, \( \ell \). Also plotted as the error bars in Figure 4 are the standard errors of the three composite mean values located within each \( \ell \)-bin.

The three composite linear combinations of the odd \( \alpha \) coefficients which are shown in Figure 4 illustrate the \( \ell \)-dependence of the frequency splittings in each of three latitude zones. In order to infer radial profile of the sun’s internal angular velocity within those latitude zones, we employed next the same simple forward analysis which we had employed previously in Ref. 1. That is, we assumed that the observed frequency splitting for each \( p \)-mode was indicative of the angular velocity at the mid-point of that mode (i.e. at a point located exactly halfway between the photosphere and the mode’s inner turning point). We next calculated the radial gradients of \( \alpha \) for those \( p \)-modes which were closest in frequency to 3 mHz and which had degrees located within a given bin. In order to calculate the modal mid-points, we employed the table of modal inner turning point radii which had previously been supplied to us by D. Gough (Ref. 35). These turning point radii were computed from Gough’s standard solar model.

Once we had computed the mid-points of the modes which corresponded to each \( \ell \)-bin, we simply averaged those mid-points to obtain an average mid-point for each bin. The result of this averaging process was a set of average modal mid-points for all of the \( \ell \)-bins which were shown in Figure 4. Employing our assumption that these modal mid-points represented the depths at which the solar angular velocity was being sampled by those modes, we re-plotted the three composite linear combinations of Figure 4 as functions of the modal mid-points instead. The assumed radial profiles of the internal angular velocity which resulted from this process are shown in Figure 5.

Figure 4. The three binned composite linear combinations plotted as functions of the degree corresponding to the center of each bin. The three composite curves were obtained from unweighted averages of the data points shown in Figures 1, 2, and 3, respectively. The error bars shown are the standard errors of the mean values. The \( \ell \)-dependent variations in the three linear combinations are described in the text.

In Figure 4 we note again the systematic decrease in the composite equatorial frequency splittings with decreasing degree below \( \ell = 95 \). We also note that the latitudinal differential rotation is roughly independent of degree between \( \ell = 110 \) and \( \ell = 40 \). Below \( \ell = 40 \), however, all three curves begin to converge. The only points which do not suggest a continued diminishing of the latitudinal differential rotation with decreasing degree are the mid-latitude and polar points for \( \ell < 10 \). As we saw in Figures 2 and 3 these points came solely from our Mt. Wilson studies (Refs. 1 and 33). We will have to await the publication of additional Legendre expansion coefficients for degrees less than 10 before we will be able to say whether the differential rotation truly does continue to converge as the degree decreases below ten.

4. ESTIMATION OF RADIAL VARIATIONS OF THE INTERNAL ANGULAR VELOCITY

The three composite linear combinations of the odd \( \alpha \) coefficients which are shown in Figure 4 illustrate the \( \ell \)-dependence of the frequency splittings in each of three latitude zones. In order to infer radial profile of the sun’s internal angular velocity within those latitude zones, we employed next the same simple forward analysis which we had employed previously in Ref. 1. That is, we assumed that the observed frequency splitting for each \( p \)-mode was indicative of the angular velocity at the mid-point of that mode (i.e. at a point located exactly halfway between the photosphere and the mode’s inner turning point). We next calculated the radial gradients of \( \alpha \) for those \( p \)-modes which were closest in frequency to 3 mHz and which had degrees located within a given bin. In order to calculate the modal mid-points, we employed the table of modal inner turning point radii which had previously been supplied to us by D. Gough (Ref. 35). These turning point radii were computed from Gough’s standard solar model.

Once we had computed the mid-points of the modes which corresponded to each \( \ell \)-bin, we simply averaged those mid-points to obtain an average mid-point for each bin. The result of this averaging process was a set of average modal mid-points for all of the \( \ell \)-bins which were shown in Figure 4. Employing our assumption that these modal mid-points represented the depths at which the solar angular velocity was being sampled by those modes, we re-plotted the three composite linear combinations of Figure 4 as functions of the modal mid-points instead. The assumed radial profiles of the internal angular velocity which resulted from this process are shown in Figure 5.

Inspection of Figure 5 shows that the systematic decrease in the equatorial angular velocity with decreasing degree below \( \ell = 95 \) which was shown in Figure 4 actually appears to correspond to a positive (i.e. \( \partial \omega /\partial \ell > 0 \)) radial gradient. Between radii of 0.6 and 0.95 \( R_\odot \), the composite equatorial angular velocity profile increases from 440 to 463 mHz. This corresponds to a radial gradient of roughly 66 mHz/\( R_\odot \) over this portion of the solar interior. The relatively small size of the standard errors around each of the composite values of \( \alpha_1 + \alpha_3 - \alpha_5 \) indicates that this radial gradient is indeed statistically significant. For comparison purposes the values of the corresponding equatorial frequency splittings which are obtained from the rotation of the photospheric gas (Ref. 36) and from the rotation of magnetic patterns in the photosphere (Ref. 37) are shown near the top of the right-hand axis of Figure 5.

Inward of a radius of 0.9\( R_\odot \), the radial profiles at middle and polar latitudes are somewhat less well-defined. At both the middle latitudes and at the pole the angular velocity appears to increase toward the equatorial value between radii of 0.9 and 0.7\( R_\odot \). In particular, by the base of the convection zone (at \( r \approx 0.7 R_\odot \)) the difference in angular velocity between the equator
and the pole has diminished from the value of 120 mHz or 36%, which is seen at \( r = 0.95R_0 \) to roughly two thirds of that value, or 80 mHz, which is roughly 18% of the corresponding equatorial velocity.

Inward of the base of the convection zone, however, neither the mid-latitude nor the polar composite profiles continue the trend toward increasing angular velocity with decreasing radius. Rather, at both latitudes the composite profiles show a decrease in the estimated angular velocity. As was mentioned earlier, these two data points both correspond to the \( \ell = 3 \) to 9 bin and were obtained solely from our own 1984 Mt. Wilson observations since no other published studies include \( \ell = 3 \) or \( \ell = 5 \) estimates for that bin.

Confirmation of the trend showing the convergence of the angular velocity at all three latitudes inward of the base of the convection zone will have to await future studies which should obtain more accurate values of the \( a_\ell \) coefficients for \( \ell < 10 \).

5. COMPARISON OF COMPOSITE RADIAL PROFILES WITH RESULTS OF INVERSIONS OF OBSERVED FREQUENCY SPLITTINGS

Because we were well aware that the simple mapping procedure which we had employed to estimate the radial dependence of the angular velocity at each of the latitude zones could be criticized as being an oversimplification of a complicated situation at best and of offering no real information of the radial behavior at worst, we also compared our composite radial profiles shown in Figure 5 with the results of several different inversions of observed \( p \)-mode frequency splittings. First, we inverted the splittings obtained from our 16-day 1984 Mt. Wilson camera observations (Ref. 1). The Mt. Wilson frequency splittings which went into this first inversion were un-averaged splittings corresponding to \( 3 \leq \ell \leq 89 \). (By un-averaged splittings we mean that they were computed with the iterative cross correlation techniques described in Reference 1. These cross correlation analyses employed all of the \( p \)-modes at each degree which had frequencies between 2200 and 3600 mHz.)

The results of an inversion computed with an iterative variation of the spectral expansion method (hereafter referred to as the ISE method) on a six-point radial grid are compared with our three composite radial profiles in Figure 6. The details of the method employed in this inversion are given elsewhere in these proceedings (Ref. 28). In Figure 6 we see that the positive radial gradient along the equator described above is also seen in the inversion as the curve indicated by the plusses. Secondly, we also see a dip in the mid- and polar latitude profiles (given by the triangles for a latitude of 45° and by the circles for 90° respectively), although the location of the dip is shifted inward relative to that shown in our composite profiles.

**Figure 5.** Same as Figure 4 except that here the composite curves are plotted as functions of the average mid-points of those \( p \)-modes which are located within each bin. The procedure which was employed in the calculation of these modal mid-points is described in more detail in the text. The top curve shows the positive radial gradient in the equatorial angular velocity which is also described in the text. The other two curves show the radial changes in the latitudinal differential rotation described there. Shown at the right side are the values of the three corresponding linear combinations which were obtained from studies of the rotation of the photospheric gas and magnetic features.

**Figure 6.** Comparison of radial profiles of angular velocity at equator, mid-latitudes, and the poles with similar profiles obtained from an inversion of the 16-day Mt. Wilson data which was computed with an interactive variation of the spectral expansion method. The equatorial inversion profile is shown as plusses; the mid-latitude profile as triangles, and the polar profile as the circles. A six-point grid of radial points was employed.
We should also point out that the points shown at \( r = 1.0 \) for
this inversion are not valid since they correspond to larger radii
than can be inverted on the basis of frequency splittings which
only extend up to \( f = 89 \). These points are merely shown for
comparison with the \( r = 1.0R_\odot \) points obtained from the other
inversions.

The second inversion we computed was also obtained with
the ISE method. However, in this case we employed the n-
averaged frequency splittings which were obtained by Tomczyk
from a total of up to 35 different days in June, July, and Au­
gust of 1984 at the 50-foot tower (Ref. 33). For \( 10 < f < 120 \)
Tomczyk’s splittings provided estimates of all three odd \( n \), co­
efficients and these coefficients and their associated standard
deviations were provided as input to the inversion procedure.
However, for \( 5 < f < 9 \) we only employed non-zero values for
the \( a_3 \) coefficients, as both \( a_5 \) and \( a_7 \) were set to zero. Fur­
thermore, for these five cases the standard deviations in the \( a_3 \)
and \( a_5 \) coefficients which were computed from the least-squares
Legendre polynomial fitting program were not used as input to
the inversion as they were for the higher degrees. Rather, esti­
mates of the uncertainties of these two coefficients were made
for \( 5 < f < 9 \) by examining the scatter in the computed values
of \( a_3 \) and \( a_5 \) over all five degrees. Since Tomczyk’s splittings
extended up to \( f = 120 \) they served to extend our analyses to
higher degrees.

The results of this ISE inversion as computed on an eight-
point radial grid are shown in Figure 7. The use of extra radial
points in this inversion was justified because of the inclusion of
the higher-degree splittings. Their inclusion did not affect the
equatorial profile (the plusses) markedly; however, the ex­
tra two radial points did result in oscillatory behavior in both
the mid- and high-latitude profiles, which are shown again as
triangles and circles, respectively. The sharpness of this oscil­
lation in these two profiles suggests that it is an artifact either
of the un-binned splittings, or of the choice of the radial points
in this particular inversion.

![Figure 7](image-url)

Figure 7. Similar to Figure 6 except that here the com­
posite profiles are compared with profiles computed from an
inversion of Tomczyk’s 1984 Mt. Wilson splittings (Ref. 33)
which was computed using the same method employed in Fig­
ure 6. An eight-point radial grid was used.

The third inversion against which we compared our com­
posite profiles is shown in Figure 8. This inversion employed
both the frequency splittings employed in Figure 7 and also
the n-averaged splittings obtained for \( 10 < f < 90 \) by Libbrecht
from a 100-day run at the Big Bear Solar Observatory in 1986
(Ref. 29). It was also an ISE inversion; however, it was com­
puted with an 11-point radial grid.

Figure 8 again shows the positive radial gradient along the
equator that we described in Reference 1. However, it shows
an increased amplitude of the oscillation at the middle and
polar latitudes which we noted in Figure 7.

![Figure 8](image-url)

Figure 8. Similar to Figures 6 and 7 except that here the
same inversion technique has been applied to a composite data
set consisting of both Tomczyk’s Mt. Wilson (Ref. 33) and
Libbrecht’s Big Bear (Ref. 29) splittings. Both data sets were
n-averaged for this inversion.

In order to compare our composite profiles with the res­
ults of additional inversions which did not include any of our Mt.
Wilson observations, we also made comparisons with two in­
dependent inversions of Libbrecht’s 100-day Big Bear splittings.
These two comparisons are shown here in Figures 9 and 10.
In Figure 9 we compare our three composite profiles with a
piecewise-constant constrained least squares inversion which
was obtained from splittings which were not averaged in \( n \) and
which extended down to \( f = 5 \). This inversion was computed
by W. Dziembowski, P. Goode, and K. Libbrecht (Ref. 25).

Figure 9 shows the positive radial gradient along the equa­
tor which we have referred to above. It also shows a “hump”
inward of \( r = 0.8R_\odot \), which is followed by a “dip” below \( r =
0.7R_\odot \). These two features are similar in amplitude and lo­
tation to those shown above in Figures 6, 7, and 8. There is a
possibility that such an oscillation in the equatorial angular
velocity is real even though our highly-smoothed composite
profile does not show such an oscillation along the equator.

For latitudes of both 45° and 90° the inversion profiles
shown in Figure 9 agree remarkably well with our composite
profiles except for the innermost bins and except for the polar
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Figure 9. Similar to Figure 6, 7, and 8 except that here a least-square constrained inversion was performed by Dziembowski, Goode, and Libbrecht (Ref. 25) on the unaveraged 1986 Big Bear splittings for 5° < \( f < 60° \).

Figure 10. Similar to Figure 9 except that the optimal averaging kernel inversion presented here was carried out by Christensen-Dalsgaard, Libbrecht, and Schou (Ref. 24) on the unaveraged 1986 Big Bear splittings for 5° < \( f < 60° \).

profile near the surface, where our composite profile is systematically above the inverted profile. It is important to note that the innermost and outermost points of the inversions shown in both Figures 9 and 10 should be viewed with caution since those two inversions were computed from a frequency splitting dataset which covered a more restricted range in \( f \) than did the datasets from which our composite curves were computed. In addition, Figure 9 does show the apparent convergence of the angular velocity below the convection zone which we also discussed above.

Figure 10 shows the results of an inversion of Libbrecht's frequencies which was computed by J. Christensen-Dalsgaard, K. Libbrecht, and J. Schou using the optimal averaging kernel method (Ref. 24). As with Figure 9 the three different profiles computed from this inversion also agree remarkably well with our composite profiles. The positive gradient which we described above along the equator is seen here again. At a latitude of 45° the inversion profile lies slightly above the composite profile, in agreement with the inversion shown in Figure 9. On the other hand, for the polar case the inversion profile lies slightly above our composite profile, in contrast to the inversion profile shown in Figure 9 which fell slightly below our composite profile. Hence, along the polar axis the last two inversions simply bracket our composite radial profile.

6. DISCUSSION

The comparisons between our radial profiles of the composite frequency splittings and the five different inversions shown here are surprisingly good. They serve to indicate several key features of the current status of internal rotation studies. First, the systematic discrepancies between the results of the various observational groups are small enough to allow a new, general picture of the sun's internal rotation to emerge. That is, these discrepancies are not so large that they prevent the development of such a picture as a function of radius and latitude. Second, the positive radial gradient in the equatorial angular velocity which extends over most of the convection zone and which we described previously in Reference 1 appears now to be well-substantiated. This is in contrast to C. Morrow who has argued that the angular velocity is constant along the equator throughout the convection zone (Ref. 30). Third, the negative latitudinal gradient (i.e., the latitudinal differential rotation) which is seen at the solar surface has been found to diminish gradient appears to decrease in magnitude inwardly over the innermost two-thirds of the convection zone and may even disappear altogether below its base. Fifth, there is a critical need for future observational studies of the lowest degree.
(15 < \ell < 60) frequency splittings. Such studies of these splittings will be necessary in order to confirm the current suggestion from the inferences that the latitudinal gradient does indeed disappear in the radiative interior. Sixth, until observations of frequency splittings are obtained which have a much smaller intrinsic observational uncertainty the simple forward approach that we have adopted herein should continue to provide useful information concerning the sun’s internal rotation.

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8. REFERENCES

1. INTRODUCTION

As is evident from the number of papers that are contained within this volume which contain estimates of the internal angular velocity of the sun, such studies are an important part of the study of helioseismology. However, with the exception of one other paper in this volume (Ref. 2), all of the other studies presented here deal with low- or intermediate-degree p-modes. By contrast, the first attempt at determining the internal solar rotation rate from the frequency splittings of acoustic oscillations employed high-degree p-modes which were only the outermost few percent of the equatorial zone of the solar interior (Ref. 3). Sectoral harmonic frequency shifts were observed in this study which implied that the rotation period increases with depth below the photosphere. However, as is reviewed elsewhere in this volume (Ref. 4), other subsequent high-degree p-mode frequency splitting analyses have yielded inconsistent results with the consequence that the variations of the sun's internal rotation are known less accurately near the solar surface than they are in the deeper regions sampled by sectoral harmonics which range in degree between 10 and 598. They were obtained from a cross-correlation analysis of the prograde and retrograde portions of a two-dimensional ($\ell - \nu$) power spectrum. This power spectrum was computed from an eight-hour sequence of full-disk Dopplergrams which were obtained on July 2, 1988, at the 60-foot tower telescope with a Na magneto-optical filter and a 1024x1024 pixel CCD camera. These frequency splittings have an inherently larger scatter than did the splittings obtained from our earlier 16-day power spectra. Consequently, the best we can say now is that these splittings are consistent with an internal solar rotational velocity which is independent of radius along the equatorial plane. The normalized frequency splittings averaged $449 \pm 3$ nHz, a value which is very close to the observed equatorial rotation rate of the photospheric gas of $451.7$ nHz (Ref. 1).

Keywords: frequency splittings, p-mode oscillations

2. THE OBSERVATIONS

Beginning in August of 1987 we altered our observational helioseismology program which is located at the 60-foot solar tower telescope of the Mount Wilson Observatory by replacing our previous moderate resolution (244x244 pixel) CDD camera system with a higher resolution (1024x1024 pixel) CCD camera. By combining this new CCD camera with a Na version of the Cacciani magneto-optical filter (Ref. 5), we have been able to obtain time series of full-disk filtergrams and Dopplergrams which will now allow us to extend our previous intermediate-degree p-mode analyses (Refs. 6, 7, and 8) to higher degrees. During 1987 we obtained such observations on 39 different days in August and September. Between September 1987 and May 1988 we employed the observations which we had obtained on August 1987 in the development of our data reduction software. Preliminary power spectra which we computed from that dataset suggested that the basic system had performed as we had anticipated that it would.

For our summer 1988 observing campaign we implemented a few minor improvements to our optics (which were mainly designed to improve the modulation transfer function of our system at high spatial frequencies) and began to obtain observations on July 1. Since that date we have obtained observations extending up to 11 hours per day on almost every clear day. A partial record of the hours of observations which we obtained during July, August, and September, 1988, is shown here as Figure 1. In this figure we show the hours of available observations for each day between July 1 (day number 183) and September 10 (day number 254). Beginning on September 11 we have obtained similar observations on 28 of the following 32 days.
The July 2 observations consisted of 488 pairs of full-disk filtergrams which were obtained through the same two NoD lines and the second in the red wing. Only five seconds elapsed between acquisition of the two filtergrams of each pair and the two Dopplergrams which were obtained at HIP rate of one pair per minute for eight hours and eight minutes. Of each pair of filtergrams the acquisition of the two filtergrams of each pair was identified and a least-squares ellipse was fit to those limb points. An ellipse was employed since the raw images were very slightly elliptical (i.e. the semi-major axis was 446 pixels, while the semi-minor axis was 443 pixels).

In addition to determining the size and orientation of the solar disk, the limb-fitting program also computed the center coordinates of the solar disk in each image. These center coordinates were then employed to "register" the two filtergrams to a common center in order to remove the slight offsets in the locations of the solar disk which were introduced by atmospheric "seeing." All of the above computer programs were based upon portions of the Jet Propulsion Laboratory's VICAR suite of image processing routines. Those routines were initially converted from VAX Fortran to Alliant Fortran at the Jet Propulsion Laboratory before they were optimized at USC for use on the FX/8 there.

Once the filtergrams had been shifted via a two-dimensional interpolation algorithm to a common center in this fashion, the Dopplergrams were obtained from each pair of filtergrams by dividing the difference between the two registered images in each pair by the corresponding sum of the same two filtergrams. For the analysis described herein we did not attempt to convert these raw Dopplergrams into actual velocities in meters per second. Such a calibration procedure will be adopted and carried out later before we combine images from more than a single day into a longer time series.

The time series of 488 full-disk Dopplergrams which resulted from the above set of steps was then transformed into a time series of spherical harmonic amplitudes through the application of the same spherical harmonic decomposition (SHD) computer program which is currently being tested for use in the GONG program. This program was supplied to us by Mr. James Pintar of the GONG project office at the National Solar Observatory headquarters. We had to make only very minor modifications to the code in order to run it on the USC Alliant FX/8.

With this program we calculated spherical harmonic amplitude coefficients for each of the 488 Dopplergrams for every even- or odd-spherical harmonic for every degree. The time series of these even and odd spherical harmonic amplitudes were analyzed as above to see which spectrum would be the most useful for determining internal rotation of the sun.

4. OBSERVED POWER SPECTRA

The output of the spherical harmonic decomposition (SHD) program was an array of time series of the spherical harmonic amplitudes which corresponded to the entire sequence of input Dopplergrams. From the time series of these even- or odd-spherical harmonic amplitudes we selected only the amplitudes of the zonal and sectoral harmonic amplitudes which corresponded to the entire sequence of input Dopplergrams for every even- or odd-spherical harmonic for every degree. The 601 one-dimensional power spectrum which resulted is shown here as Figure 2.

The corresponding sectoral $\ell - \nu$ power spectrum which resulted from the above processing is shown here as Figure 3. The frequencies of the $p$-mode ridges in the two halves of this spectrum are not identical. Rather, they have been altered by the internal rotation of the sun, as will be illustrated below.
Figure 2: Two dimensional (f - v) power spectrum for zonal harmonics. The spherical harmonic degree, f, runs horizontally from zero (radial modes) at the left to 598 at the right. The temporal frequency, v runs vertically from 0 (1Hz) at the bottom to the Nyquist frequency (8.33 mHz) at the top. A few of the p-mode ridges are visible all of the way out to f = 598 in original prints of this figure and in real time video displays of the spectrum.

Figure 3: Two-dimensional (f - v) power spectrum for sectoral harmonics. Again the degree, f, runs horizontally, from zero at the left to 598 at the right. However, since harmonic coefficients corresponding only to even values of m were computed, the sectoral harmonics (f=m) were likewise computed only for even degrees between 0 and 600. Hence, there are 300 independent power spectra (one for f=0 and one for every other f-value up to f=598) shown. The frequency axis is vertical only in this case the Nyquist frequency is located halfway up the left-hand axis. The two sets of ridges correspond to the prograde- and retrograde-traveling p-modes.

A single one-dimensional (v) power spectrum selected from those displayed above in Figure 2 is illustrated here as Figure 4. In Figure 4 the power spectrum shown is that for the f=100 zonal harmonics. Here the observed power (in arbitrary units) is shown as a function of the frequency. For the zonal harmonics the portion of the power spectrum to the right of temporal Nyquist frequency (i.e. for 8.33 < v < 16.67 mHz) is simply a reflection about the Nyquist frequency of the power spectral estimates located between v=0 and v=8.33 mHz. In this case there is no additional information contained in the right-hand portion of the power spectrum. Clearly visible in this figure are the p-modes which are strongest in amplitude at frequencies ranging between 2.5 and 4.5 mHz for f=100.

Figure 4: One-dimensional (v) power spectrum for the f=100 zonal harmonics. Here the power (in arbitrary units) is plotted vertically as a function of the frequency, v, which increases to the right. The Nyquist frequency of 8.33 mHz is plotted at the middle of the horizontal axis. The portion of the power spectrum located to the right of the Nyquist frequency is simply a reflection of the left-hand half of the figure.

The corresponding one-dimensional power spectrum for the f=100 sectoral harmonics is shown here in Figure 5. This figure is a vertical slice from Figure 3 which is located about one-sixth of the way from the left-hand edge of that Figure. Here the power again increases vertically and is displayed as a function of the frequency, v. However, in this case the left- and right-hand portions of the power spectrum are not simply mirror images of each other as they were in Figure 4.

In the case of the sectoral harmonics the left- and right-hand halves of the spectrum represent the observed power of oppositely-directed sectoral harmonic modes. While the power levels in the two portions of Figure 5 are not identical, the frequencies of the right-hand portion of that figure (i.e. those for which 8.33 mHz < v < 16.67 mHz) are simply reflections...
about the Nyquist frequency of the frequencies in the left-hand portion. Hence, in order to estimate the total frequency splitting between corresponding prograde and retrograde sectoral p-modes, the right-hand portion of Figure 5 must be flipped over, left-to-right, and then superimposed upon the left-hand portion.

5. FREQUENCY SPLITTING MEASUREMENT TECHNIQUES

The total frequency splitting between the prograde and retrograde sectoral harmonics for a single p-mode of degree equal to 200 is illustrated here in Figure 6. In this figure a portion of the retrograde sectoral power spectrum for \( \ell = 200 \) (shown as the fine dots) has been flipped about the temporal Nyquist frequency as described above and then superimposed upon the corresponding prograde portion of the same power spectrum (shown as the dashed line). Also shown for reference is the corresponding peak from the \( \ell = 200 \) zonal power spectrum (shown as the solid line). The prograde p-mode is shifted to the right (i.e. to higher frequencies) relative to the zonal harmonic peak, while the retrograde peak has been shifted to the left (i.e. to smaller frequencies) of the unshifted zonal peak. The total frequency splitting for this single \( \ell = 200 \) p-mode is then simply the difference between the frequencies of the prograde and retrograde peaks, which can be seen to be about 4 frequency resolution units, or roughly 130 \( \mu Hz \).

![Figure 6: Superposition of portions of prograde, retrograde, and zonal power spectra for \( \ell = 200 \). Power increases vertically (in arbitrary units), while frequency (in units of the frequency bin width, \( \Delta f = 32.6 \mu Hz \)) increases to the right. The unshifted zonal spectral peak is shown as the solid line which peaks near 97 (i.e. \( \nu = 3.157 \mu Hz \)) on the ordinate. The prograde sectoral harmonic is shown as the dashed line which peaks near a frequency of 98 (i.e. \( \nu = 3.19 \mu Hz \)), while the corresponding (i.e. same radial overtone number) retrograde p-mode peak is shown as the finely dotted line which peaks around 94 bin widths (i.e. \( \nu = 3.060 \mu Hz \)). To first order, the total frequency splitting between these particular prograde and retrograde p-modes is roughly 130 \( \mu Hz \).

A corresponding example to Figure 6 for \( \ell = 300 \) is shown here as Figure 7. Again in Figure 7 the zonal p-mode peak is shown as the solid line; the prograde sectoral harmonic is shown as the dashed line which peaks at the right; and the retrograde sectoral harmonic is shown as the finely dotted line which peaks at the left. Here the total frequency splitting is approximately equal to 8 frequency bins, or 260 \( \mu Hz \).

![Figure 7: Same as Figure 6 except for \( \ell = 300 \). The prograde sectoral peak is shown at the right (dashed), the zonal peak is shown at the center (the solid line which peaks near \( \nu = 100.62 \mu Hz \)), and the retrograde sectoral peak at the left (as the finely dotted line). The total frequency splitting between this pair of prograde and retrograde \( \ell = 300 \) sectoral harmonics is roughly equal to eight frequency resolution elements, or 260 \( \mu Hz \). (For both Figures 6 and 7 the power levels of the three different spectra were adjusted by arbitrary factors prior to the creation of the plots in order to have comparable amplitudes for all three peaks in each of the plots).

The two cases shown in Figures 6 and 7 were but two examples of the more than 6000 peaks which correspond to sectoral p-modes having degrees less than 600. In order to quickly obtain estimates of the total frequency splittings for as many degrees as we could in the short amount of time available to us, we employed a cross-correlation technique. Specifically, we first flipped all of the retrograde portions of the sectoral power spectra about the Nyquist frequency so that their frequency range would correspond to that of the prograde modes (i.e. \( 0 \leq \nu \leq 3.33 \mu Hz \)). We then limited the portions of the prograde and retrograde spectra which we planned to compare to the frequency range \( 2200 \mu Hz \leq \nu \leq 3000 \mu Hz \) in order to concentrate on the strongest peaks at each degree. After flipping and truncating the power spectra in this fashion, we next computed the cross-correlation function between the two power spectra. We then found the relative maximum in that function which was located closest to an initial guess which was computed on the assumption of rigid rotation with the surface equatorial gas rate. We then fit a parabola to that peak in the cross-correlation function and estimated the frequency shift between the two spectra as the maximum of that parabolic fit. Once we had calculated the frequency splittings in the above fashion, we divided the total frequency splitting at each degree, \( \ell \), by \( 2 \ell \). This normalization factor converts the total frequency splittings into estimates of the solar angular velocity as sampled by p-modes of that degree and averaged over the range of frequencies given above. Finally, the angular velocity estimates which resulted from this renormalization of the observed frequency splittings were converted from the synodic frame to the sidereal frame by the addition of 31.7 \( \mu Hz \).

6. ANGULAR VELOCITY ESTIMATES

The individual estimates of the solar angular velocity which resulted from the above analysis are shown here in Figure 8. Because only even values of \( \ell \) were computed during the spherical harmonic decomposition, as described above, we ended up with
301 estimates of the angular velocity. Due to the finite widths of the p-mode peaks in an eight-hour observing run and due to the crowding together of the p-mode ridges at low degrees, our analysis does not work well at low degrees. Consequently, we did not include in Figure 8 any estimates for \( \ell < 10 \). In addition we calculated the overall mean and standard deviation of the remaining 294 estimates and eliminated seven additional widely discordant values. (This "cleaning" procedure eliminated three points between \( \ell = 10 \) and 18, two points between \( \ell = 20 \) and 28, and two points between \( \ell = 510 \) and 518.) The remaining 287 estimates are shown in Figure 8.

The overall average of the 287 remaining angular velocity estimates was 449 \( \pm 51 \) nHz. Converting the standard deviation into the standard error of the mean we obtain 449 \( \pm 3 \) nHz. Hence, the average of our entire population of internal velocity estimates is within one standard error of the observed rotation rate of the photospheric gas at the equator, which is 451.7 nHz (Ref. 1).

In order to search for possible degree-dependent trends in our angular velocity estimates, we also binned the estimates shown in Figure 8 in 10-degree-wide bins. The results of this binning analysis are shown in Figure 9. In this figure we have also included two lines which show the photospheric rotation rate of the gas at the equator (the long dashes, Ref. 1) and of magnetic features extrapolated to the equator (the shorter dashed line, Ref. 9).

The single high value at low degrees in Figure 9 corresponds to two points having \( 10 \leq \ell \leq 18 \) (since the remaining three were "cleaned out" as described in the text). This point does not correspond to the relatively large estimates of the angular velocity for \( \ell = 1, 2, \) and 3 obtained from a study of sectoral harmonics by Duvall and Harvey (Ref. 10). Those estimates were obtained from a 17-day run rather than from a single day as is shown here. No overall \( \ell \)-dependence is evident in Figure 9.

7. DISCUSSION

Our original intention when we began the analysis described above was to try to extend our previous intermediate-degree frequency splitting analyses (Refs. 6, 7, and 8) to higher degrees. Specifically, we were interested in determining whether or not the trend we saw for the sectoral harmonic p-mode splittings to decrease with increasing degree between \( \ell = 100 \) and 170 could be confirmed and extended above \( \ell = 170 \). Those earlier results were based upon an analysis of a time series of Dopplergrams covering 14 out of 16 consecutive days of CID camera observations and the results reported herein were derived from only a single eight-hour time series of CCD Dopplergrams.

The comparison between our earlier results and the current results is given in Figure 10. In this figure the values of the equatorial angular velocity which we presented in Figure 1 of reference 2 are repeated as the two sets of points at the 1 \( \ell \) ft. The plusses give the estimates which we derived from the sum of the odd Legendre polynomial expansion coefficients (i.e. \( a_1 \)
complete analysis before we will be able to comment further on this possibility.

In spite of our inability of demonstrating any degree-dependent trends in our frequency splittings, we believe that it is significant that our overall mean splitting between $l = 10$ and 598 of $499.3 \text{ mHz}$ agrees so closely with the equatorial photospheric gas rotation rate of $451.7 \pm 0.3 \text{ mHz}$. In our analysis this result was obtained from less than one complete day of observations while in the case of the photospheric gas rotation rate the corresponding value was obtained from several years of 150-foot tower observations. It will also be important to search for evidence of any possible temporal variations by analyzing more of our existing archives of CCD observations obtained from the 60-foot tower.

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RADIAL AND TEMPORAL STRUCTURE OF THE INTERNAL SOLAR ASPHERICITY

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ABSTRACT

There are large systematic differences between the measured even-order splitting coefficients obtained from different helioseismic observations (Refs. 1-5). These data were obtained by several groups, starting about 6 years ago, and used different observing techniques (e.g. Doppler vs. intensity full disk solar observations) to derive the m-dependence of mode frequencies. Given the wide variation between the observation techniques it may not be surprising that the results differ - yet, I argued (Ref. 6) that the variation between datasets may be physically significant and directly related to solar cycle variations in the aspheric structure of the sun. This asphericity is not confined to the photosphere and is related to the solar cycle luminosity variations measured by ACRIM (Ref. 7) and the solar limb brightness observations (Refs. 6,8).

Keywords: helioseismology, solar cycle variation, asphericity

1. ASPHERICITY AND FREQUENCY SPLITTING

In relating the splitting coefficients (obtained from the observed global eigenfrequencies) to the aspheric solar structure I assume that the local sound speed \( c^2(r, \theta) \) is independent of solar longitude and that all local effects (e.g. temperature, magnetic field, 2nd order doppler shifts, oblateness, etc.) are described by this local effective sound speed. The even splitting coefficients \( a_i \) are defined by the usual expansion for the frequency of a mode of radial order \( n \), and spherical harmonic degree and order \( l \) and \( m \) as

\[
\nu_{nlm} = l \sum_i a_i P_l(-m/l)
\]

It can be shown that a quadrupolar asphericity in \( c^2 \) causes a non-zero \( a_2 \) coefficient (and somewhat smaller higher order even coefficients). Similarly a \( P_4(\cos(\theta)) \) distortion leads to non-zero \( a_4 \) (and smaller higher order even splitting coefficients). Thus we write

\[
c^2(r, \theta) = c_1^2(r)(1 + \sum_i c_i f_i(r) P_i(\cos \theta))
\]

so that \( c_2 \) and \( c_4 \) describe the (fractional) lowest order sound speed asphericity. If there is no radial dependence to the asphericity \( f_i(r) = f \) then \( a_i \propto 1/l \) and we define \( b_i = l a_i \) and directly relate the \( c_i \) to the \( b_i \) by solving the wave equation to first order in the added terms in \( c^2 \). With \( \nu_{nl} \) the unperturbed mode frequency and for \( l \geq 5 \) we obtained

\[
b_2 = -0.3 c_2 \nu_{nl}\sqrt{\pi/5}, \quad b_4 = 0.3 c_4 \nu_{nl}\sqrt{\pi/9} \quad (1)
\]

At low-\( l \) the problem does not completely separate into \( 1 - 2 \) and \( 4 \) terms, i.e. an \( l = 2 \) distortion also produces non-zero \( b_4 \) splitting coefficients. This represents at most a 50% correction for the lowest \( l \) modes considered here and, given the significant observational uncertainties, can be ignored for the purposes of this discussion.

The solar oblateness causes a non-zero \( b_2 \). One obtains \( b_2 = -8 \text{ nHz} \) from Eq.1 by assuming that the measured surface distortion (Ref. 9) describes the depth independent sound speed asphericity. Another calculation based on a different radial form to the distortion yields \( 16 < b_2 < 20 \text{ nHz} \) between \( 10 < l < 60 \) (Ref. 10). The \( 1/l \) dependence to \( a_i \) is a natural consequence of a depth independent perturbation, and approximates the observed \( l \) dependence of the data. Thus Eq. (1) is, at least, qualitatively useful for relating the splitting to the asphericity and is useful for
comparing the time variation of various global solar observations.

2. SOLAR CYCLE VARIATIONS

Year to year variations in; 1) even splitting coefficients, 2) solar limb brightness observations, and 3) total solar irradiance observations are unmistakable. That these variations are interrelated and due to real changes in the sun over a solar cycle remains an interesting question. Ref. (5) showed that \( U^2 \) was a secular variation in the \( a_l \) and \( a_t \) splitting coefficients which appeared to be related to \( l = 2 \) and 4 limb temperature changes. I argued that the magnitude and yearly trends in the temperature observations were consistent with the measured splittings, although there were frequency offsets of about 100 nHz between temperature inferred \( b_2 \) and \( b_4 \) coefficients and the actual splittings. In fact the limb data are better described (Ref. 11) by an approximately constant temperature distribution near the polar regions and a “hot band” at lower latitudes that evolves over a solar cycle timescale. It is interesting that the latter temperature component does a good job of describing the splitting data. For example, if we ignore the limb temperature at latitudes within 20° of the pole and again use Eq. 1 to calculate the temperature induced splittings we obtain Fig. 1. I’ve used weighted fits to obtain the \( b_l \) from the published \( a_l \) and y-axis error flags are 1σ fit errors, but the actual uncertainties are larger; 1) because the published errors are non-normal, and 2) because a residual \( l \) dependence in the \( a_l \) is not completely described by constant \( b_l \). The x-axis error flags correspond to the duration of the observations. The reduced \( \chi^2 \) in the fits to \( a_l \) varied between 0.25 and 3.0, in part because of the actual \( l \) dependence of the data, and because the published observational error estimates were not uniform.

Figure 1 plots the splitting implied by the limb temperature measurements obtained by assuming no radial dependence to the relative asphericity, and the usual relationship between local sound speed and temperature. The agreement in magnitude and trend during the solar cycle suggests that we’re on the right track. Note that we may not be entitled to quantitative agreement between the limb observations and the splitting results for several reasons; 1) the radial dependence we’ve assumed is convenient but probably wrong, 2) non-thermal (e.g. oblateness) asphericity components also contribute to the splitting observations.

The adhoc assumption that the constant, high latitude, temperature field does not contribute to the splittings is not satisfying, and suggests; 1) that there must be other asphericity contributions to the splittings, or 2) that a more realistic radial dependence to the asphericity is needed to describe the offset in the splitting (in particular the \( b_2 \) coefficients) data from the temperature data. Can these differences be reconciled by finding a model for the radial structure of the temperature perturbation and other non-thermal contributions to the splittings? The answer would be simple if an inversion from the splitting coefficients was possible, because then a direct comparison with the surface temperature would be informative. Unfortunately the data is too noisy and we must ask for less information from the published splittings.

3. RADIAL STRUCTURE

Given the data, an interesting question is: Can a plausible (few parameters) radial temperature asphericity model be found which is consistent with both the surface temperature measurements and the splitting observations? The physical picture I have in mind for the cause of the temperature perturbation follows Parker (Ref. 12). The outward heat flux is blocked by a toroidal field somewhere deep in the convection zone. A latitudinal pattern of excess heat flux “shadow images” the field as it emerges from the radiative photosphere. The radial structure of the temperature perturbation from this excess flux can be expected to peak in the superadiabatic region below the photosphere. A small flux excess produces a larger fractional temperature excess in the superadiabatic region. In the radiative and adiabatically stratified layers the fractional temperature variation associated
with the flux excess should be constant. A direct integration of the mixing length equations in a model solar convection zone supports this.

The simplest radial model with 2 parameters which approximates this picture assumes a constant non-zero asphericity between some limiting depth and the surface. With this model and the splitting data we can solve for the best estimate of the temperature amplitude of the perturbation, and some goodness-of-fit measure (e.g. $\chi^2$) as functions of the depth of the perturbation. In practice the problem is more complicated because the observed error estimates are non-gaussian ($\chi^2$ has only relative importance) and we do not know how to correct the splitting observations for the non-thermal contributions (e.g. the $b_2$ data should be corrected for the oblateness by an amount probably somewhere between -30 and -8 nHz).

There are too few published splitting coefficients in Refs. 1,3 and 4 to obtain any useful radial information, so we consider the data from Refs. 2 and 5. Also, since the calculation to obtain $b_2$ is sensitive to an undetermined shape correction, I consider the $b_4$ calculation to be most informative. The $b_4$ coefficients generated by a radial asphericity distribution are computed using the formalism described in refs. 6 and 13. I used a radial grid of 64 points. A series of models of increasing depth of the otherwise constant fractional temperature perturbation were considered. At each depth the best least-squares temperature amplitude was evaluated along with $\chi^2$ from fits to the $b_4$ splitting coefficients. Figures 2 A and B show the results for the Mt. Wilson 1984.6 data and the Big Bear Solar Observatory data obtained near 1986.4. These are plots of $\chi^2$ with a few corresponding best fit values for the surface temperature labeled along the curves. The horizontal axis plots the inner radius within which the constant fractional $P_4$ temperature perturbation vanishes.

Several conclusions follow from Figure 2: 1) An unreasonably large temperature asphericity is needed to (rather poorly) fit the splitting data if the perturbation is confined to the outermost layers of the model, i.e. $\chi^2$ rises rapidly as the perturbation is increasingly confined to the surface. 2) The 1986.4 data is best described by a perturbation that extends down to approximately 0.96 $R_0$. There is a local minimum in $\chi^2$ near this radius from the 1984.6 data. 3) The surface temperature for the best model in 1986.4 is about 1K and is higher than this in the local minimum $\chi^2$ model which was based on 1984.6 data.

Figure 2 supports our interpretation of the splitting. We had no apriori reason to expect the best-fit temperature from the models to agree with the observed surface temperatures. Yet the minimum $\chi^2$ model yields temperatures consistent with the surface observations. Furthermore, the depth we obtain for the asphericity is physically reasonable – it is close to the bottom of the superadiabatic layer in the convection zone. The 1984.6 data imply a higher temperature at the same depth – as were the surface temperature measurements larger in 1984 than in 1987. The global $\chi^2$ minimum, apparent in Fig 2B for small amplitude but deep temperature perturbations, is probably not physical and is unrelated to this solution. A temperature model which matches the surface measurements and is consistent with the splitting observations is possible for each dataset plotted in Fig. 1, although more believable constraints on the temperature model will depend on more extensive splitting observations.

Fig. 2. Least-squares fit surface temperature and $\chi^2$ for outer shell asphericity models. Crosses and vertical offset numbers indicate best-fit surface temperatures. A. Based on $a_4$ data from Ref. 2. B. Same for Ref. 5.
4. INTERPRETING THE ASPHERICITY

To compute frequency splittings we have assumed that the limb brightness measurements are directly related to the temperature and local sound speed. These assumptions may be questioned, especially since we know that other effects (e.g. shape distortion and magnetic fields) also modulate $c^2$.

Are the limb brightness observations measuring a global surface temperature asphericity or can they be explained in terms of surface faculae, for example? Recently we showed (Ref. 11) that the solar cycle variation in the solar constant observed by the ACRIM experiment (Ref. 7) was accounted for by the time variation in the surface temperature. A corresponding facular contribution was not only inconsistent with our temperature analysis but also yielded a solar constant which was inconsistent with our model for the ACRIM variability – there is an approximately thermal global flux asphericity which is spatially unresolved in 20" observations.

Can surface magnetic fields cause the observed splittings? An analytic calculation of the effect of fibril magnetic fields (ref. 14) on high-$l$ modes suggests that fractional frequency splittings near the level of $10^{-3}$ are possible. Yet, unless we postulate a global coherence to the fibril fields it is unlikely that the suggested coherent wave scattering could cause the observed low-$l$ splitting.

Large scale magnetic fields, like a solar dynamo field near the base of the convection zone, may be responsible for the observed splittings – but probably not because of the direct kinematic effects of the field. Previous inversion calculations which aimed to measure large scale internal magnetic fields (Ref. 15) have not yielded physically plausible results. Furthermore a field of a few kilogauss near the bottom of the convection zone is probably not detectable with that formalism (Ref. 16) and current splitting data. On the other hand the small fractional temperature perturbation associated with the energy flux perturbation should be amplified in the superadiabatic layers. Thus, it is possible that the perturbation in the temperature structure is the lowest order effect of the field and may be the only measurable signature of magnetic fields on mode splittings. From this perspective it may not be surprising that magnetic inversions have been unsuccessful.

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5. REFERENCES

SOLAR ROTATION MODELS AND THE $a_1$, $a_2$ AND $a_3$ SPLITTING COEFFICIENTS FOR SOLAR ACOUSTIC OSCILLATIONS

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ABSTRACT

I present a forward analysis of the rotational frequency splittings of solar p-modes measured by Brown and Morrow to obtain information on the variation of the Sun's angular velocity with depth and co-latitude. This approach entails computing frequency splittings for chosen models of the solar rotation and comparing them to the observed splittings. It is called a forward method to contrast it with inversion techniques, which use the measured frequency splittings directly to produce a single function for the angular velocity. An advantage of the forward approach is that it allows one to develop intuition about the sensitivities of the splitting coefficients ($a_1$, $a_2$ and $a_3$) to various changes in the nature and magnitude of the angular velocity. The results of the preliminary analysis conducted in Morrow (these proceedings) give the impression that models with surface rotation throughout the convection zone are more promising than models with angular velocity constant on cylinders. The more formal approach of this paper is able to substantiate this impression and to consider the splittings from a series of more elaborate models for the solar rotation.

Keywords: Solar rotation, frequency splittings, forward modelling, acoustic oscillation

1. THE ROTATIONAL KERNEL

A rather complete derivation of the rotational kernels $K_r$, which relate stellar oscillation frequency splittings $\Delta \nu$, to angular velocity that varies with both depth and co-latitude $[\Omega(r, \theta)]$, is given by Hansen, Cox and VanHorn (Ref 1; hereafter HCV). The frequency splitting for an oscillation mode specified by wavenumbers $n, l, m$ may be expressed:

$$2\pi \Delta \nu_{nlm} = -m \int_0^1 \int_0^{2\pi} dr d\theta \, \rho_0(r) \, \Omega(r, \theta) \left[ K_{ADV}(r, \theta) + K_{COR}(r, \theta) \right],$$

(1)

where

$$K_{ADV} = \left\{ \frac{\rho_0(r)}{I_r(n, l) I_s(l, m)} \right\} \times \left\{ \xi_n \eta_m (P_m^o)^2 + \eta_n \left( \frac{\partial P_m^o}{\partial \theta} \right)^2 + \frac{m^2}{\sin^2 \theta} (P_m^o)^2 \right\},$$

(2)

$$K_{COR} = \left\{ \frac{-2\rho_0(r)}{I_r(n, l) I_s(l, m)} \right\} \times \left\{ \xi_n \eta_m (P_m^o)^2 + \eta_n \left( \frac{\partial P_m^o}{\partial \theta} \right)^2 + \frac{m^2}{\sin^2 \theta} (P_m^o)^2 \right\},$$

(3)

and the integrals $I_r(n, l)$ and $I_s(l, m)$ are:

$$I_r(n, l) = \int_0^{R_0} \xi_n \eta_m (P_m^o)^2 + \eta_n \left( \frac{\partial P_m^o}{\partial \theta} \right)^2 \rho_0(r) r^2 \, dr$$

(4)

$$I_s(l, m) = \int_{-1}^{1} (P_m^o)^2 \, d(\cos \theta).$$

(5)

In these equations $l^2 = l(l + 1)$, $\rho_0(r)$ is the density for a spherically symmetric, equilibrium solar model, and $\xi_n(r)$ and $\eta_n(r)$ are the radial and horizontal eigenfunctions of radius, which in combination with spherical harmonics define the Lagrangian displacement vector of linear, adiabatic acoustic oscillations of the Sun (Ref 2). The kernels here are the same as those presented in HCV (Ref 1), except that they are divided into parts representing advection and Coriolis effects. Frequency perturbations caused by the Coriolis effect oppose those due to advection, but for the Sun they are generally smaller by at least two orders of magnitude, even for the lowest frequency modes. Centrifugal effects are even less important and are neglected.

2. A SPECIFIC FORM FOR THE KERNELS

By assuming a particular form for the angular velocity and performing a suitable average of kernels for each value of degree $l$, one arrives at the specific form of the rotational kernels used here to calculate rotational frequency splittings. Such splittings can then be compared with those measured by observers.

2.1 The Form of the Angular Velocity

For simplicity, one may assume a general, separable form for the angular velocity function:

$$\Omega(r, \theta) = \sum_{l=1}^{2} \Omega_l(r) \cos^{2l} \theta$$

(6)
Any radial variation of angular velocity can be represented by this expression, but the co-latitudeal dependence is clearly constrained. Nevertheless, through appropriate choices of the $\Omega_s(r)$, many physically-motivated types of solar rotation are expressible: e.g. rigid rotation, surface-like rotation, constancy on cylinders, and rotation such that specific angular momentum is conserved.

The choice of Eq 6 to represent the angular velocity permits analytic evaluation of the co-latitudeal integrals in Eqs 1-3. Examining the terms in the rotational kernels, $K_{ADV}$ and $K_{COR}$ (Eqs 2-3) in light of the chosen form for $\Omega_s(r, \theta)$ reveals the need to be acquainted with a few integrals of associated Legendre polynomials and their derivatives. Morrow (Ref 3) details one method for their evaluation. All of them can be expressed in terms of the integrals:

$$\Theta_{lms} = \frac{1}{I_{l,m}(l, m)} \int_{-1}^{1} \cos^2 \theta \left( \frac{p}{m} \right)^2 d(\cos \theta),$$  

(7)

where the $I_{l, m}(l, m)$ are defined in Eq 5.

After the co-latitudeal integration is complete, the frequency splittings may be written:

$$2\pi \Delta \nu_{lms} = -m \sum_{s=0}^{2} \int_{0}^{R_{c}} r^2 d\Omega_{l}(r) \left[ K_{ADV}^{lms} + K_{COR}^{lms} \right],$$  

(8)

where

$$K_{ADV}^{lms} = \frac{\rho_{l}(r)}{I_{l}(n, l)} \times \left\{ \left[ \xi_{l}^{2} + \left( L^2 - s(2s + 1) \right) \eta_{l}^{2} \right] \Theta_{lms} + \left[ s(2s - 1) \eta_{l}^{2} \right] \Theta_{lms-1} \right\},$$  

(9)

and

$$K_{COR}^{lms} = \frac{\rho_{l}(r)}{I_{l}(n, l)} \times \left[ -2\xi_{l}^{2} \eta_{l}^{2} + (2s + 1) \eta_{l}^{2} \right] \Theta_{lms}.$$  

(10)

Equations 8-10 are akin to the form used by my computer code to compute frequency splittings for a given set of $\Omega_s(r)$. These kernels appear in Ref 4 in slightly different form.

2.2 Averaging the Kernels

In order to calculate splittings that can be justified compared to those presented by Brown and Morrow (Ref 5), one must account for their fitting procedure, which for each $l$ implicitly averaged the splittings for the collection of radial orders $n$, in a specified frequency range. To accommodate this averaging of the data, I perform a weighted average of the rotational kernels, $K_{ADV}$ and $K_{COR}$ (Eqs 2-3) in light of the chosen form for $\Omega_s(r, \theta)$ reveals the need to be acquainted with a few integrals of associated Legendre polynomials and their derivatives. Morrow (Ref 3) details one method for their evaluation. All of them can be expressed in terms of the integrals:

$$2\pi \Delta \nu_{lms} = -m \sum_{s=0}^{2} \int_{0}^{R_{c}} r^2 d\Omega_{l}(r) \left[ K_{ADV}^{lms} + K_{COR}^{lms} \right],$$  

(8)

where

$$K_{ADV}^{lms} = \frac{\rho_{l}(r)}{I_{l}(n, l)} \times \left\{ \left[ \xi_{l}^{2} + \left( L^2 - s(2s + 1) \right) \eta_{l}^{2} \right] \Theta_{lms} + \left[ s(2s - 1) \eta_{l}^{2} \right] \Theta_{lms-1} \right\},$$  

(9)

and

$$K_{COR}^{lms} = \frac{\rho_{l}(r)}{I_{l}(n, l)} \times \left[ -2\xi_{l}^{2} \eta_{l}^{2} + (2s + 1) \eta_{l}^{2} \right] \Theta_{lms}.$$  

(10)

Equations 8-10 are akin to the form used by my computer code to compute frequency splittings for a given set of $\Omega_s(r)$. These kernels appear in Ref 4 in slightly different form.

3. FORWARD VS. INVERSE APPROACHES

The problem of ascertaining the nature of the the angular velocity $\Omega_s(r, \theta)$, given the frequency splittings $\Delta \nu_{lms}$, may be attacked via a forward or inverse approach to the integral equation in Eq 11. The inverse approach proceeds to a plausible set of $\Omega_s(r)$ by mathematically inverting the integral expression. There are many techniques available for performing this type of inversion. Methods in current use in the solar context include: 1) Backus-Gilbert numerical inversion - a technique initially applied to geophysical problems (Ref 8) and first applied in the solar context by Gough (Ref 9); 2) analytic Abel inversion using an asymptotic [high $l$] form of the rotational kernel advocated by Gough (Ref 4,20); 3) inversion based on powerful optimization techniques that employ concepts from simulated annealing and neural networks (Ref 11). Christensen-Dalsgaard has performed a Backus-Gilbert inversion and Gough an Abel inversion on the frequency splittings measured by Brown and Morrow (Ref 4). Concurrently I have explored the data via the forward approach, which involves computing the integral in Eq 11 directly for a chosen form of the angular velocity and comparing the calculated frequency splittings to the observed ones.

If a model for the angular velocity maps well into the splitting coefficients, then it may be considered one of possibly many candidates for representing the physical situation as reflected by the data. Whether the splitting data are truly reflective of the solar rotation is yet another issue. Depending on other physical considerations one might invest a greater degree of belief in a particular model for the angular velocity, but this is done at the risk of encouraging what might be a wrong interpretation of the data. Whatever the properties of the kernel, the forward approach is always good for employing believable data to eliminate possible source functions. If a physically motivated trial function does not map to within the data's error bars then the data is sufficient to eliminate this function from consideration: no stable inverse technique would produce it as a solution.
In general, use of prior information is crucial to providing solutions to both the forward and inverse problems, i.e., one must have a rough idea of the nature of the Sun's angular velocity before proceeding to refine it using discrete, noisy frequency splittings. Because neither approach inherently produces unique answers, the success of both depends on constraining potential angular velocity functions as much as possible by physical theory, observational knowledge, and common sense.

4. THE FORWARD ANALYSIS

4.1 The Data of Brown and Morrow

The Brown and Morrow splitting data are reproduced in Figure 1. They have been transformed to be coefficients of Legendre polynomials in \( r/L \) instead of in \( r/R \). This transformation is very straightforward and results in coefficients that more genuinely reflect the radial variations in the angular velocity. Recall that \( a_1(L) \) reflects the radial variation of a \( \sin^2 \theta \)-weighted co-latitude average of the angular velocity, and that non-zero \( a_2(L) \) and \( a_3(L) \) indicate the presence of latitudinal variations of the angular velocity (Ref 7 these proceedings). As shown in the figure, neither a model with purely rigid rotation nor one with pervasive surface-like rotation suits the data. If the Sun were rotating rigidly, then \( a_3 = a_2 = 0 \) for all \( L \); this is obviously not the case. If the differential rotation observed at the surface were maintained throughout the interior then \( a_3 \) should be constant with \( L \) instead of dropping off at lower \( L \)'s. Evidently, the amount of latitudinal differential rotation decreases with depth.

Figure 1: The rotational frequency splitting coefficients measured by Brown and Morrow (1987). Note the drop in \( a_3 \) at low \( L \).

4.2 Physical Motivations for the Rotation Models

In addition to using first impressions of the data, I employ prior observational interpretation and theoretical ideas to motivate choices for the rotation models considered by my forward analysis of the frequency splittings. Some of these prior notions are listed below:

1: The depth of the solar convection zone is close to \( .7 R \), the value determined by Christensen-Dalsgaard et al. (Ref 12), who employed a direct inversion technique developed by Gough (Ref 10) on the splitting data of Duvall and Harvey (Ref 13) to obtain the sound speed as a function of depth.

2: For surface rotation expressed in terms of even powers of cosine co-latitude:

\[
\frac{\Omega_2 \cos^2 \theta}{2\pi} = A + B \cos^2 \theta + C \cos^4 \theta. \tag{13}
\]

the measured values of the coefficients \( A, B, C \) for the photospheric gas are (Ref 14): \( A = 452 \text{ nHz}, B = 48 \text{ nHz}, C = 81 \text{ nHz} \), and those for the rotation of magnetic technique developed by Gough (Ref 10) on the splitting data of Duvall and Harvey (Ref 13) to obtain the sound speed as a function of depth.

3: The numerical simulations of global convection and the associated transport of angular momentum owing to the influence of rotation (e.g. Ref 16) encourage the investigation of angular velocity constant on cylinders in the convection zone.

4: Theories of a turbulent solar dynamo require the existence of a radial gradient of angular velocity in or near a turbulent region of the interior where magnetic field can be induced and propagated in a manner consistent with the observed solar cycle (Ref 17).

5: Numerical simulations of supergranulation by Gilman and Foukal (Ref 18) suggests that specific angular momentum may be constant with radius in the outer 2-3% of the solar radius, causing the angular velocity to vary as \( 1/r^2 \), and possibly offering an explanation for the variable rotation of different types of solar magnetic features. Uncertain observations of very high-degree splittings also hint at the possibility of an increase in the angular velocity in layers very near the solar surface (Ref 19).

6: Theoretical arguments involving magnetic fields (Refs 20-21) and finite amplitude shear instabilities (Ref 22) suggest that rigid rotation is the likely current state of the radiative interior. Previously derived frequency splittings (Ref 23-24) also suggest a more latitudinally independent angular velocity beneath the convection zone.

7: The frequency splittings of very low degree oscillations provided by Duvall and Harvey (Ref 13) suggest that the innermost portion of the radiative interior may be rotating as rapidly as twice the equatorial surface rate.

8: On the basis of a procedure that correlates magnetograms using specific lag times, Stenflo (Ref 25) claims to have measured the rotation rate in a layer near the base of the convection zone which rotates differentially in rotation, but with coefficients \( A, B, C \) that give a reversal in the latitudinal gradient at high latitudes.

4.3 The Radial Part of the Angular Velocity

The expression for the angular velocity given in Eq 6 can accommodate models for the solar angular velocity that reflect any of the theoretical and observational considerations mentioned above. The various types of rotation are given by the appropriate choice of the \( \Omega(L) \). For rigid rotation at \( \Omega_1 = \Omega_2 = \Omega_3 = 0 \), in this case, \( a_1(L) / \Omega_1 \) and \( a_2 = a_3 = 0 \). For pervasive surface rotation (Eq 13) \( \Omega_1 / 2\pi = A, \Omega_2 / 2\pi = B, \Omega_3 / 2\pi = C \) and thus the splitting coefficients take on the constant values (see Eqs 4 and 7 in Ref 7 these proceedings):
\[ a_1 = A + \frac{1}{5}B + \frac{3}{35}C \]
\[ a_2 = -\left(\frac{1}{5}B + \frac{2}{15}C\right) \]
\[ a_3 = \frac{1}{21}C. \]

For rotation constant on cylinders,
\[ \Omega_\theta(r, \theta) = a + b r^2 \sin^2 \theta + c r^4 \sin^4 \theta, \]
(15)

the \( \Omega_\theta(r) \) are
\[ \Omega_\theta(r) = a + b r^2 + c r^4, \]
\[ \Omega_\theta(\theta) = -\left(b r^2 + 2c r^4\right) \]
\[ \Omega_\theta(\theta) = c r^4. \]

The basic features of the Brown and Morrow splitting coefficients (Fig 1) might be produced by dividing the model of angular velocity into at least two regions: one nearer the surface rotating differentially in co-latitude and matched with the surface rotation; the other at depth, rotating more rigidly at a modest rate between the maximum and minimum rates of the surface profile. The bend in \( a_4(L) \) between \( L = 40 \) and \( L = 50 \) (Fig 1) would then be explained by the drop in latitudinal differential rotation as one moves from a region with surface-like or cylindrical rotation into a rigidly rotating deeper interior. The relative flatness of \( a_1 \) would result because the weighted co-latitude average of angular velocity would not change much with depth, even though the degree of the latitudinal variations change substantially.

4.4 Surface-Like vs. Cylindrical in the Convection Zone

Figure 2 features the computed frequency splittings for a collection of models with a combination of surface-like and cylindrical rotation in the convection zone (logically, the surface-like layer overlies the cylindrical layer). The convection zone has a depth of \( 0.5R_\odot \) and the velocities at the base are piecewise continuously connected (i.e. linearly interpolated) to a rigidly rotating interior at and below \( 0.5R_\odot \). The surface coefficients used were estimated by inverting the system in Eqs 14 and considering high-L averages of the appropriate combinations of \( a_1(L), a_2(L) \), and \( a_3(L) \). This results in a surface profile whose angular velocities lie plausibly between the photospheric and magnetic rotation rates at all co-latitudes (although they are closer to the magnetic values). The bottom curve viewed in the \( \Omega_\theta(r) \) panel represents a model with all cylindrical rotation in the convection zone; the top curve represents all surface rotation. Error bars have not been included in Figure 2, but their extent can be gleaned visually from those shown in Figure 1. The bottom curve (cylindrical model) is well outside the error bars at all values of \( L \) corresponding to the solar convection zone.

The beauty of Figure 2 is that it illustrates very clearly the fundamental problem with constancy on cylinders in the convection zone and the fundamental attractiveness of surface-like rotation for these data. If the rotation is surface-like in a particular radial layer, then co-latitude rotation is constant with depth and no variation of \( a_1(L) \) in the corresponding \( L \) range is expected. If rotation is constant on cylinders and matched to the surface rotation, then a decrease of angular velocity with depth is engendered (see Figure 3), causing the computed \( a_1 \) curve to slope downward at the relevant \( L \) values and causing the overall magnitude of the curve to be much lower than in the surface-like case. The overall magnitude of the curve to be much lower than in the surface-like case. The overall magnitude could of course be increased by changing the surface coefficient \( A \) (see Figure 3), but the equatorial rate \( (A) \) would need to be 2-3\% higher than the magnetic value to bring the curve up near the data. Regardless, the real problem is that \( a_1 \) drops by 12 nHz over the \( L \) corresponding to convection zone depths, due to the 10\% drop in angular velocity engendered by constancy on cylinders. Such a slope is inconsistent with the data.

Clearly, \( a_1 \) curves calculated for models with surface-like rotation in the convection zone are more easily reconciled with the flat distribution of measured \( a_1 \) values. Indeed, a basic 2-layer model with rotation constant on cylinders in the convection zone is not consistent with this data. Although the shape of the \( a_3 \) and \( a_5 \) curves are much the same for either type of rotation, the overall magnitudes of the curves are slightly greater for the surface-like case. Nevertheless, it is primarily through examination of \( a_1 \) curves (not \( a_3 \) curves) that one may discriminate between surface-like and cylindrical rotation in the convection zone. Brown and Morrow (Ref 5) considered \( a_3 \) only and failed to recognize the power of \( a_1 \) to discriminate between the two very different types of differential rotation. The calculated \( a_3 \) coefficients they used were taken from Christensen-Dalsgaard (Ref 20).

4.5 Complex Models for the Solar Rotation

Complex models are defined as those which have 3 or more regions containing different types of rotation. I have examined the effects on the frequency splittings of three primary models of this sort. The calculated splittings for the first two of the models are pictured and discussed by Brown et al. (Ref 4).

4.5.1 Adding a CAM Layer. The first of these models is basically an effort to salvage constancy on cylinders in the convection zone. It has cylindrical rotation in the bulk of the convection zone and rigid rotation in the radiative in-
terior. Its complexity arises from the addition of a thin layer near the surface in which the fluid conserves specific angular momentum (CAM layer) and thus in which the angular velocity varies as $1/r^2$. This addition has a rather large effect, raising the $a_1, a_2$ and $a_3$ curves in magnitude without altering their basic shapes. A CAM layer $0.02 \Delta R$, thick appears to give the best results for the magnitudes of the curves, however, the downward slopes in $a_1$ due to the cylindrical profile continues to be difficult to reconcile with the data.

4.5.2 Connecting Differential to Rigid Rotation. The second complex model is essentially an effort to refine the models with surface rotation in the convection zone by piecewise continuously connecting the differentially rotating convection zone to the rigidly rotating radiative interior. This has the effect of smoothing the bend in the $a_2$ curves since latitudinal differential rotation drops off more gradually. The data are not capable of deciding on the depth of this transitional layer since there are several combinations of that depth and the depth of the convection zone that give reasonably fits to the $a_2$ data. I fixed the convection zone depth at $0.7 \Delta R$ and sought the bottom edge of the transitional layer that fit $a_2$ best in a least squares sense. This turned out to be $0.5 \Delta R$. Note that the depth over which the rotation changes from latitudinally differential to rigid need not be identical to the depth of the overshoot layer beneath the convection zone. Indeed the Brown and Morrow data suggest that the solar angular velocity is latitudinally variable at depths deeper than those typically considered for the convection zone and overshoot region combined. For a model without a linear transitional layer, the differentially rotating layer must extend to about $0.6 \Delta R$ to fit the bend in $a_2$. In general, the deeper the layer with the full surface rotation, the shallower the linear transitional layer needs to be to fit the $a_2$.

4.5.3 Adding a Stenflo Layer. The final complex model to be considered derives from recent work by Stenflo (Ref 25), who claims that a region near the base of the convection zone may be rotating differentially in co-latitude with coefficients: $A_{ST} = 466 \, \text{Hz}$, $B_{ST} = -108 \, \text{Hz}$, $C_{ST} = 76 \, \text{Hz}$. Note that unlike typical surface values, $C_{ST} > 0$, implying a reversal in the latitudinal gradient of angular velocity at a co-latitude of about $35^\circ$ (latitude $55^\circ$). This is a sort of polar spin up. Unfortunately the data are rather insensitive to changes in $C$ (Figure 8) and do not highly resolve the depth structure of the angular velocity. Thus, the data are not able to eliminate the possibility of a model with surface rotation in the bulk of the convection zone and a thin layer near the base rotating as described by Stenflo. From Eqs 14, one may see that a change in sign of $C$ should result in a tendency for $a_2$ to change sign, but unless the Stenflo layer is quite thick, this is difficult to see. The data are capable of eliminating the model he displays in his paper, which has angular velocity varying linearly across the convection zone from its surface values to the Stenflo profile. For this model the magnitude of $a_2$ is insufficiently large at higher $L$. This implies that the degree of latitudinal differential rotation in the convection zone for this model is too small to explain the data.

5. CONCLUDING REMARKS

The most likely models for the solar rotation appear to have surface-like differential rotation pervading the bulk of the convection zone with a gradual ramp to the deeper interior, which rotates rigidly at a rate intermediate between the surface equatorial and polar rates. Simple models with rotation constant on cylinders in the convection zone were ruled out by this data (which is not to say ruled out by the Sun, until we get more data). Some observers (Refs 27-28) have presented $a_1(L)$ coefficients with more of a slope, suggesting the possibility of a more substantial radial gradient. Models with a CAM layer atop cylinders are possible but result in frequency splittings that are significantly more $L$-dependent than the Brown and Morrow data. The Stenflo profile seems unlikely, but is in any event difficult to prove or disprove at this stage. More accurate and precise measurements of $a_2(L)$ would be helpful, but this is difficult since it depends on good observations at high latitudes.

For these reasons, I chose the model with surface rotation pervading the solar convection zone for parameter optimisation in a least squares sense. Starting with the values derived from preliminary analysis: $A = 408 \, \text{Hz}$, $B = -55 \, \text{Hz}$, $C = 71 \, \text{Hz}$, $G_{SR}/2\pi = 439 \, \text{Hz}$ with a linear ramp between $0.7$ and $0.5 \, \Delta R$. I ran a 4-dimensional optimisation on $A, B, C$ and $\Omega$. The optimised values, which are also published in Ref 4, are: $A = 457 \, \text{Hz}$, $B = -49 \, \text{Hz}$, $C = 80 \, \text{Hz}$, $G_{SR}/2\pi = 436 \, \text{Hz}$.

Figure 4 shows the favored picture of internal rotation that has emerged from this forward analysis. It should be viewed cautiously since not all observations agree and since it may not be the only model that gives a reasonable fit to the splitting coefficients. Note that it has no significant radial gradient of angular velocity in the convection zone. The

ANGULAR VELOCITY vs RADIUS FOR THE SRF MODEL

Figure 4. : The SRF Model: surface rotation throughout the convection zone and rigid rotation in the radiative interior.
only significant gradients are beneath the convection zone and the sign of the radial gradient there reverses at 30° latitude. The radiative interior is shown to be rotating rigidly down to a depth of about 0.3–0.4\( R_\odot \), but the data cannot really say anything about the rotation below this depth. It is still possible that the innermost part of the radiative interior is rotating more rapidly than the outer layers.

Taken at face value, this rotational profile has intriguing implications for angular momentum transport and dynamo action in the Sun. It suggests that coupling between the base of the convection zone and the radiative interior exists and is maintaining a balance between torques exerted at high and low latitudes (i.e., continuity of angular momentum at the boundary between the convection zone and radiative interior). The profile also provokes thought about a new qualitative picture of the way in which the solar dynamo may be operating in the transitional layer. These issues are discussed more fully by Gilman, Morrow and DeLuca (Ref 31 and these proceedings).

What follows is a picture library which illustrates the sensitivities of the splitting coefficients to various adjustments in the parameters that describe the Sun's internal angular velocity. A surface profile for the convection zone and a rigidly rotating interior are used throughout. Variations to the surface coefficients \( A, B, C \), the rigid interior rate, and the depth of the latitudinally differentially rotating layer are all considered. The captions should be consulted for further information.

The work presented here was done while I was at the University of Colorado and the High Altitude Observatory in Boulder, Colorado. There I benefitted from discussions with Tim Brown, Peter Gilman and Ellen Zweibel. The paper itself was prepared at the Institute of Astronomy in Cambridge, and I gratefully acknowledge Bill Merryfield and Neil Balmforth for their invaluable assistance with the manuscript.

Figure 5 : Varying the depth of the convection zone from 0.75\( R_\odot \) to 0.55\( R_\odot \) (rightmost to leftmost \( a_2 \) curves).

Figure 6 : Varying the surface coefficient \( A \) from 444 to 462 nHz : only response in \( a_1 \).

Figure 7 : Varying the surface coefficient \( B \) from -48 to -80 nHz : greatest response in \( a_5 \).
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IS THE SUN REALLY A RIGID ROTATOR?

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ABSTRACT

Attention is drawn to observations of surface rotational periods in subgiants which indicate that a deep-seated reservoir of angular momentum must exist in these stars. This interpretation is compatible with theoretical studies of the rotational history of the Sun and sun-like stars, but is in apparent conflict with observations of p-mode splittings in the Sun. In order to understand better the source of this apparent discrepancy, we explore the predicted rotational splittings of several test rotation curves and compare them to the solar data. Finally, we discuss the implications of our findings for solar structure.

Keywords: sun, rotation, seismology, subgiants

1. INTRODUCTION

This work is part of a continuing effort to understand the history of the Sun's rotation within the context of stellar evolution theory, which is one of the scientific motivations for helioseismology. Recent models of the evolution of the rotating Sun, constrained in the pre-main sequence phase by the initial rotation suggested by the Kraft curve, indicate that a fraction of the initial internal angular momentum is still present in the Sun. These solar models, which satisfy the usual constraints of standard solar models, are also constrained to match the current solar surface rotation rate and light element abundances, and to undergo a surface rotational history compatible with the spectroscopic observations of stellar rotation in young star clusters (Ref. 1 and 2).

In this paper, we shall focus on two such observational tests of theoretical predictions of rotation in deeper layers of the Sun: (1) the rotational splitting of solar p-modes, and (2) the surface rotational velocities of sun-like subgiants. There are other tests of internal rotation in evolved sun-like stars, such as the evidence for the mixing of CNO-processed elements to the surfaces of red giants, which also merit detailed study, and will be considered in a future study. The two tests we have selected here are independent of each other, and both place clear constraints on the amount of angular momentum that may still remain in the same part of the Sun, i.e. in the outer forty per cent (by mass) of its interior (which corresponds to the outer seventy per cent by radius).

A particularly intriguing aspect of this comparison is that preliminary results suggest a discrepancy between the two tests. On the one hand, observations of the splitting of solar p-modes are usually interpreted as evidence for nearly uniform rotation in the solar interior. On the other hand, the rotational periods of sun-like subgiants indicate that the interior rotates more rapidly than the surface.

In an effort to clarify this problem, we shall: (1) briefly describe the sensitivity of the rotation periods of subgiants to the distribution of solar internal rotation, and (2) discuss calculations which illustrate the dependence of the expected p-mode splitting on the solar rotation profile. Although much work remains to be done in both areas, these comparisons should help us to understand how best to utilize these tools to probe the solar interior. As most of us are familiar with the rotational splitting data and its current interpretation, let us begin with the possibly less familiar subject, the surface rotation of sun-like giants.

2. ROTATIONAL PERIODS OF SUN-LIKE SUBGIANTS

During the subgiant evolution, from main-sequence turnoff to the base of the giant branch, the radius of a star increases rapidly, thus increasing its moment of inertia. As a consequence, if the specific angular momentum remains constant in the outer layers, the observed period of surface rotation is expected to increase. However, at the same time, the surface convection zone also deepens rapidly, encompassing a growing fraction of the total mass (from a few percent to four tenths of the total mass of the Sun). Layers previously situated well below the convection zone are now able to share their angular momentum with the surface layers on a rapid time-scale. The progressive deepening of the convection zone in subgiants thus offers a unique opportunity for testing the distribution of internal angular momentum present in these stars. If the interior rotates rigidly, the surface rotation rate is expected to decrease rapidly as the radius increases. If the interior rotates more rapidly than the surface, the surface rotation rate will
not decrease as rapidly since the convection zone will "pick-up" angular momentum from the interior. The slope of the change in surface rotation period as the star evolves through this phase of evolution is thus a measure of the rotation profile in the star's interior. Fig. 1, taken from the paper of Pinsonneault et al. (Ref. 2), illustrates this result. The dashed line is the predicted evolution of the surface rotation period assuming rigid rotation in the current Sun, and the solid line is derived from a model with internal rotation greater than at the surface. Observational points are also shown on the same diagram, taken from the work of Noyes et al. (Ref. 3). These rotational periods are based on reduced chromospheric activity data. There is some debate about the derived periods (Ref. 4 and 5), and a measure of this uncertainty is also shown on the diagram. At any rate, the data clearly favor the conclusion that some angular momentum has been transferred up to the surface of these stars. More observations are needed not only in the field, but eventually in star clusters. Observing subgiants in the cluster M67, which has a nearly solar chemical composition and age, would be particularly instructive.

Figure 1. Surface rotation period of subgiants with deepening convection zones as their effective temperature decreases. The solid line is the rotation period of the "best" solar model in Ref. 2. The dashed line is the rotation period the Sun would have if it evolved toward the giant branch as a solid body. The data points are from Ref. 3. Uncertainties are indicated by the hatched region.

3. THE PREDICTED ROTATIONAL SPLITTING OF SOLAR p-MODES: SENSITIVITY TO THE ROTATION CURVE

In order to acquire some feeling for the sensitivity of this test, we have explored the dependence of the predicted splitting of low-\(l\) p-modes to the form of the radial rotation curve. We shall plot the splitting coefficient \(B\), where \(\Delta \omega = m \beta \Delta \Omega_{\text{surf}}\) (Ref. 6), as a function of frequency for each value of \(l\). Increasing frequency thus corresponds to increasing \(l\). In the first experiment, we tested four rotation curves with increasing and decreasing rotational velocity toward the center starting from 0.2 and 0.4 \(R_\odot\). They are shown in Fig. 2, and the corresponding splitting coefficients are plotted in Fig. 3b, c, d, and e. Fig. 3a illustrates the same splitting coefficients for solid body rotation for comparison purposes. In addition, we have plotted in Fig. 4 the same coefficients for the extreme cases of Fig. 3d and 3e, together with K. Libbrecht's splitting observations for the same modes kindly made available to us by the author. On the basis of this preliminary exploration, significant conclusions are already possible: (a) Little can be inferred from the available p-mode splitting observations about the form of the solar rotation curve within 0.2 \(R_\odot\). At 0.2 \(R_\odot\) (Fig. 3b and 3c), both curves are indistinguishable from solid body (Fig. 3a) at the 1% level. (b) At 0.4 \(R_\odot\) (Fig. 3d and 3e), only increased rotation is measurable at the limit of the observed error bars (see Fig. 4). Here high-\(n\), low-\(l\) measurements are needed at the 1% level to get information on the interior. (c) It is clear that the sensitivity of the splitting coefficients to a decrease in the rotation rate toward the center is very low. One has to be very courageous to suggest that the current data imply a dip in the interior rotation curve of the Sun. In fact, the low-\(l\) splittings shown in Fig. 4 suggest the opposite.

In the second experiment, we considered three rotation curves. The original rotation curve was taken from the evolutionary calculations of Ref. 2. In order to test the sensitivity of the splitting, a second rotation curve was tested with half the original rotation rate. The third rotation curve was the same as the second, but flattened inside 0.24 \(R_\odot\). These three rotation curves, shown in Fig. 5, all have in common higher internal rotation rates than in the first tests. In particular, all three exhibit substantial rotation between 0.4 and 0.6 \(R_\odot\). The corresponding splittings are shown Fig. 6b, c, and d. They can be compared with the solid body case in Fig. 6a. We conclude from these tests that: (a) If indeed the observed splittings are due to internal rotation, the available data do test in a sensitive way the rotation profile below the convection zone and down to 0.4 \(R_\odot\). (b) Current observations cannot be used to infer the state of rotation of the inner 0.2 \(R_\odot\). (c) Current theoretical rotation curves (Ref. 2) which explain the observed subgiant rotation periods do not agree with the solar splitting data.
Figure 3. The rotationally induced frequency splitting coefficients, $\beta$, for the rotation curves defined in Fig 2. The $\ell$ values plotted are 7, 8, 9, 10, 15, and 20.

Figure 4. The low $\ell$ frequency splittings from Libbrecht are drawn with quoted errors along with the frequency splittings calculated for the two extreme test rotation curves (scaled to the surface rotation rate of the sun). Note that the frequency splittings for the test rotation curves bent at 0.2 $R_\odot$ would appear as horizontal straight lines at $\alpha = 45^\circ$ on the scale of this figure.

Figure 5. Rotation curves derived from the evolutionary calculations of Ref. 2.
4. DISCUSSION

Despite the many uncertainties, it is interesting to consider some of the questions raised by taking at face value the most straightforward interpretation of the observations. As they are, the data are not strictly incompatible, since the oscillation data do not rule out the presence of sufficient angular momentum in the deep interior of the Sun to match the subgiant periods. Most of the required hidden angular momentum could be located well below 0.4 $R_\odot$ in the current Sun. In addition, it is possible for some transfer of angular momentum from deeper-in to take place between the present and subgiant phases of the Sun. In fact, the combined data could be interpreted as suggesting just this. Such a course of events does not seem implausible following hydrogen core exhaustion in the Sun. We emphasize, however, that if the Sun is indeed currently a nearly rigid rotator in its interior, as is often courageously suggested, the two sets of data are nearly certainly incompatible, unless, of course, the Sun differs in its rotational history from other disk stars, a rather unsatisfactory conclusion on philosophical grounds.

5. ACKNOWLEDGEMENTS

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6. REFERENCES


Figure 6. The frequency splitting coefficients for the rotation curves shown in Fig. 5. Splittings are shown for $l = 7$, 8, 9, 10, 15, and 20.
OSCILLATION RING DIAGRAMS AND THE THERMODYNAMICS
OF THE OUTER SOLAR CONVECTION ZONE

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ABSTRACT

A recently developed three-dimensional Fourier analysis results in the appearance of rings in the power spectrum of solar oscillations. These rings are the cross-sections at constant temporal frequency $\omega$ of trumpet surfaces, and are the analog of the familiar ridges. The shape of the rings provides information on the local dispersion relationship of the oscillations expressed as a simple power law. The exponent and constant in the power law are related to the thermodynamics of the region in the solar interior where the waves propagate. Asymptotic expressions for high-degree modes, coupled with the assumption that the upper part of the solar envelope is an adiabatic polytrope, predict that the exponent should be 1/2. The constant should depend on the polytropic index of the envelope, and on a phase factor resulting from wave leakage. Analysis of over 5000 rings results in an observed exponent ranging between 0.3 and 0.6, a polytropic index between 1 and 7, and a phase factor between -1.5 and 5.

Keywords: Solar oscillations, solar convection zone structure

1. INTRODUCTION

Helioseismology, the study of solar oscillations, has provided estimates of physical conditions, such as rotation, in the solar interior (Ref. 1). Much of this information is derived from studying the positions and shapes of structures in multidimensional power spectra of the oscillations. Recently, a new form of power spectrum has been developed, in which the signature of the oscillations is rings, rather than ridges (Ref. 2). These rings are the cross-sections at constant $\omega$ of trumpet surfaces that, conceptually, are the result of rotating the familiar $k-\omega$ diagram about the $\omega$ axis. Here, $k = 2\pi/\lambda$ is the spatial wavenumber of the oscillations, and $\omega = 2\pi f$ is the temporal frequency. The rings appear in power spectra computed from three-dimensional Fourier transforms of Doppler data transformed to an equally spaced heliographic longitude and latitude grid. An example of the rings is shown in Figure 1.

In Ref. 2, the rings were approximated by ellipses, and in this approximation it was shown that the position of the rings reflects the magnitude of a weighted average over depth of the two horizontal components of velocity in the solar convection zone. It was also shown that the shape of the rings is a result of the local slope and intercept of the ridges. Reference 2 focused on the positions of the rings, and further work showed that the positions of rings obtained from data at different heliographic longitudes showed significant variations of 20 m/s in both rotational and meridional velocities (Ref. 3). In this paper, the shapes of the rings are used to infer the exponent and the constant of a simple power-law approximation to the dispersion relation of the oscillations. The
The dispersion relation is assumed to be a simple straight line, derived in Ref. 2. In this approximation, the unperturbed ring is an ellipse described by the intercept of the dispersion relation. It is then shown in Ref. 2, where

\[ m = m_k + 1 = p c + 1 = p c k + 1 = p m + 1, \]

that the ellipse parameters \( x_0, y_0, a, \) and \( b \) are related to the solar parameters \( m, l, U_1, \) and \( U'_1 \) by simple proportionality:

\[ x_0 = C U_1, \quad y_0 = C U'_1, \quad a = C (d - l), \quad b = -C m \]

where the constant of proportionality \( C \) is given by

\[ C = \frac{l - d}{m^2 - U_1^2 - U'_1^2}. \]

Equations (6) and (7) are not independent, so the constant \( C \) is determined by comparing the ellipse parameters for two rings with nearly equal values of \( \omega_d \). In practice, it is necessary to work with second order polynomial fits to the ellipse parameters \( a \) and \( b \) when performing this comparison.

The relationship between \( (m, l) \) and \( (p, c) \) must be established in order to estimate \( p \) and \( c \) from the ellipse approximation. Starting from the unperturbed dispersion relationship, \( \omega = c k \), the local slope \( m \) is easily seen to be given by

\[ \frac{d \omega}{d k} = m = p c k^{p-1}. \]

Substituting this expression for \( m \) into the linear approximation for \( \omega \),

\[ \omega = m k + l = p c k^{p-1} k + l = p c k^p + l = p \omega + l, \]

thus

\[ l = \omega (1 - p). \]

This now allows the determination of \( p \) from \( l \) and \( \omega_d \):

\[ p = 1 - \frac{l}{\omega_d}. \]

and, once \( p \) has been determined, \( c \) can be calculated from

\[ c = -\frac{m}{p k^{p-1}}. \]

Note that this requires a value of \( k \). This is provided by computing \( k_\alpha \), the average of \( k \) around a ring:

\[ k_\alpha = \left[ \frac{1}{2 \pi} \int_0^\pi k^2 d\theta \right]^{1/2}. \]
For an ellipse,

$$k_a = \left[ \frac{x^2 + y^2 + a^2 + b^2}{2} \right]^{1/n}$$  \hspace{1cm} (14)

but, since the dispersion relation is for unperturbed waves, \( k_a \) must be computed relative to the displaced center of the ellipse, so

$$k_a = \left[ \frac{a^2 + b^2}{2} \right]^{1/n}$$  \hspace{1cm} (15)

Since the actual unperturbed dispersion relation is not linear, there will be an error in the determination of \( p \) and \( c \) using this method. To investigate this, a simulation was performed starting with the generation of artificial rings using Eq. (4), the parameters \( p = 0.5, U_x = -1850 \text{ m/s}, U_y = -100 \text{ m/s}, \) and the calculation of \( c \) from Eq. (3) with \( \mu = 3, \epsilon = \mu/2, \) and \( \gamma = (\mu + 1)\mu. \) The rings were generated for the fundamental (f) mode \((n = 0),\) and the first four \( p \) modes over the frequency ranges available in the data. The artificial rings were then fitted with ellipses, and the apparent values of \( p \) and \( c \) were computed using Eqs. (11), (12), and (15). The results of this simulation are shown in Figs. 2 and 3. It can be seen that \( p \) is underestimated, with the error decreasing with increasing \( n, \) and that the error increases with \( v. \) The constants \( c \) are also slightly underestimated, with little dependence on \( n \) and \( v. \) These results are useful in calibrating the estimates of \( p \) and \( c \) from the data.

**Figure 2.** The predicted exponents \( p \) that would be estimated from the ellipse approximation to the rings if the actual dispersion relation exponent was 0.5. The dashed line represents the input value of \( p, \) and the estimates are labeled by the corresponding ring identification.

Once \( c \) has been estimated from the data, it is of interest to estimate \( \mu, \) the polytropic index, and \( \epsilon, \) the phase correction due to wave leakage. This can be done by comparing the values of \( c \) obtained from rings with adjacent radial orders. To do this, first define

$$a_n = \frac{\mu c_n^2}{2}$$  \hspace{1cm} (16)

where \( c_n \) is labeled by the radial order and defined in Eq. (3). Also, assume that the adiabatic exponent \( \gamma \) and the polytropic index \( \mu \) are related in the usual way: \( \gamma = (\mu + 1)\mu. \) Then, \( a_n = 2(n + \epsilon)/\mu, \) and \( a_{n+1} = 2(n + 1 + \epsilon)/\mu, \) and expressions for \( \epsilon \) and \( \mu \) can be easily obtained:

$$\epsilon_n = \frac{(n + 1)a_n - ma_{n+1}}{a_{n+1} - a_n}$$

$$\mu_n = \frac{2}{a_{n+1} - a_n}$$  \hspace{1cm} (17)

where \( \epsilon \) and \( \mu \) are now labeled by \( n, \) the lower radial order of the two rings in the comparison.

The data used in this analysis are the same as discussed in Refs. 2 and 3, and comprise a set of Doppler images obtained in the Mg I b line at 5172.6 Å. The data were obtained in February 1981 with the Echelle Spectrograph at the Vacuum Tower Telescope at NSO/Sac Peak. The spatial resolution was 2", and each image covered 128 x 512 pixels centered on the disk. One image was recorded every 65 s, and the data span the daylight hours on three consecutive days. After interpolation onto an equally spaced heliographic longitude and latitude grid, each of the rectangular rasters was split into four square images. The ring diagrams were obtained for each of the four subrasters. This mosaic allows a search for variations in the solar parameters as a function of longitude.

**Figure 3.** The predicted constants \( c \) that would be estimated from the ellipse approximation to the rings if the actual constants were the plane-parallel polytropic value with \( \mu = 3. \) The dashed lines represent the input value of \( c \) for the closest ring identification.

### 3. RESULTS

The procedure described in Section 2 has been carried out for 1295 rings at each of four solar longitudes, for a total of 5180 rings. The \( f \) and \( p_1 \) through \( p_4 \) rings were used for the current analysis. Figure 4 shows the values of \( p \) as a function of \( v \) obtained from the data for the four solar positions at each of the five ring orders. It is clear that the curves in Fig. 4 have a rather different character than the predicted curves in Fig. 2, indicating that \( p \) is not constant. The observed values of \( p \) range from about 0.3 to nearly 0.6, and they appear to depend on \( n. \) There appears to be more scatter in the observed \( p \) for
Observed Dispersion Relation Exponents

Figure 4: The values of $\rho$ obtained from the observed rings as a function of $v$. Each curve represents one longitude and one radial order, $n$.

Observed Dispersion Relation Exponents

Figure 5: The values of $\rho$ obtained from the observed rings as a function of $l$. Each curve represents one longitude and one radial order, $n$.

Normalized Observed Dispersion Relation Exponents

Figure 6: The values of $\rho$ obtained from the observed rings as a function of $l$. The curves in Figure 5 have been averaged together and normalized by the curves in Figure 2. The standard deviation of the mean is indicated by the long dashed lines on either side of the solid average curve for each radial order. The horizontal short dashed line at $\rho = 0.5$ indicates the value predicted by the high-degree asymptotic theory.

Observed Dispersion Relation Constants

Figure 7: The values of $\epsilon$ obtained from the observed rings as a function of $v$. Each curve represents one longitude and one radial order, $n$.

Normalized Observed Dispersion Relation Constants

Figure 8: The values of $\epsilon$ obtained from the observed rings as a function of $v$. The curves in Figure 7 have been averaged together and normalized by the curves in Figure 3. The standard deviation of the mean is indicated by the long dashed lines on either side of the solid average curve for each radial order. The horizontal short dashed lines indicate the values predicted by the high-degree asymptotic theory including a plane-parallel polytropic envelope with $\mu = 3$.

Figure 5 shows the observed $\rho$ as a function of $l$, the degree of the rings as determined from $a_n$. The curves now lie roughly on top of one another. In Figure 6, the four longitude curves for each radial order have been averaged together and normalized by the curves in Figure 2, producing the best possible estimate of $\rho$ using this method and this data. The errors in the normalized observed values of $\rho$ have been estimated by computing the standard deviation of the mean over longitudes for the curves in Figure 5.

Figure 7 shows the values of $\epsilon$ estimated from the data as a function of $v$. Comparison with the dashed lines in Figure 3 shows that the observed values of $\epsilon$ for the $\rho_1$, $\rho_2$, and $\rho_3$
Figure 9: The values of the polytropic index $\mu$ inferred from the data in Fig. 8. The solid curves are computed using Eqs. (16) and (17). The long dashed curves represent the errors of the curves in Fig. 8 propagated through the calculation for $\mu$. The horizontal short dashed line indicates a value of $\mu = 3$.

Figure 10: The values of the phase constant $\epsilon$ inferred from the data in Figure 8. The solid curves are computed using Eqs. (16) and (17). The long dashed curves represent the errors of the curves in Figs. 9 and 10 propagated through the calculation of $\epsilon$. The horizontal short dashed line indicates the prediction of the plane-parallel theory.

4. DISCUSSION

The results in Fig. 4 suggest that the exponent $p$ is not 0.5 as predicted by the asymptotic theory, but instead ranges from 0.3 to 0.6. It has been pointed out (Ref. 4) that, for the $p$ modes, $p$ should have a value of 0.5 only for large values of $k$. This is due to the rapid increase in sound speed with depth for more deeply penetrating modes at lower $k$, and to the break down of the plane-parallel approximation for the lower $k$ modes. When the two-dimensional $k$–$\omega$ diagram is plotted with an ordinate of $\omega/\sqrt{gk}$ instead of $\omega$, as in Fig. 2 of Ref. 4, it can be seen that, for $k \leq 1$ Mm$^{-1}$, the $p$-mode ridges are not straight lines as they would be if $p = 0.5$. The figure in Ref. 4 also shows that $\epsilon$ rises as $k$ decreases for a given ridge. Since $\epsilon = c \sqrt{gk}$, this implies that $p \approx 0.5 < 0$, or $p < 0.5$. This is consistent with the results shown in Figure 6 in the present paper, which shows that $p < 0.5$ for $l < 700$, corresponding to $k = 1$ Mm$^{-1}$. Figure 6 also shows that $p > 0.5$ for $l > 800$, but this region is not displayed in Ref. 4. It is also apparent from Fig. 6 that at a fixed $l$, $p$ roughly decreases with increasing $n$. This is consistent with the fact that at fixed $l$, the depth of penetration is an increasing function of $v$, and hence the plane-parallel approximation is less valid. The $f$ mode should have a $p$ value of 0.5 independent of $k$, but this does not seem to be the case in this analysis. This may be due to a scale error in the data.

It is rather surprising that the curves roughly coincide when the observed exponent for different rings is plotted as a function of $l$ as in Figs. 5 and 6. The depth of the turning point is the same for modes with the same horizontal phase speed $\nu l$, and one might expect that the curvature effects would be similar for modes trapped in similar cavities. This is evidently not the entire story here, as the curves do not coincide when the observed exponent is plotted as a function of $\nu l$. This can be seen in Figure 12.
The phase constant, $\epsilon$, displayed in Fig. 10 has been estimated previously by plotting the observed frequencies in coordinates of $(n + \epsilon)\omega a$ versus $\omega k$ and adjusting $\epsilon$ until the ridges coincide (Ref. 6). By doing this, it has been shown that $\epsilon = 1.5$ for $l > 150$. This value is shown in Fig. 10 as the short dashed horizontal line. A later determination (Ref. 7) found that $\epsilon = 1.6$ for $14 < l < 200$ reduced the scatter, and that a weak dependence of $\epsilon$ with $v$ further reduced the scatter about a single curve. The results in Fig. 10 suggest that $\epsilon$ decreases with both increasing $v$ and $n$, again showing the break down of the plane-parallel approximation for modes that penetrate relatively deeply and thus feel the spherical shape of the Sun. This is further indicated by Fig. 11, which shows that the ratio $\epsilon/\mu$ decreases with increasing $v$.

It appears that the results for the $p_4$ ring are somewhat anomalous. This may reflect a change in $\mu$ (or $\gamma$) in the depths probed by the modes in this ring. The anomaly could also be due to systematic errors in the fitting procedure that increase as the rings converge at high $n$ for a given $v$. It is surprising that the curves for $\mu$ and $\epsilon$ derived by comparing the $f$ and $p_1$ rings are continuous with those resulting from a comparison of the $p_1$ and $p_2$ rings. The $f$ mode constant should be $\sqrt{\epsilon}$, and this is recovered from Eq. 2 only in the plane-parallel case when $\epsilon = \mu/2$. This is apparently the case just for those $f$, $p_1$, and $p_2$ modes with identical frequencies.

It should be possible to use the results in this study to infer the depth dependence of $\gamma$. However, the kernels relating the observations to the underlying thermodynamics must be derived and computed before this can be carried out. It is also of some interest to search for longitudinal variations in the observed values of $\epsilon$ and $\mu$ that might be correlated with the observed longitudinal variations in $U_x$ and $U_y$. There does appear to be some weak evidence for variations in $\epsilon$ and $\mu$ for the $f$ and $p_4$ rings, but again these could be due to systematic errors. The answer to this question must await further analysis.

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ANGULAR MOMENTUM TRANSPORT AND DYNAMO ACTION IN THE SUN:
A report on implications of a recent helioseismic estimate of solar rotation

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ABSTRACT

Gilman, Morrow and DeLuca have recently introduced some ideas about how an emerging helioseismic picture of the Sun's internal rotation might affect the understanding of angular momentum transport and turbulent dynamo action in the solar interior. The present paper offers a brief report and commentary on these issues. Analysis of the frequency splittings of solar acoustic oscillations measured by Brown and Morrow has suggested that the latitudinal differential rotation observed at the surface persists throughout the convection zone, and that the rotation is more uniform in the outer layers of the radiative interior. Taking this picture at face value, we discuss qualitatively how angular momentum must be transported in order sustain the observed rotation, the exchange of angular momentum between the convection zone and radiative interior, and the action of a turbulent dynamo which might be operating near the base of the convection zone. Our new qualitative scenario for the solar dynamo explains both the equatorward migration of the zone where sunspots appear and the observed poleward migration of non-sunspot magnetic field.

Keywords: Solar dynamo, angular momentum transport, solar rotation

1. INTRODUCTION

Knowledge of the solar internal rotation constrains theories about the Sun's interior dynamics and magnetic cycle. The dynamics of the outer layers of the Sun are distinguished by the presence of convective motions whose interaction with rotation can generate differential rotation and helicity. Both of these are key ingredients of a turbulent dynamo, which might account for the observed solar cycle.

In the deeper, stably stratified interior of the Sun, angular momentum is transported much more slowly than in the convection zone. We shall be concerned with the mechanisms that operate there only insofar as they couple the outer layers of the radiative interior to the convection zone; our considerations of the implications of a new rotational model for angular momentum transport and dynamo action do not involve regions below about half the solar radius. We center our discussion around how numerical simulations of the rotational influence on global convection (c.g. Refs 7-8) and the associated models of turbulent dynamos operating in and at the base of the convection zone (c.g. Refs 26-31) might be reconciled with a new helioseismically derived rotational profile for the Sun.

2. THE CONSIDERED MODEL OF SOLAR ROTATION

2.1 Source of the Rotational Model

Gilman, Morrow and DeLuca (Ref 1) consider a model for the internal rotation of the Sun that has emerged from analysis (Refs 2-4) of the rotational frequency splittings of solar acoustic oscillations measured by Brown and Morrow (Ref 3). These splitting data are displayed elsewhere in these proceedings (Ref 3). Figure 1 displays a rotational model derived from a forward approach to the data (Ref 3); it shows the radial variation of angular velocity at three different co-latitudes. Figure 2 depicts how the frequency splittings were calculated using this model to fit the measured splittings. In this figure, $a_1$, $a_2$ and $a_3$ are just the odd coefficients of the expansion in Legendre polynomials that observers use to represent the rotational frequency splittings (c.g. Refs 3,5,6).

Note that the model fits the data well, but an inherent danger of the forward approach is that it is not the only one that might do so. The model seems very worthy of consideration, however, since inverse approaches to these same data yield results that are qualitatively quite similar (Ref 4). In addition, the new, higher-quality rotational splitting data offered by Libbrecht (these proceedings) also appear to be in concert with the type of rotational model considered here.

2.2 Features of the Model

2.2.1 Within the Convection Zone. Examination of the model portrayed in Fig 1 reveals that there is no radial gradient of angular velocity in the convection zone implying that the surface latitudinal differential rotation extends at least to the base of the convection zone. This tends to rule out the profile of angular velocity constant on cylinders aligned with the rotation axis, which is predicted by models of the rotational influence on global convection (e.g. Refs 7,8). Figure 3 is a plot presented in a paper by Glatzmaier (Ref 9). This paper claimed that the profile of angular velocity predicted by the models of global convection is consistent with the rotational splitting data of Duvall and Harvey (Ref 10). However, each dot depicting oscillation results is about 25 nHz thick, or 2.5 times the largest error bar on the Brown and Morrow $a_1$ coefficient (Ref 3.5). Thus with the current data, discrimination at the level of the thickness of the dots on Glatzmaier's plot is possible, and these data are not reconcilable with the predictions of the convection models. Morrow (Ref 3) directly compares the splittings for models with surface-like rotation to those for models with cylindrical rotation in the convection zone. The lack of consistency between the solar rotation

produced by global convection models and that extracted from oscillation data suggests that some crucial physics is missing from the numerical simulations.

2.2.2 Beneath the Convection Zone. Below the convection zone, the rotational model sketched in Fig 1 has angular velocity constant at a value intermediate between the maximum and minimum of the surface profile. Thus there is a transitional layer of uncertain thickness between the convection zone and radiative interior in which there are substantial radial gradients of angular velocity. Its thickness is uncertain because the forward analysis of the splitting data could not unambiguously define it (Refs 2-4). The radial gradients are of opposite sign at low and high latitudes and the reversal occurs at 30° latitude. Gilman, Morrow and DeLuca (Ref 1; hereafter GMDJ) show that the magnitudes of the radial gradients in the transitional layer are consistent with a balance between torques exerted at the interface in low latitudes and those exerted in high latitudes. They also discuss how the signs of the gradients in the transitional region make it a promising location for the dynamo action needed to produce the solar cycle. Only the general ideas along with some commentary are reported here, and thus our journal paper (Ref 1) should be consulted for a more detailed treatment.

3. TRANSPORT OF ANGULAR MOMENTUM

The considered picture of rotation for the Sun has two major implications for the transport of angular momentum in the solar interior. One involves the transport of angular momentum within the convection zone itself, and the other concerns the exchange of angular momentum between the convection zone and the radiative interior. Figure 4 (extracted from Gilman, Morrow and DeLuca: Ref 1) compares the cycle of angular momentum transport achieved by the models of global convection (Fig 4b) to the cycle implied by the new picture of internal rotation (Fig 4a). Note that the primary distinctions are 1) the new cycle requires a balance of radial transport within the convection zone that results in the spoke-like contours of angular velocity rather than the model-generated cylindrical contours, and 2) the new cycle includes coupling between the convection zone and the radiative interior, whereas the global convection models do not.

Figure 1: The solar rotation model derived from a forward analysis of the frequency splittings of acoustic oscillations (the SRF model).

Figure 2: The curves indicate the frequency splittings generated by the solar rotation model of figure 1. The points are the data of Brown and Morrow (1987).

Figure 3: Glatzmaier's comparison between the rotational profiles of Duvall et al. (1984) and of global convection models.
3.1 Transport in the Convection Zone

In the models of global convection (e.g. Refs 7-9), angular momentum transport is due to Reynolds stresses of both a diffusive and non-diffusive nature. The non-diffusive stresses are due to the Coriolis effect on the convection and serve to drive differential rotation. These stresses are explicitly computed in the models. The diffusive stresses dissipate differential rotation and are parameterized by an isotropic eddy viscosity. After about three years of simulated time, the primary net transport by non-diffusive Reynolds stresses is approximately outward, away from the rotation axis (large arrows in Fig 4b). The associated latitudinal component is directed toward the equator and leads to the latitudinal differential rotation seen at the surface; the outward radial component leads to angular velocity constant on cylinders and thus decreasing with depth. In the models, small scale turbulent diffusion counteracts these processes (squiggly arrows in Fig 4b).

3.1.1 Producing Constancy on Cylinders. Insight into the convection models' cycle and balance of angular momentum in the convection zone is achievable through closer examination of both the time-averaged solutions and the initial transient processes that work to produce the latitudinal and radial differential rotation. In the numerical model of Gilman and Miller (Ref 8) establishes the equatorward transport of angular momentum is established relatively early (after about one rotation period) in the outer layers of the convection zone; the radial transport is initially inward, consistent with the expected effect of Coriolis-induced Reynolds stresses on the radial convective velocities.

In addition to building up the latitudinal differential rotation, the increasing angular velocity in the outer layers near the equator eventually causes a significant positive radial gradient of angular velocity. This radial differential rotation then feeds back on the convection, tilting the outer part of the convective cells forward (in the direction of rotation) relative to the deeper part of the cells. The tilting causes Reynolds stresses which act to reverse the direction of the radial transport of angular momentum from inward to outward. This reversal leads to angular velocity decreasing with depth in the convection zone and approximating the limit of constancy on cylinders.

3.1.2 Avoiding Constancy on Cylinders. If some diffusive process were to counteract the build-up of the radial shear induced by the spin-up of the outer layers, then the radial transport of angular momentum could be balanced so that the cylindrical profile of angular velocity would not manifest itself. GMD claimed that a solution to the inconsistency between the cylindrical rotation produced by the convection models and the surface-like rotation suggested by the oscillation data may lie in accounting for some deficiency of the models in representing the effects of compressibility and/or unresolved small scale turbulence. Morrow (Ref 2) suggested specifically that if the turbulent viscosity (diffusive Reynolds stress) that causes the diffusive transport of angular momentum in the radial direction were to be greater than the viscosity that effects diffusive transport in latitude, then the radial shear might be prevented from developing a sufficient magnitude to significantly tilt the large scale convection. This would leave near-helioiogy (i.e. balance of Coriolis and pressure gradient forces on the Sun), and thus practically no net radial transport of angular momentum. In this case the angular velocity should not vary with depth.

Kippenhahn (Ref 11) used the sense of anisotropic turbulent viscosity that emphasizes the radial direction over the latitudinal. He argued that, unlike molecular motions, turbulent motions are generally anisotropic because they have a preferred orientation relative to the direction of gravity. Thus the associated viscosity must also be anisotropic. His model for the solar differential rotation required a meridional circulation to achieve the needed equatorward transport, but here one has non-diffusive Reynolds stresses to do the job. Schmidt and Sütz (Ref 12) have considered the idea of a generalized anisotropic viscosity model that incorporates both driving and diffusive Reynolds stresses in both latitude and radius. Such a model, with a stronger diffusive transport coefficient in the radial direction, computed consistently with the mean flow as done by Gilman or Glatzmaier (Refs 7-9) might well produce both the latitudinal differential rotation and the lack of a significant radial gradient of angular velocity in the convection zone. Several other papers have addressed the issue of anisotropic viscosity and rotation in the solar convection zone (e.g. Refs 13-15).
3.1.3 Momentum From the Radiative Interior. Whatever the mechanisms that might transport angular momentum between the convection zone and radiative interior, they are not likely to act rapidly enough to significantly affect the basic angular velocity field until the angular momentum is redistributed within the convection zone. Instead, such coupling mechanisms are a factor in the overall angular momentum content of the convection zone, much as the magnetic braking in the solar wind is a factor in this respect.

3.2 Coupling Between Convection Zone and Interior

For the proposed model of solar rotation (Fig 1), changes in angular velocity at the depths of transition between the convection zone and radiative interior offer evidence that the two regions are and have been dynamically coupled. The apparent survival of latitudinal differential rotation to depths that are likely to be beneath not only the convectively unstable layer but the overshoot layer as well, suggests that the influence of the convection zone is somehow felt more deeply than the layer in which significant mixing to the surface occurs. This hints at the existence of mechanisms that can transport angular momentum between the convection zone and radiative interior without necessarily mixing material (e.g. Ref 16). Of course transport mechanisms that do mix, such as overshooting convection, shear-induced turbulence, diffusive instabilities, and meridional circulation, may also play a role in coupling the two regions. Whatever the mechanism, it is undoubtedly related to the radial gradients of angular velocity, and one may ask simple questions about the balance and exchange of angular momentum reflected by the rotation in the two regions.

3.2.1 Criterion for Balance of Torques. In a purely one-dimensional picture of solar angular velocity, such as presented by Duvall et al. (Ref 17), a significant radial gradient of angular velocity automatically implies a rapid change of specific angular momentum and perhaps small coupling between two interior regions. Such a picture can be deceiving. A two-dimensional picture may include radial gradients of opposite signs at a single radius (e.g. Fig 1), thus allowing for the possibility that the angular momentum is continuous in a latitudinally-integrated sense. Gough (Ref 18) suggested that angular momentum might be continuous at an interface between a region rotating differentially in latitude and a region rotating more rigidly. Such continuity at the interface between the convection zone and radiative interior would imply a balance between the torques exerted at high latitudes and those exerted at low latitudes, such that no net angular momentum is exchanged between the two regions. In 1948, Starr proposed a balance for the torques for the interface between the solid earth and the zonal winds in its atmosphere. The ideas here are quite analogous to his.

Given the new picture of rotation in the transitional layer, one may consider whether such a balance holds. GMD examine a simple model in which small-scale turbulence, represented by a constant shear eddy viscosity, produces the exchange of angular momentum. The thickness of the transitional layer is assumed to be small and uniform with latitude. The rotational shear is determined at each latitude by the differences in angular velocity across the interface that are defined by forward analysis of the Brown and Morrow frequency splittings (Refs 2-4). GMD show that for a rotation model in which surface rotation pervades the convection zone, and in which the interior portion of the radiative interior is rotating rigidly at \( \Omega_0 \), the integrated criterion for a balance of torques between the two regions is

\[
\Omega_{\text{int}} = A + \frac{1}{5} B + \frac{3}{35} C.
\]

where A, B and C are coefficients of the surface rotation expressed as even powers of cosine co-latitude:

\[
\Omega_{\text{surf}}(\theta) = A + B \cos^2 \theta + C \cos \theta. \tag{2}
\]

GMD show that the rotation parameters (A, B, C, \( \Omega_{\text{int}} \)) reported from forward analysis of the rotational frequency splittings (Refs 2-4) are quite consistent with the criterion for balance of torques (Eq 1). They achieve similarly positive results when using other, independently measured values of the surface coefficients (Refs 21-22), and assuming that the surface rotation pervades the convection zone.

The co-latitude of the reversal of the radial gradient of angular velocity in the transitional layer is given by solving Eq 2 for \( \theta \) when \( \Omega_{\text{surf}}(\theta) \) (which gives the rotation at the base of the convection zone), equals the measured rotation rate of the radiative interior. GMD finds that the co-latitudes of reversal for all surface profiles are very close to 60° (i.e. 30° latitude).

3.2.2 Connection to the \( \alpha \) Splitting Coefficient. Morrow (Ref 20) shows that the rotational frequency splitting coefficient \( \alpha \) (like the one displayed in Fig 2) reflects the radial variation of \( \sin \theta \)-weighted co-latitude average of the solar angular velocity. Thus, if the angular velocity were constant at \( \theta_0 \), then \( \alpha \) would be constant at this value. If surface rotation were to pervade the solar interior, then \( \alpha \) would be constant with \( L \) at a value equal to the weighted average of the surface profile. This average is just equal to the right hand side of the criterion for the balance of torques (Eq 1). Thus if the torque balance holds between a convection zone rotating differentially like the surface and a radiative interior rotating rigidly, then \( \alpha \) should also be constant: at higher \( L \), corresponding to convection zone depths, \( \alpha \) equals the right-hand side of Eq 1, and at lower \( L \), corresponding to radiative zone depths, \( \alpha \) equals the left-hand side of Eq 1. This relationship between the balance of torque criterion and the \( \alpha \) coefficient is not surprising when one remembers that balance of torques at an interface implies continuity of specific angular momentum and that a \( \sin \theta \)-weighted integral of angular velocity is precisely what would be used to calculate the angular momentum associated with the rotational profile.

It is important to understand that constant \( \alpha \) is sufficient but not necessary for a balance of torques to hold. In the mean, general case, radial gradients may exist in the convection zone that produce \( \alpha \)-variations. However, in the mean, \( \alpha \) is a constant at all latitudes, and only small coupling is necessary for a balance of torques to hold. Nevertheless, a balance of torques at the interface is still possible, since the criterion in Eq 1 depends only on the rotational profile at the base of the convection zone.

For the current rotational model (Fig 1), in which there are no radial gradients in the convection zone, GMD show that a balance of torques implies that the angular momentum of the convection zone is the same as it would be if it were rotating rigidly at the rate of the outer radiative interior, \( \Omega_0 \). If the rigid rotation of the radiative interior were to persist to the solar center, then the angular momentum of the entire Sun would be the same as if it were rotating rigidly at the rate of the radiative interior. The relevant rigid rotation rate would be given by the constant \( \alpha \). But of course, the rotation is not rigid for the Sun even though its angular momentum might well be calculable as if it were. The radial gradients implied at the interface between surface rotation and rigid rotation (Fig 1) do not show up as \( L \)-variations in \( \alpha \), but they do show up in other combinations of the splitting coefficients, which reflect the radial variations of angular velocity at specific latitudes (Ref 20). These gradients are crucial to the new scenario for the solar dynamo proposed in the next section.
4. DYNAMO ACTION IN THE SUN

Dynamo theorists are charged with explaining the production and spatial and temporal behavior of magnetic fields over the course of the solar cycle. The famous butterfly diagram depicts the observed equatorward migration of the zone in which sunspots appear. Early in the cycle, sunspots form near 30° latitude; as the cycle progresses, the zone of sunspot formation progresses toward lower latitudes. According to Howard and LaDome (Ref 23) non-sunspot magnetic field, observed at higher latitudes, appears to migrate poleward.

4.1 The Role of the Angular Velocity

Turbulent dynamo theory (e.g., Refs 24-25) suggests that migrations of magnetic field may indicate the presence of propagating dynamo waves. The nature, location, and magnitude of variations in angular velocity play key parts in the way solar magnetic field is produced and cycled in this theory. For example, to maintain the overall field, there must exist mechanisms for converting between poloidal and toroidal fields with more toroidal generated than poloidal. A latitudinal and/or radial gradient of angular velocity stretches field lines to convert poloidal to toroidal field (the \( \alpha \)-effect). Propagation of dynamo waves requires a radial gradient of the angular velocity. Such a gradient must not only be of sufficient magnitude to produce the appropriate dynamo period, but must also be located in a region of the solar interior where fluid motions possess helicity (the scalar product of velocity and vorticity). Helicity is needed to lift and twist toroidal field into poloidal field (the \( \alpha \)-effect), and the product of its sign and the sign of the radial gradient determines the direction of dynamo wave propagation. GMD find that the \( \alpha \)-effect can also be important in converting poloidal to toroidal field, and thus consider an \( \alpha^2 - \alpha \) dynamo. Helical motions are easily produced by a combination of the Coriolis effect of rotation on the converging and diverging flows at the boundaries of large-scale convection cells (which produces the vorticity) and the downdrafts and updrafts to be found there (which provide the relevant velocities). For this reason, a turbulent solar dynamo must be located in or very near a region of convective instability.

4.2 Results From Numerical Dynamo Models

Numerical dynamo models that calculate the effect of rotation on global convection consistently with the generation and behavior of the magnetic field have struggled to produce the observed magnetic behavior (Refs 26-28). Such models manage to transport angular momentum so as to generate the observed profile of rotation at the surface, but they also predict angular velocity constant on cylinders in the convection zone and thus a rotation rate that decreases with depth throughout the convection zone. Given the sign of helicity in the convection zone, the positive radial gradients cause migration of all magnetic field toward the poles, an unacceptable result. The thin-layer dynamo located at the base of the convection zone, where the sign of the helicity may be opposite to that in the bulk of the convection zone (Ref 28, Fig 5), are just beginning to be developed (Refs 29-31). In this way, the positive sign of the radial gradient and the opposite sign of the helicity would tend to propogate the dynamo waves equatorward. This would not, however, account for the observed poleward drift of non-sunspot fields.

4.3 A New Dynamo Scenario

The new model for solar internal rotation (Fig 1) strengthens the case for seating the solar dynamo at the base of the convection zone. Only in the transitional layer do radial gradients needed for the latitudinal propagation of dynamo waves exist. The existence of both radial and latitudinal gradients of angular velocity, combined with the helical motions of counterrotating turbulence from the convection zone, provide a likely environment for the generation of magnetic fields by a turbulent dynamo. But do the signs of helicity and the radial gradient of angular velocity in the transitional layer conspire to propagate dynamo waves in accordance with the observed migration of magnetic fields? The provocative answer contributed by Gilman in GMD is that yes, indeed the dynamo dilemma is qualitatively resolved by this picture of internal angular velocity. The predicted change in sign of the helicity at the base of the convection zone together with the reversal in the sign of the radial gradient at about 30° latitude produce an extremely attractive scenario for magnetic behavior over the course of the solar cycle.

Figure 5: A schematic picture demonstrating how helicity may change sign near the base of the convection zone.

Figure 6 summarizes the various combinations of the signs of the helicity and the radial gradient that lead to particular directions for the propagation of dynamo waves. The top row of the figure illustrates the original dynamo dilemma in which angular velocity constant on cylinders implies that angular velocity decreases with depth throughout the convection zone. In this case, the dynamo waves propagate poleward. The second row shows that if one could somehow justify reversing the sign of the radial gradient in the convection zone, then equatorward migration would result. This would have caused the values at higher L of the \( \alpha \)-effect, or some combination of the coefficients for rotational frequency splitting, to exhibit an increase with decreasing L and this is not observed (Refs 3.5,20).

The final two rows in Figure 6 offer the tempting resolution to the dynamo dilemma. In the transitional layer of the rotation model proffered in Fig 1, where the sign of the helicity may be positive, the reversal of the radial gradient of angular velocity causes dynamo waves to propagate toward the equator below 30° latitude (where the angular velocity decreases with depth) and toward the pole above 30° (where the angular velocity increases inward). This would account nicely for the observed pattern of magnetic migration.

Note that this scenario for the dynamo is possible even if there were radial gradients of angular velocity in the convection zone. The lack of these serves only to further encourage moving the dynamo to the transitional layer where needed gradients do exist. As long as the rotation rate of the interior is intermediate between the maximum and minimum rates of the profile of angular velocity at the base of the convection zone, the general nature of Gilman’s dynamo scenario could be in play. The fact that the new model for the angular velocity has the reversal of the radial gradient just where it is needed (30° latitude) to explain the magnetic behavior makes it a very attractive picture for the Sun’s internal rotation.
GMD demonstrates that for a simple Cartesian, kinematic, $a^2 - w$ dynamo model, the magnitude of the shear given by the new rotation model can produce appropriate periods for dynamo waves and appropriate relative strengths of poloidal and toroidal fields. DeLuca and Gilman are proceeding to develop a more realistic spherical shell dynamo for the base of the convection zone.

5. CONCLUDING REMARKS

The implications of the newly emerging picture of the solar rotation are concentrated in the transitional region between the convection zone and the radiative interior. Both the proposed dynamo scenario and the idea of a balance of torques, which indicates a coupling between these two regions of the solar interior, are tied to the reversal of the radial gradient of angular velocity in the transitional layer. Although we have concentrated on a particular model, much of the discussion is relevant to any model of the solar angular velocity that includes a transitional layer between a convection zone that rotates differentially in latitude (such that angular velocity decreases with latitude), and a radiative interior that rotates more rigidly at a rate intermediate between the maximum and minimum rates at the base of the convection zone.

One must bear in mind, however, that the layer for overshooting convection, and therefore the layer for turbulent dynamo action, may not extend as deeply into the Sun as the transitional zone over which the latitudinal variation of angular velocity declines. DeLuca and Gilman (Ref 31) have argued that the depth of the overshoot layer might only be about $10^4$ km or $0.1R_\odot$. The radial extent of the rotation model's transitional layer is uncertain, but the data suggest that it is considerably thicker than this rather thin estimate for the overshoot layer.

The rotation model suggests that a small degree of latitudinal differential rotation may extend as deeply as $0.5R_\odot$. The depth for significant mixing is of course constrained by the observation of the depleted Li and the normal abundance of Be at the solar surface. According to current models, the temperature at $0.5R_\odot$ is above what is required to destroy both Li and Be (R. Gilliland, personal communication). This would imply that overshooting that is capable of mixing material to the surface does not extend as deeply as the differential rotation, and that some of the momentum transport needed to sustain differential rotation in these outer layers of the radiative interior is caused by transport mechanisms such as magnetic field and/or gravity waves, which do not necessarily mix material (e.g., Ref 15). If the dynamo were to be operating at the interface, then the presence of magnetic fields may well play a significant role in the transport of angular momentum, both radially and latitudinally.

The lack of radial gradients of angular velocity in the convection zone is yet to be produced with a fully consistent dynamical calculation. However, certain parameterizations of the influence of rotation on convection have achieved some success in this respect (Ref 14-15). It is possible that parameterizing turbulent viscosity more anisotropically in the dynamical models of Gilman and/or Glatzmaier might produce a more surface-like profile of rotation in the convection zone instead of generating angular velocity constant on cylinders, which is not consistent with the newest helioseismic results.

This work was done while Morrow and DeLuca were at the High Altitude Observatory in Boulder, Colorado. The paper itself was prepared at the Institute of Astronomy in Cambridge. Neil Balmforth and Bill Merryfield provided invaluable assistance with the manuscript.

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INVERSION OF THE SOLAR ROTATION RATE VERSUS DEPTH AND LATITUDE

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ABSTRACT

We have used three different inversion techniques to compute the internal solar rotation rate from several sets of \( n \)-averaged frequency splittings (Refs. 1, 2). We have used an iterative variation of the spectral expansion method (Refs. 3, 4), the optimal averaging kernel method (Ref. 5) and a piecewise constant constrained least square method to invert the data. Each computation was carried out independently. While they present similar trends, each of the solutions differs in detail. A consistent feature in all the inversions is the disappearance of differential rotation below the base of the convection zone. Also, a strong differential signature in the deeper part of the convection zone is present in most of the solutions. A slow decrease of the rotation rate with depth for the equatorial and mid-latitude curves is significant in the spectral expansion and the least square results but only marginally apparent in the averaging kernel results.

Keywords: Solar Rotation, Inversion, Solar Oscillations.

1. INTRODUCTION

Observation of solar p-mode oscillations provides a unique tool for investigating some of the internal properties of the sun. One of the properties most directly accessible to helioseismology is the internal rotation rate. The effect of the perturbation of the solar rotation on the eigenfrequencies is to split the azimuthal degeneracy of a spherically symmetric model. Such splittings will be proportional to an average rotation rate, weighted with depth and latitude, and to the azimuthal degree \( \ell \).

Using the surface rotation rate as an order of magnitude indicator, the splitting between the prograde and the retrograde sectoral modes of degree \( \ell = 10 \) is of the order of \( 3 \mu \text{Hz} \) while for degree \( \ell = 100 \) it is around \( 91 \mu \text{Hz} \). Measurement of such values is well within present observational capability.

Measurement of splittings accurate enough for a precise determination of the solar rotation rate requires observational errors several orders of magnitude smaller than the numbers quoted above. Even though such precision has not yet been achieved, especially for the low degree modes and for the high ones, recent determinations of splittings of intermediate modes (\( \ell = 5 \) to 120) have prompted us to attempt some preliminary inversion of the solar internal rotation rate.

While the remarkable potential of the inversion techniques has been successfully demonstrated with artificial data simulations (Refs 6–8), only a few attempts to invert observed splittings have been performed yet. The results of the inversion of 37 \( n \)-averaged sectoral splittings ranging in spherical harmonic degree \( \ell \) from 1 to 100, obtained from a 16-day run (Refs. 9, 10) have indicated that the solar rotation rate would decrease with depth, while the innermost part would rotate faster. From the sectoral nature of the splittings used in that work no latitude information could be extracted.

In the present work, we have used two independent data sets of \( n \)-averaged splittings. The first set was computed from a 35-day run obtained at Mt Wilson in 1984 for \( \ell = 5 \) to 120, using the iterative cross-correlation technique (Ref. 1). The second set was computed from a 100-day run at the Big Bear Solar Observatory in 1986 for \( \ell = 10 \) to 60, using a singlet fitting procedure (Ref. 2). We have averaged over \( n \) the individually identified splittings to form a set comparable to the first one. Although significant systematic differences are present between both data sets, we have also combined them into a single set to investigate the effects of more accurate data on the inversion results. An extended review and comparison of different rotational splitting measurements can be found in Refs. 11 and 12.

Inversion of rotational splittings requires solving a set of integral equations that relate each splitting to some mode dependent weighted value of the rotation rate. Since a complete set of modes is not observationally accessible and frequency measurements contain errors, there will not be a unique solution; the problem is ill-conditioned. Several methods are available to compute a solution, each of which uses some explicit or implicit assumption on the unknown variable to condition the problem.

A complete review of the inversion techniques from a terrestrial perspective can be found in Ref. 13 and from a solar perspective in Ref. 8. We have used three different methods independently: an iterative variation of the spectral expansion, the optimal averaging kernel method and a piecewise constant constrained least square method.

By using different methods and different data sets we are also attempting to investigate the merit of the different inversion techniques and to assess the limitation of each data set.

2. THE ROTATIONAL SPLITTINGS

The solar rotation will induce in the eigenfrequencies of the 5-minute p-modes a rotational splitting for each \( \{n, l, m\} \) singlet,
where \( n \) is the radial order, \( l \) the spherical harmonic degree and \( m \) the azimuthal order of the eigenfrequency.

A good approximation relating the splittings to the solar rotation rate \( \Omega (r, \phi) \) is (Ref. 14):

\[
\Delta \nu_{n,l,m} = - \frac{m}{2\pi} \int_0^{2\pi} \int_0^R K_{n,l}(r) \Omega(r, \phi) \, dz \, dr
\]

where \( \hat{P}_m \) are normalized associated Legendre polynomials, \( K_{n,l} \) are normalized rotational kernels, \( z = \cos(\phi) \), and \( \phi \) represents the colatitude.

While the rotational splittings are defined as

\[
\Delta \nu_{n,l,m} = \nu_{n,l,m} - \nu_{n,l,0}
\]

they are actually fit to a Legendre polynomial expansion where

\[
\Delta \nu_{n,l,m} = L \sum_{|m|} a_{n,l,m} P_m(\cos \phi)
\]

and \( L^2 = l(l+1) \).

### 2.1 The Mt Wilson 60-foot Tower 1984 p-mode Splittings

From the observation campaign of the summer of 1984 at the Mt Wilson 60-foot Solar Telescope, a 16-day run and a 19-day run have been fully reduced by Tomczyk for the completion of his Ph.D. dissertation (Ref. 1). Solar Doppler images have been obtained by taking a pair of full-disk images of the sun in the red and blue wings of the sodium D lines using a magneto-optical filter setup and a 244 by 246 CID camera (Ref. 15). A full spherical harmonic decomposition for all the modes for degrees up to \( l = 120 \) has been computed. Each of the two runs has been reduced independently, and both power spectra were then averaged. Due to the poor duty cycle of the second run, the averaging was performed only for \( l \) below 46.

An iterative cross-correlation method was used to compute the splittings from the power spectrum for \( l = 5 \) to 120. This method uses all the modes as well as their sidelobes, for a given \( \{ l, m \} \), by cross-correlating an iteratively built averaged spectrum for a given \( l \) from all the \( m \) with each individual \( \{ l, m \} \) spectrum. By using all the modes as well as their temporal sidelobes, the cross-correlation technique gives a good signal to noise ratio, but loses all the \( n \) information. Note that the presence of spatial sidelobes will introduce small but systematic effects in the cross-correlation results.

### 2.2 The Big Bear Solar Observatory 1986 p-mode Splittings

A 100-day run of Doppler images obtained with the dedicated heliostat telescope at the Big Bear Solar Observatory during the summer of 1986 has been reduced by Libbrecht (Ref. 2). The Doppler images were computed from a pair of full-disk images of the sun in the red and blue wing of the 6439 Å calcium line obtained with a Zeiss 0.25 Å birefringent filter in combination with a KDP electro-optical crystal. A full spherical harmonic decomposition for all the modes for degrees up to \( l = 60 \) has been carried out. Since the signal to noise ratios of the individual spectra are high due to the length of the run, an individual identification method, based on a least square fit, has been used to compute the splittings for \( l = 10 \) to 60. From the individually identified set an \( n \)-averaged set has been computed to be comparable to the Mt Wilson set.

### 3. THE INVERSION METHODS

We give here a brief abstract definition of the inversion problem. More detailed descriptions can be found in Refs. 13 and 8.

We start with a one dimensional forward representation:

\[
y_i + \sigma_i = \int K_i(r) x(r) \, dr
\]

where \( y_i \) are some observed values, \( \sigma_i \) the uncertainty on those observables, \( K_i(r) \) a set of theoretically deduced unimodular kernels, and \( x(r) \) the unknown function we are solving for. We will rescale equation (4) as:

\[
y' = \int K'(r) x'(r) \, dr
\]

where

\[
y'_i = \frac{y_i}{\sigma_i}, \quad x'(r) = \frac{x(r)}{x_0}, \quad K'(r) = \frac{K_i(r)}{\sigma_i}
\]

and \( x_0 \) is selected to bring \( x'(r) \approx 1 \). Our task is to deduce the function \( x' \) from a finite set of \( y'_i \).

#### 3.1 The Spectral Expansion

In the spectral expansion method the integral equation (5) is approximated by a matrix product \( y = Ax \) by replacing the integral with a summation.

The generalized inverse (Ref. 3) of the matrix \( A \) is used to compute the solution (since \( A \) is rectangular). The generalized inverse uses the eigenvalue and eigenvector decomposition of \( A \) to compute an inverse matrix \( H \) from the eigenvectors associated with the non-zero eigenvalues. A solution is given by applying the inverse matrix to the observables: \( x = H y = H A x \).

#### 3.2 The Optimal Averaging Kernel Method

The optimal averaging kernel method computes a solution to the
functions of the radius alone.

where \( \xi_{\text{opt}} \) are Legendre polynomials and \( X_t \)

We have parameterized the solar rotation rate as:

\[
\Omega(r) = \sum_i X_i(r) P_i(\cos(\theta))
\]  

(10)

where \( P_i(x) \) are Legendre polynomials and \( X_i(r) \) some arbitrary functions of the radius alone.

4.1 Latitude Parameterization for the Spectral Expansion Method

Since the Legendre polynomial fitting is a linear operation there is a linear expression that relates the splittings to the fitting coefficients, and we can formally invert equation (3) as

\[
a_{i,n,L} = \sum_m W_{i,m} \Delta n_{n,m}
\]  

(11)

Introducing this expression in equation (1), and replacing the integral by a summation, the problem can be reduced to a matrix product

\[
y = Ax
\]  

(12)

where, if we collapse the indices \((i,n,L)\) as \( j \) and \((t,p)\) as \( k \), with proper rescaling we have

\[
y_j = \frac{a_{i,n,L}}{\sigma_{i,n,L}}
\]

\[
A_{jk} = A(i,n,L,t,p) = \frac{1}{2\pi} K_{n,\theta} \sum_m m W_{i,m} I_{t,j,m} \frac{X_{t,j}}{X_{t,p}}
\]

\[
x_k = \frac{X_{t,p}}{X_{t,\theta}}
\]

and

\[
\hat{K}_{n,\theta} = \int K_{n,\theta}(r) \Delta(r-r_p) \, dr
\]

\[
A(r-r_p) := \text{some function centered about the radius } r_p
\]

\[
I_{t,j,m} = \int \frac{P_{m}^j(x)}{P_{m}^p(x)} P_t(x) \, dx
\]

Note that by symmetry considerations \( I_{t,j,m} \) is zero when \( t \) is odd (recall that the Legendre polynomials are antisymmetric for odd \( t \) and symmetric for even \( t \)), thus only the even terms of the latitudinal expansion need to be considered.

In the present work \( B_{i,l,t} \), the angular part of \( A \), where

\[
B_{i,k} = -\sum_m m W_{i,m} I_{t,j,m}
\]  

(13)

has been approximated by using an asymptotic expression for the \( P_t^m \).

4.2 Latitude Parameterization for the Optimal Averaging Kernel Method

Since the optimal averaging kernel method is limited to a linear formulation, the latitudinal parameterization cannot be applied in the same fashion as it is in the spectral expansion method.

Let us define the following quantities:

\[
\hat{\Omega}_{t,m}(r) = \int P_{m}^j(x) \Omega(r,\theta) \, dx
\]  

(14)

and

\[
\hat{\Omega}_{n,j,m} = \int K_{n,j}(r) \hat{\Omega}_{t,m}(r) \, dr
\]  

(15)

Introducing our latitude parametrization as defined in equation (10) we can write

\[
\hat{\Omega}_{t,m}(r) = \sum_t X_t(r) \int P_{m}^j(x) P_t(x) \, dx
\]

(16)

\[
= \sum_t X_t(r) I_{t,j,m}
\]  

(17)

where \( I_{t,j,m} \) is defined as above.

We now can rewrite equation (1) as

\[
\frac{1}{2\pi} \hat{\Omega}_{n,j,m} = -\frac{L}{m} \sum_i a_{i,n,j} P_i(\frac{m}{L})
\]  

(18)
Figure 1.a: Rotation rate versus depth and latitude, computed using an iterative variation of the spectral expansion method from the Mt Wilson \( n \)-averaged set of splittings. The splittings were computed from a 35-day run using a cross-correlation technique, for degree \( l \) from 5 to 120. The grey scale covers the range 0 to 700 nHz and contours are drawn for values between 200 and 600 nHz in steps of 50 nHz.

\[
\begin{align*}
\frac{1}{2\pi} \sum_l \int I_{l,m} \int K_{n,l}(r) X_l(r) \, dr &= \frac{L}{m} \sum_i a_{i,n,l} P_l\left(-\frac{m}{L}\right) \\
\end{align*}
\]

It is easily shown that, by using an asymptotic expression for the \( P_l \), \( I_{l,m} \) can be expressed as a polynomial in even power of \(-\frac{m}{L}\). By identifying the power of \(-\frac{m}{L}\) we obtain the following three integral equations:

\[
\begin{align*}
\frac{1}{2\pi} \int K_{0,l}(r) X_0(r) \, dr &= A_0 = a_{0,n,l} - \frac{2}{3} a_{2,n,l} + \frac{553}{840} a_{4,n,l} \\
\frac{1}{2\pi} \int K_{2,l}(r) X_2(r) \, dr &= A_2 = -\frac{10}{3} a_{2,n,l} + \frac{8}{3} a_{4,n,l} \\
\frac{1}{2\pi} \int K_{4,l}(r) X_4(r) \, dr &= A_4 = -\frac{24}{5} a_{4,n,l}
\end{align*}
\]

This formulation requires solving three linear problems independently in place of a two dimensional one.

4.3 Latitude Parameterization for the Piecewise Constant Constrained Least Square Method

We have used a linear formulation for the piecewise constant constrained least square method, where the smoothing constraint only applies to the radial variable. We have therefore used the same formulation as for the optimal averaging kernel method.

It is conceivable to introduce a second constraint on the latitude with a second Lagrangian multiplier, but an implicit smoothness constraint is already implied by the polynomial nature of the latitudinal parameterisation.

4.4 Actual Parameterization

Since the splittings used were reduced to an \( n \)-averaged set, equation (3) has to be rewritten as

\[
\Delta \nu_{n,l,m} > n = L \sum_l a_{i,l} P_l\left(-\frac{m}{L}\right)
\]
Figure 2.a: Rotation rate versus depth, plotted at the equator (dotted line), at a latitude of 30° (short dashes) and at a higher latitude of 60° (long dashes) for the solution obtained by an iterative variation of the spectral expansion method from the Mt Wilson n-averaged set of splittings. (Same solution as shown figure 1.a.)

Figure 2.b: Rotation rate versus depth, plotted at the equator, mid-latitude and at high-latitude for the solution obtained by the optimal averaging kernel method from the Mt Wilson n-averaged set of splittings. (Same solution as shown figure 1.b.)

Figure 2.c: Rotation rate versus depth, plotted at the equator, mid-latitude and at high-latitude for the solution obtained by the piecewise constant constrained least square method from the Mt Wilson n-averaged set of splittings. (Same solution as shown figure 1.c.)

Figure 3.a: Rotation rate versus depth and latitude, computed using an iterative variation of the spectral expansion method from the Big Bear Solar Observatory n-averaged set of splittings. The splittings were computed from a 100-day run, with an individual identification technique, for degree l from 10 to 60. The grey scale covers the range 0 to 700 nHz and contours are drawn for values between 200 and 600 nHz in steps of 50 nHz.

The solutions are presented only for a restricted range in depth. This range of validity was estimated for all the methods from the following two criteria: 1) only the range between the highest and lowest mode turning points was considered, where true differential information can be extracted, and 2) the resolution kernels associated with the first two Legendre expansion coefficients had to be well localized (we have ignored this criterion on the a5 coefficient since it seemed to be too conservative a criterion, compared to the small significance of the a5 coefficients in the data sets).

Two consistent and significant features can be identified: 1) the disappearance of differential rotation below the convection zone, 2) a strong differential signature in the deeper part of the convection zone. A less consistent feature, since it is only present in two of the inversions (the ISE and the PCL), is the decrease of the equatorial rotation rate with depth.

Figures 3a-c and 4a-c present results obtained using the n-averaged Big Bear Solar Observatory data set. Since the data set only comprises the degrees 10 ≤ l ≤ 60, a valid solution could only be computed for a more restricted depth range. Even though the solutions from the different methods share similar features, they present significant differences. Some hint of the same features as for the Mt Wilson splitting results can be marginally identified (i.e.: the equatorial gradient). The lack of coherence of these results prevents drawing any significant conclusions on
any feature present in these solutions. This result should come as no surprise. Indeed, by averaging over \( n \) that data set some of the radial information has been lost. The gain in accuracy obtained by the averaging process cannot compensate the lack of radial resolution of such an \( l \)-restricted data set. By using all the modes, as presented elsewhere in these proceedings (Refs. 17, 18), the radial resolution can be slightly extended, leading to a more stable result. But caution should be used as to extend it only to where the solution remains significant.

We have also combined both data sets in a unique one, using an unweighted average when both data sets overlapped. Figures 5a-c and 6a-c present the result obtained from that set. Let us point out that significant systematic differences between the two data sets for the \( q_1 \) and \( q_2 \) coefficients lead us to believe that the combined data set may not represent the best data set from an observational standpoint. Indeed, in the averaging process, we have not attempt to reevaluate the observational uncertainties on the base of the discrepancies between the data sets, when these were significant (i.e.: larger that the given uncertainty). From the inversion standpoint, by combining the data, their significance was increased (by reducing the uncertainties of the averaged values) leading to more stable results. These do not necessarily represent more realistic solutions, but are presented here to show the improvement that can be expected from larger and more accurate data sets.

The solutions from the combined data set, as shown in figures 5a-c and 6a-c, present very consistent features. The differential rotation vanishes below the convection zone while the strong differential signature in the deeper part of the convection zone is
6. CONCLUSIONS

We have shown that the inversion of the rotation rate versus depth and latitude from a large in \( l \) and accurate enough \( n \)-averaged data set leads to consistent significant results independent of the method used. On the other hand, attempting an inversion of an \( n \)-averaged \( l \)-restricted set leads to incoherent and confusing results. Differences in the significance of some features between different methods can be used as an alternate test of the significance of the solution features. Caution should be exercised in interpreting the significance of the results presented here. Significant discrepancies between the data sets point to some unidentified systematic errors present in the computation of the rotational splittings. Combining the data sets could have reinforced these systematics, especially when one keeps in mind that one data set covers less than half of the degree range of the other one. The improvement of the inversion results obtained with the combined data set leads to the obvious conclusion that more accurate measurements of the low degree splittings are present in all three solutions.

As for the results of the Mt Wilson data set, the ISE and PCL solutions for the combined set present a significant decrease of the equatorial rotation rate with depth, while no significant trend is present in the OAK solution.
Figure 6.a: Rotation rate versus depth, plotted at the equator (dotted line), at a latitude of 30° (short dashes) and at a higher latitude of 60° (long dashes) for the solution obtained by an iterative variation of the spectral expansion method from the combined data set. (Same solution as shown figure 5.a.)

Figure 6.b: Rotation rate versus depth, plotted at the equator, mid-latitude and at high-latitude for the solution obtained by the optimal averaging kernel method from the combined data set. (Same solution as shown figure 5.b.)

Figure 6.c: Rotation rate versus depth, plotted at the equator, mid-latitude and at high-latitude for the solution obtained by the piecewise constant constrained least square method from the combined data set. (Same solution as shown figure 5.c.)

needed to carry out more reliable rotation inversions.

7. ACKNOWLEDGEMENTS

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8. REFERENCES


Rotational splitting measurements of low-degree solar p-modes are calculated using different sources of data. For l=1, data obtained from years 1981 to 1985 is used to find out the stretches where each particular mode remains excited, then its splitting is measured and the mean found. Cross-correlation of power spectra confirms these findings, and this method is used to look for the l=2 splitting. For 2<l<5 only one month of data is available and superposition of peak structures gives an upper limit for their splitting.

1. INTRODUCTION

The first experimental determination of rotational splitting of solar low-degree p-modes was obtained by Claverie et al. (Ref. 2) in 1981. In this work the l=1 mode appears as a triplet and the l=2 as a quintuplet, with a separation between components that suggest a rapid internal rotation of the Sun. These observations were made with integral sunlight then, if the rotational axis of the Sun is perpendicular to the line of sight, the modes with odd l=±m could not appear due to a symmetry effect; as a consequence the structure of the l=1 p-modes must be a doublet and the l=2 a triplet, as in the numerical model described below (see figure 1).

A simple explanation about the observations described before was done by van der Raay et al. (Ref. 6). It is the amplitude modulation of the solar p-modes due to the rotation that gives rise to the frequency splitting of these modes then, all the components of the rotational splitting will be present in the observations with different amplitudes unless the mean value of the modulation wave is equal to zero.

In this work we have looked for the rotational splitting of 0<l<5. For l=1 p-modes we are looking directly at its shape in the power spectra while other techniques are used for other modes of other degrees.

Figure 1. Slice of the spectra of p-mode (n=20, l=1) obtained with the numerical simulation of a series with 88 days. a) The splitting of an l=1 and l=3 mode is clearly evident and also are its sidebands. b) The same part of the spectrum of the same series, but now, the data have changes in phase every 16 days.

2. INSTRUMENT, OBSERVATIONS AND REDUCTION

Observations have been obtained at Observatorio del Teide (Izana, Tenerife) during summer seasons of the years 1981 to 1985. Integral sunlight was used to measure accurately the radial velocity of the Sun by means of a resonant scattering spectrophotometer built at the Physics Department of the University of Birmingham and extensively described in Brookes et al. (Ref. 1) and Roca Cortes (Ref. 8). In Table 1 the data used in this analysis is shown. For each day of observation a series of 10 to 11 hours of velocity measurement is obtained, sampled at 42 seconds (from 1984 on, a 40 seconds interval was used). The raw data, showing the 24 hour period sinusoidal variation due to the spin rotation of the Earth, has to be reduced in order to look for the small variation that represent the solar
oscillations. This is done by fitting an appropriate curve as explained in Pallé et al. (Ref. 4). Once the residuals are obtained, series of several days of observation will have to be used in order to increase the poor frequency resolution in a power spectra, that can be achieved by at most 11 hours observation (~30 µHz). On the other side by putting together several days of observation a 24 hours modulation signal is generated (as during the night there are no observations). This modulation is shown in the power spectra by the appearance of side bands separated from the real peak by 11.57 µHz.

3. NUMERICAL SIMULATION OF THE EXPERIMENT

In order to understand the spectra obtained from such series a numerical simulation of the experiment has been made (Ref. 7). In such a model all velocity fields that produce a non zero contribution to the radial velocity of the Sun have been considered: lab velocity, solar differential rotation, limb shift, gravitational redshift and oscillations, p and g modes of l<3 where considered with frequencies and amplitudes as observed (Ref. 3); random phases were selected. No noise was introduced.

A grid of 1795 points were used to simulate the solar disk. The solar potassium resonance line, used in the observation, has been approximated by a gaussian profile which is good enough, provided the spectral range used. Lines produced at the lab were also approximated by gaussian profile for a simple calculation; hiperfine structure was considered. Taking into account limb darkening and foreshortening, finally the experiment can be numerically simulated. Data sampled every 60 seconds was calculated during 10 hours around local noon time to match the real raw data.

Further, data was reduced and analyzed using the same procedure as for the real data. Several days of data were put together and a harmonic
analysis was performed using an iterative sine
wave fitting procedure (Ref. 4). In figure 1a a
slice of the spectrum of 88 consecutive days is
shown. The splitting of an l=1 and l=3 mode is
clearly evident and also are its sidebands. In
figure 1b the same part of the spectrum of the
same series with only one difference: the modes
used in the numerical analysis change only their
phase every 16 days. The change, when compared to
figure 1a, is enormous and it would be rather
difficult to tell of which 1 mode we are talking
about, as rotational splitting is absolutely
masked by the lack of phase coherence of the mode.
Therefore it became evident the need to know the
mode lifetimes and the periods where anyone mode
keeps excited.

4. ANALYSIS OF SPECTRA

In order to know the time intervals where a
mode remains excited, series of 15 continuous days
were used. Figure 2a shows a set of spectra of 15
days in which each considered series starts and
ends one day after the anterior one. Amplitude
spectra are then displayed one below each other as
time increase by one day, to a total of 88;
frequency is displayed in a horizontal scale, and
amplitude varies from white to black as it
decreases. Figure 2a shows spectra from purely
numerically simulated perfect data, while figure
2b shows the same data but the phase of the mode
is changing randomly once every 16 days. Finally,
figure 2c shows the spectra of real data analyzed
in exactly the same way.

The duration of 15 days has been chosen
because the decision was easier. It is a good
compromise between too short, then no frequency
resolution is available, and too large, then no
noticeable change is apparent. When computing an
analogous phase spectra display, good correlation
between amplitude and phase changes is seen.

In figure 3a spectra of the first 32 days
together of the p-mode l=1 shown in figure 2c- is
shown. Figure 3b shows the spectra for the next 40
days together. In figure 3c is shown a l=0 p-mode
in a coherence epoch. It is possible to see that
the l=1 shows some structure in the coherence part
and the l=0 doesn't show any splitting.

To measure the rotational splitting of each
l=1 mode present in the p-mode spectrum we have
selected the coherence epochs for each of them at
each year, whenever possible. Then the spectra of
such coherence epochs has been calculated and the
frequency difference, between split peaks, has
been measured. Figures 4 and 5 show different
examples for 1981 and 1985. We see doublets and
triplets. We have assumed that when there are two
peaks in the spectrum with a separation 0.6 µHz,
they correspond to m=0 and m=±1 or -1, because
when triplets appear the difference between split
lines is higher than 0.6 µHz. The splitting found
for l=1 p-mode is (0.72 ± 0.04) µHz.

<table>
<thead>
<tr>
<th>YEARS</th>
<th>PERIOD OF YEARS</th>
<th>USEFUL DAYS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1981</td>
<td>29 - 5 → 25 - 8</td>
<td>82</td>
</tr>
<tr>
<td>1982</td>
<td>17 - 4 → 5 - 9</td>
<td>122</td>
</tr>
<tr>
<td>1983</td>
<td>11 - 5 → 2 - 8</td>
<td>75</td>
</tr>
<tr>
<td>1984</td>
<td>17 - 4 → 1 - 10</td>
<td>152</td>
</tr>
<tr>
<td>1985</td>
<td>26 - 5 → 3 - 11</td>
<td>136</td>
</tr>
<tr>
<td>1987</td>
<td>26 - 6 → 26 - 7</td>
<td>28</td>
</tr>
</tbody>
</table>

Table 1

Data used in this analysis.

Figure 3. a) Spectrum of a series consisting of
the first 32 days of the p-mode l=1 shown in
figure 2c). b) Spectrum of the next 40 days
together. c) Spectrum of 20 days of data for a l=0
p-mode where a period of observation with no
changes in phase is chosen (a phase coherence set
of days).
5. ANALYSIS OF AUTOCORRELATION FUNCTIONS.

Since in a power spectrum several 1-1 or 2 modes are present and the rotational splitting is the same for all of them, we could use the autocorrelation functions of the power spectra to detect several bumps at appropriate lags. Of course, all noise produced by the lack of phase coherence is still present but it should wash out provided enough data is considered. This method can be extremely useful for 1-2 modes.

In order to reduce noise, the power spectra have been "cleaned". This consists in putting zeros everywhere in the spectra but the peaks corresponding to the lonely sidebands of either 1-1 or 1-2. In figure 6a and 6b the autocorrelation function for 1981 is presented for 1-1 and 1-2. Figures 7a and b show the same result but for 1985. Using this analysis one finds a rotational splitting of (0.55 ± 0.05) MHz for 1-2.

6. SPLITTING MEASUREMENTS FOR 2<1<5.

A 28 days observation was achieved in 1987 observing with a circular aperture of 0.56 D diameter. When observing with such a spatial filter, the sensitivity of the modes with 2<1<5 is higher and consequently this modes can be easily identified (Ref. 5).

With such a small observing period an analysis as the one showed above is not possible and only direct superposition of the central frequencies of each multiplet can give a rough estimation of the rotational splitting of such modes. In figure 8 such superposition is showed for 1-3 and 4, for 1-5 only three clear peaks were available and not even a rough estimation can be made.

The measured splitting is calculated by measuring the full width at half depth of these graphs and divided by 2π. The results found are (0.47 ± 0.06) MHz for every one.
Figure 6. Autocorrelation of the spectra of data obtained in 1981 (60 days). We have used a spectrum a) only with one sideband of $l=1$ b) only with one sideband of $l=2$.

Figure 7. Autocorrelation of the spectra of data obtained in 1985 (60 days). We have used a spectrum a) only with one sideband of $l=1$ b) only with one sideband of $l=2$.

Figure 8. Superposition of the central frequency of the multiplet for $l=3$ and 4.
CONCLUSIONS.

The conclusions can be summarized in figure 9, where a plot of measured splittings as a function of degree is showed. From this analysis no appreciable change at the 0.1 Hz level is seen along the years observed and analysed (1981 to 1985).

Figure 9. Measured synodic splitting as a function of degree.

REFERENCES


Solar P-Mode Frequency Splittings

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ABSTRACT. I discuss here new measurements of solar p-mode frequency splittings, based on 100 days of solar Doppler observations from Big Bear Solar Observatory. Because of the long observing run, the splitting measurements have significantly smaller uncertainties than previously published values. In addition, splittings were determined accurately for individual mode multiplets for the first time, and show a dependence of splitting on radial order \( n \) as well as degree \( l \).

Two inversions, which infer the solar rotation rate as a function of depth and latitude from the measurements, are also discussed. The two inversions give surprisingly consistent results, and present the following picture of the sun's internal rotation: 1) The rotation rate in the convection zone is roughly independent of depth, showing a latitudinal differential rotation equal to that seen at the solar surface; 2) There is a relatively sharp transition zone at the base of the convection zone, where the differential rotation in the convection zone gives way to approximately solid body rotation in the radiative interior. The inversions of the present data set can provide accurate rotation information only for \( \tau/R_\odot > 0.5 \).

A comparison of the present data set with previous measurements suggests that the even index splitting coefficients change with solar cycle. This time-dependent p-mode frequency splitting is consistent with photometric measurements of the solar limb, which show that the active latitudes on the sun are significantly hotter than the quiet sun.

Introduction. If the sun were perfectly spherically symmetric, then the frequencies of solar p-modes would depend on radial order \( n \) and spherical harmonic degree \( l \) only, and would be independent of azimuthal order \( m \). Since the sun is not spherically symmetric, the mode frequencies within each \((n\ell)\) multiplet are said to be split, and the frequency splitting can be measured and used to determine some properties of the solar interior (for a review see Libbrecht 1988a).

The dominant cause of the frequency splitting is the solar rotation, and the primary motivation for measurements like this one is the possibility of determining the rotation rate of the sun as a function of both depth and latitude. Since the sun's differential rotation is not well understood theoretically, its measurement is clearly important to constrain models. Other causes of p-mode frequency splitting might include the solar oblateness, solar magnetic fields, or a latitude-dependent structure in the temperature or convection in the sun, all of which break the sun's spherical symmetry.

It has already been observed that the rotationally induced splitting is much greater than that from all other mechanisms, and indeed only the rotational frequency splitting has been unambiguously identified in the published data to date. Previous measurements of the p-mode frequency splittings include those by Brown and Morrow (1987, 1987a), Duvall et al. (1986), Libbrecht (1986), Rhodes et al. (1987), and Tomczyk (1988). A thorough inversion of the Brown and Morrow data to determine the solar interior rotation rate as a function of depth and latitude is currently in preparation (Brown et al. 1988). Much recent work in this field, both in the measurements
and inversions, can also be found elsewhere in these Proceedings.

Recently Kuhn (1988, also these Proceedings) suggested that p-mode frequency splittings have been observed which are not the result of the solar rotation, and suggested further that these splitting measurements are in rough agreement with the splittings expected from measurements of the latitude-dependent temperature of the solar limb (Kuhn, Libbrecht, and Dicke 1985, 1987). The solar limb temperature function, as well as the measured splittings, depend on the phase of the solar magnetic cycle.

The 1986 BBSO Data Set. The observations which produced the data shown here were made using the dedicated helioseismology telescope at Big Bear Solar Observatory. The telescope and data acquisition system were described by Libbrecht and Zirin (1986) and Libbrecht (1988b). In essence, a Zeiss 0.25 Å birefringent filter in combination with a KD*P electro-optical crystal was used to produce full-disk solar images in the red and blue wings of the 6439 Å calcium line. By digitizing and differencing the images from the two wings of the line, solar Doppler images were produced at a cadence of one per minute. The telescope was operated from 26 March to 2 August 1986, producing 100 days of useful data and nearly 60,000 full-disk solar Doppler images.

The data were processed by fitting each image to all spherical harmonics with \( \ell \leq 60 \), and the fit coefficients were Fourier transformed in time to produce power spectra for each \( \ell \) and \( m \). The signal/noise ratio of the individual \( \ell m \) power spectra is high, so it was possible to determine the frequency splittings for each \( \ell m \) multiplet, rather than using the cross-correlation technique which has been used by other others (e.g. see Brown and Morrow). To this effect,
the \((2\ell + 1)\) power spectra \(S_m(\nu)\) from a single multiplet with given \(\ell\) and \(n\) were fit by least squares to the functions

\[
S_m(\nu) = \sum_{\ell'} W_{m\ell'} A_{\ell'} \psi(\nu - \nu_{\ell'}) + B,
\]

where \(W_{m\ell'}\) is the calculated overlap of the projected spherical harmonics over the solar disk, \(A_{\ell'}\) is the fit amplitude of each mode, \(\psi(\nu) = \Gamma^2/(\nu^2 + \Gamma^2)\) is the assumed shape of the modes, \(\nu_{\ell'} = \hat{\nu} + \ell\sum \alpha_i R_i(m'/\ell)\) describes the frequency splitting, and \(B\) is the fit background. The parameters \(\Gamma, \hat{\nu}, \alpha_i,\) and \(A_{\ell'}\) were adjusted in the least squares fit over a range of 11 \(\mu\)Hz centered on \(\nu_{nim}\) for each \(S_m(\nu)\), while the fixed background power \(B\) was determined in a fit to the combined power over the same range. Since the fitting routine described here is rather complex, several tests were performed to check the robustness of the results. Significant observations include: 1) the fit \(\Gamma\) reproduce the results of Libbrecht (1988b), as they should, and 2) the fit \(\alpha_i\) were not sensitive to the choice of initial guesses \(a_{i,0}\) in the non-linear least-squares fit routine.

Uncertainties in the splitting parameters \(a_i\) were estimated by examining the scatter in the \(a_i\) fits, which depends strongly on frequency \(\nu\) and is roughly approximated as \(\sigma = 3 + 4(\nu - 2.5)^4\) nHz. Weighting the individual fits with \(\sigma^{-1}\), the \(a_i\) were then fit by least squares to \(a_i = a_i^* + b_i^*(\nu - 2.5)\), where \(\nu\) is in mHz, and uncertainties in the \(a_i^*\) and \(b_i^*\) were determined from the residuals in the fit, assuming the correct \(\sigma\) was proportional to the weighting \(\sigma\). Due to limitations in space, only the \(a_i^*\) are shown here in Figure 1. Also shown in Figure 1 is the above fit to \(a_{35} = a_1 + a_3 + a_5\), which is equivalent to a fit of \([\nu(m = \ell) - \nu(m = -\ell)]/2\ell\), which can be taken as an approximation to the equatorial rotation rate. Additional plots of the data, as well as a discussion of possible systematic errors in the measurements, can be found in Libbrecht (1989).

The Solar Internal Rotation. The task of inverting these splitting data to infer the solar interior rotation is not a simple one, since mathematically the inversion problem has no unique solution. The additional, sometimes implicit, constraint which is usually added to produce a solution is that the solar rotation rate be a slowly varying function of depth and latitude. Figure 2 shows a comparison of two inversions of the BBSO data set, both of which were kindly provided to me by others, and both of which I must take as somewhat preliminary results.

The first, by W. Dziembowski and P. Goode, used a technique similar to that described by Dziembowski et al. (1988). The solar rotation is assumed to be of the form \(\Omega(r, \theta) = \Omega_0(r) + \Omega_1(r)\mu^2 + \Omega_2(r)\mu^4\), where \(\mu\) is the cosine of the polar angle, and the three functions \(\Omega_i(r)\) are evaluated on a grid of radial points. To produce a unique solution the \(\Omega_i(r)\) are adjusted to minimize a quantity consisting of \(\chi^2\) (comparing the data with the calculated splittings) plus a function of the \(\Omega_i(r)\) that is related to the velocity shear from the differential rotation integrated over the solar interior. With this additional constraint, a rather fine grid of radial points can be used without changing the solution.

The second of the two inversions, by J. Christensen-Dalsgaard and J. Schou, consisting of a Backus-Gilbert type inversion, is described in detail by the authors elsewhere in this Proceedings and I will not discuss it further here.

It is clear from Figure 2 that the two results, coming from different inversion techniques but using the same data set, give very similar pictures of the solar interior rotation. Note that even a dip in the rotation rate at \(r/R_\odot \approx 0.85\) is reproduced by both inversions, as is best seen in the contour plots. Of course this dip is most probably simply a 3\(\sigma\) point in the BBSO data set, and cannot be considered significant at this point unless it were to be seen by other observers.

The results of these inversions support the conclusion reached by Brown et al. (1988), that the rotation rate in the convection zone is roughly independent of depth, showing a latitudinal differential rotation equal to that seen at the solar surface. This result was, to me at least, something of a surprise, since computer simulations of the solar convection zone (Glatzmaier 1987, Gilman and Miller 1986) gave the correct surface differential rotation and suggested that below the surface the rotation rate would be constant on cylinders.

Below the convection zone, the splitting data are best described by solid body rotation at approximately 430 nHz. This result is not firm, however, since these intermediate-\(\ell\) p-modes do not probe the deep interior very well, and the inversions give little accurate information below \(r/R_\odot \approx 0.5\). Furthermore the thickness of the transition zone at the base of the convection zone is not well determined by the inversions, since both assume that the rotation rate is a slowly varying function of radius.
Figure 2. Inversions of the BBSO data to determine the solar interior rotation rate. On the left is an inversion done by Dziembowski and Goode, and on the right is another by Christensen-Dalsgaard and Schou. The upper plots are from the raw inversions, and show the solar rotation as a function of depth for latitudes of 0, 45, and 90 degrees. These results are somewhat preliminary, and in particular the error bars on these plots have not been polished. The contour plots were cleaned up a bit by: 1) assuming the rotation gotten from the inversion for 0.55 < r/R < 0.95 only; 2) assuming the surface rotation rate is given by $\Omega(\theta) = 462 - 58\sin^2(\theta) - 84\sin^4(\theta)$ nHz, where $\theta$ is the solar latitude; 3) assuming the solar interior rotation is $\Omega = 430$ nHz. None of these assumptions is inconsistent with the data or what we know about the solar rotation. Note the similarities between the two inversions, even including the dip in the rotation rate at r/R = 0.85.
The Even Splitting Coefficients. As discussed in Libbrecht (1989), it is expected that while a host of small systematic errors can creep into the odd splitting coefficients (i.e. the $a_i$, where $i$ is odd), these problems will not affect the even coefficients. Thus we expect that the even coefficients will be relatively free from systematic errors, and so the formal error bars can be taken very literally. Furthermore to first order the solar rotation alone does not produce a non-zero $a_2$ or $a_4$. The largest second order term in the rotation is simply the solar oblateness, which produces $a_2 \approx 20/\ell$ nHz and $a_4 \approx 0$ (P. Goode, private communication).

Given this, I followed the suggestion of Kuhn (1988) and fit the measured $a_2$ and $a_4$ to a constant divided by $\ell$, $a_i = \alpha_i/\ell$, using the present data set as well as a number of previously published measurements, using a weighted least-squares fit and assuming the published formal uncertainties for the input data points. Note from Figure 1 that the $a_2$ and $a_4$ qualitatively fit this functional form. The results are shown in Figure 3. The data show a great deal of internal consistency, even though the measurements were made by a number of different observers at different sites using different techniques, and suggest that there is a real solar cycle dependence in the even coefficients.

A possible explanation of the unusual result has been offered by Kuhn (1988, and these Proceedings). Kuhn used measurements of the brightness of the solar limb (Kuhn, Libbrecht, and Dicke 1985, 1987) to infer a latitude-dependent solar surface temperature, and from this calculated the $p$-mode splittings. These calculations are in fair agreement with the existing data, including a new point in 1987 (Jeffries et al., these Proceedings). We expect additional splitting and photometric data in the next few years will explore this model further. At present there is no obvious explanation for the measured latitude-dependent solar surface temperature.

I am grateful to the cast of observers at Big Bear Solar Observatory who diligently carried out these four months of observation, particularly Bill Marquette, Randy Fear, Curtis Odell and Matt Penn. I also thank Joergen Christensen-Dalsgaard, Jesper Schou, Phil Goode, and Wojtek Dziembowski for sharing their inversion results, and Steve Tomczyk for lending the Mt. Wilson splitting measurements prior to publication. The BBSO helioseismology data were analyzed in part using the facilities of the San Diego
Supercomputer Center, and the work was supported in part by NSF ATM-8604632.

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A WAY TO EXAMINE ROTATIONAL SPLITTINGS WITHOUT EXPLICIT FORWARD OR INVERSE COMPUTATIONS

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ABSTRACT

I derive and demonstrate simple but useful techniques for extracting information about the Sun’s internal rotation from the observed frequency splittings of solar acoustic oscillations. The techniques do not require any formal forward or inverse computations and yet are significantly revealing about the radial and latitudinal variations of solar angular velocity reflected by the data. Simple linear combinations of the familiar rotational splitting coefficients \(a_1(L), a_3(L), a_5(L)\), where \(L = L(L+1)\) and \(l\) is the angular degree of an oscillation mode, allow one to gain an impression of how any chosen co-latitudinal average of the angular velocity varies with depth, or of the radial variation of angular velocity at any particular co-latitude. The techniques are easy to apply and I illustrate them using the data of Brown and Morrow; they are equally applicable to data published by others. For the Brown and Morrow data, the methods reveal a lack of a substantial radial gradient of angular velocity in the convection zone, and a reversal in the sign of the radial gradient of angular velocity at about 30 degrees latitude at depths near the base of the convection zone. When subject to more detailed forward and inverse analysis, these data have also suggested such characteristics of the solar internal angular velocity. This lends credence to the methods described here.

Keywords: Solar rotation, frequency splittings, analysis

1. MEASURED AND THEORETICAL COEFFICIENTS

Observers extract rotational frequency splittings by fitting sigmoid features in \(m - \nu\) (azimuthal order-frequency) power spectra of the Sun’s acoustic oscillations. They expand their splittings in ordinary Legendre polynomials (e.g. Ref 3):

\[
2\pi \Delta \nu_{lm} = L \sum_{i=0}^{2} a_{2i+1}(L)P_{2i+1}(-m/L),
\]

where only the odd coefficients \((a_1, a_3, a_5)\) are needed to describe frequency changes induced by the advective effects of rotation. Some observers use \(m/l\) in lieu of \(m/L\). The difference between the two is negligible for the current application, and in any event, coefficients of one expansion are easily transformed into those of the other expansion. Figure 1 recalls how \(\nu\) depends on \(m\) for no rotation, rigid rotation and latitudinal differential rotation, and it also shows which of the expansion coefficients are relevant for each case. In effect, an \(m - \nu\) spectrum for a given degree \(l\) contains information about the latitudinal variation of angular velocity in a particular depth range of the solar interior. To derive appropriately informative combinations of the measured coefficients for rotational frequency splittings \((a_1, a_3, a_5)\), one must first relate the coefficients to theoretical expressions.

![Figure 1](image)

If a theorist assumes a simple, but reasonably general form for the Sun’s angular velocity as a function of radius \(r\) and co-latitude \(\theta\):

\[
\Omega(r, \theta) = \sum_{i=0}^{2} \Omega_i(r) \cos^{2i} \theta,
\]


---

then in an asymptotic limit, he/she may express the frequency splittings as

$$2\pi \Delta \nu_{lms} = -m \sum_{n=0}^{2} R_{als} \Theta_{lms}. \quad (3)$$

where

$$R_{als} = \frac{\int_{0}^{R_{t}} \Omega_{l}(r) \left( \frac{\partial \eta_{nl}(r)}{\partial r} \right) \rho_{o}(r) r^{2} dr}{\int_{0}^{R_{t}} \left( \frac{\partial \eta_{nl}(r)}{\partial r} \right) \rho_{o}(r) r^{2} dr} \quad (4)$$

and

$$\Theta_{lms} = \frac{\int_{0}^{1} \cos^{2} \theta \left( \frac{\partial \rho_{o}(r)}{\partial \theta} \right)^{2} d\cos \theta}{\int_{0}^{1} \left( \frac{\partial \rho_{o}(r)}{\partial \theta} \right)^{2} d\cos \theta}. \quad (5)$$

In these equations $L^{2} = (l + 1)$, $\rho_{o}(r)$ is the density for a spherically symmetric, equilibrium solar model, and $\eta_{nl}(r)$ and $\eta_{n}(r)$ are the radial and horizontal eigenfunctions of radius.

The co-latitudinal integrals (Eq 5) may be evaluated analytically; they are expressible in terms of even powers of $m/L$. Distributing another factor of $-m/L$ through Eq 3 produces a polynomial with $L$ times the odd powers of $-m/L$, which is easily written in terms of the odd Legendre polynomials. By this means, the odd splitting coefficients of the observer's expansion (Eq 1) may be related directly to the radial integrals of Eq 3. Knowledge of these relationships seeds the fruits of the impending preliminary analysis.

Before proceeding to show why this is so, one must note in passing that, to the distress of theorists, Brown and Morrow extracted their data by averaging over the splittings for modes of all radial orders $n$ that appeared in a chosen frequency range of the $m - \nu$ spectrum for a particular degree $L$. Therefore, the $R_{als}$ should be averaged in an appropriate fashion over radial order, and Eqs 3-4 should be suitably re-expressed (see Refs 1-2 for details). For the current purposes, however, it is sufficient to imagine that this has been done for the radial kernels in Eq 4, and thus that the averaged $R_{als}$ become $\bar{R}_{ls}$ in Eq 3:

$$2\pi \Delta \nu_{lms} = -m \sum_{s=0}^{2} \bar{R}_{ls} \Theta_{lms}. \quad (6)$$

After the co-latitudinal integration, it is straightforward to show that

$$a_{1}(L) = \bar{R}_{00} + \frac{1}{5} \bar{R}_{01} + \frac{3}{35} \bar{R}_{11}$$

$$a_{3}(L) = -\frac{5}{7} a_{1}(L) - \frac{1}{15} \bar{R}_{22}$$

$$a_{5}(L) = \frac{1}{21} \bar{R}_{32}. \quad (7)$$

Inverting the above system of equations gives

$$\bar{R}_{00} = a_{1}(L) + a_{3}(L) + a_{5}(L)$$

$$\bar{R}_{01} = -5a_{5}(L) - 14a_{3}(L)$$

$$\bar{R}_{11} = 21 a_{5}(L). \quad (8)$$

Equations 7 and 8 enable the development of the techniques of preliminary analysis described below.

2. TECHNIQUES FOR SIMPLE ANALYSIS

2.1 Latitudinal Averages

To derive the linear combination of odd coefficients whose variation with $L$ reflects the radial variation of a co-latitudinal average of angular velocity, one first performs the desired average on the general form for the angular velocity (Eq 2). Use of this averaged velocity $\bar{\Omega}_{l}(r)$ in Eq 6 for the frequency splittings (where the $\Theta_{lms}$ are now all equal to one since the co-latitudinal dependence has been averaged out) gives an equation in terms of the $\bar{R}_{ls}$ that is transformable via Eqs 8 to a combination of the frequency splitting coefficients. Making this combination with the data allows consideration of what the data are saying about how the chosen co-latitudinal average of angular velocity varies with depth.

As an illustration of this technique, one may show that $a_{1}(L)$, as defined in Eq 1, reflects the radial variation of a $\sin^{3} \theta$-weighted co-latitudinal average of the angular velocity. For this case the latitudinally-averaged velocities are:

$$\bar{\Omega}_{l}(r) = \Omega_{l}(r) \int_{0}^{R_{t}} \frac{\sin^{3} \theta} {\sin^{3} \theta} d \theta$$

$$= \frac{3}{(2s+1)(2s+3)} \Omega_{l}(r) \quad (9)$$

By using these velocities in Eq 6, the corresponding frequency splittings in terms of the $\bar{R}_{ls}$ become

$$2\pi \Delta \nu_{lms} = -m \sum_{s=0}^{2} \frac{3}{(2s+1)(2s+3)} \bar{R}_{ls} \Theta_{lms}. \quad (10)$$

$$= -m \left[ \bar{R}_{00} + \frac{1}{5} \bar{R}_{01} + \frac{3}{35} \bar{R}_{11} \right]$$

Now using Eqs 8 to substitute for the $\bar{R}_{ls}$ reveals that the appropriate data combination to reflect the depth variation of this weighted average is just $a_{1}(L)$. Note that this result depends on the form for the expansions of both the frequency splittings (Eq 1) and the angular velocity (Eq 2). For example, if the angular velocity function had been expressed in terms of even Legendre polynomials $P_{2n}(\cos \theta)$ instead of powers of $\cos^{2} \theta$, then the $L$-variation of the coefficient of $P_{l}(-m/L)$ ($a_{1}(L)$ in Eq 1) would have reflected the radial changes in a simple, equal-area weighted average (i.e. $\sin \theta$-weighted average) of angular velocity. With the given expansion for angular velocity (Eq 2), however, the radial variations of this average are reflected by the $L$-variations of the following combination of the splitting coefficients:

$$C_{2}(L) = a_{1}(L) - \frac{2}{3} a_{3}(L) + \frac{8}{15} a_{5}(L) \quad (11)$$

For the data of Brown and Morrow (reproduced elsewhere in these proceedings; Fig 1 in Ref 1), whose $L$-range is 15-60, both $a_{1}(L)$ and the combination $C_{2}(L)$ are quite constant with $L$, indicating that neither of these latitudinal averages varies much with depth. There does, however, appear to be a slight tendency for $a_{1}$ to decline at the lowest $L$ values and for $C_{2}$ to increase there. Because the $a_{1}$ average is weighted at lower latitudes, this provides subtle evidence that angular velocities tend to decrease slightly more with depth at low latitudes than they do at higher latitudes. Such a conclusion based on this evidence would be very uncertain given the error bars on the data. Perhaps the more obvious aspect to note is that the flat distribution...
of the splittings with $L$ means that the average velocities above the convection zone are not very different from those below the convection zone, and thus rapid rotation of the radiative interior is ruled out, at least to the depth of the $L = 15$ turning point ($4R_0$).

2.2 Radial Variations at Any Latitude

In principle, the recipe described in the previous section allows examination of the frequency splittings relevant to any weighted co-latitude average of the angular velocity. Increasing the powers of $\sin \theta$ for the weighting function would concentrate the average progressively toward the equator; weighting with powers of $\cos \theta$ would emphasize the rotation at higher latitudes. Extrapolating from this idea leads to the concept of a weighting function which selects a particular co-latitude $\theta_0$. Rather than designing weighting functions, one may simply choose a co-latitude and proceed to determine the combination of splitting coefficients which reflects the radial variation of angular velocity at that co-latitude.

Equations 5 and 6 reveal that for a particular co-latitude $\theta_0$ the $\Theta_{in}$ reduce to appropriate powers of $\cos \theta_0$, and thus the frequency splittings become:

$$2\pi \Delta \nu_{(m)}(\theta_0) = -m \sum_{r=0}^{2} \cos^2 \theta_0 R_{\ell s}$$

(12)

Using Eqs 8 to substitute for the $R_{\ell s}$ in terms of the odd coefficients $a_1$, $a_2$ and $a_3$ gives the appropriate combination of frequency splittings to reflect the depth variation of angular velocity at the chosen co-latitude:

$$C_{eq}(L) = \sum_{i=0}^{2} c_{i+1}(\theta_0) a_{i+1}(L),$$

(13)

where

$$c_1 = 1$$

$$c_2 = \left[1 - 5 \cos^2 \theta_0\right]$$

$$c_3 = \left[1 - 14 \cos^2 \theta_0 + 21 \cos^4 \theta_0\right]$$

(14)

Note that for $\theta_0 = 90^\circ$, corresponding to the solar equator, the relevant combination is the familiar sum of the antisymmetric coefficients:

$$C_{eq}(L) = a_1(L) + a_3(L) + a_5(L)$$

(15)

The table displays the combination coefficients $c_{i+1}(\theta_0)$ for co-latitudes at 15° intervals between the equator and pole. Note that the tabulated coefficients depend on expressing the frequency splittings as an expansion in Legendre polynomials of $-m/L$. However, using polynomials in $-m/L$ makes no difference given the asymptotic form of the kernels used to derive the combinations. Thus these techniques are directly applicable to splitting data published by others (Ref 3-5). Splittings expressed in terms of powers of $(-m/L)$ or $(-m/L)$ (Ref 6-7) require conversion to Legendre coefficients before the derived combinations are of any use.

Figure 2 displays the data combinations for 3 co-latitudes ($30^\circ$, $60^\circ$ and $90^\circ$) using the Fourier Tachometer data of Brown and Morrow (Ref 8). At $L$ values roughly corresponding to the base of the convection zone (i.e. $L = 40$), the data seem to imply a reversal of a radial gradient of angular velocity at co-latitude $60^\circ$ (latitude $30^\circ$). The angular velocity decreases inward at latitudes below $30^\circ$ and increases inward at higher latitudes. Further inspection reveals the conspicuous lack of slopes in the higher $L$ portions

<table>
<thead>
<tr>
<th>Co-Latitude (Eq to Pole)</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$90^\circ$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$75^\circ$</td>
<td>1.000</td>
<td>0.665</td>
<td>0.155</td>
</tr>
<tr>
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</tr>
<tr>
<td>$45^\circ$</td>
<td>1.000</td>
<td>-1.500</td>
<td>-0.750</td>
</tr>
<tr>
<td>$30^\circ$</td>
<td>1.000</td>
<td>-2.750</td>
<td>2.313</td>
</tr>
<tr>
<td>$15^\circ$</td>
<td>1.000</td>
<td>-3.665</td>
<td>6.219</td>
</tr>
<tr>
<td>$0^\circ$</td>
<td>1.000</td>
<td>-4.000</td>
<td>8.000</td>
</tr>
</tbody>
</table>

Table: The coefficients for linear combinations of $a_1$, $a_3$ and $a_5$, ($c_{n+1}(\theta_0)$ in equations 13 and 14) that reflect the radial variations of the angular velocity at co-latitude $\theta_0$ of any of the displayed combinations. This suggests a lack of radial gradients in the convection zone.

Having made such combinations of the data, one may then compare them with what is expected for particular choices of the angular velocity function. This also provides a test of the validity of the cited combinations. In Figure 2, the straight lines at each co-latitude represent a model with the surface rotation pervading the solar interior; the curved lines assume angular velocity constant on cylinders throughout the Sun. Both models employ the surface rotation that is estimated directly from the high-L data: the estimate lies between the magnetic and plasma rates of Snodgrass (Refs 9-10) at all latitudes. The figure shows that the combinations for pervasive surface-like rotation are constant with $L$ at every latitude, which accurately reflects the complete lack of radial gradients in this model. The positive radial gradients inherent in the cylindrical model do indeed show up as $L$-dependent curves whose downward slope from high to low $L$ decrease with latitude - a reflection of the actual behavior of this profile of angular velocity. At the equator, the cylindrical curve reflects an overall decrease in angular velocity of about 10% from the surface to the base of the convection zone. Thus the data are certainly capable of resolving the difference between surface-like and cylindrical differential rotation in the convection zone.

3. CONCLUDING REMARKS

The techniques presented here for the preliminary analysis of frequency splitting data that contains information about the variations of solar angular velocity with depth and latitude can be quite useful, both to those in the field, and for others who would like to obtain a quick preliminary impression of what published data are saying. They also offer good points of departure for the forward modelling of frequency splittings (Refs 1-2).

For the data of Brown and Morrow, the preliminary analysis casts doubt on the data's amenability to a rotational model with angular velocity constant on cylinders in the convection zone. This model was motivated by numerical simulations of global convection (e.g. Ref 11). Indeed the results of Figure 2 might well lead one to suspect the correctness of the model proposed by Brown and Morrow (Ref 8) that has surface rotation in the convection zone and rigid rotation for the interior at a rate that is intermediate between the surface equatorial and poloidal. Though forward and inverse analysis (Ref 1,12) have sustained these initial impressions, thereby lending credibility to the use of the simple techniques presented here.
This work was done while I was at the University of Colorado and the High Altitude Observatory in Boulder, Colorado. There I benefited from discussions with Tim Brown, Peter Gilman and Ellen Zweibel. The paper itself was prepared at the Institute of Astronomy in Cambridge. I gratefully acknowledge Bill Merryfield and Neil Balmforth for their invaluable assistance with the manuscript.

REFERENCES

MEASUREMENT OF THE ROTATIONAL FREQUENCY SPLITTING OF THE SOLAR FIVE-MINUTE OSCILLATIONS FROM MAGNETO-OPTICAL FILTER OBSERVATIONS

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ABSTRACT

Observations of the solar five-minute oscillations in the photospheric velocity field were obtained during the summer of 1984 at the 60-foot solar tower of the Mt. Wilson Observatory with a magneto-optical filter. The magneto-optical filter employs magneto-optical effects in an atomic vapor to isolate narrow bandpasses in alternate wings of a spectral line. Time series of full disk velocity images having a resolution of about 10 arcseconds and a noise level of 15 m/s/pixel were obtained on 92 days between the months of May and September of 1984. A subset of two time series from this data of 18 and 19 days duration having a total of 25744 doppler images were analyzed to provide estimates of the rotational frequency splitting for spherical harmonic degrees between 5 and 120. The results of this analysis indicate a decrease in the rate of solar rotation with increasing depth inside the sun. Also, a decrease in the rate of differential rotation with increasing depth is observed.

Keywords: Rotational Frequency Splitting, Magneto-Optical Filter, Five-Minute Oscillations.

1. INTRODUCTION

During the past two decades, the solar 5-minute oscillations have emerged as a powerful probe of the solar interior. The acoustic oscillations convey information on the structure and internal rotation as functions of both depth and latitude. Measurements of the oscillation frequencies bear directly on a wide range of astrophysical problems including the structure and evolution of the sun, the nature of solar flows and dynamos, the transport of solar angular momentum, the neutrino problem, and can help constrain theories of General Relativity. Several excellent reviews of the rapid progress in the field of helioseismology have been given recently (Ref. 1-3). We present here the results of recent observations which allow the determination of the rotational rate as a function of depth and latitude over a large fraction of the solar radius.

2. INSTRUMENT

All of the data for this study were obtained at the 60-foot solar tower of the Mt. Wilson Observatory (Ref. 4). The observing system at the 60-foot tower has been described previously by Rhodes (Ref. 5-7) and more recently by Tomczyk (Ref. 8). The telescope uses a coelostat to direct the solar beam vertically down a light shaft to one of several objectives. For this study, a double lens of 3 m focal length, stopped down to an aperture of 5 cm was employed. A portion of the focusing beam was diverted to an autoguider characterized by a frequency response to about 10 Hz. and long term drift of less than 20 arcseconds over the course of the day.

The solar image was shuttered in the focal plane and fed into the magneto-optical filter (MOF) which served as the doppler analyzer for the observations. The MOF has been developed for solar observations at the University of Rome by A. Cacciani, and its operating principles have been described previously (Ref. 9-11). Figure 1 is a schematic diagram of the filter with the MOF components shown to the left and the corresponding evolution of the spectrum shown to the right. Functionally, the MOF can be considered as two units: the wing filter and the wing selector. The wing filter consists of an atomic vapor in a longitudinal magnetic field between crossed linear polarizers. The atomic vapor modifies the induced linear polarization in the wings of a absorption line through circular dichroism and circular birefringence allowing a portion of that light to pass through the second, crossed polarizer. The continuum light is unaffected by passage through the vapor and is extinguished by the exit polarizer.

The wing selector unit consists of a rotatable quarter-wave plate and a second vapor in a longitudinal magnetic field. The quarter-wave plate transforms the linearly polarized light leaving the exit polarizer into left or right circularly polarized light depending on the orientation of the wave plate axis. Inverse Zeeman absorption in the second vapor will then selectively remove either the red or blue transmission peak.

The position and width of the transmission peaks depend on both the optical depth of the atomic vapor and the strength of the magnetic field. For these observations,
atomic vapors of sodium were used in 1 kG magnetic fields. A prefilter of 25 Å allowed passage of both the Na D-lines. The instrumental profiles for the observations were characterized by transmission peaks 75 mÅ wide and separated from the rest wavelengths of the D-lines by ±1 Å.

Solar full disk images were recorded with a General Electric Model D, Charge Injected Device (CID) camera. The CID is a two-dimensional, solid-state imaging device with 244 rows by 248 columns. Each pixel is rectangular with dimensions of 47 by 35 microns. The CID system was constructed by R.S. Aikens of Photometries Ltd., who has given a detailed description of a similar CID system (Ref. 12). Although the solar image did not rotate throughout the day, an image rotator was used to align the solar rotation axis with the rows of the CID camera to an accuracy of a few tenths of a degree. Imaging the entire sun onto the camera resulted in a spatial sampling of approximately 10 arcseconds per pixel.

The wing images required an exposure time of 3 1/3 seconds each, yielding a signal to noise ratio better than 500 to 1. The corresponding noise level in the doppler images was typically 15 m/s/pixel. Image pairs were acquired every 40 seconds.

The red and blue images were registered to a common center following corrections for camera bias and gain variations and CID crosstalk signal. Doppler images were computed by dividing the sum of red and blue images by their difference. This produces a quantity which is insensitive to intensity variations and proportional to velocity to about 5 percent rms over the range of solar rotation. The linearity of the MOF is illustrated in Figure 2 where the observed doppler signal is plotted against the sine of the solar central meridian angle for equatorial pixels. A linear calibration to velocity units was performed by normalizing the daily average of the observed solar equatorial rotation rate to published values for photospheric equatorial rotation.

### 3. OBSERVATIONS

Observations of the solar velocity field were obtained throughout the summer of 1984. Figure 3 shows the observation history for that summer where the number of cloud free hours of observation is shown for each day from May to September. The two stretches of data from 16 June to 4 July and from 29 July to 13 August, shown as the cross-hatched areas of Figure 3, had the best coverage during the summer and were the only data utilized for the present measurement. During the period from 16 June to 4 July, 1248 doppler images were obtained over 19 days resulting in a duty cycle of .3, while during the period from 29 July to 13 August, 13260 doppler images were obtained over 16 days resulting in a duty cycle of .38.
4. ANALYSIS

The spherical harmonic decomposition of the doppler images was computed for all azimuthal orders for degrees from 0 to 195. The decomposition method of Brown (Ref. 13) was employed. This involved the bilinear interpolation of the doppler images onto a grid linear in longitude and sine latitude, followed by the computation of a Fast Fourier Transform in longitude and an Associated Legendre transform in latitude. The temporal transforms were computed separately for the two data sets, and the frequency splitting analysis used the average of the power spectra from the two runs for spherical harmonic degrees below 46, while the August run was used exclusively in the analysis for degrees above 46.

The observed spatio-temporal distribution of power for the 5-minute oscillations is shown in the $l$-$\nu$ diagram of Figure 4. There the power spectra for each degree have been averaged over the azimuthal order after correction for solar rotation. In the picture, the spherical harmonic degree varies linearly along the horizontal axis between 5 and 132 while the temporal frequency varies linearly along the vertical axis from 2.57 to 3.74 mHz. Note that individual modes can be distinguished over the entire diagram.

Figure 4. $l$-$\nu$ diagram showing the spatio-temporal distribution of power for the 5-minute oscillations. The spherical harmonic degree varies linearly along the horizontal axis between 5 and 132 while the temporal frequency varies linearly along the vertical axis from 2.57 to 3.74 mHz.
The observed variation of mode frequency as a function of azimuthal order $m$ is shown in the $m$-$\nu$ diagrams of Figure 5. There the $2l+1$ power spectra for a given degree are shown for degrees 5, 10, 20 and 40. The azimuthal order is shown decreasing vertically from $l$ to $-l$, and the frequency increasing from left to right from 2.57 to 3.74 mHz. The slanting of the ridges due to solar rotation is clearly visible.

The frequency splitting was estimated for each degree with a cross-correlation technique. Power spectra for each azimuthal order were cross-correlated with an average spectrum obtained by collapsing the different azimuthal orders after shifting by an estimate of the frequency splitting. The cross-correlations were limited to the region from 2.57 to 3.74 mHz. The frequency shifts were estimated from the correlation curves and were parameterized in a Legendre series, following Duvall, et al. (Ref. 14):

$$\nu(l,m) - \bar{\nu}(l) = \sum a_l P_l(-m/L)$$

where $l$ is the spherical harmonic degree, $m$ is the azimuthal order, $\bar{\nu}$ is the frequency averaged over $m$ and radial order, the $P_l$ are the Legendre polynomials and $L = \sqrt{l(l+1)}$. Legendre coefficients were computed in this manner up to fifth-order for degrees above 9, and to first order for degrees 5 to 9. After the initial cross-correlation, the collapsed spectrum was revised according to the fit curve, and the process was iterated until the Legendre $a_1$ coefficient repeated to within .1 percent or 5 iterations were completed. For high degrees, the peaks in the power spectra begin to overlap and reduce the contrast in the correlation curves. For this reason, the analysis was not extended to degrees beyond 120.

5. RESULTS

The results of this study are summarized in Figure 6 where the Legendre fit coefficients are plotted as a function of degree. Several interesting features are present in the plots. First, the $a_1$ coefficient which represents solid-body rotation, shows a monotonic decrease with decreasing degree below degree of around 80. The $a_3$ coefficient, which contains information about differential rotation, shows a significant decrease with decreasing degree for degrees below around 50. The $a_5$ coefficient is approximately independent of degree and is consistent with the value of that coefficient at the solar surface. The even coefficients which contain information about pole-equator variations in the solar structure are significantly different from zero.

A full inversion of the data is required to relate these frequency splittings to the variation of solar rotation with depth and latitude. The results of a recent inversion of these data (Ref. 15) indicate the disappearance of differential rotation below the base of the convection zone and a small decrease in the rotation rate for equatorial and mid-latitudes.

Figure 5. $m$-$\nu$ diagrams showing the variation of mode frequency as a function of azimuthal order. Temporal frequency is increasing from left to right from 2.57 to 3.74 mHz and azimuthal order is decreasing vertically from $l$ to $-l$.
Figure 6. Plots of the Legendre fit coefficients vs. spherical harmonic degree.
Figure 7. Plots of the difference of the Legendre coefficients obtained with a peak finding algorithm minus those obtained with the cross-correlation algorithm (shown in Figure 6).
6. SYSTEMATIC ERRORS

Since the spherical harmonic functions are orthogonal over the entire solar surface, observing only the visible hemisphere of the sun limits our ability to isolate individual modes. This causes the spectra to be contaminated by power from modes having nearby degree and azimuthal order. These spatial sidelobes can introduce systematic errors into the frequency measurements, especially for methods such as cross-correlations which utilize all of the power in the 5-minute band. To investigate the sensitivity of the cross-correlation analysis to systematic effects, an alternate method was also implemented. The frequencies of individual peaks in the power spectra were estimated by computing the first moment of the power in a window 1500 Hz wide, centered on the expected peak position, while neglecting power below 3.5 times the noise level. Legendre fits to the rotation curves were computed and the process iterated as before.

Figure 7 shows the difference of the Legendre coefficients obtained with the peak finding algorithm minus those obtained with the cross-correlation algorithm. The large differences for degrees greater than 110 is an artifact of the peak finding algorithm. Departures from zero for the $a_1$ and $a_3$ coefficients indicate an increased sensitivity of these coefficients to systematic errors, while the agreement of the even coefficients supports the claim (Ref. 16) that these coefficients are less susceptible to systematic errors. The magnitude of these differences is not sufficient to invalidate the conclusions of this study.

Without a deeper understanding of the source of these errors, it is difficult to ascribe these differences to either method. A comparison of measurements of Legendre coefficients of intermediate degree by various observers (Ref. 13, 14, 16, 17) employing different instruments and analyses, shows offsets and trends which are not mutually consistent within the quoted experimental errors. One is led to the conclusion that the observations of the frequency splitting of solar oscillations have advanced to a level where the systematic errors of the $a_1$ coefficient are significantly larger than the random errors.

7. REFERENCES

DIFFERENTIAL ROTATION IN THE SOLAR INTERIOR

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ABSTRACT

We present an estimate of the angular velocity in the outer half of the Sun, as a function of depth and latitude. This was obtained by applying inversion by means of optimally localized averages to rotational splitting coefficients observed by Ken Libbrecht at BBSO. The results seem to indicate that the surface differential rotation persists through the convection zone; beneath the convection zone there appears to be a transition, which occurs within our resolution width of about 0.1°, to solid-body rotation. We were unable to perform the inversion in the core, due to the lack of adequate data for modes of low degree.

Keywords: Solar differential rotation, helioseismic inversion.

1. INTRODUCTION

One of the areas in which helioseismology has provided us with significant results is in the determination of the rotation rate in the interior of the Sun (see, for example, Refs. 1, 2, 3). Most of these determinations however have only been able to give information about a fairly small part of the Sun or only about the equatorial rotation rate.

In the present study we have been able to perform an inversion for the latitude dependence of the rotation rate over a fairly large range of r and for the general features of rotation over about half the solar radius. This has been possible due to the smaller errors in the data set which was used for this inversion. Furthermore, in contrast to previous observations, splittings have been determined for individual modes, rather than as averages over frequency at fixed degree \( \ell \); this leads to a considerable improvement in the resolving power of the inversion.

2. OBSERVATIONS

The methods used for making the observations and for the reduction of the data are described in detail by Libbrecht (Refs. 4, 5). The observations utilized a dedicated helioseismology telescope at Big Bear Solar Observatory. A full-disk Doppler image was produced every minute. Each image was then fitted to all spherical harmonics with \( \ell \leq 60 \) and azimuthal order \( m \in [-\ell, \ell] \), and for each \((\ell, m)\) the resulting coefficients, as functions of time, were Fourier transformed to produce power spectra from which the individual cyclic frequencies \( \nu_{\ell m} \) of the modes were determined. For each \((\ell, m)\) multiplet the coefficients \( a_{\ell m} \), \( \ell = 1 - 5 \), in the expansion

\[
\nu_{\ell m} = \nu_{\ell m, 0} + \frac{\ell}{2} \sum a_{\ell m} \ell^2 P_{\ell}^{m}(\cos \theta)
\]

were then determined by least squares fitting, together with the standard deviations \( \sigma_{\ell m} \) on the \( a_{\ell m} \), based on the residuals from the fit.

The observations were made in the period 26 March to 2 August 1986; 100 days of useful data or about 60,000 images were obtained. The analysis resulted in splitting coefficients for 715 modes with \( 5 \leq \ell \leq 60 \), \( 3 \leq n \leq 23 \) and 1500 \( \mu \text{Hz} < \nu < 4100 \mu \text{Hz} \). These data were subsequently used for the inversion.

3. ANALYSIS

The principles of the inversion procedures are described in detail in Ref. 3, and are only summarized here. By writing the angular velocity \( \Omega \) in the form

\[
\Omega(r, \theta) = \sum_{\ell m} \Omega_{\ell m} r^\ell \sin^m \theta,
\]

where \( r \) is the distance to the centre, \( \theta \) is co-latitude and \( \mu = \cos(\theta) \), it is possible to find kernels \( K_{\ell m} \) for each \((\ell, m)\) such that

\[
2\pi \nu_{\ell m} = \sum_{\ell' m'} K_{\ell m}(r) \nu_{\ell' m'} dr.
\]

A difficulty with the expansion (2) is that all terms in equation (3) make comparable contributions to the splitting coefficients. This results in a strong correlation between the inversions for the different terms and thus between the errors in the inversions. This difficulty may be avoided by using instead of equation (2) the expansion

\[
\Omega^{(1)}(r) = \left[ \mu^2 - \frac{1}{5} \right] \Omega^{(1)}(r) + \left[ \mu^2 - \frac{2}{3} \mu^2 + \frac{1}{21} \right] \Omega^{(2)}(r),
\]

where the \( Q^{(1)} \) are linear combinations of the \( a_i \). To this expansion corresponds the expansions of the splittings

\[
2\pi a_{2j_2}(n, j) = \sum_{r_j} \int_0^R K^{(1)}(r, j) Q^{(1)}(r) dr
\]

(5)

where, asymptotically in the limit of large \( j \), the terms with \( s > j \) vanish. Hence the expansion (4) minimizes the coupling between the individual terms.

To carry out the inversion, equation (5) is rewritten as

\[
\sum_{r_j} \int_0^R K^{(1)}(r, j) Q^{(1)}(r) dr = 2\pi a_{2j_2}(n, j)
\]

(6)

Since there are only terms with \( s > j \) on the right-hand side in this expression it is possible to perform the inversions recursively: as \( s_{\text{max}} = 2 \) in the present case the equation for \( a_2 \) contains no contributions from terms of higher order in the expansion of the potential and can be inverted immediately, to obtain \( Q^{(1)}(r) \). In the inversions for the terms of lower order the integrals on the right-hand side of equation (6) are evaluated by using the \( Q^{(1)}(r) \) already determined.

Each inversion is done with the method of optimally localized averages (e.g. Ref. 6, 7) in which for each \( r_2 \) one finds an averaging kernel

\[
K(r; r_2) = \sum_{l} c_l(r_2) K_l(r)
\]

(7)

where \( l \) stands for the combination \( (n, j) \), and the kernel \( K_l \) is one of the kernels \( (1)K_l(r) \). The averaging kernels are determined such that they minimize

\[
12\cos u \int_0^R f(r-r_2) K(r; r_2)^2 dr + \lambda \sin u \int_0^R c_l(r_2) c_j(r_2) E_{lj} dr
\]

(8)

under the constraint

\[
\int_0^R K(r; r_2) dr = 1.
\]

(9)

Here \( E_{lj} \) is the covariance matrix for the observations and \( \lambda \) is a constant chosen in such a way as to make the two terms of about the same size; specifically

\[
\lambda = \left[ \frac{1}{M} \sum_{r_2} E_{ll} \right]^{-1},
\]

(10)

where \( M \) is the number of modes. Finally \( u \) is a parameter determining the trade-off between resolution (specified by the first term) and errors (specified by the second term). We have assumed that the errors in the observed splitting coefficients are uncorrelated, so that

\[
E_{lj} = \sigma(A_j)^2 \delta_{lj}.
\]

We now obtain the estimate

\[
\tilde{A}(r_2) = \sum_{l} c_l(r_2) A_l = \int_0^R K(r; r_2) \tilde{Q}(r) dr
\]

(11)

of the expansion coefficient \( Q^{(1)} \) being considered; here the \( A_l \) represent the reduced observed splitting coefficients \( 2\pi a_{2j_2}(n, j) \) (cf. equation (6)).

The standard error on \( \tilde{A}(r_2) \) can be estimated as

\[
\sigma^2[\tilde{A}(r_2)] = \sum_{l} c_l^2(r_2) \sigma(A_l)^2.
\]

(12)

To characterize the location of the determination of \( \tilde{A} \) for a given \( r_2 \) we use the centre of gravity

\[
\bar{r} = \int_0^R r K(r; r_2) dr.
\]

(13)

resulting from the inversion is correct to first order in a Taylor expansion of the actual rotation curve (cf. Ref. 7).

Unlike Brown et al. (Ref. 3) we use unnormalized kernels. This has important effects on the meaning of the trade-off parameter \( u \). We may write the minimization problem in equations (8) and (9) as that of minimizing

\[
\cos u \sum_{lj} c_l c_j + \lambda \sin u \sum_{l} \sigma(A_l)^2 c_l^2,
\]

(14)

under the constraint

\[
\sum_{lj} c_l c_j = 1.
\]

(15)

Here

\[
S_{lj} = 12\cos u K(r_2) K_l(r_2) K_j(r_2) dr
\]

(16)

and

\[
\beta_l = \int_0^R K_l(r_2) dr.
\]

(17)

The \( \beta_l \) for the inversions for \( Q^{(1)} \), \( Q^{(1)} \) and \( Q^{(1)} \) tend to 1, -1/5 and 1/21, respectively, as \( u \to \infty \). From the normalization condition in equation (15) follows that, very roughly, the \( c_l \) increase as \( |A_l|^{-1} \) with the order of the expansion coefficient increases. Since \( S_{lj} = \beta_l \), whereas \( \lambda \sigma(A_j)^2 \) is essentially fixed by the definition (10) of \( \lambda \), it follows from equation (14) that with decreasing \( \beta_j \) at fixed \( u \) the importance of the second term increases. Thus the trade-off is shifted towards limiting the error, given by equation (12), in \( \tilde{A} \).

4. RESULTS

We have performed the inversion with different values of the trade-off parameter \( u \); in each case the same value of \( u \) was used for the inversions for \( Q^{(1)}, Q^{(1)} \) and \( Q^{(1)} \). The original kernels \( K_l \) extend from the surface to an inner turning point \( r_t \), which according to asymptotic theory is determined by \( r_t c = 2\pi L \), where \( c \) is the adiabatic sound speed and \( L = L + \frac{1}{2} \). Among the modes in the present dataset, the outermost value of \( r_t \) is about 0.9R, which is fairly close to, but somewhat outside, the endpoints of the intervals where inversion was possible. Thus, perhaps not surprisingly, the upper limit of depth resolution is determined by the most shallowly penetrating mode, i.e. the mode with the highest degree and the lowest frequency.

For the purposes of computing the integrals in equations (5), displaying the results of the inversion, and computing the splitting coefficients, the \( Q^{(1)} \) have been set equal to their values at the innermost point where inversion was possible inside that point; outside the outer limit \( r_{\text{outer}} \) of inversion we used linear interpolation between \( r_{\text{outer}} \) and the surface. Except where otherwise noted, the surface rotation was assumed to be given by the plasma rotation rate as determined by Doppler measurements (Ref. 8):

\[
\Omega(R, 0) \approx \frac{(453.8 - 54.6u^2 - 75.4u) \text{ mHz}}{2\pi}
\]

(18)

The resulting rotation curves for latitudes 0°, 45° and 90° are shown in Figure 1, both for \( u = 0.1 \) and \( u = 1.0 \). To indicate the surface values the outermost sections of the linearly interpolated curves are shown.
Figure 1. Angular velocity inferred from inversion with trade-off parameter $u = 0.1$ (a) and 1.0 (b). The dashed lines indicate 1-$\sigma$ error limits. The surface values (Ref. 8) are indicated by the straight lines starting at $r/R = 1$. Results are shown for the entire range in $r$ where inversion for the lowest expansion coefficient $\ell_1^{(1)}$ (cf. equation (4)) was possible. For the higher expansion coefficients the range of inversion was more limited; the $\ell_j^{(1)}$ have been assumed to be constant inside, and interpolated linearly to the surface outside, this range. In particular the apparent constancy of the polar rotation rate interior to $r = 0.6 \beta$ has no significance.

The indicated errors are 1-$\sigma$ errors, based on standard errors for the individual $\ell_j^{(1)}$ calculated from equation (12), using the standard errors on the $a_j$ provided by Libbrecht. It is evident that the effect of increasing the trade-off parameter is to decrease the error and degrade the resolution of the inversion. Nevertheless the gross features of the results are similar in the two cases.

Examples of the averaging kernels obtained in the inversions for $\ell_1^{(1)}$, $\ell_2^{(1)}$ and $\ell_3^{(1)}$ are shown in Figure 2, for $u = 0.1$. From this figure it is obvious that the resolution gets progressively worse as the order of the expansion in latitude increases. The reason for this effect, which is associated with the decrease in the size of the integrals of the kernels, was discussed at the end of Section 3. It tends to limit the standard error in the $\ell_1^{(1)}$ determined from the inversions, although there is still some increase in $\sigma(\ell_1^{(1)})$ with $s$.

From equation (4) follows that $\ell_1^{(1)}$ enters with highest weight in the polar rotation rate, which therefore is most strongly affected by the deterioration in resolution with increasing expansion order. Thus the apparent gradual rise in the polar rotation rate with increasing depth in the convection zone may not be real, but could be due to the limited resolution. Also the increase in error with expansion order is evident in the rather large error limits on the polar rotation rate.

In Figure 3 we have shown the observed $a$, and the $a$, inferred from the rotation rates obtained from the inversion with $u = 0.1$. Evidently the fit is not satisfactory. However it should be noticed that, in contrast to e.g. a least squares method, this inversion method does not seek explicitly to fit the observations. Furthermore, no information is obtained about the rotation in the outer parts of the Sun where the kernels are large, and which therefore dominate the splitting. In order to investigate this we have adjusted the surface values of $a_0$ and $a_1$ in the expansion (1) to get a good fit to the observations. The resulting $a_0$ are shown in Figure 4, where results are also given for $a_0$ and $a_1$. It is obvious from the plot for $a_0$ that this gives a considerable improvement in the fit. The resulting crude estimate of the surface rate is
Figure 2. Selected averaging kernels, based on inversions with $u = 0.1$ for $n_1$ (a), $n_2$ (b) and $n_3$ (c). The values of $r_0/R$ in a) are 0.37, 0.50, 0.60, 0.68, 0.75, 0.82 and 0.88; in b) 0.48, 0.58, 0.66, 0.73, 0.79, 0.84 and 0.88; and in c) 0.53, 0.63, 0.70, 0.78 and 0.85. The maximum value in c) of the kernel corresponding to $r_0/R = 0.85$ is 14.6. In all cases the actual number of points where the inversion was carried out was sufficiently high to utilize fully the resolving power of the kernels. Note the change in scale between the panels; this is caused by the increased width and the resulting decreased height of the kernels (cf. equation (9)) with increasing order of the term in the $n$-expansion.

\[
\Omega(R, \theta) = \frac{471 - 77\mu^2 - 75\mu^4}{2\pi} \text{ nHz.} \tag{19}
\]

Although no great weight can be attached to these values, it is interesting that the equatorial rotation rate is somewhat higher than both the plasma rate in equation (18) and the equatorial rotation rate of small magnetic features (Ref. 9), which gives $\Omega/2\pi = 462$ nHz.

Figure 3. Observed (*) expansion coefficients $a_i$ and computed values (°) for the inversion with $u = 0.1$ shown in Figure 1a. The coefficients are plotted against $\nu/L$, which determines the asymptotic position of the turning point. To show more clearly the behaviour of the computed $a_i$, the observed points with the largest deviations from the mean have been excluded.

As an estimate of the quality of the fit Figure 5 shows the normalized residuals $|a_i^{\text{obs}} - a_i^{\text{inv}}|/a_i$, where $a_i$ is the estimated standard error in the observed $a_i$. Evidently the fit is now very good; furthermore the distribution of normalized residuals is approximately Gaussian, with a standard deviation of 1.1, indicating that the errors quoted for the observations are realistic. Similar results are obtained for $a_3$ and $a_6$.

5. DISCUSSION

As can be seen from Figure 1, the rotation in the solar convection zone appears to be roughly independent of depth; this seems to be in conflict with several numerical calculations (e.g. Ref. 10), which rather suggest that rotation should be constant on cylinders. Close to, but possibly slightly below, the base of the convection zone there is a transition to roughly latitude-independent rotation. This transition happens over an interval that is no wider than the resolution of the inversion.

Our results are roughly consistent with those obtained by Brown et al. (Ref. 3), on the basis of observations reported by Brown & Morrow (Ref. 11), but we have been able to get an inversion over a broader range in $r$. It should also be noticed that an independent inversion of the same data, using a least squares technique, has been carried out by Dziembowski & Goode (cf. Ref. 5); the results are very similar to those obtained here.

Although the analysis gives no detailed information about the rotation in the outer 10 per cent of the solar radius, it is interesting that the results suggest that the equatorial value of $\Omega/2\pi$ near the surface is about 20 nHz higher than the observed plasma rate, or about 10 nHz higher than the rate for small magnetic features. This may be related to the finding by Hill et al. (Ref. 2) from inversion of high-degree sectoral modes that at the equator $\Omega/2\pi$ increases by about 20 nHz in the outer 3 Mm of the convection zone.
DIFFERENTIAL ROTATION IN THE SOLAR INTERIOR

Figure 4. Observed (×) and computed (○) expansion coefficients \(a_1\), \(a_2\), and \(a_3\). The computed values are for the for the inversion with \(u = 0.1\) shown in Figure 1a, except that the value of \(\alpha\) at the surface has been adjusted to reduce the systematic difference between observed and computed values.

To extend the region in the convection zone where inversion is possible, data for modes of higher degree are required. Thus it is encouraging that Libbrecht is now working on the analysis of modes with \(l\) up to 100. Splitting coefficients for higher degrees were also obtained by Jeffries et al. (Ref. 12). An estimate based on the positions of the turning points suggests that with data for modes with \(l \geq 100\) it should be possible to get an inversion out to about 0.9R. To carry out inversion closer to the centre than was possible here requires more, and in particular more accurate, data for modes of low degree.

Figure 5. Normalized residuals \(|a^{\text{obs}}_1 - a^{\text{com}}_1|/a_1\) between the observations and the computations shown in Figure 4a. Here \(a_1\) is the estimate of the standard error in the observations. Note that, in contrast to Figures 3 and 4, no points have been excluded from this plot.

Acknowledgements: We thank K. G. Libbrecht for putting his observations at our disposal before publication. The computations reported here were supported by the Danish Natural Science Research Council.

REFERENCES

Session 3

Ground-based networks and techniques

Chairman: Ph. Scherrer
THE CURRENT STATUS OF THE BIRMINGHAM SOLAR SEISMOLOGY NETWORK

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ABSTRACT

Some aspects of the performance of a network of solar seismology stations are presented. Duty cycles for 1986 and 1987 are given and examples of window functions and p-mode spectra are shown. Sideband powers down to 2% of peak power have been achieved for spans of 16 days and 3% for 64 days. Noise levels of 7 (ms)$^{-1}$ Hz$^{-1}$ and 13 (ms)$^{-1}$ Hz$^{-1}$ have been achieved in regions just above and below the p-mode spectrum.

A new station in Birmingham is discussed. Its importance as a testing and training station is outlined and plans to deploy replicas around the globe are described.

Keywords: three station network, unresolved sun, duty cycles, window functions, p mode fine structure, new stations.

1. INTRODUCTION

The spectrum of solar p-modes shows fine structure with frequency spacings of the order of 10 $\mu$Hz. Measurements obtained from any terrestrial site (other than near a pole) are regularly broken by night and hence produce spectra in which real solar frequencies have sidebands 11.57 $\mu$Hz (1/day) on either side of them. These artefacts make the interpretation of p-mode spectra much more difficult. Several groups are now setting up networks of instruments distributed around the globe in order to obtain measurements as nearly contiguously as possible (GONG (Ref 1.), IRIS (Ref. 2) and ourselves). It is the purpose of this paper to describe the current status of our network, the 3 existing stations, their duty cycles and the quality of p-mode spectra so far obtained. In addition a new testing and training station in Birmingham is discussed. This station will be replicated at least 3 more sites in the next few years.

2. DUTY CYCLES AND POWER SPECTRA

At present 3 instruments contribute data in the form of solar velocity residuals of the unresolved Sun to our network. All are variants of the potassium resonant scattering spectrometer described in Brookes et al (Ref. 3.), but are operated in different ways.

### Table 1

<table>
<thead>
<tr>
<th>Year/Site</th>
<th>Tenerife</th>
<th>Hawaii</th>
<th>Australia</th>
</tr>
</thead>
<tbody>
<tr>
<td>1986</td>
<td>284</td>
<td>70</td>
<td>236</td>
</tr>
<tr>
<td>1987</td>
<td>289</td>
<td>107</td>
<td>90</td>
</tr>
</tbody>
</table>

With reference to that table:

i) Izana, Tenerife - this instrument is manned throughout the year by IAC personnel, is bench mounted and fed sunlight via a coelostat.

ii) Haleakala, Hawaii - this instrument is capable of running for 8 days before an operator is required to adjust the coelostat which feeds it.

iii) Carnarvon, Australia - this instrument is equatorially mounted and housed in an air-conditioned astronomical dome. An operator is required to carry out 1-2 hr maintenance per week.

The duty cycles and data quality of the 3 stations for 1984 and 1985 sites have been reported elsewhere (Ref 4). Table 1 shows the 1986 and 1987 figures.

Figure 1 shows six window functions for data strings which are summarized in Table 2. The first three plots show the dramatic reduction in sideband power achieved by adding in data from all three sites for a 2 week period in August 1987. The final three plots show window functions for 2 month periods in 1984 and 1987.
It is interesting to note that the sidebands in Figure 1(e) are slightly stronger than those in 1(f) despite being produced from a data set with a slightly better fill. All three sites contributed to the 1987 data in Figure 1(f) whereas the 1984 results used for Figure 1(e) were obtained before the Australian Station was built. From Table 2 it can be seen that our lowest sidebands to date have 2% of the peak power in our best filled 16 day spectrum and 3% in the best 64 day spectrum.

As an example of the quality of p-mode spectra such data sets produce, Figure 2 shows the power spectrum in the region 1.5 to 5.25 mHz for the 1987 data set with window function 1(f). Of the three instruments the Hawaiian spectrometer has the best noise performance in this region of the spectrum (it has a larger aperture and hence better counting statistics than the Tenerife instrument and Haleakala is a better site than Carnarvon). For 1984 data the Hawaiian instrument had a mean noise power of 13 ms^{-1}^2 Hz^{-1} between 1.5 and 1.8 mHz and 7 ms^{-1}^2 Hz^{-1} between 5 and 5.75 mHz.

Table 2

<table>
<thead>
<tr>
<th>Length of data set</th>
<th>Sites included</th>
<th>% fill</th>
<th>Sideband/main peak power</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 16 to 30 Aug 87</td>
<td>Tenerife</td>
<td>28%</td>
<td>59%</td>
</tr>
<tr>
<td>b)</td>
<td>Tenerife Australia</td>
<td>40%</td>
<td>13%</td>
</tr>
<tr>
<td>c)</td>
<td>Tenerife Hawaii</td>
<td>54%</td>
<td>2%</td>
</tr>
<tr>
<td>d) 19 Jun to 20 Aug 84</td>
<td>Tenerife</td>
<td>31%</td>
<td>41%</td>
</tr>
<tr>
<td>e)</td>
<td>Tenerife Hawaii</td>
<td>48%</td>
<td>8%</td>
</tr>
<tr>
<td>f) 2 Jul to 2 Sep 87</td>
<td>Tenerife Hawaii</td>
<td>44%</td>
<td>3%</td>
</tr>
</tbody>
</table>

Table 2 Data sets used to produce the window functions of Figure 1. The ratio of sideband to peak powers in the window function spectra is shown in the final column. The p-mode fine structure for data sets d) and f) is shown in Figure 3.
Figure 2: Power spectrum of the 3 station data set $1(f)$ (2 July to 2 September 1987).

Figure 3: Superimposed $p$-mode fine structure for a) data set 1d) (Tenerife only, 1984) and b) data set 1f) (3 station, 1987).
3. NEW STATIONS

It is intended to enlarge the network to contain at least six stations at good sites. The Australian station was built partly as a prototype for a series of semi-automatic observatories which would complete the network. In operating the Carnarvon system for 3 years, 2 factors have affected its duty cycle more than any others in our control:--

i) funding and time difficulties meant that much of the station – especially the control systems – were completed on site without detailed testing in Birmingham. Consequently few members of the Group have any detailed knowledge of the system. When faults occur it is not always possible to send out trained personnel quickly.

ii) data are collected on magnetic tape and then posted to Birmingham. Delays of a few weeks can occur before we are made aware of problems.

Before deploying further automatic stations abroad we have constructed one in Birmingham to provide a testing and training facility. The building is now complete and installation of the equipment should be completed in October/November 1988. The equatorial mount and spectrometer are similar to those in Australia, but somewhat larger for easier access. (The mount has a double yoke and is therefore capable of taking another instrument, e.g. a resolved Sun spectrometer). All personnel involved in the project will gain operating and trouble shooting experience with this system.

To overcome communication problems a modem is used to provide a daily status report and data transfer, and can also provide a user in Birmingham with control functions. Replicas of the Birmingham spectrometer and mount are under construction in the Main Mechanical Workshop of the School of Physics and Space Research at Birmingham University. We expect to deploy 3 such replicas in the next two years at new sites, and intend, when convenient, to upgrade the Carnarvon and Haleakala systems.

4. ACKNOWLEDGEMENTS

We should like to thank our colleagues past and present in both Birmingham and IAC solar oscillation groups. Thanks are also due to M. Andrews, S. Brookes, B. Hail, J.W. Litherland, H.K. Williams and the staffs of the Main Mechanical Workshop and Central Electronics Unit in the School of Physics and Space Research for their technical support, to A. Vorbruba for data analysis assistance and to L. Bateman and L. Heida for operational support in Australia and Hawaii respectively. Financial support from SERC and CAICYT is acknowledged.

5. REFERENCES

I.R.I.S.: A NETWORK FOR FULL DISK HELIOSEISMOLOGY

Eric FOSSAT

Université de Nice, Nice, France

ABSTRACT/RESUME

Presented in 1983 to the french "Institut National des Sciences de l'Univers", the I.R.I.S. (for Installation d'un Réseau International de Sismologie Solaire in french, or International Research on the Interior of the Sun, in english) project was first funded in 1984 by this Institute. It consists in the deployment of 7 observing stations in full-disk helioseismology, distributed in complementary longitudes and latitudes. This paper is a short presentation of the sites, teams, and calendar of this network.

Keywords: Helioseismology, Ground based network.

1. INTRODUCTION.

It is not really necessary, in these proceedings, to explain the motivation leading to the project of a worldwide network for helioseismology. In the present session, three such networks are presented, one for imaged observations, and the other two being devoted to full disk, or unimaged, measurements. IRIS is one of these two, using the sodium optical resonance spectrophotometer which was successfully operated several times in the Antarctic.

IRIS was first presented for funding request in 1983, after the first two expeditions of our group in the Geographic South Pole. These Antarctic campaigns were extremely successful to prove how important it was to obtain uninterrupted data time series. At the same time, they provided convincing evidence for the upper limit of the duration of such uninterrupted sequences in this extreme site, of the order of one week, intrinsically limited by the weather conditions. It was suggested to set a 6 or 7 stations network, which makes possible to obtain a duty cycle well over 90 percent.

The intention is to run this network during a complete solar cycle of 11 years, in order not only to obtain very long sequences of uninterrupted high quality full disk data, but also to study the solar cycle variability of all oscillation parameters, such as frequency, rotational splitting, amplitude, line shape, frequency dependence of the last two, and more. The success of such a long term cooperation is not very easy to guarantee. The basic idea of IRIS is a full participation of every local scientific team. In each selected site, there is a site representative, who is responsible for the instrument, the observations and for deciding who is IRIS member in his group. Each site representative (or the main observer in his group) is first invited for a few-month visit at Nice in order for him to acquire a good knowledge of the IRIS instrument.

At present, the structure of the IRIS scientific community is very simple. It consists in:

Project Scientist: E. Fossat.
Instrument Scientist: G. Grec.
Data distribution responsible: B. Gelly.
And an International Scientific Committee of one member per each site or participating group.

The data management and distribution policy, the publication policy and the role played by the Committee will be organized in detail during the first two IRIS workshops, to be organized in 1988 and in 1989. Such workshops will continue to be organized, successively by every IRIS group, on a once per year basis, or more frequently if necessary.

2. LIST OF IRIS SITES AND GROUPS, AND PRESENT STATUS.

La Silla, ESO, Chile (29°S, 70°W, 2400m alt).
This site has a special status, with no local team scientifically involved. D. Hofstadt, technical director of La Silla Observatory, has accepted to be responsible for the IRIS instrument operation. The first instrument was set up in May 1986. It was operated during one year as a prototype, and was stopped in spring 1987. It will be started again in definitive version in the spring of 1989.

John Wilcox Solar Observatory, Stanford, California, USA (37°N, 122°W, 1000m alt).
This site has a special status, with no local team scientifically involved. D. Hofstadt, technical director of La Silla Observatory, has accepted to be responsible for the IRIS instrument operation. The first instrument was set up in May 1986. It was operated during one year as a prototype, and was stopped in spring 1987. It will be started again in definitive version in the spring of 1989.

Kumbel, Uzbekistan, USSR (41°N, 70°E, 2300m alt).
Site representative: T. Yuldasheva.

Site representative: T. Hoeksema.
Members: R. Bogart, B. Gelly, P. Scherrer.

Members: S. Engamberdiev, and a growing group of young scientists.

Instrument setup in August 1988. This instrument was the first one to be set in a completely new mountain site without a pre-existing observatory.

L'Oukaïmeden, Morocco (31°N, 8°W, 2700m alt).

Site representative: S. Kadiri
Instrument setup before the end of 1988, in a difficult situation similar to the previous one.

Tenerife, Canary Islands, Spain (28°N, 15°W, 2400m alt).

Site representative: T. Roa-Cortes or P. Palle.
Members: P. Palle or T. Roa-Cortes.

Leemonth, Australia (30°S, 115°E, 600m alt).

Only preliminary contacts have started with the small solar physics group of this site, where a GONG site survey instrument has been operated during the last two years. The IRIS instrument should be set up there in 1990.

China. Several preliminary contacts. The definitive site will be selected in 1989 and the instrument should be set up in 1990.

Three other groups are participating to IRIS without observation.

University of Nice.

Our group is the author of the project, builds the instruments and will be responsible for the data storage, management, and distribution.

Nice Observatory.
Theoretical group working on the data analysis, the solar model and its eigenfrequencies.

Institute for Nuclear Research, Moscow.
Members: E. Gavrjuseva, V. Gavrjusev, G. Zatsepin.
Theoretical group working also on the data analysis and the solar model eigenfrequencies. This group is also working in Soviet Union on the Neutrino Galium experiment and will specially study the correlation between helioseismology and neutrino data sets.

3. CALENDAR.

1983. Presentation of the project to I.N.S.U.
1984. Project accepted and funded.
1986. Prototype instrument started at La Silla.
L'Oukaïmeden instrument started.
First IRIS workshop near Nice.
La Silla instrument.
Second workshop near Tashkent.
China instrument.
IRIS network deployment completed.
Third workshop at Stanford.
Fourth workshop near Marrakech.
SOLAR LUMINOSITY OSCILLATIONS FROM TWO STATIONS AND CORRELATION WITH VELOCITY MEASUREMENTS

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Andersen N.B., Domingo V., Álvarez M., Ledezma E.
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Instituto de Astronomía, Observatorio Astronómico Nacional, Ensenada, Baja California, Mexico.

ABSTRACT

Since 1986 the measurements of a quadruple photometer sited at the Observatorio del Teide (Izana, Tenerife) have made it possible to identify the p-mode luminosity spectrum with simultaneous velocity observations. Comparing this data, the adiabatic behaviour of solar atmosphere and theoretical expectations from solar models have been tested. Now, in order to increase the signal-to-noise ratio and reduce the sidebands due to the night-time data gaps, a second identical photometer was set-up in December 1987, at the Observatorio de San Pedro Mártir (Baja California Norte, Mexico). The first results of the observations of these two stations are analyzed and compared with simultaneous velocity measurements.

Keywords: solar luminosity p-modes, solar velocity p-modes, phase differences, adiabatic atmosphere.

1. INTRODUCTION

In a previous paper (ref. 1) the solar luminosity p-mode spectrum was identified using a quadruple photometer built at ESTEC. The instrument is the Solar Luminosity Oscillation Telescope (SLOT) and it is described by Andersen et al. in these proceedings (ref. 2). These observations were made from Tenerife and the luminosity p-mode identification was possible with the help of simultaneous velocity measurements made at the same site. The current configuration of filters in this instrument is shown in table 1. Channels 2 and 3 are centred at almost equal wavelength but they have different widths. The reason for this is to attempt to increase the signal-to-noise ratio and reduce the sidebands due to the night-time data gaps, a second identical photometer was set-up in December 1987, at the Observatorio de San Pedro Mártir (Baja California Norte, Mexico). The first results of the observations of these two stations are analyzed and compared with simultaneous velocity measurements.

Table 1. Filter configuration of the photometer.

<table>
<thead>
<tr>
<th>Channel</th>
<th>λ (nm)</th>
<th>Δλ (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>680</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>517.8</td>
<td>3.5</td>
</tr>
<tr>
<td>3</td>
<td>516.2</td>
<td>9.1</td>
</tr>
<tr>
<td>4</td>
<td>770.2</td>
<td>4.7</td>
</tr>
</tbody>
</table>

This second photometer began to work in January 1988. Good fortune during the first observations ensured that we obtained two series of five consecutive days of high quality in both observatories: in January 1988 and in February 1988.

2. ANALYSIS AND P-MODE IDENTIFICATION

Figure 1 shows the power spectra of the observational window of one of these series of five days. A) and B) correspond to the observational window of just the Tenerife data and just the Mexico data respectively. Both the first and second order sidebands appear with a power of about 70% and 20% of the central peak respectively. When we take Tenerife-Mexico data together (part C)) the improvement is obvious. Now only the first order sidebands appear and with a power of only 20% of the central peak.

Figure 2 shows one day of observation for the four channels measured as solar intensity (KHz) against time. The left side corresponds to Tenerife data and the right one to Mexico data. After removing the slow daily trend due to atmospheric extinction we obtain the residuals (in magnitudes, defined in the standard astronomical way) and a moving mean filter of 21 min is used to smooth the small low frequency trends.

The amplitude squared spectra of these series of five days was calculated for a sine wave fit procedure sampling at intervals of 0.5 µHz from 1.0 to 5.0 mHz. The intrinsic resolution of these spectra is ~ 2.5 µHz. As a first step we have calculated the power spectrum using only Tenerife data on one hand and Mexico data on the other; and the luminosity p-modes have been identified in...
Figure 1. Power spectra of the observational window of one of the five days series. A) and B) correspond to the observational window for only Tenerife data and only Mexico data respectively. C) corresponds to the observational window of the combined Tenerife-Mexico data.

Both spectra. Comparing the luminosity p-modes identified in the Tenerife spectrum and in the Mexico spectrum, the results are that they have, of course, the same frequencies and similar amplitudes, but, what is happening with their phases? If these identified peaks are effectively solar p-modes, the phase difference between them must be zero; if on the other hand they are due to noise, the phase difference between them will be different from zero. To examine this, the phase differences between the identified peaks in Tenerife and Mexico spectra have been calculated in the same frequency bins. Figure 3 shows the phase differences against frequency and for l=0, 1 and 2 modes. It is clear that the phase difference is zero; therefore, these identified peaks are effectively solar p-modes.

As a second step, we calculated the power spectra for the combined Tenerife-Mexico data and we have identified the solar p-modes. Figure 4 shows a typical p-mode identification. A) and B) correspond to the Tenerife spectrum and the Mexico spectrum respectively. In both the l=0 mode appears with its sidebands and noise peaks surrounding it. C) corresponds to the spectrum of the combined Tenerife-Mexico data. Now, the l=0 mode is clearer and the sidebands and the noise peaks have been reduced. Finally D) corresponds to the spectrum of the simultaneous velocity measurements made at Tenerife.

In this way, we have identified 34 solar luminosity p-modes with amplitudes from 3 to 14 ppm (parts per million) depending on the
Figure 3. Phase differences between the identified p-modes in the Tenerife and the Mexico power spectrum. The symbols "o" denote 1-0; "x" 1-1, and "o" the 1-2 mode.

Figure 4. A typical mode identification. A) and B) correspond to the spectrum obtained using only Tenerife and Mexico data respectively. C) corresponds to the spectrum with both Tenerife and Mexico data together and D) corresponds to the spectrum of simultaneous velocity measurements. Primes denote sidebands.

3. COMPARISON BETWEEN DIFFERENT COLOURS

A comparison between the parameters of luminosity p-modes at different wavelengths has been made. The p-modes obviously have the same frequencies and the gain between them has been calculated. The averaged gains are the following:

$$\Delta \phi_{\text{Mm}}^{680} = 1.28 \pm 0.03$$, $$\Delta \phi_{\text{Mm}}^{670} = 1.33 \pm 0.05$$

The average phase difference between the p-modes at different colours has been also calculated (in the same frequency bins) and are shown in Table 4 for all p-modes and for only 1-0, 1 and 2 modes. The main conclusion is that these phase differences are null within the errors.

<table>
<thead>
<tr>
<th>Luminosity p-modes</th>
<th>1-0</th>
<th>1-1</th>
<th>1-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>680 nm</td>
<td>-2.15 ± 1.51</td>
<td>-1.56 ± 2.63</td>
<td>-4.65 ± 2.19</td>
</tr>
<tr>
<td>670 nm</td>
<td>-2.48 ± 2.10</td>
<td>-1.65 ± 3.77</td>
<td>-4.63 ± 4.00</td>
</tr>
<tr>
<td>1-0</td>
<td>-0.91 ± 2.90</td>
<td>-0.79 ± 4.00</td>
<td>-4.65 ± 10.00</td>
</tr>
<tr>
<td>1-1</td>
<td>-4.65 ± 2.19</td>
<td>-4.63 ± 10.00</td>
<td>-4.65 ± 2.19</td>
</tr>
<tr>
<td>1-2</td>
<td>-4.65 ± 2.19</td>
<td>-4.63 ± 10.00</td>
<td>-4.65 ± 2.19</td>
</tr>
</tbody>
</table>

Table 4. Average phase differences between the luminosity p-modes at different colours for all p-modes with 1-0, 1 and 2.
4. COMPARISON OF LUMINOSITY AND VELOCITY P-MODES

The frequencies of the luminosity p-modes are coincident with the velocity ones and now we will compare the phases and amplitudes of these modes.

The phase differences between the luminosity and velocity p-modes at the same frequency bin has been calculated. A typical problem of this determination is that if the maxima of the modes are not exactly coincident, we can be comparing the frequency bin in the maximum of one peak with a corresponding frequency bin of the other peak which is near noise level, yielding an unreliable determination. For this reason we have only taken the p-modes in which the phase difference between the luminosity and velocity peaks is constant within 4 frequency bins around the central frequency bin. In the January series this determination was possible for 31 p-modes whilst in February only for 17.

Figure 5 shows the phase differences between the luminosity and velocity p-modes at 680 nm. Note that the fractional luminosity $\Delta I/I$ has been plotted rather than $\Delta m$ which we have been using so far; this fact produces a phase change of $\pi$ because of the definition of magnitude. The calculations for both series agree very well and it is clear that this phase difference is not random and it is not -90 degrees as one would expect in an adiabatic atmosphere. The phase distribution of the open points in figure 5 has its maximum between -90° and -135° as shown in figure 6. The phase difference between the luminosity and velocity p-modes is constant in the interval of 2.7 to 3.7 mHz and using the 23 best determinations.

The amplitude ratios between the luminosity and velocity p-modes have been calculated. Figures 9 and 10 show these amplitude ratios (magnitudes in ppm and velocity in m/s) for radial modes and for 1-1 modes respectively. For the other wavelengths measured the plot follows the same curve, the only difference is due to the different gains.

The amplitude ratios between the luminosity and velocity p-modes with l=0,1 and 2, in the interval of 2.7 to 3.7 mHz, and using the 23 best determinations.

The average phase difference for the January series (open points in figures 5 and 7) have been calculated and are shown in table 5 for all the p-modes with l=0, 1 and 2, only for the modes in the interval of 2.7 to 3.7 mHz, and to make the criterion for phase difference determination extremely strict, we choose only the 23 better determinations. All these average phase differences are very similar.

Table 2. Frequencies (MHz) and amplitudes (cm/s) of the identified velocity p-modes.

<table>
<thead>
<tr>
<th>n</th>
<th>1</th>
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<tbody>
<tr>
<td>12</td>
<td>1822.0 (5.6)</td>
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<td>3600.0 (15.1)</td>
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<td>4155.5 (15.1)</td>
<td>4249.5 (15.1)</td>
</tr>
<tr>
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<td>4389.5 (15.1)</td>
<td>4486.0 (15.1)</td>
</tr>
<tr>
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<td>4615.5 (15.1)</td>
<td>4712.0 (15.1)</td>
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Table 3. Frequencies (MHz) and amplitudes (magnitudes in ppm) of the identified luminosity p-modes at 680 nm.

<table>
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<td>4030.0 (15.1)</td>
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<tr>
<td>27</td>
<td>4294.0 (15.1)</td>
<td>4389.0 (15.1)</td>
<td>4486.0 (15.1)</td>
</tr>
</tbody>
</table>

Table 5. Average phase differences between the luminosity and velocity p-modes with l=0,1 and 2, in the interval of 2.7 to 3.7 mHz, and using the 23 best determinations.

<table>
<thead>
<tr>
<th>$\phi_{680} - \phi_{VEL}$ (°)</th>
<th>$\phi_{770} - \phi_{VEL}$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All: $\pm 147.3 \pm 7.5$</td>
<td>$\pm 148.3 \pm 8.2$</td>
</tr>
<tr>
<td>1-0: $\pm 144.0 \pm 13.6$</td>
<td>$\pm 151.8 \pm 15.7$</td>
</tr>
<tr>
<td>1-1: $\pm 150.3 \pm 15.9$</td>
<td>$\pm 165.4 \pm 17.8$</td>
</tr>
<tr>
<td>2 (2.7-3.7 mHz): $\pm 140.8 \pm 7.1$</td>
<td>$\pm 138.8 \pm 8.3$</td>
</tr>
</tbody>
</table>

23 better determinations: $\pm 141.6 \pm 7.7$ $\pm 145.7 \pm 8.8$
Figure 5. Phase differences between the luminosity p-modes at 680 nm and the velocity ones. "o" denotes 1-0; "□" 1-1 and "◊" 1-2. The open symbols corresponds to the January serie and black symbols corresponds to the February one.

Figure 6. Distribution of the phase differences corresponding to the open symbols on figure 5.

Figure 7. As figure 5 but for luminosity p-modes at 770.2 nm.
5. CONCLUSIONS

The solar luminosity spectrum has been identified using data from two stations (Tenerife and Mexico) at three different wavelengths (516, 680 and 770 nm) with amplitudes between 3 and 14 ppm depending on the wavelength.

The frequencies of the p-modes at different colours are exactly the same within resolution. The gains from 680 to 516 nm and from 770 to 516 nm are 1.28 and 1.33 respectively. A study of the relative phase at different wavelengths in the 5 min range, gives a null result at all frequencies in this interval.

Comparing the luminosity and velocity p-modes we conclude that the most significant average phase difference between them is of $-141.6 \pm 7.7$ degrees in the frequency range analyzed. This means that the behaviour of the solar atmosphere is not purely adiabatic and thus radiative relaxation takes place. Also, from figures 5 and 7, the phase difference between the luminosity and velocity p-modes is a function of the frequency of the mode.

Finally, the amplitude ratios between the luminosity and velocity p-modes, is also a function of the frequency of the mode and follows very closely the theoretical expectations made by Gough (ref. 4).

Acknowledgements. We are indebted to H.B van der Raay and C.R Isaak of Birmingham University for the use of the resonant scattering spectrophotometer and to J.Christensen-Dalsgaard for his useful comments. This work has been possible due to the help of our colleagues at Teide Observatory (Izaña, Tenerife) and at San Pedro Mártir Observatory (Baja California, Mexico) with the observations. Finally, the authors wish to thank the spanish CAICYT (under grant PR84-0905) for financial support.

6. REFERENCES

SOLAR INFRARED INTENSITY OSCILLATIONS

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ABSTRACT

The 5-min oscillations are found to be easily observable as intensity variations in an infrared wavelength band centered at 2.23 μm with bandwidth (FWHM) 65 nm. The observed peak to peak intensity variation is 2.4% for a circular aperture of 1 arc min and 0.8% in the full disc observations, i.e. considerably higher than in the other four observed channels between 0.67 and 1.65 μm.

In addition to the 5-min oscillation the observed full disc power spectrum shows a strong feature centered at 4.3 mHz. This frequency coincides with that of the fundamental p-mode resonance of the chromosphere. Although this identification is not proven the possibility to study the chromospheric cavity in full disc observations is interesting.

Keywords: Five-minute Oscillations, Infrared Intensity, Waves, Chromospheric Cavity.

I. INTRODUCTION

A major part of our present knowledge about the solar 5-min oscillations comes from velocity measurements, both with (Ref. 1-3) and without spatial resolution (Ref. 4-6). Important results have also emerged from intensity observations. With spatial resolution the 5-min intensity oscillations have been measured both in atomic and molecular lines (e.g. Ref. 7,8). Recently, the intensity oscillations have also been investigated by broadband measurements in the visible wavelength region (Ref. 9) and ridgets in the frequency - horizontal wavenumber diagram have been observed. In spite of difficulties caused by transparency variations in the terrestrial atmosphere several attempts (Ref. 10-13) have been made to study the low degree mode structure of the oscillations in the solar luminosity. Only by using irradiance data from the ACRIM instrument on the Solar Maximum Mission have reliable information on the 1=0, 1 and 2 p-modes been obtained (Ref. 14). One obvious way of improving the observations of intensity oscillations is through uninterrupted space observations as planned with the Solar and Heliospheric Observatory (SOHO). Here we address the question whether or not the reliability of these intensity observations may be improved by selecting one or more wavelength regions in the infrared part of the spectrum. We present data showing that one broad band infrared wavelength region centered at 2.23 μm may be of interest both for space and ground based observations of intensity oscillations.

2. OBSERVATIONS

The observations were obtained in 1987 and 1988 using a fast multichannel photometer in the solar tower of the Oslo Solar Observatory. Seven different circular apertures give spatial resolutions from 0.5 to 4.2 min of arc. The same photometer is used in the full disc observations in combination with an imaging option in the solar tower consisting of flat mirrors and a lens focusing the solar image on the largest photometer aperture.

2.1 Multichannel, multiaperture photometer

The photometer has five wavelength bands centered at 0.67, 0.88, 1.20, 1.65 and 2.23 μm with bandwidths (FWHM) of 10, 10, 11, 50 and 65 nm, respectively. The very same PbS detector is used for all combinations of five wavelength regions and seven apertures (see Figure 1). The filter wheel is turned continuously during observations, whereas the aperture wheel is moved in steps.

Figure 1. Sketch of multichannel, multiaperture photometer
Figure 2. Observed solar intensity variation, ΔI/I, with 1 arc min aperture size for a quiet region at the disc centre on May 26, 1987. The data are corrected for slow i.e. more than 20 min intensity variations. The upper curve shows the 2.23 μm intensity and the lower curve the 1.65 μm intensity.

In the observations of the disc centre a programme mode is used whereby at any one spatial resolution all five wavelength regions are measured fifteen times and the averaged values are stored before the spatial resolution is altered. The speed of the photometer is such that within 45s the five wavelength regions are measured at all seven spatial resolutions. In the full disc observations there is, of course, no need to change the position of the aperture wheel and average values over 6s are measured for each wavelength.

2.2 Observations with spatial resolution

The photometer is constructed with limited spatial resolution, but with relatively high time resolution in order to study several solar phenomena where high accuracy in relative intensity is required. The solar phenomena of special interest include the radiative flux deficit in sunspots and solar intensity oscillations. Computer programmes are written by one of us (T.L.) to observe and reduce both kinds of observations.

In our initial attempts to observe solar intensity oscillation with this photometer we concentrated on the shorter wavelengths and found as expected the oscillatory signal to be comparable with noise and with the noise decreasing with increasing wavelength. A completely new picture emerged, however, when we started to look closely into the measurements at the longest wavelength region, centered at 2.23 μm. Let us describe the characteristics by presenting the 9 hours continuous intensity observation of May 26, 1987. Figure 2 shows the 2.23 μm intensity variation, ΔI/I, versus time for a quiet region at the disc centre using a circular aperture of 1 arc min; the intensity signal has been corrected for the slow i.e. more than 20 min intensity variation throughout the day. Compared to the 1.65 μm intensity, also shown in Figure 2, the 2.23 μm intensity variation is larger and quite regular. Apart from the features at 0.5 and 2.1 h, caused by small clouds, the observing conditions were good throughout the 9h time span.

The same characteristics for the 2.23 μm intensity variations were found for the other six apertures. The amplitude did change with aperture size and we will address that topic below. Let us first consider the wavelength dependence of the observed intensity variations for May, 26, 1987. The 2.23 μm intensity (Fig. 2) shows r.m.s. variations equal to 0.61%, whereas the corresponding values for 1.65, 1.20, 0.88 and 0.67 μm are 0.06, 0.11, 0.13 and 0.13%, respectively. Shorter observing sequences on May 25 (2h and 3h) show similar behaviour.

The power spectrum (power per frequency bin) has been calculated as the square of the relative intensity oscillation, ΔI/I, per Hz. For the 2.23 μm intensity oscillation on May 26, 1987 the result is shown in Figure 3. Similar power spectra were found for the other six apertures ranging in size from 0.5 to 4.2 arc min. The observations on May 25, 1987 support this statement and show power spectra with the same characteristics and with the power concentrated in the 2.5-3.5 mHz region.
SOLAR INFRARED INTENSITY OSCILLATIONS

The power spectrum in Figure 3 appears to be similar to those found in intermediate-degree mode studies of solar velocity oscillations (Ref. 3).

A first inspection of the 1.65 μm observations shown in Figure 2 may lead us to believe that the 5-min oscillation is not observable at the other wavelength regions. It was difficult to discuss individual frequencies in the power spectra. However, we found the integrated relative power between 2.5 and 3.5 mHz to decrease with wavelength by a factor three from 0.67 to 1.65 μm. Such a decrease with increasing wavelength is usually observed in studies of luminosity oscillations (Ref. 12). Since the noise level is estimated to be comparable with the observed power at 1.65 μm, the detection of the 5-min oscillation is probable, but uncertain for these wavelengths. In comparison the 2.23 μm band shows a 50 times more power in the 2.5-3.5 mHz region than the 0.67 μm band.

If the observed oscillations are of solar origin we should expect the observed power to depend on the spatial resolution. The 9h time span does not allow us to consider individual frequencies. Thus, we show in Figure 4 the observed power integrated between 2.5 and 3.5 mHz in the 2.23 μm intensity oscillations as function of aperture size for the May 26, 1987 data. In order to estimate the uncertainty the data series were divided in shorter sequences and separate power spectra were calculated for each part. In Figure 4 a value from the full disc observations, to be discussed below, is included. A variation in observed power with aperture size could in principle be caused by transparency fluctuations in the terrestrial atmosphere. However, the angular autocorrelation function of slow atmospheric transparency fluctuations show a typical coherence size as large as 1° (Ref. 15). The observed change in integrated power with increasing aperture size may be compared with a similar study for the velocity (Ref. 21). This comparison suggests that the result presented in Figure 4 reflects the observability of different modes in the 5-min oscillation. In conclusion, the results presented in Figures 2-4 strongly indicate that the observed 2.23 μm intensity oscillations are of solar origin.

2.3 Full disc observations

Several observing sequences of the whole solar disc were carried out in 1988. Here, we present the results for the period June 8-14, 1988. Clouds prevented observations on June 10 and 13. For the remaining days the number of observing hours ranges from seven to seventeen. Also in these observations the oscillatory signal is considerably stronger at 2.23 μm than at the other wavelengths (see Fig.5).

The power spectrum for the 2.23 μm intensity oscillations, based on the full disc observations between June 8 and June 14, 1988, is shown in Figure 6 (top). Comparing Figures 3 and 6 it appears that the 5-min oscillation is present in both power spectra in the frequency range 2.5-3.5 mHz. In a further comparison we have to be careful since the magnitude of the power is so different in the two power spectra.
In Figure 6 (top) a strong feature centered at 4.3 mHz is evident and the question to its nature arises. In order to study this feature more closely power spectra for each day were compared. The feature at 4.3 mHz did change from one day to the next. In fact, the 4.3 mHz power was so strong on June 12, 1988 that the power from this one day accounted for a considerable fraction of the power in this feature. In order to illustrate this point we also show in Figure 6 the power spectrum without the June 12 data included. Comparing the top and bottom power spectra in Figure 6 it is clear that the 4,3 mHz feature is considerably reduced when the June 12 data are excluded. Thus, the power in the 4.3 mHz feature may change markedly on a daily basis.

3. CONCLUDING REMARKS

The solar 5-min oscillations are usually interpreted as the evanescent tails of standing, gravity modified, acoustic eigenmodes, trapped in a subphotospheric resonant cavity (Ref. 16,17). The observable broad band intensity variations depend on how the wave energy changes with height in and above the trapping region, the corresponding variations in continuum and line source functions as well as the changes in opacity both in continuum and spectral lines. Since the continuum intensity at 2.23 μm is formed in nearly the same layers as the intensities at 0.67 and 1.20 μm, it seems reasonable to consider molecular lines in the search for an explanation of the difference in observed oscillation amplitude between these wavelength bands. One reason for the interest in the spectral lines is that the intensity oscillations are known to reach larger amplitudes in the lines (Ref. 7,8) than in the continuum. For the wavelength band we estimate that the observed intensity may be approximately 5% lower than the continuum intensity. The wavelength band was selected such that only a part of this reduction is caused by the first overtone lines of CO from the 2.23 μm band. In order to explain the large amplitudes in the observed intensity oscillations at 2.23 μm through the influence of molecular lines, it seems likely that the molecular lines have to be formed, at least partly, in layers where nonlinearities are important. The possible importance of nonlinearities has been pointed out earlier (Ref. 18) in a theoretical discussion of the wave behaviour in different layers of the photosphere and chromosphere. Although not directly related to this study it may be of interest to note that the radiative energy balance near the temperature minimum is possibly dominated by strong infrared molecular bands. In this connection even a temperature bifurcation (Ref. 19) has been suggested for the solar atmosphere.

Depending on the molecular line in question the intensity contribution function may show one or two separate maxima; one photospheric and/or one chromospheric region do contribute to the line. The contribution from the chromosphere to the molecular line may be of interest in connection with the 4.3 mHz feature observed in the full disc power spectra, since the chromosphere is known (e.g. Ref. 18) to have a wave cavity of its own with a period close to 3 min. The detection of the fundamental chromospheric p-mode resonance has been done earlier (e.g. Ref. 20) in high spatial resolution observations. It is somewhat surprising that the chromospheric p-mode should show up in whole disc observations since the size of the chromospheric cavity depends on the temperature distribution with height, which is regarded as changing with position. One may point out, however, that the chromospheric cavity is only separated from the subphotospheric cavity by a few damping lengths and, for instance, interaction through mode "kissing" may be possible (Ref. 18). Such an interaction that would change on a daily basis may be of interest for the energy transfer problem in the solar atmosphere. This possible identification should not be regarded as proven and other possible explanations of the 4.3 mHz feature should be explored.

We are indebted to Professor G. Eriksen for constructing the photometer and Dr. B.N. Andersen for helpful discussions throughout this project.

4. REFERENCES


SOLAR LUMINOSITY OSCILLATION TELESCOPE (SLOT)

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Abstract

Low degree $l = 0$-2 solar $p$-modes have been detected with the SLOT instruments at Izana and Baja California. The main source of noise for these ground based observations is in the terrestrial atmosphere. However, the data acquisition system still has to have very low intrinsic noise. We describe how this is achieved in the SLOT instruments. We also give a general description of the design and operating principles of the photometers and data acquisition system.

Introduction

The Solar Luminosity Oscillations Experiment (SLOE) is a project started in 1983 by the Space Science Department (SSD) at ESA with the twin goals of detecting solar $p$-modes in irradiance, and to gain experience in the design of high precision solar photometry for application in space missions. SLOE now comprises two Solar Luminosity Oscillations Telescopes (SLOTs), one situated at Izana, Tenerife, and run by the IAC, and the other at the Observatorio Nacional de Mexico in Baja California.

For eighteen months the SLOT 2 instrument was operated by the Instituto de Astrofísica de Andalucía at Granada. However, due to poor weather, inaccessibility, and instrument faults, no useful data was obtained from the instrument at this site. It was therefore decided to return the instrument to Tenerife in the summer of 1987, where the instrument was overhauled, and the electronics fixed. The two instruments were then run together for six months at Izana to check that SLOT 2 was functioning properly and to compare the data from the two instruments. Since the beginning of 1988 the SLOT 2 instrument has been operating in Mexico giving daily coverage of up to 17 hours.

Instrument Description

Each of the sensor units contains four independent solar photometers, consisting of a narrow bandpass interference filter, a set of baffles to define a conic direct field of view of 1.7°, and a full field of view of 8.4°, and a silicon PIN diode detector (Oriel #7182). The arrangement is shown in Figure 2. The four photometers are held together in a rigid case, and mounted on an equatorial mount. The case also carries a Sun pointer comprising a small telescope forming an image of the solar disk on a quadrant photodiode (EG&G SGD 444-4). The optical bandwidth of the telescope is limited by a 3mm thick Shott glass VG 14 (green) filter in front of the telescope.

The diodes are run in photovoltaic mode and the output current is converted to a voltage and amplified by a low noise transconductance amplifier (AD 234 L) which is bandwidth limited to about 750 Hz with a gain of $10^5$. The signal is digitised by the combination of a voltage to frequency converter (AD460 L), and a counter.

The entire experiment is controlled by a combination of a HP85 micro computer and multiprogrammer. A temperature stabilised crystal oscillator (Thomson-CSF P5) provides an interrupt to the multiprogrammer every 13 seconds, this causes the set of four counters accumulating pulses from the voltage to frequency converter to be read, as well as an ADC that measures the temperatures of the interference filters and the electronics. This data is transferred to the HP85 via the HP-IB (IEEE488) bus, and is then stored on floppy disk.

Figure 1:
Mechanical layout of a single photometer

The outputs of each of the quadrant detector elements in the guider is amplified and sent to the multiprogrammer ADC. From these signals the HP85 then calculates the guidance error in both RA. and Dec. axis, and commands the stepper motors on the axis to correct for any error, by changing the stepping rate for the next 13s.

The major difference between the two SLOTs is the mounting. SLOT 1 is mounted on a Contraves equatorial mount built especially for the project, while the SLOT 2 is mounted on a commercial telescope mount (Lichtenknecker Optics type M 80). The major reason for the choice of the different mounts was cost. While the mechanical performance of the Contraves mount is significantly better than that of the M 80, the performance of the latter is quite adequate for the purposes of the experiment with active pointing. Typical maximum pointing errors are 20" and 45" for SLOT 1 and 2 respectively. Both these guiding errors introduce a signal variation of the order of $10^{-7}$. In the guiding signal of SLOT 1 there is a clear peak at about 4 mHz, this is not visible in the irradiance data at all.

In both cases, each axis of the mounts are driven by stepper motors. The rate of steps is controlled by the HP85 through pulse train cards in the multiprogrammer. If, due to cloud or dust, the Sun is lost from the fine guider the R.A. axis is driven at 15° per hour, and the Dec. axis drive is stopped.

**Instrument performance**

From laboratory testing of the instruments we found a typical r.m.s. noise level of the data acquisition system of $<10^{-5}$ over a 10 second integration period. This noise is mainly in the form of $1/f$ noise and the residue in the 5 minute band is of the order of 0.1 ppm/Hz. In order to estimate the instrumental noise under realistic conditions we compared the observed signals from two identical channels on SLOT 1 and from SLOT 1 & 2 when these were mounted less than two meters apart. In the first case the atmospheric effects should most certainly be the same and possibly so also in the second case. We studied the power spectra of the ratio and differences of the signals. In all cases, for the 680 nm wavelength channel and 8 hours of observation, the residues in the power spectrum were about 0.06-0.1 ppm/Hz. This is more than an order of magnitude less than the typical p-modes with peaks of 1-2 ppm/Hz.

The main part of the noise in the data is caused by the terrestrial atmosphere. The best observational days have a r.m.s. variation of the signal with 13 seconds integration of $6 \times 10^{-5}$ in the 680 nm channel. For single day observations the solar signal can generally still be marginally detected when the noise is about five times worse than this. This leaves less than 10% of the observing days where the atmosphere is quiet enough to see the p-mode oscillations.

Some of these remaining days have superior quality. Undisputable detections have been made of p-modes with degree $l=0-2$ for single and multiple day observations (Refs. 1, 2). Detailed studies of the phase relations between different wavelengths and between intensity and velocity oscillations have also been carried out (Ref. 3). Results of the two station network is currently being processed (Ref. 4).

Assessment of the sky quality at Izana has been carried out in detail (Ref. 5). These results indicate that the best days for oscillation measurements are in winter and spring. In summer many days are influenced by sand blown in from the African mainland.

Refernces


4. Alvarez M. et al. These proceedings

EFFECT OF ATMOSPHERIC EXTINCTION ON SOLAR RADIAL VELOCITY MEASUREMENTS

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ABSTRACT

Differential extinction across the Earth's atmosphere affects astronomical photometry in a well known way. Under the same circumstances when a rotating extended object is observed spectrometrically, a residual radial velocity is obtained which varies during the day. In the case of integral sunlight observations, this effect has been calculated along the day in all possible observing situations during the year. Applications to real observations are shown and discussed leading to some conclusions specially relevant to ground-based networks.

Keywords: solar oscillations, atmospheric extinction.

1. INTRODUCTION

It is well known in astronomy the effect of atmospheric extinction on the photometric measurements of stellar fluxes. However when an extended object, such as the Sun seen from Earth, is observed photometrically differential atmospheric extinction effects arise. Moreover if this object rotates, as the Sun does, when spectrometrically observed (using integrated light), there exists non-zero residual velocities coming from the differential atmospheric extinction, which will obviously change during the day. This leads to a small low frequency trend.

These residual velocities leading to low frequency trends must be removed from velocity data, because they will produce noise power in the g-mode spectral region otherwise. Moreover, a good removal of such trends is of prime importance when joining observations coming from different stations in a worldwide network (IRIS, GONG, etc) (ref.1). Furthermore, when observing with spatial resolution the problem affects differentially the observations of different pixels.

In this work this effect is computed and applied to integral sunlight velocity measurements. Application on real data is performed proving its usefulness.

2. MODELIZATION OF THE DIFFERENTIAL ATMOSPHERIC EXTINCTION PROBLEM

Let's assume we have the solar disk in a reference frame with horizontal coordinates, as shown in Figure 1. Atmospheric differential extinction is modeled by a gradient dl/dz, and solar rotation axis is in a plane perpendicular to the line of sight: that is the small angle β (heliographic latitude of the center of the disc) is not taken into account.

Velocity fields are characterized by two components: \( V_x \), the biggest, coming mainly from solar rotation and directed perpendicular to the rotation axis, and \( V_y \), much smaller (~100 times), in the direction of the rotation axis. \( \alpha \) is the angle between the axis of rotation and the celestial north projected onto the solar disk plane.

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Figure 2. Values for the geometrical parameter \( f \) calculated at several epochs (a to d) of the year for Observatorio del Teide. Two values of \( f \) are calculated, each one using the values of 0.00 and 0.05 for the parameter \( \gamma \). Dashed, dotted and dash-dotted vertical lines only show the position of an abscissa coordinate 5, 5.5, and 6 hours away from local noon.

Then the correction to the line of sight velocity arising from this component is

\[
E(v) = \left[ v_y \sin(\delta - \theta) + v_e \cos(\delta - \theta) \right] \frac{d\omega}{d\xi}
\]  
\( (e.1) \)

Most of experiments can measure \( I(t) \) and then:

\[
\frac{d\omega}{d\xi} = \frac{dI}{dt} \frac{dt}{dW} = \frac{dW}{d\xi}
\]  
\( (e.2) \)

which substituted in the equation (e.1) yields

\[
E(v) = v_y \frac{dI}{dt} f(2,4,6)
\]  
\( (e.3) \)

Where \( \phi \) is the latitude of the observatory, \( \delta \) the sun's declination, \( Z \) the zenithal angle. Finally, the parameters to fit are:

\[
\begin{align*}
\nu_y' & = K \cdot \nu_y \cdot \nu_e \quad Y = \nu_y' / \nu_e \\
\end{align*}
\]

of which \( Y < 1 \).

In Figure 2(a-d), different examples of the function \( f \), corresponding to different epochs of the year, and different values for \( f \) are shown. This parameter \( \gamma \), is certainly below 0.03 and will change throughout the year as \( \delta \) changes; moreover, its effect is very small as it can be noticed by looking at the figures. It can be seen that significant departure from its value at noon starts at 4.5 to 5 hours away from local noon. In summer when more than 12 hours can be available a singularity arises at this point.

A numerical simulation of an integral sunlight observation by a potassium resonant scattering experiment gives values for \( E(v) \) which can vary from 6 to 14 m/s along the year when \( dI/d\xi \) is set at 0.03.

3. TEST WITH REAL DATA

Raw data acquired at the Observatorio del Teide (Izaña) at different epochs of the year, using the Birmingham potassium resonant scattering spectrometer has been used to test this model.
The derivative $dl/dz$ is numerically calculated as an interesting transmission monitor provides $I(t)$ (see Figure 3a, b), as well as the $f$ value for the day and times where data exist. Then fixing a value for $f=0.02$ the correction $\Delta v$ was calculated, for several values of $v^*$ and subtracted from raw data. This data is analyzed in the same way than the raw data without correction, to yield the residuals (ref.2 and 3). The test of the method consists in looking for a minimum in the standard deviation of the residuals with and without correction.

As can be seen in Figure 4 the correction at the early morning and late afternoon works well in the sense of correcting the trends at the beginning and at the end of the observing day. Typical improvements of 5 to 20% in the standard deviation value can be achieved. On absorbent days, where this method could be more useful as trends are bigger, it shows promising results although the numerical differentiation of $I(t)$ can bring some undesired numerical problems and a fit to such a curve proves to be necessary. A few examples have been tested showing similar results to those plotted in figure 4, but no careful analysis has yet been done of the parameters found in the fit and its variation along the year and with quality of the atmosphere.

4. CONCLUSIONS

Although a much larger number of tests with real data have to be made in order to verify the quality of the correction, it can be concluded that using data beyond 5.5 hours before and after local noon time, can be very dangerous specially for signals at the low frequency band of the spectra (e.g. g-modes). On the other hand this method also offers a possibility to correct for slow intensity variations which can occur in poor absorbent days.

Acknowledgements. The help of all members of the solar oscillations group at Tenerife and Birmingham are deeply acknowledged. The authors wish to thank financial support from SERC and CICYT (under grant PRB4-0905).

5. REFERENCES

INTEGRATED SOLAR DISK OSCILLATION MEASUREMENTS USING
THE MAGNETO-OPTICAL FILTER: TESTS WITH A TWO STATION NETWORK

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ABSTRACT
The magneto optical filter (MOF) has been extensively
used to get high and intermediate \( \ell \)-modes of solar os­
cillations. For very low \( \ell \)-modes (0 \( \lesssim \) 4) the imaging
capability of the MOF is still attractive since it allows
a pixel by pixel intensity normalization. However, a
 crude attempt to get very low \( \ell \) power spectra from
dopplergrams obtained at Mt. Wilson gave noisy re­
results. This means that a careful analysis of all the fac­
tors potentially affecting high resolution dopplergrams
should be accomplished. In order to better investigate
this problem, we have considered a non imaging c • •
nef using the lock-in amplifier technique. Two
syst-

n are now operational, one at JPL and the other at Uni-
versity of Rome. Observations are in progress. They
will be used to discuss the MOF stability, the noise
level, and the possible application in asteroseismology.

Keywords : Solar oscillations, MOF technique.

1. INTRODUCTION
The Magneto-Optical Filter, first developed by A.Carc­
ciani (Refs.1,2,3), has four im portant characteristics;
1) high transmission, 2) high spectral resolution, 3)
spectral stability, 4) imaging capability.
So far the imaging capability has been considered the
most important advantage over the scattering devices
used by the french group (Nice) and the english group
(Birmingham), so that great technological effort has
been devoted to obtain high quality doppler and mag­
netic images.
The solar pictures obtainable with the MOF, and the
ongoing program at Mt.Wilson 60-foot solar tower de­
monstrate that this goal has been full achieved. How­
ever, even if the overall throughput with MOF is about
10\(^5\) times larger than with the scattering cells, a first
attempt to get power spectrum of lower \( \ell \) modes (\( \ell \leq 0,
1,2,3\)) gave noisy results for the following reasons:
- poor duty cycle: The CCD exposure time necessary
to get doppler and magnetic images amounts to a max­
imum of a couple of seconds. Each cycle is repeated
about every minute. A better duty cycle is therefore
possible to improve the S/N ratio.

Figure 1. A single magnetogram obtained with the
MOF at Mt.Wilson. The local misregistration is visible.
2. EXPERIMENTAL APPARATUS

The experimental apparatus is sketched in fig. 2.

The incoming solar light from a small aperture (1 × 2 cm) is divided into two beams by a Polarizing Beansplitter (PB).

The first beam feeds the MOF and the other is further split in two parts for continuum intensity monitor and for telescope guiding.

As usual, the system is composed by a Filter section and a Wing Selector section. The Filter section (Na vapour in a magnetic field and between two crossed polarizers P) provides two narrow spectral transmission bands tunable on the wings of the solar Na-D lines. The Wing Selector (the second Na vapour after the KDP-electro-optical-light-modulator) is able to transmit only the red or the blue wing. The used chopping frequency is about 150 Hz.

The PMT tube operates in current mode for better performance and feeds a lock-in amplifier through a logarithmic amplifier. The final signal is digitally recorded on a personal computer with a sampling rate of 3 seconds.

The logarithmic amplifier after the photomultiplier serves two requirements: a) provides an analog real time signal independent from intensity fluctuation and b) improves the linearity of the signal as a function of velocity (see figs. 3).

A similar equipment has been recently duplicated in Rome, at the Physics Department of the University "La Sapienza". It started observing on 9/21/88, so that two stations are now available to us covering 16 hours per day.

The fig. 4 show an example of the original data and fig. 5 is a power spectrum of 5 days (~8 hours/day) taken at JPL. For a quick comparison the South Pole power spectrum, measured by E. Fossat et al. in 1980, is displayed upside down on the same figure. The general agreement is encouraging, the major difference being the occurrence of sidelobes in our data due to the day, night interruptions.
The whole experimental setup shows remarkable stability, day after day, and very good coincidence with the computed sinusoidal signal due to the earth rotation. Only at the beginning and at the end of each day the signal shows a systematic red-shift due to effects of atmospheric water blends.

2.1 Thermal stability

The thermal behaviour of a two cell MOF can be summarized as follows:

- The Wing Selector is poorly sensitive to temperature changes because the working wavelengths are determined by the Zeeman effect only.
- The Filter section is sensitive to temperature changes because the transmitted wavelengths are mainly determined by Macaluso-Corbino effect.

Therefore a maximum signal is reached when the Filter temperature is such that its bandpasses match the Wing Selector wavelengths. In this condition the MOF is stable against small temperature fluctuations.

---

**Figure 4.** Diurnal variation of line of sight velocity on 9/11/88 and velocity residuals (bottom). The dashed line is the theoretical curve (earth rotation + orbital motion + gravitational red-shift).

**Figure 5.** Power spectrum of 6 days from JPL and data string (window function)
Figure 6. Temperature sensitivity of a two cell MOF.

The experimental curve in fig.6 shows this behaviour. A more extended study is in preparation using a refined MOF model and the JPL laser facility able to give high resolution transmission profiles. One example is shown in fig.7 together with a computed theoretical profile. The central peak is very sensitive to temperature changes and is usually suppressed as shown in fig.8.

Figure 7. Experimental and theoretical MOF transmission profiles.

Figure 8. Usual spectral set up of the MOF, the central transmission is suppressed.

3. CONCLUSION

The tests presented here demonstrate that the MOF displays excellent performances even for low $l$ modes. Therefore the insuccess with the Mt. Wilson data should be ascribed to detector noise, temporal lag between red and blue images and poor duty cycle as anticipated in the introduction.

Indeed our aim is to take advantage of the MOF imaging capability to normalize local intensity fluctuation, pixel by pixel, before integrating over the whole disk. This procedure will substantially reduce the noise arising from differential atmospheric extinction across the solar disk.

An additional advantage is the ability of taking into account the magnetic effect from active regions. A further program is to evaluate the possibility of taking advantage of the MOF high transmission to detect stellar pulsation. This will be done attenuating the solar flux to stellar levels. A stellar photometer is suggested in another presentation of this meeting.

REFERENCES


ACQUISITION AND REDUCTION PROCEDURES FOR MOF DOPPLER MAGNETOGRAMS

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ABSTRACT

Elsewhere in these proceedings we discuss MOF performances when it is used to detect low \ell solar modes (no image). In the present contribution we analyse defects occurred on the MOF first magnetograms, particularly we discuss the problem of the apparent contamination between velocity and magnetic fields.

We find that a correct acquisition and reduction procedure gives cleaner results. We also suggest a new vector magnetograph and compute the vector field at coronal levels using one MOF longitudinal magnetogram.

Keywords: Solar magnetic field, MOF technique, vector magnetograph, reduction procedures.

1. INTRODUCTION

The optical scheme of the existing MOF Doppler-Magnetograph at Mt. Wilson 60-foot solar tower is shown in fig.1. Four frames are needed to compute velocity and magnetic fields separately. The fig.2 gives full explanation of the reduction procedure. In the following section we discuss a few problems related to a correct determination of velocity and magnetic fields.

2. CROSS-TALK BETWEEN DOPPLERGRAMS AND MAGNETOGRAMS

The algorithm used in fig.2 that is

\[
D = \frac{(B^+ - R^+)}{(B^+ + R^+)} + \frac{(B^- - R^-)}{(B^- + R^-)} \\
M = \frac{(B^+ - R^+)}{(B^+ + R^+)} - \frac{(B^- - R^-)}{(B^- + R^-)}
\]

Figure 2. Method of velocity and magnetic field measurement. \( R \pm \) \( B \pm \) refer to a single filtergram obtained through the narrow MOF bandpasses positioned in the red (R) and blue (B) wings of the solar absorption line, in right (+) and left (-) handed circularly polarized light.

The magnetic \( \lambda/4 \) plate alternately transmits only left handed and right handed circularly light, the doppler \( \lambda/4 \) plate allows the MOF to transmits the red or the blue wing. Each of the four filtergrams are processed as shown to produce dopplergrams and magnetograms.
Figure 1. Optical scheme of the MOF in the longitudinal magnetograph-dopplergraph mode.

is equivalent to

\[ D = 2 \frac{(B^+ - R^+) + (B^- - R^-)}{B^+ + R^+ + B^- + R^-} \]

\[ M = 2 \frac{(B^+ - R^+) - (B^- - R^-)}{B^+ + R^+ + B^- + R^-} \]

(2)

when \((B^+ + R^+) = (B^- + R^-)\), that is when no spurious circular polarization is introduced by the telescope.

Figure 3. The comparison between the two images shows how spurious polarization introduces a doppler-like signal in the magnetic background level if an incorrect processing algorithm is used (right picture).

The correct formula (1) is insensitive to telescope polarization because the contributions of left handed (+) and right handed (−) circular polarizations are normalized separately. The formula (2) was adopted by the JPL technical team for the SOHO proposal in order to minimize the acquisition hardware on board and fit the constraints given by the spacecraft data rate. The telescope was supposed to be polarization free, otherwise it can be shown that the measured quantities, magnetic and doppler, are related to the true ones \((H,V)\) through the following relations:

\[ \text{Magnetic} = C_H H + \frac{K}{2} C_V V \]

\[ \text{Doppler} = C_V V + \frac{K}{2} C_H H \]

where

- \(C_V\) is a doppler calibration factor
- \(C_H\) is a Zeeman calibration factor
- \(K\) is the fraction of spurious polarized light.

Figure 5. Magnetic contamination into a dopplergram. This is due to a particular MOF spectral set up.
This means that there is a mutual contamination between velocity and magnetic fields due to spurious polarization ($K \neq 0$) and incorrect reduction algorithm. This circumstance was not recognized by Rhodes in his presentation at the AAS meeting, 1988-Kansas City. He interpreted this effect as a cross-talk to be ascribed to the MOF and removable only empirically a posteriori (fig.3). Actually no cross-talk exists if the correct algorithm is used (fig.4).

2.1 Other kind of magnetic contamination into doppler maps

One kind of contamination arises from intensity reversal of the Na line core in active regions (structures in chromospheric lines). As shown in fig.5, this effect become very pronounced if the MOF spectral transmission is non zero at line center. It is possible to block this central transmission completely to produce non contaminated doppler and magnetic maps.

Another kind of contamination arises from magnetic sensitivity of the whole profile of the selected spectral...
line.
It is known that active regions with strong magnetic
field are associated with apparent line redshift (down-
drafts). Roger Ulrich has demonstrated in this meeting
that Na D lines, among other lines, are least affected
by this effect. This result is particularly important for
the MOF because Na D and K lines are the only usable
ones.

3. MISREGISTRATION AND A PROPOSED
VECTOR MAGNETOGRAPH

Particular care should be taken to avoid misregistra-
tion between red and blue images. A difficult source
for this is the so called incoherent seeing (image dis-
torsion). Local misregistration affecting high and low
spatial frequencies are then unavoidable.
The only way to reduce this effect from the ground,
is to use fast modulation. The MOF Stokes Polarime-
ter shown in fig.6 is designed to run at video rate and
to detect the full Stokes vector in two (or four, using
Na vapour in transverse magnetic field) different wave-
lenghts on the solar line profile.
Intensity normalization is accomplished providing the
video camera with logarithmic output.
The detection of the vector field is important to com-
pute 3D maps of coronal magnetic field. Using avail-
able MOF longitudinal magnetograms we have derived
the coronal fields with current-free approximation. The
result is shown in fig.7.

4. CONCLUSION

The present study demonstrate that performances of
the MOF show excellent correspondence to the expec-
tations. The conclusion is that there is room for more
advances and improvements both on the MOF itself
and on the acquisition technology and procedure.

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NEW_THRESHOLDS_FOR_GROUND-BASED_PHOTOMETRY_OF_SOLAR_INTENSITY_OSCILLATIONS

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ABSTRACT

The analysis of new whole-disc intensity observations realized from Pic-du-Midi and Jungfraujoch reveals the presence of the solar p-modes. With a 10 hours (single day) integration time and a 10 nm optical bandpass, the resolved groups of modes show a peak amplitude between $10^{-9}$ (red channel) and $3.10^{-9}$ (blue channel), while the observed background noise power is of the order of $1.10^{-9}$/Hz at 3 mHz.

We also apply a cross-spectrum analysis to our solar data. The less than optimum data available prevent nevertheless any firm conclusion here.

Considering the possibility to transpose this technique to stars, we conclude that intensity oscillations of solar-like stars are within reach of ground-based instruments, assuming atmospheric transparency fluctuations as low as those encountered in the present work.

Keywords: photometry, Sun: acoustic oscillations

1. INTRODUCTION

During the last ten years, a few attempts have been made to detect the global intensity oscillations which are associated with the low 1-degree acoustic modes on the Sun (see Ref.12, 13, 1, 5, 6, 4, 15, 10). Until recently, ground-based observations failed to yield convincing results due to excessive amounts of atmospheric noise. In fact, the first true detection of such oscillations came from space-based radiometric measurements (Ref.16). So the conclusion that, with expected noise power levels of $10^{-9}$/Hz (Ref.7), the chances of ever detecting solar global intensity oscillations with earth-based instruments were quite limited. However, in 1987, Jimenez et al. (Ref.10) achieved positive detection from a station at Izana, owing to extremely low values of the background noise.

In this paper, we present new observations of the solar spectral irradiance, recorded during the summer of 1985 and 1986 simultaneously from the Jungfraujoch Scientific Station (JJF, alt.: 3580m) and the Pic-du-Midi Observatory (PDM, alt.: 2870m).

2. THE OBSERVATIONS

Each station is equipped with two temperature stabilized photometers, constructed from a prototype set to our disposal by Dr. D. Crommelinck at the Institut Royal Meteorologique de Belgique and installed on an equatorial mounting controlled by a solar pointing system. In each photometer, a low-noise silicon photodiode measures the luminous flux falling on the circular area of a lambertian diffuser. We thus record the global intensity of the "Sun-as-a-star". An interference filter selects a different optical wavelength for each detector (680 nm for channel #1, 398 or 486 nm for channel #2, HW: 10 nm). These wavelengths were chosen to avoid the absorption bands of water vapour and oxygen in our atmosphere. Our irradiance measurements are integrated over 10 seconds, with a sampling every 10 seconds. Synchronisation between the stations is achieved to the nearest second through standard time signals.

During both observing seasons, the instruments were in operation for a combined total of 118 days. Unfortunately, the temperate weather prevailing at both sites prevented us to accumulate multi-day data series and strongly reduced the amount of simultaneous observations of high quality. Nevertheless, even in these rather unfavorable conditions, we were able to collect several day-long continuous sequences reaching very low noise levels.

3. ANALYSIS OF INDIVIDUAL DATA SERIES

3.1 Data preparation

The rough irradiance data are first corrected for the large diurnal transparency variation due to the apparent motion of the Sun, by the application of a single numerical high-pass filter. The filtering process consists in subtracting a weighted running-mean over 129 points from the data and dividing the resulting residuals by the corresponding running-mean values in order to express the residuals in terms of the relative intensity perturbation $\Delta I/I$. The weighting used here forms an envelope similar to the Hanning-Poisson window. The cut-off frequency is 1.175 mHz (period:14min).

Figure 1 illustrates the residuals which were obtained during one of the best days of observation (20/8/85, Jungfraujoch, ch.#1: 680 nm, ch.#2: 398 nm). The standard deviations are $1.3 \times 10^{-5}$ and $2.2 \times 10^{-5}$ in relative intensity for channel #1 and #2 respectively.

3.2 Periodogram

In order to resolve the groups of acoustic modes with even and odd 1-degree using single-day data, we must prevent any loss of resolution. Indeed, Christensen-Dalsgaard (Ref.2) has shown that, in case of incomplete separation of the modes, the spectrum takes a chaotic appearance, hiding completely the presence of a coherent signal in the data. Methods like the autocovariance power spectra are thus inadequate here.

Instead, we make a direct periodogram of the data after a slight apodization (FDI apodisation function proposed by Norton and Beer, Ref.14). The Fourier transform is computed using a modified version of Deeming's DFT algorithm (Ref.11).

Figure 2 shows the periodogram for the residuals in figure 1, in the frequency interval 1 mHz - 4.9 mHz. Its main characteristic is the 1/f noise typical of the fluctuations of the extinction by the Earth's atmosphere. In order to quantify this general profile, we tried to adjust to the spectrum a law of the form:

$$P(f) = A \times \frac{1}{f^B}$$ (1)

where $P(f)$ is the power at frequency $f$.

A and $B$ are the parameters to be adjusted.

This is done by least-square fitting a straight line in the ($\log P(f)$/ $\log f$) plane. The resulting law (continuous line in Fig.2) immediately gives a mean value of the background noise near 3 mHz. For the 20/8/85 residuals, we obtain respectively $6.1 \times 10^{-7}$/Hz (ch.#1) and $2.1 \times 10^{-6}$/Hz (ch.#2). Such small values of the noise power density are of the same order of magnitude as those published recently by the ESTEC/IAC team (Ref.10).

Our observations clearly confirm that earth-based solar photometry is able to reach very low noise levels, not much larger in fact than those obtained previously from space (Ref.16) and this in a few hours time instead of months.

3.3 Detection of solar p-modes

When looking for the signature of p-modes in the periodograms of 20/8/85, we must specify significance levels as the spectrum is crowded with peaks.

We imagined to combine a classical $\chi^2$ test with the adjusted 1/f law (cf. eq.1) in order to follow the true profile of the noise spectrum. The resulting curves, corresponding to the 99%, 95%, 90% and 84% levels (from the upper to the lowest), are represented in Fig.2 by the dashed lines.

In the frequency interval 2.8 - 3.8 mHz where p-modes have their largest amplitudes, other spectra show a group of two or three peaks around 3.1 mHz which are above the 90% and even reach the 99% confidence threshold. The best way to check whether these peaks are actually p-modes is to compare the observed frequencies with those issued from many years of Doppler observations. The significant frequencies are given in table 1, along with the peak height and the proposed identification of the corresponding modes. As a matter of fact, we just observe the modes or groups of modes with the largest amplitudes. Those amplitudes range from 1 to $3 \times 10^{-9}$ and are higher in the blue than in the red. They seem also larger in restricted spectral bands (10nm in our case) than in radiometric observations, as was already suspected by Schmidt-Kaler and Winkler (Ref.15).

Other significant peaks appear elsewhere in the periodogram, especially in the low frequency range dominated by the atmospheric noise. These periodicities can be ascribed to gravity waves in the high troposphere and the stratosphere, whose typical frequencies scatter the spectrum between 1 and 3 mHz as was noted by Yerle (Ref.17). They also show strong differences between the blue and red channels.
TABLE 1.

<table>
<thead>
<tr>
<th>Frequency (mHz)</th>
<th>Power ($10^{-6}$/Hz)</th>
<th>Identification</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.047</td>
<td>3.8</td>
<td>ch.#1</td>
</tr>
<tr>
<td>3.100</td>
<td>3.5</td>
<td>ch.#2 (13)</td>
</tr>
<tr>
<td>3.165</td>
<td>2.5</td>
<td>ch.#2 (16)</td>
</tr>
<tr>
<td>3.248</td>
<td>2.2</td>
<td>ch.#1 (22)</td>
</tr>
</tbody>
</table>

3.4 Systematic variations of the i/f profile

Another effect tied to the Earth's atmosphere could be of interest for meteorologists working in the field of the dynamics of our atmosphere. While fitting expression (1) as explained in 3.2 for all our daily observations, we have noticed a significant correlation between the profile of the noise spectrum and the r.m.s. value of this noise. The plot for each day of the exponent $B$ as a function of the logarithm of the mean noise power at 3 mHz (Fig. 3) shows that, when the noise power goes down, i.e. when the effect of extinction decreases, the exponent almost vanishes linearly. In other words, the spectrum becomes progressively a white noise spectrum. This could be the flat spectrum associated with pure scintillation or, also, with the instrument itself, when the red-noise signature of tropospheric turbulent convection almost disappears. This effect was probably overlooked until now because it becomes perceptible only when a very low photometric noise is obtained.

4. CROSS-SPECTRUM AND COHESION FUNCTION

4.1 Method

After this first analysis, one must be aware that, without the preliminary knowledge of the acoustic spectrum of the Sun, we would probably have been unable to guess whether those particular peaks were produced by the star or by our own atmosphere. If we now consider the problem of the detection of the $p$-mode spectrum from the point of view of asteroseismology and want to establish the stellar origin of the peaks in the periodogram without any preliminary hypothesis on the pattern of acoustic modes, the ultimate test is furnished by a cross-correlation analysis between data coming from two independent instruments. The test is based on the fact that the solar (stellar) intensity oscillations observed at the same wavelength will appear with the same frequencies, phases and amplitudes at both sites, while the atmospheric extinction will show no correlation between these sites. To achieve this, we calculate the coherence function defined as:

$$\text{Co}(f) = \left| \frac{C_{xy}(f)}{P_{xx}(f)P_{yy}(f)} \right|$$

where $P_{xx}$, $P_{yy}$ are the individual power spectra. $C_{xy}$ is the modulus of the complex cross-spectrum.

A solar signal will then correspond to a maximum of the coherence function close to one and a phase difference close to zero, while a random noise will produce lower values of the coherence and a random value of the phase difference.

The coherence function was obtained by first computing the spectra $P_{xx}$, $P_{yy}$, $C_{xy}$ using the method described in paragraph 3.2 and then by smoothing all spectra before applying definition (2). The smoothing must run over at least four elementary frequency bands to allow for a reliable test on the coherence. The loss of resolution is thus severe and an observing time of many days is here indispensable to resolve the $p$-modes.

4.2 Application to real data

Unfortunately, as mentioned earlier, we do not dispose of such multi-day simultaneous data for the PDH and JFJ sites. Instead, we made an attempt using a three-day series of simultaneous data (29-30-31/8/86, 680nm) coming from JFJ and from the ESTEC/IAC photometers at Izana. The cross-spectrum and coherence function (Fig. 4) were computed with a rectangular smoothing over 5 elementary frequency bands. Though both data series have almost the same standard deviation, they did not benefit of the best possible conditions at both sites on all three days. As it can be seen from the cross-spectrum, the power around 3 mHz amounts to $4 \times 10^{-6}$/Hz which is still too high to allow the detection of solar modes. As a consequence, the coherence function is erratic and never approaches unity.

The failure of the test in these circumstances does not in any way question the effectiveness of the method. This only draws the attention on the importance of a careful choice of the observing sites for such future studies.
4.3 Statistics of the phase difference

To check on a statistical basis if there is any trace of coherence in our own data, we computed the cross-spectra for the six best single-days with simultaneous observations at PDM and JFJ. We then concentrated on the phase difference and, taking the uncertainty on this quantity into account, we simply counted in each 130 µHz frequency interval the number of cases where this difference is smaller than 70°. The resulting histogram is given in figure 5 with low and high confidence levels. Almost all numbers N of occurrence fall between these limits, as expected. However, in the interval between 2.8 and 5 µHz, most values are above the overall mean (±2) and a value of 5 is reached in two contiguous intervals near 3.1 µHz (the probability for this to occur by chance is 1/400). Similar results, though with some variations, are obtained when varying the threshold for phase differences.

5. DISCUSSION AND CONCLUSION

The purpose of these observations was not to make a detailed study of solar acoustic oscillations. We tried to show that it was possible to observe such small intensity oscillations of the Sun-as-a-star from the ground using an unsophisticated though carefully adapted apparatus installed at sites offering average observing conditions. We indeed found the p-modes with amplitudes near 10^-8 and thereby confirmed that thresholds in power as low as 10^-9/Hz, after an integration time of about 10 hours, are now accessible to state-of-the-art solar photometry. Table 2 retraces the large progress achieved in this field in a time-span of only ten years. We include for completeness the observations of Woodard and Hudson and of Fröhlich, though those were not made from the ground but from space and a stratospheric balloon respectively.

Table 2.

Summary of the solar full-disc intensity observations

<table>
<thead>
<tr>
<th>Publication</th>
<th>Year</th>
<th>Instrument</th>
<th>Data Length</th>
<th>Optical Bandpass (nm)</th>
<th>Noise Power (10^-9/Hz)</th>
<th>Measured Amplitudes (10^-8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Livingston</td>
<td>1977</td>
<td>spectrograph</td>
<td>8h</td>
<td>0.01</td>
<td>1000</td>
<td>&lt; 30 (* )</td>
</tr>
<tr>
<td>Nusman, Nye</td>
<td>1977</td>
<td>spectrograph</td>
<td>10.5h</td>
<td>0.2</td>
<td>1500</td>
<td>&lt; 40</td>
</tr>
<tr>
<td>Beckers, Ayres</td>
<td>1977</td>
<td>spectrograph</td>
<td>7h</td>
<td>0.4</td>
<td>1100</td>
<td>&lt; 30</td>
</tr>
<tr>
<td>Bebbner</td>
<td>1977</td>
<td>planetary Ph</td>
<td>8h</td>
<td>0.1</td>
<td>50</td>
<td>&lt; 6</td>
</tr>
<tr>
<td>Claverie et al.</td>
<td>1981</td>
<td>photometer</td>
<td>11h</td>
<td>10</td>
<td>50</td>
<td>&lt; 5</td>
</tr>
<tr>
<td>Woodard,Hudson</td>
<td>1983</td>
<td>radiometer</td>
<td>290d</td>
<td>-</td>
<td>0.4</td>
<td>= 0.3-1</td>
</tr>
<tr>
<td>Schmidt-Kaler</td>
<td>1983</td>
<td>photometer</td>
<td>30h</td>
<td>3.7</td>
<td>40</td>
<td>10</td>
</tr>
<tr>
<td>Fröhlich</td>
<td>1984</td>
<td>photometer</td>
<td>100h</td>
<td>100</td>
<td>40</td>
<td>4.5</td>
</tr>
<tr>
<td>Jimenez et al.</td>
<td>1987</td>
<td>photometer</td>
<td>4h (10)</td>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Clette</td>
<td>1988</td>
<td>photometer</td>
<td>15d</td>
<td>10</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

(* ) Symbols: < when there is no detection (upper limit) = when p-modes are identified
As a matter of fact, the present work offers a good evaluation of the possibilities and difficulties of the photometry of stellar intensity oscillations. The importance of the atmospheric perturbations is similar for both techniques. For instance, the averaging of scintillation over the solar disc (factor ~1000) is replaced by an equivalent averaging over the telescope aperture. The situation is fairly well summarized in Fig. 6 which is directly inspired from figure 3 proposed by Harvey in Ref. 9. We have included in this figure a trace corresponding to the atmospheric transparency fluctuations at PDN and JFJ, as well as a higher profile of the p-modes, in agreement with the amplitudes observed by us in the spectral irradiance.

![Figure 6. Synthesis of noise power spectra of photometric measurements.](image)

From this figure, we can conclude that solar-like intensity oscillations should be detectable using ground-based telescopes in the 2 to 4 meter class, especially if dwarfs stars slightly more massive and more evolved than the Sun, with amplitudes up to 10 times larger than the solar value (Ref. 3), are selected.

Moreover, as the dedicated use of a space-based telescope as an asteroseismological observatory cannot be considered now and in the next future, the only practical way to search for stellar oscillations at the present is to use large earth-bound telescopes. Stellar photometry will offer a complementary approach to Doppler techniques as it can give access to stars fainter than magnitude 6 and thus to a large collection of targets, using rather simple equipment. Those working in this field of research will inevitably meet the limitations described earlier. The strategy and techniques outlined in this paper should help to make their search successful.

6. ACKNOWLEDGEMENTS

First of all, I express my gratitude to Dr. A. Koeckelenbergh for his practical support during all this doctoral research and for operating the instrument at the Jungfraujoch Station. I also wish to thank Dr. D. Crommelynck for putting to our disposal the basic instrument from which the photometers are issued, Dr. B. Andersen for providing the additional data from the Izana station and Dr. P. Cugnon for carefully reading the manuscript. I am also much indebted to the Pic-du-Midi Observatory and the Jungfraujoch Scientific Station Committee for their hospitality. This research was supported during its first year by the "Institut pour l'Encouragement à la Recherche Scientifique dans l'Industrie et l'Agriculture" (IRSIA) through grant No. 83343 and permanently from its beginning, by the Royal Observatory of Belgium, through the person of its Director, Prof. Dr. P. Melchior.

7. REFERENCES

AN ATTEMPT TO UNDERSTAND THE STANFORD p-MODE DATA

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ABSTRACT

The p-mode frequencies reported by Henning and Scherrer (1986) exhibit anomalous behaviour at degree 5 and frequencies below 2 mHz. We have investigated the implications of these data, and find no plausible solar model that is consistent with them. A density inversion including the low-degree five-minute data of Jiménez et al. (1988) implies that the density of the solar core is some 10 per cent greater than it is in the standard solar model 1 of Christensen-Dalsgaard (1982). Although that result is in keeping with previous suggestions either that the sun has a greater evolutionary age than is usually supposed or that there is a cloud of weakly interacting massive particles in the solar core and its environs, the behaviour of the sound speed in the core is not consistent with either hypothesis. Both the inferred sound-speed variation, and a secondary inversion for hydrogen abundance (relying on an assumption of thermal balance), provide some evidence for material redistribution in the energy-generating core. A sound-speed inversion for the entire radiative interior, using also frequencies of low and intermediate degree compiled by Duvall et al. (1988), confirms the earlier finding that the sound speed in the sun exceeds that of a standard solar model by up to 1 per cent in a region extending some 30 per cent of the solar radius and centred at \( r \approx 0.4R \). That is consistent with, though does not necessarily imply, that the opacity in the outer layers of the radiative interior at temperatures of up to about \( 4 \times 10^5 \) K has been underestimated by some 20 per cent.

Keywords: helioseismology, stellar structure, solar core, opacity

1. INTRODUCTION

The original objective of our study was to understand the low-degree p-mode data reported by Henning and Scherrer (1986). The issue is immediately appreciated (Gough, 1986) if one plots against cyclic frequency \( \nu \) the difference \( \delta \nu \) between the observed frequencies and the eigenfrequencies of any so-called standard theoretical model of the sun, joining modes of like degree. Then one can gain some idea of where the theoretical model appears to be an inadequate representation of the sun (Christensen-Dalsgaard and Gough, 1984). As is evident in Figure 1, the discrepancy is independent of \( \ell \) for \( \ell \leq 4 \), but modes with \( \ell = 5 \) stand apart when \( \nu \lesssim 1.8 \) mHz.

The most obvious property to note is that the lower turning point \( r_1 \) of the mode with \( \ell = 5 \) and \( \nu \approx 1.8 \) mHz is about \( 0.3R \), where \( R \) is the radius of the sun: modes with \( \nu < 1.8 \) mHz are evanescent at radii \( r < 0.3R \) whereas modes with \( \nu > 1.8 \) mHz penetrate into that region. Thus one is led to question the structure of the energy-generating core of the theoretical model, which extends to about \( r = 0.2R \).

The situation is not quite so simple as it might be, for there are other modes that do not penetrate beneath 0.3R, namely \( \ell = 3 \) modes with \( \nu < 1.0 \) mHz and \( \ell = 4 \) modes with \( \nu < 1.2 \) mHz. However, these modes are of quite low order, and one might therefore have questioned whether the turning point is so sharp a demarcation between regions that do and do not have a strong influence on the modes as our asymptotic ideas might have led us to suspect. Indeed, with this in mind, Ellis and Gough (1988) compared numerically computed frequencies of a model with wimps in the core with those of a model with a standard core, but they found no similarity with Figure 1. Moreover, their result was in qualitative accord with what one would expect from asymptotic theory.

Our first attempts to understand the behaviour of the Stanford data were based on forward calculations, using models with mixed cores. To produce so sharp a turnoff as one sees in Figure 1 perhaps requires a discontinuity in the structure of the star; thus the models would differ qualitatively from the wimp model studied earlier. Moreover, in that case the asymptotic formula for the eigenfrequencies requires modification, and thus might offer hope of yielding qualitatively different eigenfrequencies. We studied asymptotic expressions, the results of perturbation theory, and the eigenfrequencies of simple models that could be expressed in terms of standard mathematical functions, but all to no avail: we could never arrange for the low-frequency \( \ell = 5 \) modes to stand apart without the modes with \( \ell = 4 \) following.

2. IMPLICATIONS OF THE STANFORD DATA

Could it be simply that our naive attempts to modify only the structure of the core were misguided? Perhaps this question can be answered by considering the sensitivity of the eigenfrequencies to perturbations to the structure in different parts of the sun. If one considers two models of the sun whose densities differ by \( \delta \rho(r) \), the difference between the frequencies of corresponding adiabatic modes can be approximated by

\[
\frac{\delta \nu}{\nu} \simeq R^{-1} \int_0^R K_\alpha(r) \frac{\delta \rho}{\rho} \, dr.
\]

(2.1)

In obtaining this expression we have assumed that the adiabatic exponent \( \gamma \) is the same in the two models, which is
constant function on a uniform dissection of the interval $Sp/p$ by first representing it as a piecewise
We determined
computed subject to the adiabatic approximation, and for
those of the reference mode], in the manner illustrated by
(by an amount $6u$ where $J$ simply labels the mode) from

The continuous lines join the nominal frequency deviations (computed from the linearized equation 2.1) associated with the model whose density deviation from the reference model is depicted in Figure 3.

Our goal was to find a perturbation $\delta p(\tau)$ that modified the low frequency dipolar modes without influencing modes of lower degree. For linearized perturbations that would be adequate to isolate the feature in the equilibrium structure responsible for the discrepancy under discussion, the remaining essentially $\ell$ independent components of the difference between the solar and theoretical frequencies would then be due to an error in the model above the turning points of all the modes, where the eigenfunctions depend only weakly on $\ell$. Therefore we needed to find a region in the star where $K_\ell$ is substantial for $\ell = 5$ yet $K_\ell$ is small for $\ell < 4$. However, there is no substantial region where that is so. Indeed, the ratio of kernels $K_\ell$ for $\ell = 5$ and $\ell < 5$ is roughly unity for $r < 0.3R$, implying that any smooth density perturbation in the core would have a similar effect on all low-degree modes. (The reason is that even in the very core the density kernels are not small, as is evident in Figure 4. We suspect that this results from their dependence on the mass conservation constraint, which explicitly relates any local density perturbation to necessary compensating perturbations elsewhere. In retrospect it might therefore have been more expedient to work with sound-speed kernels, which are not so directly influenced by this constraint. However, in the light of the findings reported below, such an exercise would be solely of educational value.)

Not totally discouraged by our observation, we then set about seeking a model with the desired properties by inverse theory. First, we demanded simply that only the frequencies of the $\ell = 5$ modes of the new model deviated (by an amount $\delta\nu_{0i}$, where $i$ simply labels the mode) from those of the reference model, in the manner illustrated by the dashed lines in Figure 2. Linear eigenfrequencies were computed subject to the adiabatic approximation, and for convenience we used a polytrope for the reference model. We determined $|\delta p/p|$ by first representing it as a piecewise constant function on a uniform dissection of the interval

$$0 \leq r < 100 \text{ subintervals}$$

and then minimizing by least squares the difference $E$ between the frequency deviations $\delta\nu_i$ implied by the new model according to Equation (2.1) and the target frequency differences $\delta\nu_{0i}$ modified by an $L_2$ norm of a finite-difference representation of the second derivative of $|\delta p/p|$ to control the smoothness of the outcome (Tikhonov and Arsenin, 1977):

$$E = \sum_{i=1}^{\ell} \left( \delta\nu_i - \delta\nu_{0i} \right)^2 + \alpha K \sum_{k=1}^{K-1} (f_{k-1} - 2f_k + f_{k+1})^2$$

subject to the mass-conservation constraint

$$\sum_{k=1}^{K} r_k^2 \delta p_k = 0,$$

where $f = |\delta p/p|$ and $k$ consecutively labels the subintervals. Except when the tradeoff parameter $\alpha$ is extremely small indeed, it is not possible to reproduce the target frequencies: the sample inversion shown in Figure 2 was obtained with $\alpha = 10^{-2}$. As is the case with our forward calculations, the $\ell = 4$ modes follow the trend demanded of the modes with $\ell = 5$.

Finally we carried out an inversion of the Stanford frequencies plotted in Figure 1. Once again we used the method of Tikhonov and Arsenin, though this time we used Christensen Dalsgaard’s (1982) standard solar model 1 as the reference. The outcome was similar to that of our previous exercise. And it was possible to reproduce the Stanford frequencies, but only when $\alpha$ was very small. The points joined by continuous lines in Figure 1 were obtained with $\alpha = 0$. In this case the only smoothing is that imposed by the dissection: a greater number $K$ of subintervals would have permitted even closer correspondence (indeed, for our amusement and as a test of the computer programme we have reproduced the Stanford data to within the thickness of the lines plotted). However, the inversion, which is illustrated in Figure 3, is inconsistent with the assumption of the smallness of $\delta p/p$ upon which the validity of the linearized expression (2.1) depends. Of course, it is possible to reduce the magnitude of $|\delta p/p|$ by increasing the tradeoff parameter $\alpha$ at the expense of degrading the frequency match. However, it is not possible to reproduce even the qualitative features of Figure 1 whilst maintaining $|\delta p/p| \leq 1$. In view

Figure 1. The dashed lines join the differences $\delta\nu$ between the cyclic frequencies reported by Henning and Scherrer (1986) and the corresponding adiabatic eigenfrequencies of Christensen-Dalsgaard’s (1982) solar model 1, which has been used as a reference model for all the inversions of solar data. The differences are plotted against the observed frequencies. The continuous lines join the nominal frequency deviations (computed from the linearized equation 2.1) associated with the model whose density deviation from the reference model is depicted in Figure 3.

Figure 2. The dashed line joins desired frequency differences $\delta\nu_{0i}$ of $\ell = 5$ modes, which exhibit qualitatively the feature of the differences plotted in Figure 1 that are presumed to result from the structure of the solar core. Continuous lines join differences associated with a model obtained by inverting those differences together with $\delta\nu_{0i} = 0$ for all other modes reported by Henning and Scherrer (1986) using the method of Tikhonov and Arsenin (1977) with $\alpha = 0.01$. 

$\delta\nu$ (mHz) 

$\nu$ (obs), mHz
and Scherrer (1985) and the whole-disk data of Jiménez et al. inversion. Our bounds on $A$ do not vary rapidly in the manner that it does in Figure 3, so our bound on $A$ is independent of $r$ (and the situation can be 'improved' still further by similarly adjusting some of the other frequencies too).

3. PERMISSIBLE DENSITY PERTURBATIONS IN THE CORE

In the light of these results we must doubt the interpretation of the Stanford data. We note, for example, that by subtracting appropriate integral multiples of one-day alias frequencies from the dotriacontapole frequencies, the frequency differences plotted in Figure 1 can be rendered more-or-less independent of $\ell$ (and the situation can be 'improved' still further by similarly adjusting some of the other frequencies too).

4. ON INVERTING OSCILLATION FREQUENCIES

The inversions we have carried out are all based on attempting to satisfy linearized observational constraints of the general form (2.1) to determine the deviation $f(r)$ of some dependent variable of a reference theoretical model of the sun from that of the real sun. Having found $f$, one could construct a new reference model and iterate the procedure, but we have not done that. The problem of finding $f$ is underdetermined, and additional (smoothness) constraints must be imposed to isolate acceptable representations of the solutions. Except where it is explicitly stated to the contrary, the results we present below have all been obtained by means of the optimal averaging procedure of Backus and Gilbert (1968, 1970: see Gough, 1985, for a discussion of its application to solar inversion).

4.1 Primary frequency inversions

Because the adiabatic oscillation equations and their associated boundary conditions depend only on the pressure $p$ and density $\rho$ of the equilibrium state, together with the coefficient $\gamma$ relating their adiabatic perturbations, a direct inversion of observed frequencies (assuming, of course, that they are adequately represented by adiabatic eigenfrequencies) can provide information only about $p$, $\rho$, and $\gamma$. The eigenfrequency equations we use are obtained by linearization about an assumed hydrostatic state, so that a differential relation between $p$ and $\rho$ is assumed at the outset. Together with the constraint on the total mass of the sun, that relation can be used to eliminate either $\delta p$ or $\delta \rho$, leaving an integral equation (which is most easily derived from the variational formulation of the eigenvalue problem) in terms of $\delta \rho/v_p$ and $\delta \gamma$. If one assumes $|\delta \rho/p| < 1$ and $|\delta \gamma/\gamma| < 1$, linearization is valid, and if one further restricts attention to the radiative interior beneath the convection zone, $\delta \gamma$ can be ignored to a first approximation and one arrives at Equation (2.1). The kernel $K_p(r)$ depends not only on the reference equilibrium state, but also on the eigenfunction of oscillation of the reference model. Therefore the constraint (2.1) on the structure of the sun is different for each mode.

Given $\delta \rho$, one can determine $\delta p$ by solving the perturbed hydrostatic equation, and hence determine also the deviations from the reference values of any function of $p$ and $\rho$. These quantities are the primary products of frequency inversions. It is a straightforward matter also, with appropriate use of the hydrostatic constraint, to transform the dependent variable of Equation (2.1) to any linear combination of $\delta p$ and $\delta \rho$ (and $\delta \gamma$), and hence invert for deviations of other functions directly, though the evaluation of the kernel usually requires the solution of a second-order
differential equation. Thus, for example, one can express \( \delta \nu \) in terms of deviations of the square of the sound speed \( c^2 \):

\[
\frac{\delta \nu}{\nu} = R^{-1} \int_0^R K_c(r) \delta c^2 dr.
\]

(4.1)

Note that because hydrostatic support is a requirement for the validity of the original eigenvalue equations, subsequent use of the hydrostatic equation in transforming the constraints in no way renders them less fundamental than the original integral constraints derived directly from the variational principle.

### 4.2 Secondary frequency inversions

To gain information about quantities other than \( \nu, \rho, \gamma \) and functions solely of them it is necessary to demand that additional nonseismological requirements be satisfied by the model. Thus in the inversion one might demand that the model inferred is in thermal balance. There is no direct evidence that the sun is in thermal balance, though we know from theory that if the main-sequence evolution of the sun were as gentle as that exhibited by so-called standard solar models* the condition would be satisfied quite precisely. Thus one imposes the additional equations of static stellar structure: the thermal energy equation relating energy generation rate to the divergence of the heat flux, and the energy transport equation relating heat flux to temperature gradient. This introduces temperature into the problem. To close the system requires supplementing the structure equations with the complete equation of state (rather than an equation relating \( \gamma \) to \( \rho \) and \( \rho \), which is all that is required for primary inversions), together with equations for the energy generation rate and the opacity (which depend on chemical composition and two independent thermodynamic state variables). Appropriate thermal boundary conditions are, of course, also required.

Secondary inversions can most easily be carried out in one of two ways. The first is simply to integrate the thermal structure equations using the results of the primary inversion. This can be carried out either by computing the new hydrostatic stratification first (by adding \( \delta \rho \) etc. to the appropriate variables of the reference model) and then integrating the full thermal structure equations, or by linearizing the thermal structure equations in deviations from the reference model (as has already been carried out on the hydrostatic equations in deriving integral constraints such as 2.1 and 4.1) with \( \delta \rho, \delta \rho \) and \( \delta \gamma \) as inhomogeneous terms, and then integrating them to determine temperature, luminosity and chemical abundance deviations: \( \delta T, \delta L \) and \( \delta X_i \). The second approach is to use the linearized thermal structure equations for the deviations between the sun and the reference model to determine integral constraints of the form (2.1) with \( \delta T/T, \delta L/L \) or \( \delta X_i/X_i \) replacing \( \delta \rho/\rho \), and then to invert for the required deviation directly; the corresponding secondary kernel can be expressed explicitly in terms of a primary kernel and a regular solution of the adjoint of the homogeneous perturbed structure equations and boundary conditions. Some examples of secondary kernels are illustrated in Figures 6 and 7.
4.3 Tertiary frequency inversions

One can proceed yet further with adding supplementary assumptions. For example, one can adopt all the assumptions of the procedure for computing a standard solar model. Thus one accepts the differential equations, initial conditions and boundary conditions of stellar evolution theory, and assumes knowledge of the sun’s main-sequence age. That uniquely determines the structure of the model now in terms of the constitutive relations, namely the equation of state, nuclear energy generation rate and opacity (that is to say, the mappings that relate thermodynamic state variables or determine the value of the energy generation rate or opacity in terms of chemical composition and thermodynamic state). It is these relations (either their mathematical form or the values of coefficients such as cross sections upon which they depend) that are brought under question. Alternatively one can accept the microphysics, and the constitutive relations they imply, and invert in the initial conditions (such as abundances $X_i(r)$ at $t = 0$). In either case the kernels now depend not only on $r$, but also on time $t$.

![Figure 8](image1.png)

**Figure 8.** The dashed line is the relative difference between the density of a simple wimp model and that of a standard model. The crosses represent optimal averages deduced from the differences between (error-free) eigenfrequencies of the two models corresponding to those modes that were used for the inversions of the solar data illustrated in Figures 10, 11 and 13. The optimal averages were obtained directly from the constraints (2.1). The horizontal components of the crosses indicate the widths of the optimal averaging kernels; the vertical components the standard errors. The latter were computed assuming the standard errors for the corresponding observed frequencies, where they are quoted, and a standard error of 0.4 $\mu$Hz otherwise.

![Figure 9](image2.png)

**Figure 9.** Difference between squared sound speed of the wimp and standard model, and optimal averages obtained from constraints (4.1). The notation is otherwise the same as in Figure 8.

5. INVERSION OF SOLAR FREQUENCIES OF LOW DEGREE

Primary inversions for $\delta p$ and $\delta c^2$ in the solar core have been carried out using 119 frequencies of modes with $0 \leq \ell \leq 5$ and $\nu > 2$ mHz obtained from Jiménez et al. (1988) and Henning and Scherrer (1986). Christensen-Dalsgaard's (1982) solar model 1 was adopted as the reference. Each inversion was carried out separately, using constraints (2.1) and (4.1) respectively. To test our procedure, we first inverted the theoretical eigenfrequencies of corresponding modes of a simple model with a cloud of wimps in its interior, obtained by integrating the hydrostatic equations subject to the condition $d\ln p/d\ln \rho = \Gamma$ using the function $\Gamma(r)$ deduced from an analysis by Faulkner et al. (1986) of a model of Faulkner and Gililand (1985), and illustrated by Ellis and Gough (1988). (Note that this is really a test of the procedure in conjunction with the mode set, rather than the optimal averaging procedure per se, for we know already (Gough 1984) that quite accurate density inversions can be achieved using low-degree $p$ and $g$ modes of low order.) The results are compared with the exact functions $\delta p/p$ and $\delta c^2/c^2$ in Figures 8 and 9, and provide the reader with a measure of credibility with which to judge our inversions of the solar data. Those inversions are shown in Figures 10 and 11. We deduce that the inner core of the sun is some 10 per cent denser than the standard solar model, with a generally lower sound speed. Notice that the form of

![Figure 10](image3.png)

**Figure 10.** Optimal averages of relative density deviation of the sun from the reference solar model, obtained by inverting the constraints (2.1) using observed frequencies of modes with $0 \leq \ell \leq 5$ and $\nu > 2$ mHz reported by Henning and Scherrer (1986) and Jiménez et al. (1988). The notation is otherwise as in Figure 8.

![Figure 11](image4.png)

**Figure 11.** Optimal averages of relative deviation in squared sound speed of the sun from the reference model, obtained by inverting the constraints (4.1) using the same frequencies as for Figure 10.
the deviation is not in accord with the wimp-infested models of Faulkner and Gilliland (1985); although the density deviations in Figures 8 and 10 are qualitatively similar, the minimum at \( r \approx 0.1 \) in the deviation of the sound speed of the reference model from that of the sun is not present in the deviation from the wimp model. Nor is it consistent simply with the sun having a greater evolutionary age, which has been suggested (e.g., Gough, 1983) as a possible way to account for earlier seismological deductions that the sun is more centrally condensed than standard theoretical models. The forms of the density and sound-speed deviations between older but otherwise standard models and a standard model with the usually accepted age are qualitatively similar to those depicted in Figures 8 and 9, though the ratio of the magnitudes of \( \delta p/p \) and \( \delta c^2/c^2 \) is only about one third of that for the wimp models. We are thus led to wonder whether our result is a symptom of some degree of material redistribution in the core, as might be suggested by comparison of our inversions with Figure 12.

To test this conjecture we have carried out a secondary inversion for possible deviations in the hydrogen abundance \( X \). It was assumed that the heavy-element abundance \( Z \) of the sun is the same as it is in the reference model. The inversion was carried out directly, using constraints of the form (2.1) with \( X \) and \( \delta X \) replacing \( p \) and \( \delta p \). The result is shown in Figure 13, and is consistent with a flatter hydrogen-abundance profile than in the standard model. Thus there is perhaps some evidence for material redistribution. However, the reader must be aware that Figure 13 is in a very real sense a secondary inference, and relies not only on the accuracy of the assumed constitutive relations but also on the assumption of thermal balance. We do not present an inversion on \( Z \) (for example, at \( \delta X = 0 \)) primarily because that depends more sensitively on the assumption of local thermal balance and on the uncertain formula for opacity, which has already been brought into serious doubt by previous inversions (Christensen-Dalsgaard et al., 1985). Nor have we even carried out any tertiary inversions, which are even more heavily dependent upon untested assumptions.

6. INVERSION OF SOLAR FREQUENCIES OF LOW AND INTERMEDIATE DEGREE

Finally, we present sound-speed inversions for the entire radiative interior. For these we used frequencies of modes with \( \ell \leq 3 \) provided by Jiménez et al. (1988) together with the frequencies of intermediate degree from the compilation by Duvall et al. (1988), selecting only those modes with lower turning points at \( r = r_1 < 0.7 R \) and with error estimates no greater than \( 0.3 \mu \text{Hz} \). The deviations \( \delta \nu \) of those frequencies from the corresponding eigenfrequencies of Christensen-Dalsgaard’s model 1 are plotted against lower turning point (in the reference model) in Figure 14. Inversions were carried out using both the optimal averaging procedure and the minimization of expression (2.2). The results are presented in Figure 15, together with an asymptotic inversion presented by Kosovichev (these proceedings). Within the estimated errors, the three inversions are consistent, and also agree well with the differential asymptotic inversion presented by Christensen-Dalsgaard, Gough and Thompson (these proceedings). The most obvious feature of the result is the greater sound speed of up to one per cent in the outer regions of the radiative zone of the sun, which was also evident from the earlier cruder asymptotic inversions of Christensen Dalsgaard et...
al. (1985), as was pointed out by Christensen-Dalsgaard et al. (1985), this feature can be accounted for by raising the opacity by about 20 per cent in that part of the radiative interior with temperatures less than about $4 \times 10^6$ K (though, of course, it cannot be demonstrated from frequency inversions that that is necessarily the correct explanation). The extent to which the model inferred from the frequency inversion by optimal averaging satisfies the data is shown in Figure 16, which is plotted on the same scale as Figure 14. The outlier is $p_{11}(l = 2)$, the lowest-frequency mode of those listed by Jiménez et al. (1988). It is interesting that that mode appears not to stand wholly in isolation, for there is evidence in Figure 16 that modes with neighbouring turning points also have anomalously low frequencies. Therefore one is left wondering whether there is some interesting localized feature in the structure of the sun’s core at $r \approx 0.15 R$.

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**Figure 15.** Relative deviation from Christensen-Dalsgaard’s model 1 of squared sound speed inferred for the sun using the frequency differences plotted in Figure 14. Continuous crosses indicate optimal averages. The dashed curve and the associated dashed error bars were obtained by minimizing $E$ defined by Equation (2.2). The dotted curve and the associated dotted error bars represent the asymptotic inversion reported separately by Kosovichev in these proceedings.

**Figure 16.** Relative frequency differences according to Equation (4.1) from the deviations between the model obtained by optimized averaging illustrated in Figure 15 and the reference model, plotted against lower turning point on the same scale as Figure 14.
THE GONG INSTRUMENT

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ABSTRACT

An instrument is being developed to provide high-quality Doppler oscillation measurements for the Global Oscillation Network Group (GONG) project. This instrument uses the Fourier interferometer principle of sweeping a squared-cosine transmission function across a limited region of the solar spectrum centered on the Ni I line at 676.8 nm. Doppler shift is detected as a phase shift between the modulated solar signal and a simultaneously modulated signal from a stabilized laser. The solar signal is measured with an array of about 250 by 250 pixels covering the full disk. Design goals include a short-term noise level at a single pixel of about 10 m/s per integration interval of 60 s and a long-term stability of better than one m/s. A breadboard model has been in operation since early 1988 and a prototype is under construction. Most of the observations as of August 1988 have been of the Doppler shift of the entire sun imaged onto a single detector. These observations easily show the global p-mode oscillations with good short-term signal-to-noise ratio but have revealed a number of interesting problems. Imaging measurements have started with promising results.

Keywords: Helioseismology, Sun, Solar Interior, Instrumentation

1. INTRODUCTION

The Global Oscillation Network Group (GONG) project was proposed in 1984 by the National Solar Observatory as a community activity. Funding for the project from the National Science Foundation started in 1986. The project is intended to provide 3 years of nearly uninterrupted helioseismograms to allow detailed study of the stratification and dynamics of the solar interior.

The primary observational goal of the GONG project is to greatly improve the current quality of helioseismology data by reducing the diurnal cycle which degrades observations from single observatories, and by providing much longer, nearly continuous observations than can be made from the South Pole. The project is a natural evolution in the observational development of helioseismology since, while a great deal remains to be done and discovered, observational limits of what can be done at single observing sites are now becoming apparent. The GONG project is complementary to other multi-site helioseismology projects already underway or planned for the near future since these involve no- or low-resolution observations of solar oscillations whereas GONG will resolve the solar disk into $>10^4$ elements. It is also complementary to the helioseismology projects planned for flight on SOHO as these will start well after initiation of the GONG observations and are constrained by data transmission bandwidth limitations which will properly require emphasis on those parts of the oscillation spectrum that are most severely corrupted in ground-based observations.

2. OBSERVATIONAL GOALS

2.1 Angular considerations

The GONG instrument is required to produce observations of solar oscillations having scales from the size of the full disk down to sizes that begin to suffer serious distortion by average terrestrial atmospheric turbulence. According to Ref. 1, 3" rms seeing, typical of many daytime observing sites, significantly distorts oscillation patterns having spherical harmonic degree $l>200$. Therefore the GONG instrument is planned for $l$ values between 1 and ~150. Analysis of angular sampling (Refs. 2-4) and practical experience indicate that the largest value, $l_{\text{max}}$, which is properly sampled by $n$ pixels along a radius is $l_{\text{max}} < 1.4 n$. This means that we require at least 220 pixels across a solar diameter.

2.2 Temporal considerations

The major improvement expected from GONG is a long duration of nearly uninterrupted observations. This is achieved by placing six identical instruments at good sites around the world (Ref. 5). A minimum duration of three years was selected to overresolve the natural width of low-frequency $p$ modes by a factor of several times. Recent observations (Ref. 6) indicate that trapped $p$ modes are detectable at a frequency of at least 7.3 mHz and this dictates sampling once per minute to produce a Nyquist frequency of 8.15 mHz. To reduce aliasing of noise from above the Nyquist frequency, sampling should be done continuously during each minute.

2.3 Sensitivity and noise

A major requirement for GONG is to reduce noise to a level below the natural background signal of non-oscillatory, solar fluctuations while using a technique which maximizes the difference between oscillatory and non-oscillatory solar fluctuations. Oscillations can be observed as Doppler shifts or as intensity fluctuations. The former approach is instrumentally more difficult but provides a significantly greater signal compared to the non-
oscillatory solar background than the latter approach (Ref. 7). Therefore a Doppler shift technique was selected for GONG. Selecting an optimum spectrum line then becomes an issue (Refs. 4,8). Criteria include freedom from blending with other solar lines and terrestrial lines, a wavelength appropriate for available array detectors, a line profile which offers a good signal-to-noise ratio detection of Doppler shift, a height of formation which maximizes the difference between oscillatory and non-oscillatory Doppler shifts, and minimal sensitivity to disk position and magnetic effects. A tradeoff study was done and we found that Ni I 6768 Å was a reasonable, if not ideal, compromise (Ref. 9).

The level of the solar background, non-oscillatory velocity fields sets the requirement for instrumental sensitivity. This level has been estimated (Refs. 3, 10) and recently observed (Ref. 11). These results show that the background power spectral density in full-image observations is about 10 m s$^{-1}$ Hz$^{-1}$ in the region of p modes. From this one may derive that an instrumental noise level of 1 in the same units would be a good goal. For a one minute integration and an assumption of a white noise spectrum for instrumental noise, the requirement is an rms noise of 0.3 m s$^{-1}$ in a whole disk integration. If there are about 200 pixels covering the solar disk then each pixel may have an rms noise of about 60 m s$^{-1}$. This naively assumes that the noise is independent from pixel to pixel. As this is unlikely, a more realistic goal for instrument noise at each pixel is 10 m s$^{-1}$.

It is also intended that the GONG instrument provide good observations of rotation and large-scale, slowly-varying velocities of importance to understanding the nature of the solar interior. The goal which has been set is that the velocity zero point of each pixel be traceable to a stable reference with an accuracy of 1 m s$^{-1}$ over the 3 y duration of the observations. Table 1 summarizes the major goals.

<table>
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</table>

The idea of using a Michelson interferometer to measure the parameters of isolated spectrum lines dates back to Michelson himself. Over the years various implementations of the idea have been advanced (e.g. Refs. 14,15). The basic idea is to use a Michelson interferometer to produce a channel transmission profile (\(\tau_{\text{sine}}\)) with a scale comparable to the width of an isolated spectrum line. Then the transmission function is modulated to produce a signal which encodes the characteristics of the line profile (strength, width, Doppler shift). Although simple in concept, only in recent years has the practical application of the idea become feasible. The major advances have been to produce designs with large angular fields and temperature-sensitivity compensation.

The basic concept has been developed for observation of terrestrial winds by Shepherd (e.g. Refs. 16,17). The use of Michelson interferometers as narrowband filter elements has been developed by Title and Ramsey (Ref. 18). Apparently the idea of using Michelson interferometers for solar Doppler shift measurements arose independently in two groups (Refs. 19,20). Practical implementation of the idea has taken different paths. Brown (Refs. 21-23) has developed an instrument he calls a Fourier Tachometer which has produced several exciting advances in helioseismology. A Soviet group has followed a somewhat different path which has led to a number of interesting velocity observations (Refs. 24,25).

Thanks to excellent cooperation from T. Brown, H. Ramsey and A. Title, the GONG project was able to start its design activities with a considerable base of experience from which to draw. Our design approach was to develop an overall concept for the instrument and then subject it to criticism from internal and external experts and the future users. This process went through about five iterations before a satisfactory approach was identified. As new problems and opportunities are identified, we discuss them with future users at the annual GONG meeting and with a Scientific Advisory Committee.

From the beginning of the instrument development activity it was realized that there were elements of the instrument which have a low risk of causing problems and other elements possessing a high risk. A parallel development course was started. Low-risk items were designed and constructed at a prototype level while high-risk items were developed at a breadboard level and subjected to careful evaluation with the expectation of major design changes. As of this writing most, but not all, of the breadboard evaluation is finished and a full prototype is being assembled for test operation at a field site a few km from the NOAO headquarters in Tucson.

4. THE INSTRUMENT

4.1 Shelter and lightfeed

The instrument is housed in a refurbished cargo container with dimensions of 2.4 by 2.6 by 6.1 m and a weight of ~3 tons. This serves both as a shipping container and as an on-site shelter. Electrical power is to be obtained from local sources at each site but a generator and uninterruptable power supply are provided to allow the instrument to continue independent operation for a few days. It is planned that the instrument be automatic with attention required by on-site personnel only briefly every few days, mainly to change recording tape. A weather station is included so that the instrument can decide if observations can be conducted. Time signals will be received from the Global Positioning Satellite system. Figure 1 shows the prototype shelter installed in Tucson.

The external part of the instrument itself is a moveable, sealed turret assembly which contains an entrance window (which is also a broadband filter) and two mirrors. These mirrors are mounted in an elevation/cross-elevation configuration and driven to track the sun under ephemeris and/or guider control. Sunlight is directed horizontally (away from the equator) to the entrance objective lens of diameter 8 cm and focal length 1 m. The servo frequency response
of the light feed system appears to be fast enough that fast, image-motion compensation can be effected without additional elements. Pending further tests, however, a provision is included to mount the second mirror on piezoelectric supports so that it can be moved rapidly over small angles. Such a fast-moving mirror system was built for evaluation and found to work well. Figure 2 shows the prototype light feed pointed to the sun.

Figure 2. The prototype of the GONG lightfeed assembly. This unit picks up sunlight and sends it horizontally to the rest of the optical system.

4.2 Fore-optics and prefilter

The light path is sealed and filled with a dry gas pressurized slightly above ambient air pressure. The objective lens and all the remaining parts of the optical system are mounted on a vibration isolation table inside the shelter. Light proceeds into a temperature-controlled oven. This oven contains an interference prefilter with a passband of 5 Å centered on the 6768 Å line. The filter consists of 3 elements with cos² wavelength transmission profiles having periods of ~8, 4 and 2 Å. The coarsest element is a single piece of calcite but the others are constructed from two pieces to have a wide field angle. Analysis of temperature sensitivity indicated that the birefringent filter has a temperature coefficient of 280 m s⁻¹ K⁻¹ (Ref. 26). This was confirmed by experiment. Two strategies have been adopted to cope with this potentially serious temperature sensitivity. First, the oven enclosing the filter provides a temperature stable to a level of <10⁻⁶ K. Second, pieces of ammonium dihydrogen phosphate crystals are paired with the calcite crystals in such a way that the temperature sensitivity of each pair cancels. Laboratory tests indicated that compensation to one part in 50 can be achieved.

4.3 Polarizing Michelson interferometer

The heart of the instrument is a polarizing, wide-field, temperature-compensated, Michelson interferometer. One of these is illustrated in Figure 4. Its function is to produce a cos² transmission profile across the part of the solar spectrum transmitted by the birefringent filter and including the 6768 Å Ni I line. This is done by splitting the incoming, linearly polarized light into two beams at a polarizing, beam-splitter coating in the middle of a cube of glass. Each beam exists the cube passing through a quarter-wave retardation plate, traverses a distance to a mirror which reflects the beam back into the cube. One arm is solid glass and the other is air. The quarter-wave plates exchange the original states of polarization, and so the beams exit the cube orthogonally in direction and polarization from their entrance state and with a substantial difference in phase. This may be expressed as

$$\phi = \frac{4\pi}{\lambda} (n_1 d_1 - n_2 d_2),$$

where \(\lambda\) is wavelength, \(d_i\) is the length of the \(i\)th arm of the interferometer and \(n_i\) is the index of refraction of the \(i\)th arm. The optical path difference of the GONG interferometer is about 30000
waves to produce a period similar to the width of the solar spectrum line (Ref. 27). The emerging beams of light are orthogonally, linearly polarized and are converted to left and right circularly polarized light by passage through another quarter-wave retarding plate.

Figure 4. A Michelson interferometer used for the GONG instrument. The cube has a side of 2.5 cm.

The interferometer is designed to have a wide angular field by making the ratios \( d_i/a_i \) identical in both arms. The interferometer is made insensitive to temperature by making the spacer which supports the mirror in the air arm from a metal which has just the right coefficient of thermal expansion to compensate thermal path difference effects in the glass arm. Constricting the interferometer is difficult. Flatness tolerances are of the order of 100 \( \mu \)m, angular tolerances are sub-arc second and certain dimensional tolerances are \(-1\) \( \mu \)m. To ensure long-term stability, optical contacting rather than adhesives are used to assemble the interferometer.

4.4 Modulator

Light from the interferometer emerges from the temperature-controlled oven and then passes through a half-wave retarding plate mounted in the hollow shaft of a motor. Following the motor is a fixed, linear polarizer. These two elements allow the orthogonally polarized light emerging from the interferometer to interfere but with a phase that depends on the angle of the rotating half-wave plate. The presence of the solar spectrum line in the light passing through the interferometer causes modulation at the rate of 4 cycles per rotation:

\[
I(t) = I_0 [1 + M \cos(4 \omega t - \phi)],
\]

where \( I_0 \) is the average intensity, \( M \) is the modulation amplitude and \( \omega \) is the rotation rate of the motor. The rotation rate is \(-5\) rev s\(^{-1}\). We integrate the signal for 120° intervals of a modulation cycle to produce three signals \( I_1, I_2, I_3 \). This integration reduces the modulation amplitude to 83% of the value one would obtain with instantaneous sampling. The three signals can be combined to find \( \phi \):

\[
\tan \phi = \sqrt{3} \left( \frac{I_2 - I_3}{I_2 + I_3 - 2I_1} \right),
\]

and the modulation amplitude,

\[
M = \left( \frac{2}{3} \right)^{1/2} \frac{1}{\tan \phi} \left( \frac{1}{3} \sum (I_i - I_0) \right)^{1/2},
\]

and, of course, the average intensity. The phase of the modulation is related to the Doppler shift of the solar spectrum line and since we can only measure it modulo \( 2\pi \) and with no absolute reference, the relation between relative Doppler velocity, \( v \), and measured phase, \( \phi \), is

\[
v = \frac{\phi}{\omega}.
\]

With the parameters selected for the GONG interferometer, velocity sensitivity is about 2000 m s\(^{-1}\) rad\(^{-1}\) (Ref. 27).

4.5 Camera

The modulated image is focussed onto a two-dimensional detector array. At present a charge-coupled-device (CCD) is used but a final selection is yet to be made. We evaluate the error propagation of Eq. 3 to find that the variance of a phase measurement is

\[
\sigma^2 = \frac{27}{8} \sigma_v^2 M^2,
\]

which allows us to find the necessary performance of the detector in order to reach a given level of velocity noise. It turns out that we need a signal-to-noise ratio of about 500 per pixel per sample in order to meet the performance goal. This is a high level of performance for CCD camera systems which we have approached but not quite met.

4.6 Calibration

Although the GONG instrument is designed for great stability, it is still essential to provide for its calibration. A two-stage calibration strategy is used. First, an auxiliary optical system can be added to the normal optics to permit integrated sunlight to be passed through the instrument instead of a solar image. This allows one to establish the velocity zero points across the field of the instrument relative to some reference value. Second, frequency-stabilized laser light is sent through the modulator and interferometer to provide the reference value. This light is modulated in the same way as the solar light and is detected and recorded in parallel. If the interferometer drifts, the phase (velocity) of the laser signal will change as will the solar signal. The solar velocity signal can thus be corrected for drifts by subtracting the apparent laser velocity signal. This two-stage process allows every point in the field of the instrument to be referenced to the frequency of the stabilized laser.

4.7 Control and data recording

A small computer with a high degree of redundancy will control the operation of the instrument. A large number of environmental and status parameters will be monitored and recorded every minute day and night. The computer will determine the position of the sun by an ephemeris calculation and, if weather conditions permit, will attempt to point the instrument toward the sun. The guiding system will take control of the pointing if clouds and other factors are unfavorable; otherwise the ephemeris calculation will control the pointing until observing conditions become favorable. The controller will synchronize the data acquisition to better than one second at all stations and perform the necessary integration of the solar images every minute. Once an hour at each station (staggered by 20 min at successive stations), a calibration program will be executed for one minute. Part of this program will include producing a magnetogram for the purpose of correcting spurious velocity signals associated with the presence of magnetic fields. A fast, ferro-electric, liquid-crystal modulator will be used as a critical element in the production of the magnetogram. Data will be recorded on helical-scan tape cartridges capable of holding \(-2\) Gb each. Four units will be located at each station; two recording identical data simultaneously and two on standby.
5. TEST RESULTS

5.1 Integrated light

A laboratory was equipped to test most of the elements of the GONG instrument in a "breadboard" fashion. Initial tests were concentrated on assessing the performance of the instrument when observing the sun using only a single detector to collect all the light from the solar image. This was technically easier than the image tests but a severe test of stability and sensitivity. A number of interesting problems were found and the designs of several elements of the instrument were changed as a result. Observations were made for several months and performance improved substantially as changes were made. Figure 5 shows results from one day. Observations like these demonstrated that an instrumental noise level below the solar background signal level has been achieved except at the lowest frequencies. At low frequency, a repeatable diurnal trend was consistently observed. This trend arises in field properties of the birefringent filter and will be greatly reduced in a new design.

![Figure 5](image1.jpg)

Figure 5. Observations of the velocity signal from the entire solar disk. The upper plot shows the velocity integrated for 50 s each 75 s and corrected for the sun-observer ephemeris velocity and a diurnal trend. The lower plot shows the amplitude spectrum of the data. The 5-min oscillation is clearly detected.

5.2 Image tests

We have not yet obtained a satisfactory camera for the GONG instrument. A large number of commercial cameras has been tested and none has adequate performance in all important parameters. One of these cameras has been modified for testing purposes and provides acceptable performance for many of the instrument evaluation tests which are required. A single, 75-s observation of most of the disk of the sun is shown in Figure 6. It shows a number of defects but also demonstrates that the basic performance of the GONG instrument promises to be satisfactory. Based on tests of this sort we are confident that the GONG instrument will provide observations which will meet the needs of the project.

![Figure 6](image2.jpg)

Figure 6. Results from a single, 75-s observation of the sun with the breadboard GONG instrument. The top image is the average intensity of the three integrated phase images. Fine fringes are caused by the entrance window on the camera and other defects are dust on certain of the optics. The next image is the modulation amplitude according to Eq. 4 expressed as brightness. This shows a number of rings and streaks associated with an imperfect calibration but also shows the decrease of spectrum line strength as one moves to the limb and also a decrease in sunspots and plages. The third image is velocity according to Eqs. 4 and 5. Solar rotation is the major signal visible. The bottom image is the same as the third image except a simple correction for solar rotation has been applied. Visible are supergranulation, the Evershed flow around some sunspots, the 5-min oscillation and a brightening toward the limb associated with the decreasing convective blue shift.

Acknowledgements.

The talents of a large number of people have been applied to developing the GONG instrument. Advice of experts from outside NOAO has been especially valuable. A list of NOAO employees who have constituted the instrument development team is included in Table 2.
Table 2. The GONG Instrument Development Team

<table>
<thead>
<tr>
<th>Name</th>
<th>Role</th>
</tr>
</thead>
<tbody>
<tr>
<td>K. Abdel-Gawad</td>
<td>thermal design</td>
</tr>
<tr>
<td>W. Ball</td>
<td>project engineer</td>
</tr>
<tr>
<td>B. Boxum</td>
<td>electronics</td>
</tr>
<tr>
<td>F. Bull</td>
<td>workstation support</td>
</tr>
<tr>
<td>J. Cole</td>
<td>drafting</td>
</tr>
<tr>
<td>L. Cole</td>
<td>lead engineer</td>
</tr>
<tr>
<td>S. Colley</td>
<td>electronics design</td>
</tr>
<tr>
<td>K. Dowdrey</td>
<td>electronics engineering</td>
</tr>
<tr>
<td>R. Drake</td>
<td>optical physics</td>
</tr>
<tr>
<td>R. Dunn</td>
<td>opto-mechanical design advisor</td>
</tr>
<tr>
<td>T. Duvall</td>
<td>data acquisition consultant</td>
</tr>
<tr>
<td>D. Farris</td>
<td>electronics</td>
</tr>
<tr>
<td>A. Green</td>
<td>instrument maker</td>
</tr>
<tr>
<td>R. Hartmeier</td>
<td>instrument maker</td>
</tr>
<tr>
<td>J. Harvey</td>
<td>instrument scientist</td>
</tr>
<tr>
<td>R. Hubbard</td>
<td>project design specialist</td>
</tr>
<tr>
<td>P. Jackson</td>
<td>electronics</td>
</tr>
<tr>
<td>D. Kucera</td>
<td>optical coatings</td>
</tr>
<tr>
<td>C. Miller</td>
<td>control software</td>
</tr>
<tr>
<td>D. Miller</td>
<td>instrument maker</td>
</tr>
<tr>
<td>A. Petri</td>
<td>instrument designer</td>
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<tr>
<td>G. Poczulp</td>
<td>opucian</td>
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<tr>
<td>J. Schwitter</td>
<td>workspace software</td>
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<tr>
<td>J. Simmons</td>
<td>optical design</td>
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<tr>
<td>R. Smartt</td>
<td>optics consultant</td>
</tr>
<tr>
<td>G. Streander</td>
<td>instrument design/fabrication</td>
</tr>
<tr>
<td>F. Vaughn</td>
<td>master optician</td>
</tr>
<tr>
<td>P. Wiborg</td>
<td>software consultant</td>
</tr>
</tbody>
</table>

8. REFERENCES


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THE GONG SITE SURVEY

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National Solar Observatory
National Optical Astronomy Observatories
Tucson, Arizona USA

ABSTRACT

The Global Oscillation Network Group (GONG) project is planning to place six observing stations around the world to observe the solar oscillations as continuously as possible. This paper describes the procedures that are being used to select the six sites. The latest results of measurements of cloud cover obtained by networks of 6 (out of 10) radiometers show a duty cycle of over 93%, with the first diurnal sidelobe in the window power spectrum suppressed by a factor of 400. The results are in good agreement with the predictions of a computer model of the expected cloud cover at individual sites.

Keywords: Site surveys, cloud cover, networks

1. INTRODUCTION

Helioseismology, the study of the solar interior using observations of solar oscillations, requires precision measurement of the frequencies of the oscillations. It has been estimated (Ref. 1) that the frequencies should be measured at least to a level of 0.3 μHz. This requires observations covering 38.6 days, during which the Sun obviously sets at a single location outside of the polar circles. The diurnal rising and setting results in the convolution of the Fourier transform of the observing window with the transform of the solar signal. In the power spectrum formed from this convolved transform, each solar oscillation spectral line is surrounded by a set of sidelobes at \( \pm 11.57n \mu Hz \), where \( n \) is an integer. The unit of 11.57 μHz is simply the frequency corresponding to a single day. The net result is to overlay a forest of spurious ghost sidelobes on an already dense and complicated solar spectrum (Fig. 1), rendering mode identification and frequency measurement an exceedingly difficult task.

A number of attempts have been made to overcome this problem either with data-processing techniques or observing strategies. The data-processing techniques include the "CLEAN" algorithm, interactive deconvolution, and maximum-entropy gap filling (Ref. 2). None of these techniques has proven to be robust enough in the context of helioseismology. More success has been achieved using alternative observing strategies. The use of the South Pole as an observing site during the Austral summer has proven to be very fruitful (c. f. Refs. 3 and 4). The experience of observers suggests that a practical maximum for an unbroken data run in the Antarctic is about 5 days, though longer runs can be achieved with the acceptance of short periods of data loss. An alternative strategy is to place a helioseismology instrument on board a spacecraft (Ref. 5). This offers the obvious advantages of freedom from the terrestrial atmosphere and the diurnal cycle, but at the equally obvious disadvantages of high cost and poor accessibility. The upcoming SOHO mission will be carrying three experiments that will provide high-quality helioseismology data (Ref. 6).

Another effective observing strategy is the development of networks of observing stations placed around the world. In this approach, the longitudes of the stations are selected to provide continual solar observations, and the effects of weather are reduced by having more than one station observing at a time. The first, and so far most successful helioseismology network is that of the Birmingham group (Ref. 7). Since 1981 this network has comprised two stations at Tenerife, Canary Islands, and Haleakala, Hawaii. A third site at Carnarvon, Australia was added in 1985. The observa-

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1A table listing the current membership of the GONG site survey team is in the Acknowledgements at the back of the paper.

2Operated by the Association of Universities for Research in Astronomy, Inc. under contract with the National Science Foundation.

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Figure 1 (left): The 3000 to 3500 μHz band of a simulated solar spectrum that might be obtained from observations covering one continuous year. (right): The same portion of the simulated spectrum as might be obtained at a single site experiencing the day-night cycle and bad weather.
tions carried out by this network are of integrated sunlight; hence the observations are sensitive only to low-degree modes. Another network of single-pixel instruments is being deployed by E. Fossat from Nice (the IRIS network). The Global Oscillation Network Group (GONG) project is planning to place a total of six instruments in the field (Ref. 8). These instruments will spatially resolve the solar velocity field, and will provide observations of modes with spherical harmonic degrees $l$ of up to 350. The instrument and data processing systems of GONG are described in other papers in these proceedings. This paper will describe the procedures used to select the sites for the network and will present the latest results from the site survey.

2. OBSERVATIONS

The GONG site selection process began with the development of a computer model for the cloud cover at various positions on the Earth (Ref. 9). This model uses a simple two-parameter model for the statistical distribution of cloud cover. The parameters are $p$, the mean probability or fraction of clear sky, and $\tau$, the mean duration of clear weather. Two seasonal values for both $p$ and $\tau$ were used to provide some simulation of the difference in summer and winter conditions. The values of $p$ for a given site were estimated from climatological maps of the average hours of sunshine in January and July. The values of $\tau$ were chosen to be either 0.5 days, meant to represent the daily orographic buildup of mountain-top clouds, or 2.5 days, to model the passage of large scale weather systems. The model predicted that a well-chosen six-site network would achieve an overall duty cycle of about 94%. It was clear that the success of the model greatly depends on the accuracy of the estimated values of $p$, thus it was decided to measure $p$ by placing a sunshine monitor at a number of candidate sites. This would also allow the GONG project to gain experience in the operation of a network of instruments at remote sites.

The instrument chosen to monitor the sunshine is a normal incidence pyrheliometer mounted on a polar equatorial clock drive. A photograph of one half of the instrument is shown in Figure 2. The other half is a similar box containing a data acquisition system and battery backup power supply. The amount of light falling on the detector is converted into a voltage that is registered by a digital voltmeter. The voltage corresponding to full sunlight depends upon the site, season, and instrument, but is typically around 8 mV. Every 10 s, the voltmeter is read by a programmable calculator, which stores the measurement and averages six data points together every minute. After thirty one-minute averages have been stored, the data is written to a micro-cassette drive. Each cassette holds 253.5 hours of data, and is usually filled in two to four weeks. The cassette is then mailed back to the GONG project for analysis. Operator intervention is required every other day to unwind a cable connecting the pyrheliometer to the drive base. The operator must also adjust the declination axis of the instrument as the Sun follows the Ecliptic. Pointing is done using a solar image formed by a pinhole on a target on the pyrheliometer. During the service events, the data is flagged to indicate that it may be corrupted by the presence of the operator. The instrument also contains a relay, which switches a circuit in the event of a power failure. When this occurs, the voltmeter measures the voltage of the backup battery power supply (typically 13 v) rather than the voltage from the pyrheliometer, providing a record of power interruptions. A more complete description can be found in Ref. 10.

![Figure 2: A photograph of the tracker half of the GONG site survey instrument.](image_url)

The candidate sites were selected on the basis of longitude, facilities, and local interest in helioseismology. It was apparent from the model that the sites should be placed so that at least two stations are potentially observing at all times. If this is not so, then the diurnal sidelobes immediately reappear as soon as the single site covering a given longitude band suffers bad weather or instrumental failure. With two or more sites covering a range of longitude, the sidelobes are greatly attenuated. Another consideration was the presence of a developed site. Established astronomical observatories are preferable to non-astronomical sites, and institutions with active helioseismologists on the scientific staff are likely to have a higher interest level. The candidate sites currently providing observations are listed in Table 1 and shown on the map in Figure 3.

<table>
<thead>
<tr>
<th>Site</th>
<th>Longitude</th>
<th>Latitude</th>
<th>Start Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Big Bear, California</td>
<td>-116° 54.9'</td>
<td>+34° 15.2'</td>
<td>26 Aug 85</td>
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<tr>
<td>Cerro Tololo, Chile</td>
<td>-70° 48.9'</td>
<td>-30° 9.9'</td>
<td>8 Mar 85</td>
</tr>
<tr>
<td>Haleakala, Hawaii</td>
<td>-156° 15.4'</td>
<td>+20° 42.4'</td>
<td>7 Dec 85</td>
</tr>
<tr>
<td>Izaña, Canary Islands</td>
<td>-16° 29.8'</td>
<td>+28° 17.5'</td>
<td>23 Sep 85</td>
</tr>
<tr>
<td>Las Campanas, Chile</td>
<td>-70° 42.0'</td>
<td>-29° 1.5'</td>
<td>7 Mar 86</td>
</tr>
<tr>
<td>Learmonth, Australia</td>
<td>+114° 6.1'</td>
<td>-22° 13.2'</td>
<td>2 Dec 85</td>
</tr>
<tr>
<td>Mauna Kea, Hawaii</td>
<td>-155° 28.3'</td>
<td>+19° 29.6'</td>
<td>10 Dec 85</td>
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<tr>
<td>Tucson, Arizona</td>
<td>-110° 36.9'</td>
<td>+32° 14.0'</td>
<td>17 Jun 86</td>
</tr>
<tr>
<td>Udaipur, India</td>
<td>+73° 42.8'</td>
<td>+24° 35.1'</td>
<td>8 Nov 86</td>
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<tr>
<td>Urumqi, China</td>
<td>+87° 38.0'</td>
<td>+43° 43.0'</td>
<td>19 Dec 87</td>
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<tr>
<td>Yuma, Arizona</td>
<td>-114° 30.0'</td>
<td>+32° 40.0'</td>
<td>16 Jul 85</td>
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</table>
Figure 3: A map of the world showing the locations of the current candidate sites for the GONG Project.

An example of the raw data for one week in July 1987 from one site (Tucson) is shown in Figure 4. The characteristic extinction curve throughout the day is readily apparent, for instance, in the mostly clear days of 7/10 and 7/12. A very cloudy day is seen at the top on 7/15; a partly cloudy day is also evident on 7/11. Service events are marked by a plus symbol. The computed times of sunrise and sunset are indicated by ellipses, allowing correction for gross clock errors. In the case of the data on Fig. 4, it is clear that both the eastern and western horizons are obscured.

Figure 4: An example of raw data from one week in July 1987 at Tucson, Arizona.

Instrumental problems are flagged in the data as they are reported in log sheets kept by each site. The main problems that have occurred are battery backup power supply failures and wind-up cable breakages. The battery backup fails because of inadequate cooling in the box containing the data acquisition system. It is less expensive and disruptive to replace the components of the system on a yearly basis, rather than redesign and redeploy all of the units in the field. The wind-up cable eventually fatigues after continued exposure, and has proven vulnerable to animal attacks. The cables are replaced as they break.

3. DATA ANALYSIS

The data analysis begins with the reading of the micro-cassettes as they are returned from the field. Although some cassettes have been delayed, only two (out of over 500) have been lost in the mails. The cassettes are read using a cassette drive and calculator identical to those in the field instruments. Quick-look plots such as those in Fig. 4 are generated, and the log sheets read to determine instrumental problems. Any such outages are flagged after the raw data has been inserted into a compressed data file which holds the entire database for a given site. Points flagged in this way are later counted as "broken" minutes during which no cloud cover information is available. Gross clock errors are also corrected at this stage.

The next step is to correct for atmospheric extinction using software developed by Harry Jones. The data from each site are grouped into segments delimited by sunrise, local noon, and service intervals. An iterative algorithm fits the brightest data points with a parabolic least-squares fit to the logarithm of intensity as a function of airmass, and a corrected data file is formed by dividing the raw data by this "upper envelope" reference curve. The corrected data are truncated at zenith angles beyond which the airmass exceeds 10—typically removing about 30 minutes of data near sunrise and sunset. The upper envelope curves are subject to several tests, and those segments which fail one or more of these criteria are flagged for subsequent interactive editing by a data analyst. A window function is formed by applying a 90% threshold to the corrected data set. An example of the corrected data for Tucson is shown in Fig. 5, corresponding to the data in Fig. 4.

Figure 5: The data of Fig. 4 after correction for airmass.

A window for a single site is created by assigning the value of 1 to each clear minute, and a value of 0 for each dark minute. The power spectrum of this function is one half of the spectrum that would surround each real solar frequency if observations were to be obtained at that site. The window for the data in Figs. 4 and 5 is displayed in Fig. 6. Next, statistics for the individual sites are computed. A 45-minute length of time at the start and end of each day is ignored in order to avoid potential lost horizons. For each day, the start and end of all clear, dark and "broken" (as indicated by the downtime flag) times are recorded, and the lengths of the intervals are computed. The daily duty cycle (fraction of clear time) is computed for the non-broken time, and the counts are accumulated for the entire database for a single site. Next, the fractions of clear, dark, and broken time for the entire database are computed, as are the proportions of clear and dark non-broken time. These last figures represent the best possible estimate of the quantity \( \rho \) for a site. Monthly averages of \( \rho \) are computed. Searches are made for the longest strings of completely clear, completely dark, and completely broken days. These are defined as \( \rho > 0.95 \) for
Figure 6: The window for the data shown in Figs. 4 and 5.

completely clear, p < 0.05 for completely cloudy, and more than 95% broken time for completely broken. The distribution and statistics (mean, standard deviation, maximum and minimum) for both clear and dark time are then computed and displayed, as is the entire window for the site.

The window for a network is created by first choosing a set of individual sites. The set of possible networks is greatly reduced over what might be expected because there are few choices at each longitude. There are twelve possible networks that can be assembled from the sites in Table 1, excluding Urumqi. Urumqi is excluded only because data from the site has just recently become available, and the airmass correction for it has not yet been performed. After choosing the set, one minute is removed from both ends of each clear period at each site. This is done to compensate for the transition as the Sun enters and leaves clouds. Next, the network window is assembled by assigning a 1 to a minute if any site is clear, and a 0 otherwise. The number of stations observing at any minute is also counted. After the network window is assembled, dark segments of one minute are turned clear, and clear segments shorter than 5 minutes are turned dark. This is to allow for simple gap filling by interpolation over one-minute dropouts, and to dispense with data segments that are too short to be of use. For a network, the effects of instrumental problems are measured by defining "broken" time to be any minute during which the network is dark and at least one daylight station was inoperative. Statistics for the networks are computed in a manner similar to that for single sites, except that no time is ignored, and no daily cycle is used to define when the statistics will be computed. Daily duty cycles are still computed, but on the basis of 24 hour days.

Power spectra of the windows for both single sites and networks are computed. To quantify the quality of the spectrum, three figures of merit have been defined. All three figures are ratios of the DC component power in the window spectrum to average power in various frequency bands. They are thus roughly equivalent to signal-to-noise ratios, and reflect the relative effect of the window on a solar frequency. The first is the ratio of the DC power to the power in the frequency band of 0 to 60 μHz. This could be thought of as the background signal-to-noise ratio. The second figure of merit is the ratio of the DC power to the power in a 1 μHz band centered on the first diurnal sidelobe. Thus, this figure measures the influence of the most prominent sidelobe. The last figure of merit is the ratio of the DC power to the average power contained in 1 μHz bands centered on the first five sidelobes.

4. RESULTS

The window for Tucson is shown in Fig. 7. In this Figure, a background grid is plotted with each space in the grid representing one hour. The grid is 72 hours or three days wide. Space in the grid is filled in if the Sun was not visible at the time. Night is thus clearly seen as the three black swaths running horizontally across the plot. The seasonal variation of the length of the day is readily apparent.

The analysis for Tucson is shown in Figure 8. The monthly value of p is shown in this figure, along with various statistics. Tucson is measured to have clear skies 63.21% of the time, considerably lower than the "Chamber of Commerce"
Figure 8: The measured clear time fraction $\rho$ for Tucson. The figures on the right hand side are NDAYS: the number of days in the database, POS: the total possible number of sunshine hours, CPCT: the fraction of clear time for the entire period, DPCT: the fraction of dark time for the entire period, BPCT: the fraction of instrumental downtime for the entire period, CWPPCT: the fraction of clear time after removal of instrumental downtime, DWPCT: the fraction of dark time after the removal of instrumental downtime, CLR DAYS: the number of completely clear days, DK DAYS: the number of completely dark days, BK DAYS: the number of days completely lost to instrumental downtime, LSCD: the longest string of consecutive clear days followed by the date it started on, LSDD: the longest string of consecutive dark days and the starting date, LSBD: the longest string of consecutive broken days and start date, and MTBF: the mean time between instrumental failures in days.

Figure 9: The power spectrum of the window in Fig. 7. The power is normalized to the DC component, and the logarithm taken. The figures at the right of the spectrum are: SNRB, the ratio of the DC power to the average power in the band 0 to 60 $\mu$Hz, SNFSL, the ratio of the DC power to the average power in a 1 $\mu$Hz band centered on the first diurnal sidelobe, and SNASL, the ratio of the DC power to the average power in 1 $\mu$Hz bands centered on the first five diurnal sidelobes.

The power spectrum of the first 60 $\mu$Hz of the window at Tucson is shown in Fig. 9. The power has been normalized to the power in the DC component, thus the height of the diurnal sidelobes represents their relative strength compared to any solar amplitude. The sidelobes are very prominent, with the first sidelobe having a log normalized power of -0.3, representing a 5% amplitude relative to any solar spectral line. The background noise is at a level of about $10^{-4}$.

The window for one of the twelve possible six-site networks is shown in Figure 10. The day-night cycle has virtually vanished, leaving only random weather. The monthly duty cycle for this network is shown in Figure 11. The distribution of clear time for this network is shown in Fig. 12, and the distribution of dark time in Fig. 13. The statistics show a maximum length of unbroken clear time of 416.23 hours, some three times longer than that achieved at the South Pole. The maximum length of dark time was only 12.82 hours.
The observed duty cycle for the six-site networks of about 93% compares remarkably well with the model prediction of 94%. The model does not reproduce the seasonal variations apparent in Fig. 11. The use of only two values for $p$ is too coarse to allow the modeling of the seasonal fluctuations due to the unsettled Boreal spring weather in April and the widespread monsoons in the late Boreal summer. The model will be improved to allow the assignment of a monthly value of $p$.

Observations will begin at two additional sites in Saudi Arabia and Morocco in the near future. The final selection of the sites comprising the GONG network will be made about one year before the Doppler instruments are placed in the field. Until that time, the data for the individual sites are returned only to the owners, so that they may control its use. The site survey data will also be used by GONG to estimate the spectrum of transparency variations in various frequency bands that will impact the observations. The site survey has proved to be of enormous benefit, not only in the validation of the model, but also in the establishment of international cooperation essential to the success of GONG.
ACKNOWLEDGEMENTS

The GONG project is composed of many people. At last count, more than 120 people around the world were contributing to the site survey alone. While it is not possible to list them all, the following table names the more significant contributors.

<table>
<thead>
<tr>
<th>Name</th>
<th>Site</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ashok Ambastha</td>
<td>Udaipur</td>
</tr>
<tr>
<td>Warren Ball</td>
<td>NSO</td>
</tr>
<tr>
<td>Oscar Duhalde</td>
<td>Las Campanas</td>
</tr>
<tr>
<td>Don Farris</td>
<td>NSO</td>
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<td>George Fischer</td>
<td>NSO &amp; Utah</td>
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<td>Les Hieda</td>
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<td>Frank Hill</td>
<td>NSO</td>
</tr>
<tr>
<td>Huang Zhen</td>
<td>Urumqi</td>
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<tr>
<td>Bob Ingram</td>
<td>Yuma</td>
</tr>
<tr>
<td>Patty Jackson</td>
<td>NSO</td>
</tr>
<tr>
<td>Harry Jones</td>
<td>NSO &amp; NASA</td>
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<tr>
<td>Wayne Jones</td>
<td>NSO</td>
</tr>
<tr>
<td>John Kennewell</td>
<td>Learmonth</td>
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<tr>
<td>William Kunkel</td>
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<td>Renate Kupke</td>
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<tr>
<td>Barry LaBonte</td>
<td>Haleakula &amp; Mauna Kea</td>
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<td>Ken Libbrecht</td>
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<td>Wayne Lu</td>
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<td>Pere Pallé</td>
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<td>Tuck Stebbins</td>
<td>NSO &amp; JILA</td>
</tr>
<tr>
<td>Xiao Suming</td>
<td>Urumqi</td>
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</table>

REFERENCES


THE GONG DATA REDUCTION AND ANALYSIS SYSTEM

James A. Pintar and the GONG Data Team

National Solar Observatory
National Optical Astronomy Observatories
Tucson, Arizona, USA

ABSTRACT

Each of the six GONG observing stations will produce three, 16-bit, 256x256 images of the Sun every 60 seconds of sunlight. These data will be transferred from the observing sites to the GONG Data Management and Analysis Center (DMAC), in Tucson, on high-density tapes at a combined rate of over 1 gigabyte per day. The contemporaneous processing of these data will produce several standard data products and will require a sustained throughput in excess of 7 megaflops. Peak rates may exceed 50 megaflops. Archives will accumulate at the rate of approximately 1 terabyte per year, reaching nearly 3 terabytes in three years of observing.

Researchers will access the data products with a machine-independent GONG Reduction and Analysis Software Package (GRASP). Based on the Image Reduction and Analysis Facility (IRAF), this package will include database facilities and helioseismic analysis tools. Users may access the data as visitors in Tucson, or may access DMAC remotely through networks, or may process subsets of the data at their local institutions using GRASP or other systems of their choice.

Elements of the system will reach the prototype stage by the end of 1988. Full operation is expected in 1992 when data acquisition begins.

1. INTRODUCTION

The Global Oscillation Network Group (GONG) is a helioseismic data acquisition, processing, and analysis project. The project is actively supported by the GONG research community that consists of over 150 members who are located throughout the world. GONG will establish a six-site, world-wide network of automatic solar oscillation observing stations, and a data reduction and analysis facility to process the acquired data.

Each station in the six-site network will produce three, 16-bit, 256x256 images of the Sun every 60 seconds of sunlight. Each of these three images represents one phase of a three-sample Fourier Tachometer (Ref. 1, 2). Each triplet of phase intensity images is transformed to velocity, intensity, and modulation (line strength) images of the sun.

Data acquisition is scheduled to begin in 1992 and continue for 3 years. Data will be transferred to the analysis center in Tucson on high density tape at the rate of about 1.5 gigabytes per day. Pipeline data reduction will proceed contemporaneously with the data acquisition. The pipeline processing will produce several standard Data Products. These data along with the field data will be archived in Tucson. Data in the archive (summarized below for 3 years of acquisition) will accumulate at a rate of about 1 terabyte per year.

<table>
<thead>
<tr>
<th>Data Product</th>
<th>Gigabytes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw Field Data</td>
<td>1500</td>
</tr>
<tr>
<td>Intensity Time Averages (600s)</td>
<td>150</td>
</tr>
<tr>
<td>Merged Velocity, Intensity, Modulation</td>
<td>500</td>
</tr>
<tr>
<td>Magnetograms</td>
<td>2.5</td>
</tr>
<tr>
<td>Velocity Time Averages (1-hour)</td>
<td>2.5</td>
</tr>
<tr>
<td>Day, Month, &amp; 3 Year Time Series</td>
<td>500</td>
</tr>
<tr>
<td>Day, Month, &amp; 3 Year Power Spectra</td>
<td>250</td>
</tr>
<tr>
<td>Day Mode Frequencies</td>
<td>10</td>
</tr>
<tr>
<td>Month Mode Frequencies</td>
<td>0.75</td>
</tr>
<tr>
<td>3 Year Mode Frequencies</td>
<td>0.025</td>
</tr>
<tr>
<td>Total</td>
<td>2900</td>
</tr>
</tbody>
</table>

The time-averaged intensity data has been added to the Data Products list since the correction and merging for the analysis of medium frequency p modes may not be appropriate for low-frequency p modes and g mode searches.

Currently, potential sites are being surveyed to determine a global network for optimum observing of the sun; science teams have been organized and are actively supporting the data acquisition and processing projects and preparing for the analysis of the processed data products; the development of the data acquisition instrument is nearing the prototype stage; and the development of the data reduction, analysis, and archiving facility is underway.

The GONG Data Reduction and Analysis project will acquire and operate the computer hardware for the Data Management and Analysis Center (DMAC), develop and support the data reduction and analysis software, process the GONG field data to produce the Data Products, and support the GONG community's access to and use of the archived data (Ref. 3).

This report provides a summary of the GONG computer system project including a discussion of recent project activity, and additional details that pertain to those aspects of the system that are currently being developed.
2. PROJECT SUMMARY

2.1. Hardware

The DMAC will be a dedicated, central, data reduction and analysis facility at NSO-Tucson consisting of:

- Workstations for interactive analysis and display of the archived data and quality assessment of products from the pipeline.
- Mini-computer system(s) for reading the field data tapes.
- Vector processor(s) with adequate IO and arithmetic capacity for GONG's pipeline processing.
- An accessible data archive, the associated removable media library, and off-site storage of copies of the data.
- Local network interconnecting the various processors.
- Access to global computer networks.

2.2. Software

The software products which are being developed for GONG include:

- GONG Reduction and Analysis Software Package (GRASP) which will be a portable, interactive, image processing package including the interface to the Data Products in the ARCHIVE, a collection of data display and analysis tools, the image processing functions of PIPE, and a user friendly layer of software with menus and formatted screens to provide a level of integration above the highly functional image processing modules.
- The processing software for the pipelined, bulk reduction of GONG field data (PIPE). PIPE generates the Data Products for the ARCHIVE.
- The software which supports the storage, maintenance, and access of the raw data and Data Products in the ARCHIVE.

2.3. Processing

The project intends to process 1.5 terabytes of raw data in 3 years. To do so, a processing team will need to be assembled; processing procedures with realistic quality control will need to be developed; and appropriate equipment will need to be acquired. These items will probably be addressed in the 1991 time frame. Presumably, processing will include:

- Receiving the removable media containing raw data from the field sites.
- Initial raw data tape read and quality assessment.
- Processing the raw data to produce the mode frequencies and other data products.
- Maintaining the data storage facility.
- Copying and shipment of data products for off-site storage.
- Copying and shipment of data products to remote users.

2.4. Support of Analysis

The project will support both local and remote user access to all of the hardware and software resources of DMAC for both batch and interactive use. To reduce the amount of duplicated effort in writing programs, to facilitate comparisons of different algorithms, and to generally make available programs that are applicable to helioseismology; the project will support both local and remote user access to the libraries of reduction and analysis software; and the transfer of any of this software to the user's site via mail and electronic file transfer. These will include:

- Software supported by NSO representing a "one each" selection of basic reduction and analysis programs.
- Unsupported self-documented source code contributed by members of GONG and provided as is.

The project will support the transfer from DMAC to user sites of raw data and Data Products via mail (shipped removable media) or electronic file transfer.

For GONG members who elect to use DMAC locally, the project will provide various types of visitor assistance including lodging, space and supplies, computer resources: time and disk space, system and application software documentation, and assistance in the use of the system.

2.5. Recent Activity

One of this year's objectives is to begin assembling a usable, prototype version of GRASP inside NOAO's Image Reduction and Analysis Facility (IRAF). (In this context, GRASP means an interactive version of the data reduction capability that will eventually become the pipeline. This will be one or more packages inside IRAF. It will also include defining the GRASP data base(s) for raw data and data products inside IRAF, and some work on defining the files in the ARCHIVE and the interface for loading the GRASP data base from the ARCHIVE.) This is being driven by the activities of the GONG Artificial Data Science Team, interest from the GONG Inversion Science Team in the file definitions for the downstream data products, and images from the GONG breadboard instrument.

The Artificial Data Team has produced detailed artificial images of the sun including oscillation and (nearly) steady-flow velocity fields (Ref. 4). These images have been distorted by simulations of light scattering, atmospheric affects, and instrumental affects for two observing sites: Sac Peak and Haleakula. Prototype modules (inside the IRAF environment) have been assembled to determine the solar limb ellipse parameters; to convert the three intensity images to velocity, intensity, and modulation; and to calculate and apply ephemeral velocity corrections.

Since one of the principal products of the GONG data reduction will be tables of mode frequencies, these tables, rather than the move voluminous intermediate Data Products, will likely be the focus of much of the attention of the GONG scientific community. A software package developed by Space Telescope Science Institute (STScI), STTABLES, has been used for the mode frequency database inside IRAF. Functions include storage, inspection, editing, plotting, arithmetic, and selection of mode frequencies. In July 1988, this implementation of the mode frequency database, along with IRAF, was installed and demonstrated to the participants at the GONG Inversion Workshop at JILA in Boulder. If the capacity and performance of STTABLES can be increased to accommodate GONG-sized tables, this implementation of the mode frequency database will become the first fully usable feature in GRASP. Other alternatives are the IRAF database that may be available in the future or a proprietary database: ORACLE, INGRES, etc. STTABLES has the advantages of being available and usable now, and of being public domain software. While this matter is being resolved, work

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ORACLE is a registered trademark of Oracle Corporation.
is proceeding with the development of the code for extracting mode frequencies from the power spectra.

A second objective is to begin the development of the data storage and management facility at DMAC (ARCHIVE). Since the ARCHIVE will make extensive use of removable media, many of the software and operational procedures for the ARCHIVE will be hardware dependent. The appraisal of media alternatives and the development of the systems which depend on removable media needs to begin.

Current efforts are directed toward establishing that Exabyte drives and 8mm cartridges will provide the reliability adequate for GONG's data acquisition. The cost of the drives, the media cost/byte of recorded data, data density, and data transfer rates appear to make Exabyte an ideal device for GONG's data acquisition. Read rates of 60-120 KBytes/s (KB/s) (which is adequate for the tape reader system) are routinely achievable on a SUN 3/160. Only the reliability issue remains unanswered. The hardware for a system to read the field acquired data tapes is being assembled. It will be used to verify that the prototype field tape reader system provides adequate performance and reliability and to develop the software for reading and assessing the quality of the field data tapes.

The issues of removable media for processing the data, for storage of the Data Products in the ARCHIVE, and for shipment of Data Products to remote sites will be addressed in the future.

3. ARCHIVE

A system will be developed to store, maintain, and access the Archivable Data Products. This system will consist of three major components.

The first component is the DMAC Entry and Quality Assessment (QA) of the field data; i.e., the Field Tape Reader, which will include the manual procedure for a site tape arriving at DMAC and the initial tape read and QA of the field data plus utilities for diagnosing problem tapes. The QA parameters calculated during the read will be archived.

The second component includes items related to maintaining the data storage facility:

- Staging and Destaging of Data Products for PIPE.
- Staging of Data Products for GRASP.
- Backup of On-line files other than Data Products; e.g., GRASP user files.
- Reports on Stage and Destage activity.
- Reports on contents of the ARCHIVE: What's on-line, near-line, off-line.
- Copying and shipment of Data Product tapes to off-site storage.
- Reports on the contents of off-site storage.

The third component is the DMAC Exit which will copy subsets of Data Products and ship the tapes or electronically transfer these files to remote sites.

3.1. Field Tape Reader Function

The Field Tape Reader System will perform three basic functions: certify the Exabyte tapes (which will be eliminated if proven to be unnecessary), supply the field sites with tapes, and the tape read and QA of the field data.

Exabyte tapes may need to be certified before being distributed for use at the field sites and for use in the DMAC. This results from the lack of operational experience with the Sony 8mm video tapes as a digital recording media (and consequent low level of confidence) and the dire consequences of supplying a field site with a batch of bad tapes. Some field sites may not have convenient access to retail video supply stores. Certification might consist of writing to and then reading from the entire volume. In addition, it may be necessary to label each tape with a volume serial number by placing a label on the cartridge and by writing a matching, machine readable label on the tape.

In the description of the DMAC Entry function which follows, significant data files are identified, once, by bold type.

Each site will record 2 copies of each field tape. One copy will be mailed to DMAC, the other will be retained at the site. The field tapes from each site should arrive on a regular schedule, say, once a week. If a field tape from a site does not arrive on schedule, a request will be made to the site to copy the field tape retained at the site and mail the copy to DMAC. Once the field tape arrives, an entry will be made in the field tape arrival log. Field acquisition observer reports may need to be reviewed and keyed to be machine readable.

The tape will probably need to be inspected for physical damage. (Although there have been no physically damaged tapes from the GONG site survey, the possibility of physical damage cannot be overlooked, since the project will produce and mail many field tapes.) If the tape is physically damaged or otherwise unreadable, a request will be made to the site to copy and remail the tape. If site tape copy is not possible, bad tape fix-up capability will be needed to salvage usable data from the tape. Utilities to diagnose problem tapes will probably be needed.

A field tape copy will be made. The copy will be cataloged in the off-line catalog where it will be retained. The site tape will be subsequently sent to off-site storage with an entry into the off-site catalog. A disk copy of the field tape will be made and the headers will be extracted. QA parameters will be calculated from the image data. The field tape headers and QA parameters will be copied to on-line storage in the on-line ARCHIVE for permanent retention with an entry in the on-line catalog. The disk copy of the field tape, the headers, and the QA parameters will be temporary files for human appraisal of the field tape and will be deleted after the appraisal.

The calculated QA parameters may include rms, minimum, and maximum pixel values for rows, columns, or images and the limb ellipse parameters. A machine examination of the header and QA parameters may be instrumental in automatically identifying a malfunctioning instrument or poor sky conditions that could produce unusable images.

The human aspects of the field tape QA will involve examining hardcopy or electronic prints of header and QA parameters, graphic displays of header and QA parameters, displays of field tape images (random selection, selection at a fixed time increments, and short "movies". These can be provided by IRAF). The output from this activity would be frame and pixel kills that will become part of the processing "deck" for GONG Corrections in PIPE.

3.2. Field Tape Reader Hardware

The hardware should have enough IO and CPU capacity to do the processing twice in a 40 hour week (about 10 times the real time rate; i.e., 3 years of raw data are processed in
This is based on the assumptions that rarely are non-trivial technical tasks performed correctly the first time; that the machines will not operate themselves, so humans are necessary; and few humans work 40 hours a week after subtracting normal distractions like vacation, holidays, illness, etc.

For the non-processing operation (i.e., tape certification), the equipment must have enough IO and CPU capacity to do the required operations once in a 40 hour week. This is about 5 times the real time rate.

Exabyte media and drives have a capacity of 2.3 GBytes (GB) and a transfer rate of 256 KB/s. Assume that during use they provide 128 KB/s with a capacity of 1.5 GB.

The field data acquisition network will record 1.5 TB of data in 3 years that will be shipped to the DMAC on 1000, Exabyte tapes (the tapes are about 50% utilized with one tape per site per week). The Field Tape Reader system will read and QA the field data tapes and will certify the Exabyte tapes for field acquisition (2000 tapes) (1000 shipped to DMAC, 1000 retained at sites), and for the field tape copies during the field tape read (1000 tapes, again assuming 50% capacity utilization). A total of 3000 tapes will be certified.

The tape certification could be performed at night. 3000 Exabyte tapes need to be certified, with 2 GB per volume. Since each tape is written then read, the Exabyte tape IO is 12 TB at 5x's the real time rate or 600 KB/s of tape IO.

This would amount to 4 concurrent jobs with 1 Exabyte drive per job and 4 Exabyte mounts per night.

The field tape read and QA would be performed during the day and would consist of reading 1.5 TB from Exabyte tapes, writing 1.5 TB to Exabytes for ARCHIVE copies, writing 1.5 TB to disk for QA, calculating the QA parameters that may require 10 Floating Point Operations (FLOP’s) per pixel, and performing the human QA tasks that may be equivalent to a disk to disk copy of the data. The above items result in 300 KB/s of Exabyte tape IO, 450 KB/s of disk IO, and 0.8 MFLOP/s of arithmetic at 10x’s the real time acquisition rate. This might be considered as two concurrent jobs: a field tape read requiring 2 Exabyte drives and 2 GB of disk, and a series of tasks to QA the previous field tape requiring 2 GB of disk. The tape read and copy might require 6 tape mounts per day.

<table>
<thead>
<tr>
<th>Function</th>
<th>MFLOP/s</th>
<th>Tape KB/s</th>
<th>Disk KB/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tape certification</td>
<td>0</td>
<td>600</td>
<td>0</td>
</tr>
<tr>
<td>Tape Read &amp; QA</td>
<td>0.8</td>
<td>300</td>
<td>450</td>
</tr>
</tbody>
</table>

Equipment to provide this capacity might consist of two similar systems each consisting of a SUN 3 (with 8MB, Floating Point Accelerator, and monitor for job control and QA functions), Ethernet connection, 2.5GB of disk, 2 Exabyte drives, and a printer/plotter, and 3000 Exabyte tape cartridges and spare Exabyte drives.

3.3. Field Tape Reader Development Plan

The immediate objective is to develop a high degree of confidence that the Exabyte drives and 8mm cartridges will be able to provide the necessary performance, capacity, and reliability. By mid-89 the results of hands-on experience, reports from other helioseismic acquisition projects, and reports from other applications should provide enough information to instill confidence in the use of these devices or cause them to be discarded. In this regard the project has acquired two drives (from Perfect-Byte) which are being routinely exercised. In addition, the project will need to obtain a failure rate estimate for the drives and media. If the media proves to be highly reliable, tape certification will be eliminated with a corresponding reduction in hardware.

The next objective is to obtain a realistic estimate for the computer resources needed for the Field Tape Reader system. A complete Field Tape development system will be assembled. A prototype version of Tape Read & QA will be developed and exercised to determine a realistic hardware specification. Ideally, no additional hardware beyond the development system will be needed. If this is not the case, additional equipment will need to be installed before the start of data acquisition.

4. GRASP

GONG's Reduction and Analysis Software Package (GRASP) will be an interactive, image processing application that will be available for both local and remote use at DMAC, and for use by scientists at their home institutions.

GRASP will provide the following features:

- IRAF/GRASP; i.e., data and tasks inside of IRAF including access to the on-line Data Products.
- The capability to access the archived data products and to create and manage a local copy of a subset of the archived data products.
- A user friendly layer of software which would provide a level of integration on top of the highly functional components found in IRAF/GRASP and access to the archived data products, maybe with menus and formatted screens.

IRAF/GRASP will be an interactive image processing application within the IRAF software environment:

- Reduction Software - data reduction, image processing, software. IRAF/GRASP Reduction Software will overlap functionally with (be a superset of) the image processing capability of PIPE.
- Directory - facility for inspecting what is in the GRASP data base(s) (database(s) inside IRAF).
- Facility for loading the GRASP data base(s) from the ARCHIVE.
- Facility for unloading the GRASP data base(s) to non-IRAF dependent formats.

4.1. IRAF/GRASP Reduction Software Functional Summary

IRAF/GRASP Reduction Software will be one or more packages of interactive image processing modules inside IRAF that convert GONG raw data into mode frequencies. Possible uses of the package would include the scientific analysis of data products produced by PIPE; and the development of algorithms and processing procedures to be integrated into PIPE.

IRAF/GRASP Reduction Software will be an interactive application providing functionality and flexibility and oriented towards processing and examining single images or short sequences of images. As an aid to reconstructing processing sequences, processing histories (the list of modules and parameters through which the data has been processed) will be retained in the IRAF image headers.

The Reduction Software consists of 3 packages of modules: GONG Corrections, Spatial Transforms, and Time Series
Processing; and the Mode Frequency Data Base. Each module in each package processes one or a series of disk resident images or time series. The packages and modules will be consistent in style and usage with other image processing elements of IRAF. This includes help screens, parameter lists, and parameter editing menus.

GONG Corrections will calculate and apply photometric and geometric corrections and merge the images, recorded contemporaneously at multiple sites, into a single time series of velocity, intensity, and modulation images. This package will also provide the capability to extract, correct, and merge magnetograms.

The Spatial Transforms package will provide time filtering and resampling for producing time averaged velocity images, the spherical harmonic transform, and a 2D Fourier transform. The transposition of a time series of complex valued images to a collection of complex valued time series would also be part of this package.

The Time Series Processing will fill time gaps, compute power spectra, and identify the mode frequencies (frequency, power, width). A modeling programs for computing synthetic time series from power spectra and synthetic power spectra from mode frequencies may also be included in this package.

The Mode Frequency Data Base will provide the facility for examining the mode frequency parameters. This will include data base queries, listings and graphical displays, and arithmetic.

4.2. IRAF/GRASP Reduction Software Development Plan
The items currently being developed are GONG instrument and atmospheric corrections, the time series processing (the beginning and the end of GRASP’s image processing capability), and the mode frequency database.

The Time Series Processing Package involves three Data Products: complex-valued time series, power spectra, and the mode frequencies, and image processing to produce the mode frequencies from the time series.

The mode frequency data base may be used in several different ways. Standard Solar Model modeling programs could be used to produce theoretical mode frequencies that might be stored in the data base for later comparison with real mode frequencies. Also, theoretical mode frequencies might be used as input to test the effectiveness of inversion algorithms. Lastly, this data base would hold the real mode frequencies for access by the inversion algorithms that would produce physical parameters describing the interior of the sun.

The file format for the mode frequencies in the ARCHIVE will be ASCII with some text to identify the source of the frequencies and to identify the data elements. The use of ASCII has two large advantages over binary. ASCII files can be copied between computers via SPAN or by using FTP via INTERNET or by using email without the nuisance of encode and decode. The file format should be self-evident if the file is printed. ASCII files present minimal barriers to access and transfer.

The 3-year Mode Frequencies will be a 25 MB ASCII file, stored on-line in the ARCHIVE. A user at DMAC could easily transfer this file via Ethernet to a workstation with a hard disk. A user at a remote site may have to request a tape copy unless science networks improve significantly in the future.

After the file is resident on the user’s machine, the user might analyze or invert the ASCII file, as is, or to inspect, edit, and extract subsets using editing tools available from the native OS (e.g., vi, sed, awk for UNIX). Alternatively, the user might enter the IRAF environment, load the Mode Frequency Data Base, edit and inspect (graphs and listings) the mode frequencies, and extract subsets using STTABLES, then unload the subsets to ASCII files before proceeding with the inversion. Since IRAF supports scripts and escapes to the native OS (e.g., to compile and execute the inversion algorithm) the latter scenario can be very efficient for repetitive editing and inversion.

5. SUMMARY
The GONG project is actively developing a computer system for reducing and analyzing the helioseismic data which will be acquired by a world-wide network of observing stations. Application software is being developed and hardware will be installed such that this computer system will be operational at the start of data acquisition in 1992. The following schedule provides key dates for the project.

- Jan. 1988 Begin GRASP development
- Jan. 1990 Begin PIPE development
- Oct. 1991 DMAC Hardware Ordered
- Mar. 1992 Data Center Operational
- Oct. 1992 Begin GONG Data Acquisition
- Sept. 1995 Data Acquisition Complete

Acknowledgements. The following table lists the names and institutions of those who have actively contributed to the work described in this report. This list includes members of the GONG Science Team for Data Reduction and Analysis and members of the GONG Project programming staff.

<table>
<thead>
<tr>
<th>Name</th>
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6. REFERENCES
2. J. W. Harvey, The GONG Instrument, this volume.
A DETECTOR OF SMALL GRADIENTS OF TRANSPARENCY OF THE TERRESTRIAL ATMOSPHERE

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ABSTRACT

In the study of low $\ell$ $p$-modes of the Sun in integrated sunlight with ground based spectrometers even small ($10^{-3}$) gradients in transparency across the solar disk give rise to displacements of centroids of spectral lines at the m/s level.

We describe a prototype instrument which measures this atmospheric gradient continuously and thereby makes it possible to subtract these artefacts from the measurements.

Keywords: Solar Oscillations, atmospheric artefacts, Extinction gradient, Monitor.

1. THE EFFECTS OF THE TERRESTRIAL ATMOSPHERE

The effects of the Earth's atmosphere on solar observations are only too well known to ground based observers. Many of the more adverse effects can be avoided by siting on high mountains at exotic locations. However, even some of the highest of observing stations, such as Haleakala, still have 60 - 70% of the Earth's atmosphere, by mass, above them. With the Sun's equatorial velocity of 2 Km/s a gradient of only 0.05% across the solar disc will give rise to a 1 m/s displacement in the centroids of velocities of the Solar Fraunhofer lines. These spatial variations will therefore add to the noise level on any Doppler velocity measurements of the unresolved Sun. It is even possible that artefact frequencies could result if the spatial structure of extinction is periodic, as in certain types of cirrus and alto-cumulus cloud formations or in skies criss-crossed by barely detectable vapour trails. This consideration was of concern during data taking at Pic du Midi in 1974 and in the rejection of Flagstaff, Arizona, as a potential observing site.

Another, more predictable, atmospheric effect is the natural gradient in the hours after sunrise and before sunset due to the path differences through the atmosphere of the 'upper' and 'lower' portions of the disc. See figure 1. This gives variations in the velocity residuals as shown in figure 2, of sample data. Numerical corrections are possible giving results such as in figure 3. These, however, rely on assumptions of a steady laminar atmosphere whereas inspection of the sky often reveals the presence of a stratified structure due to dust or haze layers.

2. THE APPARATUS

There are several stages of design imaginable for such a device. The simplest, as described here, involves the measurement of the mean gradient along one axis of the Sun. In a final version this axis would be the maximum velocity axis, i.e., perpendicular to the solar rotation axis. At present, while under development, the device measures along the R.A. axis, mounted on a servo-controlled equatorial mount with, thus, a +/- 26 degree variation from the maximum velocity axis during the year. A later stage of design would involve the measurement of the mean gradient along two orthogonal axes.

There are three conceivable methods to measure the gradient of transparency across the solar disc.

i) Having a stationary image of the Sun and a line of detectors.

ii) A stationary image and a single scanning detector.

iii) A single stationary detector with a scanning image of the Sun.

Use of a single detector was chosen because the intensity variations to be measured are of the order of $10^{-5}$, and continuous intercalibration of the different detectors in any array of this level would be difficult. Using a fixed single detector and a rapid sweep of the solar disc image across this avoids these instrumental problems. Thus method (iii), is used.
Figure 2. Example data for one day showing: i, the relative radial velocity between the sun's surface and the observer, as measured by a spectrometer, ii, the residual velocity once corrected for orbital and rotational velocities. Thus showing the effects of extinction near sunrise and sunset.

Figure 3. The same data after numerical correction for extinction.

The device at present under development is based upon a commercially built mechanical scanner. A steel pin is driven to oscillate, in a twisting manner, at its resonant frequency of 800 Hz and at an amplitude of twist of about 14° in angle. A mirror on top of this scanner collects sunlight from an aperture in an otherwise light tight box. The light is then directed through a system of lenses to form an oscillating image on the detector plane. The image falls symmetrically upon a silicon photodiode framed by two knife edges, perpendicular to the direction of scan, to give a narrow detector slit.

The signal, resulting from the sinusoidal sweep of a portion of the Sun's disc over the central detector, is fed together with a reference signal, to a phase sensitive detector (PSD). The PSD effectively compares the two halves of the sweep, any difference in intensity between these two halves indicates a gradient across the disc, provided the scanning falls exactly symmetrical across the detector. This in turn indicates an atmospheric extinction gradient (assuming, reasonably, that the Sun's intensity profile is symmetrical). There will also be a component of the signal at twice the frequency due to limb darkening, which can be separated by the PSD. At present the signal from the PSD is averaged and stored over 1 second intervals by a microcomputer.

Stability in the amplitude of scan is important in such a system. A feedback from the scanning device allows a phase locked loop, generating the driving signal, to lock onto the resonance frequency of the scanner and follow it through ambient temperature changes. This frequency servocontrol alone gave high stability in amplitude. Further amplitude servocontrol has been attempted using two secondary detectors equally spaced on either side of the central detector. With the edge of the Sun's disc just touching them at the limits of its sweep, the signal from these side detectors has a high sensitivity to the amplitude of swing and is used to control the gain of the driving signal.

Signals from the side detectors will also be used to feed an image positioning servo. This will allow for shifting in the image position caused by variation in the refraction through the Earth's atmosphere or due to the equatorial mount servos and other effects.

A block diagram of the control electronics is given in figure 4.

The amplitude of the driving signal is set, by the positioning of the two side detectors, to scan 2/3 of the Sun's disc across the central detector. This is a compromise between measuring the true gradient across
A DETECTOR OF SMALL GRADIENTS OF TRANSPARENCY

3. RESULTS

The device has been run in the laboratory, with illumination by laser, to test the stability of the scanning amplitude. With both frequency and amplitude servos in operation the amplitude of swing of the reflected beam showed a drift of less than 8" in a total swing of 27" during a 15 hour run. This is sufficiently small. However, problems were met with the amplitude servoing system when overall incident intensities varied, as would be the case with the Sun, and an alternative, non optical, system is being considered.

Stability in the image positioning servo will be particularly important but this is still being developed.

Unfortunately the climate of Birmingham has not allowed any preliminary measurements of variation in extinction gradients near sunrise, sunset or in the vicinity of tenuous clouds. Thus there are, as yet, no interesting results to present.

4. USE IN PARALLEL WITH A SPECTROMETER

In the final version the gradient monitor would work alongside one of our spectrometers utilising the same data storage system. Since the monitor has to correct for atmospheric effects that affect the spectrometer, it should operate at the same central wavelength and a filter will be introduced into the system.

Calibration of the system will involve comparison of the A.C signal, given by a gradient as the disc is scanned back and forth across the detector, (and extracted by the PSD), with the D.C. signal representing the overall solar intensity. Calculation will then give the velocity shifts induced by the terrestrial atmosphere. Rough tests of the system will be carried out using glass plates with known gradients of transmission.

5. CONCLUSION

As mentioned earlier, correction of Doppler velocity measurements for atmospheric effects by use of a gradient monitor has two major benefits;

i) By correcting the effects produced near sunset and sunrise longer usable data stretches could be obtained each day.

ii) By correcting for random (and also potentially periodic) effects during the day the background noise level on the power spectra achieved could be reduced.

This is particularly important at low frequencies where the noise level, at present, may be hiding the low order acoustic, and possibly gravity, modes. These modes, reaching far deeper into the solar interior, would provide additional constraints on solar models.

The amplitude for the fundamental p mode is expected to be of the order of 1 to 5 mm/s and may show up when the noise level is reduced to a sufficiently low level.

![Figure 4. Schematic diagram of the Gradient Monitor system](image)
THE STABLE SOLAR ANALYZER

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ABSTRACT

This paper presents a progress report on the development of an instrument with very high (1·10^-10) wavelength stability designed to measure solar surface velocities and magnetic fields. The instrument determines Doppler and Zeeman shifts in solar spectral lines by a 6-point weighted average. It is built around an electrically tunable solid lithium-niobate Fabry-Perot étalon that is stabilized against a diode laser which itself is locked to a resonance line of Cesium 133. Key features are the unique étalon, which acts as a wide-angle 0.017-nm solar filter, the camera with a specially stabilized shutter, and the instrument control and data collection system. Use of the instrument in helioseismological research is emphasized.

Keywords: Solar Observations, Helioseismology, Fabry-Perot Filter, Astronomical Instrumentation.

1. SCIENTIFIC OBJECTIVES

1.1 Emphasis on Helioseismology

The scientific studies that may be undertaken with the recently redesigned and rebuilt Stable Solar Analyzer (SSA) range from the internal structure of the sun to chromospheric (He 10830) signatures of the structure of the solar corona. But our attention in the next few years will likely focus on the problems of the deep interior solar structure and magnetic field evolution. By measuring the surface oscillations and the surface magnetic flux distribution, we hope to achieve a better understanding of large-scale magnetic field distribution below the photosphere.

For helioseismological studies the SSA should provide unique data at the lowest global oscillation modes, which are difficult to detect in an imaging instrument unless it has extremely high wavelength stability. The objective is to improve our understanding of the structure and dynamics of the deep solar interior.

Non-imaging instruments can provide low-l data, but they cannot always measure individual m components. When the frequency components associated with separate m components are distinct, integrated sunlight measurements provide the needed information. But, for modes whose lifetime is so short that the individual frequency peak width becomes comparable to the peak separations, the m components will be blended. The SSA images will be filtered to favor individual m components so that precise frequencies can be obtained at all l.m. Such precise frequency measurements should allow study of the depth dependence of differential rotation near the solar core.

1.2 Spectral Line Selection

With a Fabry-Perot filter, one is free to select practically any line of the solar spectrum for Doppler and Zeeman effect measurements. For helioseismology the main criterion is to minimize spurious Doppler shifts. The line formerly used by the SSA is a Ca I line at 610.27 nm which is useful for magnetic field measurements because it has a large Lande' g-factor, but it is not an optimum choice for helioseismology. Also, there are many other strong lines in the adjacent spectrum. It would be better to have a flat continuum surrounding the line, in order to minimize spurious effects from lines on the shoulders of the Fabry-Perot blocking filter.

After having examined the entire solar spectrum, for lines which are broad enough for study with our filter and which have a small g-factor, so as to minimize spurious Doppler shifts, we found that the cleanest (in terms of long stretches of clean, surrounding continuum) portion of the solar spectrum contains two lines with ideal properties for our scientific objectives. One is Fe I 778.057 nm which has an excitation potential of 4.47 eV and a relative line depth of about 61%. The other line is Ni I 778.893 nm which has an excitation potential of 1.95 eV and a relative line depth of about 54%. This line is very similar in its characteristics to the line selected for the Global Oscillation Network Group (GONG) project. Ni I 676.778 nm, which has an excitation potential of 1.83 eV and a line depth of about 62%. It was selected for GONG largely because of its good line bisector behavior.

The Lande' g-factor of the Fe I line is 0.833, and the Ni I line has a Lande' factor of 1.5, which scales to 2.34 at 500 nm. For the GONG Ni I line, it is also 1.5, scaling to 2.63 at 500 nm.

The spectral region in which our two lines are located contains, apart from these two lines, only entirely clean continuum over a range of about 2.2 \text{ nm}, a unique situation in the solar spectrum. For this reason the demands on blocking filter stability and bandwidth can be greatly reduced, since several surrounding Fabry-Perot orders sampling only pure continuum can be admitted within the blocker passband. These continuum passbands will add some random noise, but we feel this is more than compensated for by the greatly reduced temperature drift noise of the blocking filter.

2. INSTRUMENT DESCRIPTION

Early work on the SSA focused principally on instrument definition and on research with key components, i.e., electro-optic light modulators, optical filters, and telescope design (Ref. 1,2). Progress in 1985 and 1986 included construction of the APL Solar Observatory and installation of a photoelectrically-guided telescope donated by NASA GSFC. The first Li NbO\(_3\) etalon, which had a 50-mm aperture, was stabilized with \(\Delta \nu/\nu < 3 \times 10^{-9}\) for 30 min. In October 1985 at the Sacramento Peak Facility of the National Solar Observatory, we made our first successful observations of the solar oscillations.

Commencement of observations with the 75-mm etalon described below had been planned for 1987, but difficulties with the telescope guiding and the diversion of effort into preparation of an SSA proposal for the SOHO mission delayed the program. Progress in the instrumentation program and the results of a six-day observing run in early 1986 at Sacramento Peak were reported at the IAU Symposium on Advances in Helio- and Asteroseismology (Ref. 3,4). Figure 1 is an example of the data obtained. These data provided estimates of the interior equatorial rotation rate which are consistent with a rate that decreases with depth between 15,000 and 60,000 km below the photosphere, which are consistent with a rate that decreases with depth between 15,000 and 60,000 km below the photosphere, which are consistent with a rate that decreases with depth between 15,000 and 60,000 km below the photosphere.

Since the production of a second 75-mm diameter etalon, the CSIRO Division of Applied Physics in Australia has significantly improved the performance of its phase-shifting digital interferometer (PSDI). Using the improved PSDI it is possible to measure a phase map to an accuracy better than 1°. This corresponds to \(\lambda/720\) in the medium (\(\lambda/1600\) in actual thickness with \(n = 2.29\)).

Recent theoretical work and modeling of etalon properties at CSIRO has allowed more accurate prediction of etalon performance from the data acquired during etalon manufacture. Figures 2a and 2b show the relationship between the macroscopic irregularities of thickness as measured by the DPSI and the finesse, as determined by angle-tuning measurements, for various apertures for the 75-mm etalon. The profile of the etalon is shown in Figure 3.

2.1 Fabry-Perot Etalon

The key component of the Stable Solar Analyzer is the lithium niobate Fabry-Perot etalon which makes possible very accurate measurements of Doppler and Zeeman shifts in solar spectral lines. Lithium niobate, first grown in 1966, is a highly transparent crystalline material whose index of refraction changes in proportion to voltage applied parallel to the crystallographic c-axis. Thus, an etalon of it can be tuned to any point in a line profile to detect lineshifting (Ref. 5).

Simultaneously, the etalon can also be tuned to a standard wavelength reference source, such as a stable laser, so that the absolute wavelength of the solar line can be determined (Ref. 6).

2.1.1 Etalon Flatness Measurement and Manufacture. The fundamental characteristic of interest in a Fabry-Perot filter or of any filter used for spectroscopic imaging is the ratio of the free spectral range to the full width at half maximum transmission. This is called the finesse. The higher the finesse, the easier it is to block unwanted orders.

Realizable finesse depends on coating reflectance, microscopic deviations in surface flatness, irregularities in the substrate, convergence of the beam, and surface parallelism (Ref. 5). It now appears possible to make lithium niobate etalons with rms macroscopic deviations of actual thickness in the range \(\lambda/1000\) to \(\lambda/600\). It appears that quality will be limited only by the achievable level of optical homogeneity in lithium niobate single crystals. Because of the rapidly increasing interest in lithium niobate as a data storage medium, the outlook for obtaining improved material is good.

Since the production of a second 75-mm diameter etalon, the CSIRO Division of Applied Physics in Australia has significantly improved the performance of its phase-shifting digital interferometer (PSDI). Using the improved PSDI it is possible to measure a phase map to an accuracy better than 1°. This corresponds to \(\lambda/720\) in the medium (\(\lambda/1600\) in actual thickness with \(n = 2.29\)).

Recent theoretical work and modeling of etalon properties at CSIRO has allowed more accurate prediction of etalon performance from the data acquired during etalon manufacture. Figures 2a and 2b show the relationship between the macroscopic irregularities of thickness as measured by the DPSI and the finesse, as determined by angle-tuning measurements, for various apertures for the 75-mm etalon. The profile of the etalon is shown in Figure 3.

2.1.2 Materials. Lithium niobate is a robust material with mechanical properties not too dissimilar from glass in its ease of handling. It is chemically resistant to attack by strong and weak acids and alkalis. An etalon 75 nm in diameter and only 200 \(\mu\text{m}\) thick obviously needs protection from shock and rapid temperature...
2.1.3 Stability. The problem of temperature sensitivity in the solid étalon was solved by illuminating it with laser light of a precisely known wavelength and sampling the light reflected in the FP order that falls at the reference wavelength. The new SSA has an integrating servo system that locks the FP to the laser line to achieve a 2 x 10^-10 precision in $\Delta \lambda / \lambda$. Our method of stabilizing the diode laser drew upon techniques developed in an on-going program to improve optically-pumped cesium beam frequency standards (Ref. 6). In the device made for the SSA, cesium-133 vapor is held at 40 °C in a small glass cell. Part of the laser beam passes through the cell to a small detector. The amount of light passing through the cell depends upon how close the laser's wavelength is to the cesium D2 line. The laser can be servoed to the line at 852.126 nm since its operating wavelength is proportional to the current applied to it. The cesium line is an NBS secondary wavelength standard.

A recent study (Ref. 7) indicates that rubidium vapor, which has a resonance absorption line at 780 nm, may be a superior standard since the resonance line is near two solar lines, Fe I at 778.057 nm and Ni I at 778.893 nm, which are ideally suited for helioseismological studies. Wavelength proximity between the target solar lines and the standard line is important because the index of refraction of lithium niobate is a function both of wavelength and temperature.

2.2 Optical Layout

Figure 4 shows the instrument assembled in our laboratory. The optical path is quite straightforward. Unfocused sunlight is relayed by a guider mirror through an infrared absorber, a circular polarization analyzer and a 0.6-nm thin-film filter to the FP étalon. The étalon and the 0.6-nm filter are in temperature controlled housings. Next the solar image is focused on the detector array by a 600-mm lens. The scale at the detector is 20 arcsec per pixel, so the entire solar disk is accommodated on the 100 x 100 pixel array.

The SSA must average many frames to achieve the required signal-to-noise ratio. Any contributions to the uncertainty of the data by the instrumentation must be minimized. For this reason, exposure rates as well as exposure intervals must be well controlled from frame to frame. It is desirable that both be stable to better than 1000 ppm. In the new SSA, a hybrid shutter consisting of a mechanical chopper wheel...
Figure 4. The Stable Solar Analyzer in the test laboratory. The optical module in the foreground consists of a guider mirror at A to direct sunlight (black trace) to the beamsplitter B. From there most of light passes into the thermally controlled étalon housing C. The housing includes the 75 mm étalon, the blocker filter and a 600 mm lens to bring the sun into focus on the Reticon D. The étalon is locked to the reference wavelength of the laser E which illuminates the étalon via mirror F. After reflection at the étalon, the laser beam (white trace) is relayed from mirror G to position-sensitive detector H. Lens I in the guider loop focuses the sun on a quadripartite cell J from which signals flow to the amplifiers and servo circuits in rack K. Rack L holds the IBM PC/AT and a 9-track tape drive. Two other racks (not shown) hold data encoders, the camera controller and data processors.
In series with a liquid crystal light valve controls the exposure rate and interval. The liquid crystal shutter provides a precisely timed exposure with a moderate extinction ratio whereas the mechanical shutter provides an excellent extinction ratio but only a moderately timed exposure. By operating the liquid crystal shutter within the window of the mechanical shutter, the best of both shutters is gained.

The blade shutter is an HMS Light Beam Chopper 220 marketed by Ithaco, Inc. It consists of an 11-pole DC brush motor with a 20:1 gear reduction drive to turn the conventional dual-aperture chopper. The motor is driven open-loop by a precision voltage-sourced power amplifier. The gear reduction drive has about 2.7° of play. Shutter open/close cycles can be monitored with a transmission type IR detector. Using this detector, we observed that the exposure time stability at a nominal 1 Hz exposure frequency was about ±2 ms peak to peak. This variation is suspected to be due largely to the gear reduction drive and its 2.7° degrees of play.

The liquid-crystal shutter is a type LV500A Light Valve from Displaytech, Inc. This device uses a ferroelectric liquid-crystal display film which functions as a voltage switchable halfwave plate. With HN38 polarizers, a 200:1 extinction ratio was observed with an open-valve transmittance of 0.7, not including the polarizers. Switching times of better than 200 µs were measured.

Physically, the liquid-crystal shutter sits between the blade shutter and the Image sensing array detector. For proper operation, the blade shutter must completely pass the 6-mm diam beam before the liquid-crystal shutter is opened for a precisely timed interval. The blade shutter must not start blocking the beam until the liquid-crystal shutter is closed.

In order to achieve the required coordination between shutters and precision timing, a small microcomputer, model HMIX-0023 from New Micros, Inc., was used. Figure 5 shows the additional electronics added at APL to the microcomputer to interface the blade shutter motor amplifier and the liquid-crystal device. As the blade rotates, a motor velocity control loop, implemented in software, counts the interval between blade closures. This count is compared to the exact interval count one would expect at the desired
velocity. The difference count represents an angular difference from the ideal count per shutter interval. These differences are summed to form a cumulative angular difference which the servo attempts to drive to zero. The digital integrator SI (Fig. 5) ensures a zero long-term angle error. Gains Gp and GI are proportional and integral gains which were chosen to best stabilize the control loop at each exposure rate.

The shutter and the readout cadence are referenced to a 57624 Hz crystal oscillator which provides the precise time base needed for helioseismological observations. Further details are given in Ref. 8.

2.3 Data Collection System

Since the various data collection systems that were used with the SSA in 1985-86 proved to be inadequate for our purposes, we designed and built a data collection system around low-cost commercial components (Ref. 9) and integrated this with a 100 x 100 photodiode detector array. The SSA is now an integrated package consisting of four electronics racks and an optical module that can operate at any U.S. or European observatory that offers a steady beam of sunlight.

Figure 6 summarizes the data flow path in the data collection and preprocessing system. A modified Sony digital audio PCM (pulse code modulation) processor digitizes data from the Reticon array at 44,000 pixels/s with 16-bit precision. The Reticon array data are encoded on the "left" channel while engineering data are encoded on the "right" channel. Both channels have 44,000 word/s capacity.

Figure 7 is a block diagram of the SSA data collection system. The detector array is an EG&G Reticon RA 100x100A chip. It is read out at a rate of 4 images/s. The chip is contained in an evacuated chamber which is cooled to hold down dark current effects. The camera controller was built at APL. A Peltier-effect device cools the Reticon array. The cooler controller is a Marlow Industries Model SE1100.

The PCM Encoder is a modified Sony 501ES digital audio processor used to: 1) digitize analog image and engineering data; 2) serially write the data to the serial/parallel interface of CPU 1; 3) send the data to the VCR. The modifications to the PCM included tapping into the serial digital output stream to send the data to the serial/parallel interface and removing antialias filtering of the input data.

CPU #1 is the first processor in the SSA pipeline. It is a Heurikon HK-88 package containing a Motorola 68000 processor, 1 Mbyte of RAM, DMA (direct memory access) chip used for input from the serial/parallel board, a clock, and 4 serial I/O ports. The software for CPU #1 is written in FORTH and 68000 assembly language. CPU #1 separates engineering data from imaging data and formats the images for CPU #2.

The second processor in the SSA pipeline, CPU #2, uses the same hardware package as CPU #1. It performs the bulk of the processing and outputs reduced data to the IBM PC/AT. It also scales and displays image data using pseudo color. The image processor is from Image Technology, model FC-100-S12-30MP. CPU #2 currently has eight processing modes. The first four modes are for data acquisition at various spectral sampling intervals and averaging rates. The last 4 modes produce various plots to aid in the setup of the SSA.

The IBM PC/AT is used primarily as a disk storage device and monitor for the rest of the data control system. The observer selects the processing modes and parameters at the PC keyboard.

The principal mode of operation is called the 6-point enhancement, which uses six weighted frames and a continuum frame. Each frame in the enhancement (including the continuum frame) is assigned a weight for the centroiding calculations. After accumulating all 6 frames in an enhancement, the sum and weighted images are subtracted from the weighted continuum frame to produce, for each pixel, the line centroid signal:

\[
\mathcal{N} = \frac{\sum_{i=1}^{6} w_i C - \sum_{i=1}^{6} w_i I_n}{\sum_{i=1}^{6} w_i}
\]

The SSA collects approximately one gigabyte of image data each day. Raw data are recorded in real time on video tape cassettes. Each Beta video tape can hold up to three gigabytes of data. Simultaneously with the video tape recording, the data are processed into Dopplergrams and compressed by a factor of 30 before being output to standard nine-track tapes. We decided to process the data in real time to avoid having to read back the video tapes into a separate processor afterward. But since the raw data are archived, alternative processing algorithms may be used later if necessary. Either raw or processed data may be routed to a video display so that the observer may monitor image quality and instrument performance either in real time or in playback mode.

3. Calibration

To carry out the scientific program, the SSA must make highly accurate measurements of the line shifts. The line shift measurements will be derived from the computed solar spectral line centroid for each pixel. Sampling the line profile at six points will yield a minimum static velocity error, i.e., careful calibration can yield corrections for all the non-linear instrumental effects to within \(\pm 1\%\). Even without calibration, the line centroiding approach is highly linear, i.e., with six-point sampling, the maximum deviation from linearity is only \(5\%\). The accuracy is significantly worse for fewer samples. Accurate calibration will be achievable because the instrumental line shifts are known for each pixel on the sun. They are a smooth function of the angle between the projected pixel position and the optical axis of the instrument. Figure 8 shows the calculated and the observed distribution of the steady solar velocities convolved with the Fabry-Perot angular response function.
Figure 8. (Left) Theoretical velocity map on the sun in km/s taking into account differential rotation, limb-shift and the velocity effect of the Fabry-Perot. The Fabry-Perot is tilted 24 arcmin towards the west canceling the solar rotation effect. The velocity variation is 1.1 km/s. (Right) Measured velocity map in km/s obtained at the APL Solar Observatory. This is an average of 36 pictures taken every 56 s. The Fabry-Perot was tilted 24 arcmin towards the west. The velocity variation is about 1.2 ± 0.1 km/s, as expected. The most remarkable effect is the symmetry with respect to the solar equator, and the asymmetry with respect to the polar axis. This is due to the differential rotation which has these properties.

4. CONCLUSION

Our plans for further work reflect the maturation of a project that, until recently, has been devoted almost entirely to instrument development. We now have an integrated instrument with which solar observations can be made at the research frontier. Consequently, we plan to undertake a vigorous observing and data analysis program on solar oscillations and magnetic fields. Since the sky conditions at our plant in Maryland are unsuitable for helioseismology - the APL Solar Observatory has always been used only for instrument test - we plan to move the SSA to an established observatory site. Preliminary discussions have been held with several observatories regarding this prospect.

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ABSTRACT

We present 37 GHz observations of solar oscillations made during August 1983 and May 1988 at Metsähovi Radio Research Station. Altogether we had data from ten clear days which could be used for further analysis. Five quiet regions were selected from the Sun and the amplitude differences were monitored during the day. Due to large amount of data only typical examples are presented. During favourable conditions the amplitude at the period of 160 minutes would increase in respect with the other peaks in the power spectrum. The data would also give some suggestions that a higher degree g-mode oscillation $l \geq 5$ could be possible.

Keywords: oscillations of the Sun, solar radio radiation

1. INTRODUCTION

Long period solar oscillations first reported by (ref. 1,2) have been studied at the Metsähovi Radio Research Station since 1980. The observations have been done with the 13.7 m radio telescope at frequencies of 22 and 37 GHz. In this paper part of the observations made during August 1983 and May 1988 are presented. The purpose of these studies has been to confirm whether we could resolve the degree of oscillation at the period of 160 minutes.

2. INSTRUMENTATION

The 13.7 meter telescope of the Metsähovi Radio Research Station was used for the observations. A more detailed presentation of the telescope performance is given in (ref. 3). The telescope is alt-azimuthal mount and is covered by a radome. The transmittance of the radome is about 0.76 at the used frequency 37 GHz. The radome protects the antenna surface from rain and also reduces the effect of solar heating to the antenna. The receiver is an uncooled mixer front-end receiver with a noise temperature of 1200K DSB during 1983 and about 500K DSB during 1988 including losses from radome and atmospheric noise. The IF-bandwidth is 1000 MHz. The antenna HPBW is 2.4 arcmin at the used frequency. The receiver is Dicke-switched with a rate of 25 Hz between the signal horn and a constant temperature reference load. The data was sampled with one second intervals and stored on magnetic tape for further analyses.

3. MEASUREMENTS

During both years 1983 and 1988 the observing method was similar: we choose four or five quiet regions from the Sun and used one point off the Sun. These points where integrated in such a way that during 5 minutes a complete revolution was performed. To eliminate long-term drift in receiving system gain, the test figure $A_{n-m}$ was calculated according to formula (1)

$$ A_{n-m} = 2 * (A_n - A_m) * \frac{1}{\tau \sin(\varepsilon_l)} $$

where $A_n$ and $A_m$ are amplitudes on the Sun respectively, $\tau$ is the optical depth during the measurement, $\varepsilon_l$ is the elevation angle during the observation and $A_{off}$ is the off-Sun amplitude.

In the beginning of every morning observations one map of the hole Sun was measured to select quiet regions far from active regions. The order of points for measurements on the Sun was selected in such a way that the elevation difference between points would be minimized. After noon the order of the points was reversed.

During both August 1983 and May 1988 we had five runs longer than 8 hours.

A polynomial of second order was removed from the test figure array and then the power spectra was calculated according the method in (ref. 4).

4. RESULTS AND CONCLUSIONS

4.1 Measurements during August 1983

From the five days observations 50 amplitude differences and power spectra can be calculated. Some typical observing runs will be presented: in figure 1 there is a map of the Sun measured 7 August 1983 with the points selected for oscillation measurements marked with circles. From the ten possible amplitude difference combinations three are presented in figure 2. The solid line in figure 2 represents the test figure series and the broken line is the second order polynomial baseline fitted to it.

The criteria for a good data array is a low amplitude difference along the day. When this criteria is fulfilled, we have a better chance of power spectra peaking near 160 minutes. The resulting power spectrum is presented in figure 3. It is to be noted that points 2 and 5 are measured at the same position. This was done for checking the data handling process. The amplitude change 2-5 is clearly the smallest of the combinations presented. Also the power spectra has no peak around 160 minutes like the other combinations presented.

The other example is from 9 August 1983. This days measurements have been analysed in a different way in ref. 5) by treating the points separately and performing the power spectra analysis to the integration points and not to the differences. That analysis suggested a higher degree 1> 5 oscillation.

In figure 4, is the solar map measured on 9 August 1983 with the chosen integration points marked within. In figure 5, the resulting amplitude differences with the second order polynomial baseline are presented. In figure 6, there are the calculated power spectra. Our earlier observations (ref. 6) where we looked for 160 minute oscillations and calculated phase differences between different positions, would give justice...
to assumptions of having a higher order oscillation at the period of 160 minutes. If the amplitude difference is having a big change during the day, the resulting power spectra is more or less random as the amplitude variations are mainly due to unwanted phenomena like active region developments in the sidelobes and unfavourable selection of integration points.
Even when the test figures look promising like in the case of figure 5, 1-4 difference there is no assurance that we would get a big amplitude at 160 minute oscillations. To really prove the existence or the nonexistence of the oscillations we should have a much better signal to noise ratio. Anyway most of the noise is coming from the Sun itself, so comparison between observations made during the different phases of the solar activity cycle could give some hints what would be the best strategy for observations.

Figure 6. Power spectrum for baseline reduced test figure series presented in figure 5.

4.2 Measurements during May 1988

The May 1988 run is quite similar to the August 1983 run and only one example of the five days measurements is given. In figure 7 is the solar map measured at Metsahovi on 14 May 1988. The integration areas are again marked with circles in the map. In figure 8 is the test figure array with second order polynomial baseline fitting. In figure 9 the power spectra calculated from the residual is presented.

It can be seen from the map in figure 7 that the Sun has now a lot of different features which makes the finding of quiet regions difficult.

Figure 7. Solar map at 37 GHz measured on 14 May 1988. Selected beam positions are marked with circles.

Figure 8. Test figure series 3-4 from 14 May 1983.

Figure 9. Power spectrum for baseline reduced test figure series presented in figure 8.
4.3 Improvements to measuring system.

The main disadvantage in our receiving system is to not have an equatorial mount telescope. The pointing accuracy of our telescope should be around 11 arc seconds and this might also give sufficient distorting amplitudes if the chosen points were near active regions. A higher frequency in the infrared region could be used as according to our own observations the Sun is considerably less active even when going only from 37 to 81 GHz. The high atmospheric attenuation would then require space born observations. The longest continuous observation times at Metsähovi are less than 12 hours, which gives only four full periods of 160 minutes. This makes the lower frequency end of the power spectra dependent of the baseline configurations. Basic improvements to the system would be an equatorial mount telescope and longer integration times which is only possible closer to the poles.

4.4 Conclusions

Our observations at 37 GHz suggest that during favourable conditions the solar 160 minute oscillations can be detected by observing brightness differences between different parts of the Sun. The variations in oscillating amplitude locally could be explained by higher degree (l=5) g-mode.

5. REFERENCES

VISIBILITY OF NONRADIAL PULSATION MODES IN SOLAR CONTINUUM 
INTENSITY MEASUREMENTS

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Abstract: A calculation of the continuum emission integrated over the solar disk, and of its perturbations arising from different nonradial pulsation modes, is presented. In addition to geometrical effects (variations of the emitting areas) and Planck function variations studied in previous similar calculations, we examine the influence of opacity changes in H- (the principal absorber at the wavelengths under consideration). The computations are carried out numerically for 500 and 826 nanometers, which correspond to different channels of the IPHIR/PHOBOS experiment. The relative importance of these different processes of flux modulation is discussed as a function of oscillation period and modal type.

2. ORIGIN OF FLUX VARIATIONS

Two kinds of effects produced by a nonradial oscillation can modify a measurement of the integrated flux of a star.
- A geometrical effect caused by a distortion of the surface of the star.
- An intensity effect induced by physical variations of the stellar matter.

We are going to explicit them.

2.1 Geometrical effect.
A nonradial perturbation creates a distortion of the star surface. Consequently, it generates also an integrated flux variation, because the measurements are sensitive to area variation and modification of the local surface orientation.

We derive here an analytic expression of this effect. If we define the flux as:

\[ F = \int_{\text{half sphere}} I_u(\mu) \overline{e}_z \cdot \overline{n} \, dS \]  

(2.1)
with \( I(\mu) \) : emergent intensity
\( \mu \) : \( \cos(\theta) \)
\( (r,\theta,\phi) \) : the usual spherical coordinates
in the observer frame
\( \hat{e}_z \) : z axis unit vector
\( \pi \) : surface normal
\( d\Omega \) : \( r^2 \sin(\theta) d\theta d\phi \)

Hence we can obtain the geometrical effect by expanding \( I \) and \( \pi \) around their equilibrium values (indicated by a 0 subscript). That is to say:

\[
I_v(\mu) = I_v(\mu_0) + (\mu - \mu_0) \frac{\partial I_v}{\partial \mu}
\]

with \( (\mu - \mu_0) = (\hat{m} - \hat{m}_0) \cdot \hat{e}_z \)

and \( \hat{m} = \hat{m}_0 + \delta \hat{m} \)

where \( \delta \) in the next indicates a Lagrangian variation.

We can introduce now the perturbation in terms of \( \delta R \) which is the radius perturbation. For this we use the relation:

\[
\hat{m}/R = (\hat{e}_r + \delta \hat{e}_r) \cdot \hat{e}_z
\]

joined with

\[
\hat{R} = (R + \delta R) \hat{e}_r
\]

where \( \hat{e}_r \) is the radial unit vector which gives \( (\hat{m} - \hat{m}_0) \) in terms of \( \delta R \). Now we insert this in (4.4). The last step is a projection of \( \delta R/\hat{R} \) on the spherical harmonics basis.

\[
(\delta F)_{\text{geo}} = R^2 \delta \epsilon_{\text{I}}(r) \frac{\partial \epsilon_{\text{I}}}{\partial \mu} (\theta^*, \phi^*) e^{i \mu \mu_0}
\]

\[
\times \int_0^1 \left[ 3 \sum_{l=1}^{l=d} (l+1) (l+1 - \mu^2 I_{l+1} + (l+1)(l+1 - \mu^2 I_{l+1}) \right] d\mu
\]

\[
+ \frac{dI_v}{d\mu} (l+1)(l+1 - \mu^2 P_{l+1}) \mu \right] d\mu \tag{2.2}
\]

in that case \( \delta F \) has the next simple expression

\[
\frac{\delta F}{\delta I} = R^2 \delta \epsilon_{\text{I}}(r) e^{i \mu \mu_0} \frac{\partial \epsilon_{\text{I}}}{\partial \mu} (r, \theta^*, \phi^*)
\]

\[
\times \left( \mathcal{L} - \ell (l+1) \right) \sum_{p=0}^P a_p \int_{\mu_0}^1 I_{\ell p+1} d\mu
\]

where \( I_{\ell p+1} = \int_0^1 R^2 P_{\ell p+1} d\mu \)

Thus one can obtain a semi-analytic expression for the geometrical effect without any restrictive hypothesis.

2°) Intensity effect.

It includes the whole of effects induced by physical modification of the stellar matter. If \( \delta I_v \) is the Lagrangian variation of \( I_v \), then:

\[
(\delta I_v)_{\text{int}} = \int_{\Omega_{\text{eff}}} \delta I_v \cdot \hat{e}_z \cdot \hat{m} d\Omega
\]

The linear adiabatic theory of non radial pulsation gives in the COWLING approximation simple relations between \( \delta R, \delta T, \delta P, \delta \phi \). Hence, having \( \delta I_v \) in terms of \( \delta F \) and \( \delta \) one can write:

\[
(\delta I_v)_{\text{int}} = \int_{\Omega_{\text{eff}}} \delta F \frac{\partial I_v}{\partial \mu} \frac{\partial R}{\hat{R}} \mu d\mu d\phi
\]

where \( X \) represents the whole of the useful physical parameters. So, to derive this effect it is enough to use the previous method.

Named, to project \( \delta R/\hat{R} \) on the spherical harmonics basis, then:

\[
(\delta F)_{\text{int}} = R^2 \delta \epsilon_{\text{I}}(r) e^{i \mu \mu_0} \frac{\partial \epsilon_{\text{I}}}{\partial \mu} (r, \theta^*, \phi^*)
\]

\[
\times \int_0^1 f(x, \mu) P_{\ell p} \mu d\mu
\]

The computation of \( \int I_v d\Omega \) is detailed in the following section.

3. INTENSITY VARIATIONS

For this preliminary study, we limit ourselves to the adiabatic case. In addition, the calculation of opacities is restricted to the main contributor which, at the wavelengths under consideration, is the negative hydrogen ion. Finally, local thermodynamical equilibrium is assumed throughout the present computations.
Oscillations produce intensity variations via two processes:

i) Source function variations which, in LTE, only depend on temperature perturbations, \( S \) being equal to the Planck function:

\[
S = \mathcal{B}_\nu(T) = \frac{2}{c^2} \frac{h \nu^3}{e^{h \nu/kT} - 1}
\]

Hence, its perturbation is:

\[
\Delta S = \frac{h \nu/kT}{1 - e^{-h \nu/kT}} \Delta T = R_\nu(T) \Delta T
\]  

ii) Opacity variations arising from both changes of temperature and density. When opacity increases, the region of formation of the continuum is shifted upwards, which results in a decrease of emitted intensities, as a result of the negative temperature gradient of the solar photosphere.

The absorption coefficient per unit mass due to the \( H^- \) ion is:

\[
\kappa = \frac{\alpha_{H^-}(\nu)}{\rho} \left[ 1 - e^{-h \nu/kT} \right]
\]

where \( \alpha_{H^-}(\nu) \) is the absorption cross-section and \( \alpha_{H^-} \), the number density of \( H^- \), is given by:

\[
\alpha_{H^-} = \alpha_e \mu_e \exp\left( \frac{h \nu}{kT} \right) \frac{U_{H^-}}{2 U_H} \left( \frac{h^2}{2 \pi m_k T} \right)^{3/2}
\]

\( \alpha_e \) and \( \mu_e \) are the number densities for electron and total hydrogen, respectively, \( \nu_p \) the frequency of photodetachment of \( H^- \), \( U_{H^-} \) and \( U_H \) partition functions. Other symbols have their usual meaning.

Since the atmosphere is weakly ionized, the ratio \( \mu_e / \mu \) is approximately constant, so that \( \alpha_e \) is proportional to:

\[
\alpha_e \propto \mu_e T^{-3/2} \left( e^{c_1/T} - e^{c_2/T} \right)
\]

with \( c_1 = h \nu_p / k \) and \( c_2 = h (\nu - \nu_p) / k \).

In the layers of the Sun under consideration, electron density depends principally on the ionization of hydrogen, except near the temperature minimum region, where it is restricted to a residual contribution of metals. It may be approximated by

\[
\alpha_e = A \mu_e + \mu_p
\]

where \( \mu_p \) is the proton density and \( A \) a constant (\( A \approx 4.2 \times 10^{-4} \)).

Combining this expression with the Saha law:

\[
\frac{\alpha_{H^-}}{\alpha_e} = \frac{g_1}{g} \exp\left( \frac{h \nu}{kT} \right) \left( \frac{h^2}{2 \pi m_k T} \right)^{3/2}
\]

and eliminating \( \alpha_{H^-} \), we can express \( \alpha_e \) as a function of \( \mu_e \) and \( T \), and subsequently \( \Delta \alpha \), by substituting this function into equation (3.2). By differentiating the expression so obtained, the opacity perturbation may be expressed as a function of those of density and temperature:

\[
\frac{\Delta \alpha}{\alpha} = \left( \frac{\partial \alpha}{\partial \rho} \frac{\Delta \rho}{\rho} + \left( \frac{\partial \alpha}{\partial T} \right) \frac{\Delta T}{T} \right)
\]

In the adiabatic approximation, these two perturbations are related to each other by

\[
\frac{\Delta \rho}{\rho} = \frac{1}{\gamma - 1} \frac{\Delta T}{T}
\]

which allows to express \( (\alpha / \rho) \) as a single function of \( (\Delta T / \rho) \):

\[
\frac{\Delta \alpha}{\alpha} = R \left( T, \rho \right) \frac{\Delta T}{T}
\]

The emergent intensity as a function of \( \mu \) (cosine of the angle between the local vertical and the direction of the observer) is given by the Laplace transform:

\[
I(\mu) = \int_{0}^{\infty} S(t) \exp\left( -\frac{t}{T} \right) \frac{d\tau}{\tau}
\]

where \( \tau \) is the optical depth:

\[
\tau = \int_{0}^{\mu} \alpha(\mu') d\mu'
\]

\( m : \) column mass.

Since the model atmospheres necessary to compute the intensity are given numerically, it is convenient to introduce a mesh \( (m_1, m_2, \ldots, m_l) \) in order to discretize the preceding equations. Equation (3.8) yields:

\[
I(\mu) = \sum_{\alpha=1}^{l} L_\alpha(\mu) S_\alpha
\]

with:

\[
L_\alpha(\mu) = \frac{W_\alpha}{\mu} \exp\left( -\frac{t_\alpha}{\mu} \right) \alpha_\alpha
\]

where \( \alpha_\alpha \) are quadrature coefficients, and (3.9) becomes:
where the \( p_{\alpha} \) constitute another set of quadrature coefficients, which vanish for \( \beta > \alpha \).

Using equations (3.11) and (3.12), we can express the \( \xi_\lambda \) as functions of the set \( (\xi_{\alpha_{1}}, \ldots, \xi_{\alpha_{L}}) \) and thus deduce the partial derivatives \( \xi_{\alpha_{\mu}} = \delta \xi_{\lambda} / \delta \xi_{\alpha_{\mu}} \) analytically.

The intensity perturbation is obtained by differentiating (3.10):

\[
\delta I(\mu) = \delta_\xi I(\mu) + \delta_\xi I(\mu) \tag{3.13}
\]

with

\[
\delta_\xi I(\mu) = \sum_{\alpha=1}^{L} s_{\alpha} \delta L_{\alpha}(\mu) \tag{3.14}
\]

and

\[
\delta_\xi I(\mu) = \sum_{\alpha=1}^{L} L_{\alpha}(\mu) \delta s_{\alpha} \tag{3.15}
\]

Expressions (3.14) and (3.15) represent the variations of intensity arising from opacity and source function changes, respectively.

If \((\delta T / T)\) may be assumed constant within the region of formation of the continuum, these perturbations may, using (3.1) and (3.7), be expressed in terms of the temperature perturbation. This finally yields:

\[
\delta_\xi I(\mu) = \left[ \sum_{\alpha=1}^{L} s_{\alpha} \sum_{\rho=1}^{L} L_{\alpha}^{\prime}(\mu) R_{\alpha}(\mu, \rho) \right] \frac{\delta T}{T} \tag{3.16}
\]

and:

\[
\delta_\xi I(\mu) = \left[ \sum_{\alpha=1}^{L} L_{\alpha}(\mu) \delta s_{\alpha} \right] \frac{\delta T}{T} \tag{3.17}
\]

Thus, the intensity perturbation relative to the intensity \( I_c \) at the center of the disk may be expressed as:

\[
\frac{\delta I(\mu)}{I_c} = \left[ \xi_{\alpha}^{\prime}(\mu) + f_{\xi}^{\prime}(\mu) \right] \frac{\delta T}{T} \tag{3.18}
\]

It should be noticed that, in the adiabatic approximation, the functions \( \xi_{\alpha}^{\prime} \) and \( f_{\xi}^{\prime} \), which describe respectively the influences of opacity and source function perturbations, are independent of the oscillation mode considered. They are represented on Fig. 4, for a mean solar atmospheric model and a wavelength of 0.5 microns. For low values of \( \mu \), \( \xi_{\alpha}^{\prime} \) is negligible with respect to \( f_{\xi}^{\prime} \).

Near the center of the disk, \( \xi_{\alpha}^{\prime} \) and \( f_{\xi}^{\prime} \) have comparable magnitudes and opposite signs. In this region, neglecting \( \xi_{\alpha}^{\prime} \) causes an overestimation of the intensity perturbation by a factor of about 3.

Figure 1. Limb to center variation of an intensity variation corresponding to a constant temperature perturbation. The dotted line represents the opacity effect, the dashed one the source function effect, and the solid one the result of the two.

4. RELATIVE MAGNITUDE OF DIFFERENT PROCESSES OF FLUX PERTURBATION

Relative flux variations, arising from the different processes described above, have been computed for two different oscillation periods and different values of \( I \) and \( m \). We take as reference the perturbation \( \delta F / F_0 \) related to the source function (generally called "temperature effect" in previous papers).

Figure 2 is a comparison of geometrical and source function effects versus \( I \), for a 5-minute mode (for clarity, the comparison is restricted to \( m=0 \)). The same comparison, for a longer period (2h 47mn) expected for g-modes, is shown on Fig.3. (In both cases, the optical wavelength is 0.5 microns; the corresponding pictures for 0.826 microns are very similar).
Figure 2. Comparison of relative flux perturbations arising from geometrical (dotted line) and source function (solid line) effects, for an oscillation period of 5 minutes.

Figure 3. Same as Fig.2, but for an oscillation period of 2h 47mn.

Figure 4. Comparison of the modulus of relative flux perturbations arising from opacity and source function effects, for $l = 0$ to 4, and $m = 0$ to $l$. The sum of the two effect is also indicated.
For 5-minute oscillations, source function effects clearly dominate geometrical ones whatever the value of I. For long periods, the geometrical effect is no more negligible, whereas its amplitude is yet lower than that of the source function effect.

The effects of opacity and source function variations have been compared in the preceding sections for intensities. The corresponding flux variations also have relative amplitudes independent of the frequency of oscillation, but dependent on I, through the integral:

\[ \int_\rho \rho_{el}(\nu) \cdot \frac{\partial f(\nu)}{\partial \nu} \cdot \rho \cdot d\nu \]

(\( f \) being either \( f_\infty \) or \( f_\nu \), and \( \rho_{el} \) the Legendre polynomial of degree \( I \)). This is illustrated on Fig. 4, which represents the modulus of relative flux variations, for a constant perturbation of temperature, when \( I \) varies from 0 to 4. Opacity and source function effects always give perturbations of opposite sign, and their combination has a significantly lower magnitude. The source function component is generally slightly greater than the opacity one, with some exceptions (e.g. for \( I=3 \)).

5. Conclusion

This is a preliminary study. It is certain that the introduction of nonadiabatic effects, as studied by Berthomieu and Provost (Ref. 3) will deeply modify the preceding results. NLTE radiative transfer effects in the H- ion should also be investigated. What should be principally retained of this study is that the effect of opacity variations is not negligible with respect to that of source function and should be taken into account in future computations.

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References:


PERIODICAL ASYMMETRY OF SOLAR LIMB DARKENING

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Summary - Periodical brightness gradient differences between extreme ($\mu < 0.2$) West and East limb darkening profiles of the Sun are shown from diametrical high speed scans obtained by the use of the Heliotrope of the Pic-du-Midi Observatory. Time series of data display, in the five-minute range, several frequencies with amplitudes of difference corresponding to 2-4% of the mean brightness gradient value in the 678 nm wavelength range. One of these periods (270 s) was previously exhibited by Hill et al. (1982) and later by Koutchmy et al. (1983). These results might be related to asymmetries of solar limb darkening observed by Neckel and Labs (1987). A solar origin is discussed.

Key words: Oscillations of the Sun - Photosphere - Seeing.

The understanding of the global modes of the Sun still rests to some extent on their behavior in the photosphere. As it was recently re-emphasized (Stebbins and Goode, 1987) all methods of observation detect their manifestations in this layer. Thus Christensen-Dalsgaard (1984) has suggested that some of the discrepancies between observed and theoretical frequencies may be attributed to an incorrect upper boundary conditions in pulsation models. Using this idea, independently of well known regular patterns of the Sun's surface, several results [Hill et al. 1982; Koutchmy et al. 1986] show that it is possible that the periodical behaviour of this photosphere should not be entirely accounted to global modes of the Sun. More precisely the data obtained from limb darkening analysis of the photosphere (Brown et al. 1978; Yerle, 1981) were so surprising, even disappointing, that we suspected that the Earth's atmosphere had lead us to produce false results (Yerle, 1984; Kobanov, 1985; Yerle, 1986; Severny et al. 1987). As a matter of fact, it is still not certain that the results are unaffected by it. And the unexplained systematic frequency differences between Doppler shifts and intensity fluctuations recently observed (Duvall et al. 1988) may give further proof of the atmosphere's interference. But, on the one hand, the spatial filter estimates for limb darkening data cannot be easily calculated and therefore it is not inconceivable that the sensitivity of the limb brightness observations is, for several frequency ranges, increased in relation to the line shift data (Christensen-Dalsgaard, 1982). On the other hand, looking at the line shift and limb brightness observations, two very different and complementary aspects of the oscillations, we cannot seriously neglect the latter. So, prudently, we publish without comment these results obtained using the partly finished scanning Heliotrope at the Pic-du-Midi Observatory (Rösch, J. 1988).

Instrumentation

Diametrical high speed scans (3200°/s) of the Sun were obtained throughout several consecutive days in July 1987 using, for the first time, the scanning Heliotrope developed at the Pic-du-Midi Observatory (Rösch et al. 1983; Rösch, 1985). Scans of extreme ($\mu < 0.2$) opposite West and East limb darkening profiles were sampled (4000 readings per time-sec.), stored and processed. One of these profiles obtained in the 678.7 nm (bandwidth 1.7 nm) wavelength range is displayed in Figure 1 as an exemple. Intensity data are related to the middle area of each profile; the position of the two peaks flanking the solar image and the two sharp edges of a CERVIT rod cut (see Rösch and Yerle, 1983) give scanning speed references. In this way two sufficiently long time series of profiles, recorded at 30 s intervals, were obtained on July 27th (820 profiles) and July 28th (420 profiles). Unlike those processed between 1979 and 1981, these data were not stored with very high image quality and in any case the spreading function (blurring) exceeded 1.5°.
However we studied them again, this time from two opposite limbs of the Sun, using a lot of information presently examined, concentrating solely on the oscillations of the brightness gradient at the inflexion point shown during and since 1979 from only one solar edge (Yerle, 1981).

Analysis

From each profile, the maximum value of the apparent brightness gradient is calculated:

\[ m = \frac{1}{F} \max \left| \frac{dF}{dx} \right| \]

\( F \) being the mean intensity value at disc center calculated from the smoothed middle area data, \( \frac{dF}{dx} \) being corrected of possible scanning speed fluctuations, but not of stray light and seeing, after substraction of the dark current.

Firstly, the power spectra from the W and E string values are calculated separately, for each day. These power spectra do not display, at the 2σ confidence level, frequencies other than those linked to the corresponding window functions. Then the same kind of analysis is applied to each of the W-E time series. The Figures 5 and 6 show the corresponding power spectra. In the July 28th time serie, after substraction of a well-fitted parabolic daily pattern, the power spectrum was calculated again (Figure 7). Clearly, each analysis of whatever character, showed periodical differences, in the 5-8 min range, of the brightness gradient at the inflexion point between the West and East limbs of the Sun. Neither window functions, scanning speed fluctuations or off-axis effects could explain the shape of these power spectra. In addition, to exclude asymmetries linked to possible optical or electronic effects, we calculated the difference of the respective brightness gradients at the inflexion point of the

Results

Thus the data, and more exactly the amplitude of oscillations, can now be quantitatively analysed with precision. This m value is calculated from each West (W) and East (E) limb: e.g. Figure 2 for July 27th. Moreover, to study possible global oscillations, the W-E difference is calculated for each profile (Figure 3 and 4). These data form time series which may be processed using standard power techniques.
two peaks flanking the solar image, for each profile. Thus, the window function as also the method of analysis are identical. As an example, the figure 8 displays for the July 28th time serie the corresponding power spectrum which does not exhibit one frequency only. Besides, these oscillations are visible, at the first glance, in the W-E time series (Figure 3 and 4). If the negative W-E mean value after 11.00 UT each day could be linked to improbable selective defocusing effects, we would not be able to explain in this way the periodical changing values after 10.36 UT on July 27th and 11.39 UT on July 28th, for example.

In brief with average seeing conditions (1.5"-3.5" FWHM) we do not observe, in 1987, the solar limb darkening oscillations which were displayed between 1979 and 1981 with very high image quality (0.3"-1.0" FWHM). But the same techniques show periodical brightness gradient differences, probably by increasing the signal to noise ratio level, between opposite West and East limbs of the Sun which tends to favour global mode interpretations. We notice that the two periods of measurement correspond to extrema of the solar cycle. In other respects the July 28 power spectrum corresponding to relatively poor seeing conditions (the mean brightness gradient value is 0.053 arc sec^{-1} on July 27th and 0.042 arc sec^{-1} on July 28th) shows, in the five minute range, two peaks near the 512 (1.953 mHz) and 274 s (3.649 mHz) periods with respective amplitudes corresponding to 3.1% and 3.6% of the mean brightness gradient. We also found significant (2σ) periods close to 427 (2.342 mHz) and 265 s (3.773 mHz) with amplitudes of about 1.9% and 1.6% in the July 27th data. Amplitudes calculated by Schmieder et al. (1986) for high degree p-mode blendings (probably odd in this case), coincide with the above results though it is rather surprising to observe near the 270 s period that the power level increases as seeing gets worse. But bearing in mind that the complete spatial filter estimates must take seeing effects into account, this apparently disappointing results is not inconsistent with global mode interpretations.

Notice that a completely different interpretation could explain these data. For example a modulation of the transmission of the atmosphere by gravity waves which are, after all, density waves. As the quantity m is normalized in relation to the intensity at the center of the solar disc and the Sun has an apparent diameter which is not negligible, we would observe apparent oscillations of the gradient difference between the West and East edges of the Solar disc, close to the Brunt-Väisälä periods of the Earth's atmosphere (Yerle, 1986).
Generally speaking, it is nowadays beyond doubt that powerful periodical redistributions of the brightness gradient and asymmetries can be observed at the limb darkening of the Sun. In the five-minute range we might possibly see acoustic p-modes through changing real spatial filter functions, balanced by seeing effects. But these seeing effects are linked to turbulence which is linked to gravity waves in the middle atmosphere (Hines, 1988). And these gravity waves have Brunt-Väisälä periods in the 5-minute range.

Acknowledgements - I am greatly indebted to Pr. J. Rösch who permitted this study to F. Chauveau, F. Beigbeder and the technical staff of the Pic-du-Midi Observatory who have given much time and talent for the development of the Heliometer. Mrs J. Toulouse-Jobard was the typist of this asymmetrical story translated by Emily Davis.

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THE GRANULATION SENSITIVITY OF HELIOSEISMOLOGY LINES

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ABSTRACT

We address the sensitivity of the Ni I 676.78 nm GONG line and the K I 769.9 nm resonance line to the temperature fluctuations present in the solar granulation. The temperature contrasts due to granulation are probably small in the upper photosphere where the cores of these two helioseismology lines are formed. However, the cores are sensitive also to the granulation temperature contrasts in the deep photosphere, through non-local NLTE effects in their formation. The largest effects are due to the ultraviolet radiation field, which is strongly modulated by the granulation in the deep layers where it escapes and carries these contrasts upwards to the line formation height. We discuss the resulting NLTE mechanisms and their influence on the two lines.

Keywords: Helioseismology, NLTE Line Formation, Solar Photosphere, Granulation, Ni I Lines, K I Lines, Fe I lines.

1. INTRODUCTION

A matter of concern in the helioseismological use of solar Fraunhofer lines is to what extent they are influenced by solar noise from other dynamic phenomena than oscillations. A prime source of noise is the solar granulation. We discuss the sensitivity of Fraunhofer lines to the temperature fluctuations imposed on the solar photosphere by the granulation, concentrating on the GONG Ni I 676.78 nm line and the K I 769.9 nm resonance line.

The reason to study temperature variations specifically is that these have a large influence on the emergent continuous radiation field, which through its non-local nature may affect line formation in unexpected manners.

We have performed detailed NLTE syntheses for granular models, respectively for the K I line at Naples and for the Ni I line at Utrecht. In both cases, model atoms of sufficient detail were constructed and used as input to the Carlson (1986) NLTE radiative transfer code (which implements the particularly efficient method of Scharmer and Carlson 1985), for different components of a granulation model taken from the literature. The effects that we find in this “1.5-D”-type computation are indicative of what happens with these lines above real granules and real intergranular lanes.

In this contribution we skip the details of the computations, which will be published elsewhere. Instead, we employ the results to provide a general explanation of the NLTE effects that may be expected in helioseismology lines.

2. GRANULATION MODELS

A first issue is to which height the granulation actually penetrates. Nordlund's well-known anelastic simulation (e.g. Nordlund 1984, 1985b) predicts that the contrast between granules and intergranular lanes remains very large all the way up to the upper photosphere (righthand panel of Figure 1). Nordlund explains this high penetration as due to the large amount of radiative heating that the rising granules undergo by absorbing ultraviolet photons from below, in the numerous iron lines that together make up the so-called line haze (see also Rutten 1988). In contrast, Steffen's 2-D simulation (see Steffen and Muchmore 1988) produces granules of which the temperature contrast quickly vanishes with height, and even reverses in the middle photosphere (lefthand panel of Figure 1). Observations indicate that this reversal indeed exists (Evan and Catalano 1972, Suemoto et al. 1987, Holweger and Kneer, in press), and therefore that this description may be closer to reality for upper photosphere than Nordlund's results, although the latter are based on a more sophisticated treatment of the radiative heating than the grey approximation used by Steffen.

Figure 1: Models of the solar granulation. Results from Steffen's simulation (left) and Nordlund's simulation (right) are compared. For each, the temperature is plotted against height for a number of grid points covering the granulation. The scales are identical.
We discuss experiments here for granules that vanish quickly with height, like Steffen’s. We do this because this type of granule most clearly demonstrates the non-local effects that we describe below. These effects exist for Nordlund-type granules as well, but then act in addition to the large temperature contrast already locally present. It is more illustrative to show that granules affect lines formed in the middle and upper photosphere even if no granule persists to that height.

3. NLTE EFFECTS

There are three specific NLTE effects to be explained in this context. All three have to do with the differences between the mean continuous intensity $J_r$ and the Planck function $B_\nu$ as shown in Figure 2 for a standard model atmosphere. (They are shown in the form of radiation temperatures rather than energies in order to have a single curve for $B_\nu$ at the three wavelengths.) The height of escape at the three wavelengths shown is about the same, and the surface value of $J_r$ is therefore similarly set by $J_r = 0.3B_\nu(\tau_\nu = 1)$ at the same geometrical depth for the three wavelengths. However, the changes in the temperature sensitivity of the Planck function throughout the spectrum cause $J_r$ to exceed $B_\nu$ appreciably in the upper photosphere in the blue and ultraviolet, while $J_r < B_\nu$ at longer wavelengths. Thus, radiative $b_\nu$ and $f_\nu$ transitions that use $300$–$400$ nm photons (corresponding to 4–3 eV transitions) can feed on a strong NLTE radiation field in the upper photosphere, whereas NLTE imbalances can also occur for transitions of 1.5 eV or less from insufficient photoionization and photoexcitation. In the first case, overpopulation of the upper state follows; in the second case, overpopulation of the lower state.

The first NLTE effect discussed here is that of ultraviolet overionization, which typically occurs for minority species with sufficient atoms in levels that are about 1 eV from the continuum. (It is not important for majority species, because ionization imbalances do not affect their overall populations unless the balance is completely upset.) Ultraviolet overionization has been extensively studied for the case of Fe I (Lites 1972, Rutten and Kostik 1982, Nordlund 1984, see also Rutten 1988). Figure 3 contains a simplified Fe I Grotrian diagram which demonstrates that there are numerous levels from which ultraviolet ionization can take place. As a result, Fe I is appreciably underpopulated in the upper photosphere wherever there is a strong ultraviolet radiation field.

The second NLTE effect is that of ultraviolet overexcitation of lines near 300 nm. Such pumping has been studied mainly for Fe II (Cram et al. 1980, Watanabe and Steenbock 1986). Overpopulations of the upper levels of 300 nm lines (which are typically resonance lines in neutral and singly ionized metals) can lead to large effects in the source functions of lines at longer wavelengths that share upper levels with these pumping lines. The source function changes are large at longer wavelengths because there the fractional change of a line source function as specified by the upper level population departure is much larger when measured in terms of the local temperature sensitivity of the Planck function (see Rutten 1988, p. 196). For example, pumped long-wavelength lines may turn into emission while the ultraviolet pumping line itself remains in absorption.

The third NLTE effect is that of overrecombination at long wavelengths. This can be important for those minority species that have well-populated levels at about 1 eV from the continuum. The photoionization from such levels is insufficient to balance the LTE recombination because of the deficit shown in Figure 2.

How do these NLTE processes depend on the solar granulation? A granule is (in any model) considerably hotter in the deep photosphere than the intergranular lanes are. Therefore, ultraviolet overionization and overexcitation are enhanced above a granule by the extra-hot radiation from below. Pumping phenomena are also enhanced. Red and infrared overrecombination occur less than for the averaged atmosphere, however. Above an intergranular lane, the three effects are reversed.

4. NLTE EFFECTS FOR Fe I

Figures 3–5 show Grotrian diagrams for Fe I, Ni I and K I. Their vertical scales are identical to enable comparison. The Fe I diagram is the simplified basic diagram of Lites (1972), while the Ni I and K I diagrams are reasonably complete.

The Fe I diagram is given here as a reference. An extensive and illuminating discussion of Fe I line formation within the solar granulation has been presented by Nordlund (1984, 1985a). In summary, the line opacities are sensitive to the ionization equilibrium. Since this depends sensitively on the ultraviolet radiation field coming from below, the line opacities vary across granules with the temperature in deeper layers. Thus, Fe I is strongly affected by NLTE effect 1 above.

In addition, one may expect pumping to take place, for example in the 1–14–3 and 1–15–3 triangles of Figure 1. Such phenomena have not been studied yet. However, since there are many more levels and lines present in Fe I than shown in Figure 3, pumped overpopulations may well be washed out by collisional redistribution over neighbouring levels and by crosstalk in lines, so that the populations are effectively equalized.
Thus, for Fe I lines to be used as diagnostics one may well assume that the line source functions are locally in LTE. The opacities, however, will be out of LTE and very dependent on the ultraviolet radiation field from below.

\[
\text{Fe I, } \lambda = 7.87 \text{ eV}
\]

**Figure 3:** Grotrian diagram for Fe I. After Lites (1972).

\[
\text{Ni I, } \lambda = 7.635 \text{ eV}
\]

**Figure 4:** Grotrian diagram for Ni I.

5. NLTE EFFECTS FOR Ni I

Ni I is quite like Fe I except that there are far fewer levels and lines in the Grotrian diagram (Figure 4). The Ni I ionization energy is nearly equal to the Fe I ionization energy so that the ultraviolet overionization should be very similar. In addition, ultraviolet pumping may be expected. The GONG line, which connects levels 5 and 7, is a prime candidate for NLTE effect 2.

Our computations confirm these expectations. Figure 6 shows results. (Note that the abscissae measure column mass density and run reversely from the height scales of Figures 1 and 2.) The top panel shows the input models, consisting of the standard HSRA plane-parallel model atmosphere and hot and cool components from Steffen's simulation. The middle panel shows NLTE departure coefficients of the levels of Figure 4 that result for the HSRA. Their pattern closely mimics Lites' (1972) results for Fe I. Levels 13–15 drop away from the 1–6 curves deeper in the photosphere than levels 7–12 because their downward lines are weaker and become optically thin at lower height than the downward transitions from levels 7–12. Levels 1–6 do not experience photon losses at all, and simply follow the ultraviolet overionization which is produced through NLTE effect 1, primarily from levels 7–12.

The bottom panel shows the departure coefficients of the upper and lower levels of the GONG line, computed for Steffen's hot and cool granular components. The differences are large. The outward increasing split between the two lower-level curves (5) is due to the difference in ultraviolet overionization between the two granular components. The resulting opacity difference is given by the separation between the two tick marks which specify the locations of optical depth unity.

In the case of the cool component (dashed), the upper level departure coefficient (7) drops away in standard fashion from the lower-level departure coefficient due to photon losses. However, for the hot model this does not occur until \( m = 0.6 \text{ g cm}^{-2} \), and in fact there is a population inversion below that height. This is due to ultraviolet pumping in line 2–7.

In summary, the core of the GONG line is affected by the granular modulation of the ultraviolet radiation field in both its opacity and its source function. The effects combine: both the NLTE decrease of the line opacity and the reduction of the NLTE photon losses by the NLTE pumping increase the line core intensity above a hot granule.
6. NLTE EFFECTS FOR K I

The Grotrian diagram for K I (Figure 5) differs from the other two, illustrating the difference between alkalids and metals. The ionization energy is much lower. Only the ground state ionizes by using the hot ultraviolet radiation field. Due to spin-orbit interaction, its ionization cross section is unusually small. Therefore, NLTE effect 1 is less important here.

There is a blue transition that is pumped, however, namely the 1-5 line in Figure 5. The pumped-up atom cascades down along levels 3 and 4 to level 2 and back to the ground state. This pumping partially offsets photon losses in the resonance line.

The largest effect for K I is NLTE effect 3. It causes large over-recombination to level 4, which results in slight overpopulation of the K I atom in the deep photosphere and partially cancels the ultraviolet overionization higher up. As a result, the K I ground state is close to LTE throughout the photosphere for the HSRA—which is quite surprising for such a minor stage of ionization.

What happens above a hot granule? The ultraviolet ionization from level 1 increases and the long-wavelength recombination to level 4 decreases; both changes reduce the K I population. In fact, the ground state becomes underpopulated throughout the photosphere in our computation. In addition, the 1 to 5 pump is enhanced so that the resonance line photon losses are compensated somewhat more and the line source function increases. Both effects again produce a higher core intensity above a granule than there would be without NLTE phenomena.

CONCLUSION

The modulation that a granule imposes on the emergent radiation field affects the formation of helioseismology lines, even though their height of formation is above the layer of granular overshoot. Both the line opacity and the line source function can be affected.

For changes that are due to the ultraviolet radiation field, ultraviolet Planck function sensitivity is thus translated across the spectrum to the line wavelengths. This is a phenomenon similar to the Zanstra planetary nebulae mechanism, in which optical Balmer lines display ultraviolet radiation quality. An important difference between the Ni I line (and similar Fe I and Fe II lines) and the K I line is therefore that the former line displays ultraviolet sensitivity, while the latter line is dominated by recombination operating in the red. Thus, the granular signature in the core of the K I line should be less evident than in Ni or Fe lines that are formed at the same height in the photosphere.

ACKNOWLEDGEMENTS

We are indebted to Mats Carlsson for his helpfully spreading around of his efficient and elegant radiative transfer code, and to Matthias Steffen for emailing us results from his granulation simulation as computer-ready files immediately upon our request.

Figure 6: Results for the Ni I 676.78 nm GONG line.
Top panel: models used.
Middle panel: NLTE departure coefficients of the Ni I levels shown in Figure 4 for the HSRA standard plane-parallel model atmosphere. There are three groups of curves, respectively for levels without downward radiative transitions (1-6), medium-energy levels with strong downward transitions that become optically thin in the upper photosphere (7-12), and high-energy levels with weaker downward transitions that cause photon losses already in the deep photosphere (13-15).
Bottom panel: population departures for the GONG line, above a hot granule (dot-dashed) and above a cool intergranular lane (dashed). Tick marks indicate the locations of optical depth unity at line center. The line optical depth scales with the lower-level departure coefficient, while the line source function scales with the ratio of the upper-level and lower-level departure coefficients. (Note that the abscissa is reversed from the abscissae of Figures 1 and 2: height runs to the left here.)
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Session 4
Solar cycle, magnetic fields and oscillations
Chairman: J. Leibacher
Using oscillation data to probe the internal rotation and magnetism of the Sun.

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Abstract

Integral equations relating the rotation rate and the toroidal field intensity to the parameters describing fine structure in the frequency spectra are discussed. Solutions of the inverse problem for the rotation are presented. Features which may be relevant to the magnetic activity were found in the radial dependence of the rotation rate. Some evidence exists for changes with the solar cycle. There is no clear signature of direct effect of magnetism in the observed fine structure.

Keywords: Sun, Oscillations, Rotation, Magnetic Field, Activity

1. Introduction

The theory for stellar rotation and magnetism is enormously complicated. Possibility of measuring these quantities inside the Sun by means of helioseismology offers certainly the best chance for progress in this field. Knowledge of spatial and, perhaps, temporal variations in the rotation rate is crucial for understanding how the solar dynamo works and how the angular momentum is resupplied to the outer layers. In this review I will outline a theoretical background of the helioseismological method, present some recent results concerning solar rotation, and discuss their implications.

Deriving equations that relate the r-dependent coefficients in the expansion of the rotation rate into a cos series to the observable quantities is the easy part of the task. The difficult part is to obtain the solution and assess its reality. Our problem shares all well-known difficulties of inverse problems.

Magnetic fields that could lead to an observable effect in the frequency splitting must be quite strong - ranging from 0.1 MG in the lower convective zone to 100 MG near the center. Mestel et al. (Ref. 1) and Spruit (Ref. 2) argue against such strong fields in the radiative interior, but their reasoning directly applies only to steady-state fields of specific geometry. Therefore the problem may still be regarded as open.

For the toroidal field I will present formulae which are analogous to those for the rotation rate. It cannot be expected that we soon will be able to solve the inverse problem. However, plots of relevant kernels facilitate prediction of observable effects.

2. Observational data

Information about internal rotation and magnetism is contained in the observed dependences of frequency on the azimuthal order $\nu(n)$ for modes of various radial orders, $n$, and spherical harmonic degrees, $\ell$. The first data of interest due to Duvall and Harvey (Ref. 3) provided only $\nu(\ell) - \nu(-\ell)$ - values that are sensitive mainly to the equatorial rotation rate. Subsequent data (Refs. 4-8) provide functional description of the dependence. This is given in terms of coefficients $a_j$ in the following Legendre polynomial series

$$\nu(n) = \nu(0) + L \sum_{j} a_j \frac{P_j(-m/L)}{L}$$

where various authors use $L = \sqrt{\ell(\ell + 1)}$ or $\ell$. In the next section I will argue that the former choice is somewhat wiser.

The values of the $a$-coefficients are given as averages for all modes at given $\ell$ or ranges of adjacent $\ell$'s. This leads to a significant reduction of the noise but also to some loss in resolution. It is, thus, always preferable to analyze data for individual modes letting the inversion procedure to do its own averaging.

3. Formalism

3.1 Linear effect of rotation

We consider here the differential rotation law given in the form of the following expansion

$$\Omega(r, \theta) = \sum_{n=0}^{s} \Omega_n(r) \mu^n$$

where $\mu = \cos(\theta)$. It should be noted, however, that possible anti-symmetric terms make no contribution to the frequency splitting that is linear in $\Omega$. Using Eq. (2) and writing the displacement for an oscillation mode as

$$\xi = Re \left( (\nu_0(r) Y_{\ell m}(\theta, \varphi) e^{\tau} + \nu_1(r) \nabla_r Y_{\ell m}' ) \exp(2\pi\nu_0 t) \right)$$

one gets from the linear perturbation theory (see e.g. Refs. 9-10)

$$\nu_{0\ell m} - \nu_{1\ell m} = -\frac{m}{2\pi} \int_0^R \Omega_n \left( \frac{\kappa_{\ell m} - s(2s + 3)J_{\ell m}}{\kappa_{\ell m} - s(2s - 1)J_{\ell m}} dr \right)$$

where, dropping $n$ and $\ell$ subscripts.

---

I — oo give good approximation to the actual values at considerably lower if we use $\sqrt{X} = \sqrt{\ell + 1}$.

The angular integral $Q_{lm}$ is a polynomial in $m^2$ of degree $s$. Its explicit form is most easily obtained with the use of the following recursion relation

$$Q_{lm} = \frac{1}{4A + 4 - 4s^2} \left[ 2s - 1 \right] \left[ (Q_{l-1,m} - 2A - (2s - 1)^2 - 2m^2) + Q_{l-2,m}(2s - 3)(s - 1) \right]$$

The final step in connecting $Q_{lm}$ to $a_j$ coefficients occurring in Eq. (1) consists in expressing $(m/L)^{2s+1}$ in terms of $P_{2s+1} \cdots P_t$. If one limits oneself to $S = 2$ then for the observed solar p - modes it is an excellent approximation to neglect $J$ in comparison with $K$ in Eq. (4) and obtain:

$$Q_{lm} = a_0 H_{2s+3} + H_{2s+1},$$

where $< \ldots >$ denotes an average over $r$ weighted with $K(r)$, and $H_{2s+3}$ are certain $r$-dependent coefficients. In Figure 1 an example of $K$ for an individual mode is shown and compared with $K$ averaged over range of $n$ typically present in observational data.

3.2 Quadratic effect of rotation

In an application to the solar oscillation these effects were calculated by Gough and Taylor (Ref. 11) and by Dziembowski and Goode (Ref. 12). Considering only the case of $r$-dependent rotation they found that the frequency perturbation may be quite large - comparable with that due to the linear effect for low- $\ell$ modes — if there is a steep $\Omega$-gradient occurring in the subphotospheric region. Helioseismological data obtained later excluded such a possibility. Nevertheless quadratic effects of rotation make significant contribution to the even-order $a_j$-coefficients in Eq. (1). P. Goode and I derived a formula for these coefficients for the rotation law given by Eq. (2). This together with some numerical results will be published elsewhere.

If the expansion of is terminated at $S = 2$ then all even terms up to $4S + 2$ are present in Eq. (1). For the rotation rate derived from odd-order $a_j$'s, however, only the contribution to $a_0$ may be regarded significant as may be seen in Figures 11 and 12 where evaluated $a_2$ and $a_4$ are plotted against $l$ and compared with observational data.

3.3 Large scale magnetic fields

Effects of such fields on the solar p-mode frequencies were studied in a number of papers (see e.g. Refs. 11-12 and several contributions in Ref. 13). Here I will present some new results obtained in collaboration with P. Goode, which in full length will be published later.

The toroidal and poloidal components make separate contributions to the frequency splitting. To obtain an expression in the form of Eq. (1) one should insert into general perturbation formula an expansion for the magnetic field component in terms of $\mu^2 \phi$ and $P_{2s}$, Eq. (3) for the eigenvector and perform the angular integration. It turns out that all the integrals become either $Q_{lm}$ or $m^2 Q_{l-1,m}$ and therefore can be expressed in terms of $P_{2s}(m/L)$ of even orders. The explicit expressions for $a_j$ is extremely complicated and therefore will not be reproduced here.

For the toroidal field one may obtain an analogue of Eq. (4). Writing the square of the field in the following form

$$B_\phi^2 = 4\pi p(r) \sin^2 \theta \sum_{s=0}^{L} \beta_s(r) \mu^{2s},$$

where $p$ is the gas pressure, we have

$$a_j = \sum_{s=0}^{L} \frac{1}{\beta_s(r)} \int_{r_{min}}^{r_{max}} (T_{2m} J_s + D_s \frac{d \beta_s}{dr}) \sin^2 \theta \sin \theta d \theta d\phi,$$

where $j = 2, 4, \ldots, 2S + 2$.

Examples of $T_{2m}$ and $D_{2s}$ functions are shown in Figures 2-5. They can be compared with the $K(r)$ — the kernel for rotation shown in Figure 1. Integrating by parts we could, in principle, eliminate the derivative of $\beta$ from Eq. (7) to obtain a single kernel but it would exhibit a very erratic behaviour resulting from differentiation of $D$.

In Table 1, values of $H_{2s}$ are given for selected $L$s and for two choices of $L$. We can see that the asymptotic values corresponding to $l$ — oo give good approximation to the actual values at considerably lower $l$ if we use $L = \sqrt{\ell}$.

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Table 1

![Figure 1](image1.jpg)

Figure 1. The individual and average kernels $K$ for calculation of the frequency splitting due to the linear effect of rotation plotted against the fractional radius.

In Table 1, values of $H_{2s}$ are given for selected $L$s and for two choices of $L$. We can see that the asymptotic values corresponding to $l$ — oo give good approximation to the actual values at considerably lower $l$ if we use $L = \sqrt{\ell}$.

![Figure 2](image2.jpg)

Figure 2. The individual and average kernels $T$ (conf. Eq. 6) for calculation of $a_2$ coefficient due to the toroidal field corresponding to $s = 1$ (conf. Eq. 7) plotted against the fractional radius.
For not too rapidly changing field it is more revealing to use a spline function representation of $\beta$ and write $a_j$ in the following form

$$a_j = \sum_i \sum_{\ell} b_{i,\ell} \rho_{i,\ell}(x_i)$$

where $x_i$ are the values of $r/R$ at selected mesh points. In Figures 6 and 7 behaviour of $F_{s3}$ and $F_{s4}$, respectively, is shown for a few values of $\ell$. It should be noted that (i) an effect at the level of 1 nHz may be expected if $\beta$ exceeds 0.001 in some range of $r$ (see Table 2 for corresponding field), (ii) expected $a_2$ and $a_4$ are of the same order, (iii) their sign should vary with $\ell$.

<table>
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### Table 2

Maximum $B$ [MG] if $\beta = 0.001$

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</thead>
<tbody>
<tr>
<td>$B$</td>
<td>55</td>
<td>12</td>
<td>3.0</td>
<td>0.72</td>
<td>0.28</td>
</tr>
</tbody>
</table>

### 4. Internal rotation obtained as a solution of an inverse problem

Various methods of the solution of the inverse problem for the solar rotation were thoroughly discussed by Gough (Ref. 14). Methods involving a regionsisation (for mathematical background see e.g. Ref. 15) seem particularly suitable for our main aim which is a search for radial gradients in the rotation rate. I will start with a brief description of an algorithm recently applied (Ref. 16) to the data obtained by Libbrecht (Ref. 6) containing $a_j$ coefficients for nearly 700 individual modes. Then I will present the results and compare them with those obtained earlier.

The essence of our approach consists in looking for $\Omega_i(r/R)$ which minimizes

$$h_s + \lambda \int \left( \frac{\partial \Omega_i}{\partial x} \right)^2 dx$$

where

$$h_s = \frac{1}{N} \sum_{k=1}^{N} \frac{<\Omega_{\gamma k} - W_k>^2}{\sigma_k}$$

(9)
\( k \) is an index identifying \( n \) and \( \ell \), \( W_{\ell k} \) is an appropriate r.h.s. in Eq. (2) evaluated from the data and \( \sigma \) is its error. The choice of the regularisation parameter, \( \lambda \), is a subtle problem. Setting it large implies calculation appropriate averaged values. Its decrease results in improvement of the fit, \( h \), decreases, but simultaneously the errors on \( \Omega \) increase. At some point the results become clearly meaningless.

In our calculations we assumed \( \Omega \) constant below \( x = 0.4 \) because the data have very little resolution below that point, while above we let \( \Omega \) vary linearly between consecutive mesh points separated by 0.03. We used accurate Eq. (4) rather than approximate Eqs. (5) and (6) following three features visible at \( \lambda = 10^{-7} \) are quite stable to changes in \( \lambda \) and in choice of the mesh points: (i) an algebraic decrease of \( \Omega \) and \( \Omega \) between the radiative interior and the convective zone, (ii) a decrease of \( \Omega \) with \( x \) for \( x = 0.4 \) and 0.8, (iii) the dip in \( \Omega \) at \( x = 0.4 \). Thus, they deserve attention.

Brown et al. (Ref. 10) used various methods to determine the internal rotation from the data obtained by Brown and Morrow (Ref. 7). They found similar, though seemingly less steep, increase in the equatorial rotation rate between the radiative interior and the convective zone and also the tendency to smaller difference between the equatorial and the polar rotation rate.

There are more significant differences between the rotation curves shown in Figure 8 and that obtained by Duvall et al. (Ref. 17) from Duvall and Harvey (Ref. 3) observations. The latter runs significantly lower in the whole radiative region above \( x = 0.4 \). Good (private communication) who applied the method described in this section to the same data finds that the difference exceeds 30 mHz - more than twice the sum of errors.

5. Origin of the symmetric component in the frequency splitting

In Figures 11 and 12 which are taken from Ref. 16 the averaged values of \( a_2 \) and \( a_4 \) obtained by Libbrecht (Ref. 8) are plotted against \( \ell \). The sign of \( a_2 \) is consistently negative and that of \( a_4 \) positive, but the values are well below the dispersion. Libbrecht (Ref. 8) comparing these data with previously obtained ones finds a systematic decreasing over a four-year period.

Continuous lines in these plots show evaluated values due to the second order effect of rotation assuming \( r \)-independent \( \Omega \) as derived from the odd \( a \)'s in the same data set. It is interesting to observe that this explains large part of the observed effect in \( a_2 \), but has negligible influence on \( a_4 \).

The residual effect may be due to the latitude dependence in temperature in photospheric region linked to the local magnetic field as suggested by Kuhn (Ref. 18) but it also may be caused by a large scale magnetic in the interior. With more accurate data one should be able to distinguish between the two possibilities on the basis of different \( \ell \)-dependence.
6. Conclusions and discussion

I will now restate the major conclusions obtained from analyses of the fine structure in the frequency spectra for solar oscillations. The order reflects my personal opinion about credibility of the findings:

(i) rotation rate in the whole convective zone is similar to that observed in photosphere

(ii) there is a downward decrease in the equatorial rotation rate, $\Omega_0$, by about 20 mHz occurring in some vicinity of the lower boundary of the convective envelope

(iii) the latitude dependence of the rotation rate in the radiative interior is weaker than that in the convective envelope if at all existent.

(iv) There is a dip in $\Omega_0(r)$ occurring in the middle of the convective zone

(v) $\Omega_0(r)$ increases with depth in the inner part of the radiative interior

(vi) second order effects of rotation do not account for observed symmetric component in the frequency splitting

(vii) there is evidence for the changes with solar cycle

The small difference in the angular velocity of rotation between the radiative interior and the convective envelope implies that there is an upward angular momentum flow. Dziembowski et al. (Ref. 19) and Spruit (Ref. 2) suggest that this is caused by a magnetic torque. Somewhat greater equatorial rate in the interior does not imply greater angular momentum density because at higher latitudes the rate is most likely lower. The Kelvin-Helmholtz instability occurring in the radiative region should result in reducing the difference between the polar and the equatorial angular velocities to much smaller value than that determined for the convective envelope (Ref. 20).

It is still impossible to determine accurately the extent of the zone where the main changes in rotation take place. The data are consistent with jump-like changes occurring in the thin overshooting region where according to currently prevailing ideas the solar dynamo operates.

We do not know whether the dip in the middle of the convective zone is a real feature. If so then it must be a manifestation of the solar activity and should exhibit changes with the cycle.

The changes in the deep interior rotation, if real, would have far reaching consequences for our understanding of the magnetic activity of the Sun.

7. Acknowledgements

I am grateful to Phil Goode and Ken Libbrecht for allowing me to use their results before publication and to my colleagues from CAC Maciek Kozlowski and Jan Zalewski for their kind assistance in preparing the manuscript.

8. References

On The Radiative Damping Of P-Modes
In Solar Magnetic Flux Concentrations

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Abstract:

In a generalization of a work by Mihalas & Mihalas (1983) describing the propagation of compressive disturbances in a radiating fluid, we include the dynamical influence of a uniform magnetic field. The radiating fluid is treated to be gray, in LTE, and assumed to obey the Eddington approximation. We apply these results to the interaction of solar p-modes with sunspots in the context of a simple model developed by Abdelatif & Thomas (1987). For physical conditions representative of the solar envelope and for a variety of embedded magnetic structures, the temperature fluctuations associated with compressive waves are reduced inside the magnetic regions. Hence, radiative damping of these disturbances is suppressed to an extent that depends upon the nature of the mode (fast or slow magnetoacoustic) and the propagation direction relative to the uniform background field.

This conclusion raises some interesting predictions concerning the observational signatures of compressive waves in the solar envelope.

Subject Headings: Hydromagnetics - Radiative Transfer
Sun: Sunspots— Oscillations.

Introduction

Braun et al. (1987) reported the absorption of p-modes by sunspots. One possible mechanism for this absorption might be strong radiative damping due to the much lower opacities in sunspots as compared to the undisturbed photosphere at the same geometrical depth. Abdelatif & Thomas (1987) have developed a model for the interaction of p-modes with sunspots in the limit of infinite Boltzmann-number Bo, which measures the ratio of fluid enthalpy flux to the radiative flux. Hence, radiative damping of p-modes is excluded in their model. Typical values for Bo range from 10 in the subphotospheric layers of a sunspot to 0.5 high in its atmosphere. Therefore we have extended the work of Mihalas & Mihalas (1983), describing the propagation of compressive disturbances in a radiating fluid to account for the presence of a magnetic field. The complete description of this generalization together with conclusions for a wide variety of astrophysical objects is given elsewhere (Bogdan & Knöller, 1989). In this contribution we restrict our attention to the effects of radiative damping on the interaction of p-modes with vertical magnetic fields in the surface layers of the sun.

Basic Equations

The equations for the magnetized material are:

**Continuity**:

\[
\frac{\partial \rho}{\partial t} + \rho \nabla \cdot \mathbf{v} = 0 \tag{1}
\]

**Momentum**:

\[
\rho \frac{\partial \mathbf{v}}{\partial t} = -\nabla (\rho + P) + \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B}, \tag{2}
\]

**Induction**:

\[
\frac{\partial \mathbf{B}}{\partial t} = (\mathbf{B} \cdot \nabla) \mathbf{v} - \mathbf{B} (\nabla \cdot \mathbf{v}), \tag{3}
\]

and **Energy**:

\[
\frac{D \rho}{D t} + C_S^2 \frac{D P}{D t} = 4\pi(\gamma - 1)\kappa (J - S), \tag{4}
\]

where \(\rho\) denotes density and \(\mathbf{v}\) velocity, \(P\) gas pressure, \(P\) radiation pressure, and \(\mathbf{B}\) magnetic field. The ideal gas law is assumed, \(C_S\) is the adiabatic sound speed, and \(S\) is the source function, which is set equal to the Planck function.

---

The corresponding equations for the radiation field are:
\[
\frac{1}{c} \frac{DH}{Dt} + \frac{1}{3} \nabla J = -\kappa H,
\]
where \( H \) is the Eddington flux, and \( c \) is the speed of light.

Since we adopt the Eddington approximation, the radiation pressure \( P \) is equal to \( 4\pi c^2 J / 3c \). where \( J \) is the mean intensity, \( H \) is the Eddington flux, and \( c \) is the speed of light.

**Linearization**

We linearize equations (1) - (6) about a uniformly magnetized, gray LTE atmosphere, of constant temperature, density, sound-speed, and Alfvén-speed. Hence, the Eddington flux is zero in the equilibrium state.

We seek plane wave solutions of the form
\[
\exp[i \omega t - k(z \cos \theta + x \sin \theta)]
\]
where \( k \) is the wavenumber, \( z \) is the axial direction, and \( \theta \) is the angle between the wave vector and the magnetic field. The transverse Alfvén mode is unaffected by the radiation field and can therefore be factored out. The dispersion relation reduces then to a cubic equation in \( \tau \) with a, \( \tau_c \), and \( \tau_e \) as the optical thicknesses of one wavelength of a disturbance with frequency \( \omega \) traveling with the sound speed or the speed of light, respectively, and \( \tau_e = \rho_0 c \gamma \left( \frac{\gamma - 1}{\gamma - 1} \right) \rho \) is the Boltzmann number.

Three modes can be easily identified in the limit of \( \tau_e \to \infty \), where the material decouples from the radiation. They are the fast and slow magnetoacoustic waves and a radiation diffusion wave. In general, one solves the (complex) cubic equation for \( \tau \) numerically in order to obtain the propagation characteristics of the three modes as a function of the independent variable \( \tau_e \). A thorough discussion of the solution characteristics of this dispersion relation can be found elsewhere (Bogdan & Knolker, 1989). Here we restrict ourselves in applying this relation to a special case.

**Application to a sunspot model**

We employ the dispersion relation (7) to study the effects of the radiation field on the theoretical model of Abdelatif & Thomas (1987) describing the interaction of \( p \) modes with a sunspot. In their model, the sunspot is represented as a uniformly magnetized cylinder surrounded by field-free gas. Gravity and vertical stratification are neglected, so trapped modes are obtained by imposing a pair of rigid horizontal plates. The nature of the excited magnetoacoustic wave depends upon the value of \( \omega / k_c \) for the incoming acoustic wave, since the excited and scattered waves must have the same \( \omega \) and \( k_c \) as the incident disturbance. Let \( C_S, C_A \) be the sound speed and Alfvén speed within the magnetized cylinder, and let \( C_{ext} \) be the external sound speed. In a typical sunspot, these velocities have the relation \( C_S < C_{ext} < C_A \), and moreover \( \omega / k || > C_{ext} \). Regarding both \( \omega \) and \( k || = k_c \) as real and fixed, the dispersion relation (7) may be rewritten as

\[
k^2 \equiv k^2 \sin^2 \theta = \left( \frac{\omega^2 - k^2 (C_S^2 + C_A^2)}{(C_S^2 + C_A^2) \omega^2 - k^2 || C_S^2} + O\left( \frac{1}{\rho_0} \right) \right),
\]

for the magnetoacoustic mode, and for the radiation-diffusion mode

\[
k^2 = -3\kappa^2 (1 + \frac{i}{\tau_e})^2 - k^2 || + O\left( \frac{1}{\rho_0} \right),
\]
in the limit of large Boltzmann number. For \( \omega / k || \gg C_A \), a magnetoacoustic wave that propagates nearly perpendicular to the magnetic field is excited. If \( \omega / k || \geq C_A \) this excited fast mode propagates nearly along the magnetic field. When \( C_{ext} < \omega / k || \) the excited fast mode is laterally evanescent. A slow magnetoacoustic wave cannot be excited until \( \omega / k || < C_S \) and this is not possible unless the flux tube is hotter than its surroundings: \( C_{ext} < C_S < C_A \). Thus, the incident acoustic wave excites the fast wave inside the cool magnetized cylinder (Abdelatif & Thomas 1987). Moreover, this excited fast
magnetoacoustic wave may be horizontally evanescent if the acoustic wave propagates predominantly in the vertical (e.g., field-aligned) direction.

Now we adopt representative values of $C_A, C_S, k, \gamma$. Both from a numerical model of a sunspot and its nonmagnetic surroundings. The sunspot model was "synthesized" by taking density, pressure and magnetic field strength values from Pizzo's (1986) sunspot model, and then computing the degree of ionization and the opacity with depth using subroutines from Denzer et al. (1984). For the incident acoustic wave we specify $\gamma = 2.09 \times 10^{-5} \text{s}^{-1}$ (representing p-modes in the 5-minute band) and a real value for $k_0$. From the dispersion relation, now a quadratic equation in $k^2 - \text{the (complex) values of } k_0$, in the unmagnized fluid and along the sunspot central axis are obtained. Since the radiation-diffusion wave is heavily damped for wave periods of order 5 minutes, we restrict our attention to the root that describes the (magneto) acoustic wave.

![Depth (km) and LOG (λ/cm)](image)

In figure 1 we plot the real and imaginary parts of $k_0^{-1}$ inside and outside the sunspot model versus the geometrical depth $z$. The non-magnetic solution exhibits the expected variation of sound-speed ($\lambda = 2\pi / \text{Re}[k_0]$ and opacity ($\kappa = 1 / \text{Im}[k_0]$)) with depth in the surrounding solar envelope. The radiative damping is severe in the vicinity of $\eta_{5000} = 1$ where the acoustic wave attains unit optical depth. In the sunspot the fast magnetoacoustic wave attains unit optical thickness ($\kappa \approx 1$) near the bottom of the Wilson depression of about 450 km. Here $L$ goes through a local minimum. However, the minimum value of $L$ is a factor of 10 larger than the value possible near $\eta_{5000} = 1$ in the unmagnized atmosphere. This suppressed radiative damping is directly attributable to the presence of the strong magnetic field (Bogdan \& Knöller, 1989). Near $z \approx -100$ km in figure 1 the excited fast magnetoacoustic wave switches from being laterally propagating to laterally evanescent. When $\omega / \kappa \approx C_A$ both $\lambda$ and $L$ are very large, so the transition between horizontally propagating and horizontally evanescent waves is marked by a "spike" in fig. 1. Above the Alfvén spike, the horizontal evanesence of the fast magnetoacoustic wave indicates that the incident acoustic wave suffers almost complete reflection from the magnetized cylinder when $L$ is less than the cylinder radius.

On the whole these results indicate that the significant magnetic fields that accompany the reduced opacities in the flux tube Wilson depressions produce interesting levels of radiative damping of acoustic waves. This conclusion follows from the nature of the acoustic excitation: the incident p-mode generally couples to a fast magnetoacoustic mode that propagates nearly field-aligned, but not near the whole propagates more nearly along than across the (vertical) magnetic field. We have the possibility of significant radiative damping only if the exciting p-mode can couple to a slow magnetoacoustic mode that propagates nearly field-aligned. This situation may arise if the internal flux tube sound speed is slightly greater than the external sound speed, $C_s > C_{\text{eff}}$. However, such a scenario appears to be ruled out for sunspots and pores where this radiative damping could have observable consequences.

In addition to the above application we have also considered a variety of flux tube models given by Knöller et al. (1988) and by Knöller \& Schüssler (1988). As in the sunspot, we find that the maximum damping occurs at about $\eta_{5000} = 1$, i.e. that it depends on the Wilson depression. Furthermore, the higher layers show laterally evanescent fast magnetoacoustic modes with multiple Alfvén-spike like the one in the sunspot shown in figure 1. Hence, all flute models considered show less radiative damping of the fast magnetoacoustic mode coupling to the solar p-modes as the p-mode damping in the undisturbed photosphere itself. It would appear, therefore, that the interplay of the magnetic field and the nature of the acoustic excitation produce fairly insignificant finite Boltzmann number corrections to the Abdelatif \& Thomas (1987) analysis. The $L = -500$ km layer of the sunspot model suggests these corrections probably do not exceed 5 - 15 %. Further, in so far as the neglect of gravity and vertical stratification inherent in the Abdelatif \& Thomas (1987) model can be justified, we would tentatively conclude that radiative damping is probable not the cause of the p-mode absorption by sunspots that Braun et al. (1987) have reported.

Conclusion

The above findings suggest a possible solution to a couple of outstanding problems concerning the interaction of p-modes with the lower photosphere.

Title (1988) notes that the observed p-mode induced intensity (temperature) fluctuations are greatly reduced inside magnetic plage relative to their level in the quiet photosphere. From the above discussion, it is readily seen that suppressed intensity / temperature / source function fluctuations are a natural consequence of the incident p-modes exciting quasi-parallel propagating fast magnetoacoustic modes in the vertical plage magnetic fields. It will be interesting to see if this simple conceptual explanation holds up under more detailed quantitative scrutiny.
As yet another issue, consider various calculations of the radiative damping of the solar p-modes, i.e., Christensen-Dalsgaard & Frandsen (1983); Kidman & Cox (1984), that ignore the influence of photospheric magnetic fields. That the omission of magnetic effects clearly overestimates the damping in and above magnetic plage can be supported both observationally (Title 1988) theoretically (this contribution). It remains to be seen whether this reduced damping in plage might have an impact upon the global p-mode lifetimes.

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Oscillations in Presence of Local Magnetic Fields

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ABSTRACT

Oscillatory Doppler shifts in sunspots and in small fluxtubes are observed using either the intensity profile of lines with negligible magnetic splitting or the circular Zeeman profile (Stokes-V inversion point) of lines with strong Zeeman splitting. Sunspot umbrae show local 5 min oscillations of reduced amplitude as compared to the neighbouring photosphere. Occasionally we find 5 min oscillations of opposite phase at both outer penumbral borders. This could be interpreted as tilting and bobbing of the spot as a whole on the oscillating solar surface.

In contrast to these larger scale sunspot fluxtubes the gas in the small fluxtubes of a few hundred km diameter seems to fully participate in the photospheric oscillations without any interaction with the magnetic field.

Prominence magnetic fields show sometimes a reaction to the photospheric 5 min and to the chromospheric 3 min oscillations, in addition several other periods are found. Indication is found for eigenmodes of Alfvén waves.

1. INTRODUCTION

The investigation of a possible interaction between oscillations and local magnetic fields so far plays only a minor role within the field of solar seismology. Nevertheless, we find it interesting to present some observational results.

On the solar surface local magnetic fields seem to occur either at larger scales in sunspots (e.g. 5=35.000km) or at small scales (typically 300km) in isolated fluxtubes responsible for the solar filament and CαK bright features.

2. SUNSPOTS

Oscillations in sunspots have been observed since 1972 by numerous authors who report a large variety of periods (for reference see e.g. Balthasar, Küevers, Wiehr, 1987). Recent observations by the authors, using an extended slit across the spot (Fig. 1a) and a diode array as sensor in the spectral plane, show the existence of 5 min oscillations of reduced amplitude with respect to the spot's neighbourhood. This is well visible in the unprocessed Doppler data (Fig. 1c) but also in the power contours Fig. 1b which additionally show that in photospheric layers the spots do not exhibit particular periods different from those in their neighbourhood. (This does not hold for chromospheric levels, where 3 min periods are found; see e.g. v. Uexküll et al., 1983, Thomas et al., 1987).

The above finding indicates that the spot participates almost passively in the photospheric oscillations except for the reduced amplitude. (It shall be mentioned that Abdelatif et al., 1986, find indication for the suppression of some periods in the spot). We occasionally find that the spot's

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Fig. 1: Sunspot oscillations along one spatial direction
a) slit jaw image in white light continuum
b) iso-power contours vs. frequency and slit direction
c) raw Doppler data vs. time for the 30 scan rows

neighbourhood oscillates in opposite phase at both sides along the slit direction. The observed penetration of oscillating regions into the penumbra (Fig. 1b) then implies that the spot itself oscillates together with its surroundings. The high conductivity implies that the highly inclined penumbral magnetic field might tilt and bob on an oscillating underground.

3. SMALL FLUXTUBES

Possible oscillations in the small fluxtubes are rather difficult to distinguish from those of their neighbourhood due to their very small size far below one arcsec (e.g. Wiehr, 1978). Observing, however, the Zeeman profile (Stokes-V), the unmagnetic surroundings cannot interfere. Such a measurement was first realized by Wiehr (1985) using a Doppler compensator. He found a perfect co-oscillation of the Stokes-V profile together with its corresponding intensity profile indicating that the gas inside the fluxtube oscillates with that outside.

These results have been verified at much higher spatial resolution at the 'Observatorio del Teide' / Tenerife, using the evacuated Gregory Coudé telescope shortly after the installation of its large spectrograph (Wiehr, 1986a). In the course of first test runs, a preliminary device was used consisting of a polaroid behind a 1/4 plate alternatively oriented in two perpendicular directions. The difference of the thus obtained left and right handed circular polarization spectra gives the Stokes-V profile, the zero crossing point of which was used as a measure of the Doppler motion inside the fluxtube. First rough evaluation (Wiehr, 1986b) showed a temporarily varying redshift with respect to the unmagnetic neighbourhood. In Fig. 2c we present the result of a detailed analysis showing a well defined oscillatory period. These data, however, still suffer from the non-simultaneous exposures of the two left and right handed polarization spectra. We therefore repeated these observations using the circular polarization analyzer from Semel (1933) which yields strictly simultaneous polarization spectra of 18 arcsec height (Fig. 2b). A single Ca II bright point (Fig. 2a) was observed in 20 steps over nearly 1/2 hour. The spatial resolution achieved ranges between 0.8 and 1.5 arcsec. The results, also shown in Fig. 2c, established the participation of the gas inside the fluxtube in the
oscillatory motion. We do not find a significant modification of the amplitudes as compared to those known from the unmagnetic photosphere in contrast to our above results for sunspots. Here we suppose a fundamental difference between the small fluxtubes and the large ones in sunspots as far as their influence on the solar photospheric oscillation is concerned.

4. Prominences

Prominences are generally considered to be suspended by a weak magnetic field of the order of a few Gauss (e.g. Leroy, 1987). It seems obvious to assume a high sensitivity of those field configurations to disturbances from the photospheric and chromospheric oscillations. Indeed, one occasionally finds five and three minute periods in the Doppler shifts of prominence emission line (for references see Balthasar, Stellmacher, Wiehr, 1988). However, numerous other periods are found which strongly vary spatially and temporarily as can be seen from CCD measurements taken at Sac Peak of the infrared Ca\(^{+}8542\) \(\AA\) emission as well as in the raw Doppler data (Fig. 3b) and in the corresponding power spectra (Fig. 3c). This might indicate that the prominence exhibits quasi-periodic Doppler motions different from those of the underlaying layers. However, one also has to consider possible influences from the development of the prominence as a whole.

Another prominence was observed under completely different conditions: Gregory telescope at Tenerife (instead of Sac Peak tower), 256*256 diode array behind a proximity-focussed image intensifier (instead of CCD camera), Ca\(^{+}\)H simultaneously with He (instead of Ca\(^{+}\)8542) emissions. The duration of this series was more than 5 hours. The raw Doppler data (Fig. 4a) show the high correlation

![Prominence Nov. 10, 1987 (Ca+ 8542)](image)

![Prominence Nov. 14, 1987: solid line Ca+ H, dotted line H epi)](image)
between those two emission lines in spite of their very different optical depths of more than 10 and, respectively, less than 1.0. These data show a similar behaviour as those from the Sac Peak measurements, i.e. a spatial and temporal 'chaos' of periods.

Besides this general verification of other observations, we find for the upper three rows harmonic frequencies at 0.7 mHz, or \( n = 1, 2, 3, 4 \), and probably also 5. Since similar harmonic frequencies are not known from the undisturbed solar layers, we are inclined to interpret these frequencies as eigenmodes of the prominence. A rough estimate of Alfvén waves in a prominence magnetic field of 10 Gauss, assuming a density of \( 10^{-13} \) g/cm\(^3\) and a typical length of 50,000 km, yields \( 1 \) mHz which is close to our observed 0.7 mHz.

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Wiehr, E.: 1986, in: The Role of Fine-Scale Magnetic Fields on the Structure of the Solar Atmosphere (E.H. Schröter, M. Vázquez, A.A. Wyller, eds.) p. 354 (=a) and p. 91 (=b)
Abstract

Using sunspot-spectrograms near the solar limb it is possible to determine the height of the formation of selected strong (chromospheric) lines by micrometric distance measurements. We have analysed the time sequences of $\text{H}\alpha$ and $\text{Na D}$ linespectra to check the possibility of a time variation in the height of formation of these lines. We have found periods in the 3 minutes range, but also longer periods. The amplitude of the height variation is in the range $\Delta h(\text{rms})=100-200$ km.

This is a new method to study oscillations in sunspots, without making use of intensity or velocity variations.

Keywords: oscillation, sunspots.

1. Introduction

Most of the information on oscillations in the solar atmosphere is based on the analysis of intensity- and velocity-variations in selected Fraunhofer lines. For studying the oscillations in sunspot chromospheric regions a new method can be used. It will be described below.

It is well known that compared to the nearby continuum the cores of Fraunhofer lines originate in higher atmospheric levels. Thus the solar diameter in chromospheric lines looks somewhat larger than in the continuum. So the position of a sunspot near the solar limb in white light is not the same as in a chromospheric line. This is demonstrated at the top of Fig. 1. When the slit of a spectrograph over the sunspot is perpendicular to the solar limb the spectrum looks like the schematic drawing at the bottom of Fig. 1: The center of the line is shifted towards the solar limb. This observational effect is well known for more than 30 years. It gives the possibility to determine the height of formation of the observed line compared to the continuum by purely geometrical distance measurements, see also Ref 1,2. For $\text{H}\alpha$ the difference between $x_2$ and $x_3$ (see Fig. 1) is of the order of $2000$ km or $3$ arcsec, respectively. This value is in good agreement with chromospheric sunspot models Ref 3)

The question appears now: Is the height of formation of the line-core time independent, or does it change with time, particularly with periods of the well known chromospheric oscillations?

To check this we observed several sunspots in $\text{H}\alpha$ and in the sodium lines $D_1$ and $D_2$ with the domeless Coudé-refractor in Capri. The time interval was only 20 minutes. These observations were carried out only for searching for oscillations, not to study or analyse them, if there are any.

1. Each of the data sets $|x_2 - x_1(t)|$, $|x_2 - x_3(t)|$ or $|x_2 - x_3(t)|$ show a nonstochastic, oscillatory structure. It is remarkable, that the position of the continuum (compared to the unchanged solar limb) also varies considerably. The amplitudes are about 200 km. In all cases the $\text{rms}$-values of the line cores are larger than the $\text{rms}$-values of the continuum.

2. In order to check the method and the accuracy of the data we compared the independent data sets of the continuum variation...
We found a remarkable power in the well known frequency region. But more important for this analysis is the fact, that the coherence is about 0.7—0.8 and the phase difference is nearly zero. This is a strong indicator for a real oscillation.

3. The Hα power spectra \([\nu - \nu_1]\), corresponding to the height difference between the core and continuum of Hα, show two power regions. One is between 4 and 7 mHz, that is the 260 to 140 sec region, the other is near 1.7 mHz, that is about 600 sec. There is no distinctive power in the 5 min region.

4. The overall results are shown in Fig. 2. Although these mean power spectra where obtained with a low frequency resolution, we found oscillations in the height of formation of the line cores and in the continuum as well. It is indicated, particularly in the individual powerspectra, that two frequency regions are predominant:

1. \(1.8 \pm 0.3\) mHz or 550 sec (600-450)
2. \(5.8 \pm 0.7\) mHz or 170 sec (200-150)

The power in the low frequency region is remarkably higher than in the higher frequency region. The 3-minute-oscillation in the sunspot chromosphere is well known. There are indications by v. Uexküll et al (Ref. 3) that oscillations with longer periods appear in the observations too.

5. The coherence and phase relations show, that there is a clear coherence between the D_1 and D_2 cores, and between the cores and the continuum. That means, the whole sunspot atmosphere oscillates coherently. But the amplitudes in the lines, that means the amplitudes of the oscillations in the higher atmosphere, are remarkably larger, about a factor 1.5.

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SOLAR P-MODES FREQUENCY VARIATIONS BETWEEN 1980 AND 1986

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ABSTRACT / RESUME

Using a method of power spectra cross-correlation, we were able in 1986 to confirm a 0.39 $\mu$Hz decrease of solar low degree p-mode frequencies between 1980 and the end of 1984. With the same method applied to our 1984/85 South Pole data and to all the ACRIM data obtained until April 1986, we again confirm the value of this decrease. Moreover, we find an extreme stability of these frequencies from spring 1984 to spring 1986 with a relative accuracy of 0.02 $\mu$Hz.

Keywords: low degree p-mode, frequency change.

1. INTRODUCTION

The idea of tracking down long-term changes in the solar interior through seismology has long been discussed. Perhaps one of the most simple helioseismological measurement on which investigation are the p-modes eigenfrequencies. The accuracy obtained on the measurement of any individual p-mode frequency is only limited by its amplitude and phase lifetime. This lifetime is strongly frequency dependent (Ref. 1, 2, 3) but in the 3 mHz frequency range, where amplitudes are maxima, the width of each peak in a Fourier spectrum is of the order of 1 $\mu$Hz. Assuming perfect data, 10 to 30 lifetime of integration will then give a r.m.s. deviation of the order of 0.2 to 0.3 $\mu$Hz around the true frequency, which means a relative accuracy of $10^{-4}$ (Ref. 4, 5). If we now use more than one, say $N$, eigenmodes, we can track any change on this parameter with a relative accuracy of about $10^{-4}/\sqrt{N}$.

2. FORMER RESULTS

Analyzing the ACRIM solar intensity data, Woodard and Noyes (Ref. 6) reported a measured 0.4 $\mu$Hz decrease of low degree p-modes frequencies between 1980 and 1984. This result was obtained by comparing individual frequencies of 13 eigenmodes and resulted in a final uncertainty of the order of 0.15 $\mu$Hz. It was followed by many other attempts providing conflicting results for the low 1 p-modes ($f = 0 - 6$) (Ref. 7, 8, 9).

Figure 1 presents our 1987 result (Ref. 10). It shows the central peak of the correlation between ACRIM 1980 data and our 1984/85 South Pole data. Once corrected for the small bias introduced by the difference in visibility between the two experiments, this curve shows a decrease in p-modes frequencies of about 0.39$\mu$Hz.

Figure 1 presents our 1987 result (Ref. 10). It shows the central peak of the correlation between ACRIM 1980 data and our 1984/85 South Pole data. The peak is obviously displaced from the zero of the correlation. Careful measurements, involving several methods as the determination of the centroid, determination of the median point at different heights, determination of the highest point, removal of background trend, lead us to reduce the uncertainty on the determination of this shift. The value was found to be 0.39 \pm 0.04 $\mu$Hz.

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Simultaneously, Duvall et al. (Ref. 11) and Rhodes et al. (Ref. 12) presented results for high 1 (10 - 200) frequencies changes consistent with a null variation along the period 1981 - 1987.

3. METHOD AND DATA

One can find easy to produce a mean difference between two frequency sets. This involves three operations: two determinations of a frequency set and one comparison. It happens that determining frequencies with peak finding methods leads to systematic errors due to the presence of noise and of a window function introducing sidelobes in the Fourier spectrum. The situation is even worse when this is to be done twice. For this reasons we prefer to avoid this problem and use a cross-correlation analysis for a statistical frequency comparison. This is a one operation method. As an example, figure 2 shows the peak of correlation between ACRIM 1984 and ACRIM 1985-86.

Table 1

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Frequency Shift Since 1980, in µHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACRIM 80</td>
<td>0.00</td>
</tr>
<tr>
<td>ACRIM 84</td>
<td>-0.356</td>
</tr>
<tr>
<td>South Pole 84/85</td>
<td>-0.394</td>
</tr>
<tr>
<td>ACRIM 85</td>
<td>-0.350</td>
</tr>
<tr>
<td>ACRIM 85/86</td>
<td>-0.368</td>
</tr>
</tbody>
</table>

We have ten numbers which are not independant. For instance, we should have (ACRIM 84 - ACRIM 80) = (ACRIM 84 - South Pole) + (South Pole - ACRIM 80), and this is not the case. Using all such equations, we can have a reasonable confirmation of our estimated uncertainties, and we can also calculate the average frequency shift for each data set referred to the 1980 ACRIM initial frequencies. In this case, the final uncertainty is reduced at about ± 0.02 µHz, and the results are:

4. DISCUSSION

Our result is plotted on figure 3. Given ± 0.02 µHz as the one sigma error on these numbers, it must be noted that these numbers are consistent with a change of 0.0 ± 5 10⁻⁶ of the solar p-modes frequencies during a two year period extending from may 1984 to april 1986. However these frequencies are smaller by 18 sigmas than they were four years earlier.

Figure 3. The thick horizontal line in the upper left corner is the frequency reference taken from the 1980 ACRIM power spectrum. In the four boxes shown between 1984 and 1985, the horizontal extension is the duration of the analyzed time string, and the vertical extension is the uncertainty bar. It appears that despite the surprising stability found for the p-modes frequencies during two years around the solar minimum, they are consistent with an 11-year variation in phase with the magnetic cycle. This will have to be confirmed by more measurements obtained after may 1986.
5. CONCLUSION

The mean frequencies of the solar low degree p-modes can be compared with an accuracy better than $10^{-5}$ when measured with different datasets. They have now been shown to be smaller by an amount greater than this uncertainty, during a two year period centered around the solar cycle minimum. Their very high stability observed during these two years has been shown to be consistent with an 11 years variation of about $10^{-4}$ peak to peak relative amplitude. The results of the Tenerife group are quite consistent with a variation of these frequencies in phase with the solar magnetic cycle. It is now of the greatest interest to gather information on the behaviour of this phenomenon. It is then tempting to compare this decrease to another solar observable like the solar radius, which is measured to a comparable precision by astrometrists. Although these measurements have been shown controversial (Ref. 16, 17, 18, 19), the variation in diameter implied by the seismology (0.2 arcseconds) is close enough to the 0.3 arcseconds variation reported by Laclare (Ref. 18.). The phase of the two phenomena is opposite, so that the sun would be bigger by 200 Km at minimum cycle, when its eigenfrequencies are smaller.

6. REFERENCES

HELOSEISMOLOGY FROM THE SOUTH POLE: COMPARISON OF 1987 AND 1981 RESULTS

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ABSTRACT

Full disk images with -10 arc sec pixels and filtered to a -7 Å pass band centered on the Ca II K line were obtained from the geographic South Pole in 1981 and 1987. In 1981, about 50 hours of essentially uninterrupted data were obtained. In 1987, three such runs were obtained over a period of 325 hours for a duty cycle of about 47%. The 1987 observations are characterized by a much lower level of solar activity than 1981, a much improved CCD camera, considerably better image stability and a varying amount of instrumental scatter. The 1987 data have a substantially better, signal-to-noise ratio than the 1981 data so that oscillations with degrees from 0 to 150 and frequencies from 2 to 7 mHz are well observed. The observations were reduced to spectra in \( f, m \) and \( v \). This paper presents a comparison of \( p \)-mode frequencies measured in 1981 and 1987 and coefficients of Legendre polynomial expansions of frequency shifts caused by solar rotation. We also study the time behavior of systematic frequency shifts which depend upon \( m \) but which do not arise from rotation.

Keywords: Helioseismology, Sun, Solar Interior, Solar Observations

1. INTRODUCTION

In December 1981 we made observations of solar oscillations from the geographic South Pole (Refs. 1-9). This observing site was used to obtain a long sequence of uninterrupted observations so that oscillation spectra with minimal sidelobe corruption could be computed. The full solar disk was recorded with -10 arc sec pixels every 90 s. A filter centered on the Ca II K line and having a passband of about 7 Å allowed us to detect intensity oscillations around the temperature minimum region of the solar atmosphere. Although about 160 hours of observations were obtained, the weather was not favorable and eventually we analyzed a 50-hour period having one 4-hour break due to clouds. Analysis of these observations provided accurate measurements of frequencies of \( p \)-mode oscillations over a spherical harmonic degree range of 6 to 98 (Ref. 9) and measurements of angular-order-dependent frequency splitting over a degree range of 20 to 100 (Ref. 4). We measured the line widths and amplitudes of \( p \)-mode multiplets (Ref. 8) and the observations were also used to confirm the absorption of \( p \) modes by sunspots (Ref. 7).

2. 1987 OBSERVATIONS

Spectral activity reached a minimum level in September 1986. Our new observations were made in November 1987 while activity was still at a fairly low level. It was not possible to exactly reproduce the instrumental setup used in 1981 but we tried to match the essential characteristics. Figure 1 shows the instrument on site about 7 km from the South Pole and about 200 meters from the 1981 site.

In 1981, the telescope and instrument package were fed by a rotating mirror while in 1987 the telescope and instrument were attached to a moving platform which tracked the sun. In both cases the image was essentially stationary on the detector, however, in 1987, the rotation axis of the sun was not aligned parallel to the pixels of the detector as it had been in 1981. The mirror drive in 1981 exhibited a small tracking error with a period of 8 minutes. In 1987, the rotation axis of the sun was not aligned parallel to the pixels of the detector as it had been in 1981. The mirror drive in 1987 has a substantially better, signal-to-noise ratio than those used by others and it is possible that some systematic error affects our method, the methods used by others, or both. For these reasons, we decided to repeat our 1981 measurements at a time of low solar activity to search for the origin of the discrepancies.
The CID exhibited a nonlinear response to light level and this factor plus an electrical component failure caused the camera systems used in both passbands to be blueward of the K line by about 3 Å which reduced oscillations down to degree 0.

The K-line observations alone in 1987 were quiet enough to show transparency fluctuations change with wavelength enough to produce satisfactorily. For each run a composite constant exposure was created by removing the low-spatial frequency trends from both the broad and narrow passband filters whereas instrumental and atmospheric fluctuations should be the same. Thus a suitable ratio of signals from the broad and narrow passband filters should cancel non-solar oscillations and leave a solar signal. This idea did not work out because atmospheric transparency fluctuations change with wavelength enough to produce substantially different signals in the broad and narrow passbands. As we shall see, the strategy was not needed because the K-line observations alone in 1987 were quiet enough to show oscillations down to degree 0.

The K-line filter showed aging effects between 1981 and 1987. In 1987 we added an auxiliary detector fed by a beamsplitter to help detect low-degree oscillations. This detector, a silicon quadrant cell, was located at an auxiliary focal plane and was filtered by the same 400 Å passband prefilter used in conjunction with the K-line filter. The idea was that solar oscillations should be weaker seen with the broad passband compared with the narrow passband filter whereas instrumental and atmospheric fluctuations should be the same. Thus a suitable ratio of signals from the broad and narrow passband filters should cancel non-solar oscillations and leave a solar signal. This idea did not work out because atmospheric transparency fluctuations change with wavelength enough to produce oscillations extending to a frequency of about 6.2 mHz, well above the Nyquist frequency of 5.57 mHz. To avoid this aliasing problem in 1987, we raised the Nyquist frequency by integrating 1 s exposures for 75 s before recording. As we shall see, the 1987 observations surprisingly show that oscillations extend to frequencies above the new Nyquist frequency of ~6.67 mHz so we were not successful in entirely avoiding aliasing problems in 1987.

Thanks to the efforts of support personnel, we were able to reach the South Pole in early November and start observations while skies were relatively clear. We recorded about 300 hours of observations and stopped, having used all the tape we brought. The observations are concentrated in 4 blocks of about 60 hours duration each. In this paper we consider the results from the first of these blocks; we defer a discussion of the fth data set to a future paper.

In both 1981 and 1987 we had a problem with scattered light in the observations. The source of the problem was the K-line filter which contained a number of elements oiled together to prevent multiple reflections. Some of this oil vaporized within the heated filter housing and escaped to condense on nearby cold surfaces. In 1981 this surface was unfortunately the last low in the optical system. In 1987, we took a number of precautions to prevent the problem. First, a different oil which should not have vaporized was used in the filter. Second, the filter was located away from cold optical surfaces. Third, the system was operated in Tucson for many days with no sign of trouble before shipment. In the end these precautions were not sufficient and the 1987 data suffer from a scattering 1.5° above the limb which ranges from a few percent of the disk center intensity to ~30%. A strategy for correcting the effects of this scattered light has been devised and found to be a satisfactory way of coping with the problem. The details will be given in a future paper.

In both 1981 and 1987, the first step in the reduction of the data is to correct for photometric errors. Dark exposures are subtracted from all exposures and then the response of each pixel is normalized to a constant level for a constant input light flux. In 1987, we used two methods to produce a uniform light flux on the detector. First, we integrated 10 s exposures for 90 s before recording. As we shall see, the 1987 observations surprisingly show that oscillations extend to frequencies above the new Nyquist frequency of ~6.67 mHz so we were not successful in entirely avoiding aliasing problems in 1987.
between rows and columns was corrected. In 1987, there were no bad pixels but there was a smearing of the image due to the lack of a shutter during readout. There was also a secondary image displaced from the main image which annealed in the K-line filter. The secondary image was a distorted version of the main image, slightly larger but also geometrically warped. A nonlinear deconvolution routine was developed to suppress the secondary image.

The third reduction step is to remap the images onto a standard grid with uniform steps in the sine of the latitude and longitude difference from the central meridian. This involves finding the coordinates of the limb of each image. In 1981, a simple first-derivative edge detector and a least squares ellipse fit were used to define the limb. One of us (SJ) has studied limb finding routines and learned that the simple 1981 approach produces a systematic diameter error of about 0.14% in good accord with the empirical finding (Ref. 9). To reduce this slight error by a factor of three, for the 1987 data we used a more accurate routine, based on a second derivative. This will be described in detail elsewhere. Interpolation for the remapping was done with a cubic spline interpolator. We produced maps having 400 elements of 0.3516 degrees in longitude and 226 elements of 0.00841 size in sine latitude. This range excludes noisy pixels close to the limb. No spatial apodization was used.

The fourth step is to remove low-frequency fluctuations from the time series of maps. This was done in 1981 by normalizing each remapped image to a low-order spatial function. Limb darkening is hard to remove this way. For the 1987 data we adopted the simple strategy of normalizing each map to a running mean of the maps. The running mean was produced from 15 maps centered on the map to be corrected and using equal weights. The resulting frequency response function is well defined with minor fluctuations in the frequency range of the oscillations which are easily corrected after spectra are computed.

The fifth step is to compute time series of spherical harmonic coefficients. This was done by a fast Fourier transform of the maps in longitude followed by a Legendre transform in sine latitude. Finally, power spectra are produced by Fourier transforming the time series of spherical harmonic coefficients. For both 1981 and 1987 data, we filled gaps with zeros and did not apply weighting to the ends of the data strings. The time series were extended with zeros to equal a power of two number of elements before the temporal transform.

The frequency scale of the final spectra has to be corrected for clock rate errors. This correction for both 1981 and 1987 data is estimated to be good to better than one part in a million.

4. RESULTS AND ANALYSIS

Figure 2 illustrates the power spectrum of oscillations based on 59 hours of observations with a duty cycle of about 95.3%. Computed with a similar spectrum from 1981, there are far fewer instrumental periodicities associated with tracking errors or sensor nonuniformities. This provides a cleaner background in the 1987 data.

In both 1981 and 1987 data the sun seems to set the basic background. This provides a cleaner background in the 1987 data. Limb darkening is hard to remove this way. For the 1987 data we adopted the simple strategy of normalizing each map to a running mean of the maps. The running mean was produced from 15 maps centered on the map to be corrected and using equal weights. The resulting frequency response function is well defined with minor fluctuations in the frequency range of the oscillations which are easily corrected after spectra are computed.

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The frequency scale of the final spectra has to be corrected for clock rate errors. This correction for both 1981 and 1987 data is estimated to be good to better than one part in a million.

Figure 2. Spectrum of p-mode oscillations based on 59 hours of observations made in November 1987. This spectrum is an unweighted average of spectra at all m values after removing frequency splitting according to Eq. 1. The display has been treated to approximately equalize contrast between the background and the oscillations over all frequency and degree ranges. The degree range is 1 (left) to 150 (right). The frequency range is 1.567 (bottom) to 6.667 (top) mHz. Note aliasing of high-frequency oscillation ridges at the upper right.
4.1 Frequency changes

For both 1981 and 1987, frequencies of mode multiplets were determined in the same way which is described in detail in Ref. 9. Briefly, we remove from each \((l, m, n)\) spectrum a dependence of frequency on \(m\),

\[
\Delta v = \sum_{i=1}^{l} a_i P_i\left(-m/L\right),
\]

where \(L = [(l+1)]^{1/2}\), \(P_i\) are the Legendre polynomials of degree \(i\) and the coefficients (in nHz sidereal) are \(a_1 = 444.6\), \(a_2 = 7.8\), \(a_4 = 21.2\), \(a_5 = 2.9\), and \(a_3 = -4.1\). Then the spectra are averaged in \(m\) with equal weight. The resulting spectral features are degenerate multiplets with frequencies approximately identical to those for a non-rotating sun. The spectrum in Figure 2 is averaged in this way.

Lorenz profiles were fit to the spectral features of the spectrum shown in Figure 2. This provided estimates (and errors) of frequencies, amplitudes, and line widths of the mode multiplets. We subtracted 1981 frequency measurements from 1987 frequencies and found the standard deviation of an average difference measurement to be ±0.6 nHz. We rejected 42 differences in excess of three times this value. A comparison of the remaining 712 frequencies measured with the 1981 and 1987 data is shown in Figure 3.

4.2 Frequency splitting

The dependence of mode frequencies on \(m\) contains information about rotation and structural asymmetry within the sun. This dependence can conveniently be represented by Eq. 1. We described the procedure we used for the 1981 data in Ref. 4 and we used the same technique for the 1987 data discussed here. Briefly, at a given degree we cross correlate the \(m\)-averaged spectrum with the spectra at other \(m\) values. Frequency shifts corresponding to the maximum correlations are then used to determine the coefficients by least squares. An iterative procedure is used to improve the \(m\)-averaged spectrum. We then calculate the weighted mean (and standard deviation of the mean) in ten-degree bins. Results below \(\ell=20\) are quite noisy and are not included here. Figure 5 shows a comparison of 1981 and 1987 results for odd coefficients and Table I lists the results.

Table I. Frequency splitting coefficients and standard deviations (nHz)

<table>
<thead>
<tr>
<th>(\ell)</th>
<th>(a_1)</th>
<th>(a_2)</th>
<th>(a_3)</th>
<th>(a_4)</th>
<th>(a_5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20-29</td>
<td>446.1±2.7</td>
<td>18.2±2.3</td>
<td>20.8±2.1</td>
<td>0.4±2.7</td>
<td>-5.2±2.5</td>
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<tr>
<td>440.6±1.6</td>
<td>7.8±2.1</td>
<td>18.1±2.3</td>
<td>-3.9±2.9</td>
<td>-2.0±2.3</td>
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</tr>
<tr>
<td>30-39</td>
<td>444.5±1.5</td>
<td>4.5±0.9</td>
<td>20.1±2.4</td>
<td>8.2±2.7</td>
<td>-6.3±2.6</td>
</tr>
<tr>
<td>439.3±1.0</td>
<td>-2.4±1.3</td>
<td>22.3±1.5</td>
<td>-3.1±1.7</td>
<td>-4.3±1.9</td>
<td></td>
</tr>
<tr>
<td>40-49</td>
<td>445.4±1.1</td>
<td>8.2±1.5</td>
<td>20.2±1.8</td>
<td>3.4±2.0</td>
<td>-1.8±2.2</td>
</tr>
<tr>
<td>441.1±0.7</td>
<td>10.0±0.9</td>
<td>23.5±1.0</td>
<td>-2.4±1.2</td>
<td>-5.1±1.3</td>
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<tr>
<td>50-59</td>
<td>442.3±1.0</td>
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<td>-7.4±1.9</td>
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<tr>
<td>441.7±0.6</td>
<td>-1.20±0.8</td>
<td>22.0±0.9</td>
<td>-1.5±1.0</td>
<td>-3.8±1.1</td>
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<tr>
<td>60-69</td>
<td>444.2±0.9</td>
<td>8.6±2.1</td>
<td>20.8±1.4</td>
<td>4.0±1.6</td>
<td>-3.5±1.8</td>
</tr>
<tr>
<td>441.5±0.4</td>
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<td>22.3±0.7</td>
<td>-1.5±0.8</td>
<td>-3.9±0.9</td>
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</tr>
<tr>
<td>70-79</td>
<td>444.4±0.8</td>
<td>4.4±1.0</td>
<td>21.4±1.2</td>
<td>3.5±1.4</td>
<td>-3.9±1.5</td>
</tr>
<tr>
<td>441.8±0.4</td>
<td>1.1±0.5</td>
<td>20.8±0.6</td>
<td>-0.4±0.7</td>
<td>-3.5±0.8</td>
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<tr>
<td>80-89</td>
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<td>4.0±1.1</td>
<td>20.0±1.4</td>
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<td>-3.0±1.7</td>
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<tr>
<td>441.7±0.4</td>
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<td>23.0±0.6</td>
<td>-1.2±0.6</td>
<td>-2.7±0.7</td>
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<tr>
<td>90-98</td>
<td>444.0±1.0</td>
<td>6.5±1</td>
<td>22.9±1.5</td>
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<td>-1.7±1.9</td>
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<tr>
<td>443.1±0.4</td>
<td>0.9±0.5</td>
<td>21.3±0.6</td>
<td>-0.4±0.6</td>
<td>-4.7±0.7</td>
<td></td>
</tr>
</tbody>
</table>

Note: 1981 and 1987 results are in successive rows.
Figure 5. Comparison of odd coefficients in Eq. 1 measured with 1981 (open boxes and dashed one-sigma error bars) and 1987 (solid boxes) observations. Frequency units are nHz in a sidereal reference frame. The lower panel shows the sum of the odd coefficients which is an estimate of the sectoral splitting and approximates the rotation rate near the equator. The dotted lines indicate the rotation of surface magnetic patterns (Ref. 28).

Notable in Figure 5 is the significant decrease of values of $a_1$ between 1981 and 1987. All the 1981 measurements of $a_1$ are faster than the magnetic rate while all but one of the 1987 results are slower. There is no clear trend in the differences between 1981 and 1987 results for $a_{2N}$ coefficients except in the lowest degree bin. In general the coefficients are clustered about the magnetic rotation rates within the level of statistical error. In the lowest degree bin, we find $a_3$ in 1987 to be less than the 1981 value and $a_5$ to be more negative. This suggests the possibility of a change of the differential rotation profile between 1981 and 1987 at depths sampled by degrees 20-29, but more likely one or both data sets are simply too noisy for a good determination. We plan to reanalyze the 1981 data and to extend the 1987 analysis to the full 325 hours available before drawing final conclusions. The lower panel in Figure 5 shows the sectoral splitting frequency estimated as $a_{135} = a_1 + a_3 + a_5$. The 1987 results tend to be slightly smaller than the 1981 results, primarily because of the smaller values of $a_1$. The smaller value of sectoral splitting in the degree 20-29 bin in 1987 is in much better accord with other measurements than was our 1981 measurement.

The even-indexed coefficients of Eq. 1 are shown in Figure 6. Rotation which is symmetric about the equator should produce quite small values for these coefficients. Our 1981 values appear to be significantly non-zero, suggesting a non-spherical sound speed variation, but later measurements at lower levels of solar activity are much closer to zero.

In Figure 6 we see 1981 measurements of $a_2$ as significantly positive while 1987 results are essentially consistent with zero. It is curious that the variations of $a_{2N}$ with degree are nearly identical in 1981 and 1987. The values of $a_4$ are significantly different with positive values in 1981 and negative values in 1987. Brown and Morrow (Ref. 14) found a $t^{-1}$ dependence in measurements of $a_{2N}$. Thus one defines $a_\phi = a_i / t$ and we find 1987 values of $a_3 = 28\pm15$ and $a_4 = -84\pm20$ nHz. In Figure 7 we compare these measurements with other results (Ref. 11).

Figure 6. Measurements of $a_{2N}$ for 1981 (open boxes) and 1987 (filled boxes).

Figure 7. Estimates of $a_{2N}$ and standard deviations vs. time.

5. DISCUSSION

The results in this paper are not final. We anticipate significant improvements from a detailed analysis of the complete set of 1987 data and we plan to reanalyze the 1981 observations. Furthermore, we have prepared a new instrument for operation at the South Pole starting in November 1988 which should produce better data, especially at degrees above 100.

We have not yet obtained a clear answer to the question of a solar cycle dependence of the frequencies of intermediate degree p modes. The possibility of subtle systematic errors in the data reduction makes our results uncertain at the 100 nHz level. We plan to reanalyze the 1981 data to reduce this potential problem. It is possible that the changes observed so far are truly solar but, if
so, a clear association with common indicators of solar activity is not obvious. A safe conclusion is that there is no clear evidence for frequency changes of intermediate-degree p modes between 1981 and 1987 larger than 100 nHz. This conclusion seems hard to reconcile with some evidence of a frequency decrease of low-degree p modes of about 350 nHz between 1980 and 1985.

The main change we find between 1981 and 1987 measurements of rotational frequency splitting is a decrease of the $a_1$ coefficients by about 1%. It is hard to understand how this could arise from a known systematic diameter error of 0.14% but the planned reanalysis of 1981 data should resolve that possibility. Our 1987 results are in considerably better agreement with other measurements than were our 1981 results, particularly in the 20-29 degree range.

Frequency splitting of even index shows a significant change between 1981 and 1987. Our 1987 results agree more closely with other measurements and we have thus verified our observational and reduction approach but have not discovered any reason to doubt the 1981 results. Regarding Figure 7, one notes that a minimum in $a_2$ was reached at the same time as the minimum of solar activity (September 1986) and that $a_2$ reversed sign at about the same time. The minimum in $a_2$ is about the same value and sign as one would expect from the oblate distortion of the sun caused by its rotation (Ref. 11). The temptation to associate these variations with the solar activity cycle is almost irresistible. Kuhn (Ref. 29) suggested that the variations are related to changes in the internal thermal structure and are correlated with observed changes of the surface temperature distribution. Although there is some question of the magnitude of the effect (Ref. 30), it is noteworthy that Kuhn's 1987 observations of limb temperature yield a splitting prediction consistent with our observations (Ref. 31).

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FREQUENCY SHIFT OF SOLAR P MODES AS SEEN BY CROSSCORRELATION ANALYSIS


ABSTRACT

Data obtained at the Observatorio del Teide (Izana, Tenerife) during the years 1978 to 1988 using a resonant scattering spectrophotometer, is analyzed to look for variations of the low degree p-mode frequencies along the solar cycle. The analysis based on the cross-correlation of power spectra leads to the conclusion that the variation of its maximum correlation, correlates well with the solar cycle. An overall variation, from minimum to maximum solar activity, of 0.5±0.1 kHz is found, when low l modes are considered. Moreover, this effect depends on the l values of the modes being absent for l=0 and of 0.7±0.1 kHz for l=1. Therefore, other interpretations than a simple frequency shift are plausible, such as different amplitudes between modes in the same multiplet or an asymmetric change of the splitting along the solar cycle.

1. INTRODUCTION

The solar activity cycle can alter the structure of the Sun's layers and thus the frequencies of the acoustic p-modes may change. Observational evidence for this effect was first noticed by van der Raay (Ref. 10) based on the shift of one line measured in 1981 and 1983. Later, Woodard and Noyes (Ref. 11) from the analysis of the ACRIM data taken in 1980 (before the breakdown) and in 1984 (after the breakdown), found a mean variation, using only 9 discrete frequencies, of 0.42±0.14 kHz, the frequencies being higher in 1980.

A preliminary analysis of data from 1980 to 1984 taken at Izana (Tenerife) and Haleakula (Hawaii), using 10 l, and 10 l, lines (Ref. 6), shows that no statistically significant trend could be measured. In a later analysis (Ref. 7), this result is basically confirmed.

Gelly et al. (Ref. 5) reported a frequency shift by comparing the ACRIM 1980 frequency table with the centroids found in the spectra of velocity measurements obtained in 1984/85 at the South Pole. Later on, Fossat et al. (Ref. 3) by a cross-correlation analysis of power spectra of intensity and velocity observations (same data than before) found a value of 0.39±0.04 kHz for the frequency shift, which agrees well with previously found values. Recently, Fossat (Ref. 4) adding the data from 1985 and 1986 of ACRIM to its previous one, confirmed the same result. Finally, Woodard (Ref. 12) used individual frequency determinations (4 l, 5 l, and 4 l, modes) to arrive at the result that the shift is of 0.41±0.26 kHz for l=0 modes, 0.4±0.11 kHz for l=1 modes and 0.03±0.16 kHz for l=2 modes.

In this work, velocity data obtained at Izana from 1978 through 1988 is crosscorrelated and analyzed to check whether there is a significant frequency shift which could be correlated with the solar activity cycle.

2. OBSERVATIONAL DATA

Data has been gathered at the Observatorio del Teide (Izana, Tenerife) during summer observing seasons from 1978 to 1988 with the sole exception of 1979; moreover from April 1984 on, there has been only two months (October 1984 and June 1986) where observations had been stopped. The observations have been carried out using optical resonant scattering spectrometry of the full solar disc, to measure the line of sight velocity of the Sun relative to the laboratory (Ref. 1-2). Table 1 lists the data used in this analysis. As we have previously noted (Ref. 8) data obtained in 1983 is more noisy due to technical problems with the electronics. Series of 60 natural days length were formed, whenever long enough series were achieved, because this seems to be the longest coherence time found for any p-mode in the 5 minutes range (Ref. 7).

Data is analyzed to yield the daily residuals as indicated by Pallé et al. (Ref. 8). In order to calculate the power spectra an iterative sine wave fitting procedure is used (Ref. 9). At intervals of 0.1 kHz the amplitude squared for each frequency from 2 to 3.8 mHz is calculated.

Table I. Observed series used in this analysis. An asterisk (*) denotes the series used in the analysis shown in figure 1.

3. CROSS-CORRELATION FUNCTIONS OF POWER SPECTRA

Having the power spectra of each series, cross-correlation functions between two of them can be obtained in order to see whether or not a frequency shift of the signals present, has taken place. This technique has the advantage of using all information contained in the power spectra, but all noise is also included. In order to "clean" the spectra of noise, the frequency range from 2.0 to 3.8 mHz has been divided in 9 intervals, in which linear fits to the spectra have been made after signals present (including first order side-bands) have been ignored; the straight lines are then subtracted from the spectra.

In Figure 1 the cross-correlation functions of the summer months for each year when data is available, respect to 1981. Our interest is now focused in obtaining a measure of the position of the cross-correlation function peak. The centroid of such peak seems the best parameter to calculate but, when it is done, it shows a dependence on the width of the interval used to calculate it. The centroid is more and more negative as the interval increases, clearly indicating a shifted or asymmetric profile.

If the centroid is calculated for the same intervals but starting at a lag \( k < 0 \), then the centroids become less negative than before; eventually for a given value of \( k = k_0 \), the centroids will be constant, irrespective of the interval width, and, if \( k > k_0 \) then the sign of the asymmetry is reversed. If such \( k_0 \) exists, will show a real shift rather than only an asymmetry and defines the shift wanted. It can be further checked because if the left part of the cross-correlation function (see figure 1) is folded on to the right part, a null difference should be obtained. The centroid is then calculated over the interval \((-4.4)\) mHz, although as said before it does not change as the width of this interval increases.
The results for the series shown in figure 1 are shown in figure 2 in which a clear variation over the years can be seen. The white dots are the centroids calculated from the cross-correlation of the amplitude spectra rather than the power. The reason for trying it is that in power spectra the weight of the central peaks respect to the ones in the extremes of the 5 min interval is greater than in the amplitude one. As it can be seen the results from both analysis are identical (within errors) and from now on will be used power spectra to cross-correlate.

4. RESULTS AND DISCUSSION

In figure 3 all data available as in table 1 has been analyzed in the same way explained before and the centroids are plotted. White dots stand now for the poorer series (duty cycle less than 25%); the line drawn, joints the results for the same series used in figure 2. The scatter is a measure of the error in such analysis. The variation is as clear as before and the turnover at the solar minimum 1986, is particularly evident. The one at maximum (-1981) is also important but data before then is scarce and of less quality.

This result shows that the cross-correlation between power spectra of data observed at different epochs, is maximum for different lags. Moreover this variation is well correlated with the solar cycle and amounts to 0.5±0.1 μHz. This fact can lead to the conclusion that frequencies of the p-modes shift to lower values from maximum to minimum of solar activity. However, this is not the only explanation for the observed effect and other facts, such as rotational splitting changes or even changes in the amplitude of split peaks within the same multiplet, can also explain the effect. Moreover, this possible interpretations could explain partially why there has been different results, from similar data, using different analysis (see paragraph 1).

If any of the last explanations is correct then this effect should depend on l. In order to check such dependence, the obtained power spectra have been cleaned of the l=0 and 2 peak groups with its sidebands by including zeros where appropriate. The same exercise was made with the group of peaks of l=1 and 3. The results for the centroids of the cross-correlation functions are shown in figure 4; in them it can be seen that when only modes with l=1 and 3 (although l=3 weight is a factor of 10 less than l=1) are considered the variation of the cross-correlation centroid is higher and clearer. However, when l=0 and 2 are kept (they have about equal weight) the variation is smaller if any at all. On both, the scatter is somewhat higher, as expected, since the information contained in the power spectra now has been reduced by a half.
Figure 5.- With the same source of data for figure 1, power spectra "cleaned" of all peaks but 1-0 (a), 1-1 (b), 1-2 (c), 1-3 (d) "free" first order sideband have been obtained and the centroids of their cross-correlation functions (respect to 1981) calculated, (see text).

To go even further we have tried to isolate individual 1 modes. Due to their proximity the splitting structure and the first order sidebands the task is very hard. Fortunately, all modes have at least one "clean" sideband in the sense that can be isolated without problems. However, in doing so the resulting power spectra contains only one-twelfth of the information than before; this will, therefore increase the noise in the centroid determinations. Figure 5 plots the centroids obtained from the cross-correlation functions calculated for this exercise. The first striking feature is that for 1-0 modes there seems to be no variation at all, while for the others some variation, specially clear for 1-1 can be concluded. Moreover the scatter found for 1-0 is compatible with the lifetimes associated to them (Ref. 7).

5. CONCLUSIONS

The results obtained in this work, from Figure 2 and 3 shown virtually identical results as the ones obtained by Fossat et al. (1987); identical in sign and quantitatively equal within errors. Moreover, from Figure 4 and 5, it is showed that this effect is dependent on the 1 value of the modes. To be more precise for 1-0 modes no variation is found while for 1-1 is of -0.7 mHz, being somewhat smaller for 1-2 and, although with larger scatter, also for 1-3.

When this result is compared with the ones found by Isaak et al. (Ref. 6), Jeffries et al. (Ref. 7) and Woodard (Ref. 12), in which using single peak frequency identifications found no evidence for a shift in frequencies of the p-modes, it may be concluded that other phenomenon than simple frequency shift can be its cause. This phenomenon can be related with the split lines in a multiplet, either asymmetrically changing its splitting or changing the relative amplitudes of the modes. This effect deserves a new physical interpretation.

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The Influence of the Toroidal Roll Pattern on the Sunspot Activity

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Abstract

The latitude distribution of active regions with respect to the toroidal roll pattern has been investigated for the period 1978-1982. It is shown that the sunspot activity is not concentrated only at the boundaries of rolls. The "downdraft", at the convergence of adjacent rolls is however, more favourable than the "updraft" to the magnetic flux expulsion. The roll motions are probably not the hydrodynamic forces controlling the flux emergence. They, however, influence the magnetic complexity of the active regions, each torus acting as a selective amplifier.

Introduction

The presence of toroidal rolls associated with the sunspot activity accounts for a number of observational facts: reversal of the polar magnetic field near sunspot maximum, equatorwards latitude drift of new rolls outlining the migration of solar activity, modulation of the surface rotation "Torsional oscillation", through the action of the solar cycle (Ref. 1-2). Thus, toroidal rolls could well be the natural convective response of a magnetically-dominated fluid. The question arises as to whether the rolls are responsible for the magnetic flux emergence. One would expect that upward motions, at the border of diverging flows, help to the flux expulsion (Ref. 3). On the other hand, the downdraft, at the boundaries of converging rolls, is favourable to the field concentration (Ref. 4). To address this question, I have studied the latitude distribution of active regions with respect to the border of rolls, in order to see the effect of rolls on the magnetic flux emergence and complexity.

I have considered active regions visible on the Meudon FIV spectroheliograms (taken in the wing of the 3933 A Ca II line, at 1.25 A from the line center), during the years 1978 to 1982. In spite of the unavoidable winter gaps, about 681 active regions have been found. The boundary of the rolls has been taken as the latitude of reference, because the boundary of two adjacent rolls changes rapidly with time. The zonal average magnetic field measured at Stanford (Ref. 5) has been used to define the rolls' boundaries, as a direct measurement is more accurate than the empirical method of using the H filaments. The distribution of active regions with respect to the closest boundary of the rolls is shown in Figure 1. It is clear that most active regions (70 per cent) occur within the roll corresponding to the dominant polarity of the solar cycle. That is to say, the active regions occur preferentially in positive magnetic torii (in the northern hemisphere), the leading polarity being positive in the northern hemisphere for cycle 21 (an odd cycle). The situation reverses for even solar cycles, that is, most active regions (the leading polarity being negative) occurring in negative torii. There is a concentration of active regions near the border of rolls (16 per cent), the occurrence being 4 times more near the converging rolls or "downdraft". The "updraft" is less favorable to the emergence of active regions, the upflows dispersing the magnetic fields rather than concentrating them. It is then reasonable to think that magnetic torii could be a reservoir for newly emerging regions. If this were the case, one might expect the leading polarity of the active regions to be conditioned by the polarity of the corresponding magnetic torus. Active regions could, then, exhibit opposite leading polarities, in the same hemisphere, for a given cycle. Such a situation has never been found. Therefore, it is likely that the process governing the polarity rule of the active regions over the 11-y cycle is independent of the large-scale circulation. In other words, the rolls are passive towards the generation of active regions. If the rolls do not contribute to the generation of the toroidal loops, one may wonder whether they play a role in the evolution of the magnetic complexity. It is well-known that most active regions of Sp type, that is, with a predominant leading polarity. There exist, however,
FIGURE 1.
- Histogram showing the distribution of the active regions with respect to their distance to the border of converging ("down") and diverging ("up") convective rolls.

FIGURE 2.
- Histogram showing the distribution of the active regions according to their magnetic classification: most $\beta_f$-type active regions (solid lines) occur in the torus, the magnetic polarity of which is the same as the leading polarity of the active region (i.e., plus in the Northern Hemisphere, for an odd cycle). On the other hand, most active regions of the $\beta_f$-type (dashed line) develop in negative torus in the Northern hemisphere (and positive torus in the Southern hemisphere), the magnetic polarity being that of the following polarity of the active region.
active regions of the $\delta p$ type, with the following polarity more developed than the leading one. 

Some active regions have complex mixed polarities and are denoted as delta type in the Mount Wilson magnetic classification. The latter magnetic configurations are subject to large flaring activity. It is interesting to check whether the rolls influence the magnetic development of a newly emerging region. In the histogram of figure 2b, one sees clearly that the location of the active regions within a roll will condition their magnetic evolution. 430 of the classical bipolar groups $\delta p$ (over 52 in our sample) are born within a roll having the same polarity as the leading sunspot. 64 of the $\delta p$ type groups (out of 92) are born in the roll of the same polarity as the following sunspot polarity. The number of exceptions for the $\delta p$ type of active regions is large. Ther is always a possibility that the sunspot identified as a $\delta p$, in fact, of the following polarity, as it has not always been possible to check its polarity.

Another explanation has been proposed : Martres (Ref. 6) noticed that the birth of an active region located at the west of a pre-existing region favours the evolution of an active region in the $\delta p$ type. Most of the complex active regions are born on either side of adjacent roll. The latter situation enhances the development of equally-distributed polarities (see also Ref. 7).

This result implies that 1) the solar activity may not necessarily occur at the boundary of adjacent rolls, but over a large latitudinal range centered around + 20°, 2) the roll "filters" the emerging magnetic loop, by amplifying and / or cancelling the corresponding polarity. The rolls act as a selective amplifier of the initial emerging loop.

It has been shown independently that the flux emergence occurs near singularities of rotation (Ref. 8 - 9). These singularities, denoted as "pivot-points", are places where the rotation exhibits the Carrington velocity, (a synodic rate of 13.2° per day), irrespective of the latitude. This again suggests that the solar activity originates from layers in rigid rotation. If the seismology results of Brown and Morrow (Ref. 11) are confirmed, there is a source of toroidal field in the radiative layers.

In this new view of the dynamo, the roll motions are probably not the hydrodynamic forces controlling the flux emergence as required by Parker (Ref. 12). The intrusion of the toroidal field in the convective layers probably implies another type of instability. The large-scale convective motions play a role, however, in coalescing the magnetic fibrils in sunspots near the downdraft of converging rolls. They also influence further development of active magnetic regions.

III. CONCLUSION

The rolls do not seem to be the driving mechanism for transporting the toroidal field to the solar surface. The active regions occur rather randomly within rolls having the same polarity as the preceding polarity of the active regions, although a significant fraction concentrate near the border of rolls. Moreover, the downward situation provided by a converging system of rolls is much more favorable to the flux emergence than the upward situation. This situation would be reversed if the rolls were the driving force of the flux expulsion. The rolls seem to be rather passive in this context.

Increasing observational evidence indicates that the dynamo source is located in rigidly-rotating layers and cancels the magnetic field. Although it has to be confirmed, the rolls together with some puzzling observations of the internal rotation, seem to challenge the current picture.

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The dynamical oscillation and propulsion of magnetic fields in the convective zone of a star. I. General considerations.
We consider the propagation of Alfvén waves in a viscous and resistive atmospheric layer, in isothermal magnetohydrostatic equilibrium, with constant rate of ionization, under an horizontal magnetic field, either uniform or decaying with altitude. The wave equation relevant to these two cases is derived, and its solution in terms of hypergeometric functions is used to plot wave amplitude and phase versus altitude for several values of wave frequency, horizontal wavenumber and viscous and resistive damping. It is shown that:

(i) the amplitudes initially grow due to reducing mass density and then decay due to dissipation, i.e. go through a maximum; (ii) the phases also go through a maximum, i.e., upward propagating waves are reflected downward; (iii) amplitude and phase changes are smaller in a flux tube than in an uniform magnetic field, because in the former case the magnetic field decays, as the flux tube fans-out with height.

1. INTRODUCTION

The simplest hydromagnetic mode in a compressible and ionized atmosphere is an Alfvén wave. It is simpler than the slow and fast modes (Campos 1987) because: (i) it is a transverse wave, hence decoupled, at linear level, from compressibility effects; (ii) it is not affected by thermal conduction and radiation, so that dissipation is due solely to fluid viscosity and electrical resistance. Alfvén waves in an atmosphere are affected by three aspects: (i) since Alfvén modes propagate along magnetic field lines, it is essential to specify the topology of the magnetic field, e.g. uniform or concentrated in flux tubes; (ii) gravity affects Alfvén waves indirectly, through the mass density, which causes the wave speed to vary with altitude, implying that the waves are non-sinusoidal; (iii) the viscous and resistive diffusivities may also vary with altitude, in a way determined by the atmospheric mean state, leading to non-exponential wave damping.

Alfvén waves were first studied for homogeneous media, as concerns propagation (Alfvén 1942, 1948), generation (Campos 1977), forcing (Ionson 1982), and dissipation (Alfvén 1947, Hoffatt 1976). The first exact solutions of the Alfvén wave equation in a non-uniform medium were obtained for an isothermal atmosphere under an uniform magnetic field (Ferraro & Plumpton 1958), assumed to be vertical; this result has been refined by several authors (Hollweg 1978, Leroy 1980, Campos 1983a), and extended to oblique magnetic fields (Schwartz, Cally & Bel 1984, Campos 1988). Alfvén waves have also been considered in homogeneous magnetic slabs and tubes (Roberts & Webb 1978, Spruit 1982), and non-homogeneous magnetic structures (Heyvaerts & Priest 1983, Nocera & Leroy & Priest 1984), neglecting gravity effects in both cases, and including dissipation in the latter pair of references. The effects of gravity and dissipation have been considered (Campos 1983b,c, 1988) for Alfvén waves in atmospheres under uniform magnetic fields, and in the present work we compare with the case non-uniform magnetic field.

2. ISOTHERMAL ATMOSPHERES IN ONE-DIMENSIONAL MAGNETO-HYDROSTATIC EQUILIBRIUM

The magnetohydrostatic equilibrium of an atmosphere is one-dimensional (Campos 1987, 1988), i.e., all quantities depend only on altitude, in two cases: (i) an uniform magnetic field of arbitrary direction; (ii) an horizontal magnetic field varying with altitude. The case of uniform external magnetic field would apply in the high-chromosphere, after the magnetic flux tubes have merged, and in the corona. It corresponds to an atmosphere in hydrostatic equilibrium, which for an isothermal perfect gas leads to a density and pressure decaying exponentially (la) with altitude:

\[ \rho(z)/\rho_0 = e^{-z/L} = p(z)/p_0, \quad L = RT/g, \]  

(1a,b)

on the scale height (lb), specified by the gas constant R, temperature T and acceleration of gravity g. This implies that the Alfvén speed:

\[ A(z) = B(z)\sqrt{4\pi\sigma_0(z)} = a e^{z/L}, \quad B(z) = B + \frac{z}{x}, \]  

(2a,b)

increases exponentially with altitude on twice the scale height (lb), from an initial value a (2a). In the case of a uniform magnetic field (2b); we take the latter to be horizontal, for comparison with the case (ii), of non-uniform horizontal magnetic field.
In the case of a non-uniform magnetic field, the atmosphere is in magnetohydrostatic equilibrium, and the balance of gas (1a) and magnetic $\mu B^2/8\pi$ pressures, requires the magnetic field to decay exponentially (3a) with altitude on twice the scale height:

$$B = B_0 e^{-z/2L}, \quad A(z) = B_0 \sqrt{\mu B^2/8\pi} = a,$$

leading to an uniform Alfvén speed (3b). In the solar atmosphere there is a net magnetic flux (Stenflo & Harvey 1905) corresponding to a vertical component of the magnetic field. In the case (i) of uniform field, the atmospheric state remains of one-dimensional hydrostatic equilibrium, and the properties of Alfvén waves can be studied similarly for oblique (Campos 1988) and horizontal (present work) fields; in the case (ii) of non-uniform magnetic field, the presence of a vertical component, would lead to two-dimensional magneto-hydrostatic equilibrium, i.e. to an horizontal granular structure, whose periodicity we represent (Berton & Heyvaerts 1987) by an horizontal wave-number. In both cases (i) and (ii), wave damping is associated with the viscous (4a) and resistive (4b) diffusivities:

$$\eta(z) = \eta_0, \quad \chi(z) = \chi_0 = c^2/4\pi\mu_0\nu, \quad (5a,b)$$

is the resistive diffusivity which depends on constant quantities, like the speed of light in vacuo $c$ and magnetic permeability $\mu$, and on the Ohmic electrical conductivity $\eta(z)$, which is also constant $\eta_0$ for an isothermal layer with constant ionization.

### 3. Alfvén Wave Equation with Non-Uniform Propagation Speed and Damping Rates

The geometry of the wave problem which we have defined is illustrated in Figure 1, viz. we have an atmosphere stratified in the $z$-direction opposite to gravity $g$, under an uniform or non-uniform external magnetic field $B$ in the $x$-direction, with velocity $v$ and magnetic field $h$ perturbations in the $y$-direction, corresponding to an Alfvén wave, which is an unsteady motion, i.e.,

$$\nabla \times E = 0, \quad \nabla \times B = 0,$$

perturbations in the $y$-direction, corresponding to an Alfvén wave, which is an unsteady motion, i.e.,

Substituting the total velocity (6a) and magnetic field (6b), in the equations of induction and momentum, we obtain respectively:

$$\frac{\partial v}{\partial t} - B \frac{\partial v}{\partial x} = \chi_0^2 \nu \frac{\partial^2 v}{\partial z^2} - \frac{B}{c^2} \frac{\partial B}{\partial x}, \quad (7a)$$

$$\frac{\partial h}{\partial t} + \left( \frac{A^2}{B} \right) \frac{\partial h}{\partial x} = \chi_0^2 \nu \frac{\partial^2 h}{\partial z^2} + \frac{A^2}{B} \nu v, \quad (7b)$$

where $A$ is the Alfvén speed $c$ in the resistive (5b) and viscous (5a) diffusivities, and $\partial^2/\partial x^2 + \partial^2/\partial z^2$ denotes the two-dimensional Laplacian in the $(x,z)$-plane.

Eliminating between (7a,b), we obtain the wave equations: (i) for the velocity perturbation:

$$\frac{\partial^2 v}{\partial z^2} - A^2 \frac{\partial^2 v}{\partial x^2} = \eta_0 v \frac{\partial^2 v}{\partial t^2} + \left( \frac{A^2}{B} \right) \frac{\partial v}{\partial x} + \chi_0^2 \nu \frac{\partial^2 v}{\partial z^2} + \frac{A^2}{B} \nu v + \frac{\eta_0 \nu v}{\chi_0^2 \nu} \frac{\partial^2 v}{\partial t^2}, \quad (8a)$$

where $\chi = \text{const}$ and $A, B, \eta$ may depend on altitude; (ii) for the magnetic field perturbation:

$$\frac{\partial^2 h}{\partial z^2} - A^2 \frac{\partial^2 h}{\partial x^2} = \chi_0^2 \nu \frac{\partial^2 h}{\partial t^2} + \left( \frac{A^2}{B} \right) \frac{\partial h}{\partial x} + \frac{\eta_0 \nu v}{\chi_0^2 \nu} \frac{\partial^2 h}{\partial t^2}, \quad (8b)$$

which is well-known (Cowling 1960). This equation (8) has been applied erroneously to dissipative Alfvén waves in an atmosphere with non-uniform density and magnetic field profiles (Heyvaerts & Priest 1983; Nocera, Leroy & Priest 1984); the correct equation (8a) has several extra terms, because the dissipative terms, even for uniform diffusivities, are affected by non-uniform propagation speed (Campos 1983b,c). Note that the viscous $\eta$ and resistive $\chi$ diffusivities are interchangeable in a homogeneous medium (9), but not in an atmosphere (8a,b).

### 4. Nodes of a 'Flux Tube' in Magnetohydrostatic Equilibrium

Since the atmospheric properties depend only on altitude $z$, we may use a Fourier decomposition in the other coordinates, namely, horizontal distance $x$ and time $t$.

$$v(x,z,t) = \int V(z,k,\omega) \exp(ikx - \omega t) \, dk\, d\omega, \quad (10)$$

where $V(z,k,\omega)$ is the velocity perturbation spectrum, for a wave of frequency $\omega$ and horizontal wavenumber $k$ at altitude $z$; its dependence on altitude is specified substituting (10) into (8a), where we neglect the last term for weak diffusivities ($\eta_0, \chi$ small) leading to:

$$\omega^2 + \frac{2A^2}{B} \left( \frac{A^2}{B} \right) \omega + \chi_0^2 \nu \frac{\partial^2 v}{\partial t^2} + \frac{A^2}{B} \nu v = 0, \quad (11)$$

where prime denotes derivative with regard to altitude $z$, $d/dz$. In the absence of dissipation ($\eta = 0$), the equation (11) reduces to $\omega^2 - k^2 A^2/\omega$ $= 0$, which is the dispersion relation $\omega = kA$ for horizontal Alfvén waves. In the presence of dissipation, the velocity perturbation spectrum $V$ is specified by a second-order linear differential...
equation, with variable coefficients depending on the non-uniform atmospheric mean state.

In the case of an isothermal atmospheric layer (5a), with constant rate of ionization (5b), in magnetohydrostatic equilibrium (3a,b), which will henceforth be designated 'flux tube' for brevity, the wave equation (11) becomes:

\[(1 + c/\delta) e^{-z/L} L^2 \psi'' - 2(c/\delta) e^{-z/L} L \psi' + \{c(1/K^2 - \delta/2) - i(\delta/2 - K^2)\} \psi' = 0, \quad (12)\]

involving four dimensionless parameters:

\[\Omega \equiv \omega L/a, \quad K \equiv K_L, \quad \delta \equiv \eta w/a^2, \quad \beta \equiv n w/a - (I3a-d)\]

and also perform a change of dependent variable (14b), the equation for dissipative Alfvén waves in a magnetic flux tube (12), transforms to an hypergeometric type:

\[(1 - \delta) \psi'' + ([2K - 2(1 + K)\xi] \psi' - [K(1 + 1/2) - K^2\delta/\Omega + 1(\delta^2 - K^2)/\Omega] \psi = 0, \quad (15)\]

where prime denotes derivative with regard to \(\xi\), viz. \(\psi' = d\psi/d\xi\).

5. WAVES IN AN ATMOSPHERE UNDER AN UNIFORM MAGNETIC FIELD

In the case of an isothermal atmospheric layer with constant rate of ionization, under an uniform magnetic field, which we will henceforth designate 'magnetic atmosphere' for brevity, we substitute (5a,b) and (2a,b) in the wave equation (II), which takes the form:

\[(1 + (c/\delta) e^{-z/L}) L^2 \psi'' - 2(c/\delta) e^{-z/L} L \psi' + \{c(1/K^2 - \delta/2) - i(\delta/2 - K^2)\} \psi' = 0, \quad (16)\]

involving the same four dimensionless parameters (13a-d) as in the case (12) of the magnetic flux tube. In the present case of a magnetic atmosphere (16), the same change of independent variable (14a), multiplied by a constant:

\[\xi = (c/\delta) e^{-z/L}, \quad \psi(z;k,\omega) = \xi^\nu \psi(c), \quad (14a,b)\]

and also perform a change of dependent variable (14b), the equation for dissipative Alfvén waves in a magnetic flux tube (12), transforms to an hypergeometric type:

\[(1 - \xi) \psi'' + ([2K - 2(1 + K)\xi] \psi' - [K(1 + 1/2) - K^2\delta/\Omega + 1(\delta^2 - K^2)/\Omega] \psi = 0, \quad (15)\]

which interchange between themselves the three regular singularities 0,1,∞ of the equation.

6. COMPARATIVE EFFECTS OF FREQUENCY, COMPACTNESS AND DAMPING RATES

Since \(\zeta < 0\) in (14a) the variable \(\xi = \zeta/(\zeta - 1)\) in (22) satisfies \(0 < \xi < 1\), so that a solution of (19) satisfies hypergeometric functions of this variable:

\[\psi(\xi) = C_0 (1 - \xi)^{-\nu} F(a, 1 + 2\nu - \beta; 1 - \beta; \nu + 1 - a; \xi) + \psi(0) = \psi(0), \quad (23)\]

converges at all altitudes. We take the first term of (23) by setting \(C_0 = 0\) the spectrum (17b):

\[V(z;k,\omega) = C_0 \xi^\nu (1 - \xi)^{-\nu} F(a, 1 + 2\nu - \beta; 1 - \beta; \nu + 1 - a; \xi), \quad (24a)\]

\[F(\xi) = F(a, 1 + 2\nu - \beta; 1 - \beta; \nu + 1 - a; \xi), \quad (24b)\]

The remaining constant of integration \(C_0\) can be determined from the initial velocity perturbation spectrum \(V(0;k,\omega)\) at altitude \(z = 0\):

\[V(X) = e^{-UX} (Y / Y_o)^\nu \{F(c/Y)/F(c/Y_o)\}, \quad (25)\]

where:

\[Y \equiv \epsilon + \delta e^X, \quad Y_o = \epsilon + \delta, \quad (26)\]
and the velocity perturbation has been normalized with regard to its initial value (27a):

$$U(x) = \frac{V(z;k,u)}{V(0;k,u)}, \quad x = z/L,$$  \hspace{1cm} (27a, b)

and the altitude with regard to the scale height (27b).

In Figures 2 to 5 we plot separately, against dimensionless altitude (27b), on the top or l.h.s.: $|U|$ which is the ratio (28a) of wave amplitudes at altitude $z$ and $0$:

$$|U| = \frac{|V(z;k,u)|}{|V(0;k,u)|},$$  \hspace{1cm} (28a)

$$\arg(U) = \arg(V(z;k,u)) - \arg(V(0;k,u)), \hspace{1cm} (28b)$$

and $\arg(U)$ which is (28b) the phase shift between altitudes 0 and $z$. We take as reference values for the four dimensionless parameters (13a-d) the following:

$$\Omega = 2, \quad K = 1, \quad \zeta = 0.3, \quad \delta = 0.5; \hspace{1cm} (29a-d)$$

we then allow each parameter to take in turn smaller and larger values:

$$\omega = 1, 2, 5; \quad K = 0.5, 1.2; \quad \zeta = 0.1, 0.3, 0.5; \quad \delta = 0.3, 0.5, 1.1, \hspace{1cm} (30a-d)$$

plotting in (Figure 2) the effects of dimensionless frequency $\omega = \omega_L/a = 2\pi n/t_a$ for periods $t$ comparable to the time $t = t_L$ a wave takes to transverse a scale height; (Figure 3) compactness $K = kL_2$ corresponding to an horizontal wavelength larger than the scale height; (Figures 4 and 5) moderate resistive $\zeta$ and viscous $\delta$ damping, such that their product remains small $\zeta \delta << 1$.

6. DISCUSSION

In each Figure 2 to 5 the solid (dotted) lines correspond to an viscous and resistive Alfven waves, propagating in an isothermal layer of a perfect gas, with constant rate of ionization, in hydrostatic (magnetohydrostatic) equilibrium under an uniform (non-uniform) horizontal external magnetic field, which we designate in a brief but not totally accurate (see §1) way as a magnetic 'atmosphere' ('flux tube'). The plots of amplitude $|U|$ show that waves initially grow as they propagate upward into less dense layers of the atmosphere, and then start to decay as dissipation effects start to dominate. The plots of phase show that it is initially an increasing function of altitude, i.e., waves propagate upward, but then the phase reaches a maximum and decreases, indicating that waves are reflected and change to downward propagation, due to the rapid increase in Alfven speed and damping with altitude, which implies that the medium becomes more refractive as higher layers are penetrated. These general qualitative features apply to dissipative Alfven waves both in a magnetic atmosphere and flux tube, and for all values of the parameters.

Turning to an analysis of the effects of each parameter, the amplitude grows faster for higher frequencies (Figure 2), smaller horizontal compactness (Figure 3), larger resistive damping (Figure 4) and smaller viscous damping (Figure 5). The interpretation is that: (i) the local vertical wavenumber $k_Z = \sqrt{\omega^2 c^2 - K^2 L^2}$ increases with frequency $\omega$ and decreases with local compactness $K$, so that they have opposite effects on amplitude and phase; (ii) increasing viscous damping causes a smaller amplitude growth, but larger resistive diffusivity helps the magnetic field spread outward and causes a local or temporary (Moffatt 1976) amplitude growth. The phase increases more rapidly for higher frequency (Figure 2), larger compactness (Figure 3) and smaller resistive (Figure 4) and viscous (Figure 5) damping; the interpretation is that phase changes increase for shorter vertical and horizontal wavelengths, and are reduced by stronger dissipation, which opposes large waveform gradients. In all cases, for the same values of parameters, amplitude and phase changes are larger in an atmosphere under an uniform magnetic field (dotted lines), than for a magnetic flux tube (solid lines), because the latter fans-out with altitude, i.e., corresponds to a magnetic field decaying with altitude. In general Alfven waves are reflected in a dissipative atmosphere, and multiple reflections (Figure 2, top) may occur for high-frequencies.

The computer programs for the plots in Figures 2 to 5 were done by Mr. P.M. Mendes.

REFERENCES


LEGENDS FOR THE FIGURES

FIGURE 1 - Dissipative Alfvén waves in an atmosphere.

FIGURE 2 - Amplitude (U) and phase arg (U) as a function of dimensionless altitude X, for a magnetic atmosphere (dotted line) or flux tube (solid line), effect of dimensionless frequency (8a).

FIGURE 3 - As for Figure 2: effect of horizontal compactness (8b).

FIGURE 4 - As for Figure 3: effect of resistive damping (8c).

FIGURE 5 - As for Figure 2: effect of viscous damping (8d).

mean state: A, B, x, \eta(x), waves: \vec{\eta}(x, z, t)

FIGURE 1

FIGURE 2
LOCAL EFFECTS OF A MAJOR FLARE ON SOLAR FIVE-MINUTE OSCILLATIONS

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1Operated by the Association of Universities for Research in Astronomy, Inc. under contract with the National Science Foundation.

ABSTRACT

Doppler velocity images of the full Sun were obtained both during and after a major white-light flare. These velocities were interpolated onto a cylindrical coordinate system centered on the flare and decomposed into radially propagating waves defined by Hankel functions. For a similar analysis of quiet Sun regions we find fairly comparable power in incoming and outgoing waves irrespective of the presence of the flare. However, for the flaring region, there is 14% greater power in incoming as opposed to outgoing waves when there is no flare, but 5% greater power in outgoing than in incoming waves during the flare. This result suggests that the flare may have excited outgoing waves which counteracted the more usual absorption of incoming acoustic waves by sunspots.

Keywords: solar oscillations, flares

1. INTRODUCTION

The excitation and decay of the solar five-minute oscillations are still a subject of debate. Two possible mechanisms are stochastic excitation of the acoustic modes by intense turbulence within the convection zone (e.g. Refs. 1-3) or self-excitation by either a $\kappa$ or $\gamma$-mechanism (e.g. Ref. 4). These processes perhaps explain the presence of the oscillations in general, but they do not preclude the possibility of an impulsive event such as a comet impact (Ref. 5) or a large solar flare (Ref. 6) contributing to the excitation of the oscillations.

A major solar flare can deposit a substantial amount of energy ($\sim 10^{32}$ ergs) in a fairly short time (Ref. 7). The energy in a single oscillation mode, however, has been estimated to be $\sim 10^{27}$-$10^{28}$ ergs and the energy in all the modes $\sim 10^{24}$ ergs (Ref. 8). Thus, even if the efficiency of converting flare energy into acoustic disturbances is small, and this is still uncertain since it depends significantly on whether the energy release is confined to the atmosphere or extends into the convection zone, we might still see some of the modes affected.

A major white-light flare occurred on 24 April (Day 115) 1984 while we were observing solar oscillations with a new setup of the Universal Birefringent Filter (UBF) on the Vacuum Tower Telescope at the National Solar Observatory at Sacramento Peak (NSO/Sac Peak). The flare (of class X13/3B) was one of the largest of the past solar cycle, releasing an estimated $3\times10^{31}$ ergs in continuum radiation (Ref. 9), with the entire flare producing upwards of $4\times10^{32}$ ergs (Ref. 10). As shown in Figure 1, the flare occurred near the limb, starting at 23:56 UT in Hα and reaching its maximum phase at 23:59. The flare lasted for over 2 hours and had several eruptive centers (Ref. 11).

Figure 1. Hα photograph showing scale and location of the major white-light flare which occurred on 24 April 1984 starting at 23:56 UT.
Observational evidence of the impact of a major flare on the oscillations has so far only shown the flare's impact on polar and equatorial sectoral modes (Ref. 12, hereafter referred to as Paper I). After the flare there was an increase in power of the global oscillations in the $p_3$ ridge, for waves travelling west-east, of 185% (or about three sigma over the daily variation between ridges for days of similar atmospheric quality). However, the overall wave power only increased by about 4%, which, given the errors in the analysis, is probably not significant. The increase in the $p_3$ ridge suggests that the flare preferentially excited modes characterized by having 5 nodes in the radial direction regardless of their frequency or wavenumber, a somewhat puzzling result. If this increase in power is actually caused by the flare, then there is some mechanism which translates flare energy into acoustic energy and it should be possible to see evidence of this near the flare as well as globally. Conversely, if no effects of the flare are seen on a local scale, then the interpretation of the increase in power in some of the global modes as being due to the flare would have to be reconsidered.

The flare occurred late in the day but we still have at least 50 min of data taken immediately after its start. Using this data we now search for any local effects of the flare, i.e. responses in the oscillations that are short-lived and, hence, undetectable in the analysis presented in Paper I. There are two principal signatures to look for: 1) a wave pulse emanating from the flare, and 2) differences in power between waves travelling radially towards and away from the flare.

In the first case, we might look for a pulse having characteristics of the ripples generated when a pebble is dropped in a pond. Since the Sun is a dispersive medium this might be seen as a pulse whose longer wavelengths travel faster than its shorter wavelengths. However, if many modes were to be excited, the pulse signature would likely be quite complicated and could be difficult to discern. To search for such a pulse, we interpolate the data onto a cylindrical grid centered on the flare and then average along annuli extending radially away from the flare. This averaging should diminish the contribution of steady background velocity fields due to granulation and supergranulation, giving us a better opportunity to distinguish a pulse.

To look for the second type of signature we perform a harmonic analysis on the data in order to differentiate between incoming and outgoing waves. In this case, the data are also interpolated onto a cylindrical grid whose polar axis passes through the flare site and then averaged along annuli of constant radial distance from the flare. However the next step consists of a Hankel function decomposition scheme. This consists of describing the two-dimensional general velocity field as a sum of eigenfunctions, $V_{mk}$, which are in turn composed of Hankel functions of the first and second kind: $H_{mk}^{(1)}$ and $H_{mk}^{(2)}$, where $m$ is the polar azimuthal order. In this description, a homogeneous medium would be characterized by eigenfunctions made of equal amounts of the different types of Hankel functions. If, on the other hand, $V_{mk}$ is composed of more $H_{mk}^{(2)}$ than $H_{mk}^{(1)}$, then there is more power in the outward travelling waves and we would conclude that the flare is acting as a source of acoustic waves.

2. OBSERVATIONS AND ANALYSIS

Observations of intermediate-degree solar oscillations proceeded throughout Day 115, ending with the flare, and the following Day 116; however, only a 106 min segment around the time of the flare and another such segment the next day are used in the present analysis. The UBF was used as an imaging Doppler analyzer, providing simultaneous images of the full Sun on two sides of the Fe I $\lambda$ 5576 spectral line. These intensity images were recorded at 60 s time intervals on 35 mm film and later digitized with 8° spatial resolution. Doppler velocities were formed by taking ratios of the difference of a pair of intensity images to their sum.

Before further analyzing the data, we found it important to remove the effects of solar rotation so that they would not systematically bias the results. The functional representation of latitudinal differential rotation is basically sinusoidal and may be approximated by appropriate polynomials. We 'flattened' the Doppler velocity data by fitting fifth-order polynomials in both the east-west and north-south directions to velocity data which had been averaged over narrow strips oriented along the solar equator and central meridian. These polynomials were then subtracted from the entire image. This technique also corrects for any low order inhomogeneities in the filter, however we also lose information about waves having low wavenumbers.

The flattened velocity data are next interpolated onto several different grids: one centered on the flare, the others on regions of quiet Sun. The signal is averaged over 180° along annuli centered on the flare in order to average out signatures of the convection zone such as granules and supergranules and to isolate cylindrical waves about the flare by filtering out plane waves travelling in arbitrary directions. In general, one would like to average over 360° rings; however, the position of the flare on the disk precluded use of the 180° to the east of the flare.

To search for radially incoming or outgoing waves a Bessel-type transform was performed using the asymptotic forms of the cylindrical Hankel functions. This was accomplished by multiplying the averaged data by $\sqrt{ks}$, where $ds = R d\theta$, $k$ is the horizontal wavenumber, $R$ is the solar radius, and $\theta$ is the colatitude measured away from the flare. The resulting data are then Fourier transformed in space and time. This type of analysis is similar to that carried out by Braun, Duvall & LaBonte (Ref. 13) using a technique reported by Candel (Ref. 14). The Hankel transform was performed on annuli corresponding to $\theta$ values of 16°-72° radiating away from the flare center. The lower limit was set to avoid other sunspots, the upper limit to avoid going outside the solar image. Figure 2a shows the grid used in analyzing the flare.

To separate the effect of the flare on five-minute oscillations from non-periodic sources we used 106 minutes of data composed of 53 min before the start of the flare and 53 min afterward. This increased the temporal resolution and stabilized the results. This may in some sense dilute the effects of the flare but it is otherwise difficult to distinguish the five-minute acoustic waves. Power in outgoing waves was determined by integrating the power in negative
local effects of a major flare on solar 5 minute oscillations

Figure 2. Polar grids employed in local analysis. a) flare grid centered at (φ = -44°, θ = 103.5°).
b) alternate site 2 (25°, 125°).

frequencies within the frequency band 2.0-4.0 mHz at each k and then integrating in k, while power in incoming waves was determined from positive frequencies in the same manner. This analysis was also performed for two quiet Sun sites on the disk and at the same sites for 106 min on the following day to act as a control. An example of an alternate site is shown in Figure 2b. The locations of the various sites are given with respect to the usual solar grid with the equator at 90° colatitude and the central meridian at 0° longitude. In this system the flare is located at (φ = -44°, θ = 103.5°) (Fig. 2a) where φ is the longitude and θ is the colatitude, alternate site 1 is at (-30°, 60°), and alternate site 2 (Fig. 2b) is at (25°, 125°).

4. RESULTS

The ratios of the total power in five-minute oscillations travelling outward from a given site to those travelling inward are shown in Table 1 for each of the different sites. A ratio of 1.00 means that there is no difference in power between outgoing and incoming waves, a ratio less than one means there is more power in inward propagating waves, and a ratio greater than one means there is more power in outward travelling waves. In general the ratios are functions of k, however, the approximations to the Hankel functions used here are only valid for large kr, where r is the radial distance from the flare. Because of this, the values shown in Table 1 have been summed over 30 k bins starting from bin 6. This corresponds to starting at k = 0.44 Mm⁻¹ which is large enough so that the approximate form of the Hankel transform is good to within 4% in the worst case. We estimate that this is the largest of the errors associated with this analysis, so that differences in power of 4% are probably not significant in determining whether there is more power in outward travelling waves than in incoming waves. At the flare site itself there is 5% more power in outgoing than in incoming waves during the flare, but 14% less power in outgoing waves the day after the flare.

The local analysis of the flare site the day following the flare, is basically the study of waves travelling inward and outward from a sunspot region. The 14% greater power seen in inward travelling waves is evidence of some type of absorption by the spot. This apparent absorption of oscillations by sunspots may be due to several phenomena: scattering of the waves by intense magnetic fields into spatial scales below the observational detection limit, transformation of mechanical wave energy into magnetohydrodynamic waves, or actual damping of the waves as they pass through the sunspots.

Braun, Duvall & LaBonte (Ref. 13) obtain similar results in the absence of a flare, in that they also observe more power in waves propagating towards sunspots than propagating outwards. However, while they find the absorption of oscillations increasing linearly with k up to 50%, we see no such trend. This may simply be due to the fact that the present data do not reach high enough k values. For quiet Sun regions they find no difference in power between incoming and outgoing waves. In this analysis, for alternate site 1, we also find no real difference in power between the two types of waves regardless of the flare. However, for

Table 1.

<table>
<thead>
<tr>
<th>Site</th>
<th>Day 115</th>
<th>Day 116</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Out/In</td>
<td>Out/In</td>
</tr>
<tr>
<td>Flare</td>
<td>1.05 ± 0.04</td>
<td>0.86 ± 0.04</td>
</tr>
<tr>
<td>Alt 1</td>
<td>1.03 ± 0.04</td>
<td>1.01 ± 0.04</td>
</tr>
<tr>
<td>Alt 2</td>
<td>1.07 ± 0.04</td>
<td>1.03 ± 0.04</td>
</tr>
</tbody>
</table>
alternate site 2, there appears to be more power in waves propagating outward than in those propagating toward the site center (even allowing for a 4% uncertainty). On the other hand, the ratios of the power presented for alternate site 2 in Table 1 do not vary appreciably from Day 115 to Day 116 (~4%); they appear unaffected by the flare. At the flare site, as mentioned above, the variation in power between the two days is closer to 20%. It is this difference in power that we feel is noteworthy.

This result may suggest that energy from the flare excited oscillations travelling outward from the sunspot which largely cancelled out its absorption effects. Or it may suggest that the mechanism responsible for the appearance of acoustic wave absorption by sunspots is damped in the presence of the flare. If the first view of the flare interacting with the oscillations is correct, then further theoretical research is needed on a mechanism for coupling the energy of the flare to pressure waves, perhaps by local heating of the atmosphere, by interaction of the magnetic field with the gas, or by a strictly mechanical impulse.

We have been unable to detect any obvious pulses when simply looking at spatially-averaged time series. This could be due to the low signal-to-noise level of this data. More likely, is that the geometry and time structure of the flare preclude such a possibility. As seen in Figure 1, the flare does not have a circular shape; it looks more like a ribbon. It also varies substantially in time, although the white light portion of the flare is quite short. The complexity of flares in general may make a search for a distinct pulse in time difficult or impossible even with high-quality data. A Fourier-Bessel analysis may yet be the best way to investigate the localized effects of a flare on acoustic oscillations.

Since the data are on film, they have also been digitized with higher spatial resolution. We are currently reducing this data to determine whether we see the same increase in absorption with wavenumber noted by Braun, Duvall, & LaBonte (Ref. 13). It may also be easier to search for some kind of pulse with the higher resolution data.

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5. REFERENCES

VISCOUS MAGNETOHYDRODYNAMIC MODES
AND P-MODE ABSORPTION BY SUNSPOTS

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ABSTRACT

We investigate properties of small compressible magnetohydrodynamic (MHD) perturbations in the presence of viscosities. Background physical variables in static equilibrium are assumed to vary continuously only with radial coordinate in cylindrical geometry. The well-known ideal MHD singularities are removed by viscosities. We discuss novel features of viscous MHD modes and the viscous diffusive mode conversion process. It is shown that this diffusive mode conversion process can be very effective in the presence of MHD singular layers. We suggest a possible mechanism relevant to the observed p-mode absorption by sunspots in the solar atmosphere. An upper limit for "turbulent viscosity" is inferred from the observed linewidths of p-modes.

Keywords: Viscous Magnetohydrodynamics, Viscous Diffusive Mode Conversion, P-Mode Absorption by Sunspots, Solar Oscillations, Alfvén and Cusp Singular Layers

1. INTRODUCTION

We examine some general characteristics of linear compressible magnetohydrodynamic (MHD) perturbations in the presence of viscosities and introduce the concepts of viscous Alfvén shear mode, viscous MHD shear mode, and viscous MHD fast and slow mode. We further investigate properties of these viscous MHD modes in a continuously varying background equilibrium, describe the so-called "viscous diffusive mode conversion" process, and discuss the effectiveness of this process in the presence of MHD singular layers. We then construct a model example in the context of a typical sunspot in the solar atmosphere and relate the results of our model calculations to the observed acoustic p-mode absorption by sunspots (Refs. 1, 2). In other words, we suggest that the apparent, substantial absorption of p-mode by sunspots could be partially relegated to the viscous diffusive mode conversion process in the presence of MHD singular layers. There have been considerable observational and theoretical interests in the understanding of the interaction of solar p-modes with a sunspot in order to probe the internal structure of sunspots (Refs. 3 - 8).

The concrete model we shall consider is a straight cylindrical magnetic flux tube in magnetostatic equilibrium with its surrounding. Equilibrium physical variables are assumed to vary continuously with radial coordinate only. Gravitational acceleration is ignored purely for the sake of mathematical simplicity. It is well known that in the ideal MHD limit, when small perturbations are superposed onto such a continuously varying background, the Alfvén and the cusp singularities occur when certain conditions are met (Refs. 9, 10). To avoid infinities at these singularities, it is generally agreed that diffusive and/or kinetic effects should be included (Refs. 11 - 14). In the present treatment, we single out effects of viscosities for a detailed analysis and the upper limit for the "turbulent viscosity" at the photospheric level (in the phenomenological sense) is inferred from the observations for linewidths of solar p-modes (Refs. 15, 16).

In this context, we note two recent investigations (Refs. 17, 18) on viscous MHD perturbations in inhomogeneous media. However, in (17), perturbations are assumed to be incompressible and in (18), the compressibility of perturbations is not handled consistently. There exist vast literature on various versions and applications (e.g., "phase mixing", "critical layers", "resonant heating", "resonant absorption") in the contexts of fusion plasma and coronal loop heating mechanisms, and propagation of gravity waves in shear flows) of one essential physical problem in common, viz., what is the physical implications and consequences of singularities encountered in the usual idealized mathematical descriptions for small perturbations propagating in an inhomogeneous medium and how can we provide adequate and appropriate mathematical treatment for such problems in various physical contexts? The reader is referred to the references contained in (9 - 11), and a recent review (Ref. 22).

2. A GENERAL FORMULATION

In this section, we provide a general formulation of the physical problem and the mathematical treatment. We emphasize the basic concepts of viscous MHD modes which are natural generalizations of viscous hydrodynamic modes and ideal MHD modes for small perturbations and establish the so-called viscous diffusive mode conversion process.
2.1 The background equilibrium

The background magnetic field \( \vec{B}_0(r) \) is in the vertical \( \hat{z} \)-direction. We assume that all background variables depend only on radial coordinate \( r \) in cylindrical geometry. The radial pressure balance and the ideal gas law are

\[
\rho \frac{\partial \vec{v}}{\partial t} = -\nabla p + \frac{\nabla \times \vec{B}_0 \times \vec{B}_0}{4\pi} + \frac{\nabla \times \vec{B}_0 \times \vec{B}_s}{4\pi} + \eta \nabla^2 \vec{v} + \zeta \nabla(\nabla \cdot \vec{v}),
\]

(3)

\[
\frac{\partial \rho}{\partial t} = \nabla \cdot \vec{v},
\]

(4)

\[
\frac{\partial \rho}{\partial t} + \rho_0 (\nabla \cdot \vec{v}) + (\vec{v} \cdot \nabla) \rho_0 = 0,
\]

(5)

\[
\frac{\partial \rho}{\partial t} + (\vec{v} \cdot \nabla) \rho_0 + C_s^2 \rho_0 (\nabla \cdot \vec{v}) = 0,
\]

(6)

where \( \vec{v} \), \( \rho \), \( \vec{B} \), and \( \rho_0 \) are the perturbations in velocity, gas pressure, magnetic field, and density, respectively; \( \eta \) and \( \zeta \) are two coefficients of dynamic viscosity; \( C_s^2 \equiv \eta \rho_0 / \rho_0 \) is the square of sound speed with \( \gamma \) being the ratio of specific heats. Since the background equilibrium depends only on \( r \), the set of viscous MHD wave eqs. 3 - 6 can be reduced to a set of ordinary differential equations of order six by assuming \( \exp(\imath k r + \imath m \theta + \sigma t) \) dependence for perturbations, where vertical wavenumber \( k \) is real, azimuthal order \( m \) is an integer, and \( \sigma \) is complex in general. It is tedious but straightforward to obtain a single ordinary differential equation of order six for \( \nu \) in which no singularities appear.

2.2 Linear equations for small MHD perturbations

The linearized MHD perturbations for a viscous fluid are described by the following equations, where the background variables are understood to have radial dependence, viz.,

\[
\frac{\partial \vec{v}}{\partial t} = -\nabla p + \frac{\nabla \times \vec{B}_0 \times \vec{B}_0}{4\pi} + \frac{\nabla \times \vec{B}_0 \times \vec{B}_s}{4\pi} + \eta \nabla^2 \vec{v} + \zeta \nabla(\nabla \cdot \vec{v}),
\]

(3)

\[
\frac{\partial \rho}{\partial t} = \nabla \cdot \vec{v},
\]

(4)

\[
\frac{\partial \rho_0}{\partial t} + \rho_0 (\nabla \cdot \vec{v}) + (\vec{v} \cdot \nabla) \rho_0 = 0,
\]

(5)

\[
\frac{\partial \rho_0}{\partial t} + (\vec{v} \cdot \nabla) \rho_0 + C_s^2 \rho_0 (\nabla \cdot \vec{v}) = 0,
\]

(6)

where \( \vec{v} \), \( \rho \), \( \vec{B} \), and \( \rho_0 \) are the perturbations in velocity, gas pressure, magnetic field, and density, respectively; \( \eta \) and \( \zeta \) are two coefficients of dynamic viscosity; \( C_s^2 \equiv \eta \rho_0 / \rho_0 \) is the square of Alfvén speed (see eqs. 7, 9 for comparison). It is clear that the solution for \( \vec{v} \)-component vorticity \( \xi \) of eq. 10 describes the so-called viscous Alfvén shear mode. The now coupled eqs. 8 and 11 contain a solution for viscous MHD shear mode and a solution for MHD fast and slow modes modified by viscosities. We shall call the latter solution as viscous MHD fast and slow mode (see Appendix). Various limiting cases can be readily recovered in a straightforward manner.

2.5 Viscous diffusive mode conversion process

Having discussed the novel features of viscous MHD wave modes, we now introduce the concept of viscous diffusive mode conversion process. For an inhomogeneously magnetized medium, these viscous MHD modes can only be identified locally. In general, they are coupled within a finite spatial domain of background inhomogeneity. Suppose an inhomogeneously magnetized, viscous compressible medium confined within certain radius is embedded in an otherwise nonmagnetized, homogeneous viscous compressible medium. In the exterior homogeneous medium, the viscous acoustic mode and viscous shear modes with two possible "polarizations" can be readily distinguished. When viscous acoustic waves are incident towards the interior inhomogeneous region, they are partially converted into viscous Alfvén shear mode and viscous MHD shear mode (via the coupling to viscous MHD fast and slow mode in the integrated sense) gradually due to the background inhomogeneity. Upon exit from the interior inhomogeneous region, the net effects are the appearance of outgoing viscous shear modes (via viscous Alfvén shear mode and viscous MHD shear mode) and the reduction of amplitude for outgoing viscous acoustic waves (via viscous MHD fast and slow modes) in addition to normal viscous damping in time. Since outgoing viscous shear modes damp exponentially (in space as well as in time) in the direction of propagation, the finite inhomogeneously magnetized region appears to absorb the incident viscous acoustic waves. We shall re-
for this mechanism as viscous diffusive mode conversion process.

3. MODEL, NUMERICAL PROCEDURE, AND NUMERICAL RESULTS

As an application of the physical concepts discussed in the preceding section, we provide model calculations in the context of a simple, idealized sunspot in this section. In particular, we relate the integrated effects of the viscous diffusive mode conversion process in the presence of MHD singular layers to the observed p-mode absorption by sunspots. Since high-degree p-mode oscillations are strongly influenced by the physical conditions of outer solar atmosphere, we attribute the observed linewidths of p-modes (which are indicators of damping for p-modes) as due to the mean “turbulent viscosities” there. Because the size of an isolated sunspot is much smaller than that of the Sun, we imagine a cylindrical magnetic flux tube imbedded in a horizontally infinite, turbulent fluid slab and adopt a cylindrical coordinate system for the convenience of analysis. The incident (cylindrical) viscous acoustic waves towards a sunspot is partially converted into viscous shear modes and the reflected viscous acoustic wave has reduced amplitude in addition to normal viscous damping. We thus observe an apparent absorption of p-modes due to the presence of a sunspot. In the final analysis, those viscous shear modes are damped and their energy is absorbed by a sunspot and its ambient atmosphere. We resort to numerical integration to solve the coupled ordinary eqs. 3 - 6 of six order. It is natural for us to choose a background equilibrium to represent a typical sunspot as close as possible.

3.1 Observations

A typical isolated sunspot has an radius ranging from 4 to 10 arc seconds (one arc second ~ 700 km). The temperature at the center of a sunspot is around 3500 ~ 4000°K; the temperature of the surrounding photosphere is 5500 ~ 6000°K. The magnetic field strength in a sunspot can be as high as ~ 3000 gauss. At the upper photospheric level, the density inside a sunspot is less as compared to that of its surrounding at the same level. The Sun is oscillating as a gigantic resonant cavity which traps acoustic waves. The observed power of solar acoustic p-mode oscillations is peaked at five minute band (3.3 mHz). The observed viscosity of solar p-mode oscillations is at peak five minute band (3.3 mHz). The observed p-mode absorption by sunspots (Refs. 1, 2) are in the frequency range 2.5 ~ 4.1 mHz, in the horizontal wave number range 0.2 ~ 1.5 x 10^-5 cm^-1, in the azimuthal order range -5 < m < 5 (Ref. 1), 0 < m < 20 (Ref. 2). The amount of absorption averaged over m at given frequency and horizontal wave number ranges from 10% to 50%. A recent paper by Libbrecht (Ref. 15) provides the observed linewidths of solar p-modes in the frequency range 1.6 ~ 5.5 mHz. The observation seems to indicate that linewidth depends only on frequency. Around 3.3 mHz (five minutes) band, the linewidth is of order of a few µHz.

3.2 Background model

We prescribe a model background for a cylindrical magnetic flux tube to represent an idealized sunspot. The background temperature profile T_b(x) is given by

\[ T_b(x) = T^* \left[ 1 + 0.5 \exp \left[ -\lambda (x - 1)^2 \right] \right]. \]  
\[ \rho_b(x) = \rho^* \left[ 1 + \exp \left[ -\lambda (x - 1)^2 \right] \right]. \]  
\[ r = r_{RS} \quad (0 \leq r \leq 1). \]  

where \( r_{RS} \) is the radius of a sunspot and \( \lambda \) is a parameter controlling the sharpness of transition between magnetized and nonmagnetized regions. The temperature \( T^* \) and the density \( \rho^* \) at the center of a sunspot are chosen to be 4000°K and 2.5 x 10^{-7} g cm^{-3}, respectively. For \( x \geq 1 \), a nonmagnetized homogeneous viscous, compressible medium surrounds the inner magnetized region. The ratio of specific heats \( \gamma \) assumes a value of 5/3. The total gas pressure \( P_T \) takes a value 2.4 x 10^{10} dyne cm^{-2} at the photospheric level. Given above parameters, the magnetic field strength on the axis of the flux tube is ~ 2000 gauss. We estimate the upper limit to the value of \( \eta \) and \( \zeta \) by assuming that the observed linewidths of p-modes of solar oscillations are due to “turbulent viscosities” of solar atmosphere in a phenomenological sense. For viscous acoustic waves with real wave numbers, the damping rate -\( \sigma_w \) in time is given by

\[ -\sigma_w \approx \frac{a^2 \sigma w + \zeta}{2 \nu_T} \]  

for small viscosities, where \( \sigma \) is the angular frequency (see Appendix). We shall take \( \eta = 3.0 \times 10^2 \) g cm^{-1} s^{-1} and \( \zeta = 2.0 \times 10^2 \) g cm^{-1} s^{-1}, so that for frequency of 3.3 mHz, the linewidth given by eq. 15 is 2.54 µHz (Ref. 9). We also note that given certain mean physical conditions at the photospheric level, eq. 15 indicates a simple frequency dependence of damping rate and thus linewidth of p-mode.

3.3 Boundary conditions

The origin \( r = 0 \) is a regular singularity of eqs. 3 - 6 by assuming zero first derivatives of background variables there. By requiring a regular solution for perturbations at the origin, we obtain

\[ v_x \sim r^{m+1}, \quad v_y \sim r^{m+1}, \quad v_z \sim r^m, \]  

where \( m (\neq 0) \) is the azimuthal order of small perturbations. For axisymmetric perturbations (\( m = 0 \)),

\[ v_x \sim r \quad v_y \sim r \quad v_z \sim r^0 \]  

at the origin.

Far away from the axis, the boundary is a homogeneous viscous acoustic medium where incident and outgoing viscous acoustic waves, and outgoing viscous shear modes are present. The explicit analytic expressions for these modes can be obtained so that matching conditions at the other end of integration are known. We use the representation of incoming and outgoing cylindrical waves (Hankel functions) for convenience of comparison (Refs. 1, 2). We normalize the amplitude of a given incoming viscous acoustic wave as unity. The definition of the absorption coefficient \( a_A \) for a particular incident viscous acoustic wave with given frequency, horizontal wave number (far away from the axis), and azimuthal order is simply one minus the square of the amplitude of the corresponding outgoing viscous acoustic wave (see Appendix).

3.4 Numerical method

In the presence of small viscosities, eqs. 3 - 6 are quite singular for the purpose of numerical integration. This nu-
merically singular behavior is due to the presence of (unwanted) fast exponentially growing solutions in both directions of integration. Simple-minded shooting method does not work due to loss of numerical accuracy when viscosities are sufficiently small. Relaxation method could work but would be less efficient due to the high resolution required to resolve the subtle features across MHD singular layers. We used a package routine developed by Scott and Watts (Ref. 21). The method is the so-called “piece-wise recorthornormalization procedure” which handles this integration satisfactorily.

3.5 Numerical results

Since we are interested in the response of a magnetic flux tube to incident viscous acoustic waves for a temperature profile given by eq. 12, the cusp singularity never occurs (Refs. 9, 10).

For axisymmetric \((m = 0)\) perturbations, the Alfvén singularity never arises (Refs. 9, 10). Therefore for axisymmetric perturbations, the fractional change in the amplitudes of incoming and outgoing acoustic waves is estimated to be in the order of \((\eta + \zeta)l_H / l_T\). This is verified by sample numerical calculations. In order to have 10% to 40% of power absorptions, dynamic viscosities should be, at least, in the order of \(10^3\) \(g\) \(cm\) \(^{-1}\) \(s\) \(^{-1}\) which implies a much larger “turbulent viscosities” inside a magnetic flux tube. This required enhancement of “turbulent viscosity” inside a sunspot (in order to account for substantial absorptions) is somewhat contrary to our intuition. Thus, the question remains open.

For nonaxisymmetric \((m = 1, 2, 3 \text{ etc.)})\) perturbations, the Alfvén singular layer (or “resonance”, or “critical layer”) which is “smeared” across the singular point by viscosities, plays a unique role. The Alfvén singular layer is characterized by pronounced, fast oscillations and sharp transition of the eigenfunctions. The larger the viscosities are, the smoother the transition across the singular layer is. Numerical results for various combinations of parameters of our model calculations indicate substantial reductions in the amplitudes of outgoing viscous acoustic waves with the dynamic viscosities limited by the observed p-mode linewidths. Heuristically, this seems to suggest that the Alfvén singular layer is quite effective in partially converting incident viscous acoustic waves (via the coupling to viscous MHD fast and slow modes) into viscous shear modes (via viscous Alfvén shear mode and viscous MHD shear mode).

For reference, we report several major numerical results in the following. For example, when \(m = 1\), \(\lambda = 5.0\), \(\sigma_1 = 2 \times 10^{-3}\) \(s\) \(^{-1}\), \(\eta = 3.0 \times 10^5\) \(g\) \(cm\) \(^{-1}\) \(s\) \(^{-1}\), \(\zeta = 2.0 \times 10^3\) \(g\) \(cm\) \(^{-1}\) \(s\) \(^{-1}\), horizontal wavenumber \(k_x = 1.0 \times 10^{-4}\) \(cm\) \(^{-1}\), then \(f = 2.0 \times 10^{-3}\) \(cm\) \(^{-1}\), \(\sigma_n = -2.5\mu Hz\); for \(R_S = 4.2, 5.0, 6.3, 7.0 \times 10^4\) \(cm\), the corresponding absorption coefficients \(a_k\) are 30%. 43.7%, 46.9%, 48.6%, 47.6%, respectively. The absorption coefficient \(a_k\) is quite sensitive on the sharpness transition parameter \(\lambda\) for given radius \(R_S\). With \(R_S = 4.2 \times 10^4\) \(cm\), for \(\lambda = 2.0, 2.5, 3.0, 5.0, 10.0, 20.0, 50.0\), the corresponding absorption coefficients \(a_k\) are 0.3%, 18.2%, 36.7%, 38.0%, 18.5%, 10.0%, 5.4%, respectively. We now illustrate the dependence of \(a_k\) on the horizontal wavenumber \(k_x\). With \(\lambda = 5.0\), \(R_S = 4.2 \times 10^5\) \(cm\), for \(k_x = 0.5, 1.0, 1.2 \times 10^{-8}\) \(cm\) \(^{-1}\), the corresponding absorption coefficients \(a_k\) are 18.4%, 38.0%, 37.9%, respectively. Finally, we provide a few examples to show the dependence of \(a_k\) on the azimuthal order \(m\). With \(\lambda = 5.0\), \(R_S = 4.2 \times 10^5\) \(cm\), \(k_x = 1.0 \times 10^{-6}\) \(cm\) \(^{-1}\), for \(m = 1, 2, 3\), the corresponding absorption coefficients \(a_k\) are 38.0%, 17.2%, 12.5%, respectively.

4. SUMMARY AND DISCUSSION

We discussed the characteristics of viscous, compressible MHD perturbations in a uniformly magnetized medium and the viscous diffusive mode conversion process in the presence of an inhomogeneously magnetized region embedded in an otherwise nonmagnetized homogeneous medium. We then relate this viscous diffusive mode conversion process to the observed p-mode absorption by sunspots. For axisymmetric perturbations, the viscous diffusive mode conversion is not very effective and is in the order of \((\eta + \zeta)l_H / l_T\).

For nonaxisymmetric perturbations, the viscous diffusive mode conversion process can be very effective due to the presence of Alfvén singular layer. Therefore more realistic, comprehensive treatment of diffusive MHD wave propagation in an inhomogeneous medium should be pursued further (Refs. 2, 23). The observed p-mode absorption by sunspots for axisymmetric perturbations (Ref. 2) cannot be explained by the present model; however, we note that for a twisted flux tube in equilibrium, MHD singularities occur also for axisymmetric perturbations (Ref. 10) . and the actual magnetic field in a sunspot may be twisted. There are other important diffusive processes within a sunspot yet to be explored, viz., resistive, thermal, and radiative (Ref. 8, 10). These diffusive processes are possible mechanisms to remove ideal MHD singularities in an inhomogeneously magnetized medium and present new features of diffusive mode conversion. The magnitudes of those diffusive coefficients may not be limited by the linewidths of global p-modes, because those diffusive processes are local to a sunspot. Another important aspect of an actual sunspot is the gravitational stratification and this lends the problem even more difficult to handle because the background equilibrium is at least inhomogeneous in two spatial dimensions. We need to resort to robust numerical routines to solve a set of partial differential equations.

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5. APPENDIX

We accumulate several useful equations and solutions in this Appendix for reference. Taking the time derivative of eq. 3 and making use of eqs. 4, 6, we obtain the following equations in cylindrical coordinates \((r, \phi, z)\), viz.,

\[
\frac{\partial \psi}{\partial t} = \frac{\partial}{\partial r} \left[ C_3 \rho \left( \nabla \cdot \psi \right) \right] + \rho \frac{C_3^2}{2} \frac{\partial \psi}{\partial r} + \frac{\partial}{\partial r} \left( \rho \frac{C_3^2 \Delta}{2} \right)
\]

\[
+ \frac{\partial}{\partial r} \left( \nabla \psi \right) + \frac{\partial}{\partial r} \left( \psi \frac{\partial \psi}{\partial r} \right) - \eta \frac{\partial^2 \psi}{\partial r^2} + \frac{2 \partial^2 \psi}{\partial r^2}.
\]

(18)
In the following discussion of solutions for \( v_0, v_0, v_0 \), the factor \( \exp(ikz + \imath t \theta + \imath \eta r) \) is not written out explicitly.}

For a nonmagnetized homogeneous, viscous compressible medium far away from the origin, we obtain general solutions describing incident and outgoing viscous acoustic cylindrical waves and outgoing viscous shear waves with two possible "polarizations," viz.,

\[
v_0 = - \left\{ 1 + \frac{k^3}{c^2} \left[ C_0^2 + \frac{(\eta + \zeta) \eta}{\rho_0} \right] \right\} \frac{1}{k^2} \left[ \frac{dH_m(1)(k,r)}{dr} \right] + \frac{B}{k^2} \frac{dH_0^2(k,r)}{dr} - \frac{mC}{k^2} H_m(k,r).
\]

We note that coefficients \( A, B, C, D \) are constant coefficients, \( H_m(\eta) \) is the Hankel function of order \( m \) with complex argument \( \eta \).

\[
v_0 = -i \left\{ 1 + \frac{k^3}{c^2} \left[ C_0^2 + \frac{(\eta + \zeta) \eta}{\rho_0} \right] \right\} \frac{1}{k^2} \left[ \frac{mH_m(1)(k,r)}{k^2} \right] + iD \frac{dH_m(1)(k,r)}{dr}.
\]

where \( A, B, C, D \) are constant coefficients, \( J_m(\eta) \) is the Bessel function of order \( m \) with complex argument \( \eta \).

\[
v_0 = - \left\{ 1 + \frac{k^3}{c^2} \left[ C_0^2 + \frac{(\eta + \zeta) \eta}{\rho_0} \right] \right\} \frac{1}{k^2} \left[ \frac{mH_m(1)(k,r)}{k^2} \right] + iD \frac{dH_m(1)(k,r)}{dr}.
\]

We note that coefficients \( A, B, C, D \) are associated with the incident and outgoing viscous acoustic waves, respectively, and coefficients \( C \) and \( D \) are associated with the two independent outgoing viscous shear modes, respectively. In eqs. 29 - 31, the notation \( \pm \) (or \( \mp \)) without specifying first or second kind is adopted to represent outgoing cylindrical viscous shear modes for the appropriate complex argument. Since we shall not consider "forced oscillations," and also general perturbations are finite in the spatial domain, we thus require both \( k^* \) and \( k^2 \) to be real and positive. By writing \( \sigma \equiv \omega + i \eta \), we have

\[
\sigma_R = - \frac{C_0^2 + \frac{(\eta + \zeta) \eta}{\rho_0}}{k^2} \eta k^2 \left( k^2 - \frac{\sigma^2}{\eta^2} \right)^{1/2}.
\]

For a particular \( m \)-mode with given angular frequency \( \sigma \) and horizontal wavenumber \( k_0 \), the absorption coefficient \( \alpha_m \) is defined as

\[
\alpha_m = \frac{|\sigma^2 - \omega^2|}{|\sigma^2|}.
\]

6. REFERENCES


THE INFLUENCE OF A CHROMOSPHERIC MAGNETIC FIELD ON p- AND f-MODES

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ABSTRACT

The influence of a magnetic chromosphere on p- and f-modes of oscillation is explored theoretically, modelling the chromosphere by an isothermal atmosphere permeated by a horizontal magnetic field and the field-free convection zone by a polytrope. It is pointed out that active regions, characterized by magnetic field strengths of the order of 100 gauss, have a significant effect on the frequencies of high degree modes. The frequency of the f-mode is increased by the presence of a magnetic field; the frequencies of p-modes are decreased. In an evolving active region the frequencies of high degree p- and f-modes are systematically split apart by the changing magnetic field. These effects should be apparent in a comparison of high resolution data sets for active regions with either low resolution sets or high resolutions sets for quiet regions.

Keywords: p- and f-modes, magnetic effects, atmospheric corrections, active and quiet regions.

1. INTRODUCTION

The solar p-modes are sound waves trapped in the solar interior within a cavity formed at its lower level by the increasing sound speed in the convection zone and at its upper level by the reflective properties of the photosphere and chromosphere. The upper levels of the acoustic cavity are threaded by magnetic field and so variations in this field must be reflected in frequency changes in the p-modes. Similarly, the f-mode - a surface mode with a frequency that is independent of the thermal stratification within the medium - must also be influenced by magnetic effects.

In the photosphere the solar magnetic field is organized into discrete flux tubes which are moved around and buffered by granules and supergranules. These tubes are extremely small (some 100km across) though magnetically intense with a field strength of some 1-2kG. See Refs 1 and 2 for a recent discussion. Such intense flux tubes make up the bulk of the solar photospheric magnetic field outside of sunspots. Sunspots, with field strengths of about 3kG, comprise yet more magnetic flux in the photosphere; whether sunspots are simply a collection of many intense tubes or whether they are a single monolithic tube is undecided.

Photospheric concentrations of magnetic field, be they isolated intense flux tubes or sunspots, rapidly expand out with height so that by the mid-chromosphere the magnetic field permeates the whole of the solar atmosphere. The result is that the photosphere and upper convection zone is covered by a cathedral-like canopy of magnetic field, supported on flux tube 'columns' (Ref 3). The field strengths in the chromospheric layers of such cavities depend upon the concentration of flux tubes and sunspots that support the canopy (Refs 4, 5). Regions of the chromosphere where such concentrations are thin and sunspots are absent are referred to as quiet regions; where concentrations are high and sunspots are present are called active regions. We characterize quiet regions by a magnetic field strength of 10G, and active regions by a field strength of 100G.

What effect does a magnetic atmosphere have on the frequencies of p- and f-modes? Clearly, the effect on low degree modes is likely to be negligible, since such modes penetrate deep into the Sun and consequently their frequencies are largely determined by the properties of the solar interior. (Low degree modes may of course be influenced by magnetic fields within the solar interior; see Refs 6 and 7.) High degree modes, however, are influenced by the photospheric and chromospheric layers of the Sun, and so may be expected to be affected by atmospheric magnetic fields. And since such magnetic fields evolve, as active regions are born, develop and decay, the magnetic influences on oscillation frequencies are likely to change on a (perhaps) day by day basis, and also over the solar activity cycle. Indeed, it may be possible to use mode frequencies as a diagnostic of magnetic field strengths in the chromosphere.

High degree modes, then, are likely to be the most strongly influenced by chromospheric magnetic fields. In this context we note a recent analysis of p-mode data for the years 1981 and 1985 carried out by Duvall et al (Ref 8) which indicated a systematic frequency change of less than 0.1Hz at low % to 0.6Hz at % 99. However, Rhodes et al (Ref 9) found no significant change in frequency between data sets with % 8 and % 99 for late 1981 and mid 1984. At higher degrees, Libbrecht and Kaufman
remark that the frequency of the f-mode was as much as 13.4μHz above its theoretical (non-magnetic) value. Systematic errors may account for much of this (Ref 10) but (as we argue here) magnetic effects may also be responsible. High accuracy measurements of oscillation frequencies for high degree modes is awaited with interest.

In this paper we investigate the effect of a horizontal magnetic field on p- and f-mode frequencies. For simplicity, we consider a polytropic non-magnetic atmosphere above which is an isothermal gas permeated by a magnetic field. The magnetic atmosphere is taken to have a constant Alfvén speed, implying that the magnetic field strength e-folds in two scale-heights of the atmosphere. The assumption of a constant Alfvén speed simplifies the mathematics of our model but tends to underestimate the effect of a magnetic field. (A model in which the magnetic field is uniform is currently under consideration (Ref 11) and in fact indicates stronger magnetic effects than those to be presented here.)

2. DISPERSION RELATIONS

Consider a plane-parallel atmosphere with gas pressure \( p_g(z) \) and density \( \rho_g(z) \) within which is embedded a horizontal magnetic field \( B(z) \). The equilibrium satisfies

\[
\frac{\rho_0 + B^2}{2\rho_0} = \rho_g, \tag{1}
\]

where \( \rho_0 \) is the gravitational acceleration and the dash (') denotes differentiation with respect to depth \( z \). The gas law is

\[
\rho_0 = \frac{K_B}{m} \rho_g |_0, \tag{2}
\]

for temperature \( T_0(z) \), Boltzmann's constant \( K_B \), and mean particle mass \( m \).

Perturbations about the equilibrium given by Eqs (1) and (2) are described by the equations of ideal magnetohydrodynamics for an isentropic gas. We consider two-dimensional motions Fourier analysed in time and the horizontal direction (the x-axis)

\[
v = (v_x(z),0,v_z(z)) \exp i(\omega t - kx), \tag{3}
\]

for frequency \( \omega \) and wavenumber \( k \). The equations of ideal Mhd may then be combined to yield

\[
\left(1 - \frac{k^2 c_s^2}{\omega^2}\right) \Delta = \frac{d^2 v_z}{dz^2} + \frac{gk^2}{\omega^2} v_z, \tag{4}
\]

\[-(k^2 c_s^2 - \omega^2) v_z + g \frac{d v_z}{dz} = -c_s^2 \frac{d A}{dz} - (\gamma - 1) g \Delta + \frac{2 \Delta}{(2\pi)^2} A, \tag{5}
\]

where \( \Delta = \text{div} v_x \), and \( c_s(z) = (\gamma p_0/\rho_0)_{\text{iso}} \) and \( v_A(z) = (B^2/\mu_0 p_0)_{\text{iso}} \) are the sound and Alfvén speeds.

The variable \( \Delta \) may be eliminated between Eqs (4) and (5), with the result (Refs 12-14)

\[
\frac{d}{dz} \left\{ \frac{\rho_0 (c_s^2 + k^2) (\omega^2 - k^2 c_s^2)}{(\omega^2 - k^2 c_g^2)} \frac{dv_z}{dz} \right\} = \left\{ \frac{\rho_0 k^2}{\omega^2 - k^2 c_g^2} - \frac{\rho_0 (\omega^2 - k^2 c_s^2)}{\omega^2 - k^2 c_s^2} - gk \left( \frac{c_s^2}{\omega^2 - k^2 c_s^2} \right) \right\} v_z, \tag{6}
\]

where \( c_s^2 = c_g^2/(\omega^2 - k^2 c_s^2) \). It is also possible to eliminate \( v_z \) in preference to \( \Delta \), but, unlike the non-magnetic case, this is generally less useful (but see Ref 6).

Equation (6) may be solved in the two regions of our model, namely in the magnetic atmosphere and the field-free 'convective zone'. We take the 'convective zone' to be adiabatically stratified with sound speed \( c_s(z) \)

\[
c_s^2(z) = c_s^2(1 + z/z_0), \quad z > 0, \tag{7}
\]

for sound speed \( c_s \) at the interface \( z=0 \) between the field and the convection zone. The constant \( z_0 \) and the polytropic index \( m \) are given by

\[
m = \frac{1}{\gamma - 1}, \quad z_0 = \frac{m a^2}{g} \tag{8}
\]

where \( \gamma \) is the adiabatic index. In the non-magnetic region \( (z > 0) \) Eqn (6) has a solution in terms of the confluent hypergeometric function \( U \) and its derivative. This solution is most conveniently obtained by first solving Eqns (4) and (5) with \( B = 0 \) for \( \Delta \), rather than \( v_z \), much as first carried out by Lamb (Ref 15; see also Refs 16-18). In fact,

\[
\Delta = \exp \left( \frac{k(z + z_0)}{2} \right) U(-a,m+2,kz+2kz_0), \quad z > 0, \tag{9}
\]

where the parameter \( a \) is defined by

\[
a = \frac{m k}{\gamma - 1}, \quad z_0 = \frac{m a^2}{g} \tag{10}
\]

This solution satisfies the requirement that the kinetic energy density of the motion in \( z > 0 \) is finite at infinity.

In the magnetic atmosphere \( (z < 0) \), the assumptions of isothermality \( (c_s(c_s=A, \quad z < 0) \) and constant Alfvén speed \( v_A \) (so \( B(z) \sim \rho_g(z) \)) permit us to solve Eqn (6), viz.

\[
v_z(z) = C \exp \left( \frac{\lambda z}{2A} \right), \quad z < 0, \tag{11}
\]

where \( 4\Delta H_g < 1 \) and \( C \) is an arbitrary constant. The magnetically-modified pressure scale-height \( H_B \) is defined by

\[
H_B^{-1} = \frac{\gamma g}{c_s + \frac{4}{3} \lambda v_A}, \tag{12}
\]

and \( \lambda \) is given by

\[
\lambda = \frac{(c_s^2/gH_B - 1) k^2 q^2 + (\omega^2 - k^2 c_s^2)(\omega^2 - k^2 c_g^2)}{(c_s^2 + \omega^2)(\omega^2 - k^2 c_s^2)} \tag{13}
\]

The two regions \( (z > 0) \) and \( (z < 0) \) are matched by requiring that \( v_z \) and
Frequency shifts for the p-modes may also be magnetic field. Note that $f \gg 0$, and is zero in the absence of a $Q_0$ atmosphere (Ref 19); explicitly, for $kz \ll 1$, an investigation of the dispersion relation (14) general a solution of Fqns (4) and (5). In fact, $u = gk$ with $v_z$ satisfies the differential equation whatever the thermal stratification, $\Omega^2 = (kz^2 + \frac{v_z}{(v_z^2 + \frac{\rho}{\rho_A})}) \Omega_{mag}$, given approximately by

$$\Omega^2 = \frac{(gk^2 \rho_z^2)}{(\omega^2 - k^2 \omega_0^2)} \frac{v_z}{v_A^2},$$

where $\omega$ and its derivative $\omega'$ are evaluated at argument $2kz$.

The restriction $4AHg < 1$, imposed on Eqn (11), implies a cutoff frequency for propagation in the magnetic atmosphere. For small wavenumber $k$ this yields a cutoff frequency, $\Omega_{mag}$, given approximately by

$$\Omega^2 = \frac{c_S^2 (v_z^2 + 2v_A^2)}{(v_z^2 + \frac{\rho}{\rho_A})} \Omega_{mag},$$

where $\omega = \omega_0/(2c_S)$ is the acoustic cutoff in the field-free isothermal atmosphere (Ref 19). Frequencies above $\omega_{mag}$ propagate; those below are evanescent. Equation (15) predicts a reduction in $\omega_{mag}$ with increasing field strength, though eventually corrections at order $k^2$ arise causing $\omega_{mag}$ to increase with $v_A$. The net result is that a mode may change its character under the influence of an evolving magnetic field, being trapped at one field strength but propagating at a different field strength. Consequently, the presence of a magnetic atmosphere introduces a 'window effect' (Ref 19) for p-modes.

3. FREQUENCY SHIFTS

The dispersion relation (14) may be solved for frequency $\omega$ and the results examined for frequency shifts induced by the magnetic field. Consider, first, the f-mode. This is a surface mode in that its amplitude declines exponentially either side of the interface $z=0$. In the absence of a magnetic field, inspection of Eqs (4) and (5) reveals that $\omega^2 = gk$ with $v_z = e^{-kz}$ satisfies the differential equations whether the thermal stratification producing $c_S(z)$. This is the f-mode, the frequency of which is accordingly independent of $c_S(z)$. In the presence of a magnetic field, $\omega^2 = gk$ is not in general a solution of Eqs (4) and (5). In fact, an investigation of the dispersion relation (14) for small $k$ reveals that the f-mode suffers a frequency shift due to the presence of a magnetic atmosphere (Ref 19); explicitly, for $kz_0 \ll 1$,

$$\Omega^2 = 1 + f_0 (kz_0)^{m+2},$$

where $f_0 = 1 + 2n/m$, the coefficients $p_n$ (8) are involved and will not be given explicitly (see Ref 19 for details). In the presence of a magnetic atmosphere the coefficients $p_n$ are negative for all $n$. In the presence of a sufficiently strong field, however, $p_n$ becomes positive.

How large are the corrections introduced by an atmosphere? In Tables 1 and 2 we display the atmospheric corrections determined by Eqs (15) and (16), expressing the results in terms of corrections to the cyclic frequency $\nu = \omega/2\pi = f_0(kz_0^{m+2})$ for various magnetic field strengths $B_0$ ($= B(z=0)$) at the base of the magnetic atmosphere and degree $k$, where $kz \ll \Omega/(c_S^2)^{1/2}$. The base of the magnetic atmosphere is taken, for convenience, to coincide with the temperature minimum,

$$\Omega = \frac{\sqrt{\Omega_n^2 + (kz_0)^2}}{n}, \quad n = 1, 2, 3, \ldots \ldots$$

Table 1 Frequency shifts (in $\mu$Hz) in the cyclic frequency of the f-mode caused by the magnetic atmosphere, for a quiet Sun ($B_0 < 10^4$) and for an active Sun ($B_0 > 10^4$). We have set $\gamma = \gamma_k/3$ in $v_A^2$ and taken $c_S = 6.76 \text{ km s}^{-1}$, corresponding to the sound speed at the temperature minimum where the field $B_0$ is taken to reside.

<table>
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<th>$10^5$</th>
<th>$200G$</th>
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<td>0.04</td>
<td>30.21</td>
<td>127.13</td>
</tr>
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</table>

It may be seen from the tables that frequency shifts are negligible at intermediate $k$ (say $k=100$) but become more significant at high $k$.

What happens if the magnetic field evolves? We are thinking of the possibility of high resolution data sets gathered from a region of the Sun at different times, or of a comparison of low resolution sets (for which the field is effectively 'quiet') with high resolution ones. In such circumstances tables 1 and 2 make it clear that frequency differences are to be expected. For example, measuring the f-mode in an active region should yield a higher frequency than measurements for a quiet region, a frequency increase of some tens of $\mu$Hz being expected for high degree ($l > 1000$) modes. We note that this is consistent with the recent results of Libbrecht and Kaufman (Ref 10). Similarly, in any comparison of data sets for p-modes gathered at different times we may anticipate frequency differences between the sets. Consulting Table 2, we see that even for a field change from $B_0 = 200G$ at one time to $B_0 = 0$ at another changes in the low and intermediate ($k < 100$) modes, due to an evolving atmosphere, are virtually negligible ($< 0.1\mu$Hz). This is consistent with the constancy of p-modes for $6 < k < 89$ reported by Rhodes et al (Ref 9), and in contradiction with the change of as much of $0.6\mu$Hz reported by Duvall et al (Ref 8) for modes with $4 < k < 99$. For higher degree modes, however, frequency changes at the $\mu$Hz level are to be expected.

The above results make it clear that in any monitoring of solar oscillations at high degree $k$, it is important to be aware of the influence of...
Table 2: Frequency shifts (in \(\mu\)) for the \(p\)-modes caused by the magnetic atmosphere. There is a little difference between the quiet Sun case (of 10G) and the zero field case.

<table>
<thead>
<tr>
<th>(B_0=10G)</th>
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4. REFERENCES


OBSERVATIONS OF F- AND P-MODE OSCILLATIONS OF HIGH DEGREE
(500 < \ell < 2500) IN QUIET AND ACTIVE SUN

T. Tarbell, M. Peri, Z. Frank, R. Shine, A. Title
Lockheed Palo Alto Research Labs

ABSTRACT

This poster presents spectra (f - \nu diagrams) from high resolution observations taken at the Vacuum Tower Telescope (NSO/Sunspot). The raw data are CCD images taken through the SOUP narrowband filter in Fe I 5576 Å. Four filtergrams spaced through the spectral line are combined to form velocity movies. Spectra for 80 minutes of data with 0.5 - 1.5 arcsecond resolution are presented for the entire field-of-view and for quiet and magnetic (plage) subregions. Ridges f and p₁ - p₃ are evident in velocity spectra, extending to \ell = 2500 (f), \ell = 1800 (p₁), and \ell = 1200 (p₃). Much less power is seen in the magnetic region than in the quiet sun. Three-dimensional Fourier filtering shows that oscillation velocity amplitude drops sharply at the boundary of the active region for each family of modes considered.

Keywords: Solar Oscillations, F-Modes, High Degree, Magnetic Fields, 3-D Fourier Filtering

1. INTRODUCTION

Observations of high-\ell (short-wavelength) oscillations in the solar atmosphere are difficult from the ground because of atmospheric seeing. The SOUP experiment on the Spacelab 2 shuttle flight detected f-modes up to \ell = 3500 but saw no clear evidence for p-modes in white light. This poster presents a first look at Doppler observations of f- and p-modes whose horizontal wavelengths approach the size of solar granulation. Because of their short wavelengths, these oscillations are affected by fine structure in the upper convective zone and atmosphere, in particular by the magnetic flux tubes in an active region. Observers have known for a long time that the amplitudes of the five minute oscillations are reduced in magnetic regions (Refs. 1-3). More recently, direct measurements of the absorption of p-modes by sunspots (Ref. 4) have suggested that sub-surface magnetic structures may be mapped in some detail by studying their effects on oscillations of different wavelengths. Although our dataset has very limited duration and field-of-view, it shows that maps of the spatial structure of oscillation amplitudes also outline the active region magnetic field to high accuracy.

2. OBSERVATIONS

Our observations were made at the Vacuum Tower Telescope (NSO/Sunspot) on August 6, 1987. The raw data consist of 512 x 512 pixel images (90 x 90 arcsec) taken with the HRSS board CCD camera through the SOUP tunable filter. In an 82 second cycle time, images at 13 different wavelengths and/or polarizations were taken sequentially. The results presented here use images in Fe I 5576 Å (a nonmagnetic line: g = 0) and nearby continuum. A total of 59 cycles (80 minutes) were recorded, during which the seeing was usually better than 1 arc second. The data can be viewed as 59-frame movies in different wavelengths, all taken effectively simultaneously.

Figure 1 shows a continuum image of the region of the sun studied with a contour from a simultaneous magnetogram superimposed. Analysis was done on the full region, and on magnetic (plage) and nonmagnetic (quiet sun) subregions. The magnetic and nonmagnetic subregions are indicated by the black boxes. Figure 2 shows a Doppler velocity image of the region shown in Figure 1, with magnetic and nonmagnetic subregions indicated. Four filtergrams evenly-spaced in wavelength through the line were combined to form the velocity and Doppler-compensated line-center intensity images. The velocity measurement technique was devised for the SOHO Michelson Doppler Imager (Ref. 5). It is accurately calibrated in meters/second, linear over a wide dynamic range, and insensitive to variations in line depth and width.

3. ANALYSIS

A 3-D Fourier transform was performed on the velocity data, converting coordinates x, y, t to kₓ, kᵧ, \nu in frequency space. At each temporal frequency \nu, the f and p-mode ridges appear as elliptical power concentrations in the kₓ - kᵧ plane. Conversion of 3-D data to a 2-D f - \nu diagram was accomplished by integrating along the ellipses to compensate for forshortening (the region was viewed at a disc position of \mu = 0.8), thus converting kₓ and kᵧ to a kₓₜₙₑᵣₑ spatial wavenumber.

Figures 3-5 show the $\ell - \nu$ diagrams with enhancement to emphasize the extent of the ridges. In the full region (Figure 3) $\ell - \nu$ diagram ridges can be seen to $\ell = 2500$ (f), $\ell = 1800$ ($p_1$), and $\ell = 1200$ ($p_2$). The magnetic subregion (Figure 4) exhibits much less power than the nonmagnetic subregion (Figure 5) with the ridges appearing less distinctly. Also, f is suppressed relative to $p_1$ in the magnetic subregion as compared to the nonmagnetic (quiet sun) subregion.

Positions of the ridges in the $\ell - \nu$ diagrams were visually determined, with a square-root curve fit to the f ridge and straight lines fit to each of the p-mode ridges. Positions of the power minima above and below each ridge were similarly determined. The intensity in the power minima above and below each ridge were averaged to calculate a background power value corresponding to each point along the ridge. The relative power in the ridge (solid line) and the background value (dashed line) are plotted against $\ell$ in Figures 6 and 7 for f and $p_1$, respectively. The signal-to-noise ratios for each ridge as a function of $\ell$ and of $\nu$ are highlighted separately for the full region and for the magnetic and nonmagnetic subregions. The residual power, remaining after subtraction of the background power value, is given as a function of $\ell$ and $\nu$ in Table 1. Power in the magnetic region is typically reduced by a factor of 2-3 relative to the nonmagnetic region.

To display this effect in another way, we have made new velocity movies which show only a single family of modes (f or $p_1$, for example). The mode isolation was accomplished by zeroing all components in the 3-D Fourier transform of the velocity movie, except those components on the ellipse in the $k_x - k_y$ plane corresponding to the desired mode at each value of $\nu$. Inversion of the resulting transform yielded a new velocity movie showing oscillations only of the selected family of modes. At each point in the field-of-view, the rms velocity fluctuation in time was calculated, and these values form an image of the oscillation amplitudes throughout the active region and surrounding quiet sun. Figure 8 shows these images for f and $p_1$, as well as for the original movie (containing all oscillations and convective flows) and for a "sub-f-mode" filtered movie (containing no f- or p-modes).

Figure 8 shows that the magnetic contour of the active region can be located quite precisely by the suppression of both oscillations and convective flows. The upper extent of a ridge in $\ell$ implies limits to the spatial resolution that can be obtained in an rms image of a filtered movie. The f and $p_1$ images in Figure 8 exhibit resolution that is consistent with the spatial frequency extent of the ridges in Figures 6 and 7.

4. ACKNOWLEDGEMENTS

The observing staff of the Vacuum Tower Telescope and Harry Ramsey of Lockheed were very helpful in obtaining these observations. This work was supported by Lockheed Independent Research Funds and NASA contracts NAS8-32805 (SOUP) and NAS5-26813 (HRSO).
Figure 3. $\ell - \nu$ diagram of the power spectrum of the full region, enhanced to show the extent of the ridges.

Figure 4. $\ell - \nu$ diagram of the power spectrum of the magnetic (plage) subregion, enhanced to show the extent of the ridges.

Figure 5. $\ell - \nu$ diagram of the power spectrum of the nonmagnetic (quiet sun) subregion, enhanced to show the extent of the ridges.

Figure 6. Relative power in the ridge (solid line) and the background power value (dashed line) are plotted against $\ell$. The signal-to-noise ratios are highlighted for the full region (upper plot) and for the magnetic (lower curve, lower plot) and nonmagnetic (upper curve, lower plot) subregions.
Table 1. The residual power (after subtraction of the corresponding background power values) in ridges \( f \), \( p_1 \), and \( p_2 \), is given in arbitrary units as a function of \( \ell \).

<table>
<thead>
<tr>
<th>( \ell )</th>
<th>( V_{\text{full}} )</th>
<th>( V_{\text{mag}} )</th>
<th>( V_{\text{nonmag}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>400 - 800</td>
<td>53.115</td>
<td>38.402</td>
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<td>27.392</td>
<td>16.250</td>
<td>53.012</td>
</tr>
<tr>
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<td>9.783</td>
<td>4.614</td>
<td>18.866</td>
</tr>
<tr>
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<td>3.874</td>
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<td>5.405</td>
</tr>
<tr>
<td>2000 - 2400</td>
<td>1.610</td>
<td>0.344</td>
<td>1.625</td>
</tr>
<tr>
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<td>91.450</td>
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<tr>
<td>2000 - 2400</td>
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</tbody>
</table>

Figure 7. Relative power in the \( p_1 \) ridge (solid line) and the background power value (dashed line) are plotted against \( \ell \). The signal-to-noise ratios are highlighted as in Figure 6.
Figure 8. Images of the rms velocity fluctuation over four 80 minute Doppler movies processed with different 3-D Fourier filters: a) raw velocity (no filter); b) Sub-f-mode filter, which passes only frequencies below the f-mode ridge; c) $p_1$-mode ridge isolation filter; d) f-mode ridge isolation filter.
EVIDENCE FOR A THIN PERTURBATIVE LAYER
NEAR THE BASE OF THE SOLAR CONVECTION ZONE

M. J. Thompson

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P.O. Box 3000, Boulder, CO 80307, U.S.A.

ABSTRACT

Low-degree solar p-mode frequencies hint at the presence of a thin perturbative layer at the base of the Sun's convection zone. Magnetic flux stored in an overshoot region could constitute such a perturbative layer.

Keywords: Helioseismology, Magnetic fields, Solar dynamo, Overshoot region

1. INTRODUCTION

A magnetic layer, of width of the order of one pressure scale height and located in an overshoot region at the base of the convection zone, is invoked in some solar dynamo theories as the seat of the Sun's magnetic activity cycle (Ref. 1). Vorontsov (Ref. 2) and Gough & Thompson (Refs. 3, 4) have investigated the effect of a thin layer of magnetic flux located at the base of the convection zone on the frequency of a solar p mode. The resulting frequency perturbation depends on the spatial phase of the mode's eigenfunction in the vicinity of the perturbing layer. Indeed, this will generally be true for any similarly tightly confined perturbation, magnetic or otherwise: for example, Bogdan and Cattaneo (Ref. 5) obtain a similar effect with suitable configurations of magnetic fibrils.

Let us specialize to low-degree, five-minute solar p modes, which are well-described by \( n/l \gg 1 \) asymptotics - where \( l \) and \( n \) are the degree and radial order of the mode. Specifically, in the propagating region and far from either turning point, the displacement eigenfunction is then essentially in the radial direction and has radial dependence

\[ \xi(r) \sim \frac{A}{(pc)^{1/2}r} \cos(\omega r + \phi) , \]

where \( r \) is the radial variable in a spherical coordinate set \((r, \theta, \phi)\), \( p(r) \) and \( c(r) \) are the local density and adiabatic sound speed respectively, \( \omega \) is the angular frequency of the mode and

\[ \pi(r) = \int_{r}^{R} \frac{dr}{c} \]

is the acoustical depth; \( R \) is the solar radius, \( A \) is a constant amplitude and \( \Phi \) is a constant (or more generally a function of \( \omega \)) that is determined by conditions near the Sun's surface. Thus the spatial phase in this region, and hence also the frequency perturbation due to a thin perturbative layer, are cyclic functions of \( \omega \).

Vorontsov (Ref. 2) analysed the \( 5 \leq l \leq 20 \) frequencies reported by Libbrecht & Zirin (Ref. 6) to seek any such cyclic frequency perturbation, but concluded that the accuracy of the data was insufficient to reveal the possible presence of a thin magnetic layer. Here I analyse some low-degree data from the compilation of Duvall et al. (Ref. 7) which do hint at the presence of a thin perturbative layer of some kind at the base of the convection zone. It may be significant that the data used by Vorontsov were collected in 1985, quite close to solar minimum, whereas those I use include the data of Jiménez et al. (Ref. 8) which are averages over the period 1977 - 1985.

2. EXAMPLE OF A MAGNETIC FIELD

For a specific numerical example, we consider an axisymmetric toroidal field

\[ B = B(r) \frac{dF_{k}}{d\theta} (\cos \theta) e_{\theta} , \]

with

\[ B(r) = B_{0} [1 - (r - r_{0})^{2}/d^{2}]^{2} \]

for \( |r - r_{0}| < d \), and \( B(r) \) zero elsewhere. Here \( F_{k} \) is a Legendre polynomial and \( e_{\theta} \) is a unit vector in the \( \theta \)-direction. Assuming that the observations are equally sensitive to all values of the azimuthal order \( m \) and that all modes of like \( n \) and \( l \) have the same physical amplitude, the observed average frequencies of each \((n, l)\) multiplet are insensitive to the non-spherically symmetric distortion of the equilibrium state (Ref. 4); and for a field of the form (3) with \( r_{0} = 0.7R \) and \( d = 0.05R \) Gough & Thompson found that the effect of the spherically symmetric distortion was also negligible: thus the dominant contribution to
the frequency perturbation $\delta \omega$ comes from the direct effect of the term in the oscillation equation arising from the change in the Lorentz force per unit mass due to the oscillation. For $n/l \gg 1$ and for a toroidal field this is approximately given by

$$\delta \omega \approx \frac{1}{\tau} \int \frac{v_A^2}{c^2} \sin^2(\omega t + \Phi) \, dt,$$

where the integral is over the magnetic region, $v_A = (B^2(r)/\mu_0)^{1/2}$ is the Alfvén speed (aside from an angle-dependent factor) and $\mu_0$ is the magnetic permeability. Assuming that $v_A$ and $c$ can be taken as approximately constant over the thin layer and that $\tau$ can be approximated linearly, Eq. 4 gives $\delta \omega/\omega$ proportional to

$$\frac{1}{\tau} \left[ \int \left( \frac{2v_A^2}{c^2} \right) \sin(2\omega t + \Phi) \, dt \right],$$

where if $v_A$ is evaluated at the peak field strength then $d$ is some effective half-width of the layer. The subscript "0" denotes values where the layer is located. Note that for a wave propagating perpendicularly to the field the sound speed is essentially $c^2 + v_A^2$, so $1/2(v_A^2/c_0^2)$ is the relative perturbation to the sound speed; thus the factor outside the square brackets in expression (5) is the magnitude of the perturbation to the acoustic width of the layer divided by the acoustical radius of the star. Note also that this factor is proportional to $B^2 d$ (hence at constant flux $\propto B d$, the perturbation increases with decreasing $d$, as is often pointed out with regard to fibril fields). The factor in curly brackets gives the degree of smoothing out of the oscillatory frequency perturbation due to averaging the eigenfunction's phase over the finite width of the layer.

To a first approximation $\Phi$ is constant, so $\cos^2(\omega t + \Phi)$ is a cyclic function of $\nu = \omega/2\pi$ with cycle length $1/2\pi$. Thus a determination of the cycle length gives the acoustical depth at which the perturbative layer is located. Since the internal sound speed of the Sun has been measured except in the core (Ref. 9), this in turn gives the physical depth. This calculation can be refined to take into account the variation with frequency of $\Phi$, which is related to the constant in the Duvall law, but I shall not consider this further here.

3. ANALYSIS OF FREQUENCIES

In order to look for a cyclic frequency perturbation, at each $l$ the asymptotic relation for the frequencies of high-order modes of a smoothly varying model was crudely taken into account by fitting (in a least-squares sense) a quadratic polynomial in $n$ to the mode frequencies, and

![Figure 1](image-url)
A THIN PERTURBATIVE LAYER NEAR CONVECTION ZONE BASE

the residuals

\[ R_{nl} = \omega_{nl} \] (least squares fit)

were computed in turn for \( l = 1, 2, \ldots, 5 \). The residuals for all the \( t \) values combined, considered as functions of frequency \( \omega_{nl} \), were then Fourier-transformed:

\[ \tilde{R}_{nl}(\tau) = \int R_{nl} e^{-2\pi \tau \omega} \, d\omega. \]

(The hat will be dropped henceforth). The results \( |R_{nl}|^2 \) as a function of \( \tau \) are shown in Figure 1 for three cases: (a) the frequencies reported in the compilation of Duvall et al. (Ref. 7); (b) the corresponding frequencies of Model 1 of Christensen-Dalsgaard (Ref. 10); (c) the corresponding frequencies of Model 1 but including the perturbation due to a field of the form (3) with \( k = 1 \), \( B_0 = 10^4 G \), \( r_0 = 0.7R \) and \( d = 0.05R \). The Fourier transform code used is based on the algorithm of Deeming (Ref. 11). The window function is shown in Figure 2 (see Ref. 11 for details).

The substantial amplitude of \( |R_{nl}| \) at small \( \tau \) is not surprising since the quadratic fit is only a very crude approximation to the frequency. It is very interesting that aside from power at small and large \( \tau \) the highest peak is at \( \tau \approx 4000 \), a depth corresponding to the base of the convection zone. There is also perhaps a small peak at a similar value of \( \tau \) in the analysis of the Model 1 frequencies, possibly reflecting the abrupt change in \( \partial \omega/\partial r \) at the base of the convection zone. As one expects the magnetic case shows \( |R_{nl}| \) greatly enhanced at \( \tau \approx 4000 \).

In an attempt to reduce the "power" at small \( \tau \), the analysis was repeated identically but doing a least squares fit to the difference between the frequencies under consideration and the corresponding frequencies for Model 1, rather than to the frequencies themselves:

\[ \tilde{R}_{nl} = (\omega_{nl} - \omega_{nl}^{\text{Model 1}}) \] (least squares fit).

The results are shown in Figure 3 for (a) the observational data used for Figure 1a; and (b) for the magnetic data used for Figure 1c. The peak at \( \tau \approx 4000 \) is even more pronounced.

Figure 3a should be interpreted cautiously, however, as illustrated by Figure 4. Here frequencies have been computed for a model with a partially mixed core, described by Christensen-Dalsgaard (Ref. 12), and the differences

between these frequencies and those of Model 1 have been analysed as for Figure 3. Neither model has a perturbative layer at the base of the convection zone. However, they require slightly different values of the helium abundances and mixing length parameter to each match the solar radius and luminosity; thus their convective regions extend to slightly different depths, so that the relative sound speed difference between the models changes from essentially zero where both models are convective to a value of about 1% at \( \tau \approx 4000 \), the change taking place over about 0.02 solar radii (see Ref. 13). This difference occurring over a short distance causes the peak in Figure 4. Nonetheless the peak amplitude of \( |R_{nl}| \) corresponding to the base of the convection zone is smaller by a factor of 8 than that in Figure 3a. It appears unlikely that a simple discrepancy in the depth of the convection zone could account for anything but a small fraction of the peak in Figure 3a. Nor would this explain the corresponding peak seen in the data not referred to any model (Figure 1a).

Figure 2. Window function for the Fourier transforms.

Figure 3. Squared modulus of the Fourier transform \( \tilde{R}_{nl} \) for (a) observational data; (b) Model 1 with a magnetic field at the base of the convection zone.
4. DISCUSSION

If we suppose that a magnetic field is responsible for the peak in Figure 1a, what strength field would be required? This is most easily calculated from Figures 1a and 1c; for the same magnetic geometry as used in this example it would require a peak field strength of $8 \times 10^6$ G. Different latitudinal dependence could change this a little, but it is unlikely to reduce this estimate by much. The field strength required could be reduced by having a uniform field and reducing the width to reduce the effect of averaging over the layer; but at an optimal width of $\sim 0.03 R$ the field strength required is still $\sim 5 \times 10^6$ G. It is of course possible that the magnetic field might have localized effects on the thermal structure - for example, by inhibiting convection - not considered here, and these might enable the required field strength to be reduced still further. Also, if modes with different values of $m$ do not contribute equally to the average multiplet frequency, the non-spherically symmetric distortion may substantially increase the oscillatory frequency perturbation for a given magnetic field.

5. ACKNOWLEDGEMENTS

I am indebted to Douglas Gough and Jorgen Christensen-Dalsgaard for many helpful discussions, and to Søren Frandsen for providing me with a program to perform the Fourier transforms. NCAR is sponsored by the National Science Foundation.

REFERENCES

The cause for the magnetically induced line shifts is not explained by Robillot et al. (Ref. 4) as probably being the result of the traversal of active regions across the solar disk. The transit of active regions is only one way that the magnetically induced doppler shift can produce a variable signal. For $\lambda 5250$ the correction to be added to the observed velocity is $-0.9 \pm 0.1 \left( \nu_{\lambda 5237} - \nu_{\lambda 5250} \right)$.

Keywords: Magnetic Downdrafts, Long Period Solar Oscillations, Stokes V Asymmetries.

1. INTRODUCTION

The measurement of solar velocities in order to detect and study long period oscillations or persistent circulation patterns is complicated by the fact that active regions are known (Ref. 1) to be associated with apparent downdrafts. Asymmetries in the Stokes V line profiles are related to the apparent doppler shifts in observing systems which measure the spectrum in a pair of pass-bands centered on the spectral line. These shifts can contaminate the observed solar velocity and interfere with the detection of low amplitude, long period oscillations. We describe here a method which will allow the calibration of the magnetically induced velocities and permit an improvement of the information from various planned solar oscillation experiments. Based on the differential sensitivity to the magnetic effects of various spectral lines we derive a correction function giving the doppler shift in each line as a function of the difference between shifts of the lines. For $\lambda 5250$ the correction is $-0.9 \pm 0.1 \left( \nu_{\lambda 5237} - \nu_{\lambda 5250} \right)$.

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Keywords: Magnetic Downdrafts, Long Period Solar Oscillations, Stokes V Asymmetries.

1. INTRODUCTION

The measurement of solar velocities in order to detect and study long period oscillations or persistent circulation patterns is complicated by the fact that active regions are known (Ref. 1) to be associated with apparent downdrafts. Asymmetries in the Stokes V line profiles are related to the apparent doppler shifts in observing systems which measure the spectrum in a pair of pass-bands centered on the spectral line. These shifts can contaminate the observed solar velocity and interfere with the detection of low amplitude, long period oscillations. We describe here a method which will allow the calibration of the magnetically induced velocities and permit an improvement of the information from various planned solar oscillation experiments. Based on the differential sensitivity to the magnetic effects of various spectral lines we derive a correction function giving the doppler shift in each line as a function of the difference between shifts of the lines. For $\lambda 5250$ the correction to be added to the observed velocity is $-0.9 \pm 0.1 \left( \nu_{\lambda 5237} - \nu_{\lambda 5250} \right)$.

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sun observations, the oscillation induced shifts are larger than the magnetically induced shifts. Robillot et al have attempted to minimize this problem by using a scanning period of 7.5 minutes so that the oscillatory component of the velocity has an opposite phase in consecutive scans. While this method reduces the amplitude of the oscillatory motions, the range of oscillation periods is from 3 to about 7 minutes so that the cancellation is not perfect.

The Mt. Wilson 150-foot tower system permits a better approach to the isolation of the magnetically induced velocities since as was described by Howard et al in Ref. 16 it permits the simultaneous measurement of two spectral lines. Because distinct spectral lines have differing sensitivity to the magnetic effects but the same sensitivity to the oscillation doppler velocity, the difference of the velocity between the two lines will cancel the oscillation signal and isolate the magnetic effects.

We can make the description quantitative by writing:

$$v_{\lambda}^{\text{total}} = v_{\lambda}^{\text{mag}} |B| + v_{\lambda}^{\text{osc}}$$

where $v_{\lambda}^{\text{total}}$ is the total apparent doppler velocity of a particular region on the solar surface, $c_{\lambda}^{\text{mag}}$ is the sensitivity coefficient for the line at wavelength $\lambda$ and $|B|$ is the absolute value of the magnetic field. We will not actually use the measured value of $|B|$ in this equation since the dependence should include an unmeasured filling factor. Finally equation 1 includes $v_{\lambda}^{\text{osc}}$ which is the oscillatory velocity and is assumed to be independent of $\lambda$. As long as the spectral lines are formed in nearby layers of the solar atmosphere, this is likely to be a good assumption.

If both exit slit assemblies are used to measure solar velocities, we can calculate the difference between these two velocities and largely cancel the oscillatory component of the signal. We will refer to the velocity difference measured simultaneously in two different spectral lines as the $\lambda$ differential velocity. Apart from the active region effects that we seek to study, this $\lambda$ differential velocity will depend on the limb shift function. In the final analysis we should determine limb shift functions for both lines and correct the signal for each line separately. We have recently discussed the limb shift question at length recently (Ref. 18), and found that the limb shift functions can be time dependent. The determination of the proper way to reduce both lines will require careful study. As a preliminary approach, we have used the same limb shift function for both lines and used the reduction procedure described by Howard et al (Ref. 17) to eliminate the larger velocity fields. Within the context of the uncertainty over the limb shift and the gradient of oscillatory amplitude between the depth of formation of the two lines, we should be able to isolate the magnetic effects on the apparent velocity by forming an image consisting of the difference between the apparent velocity in the two lines. This is the $\lambda$ differential velocity which we define to be $v^{\text{diff}}$ and calculate it from:

$$v_{\lambda}^{\text{diff}} = v_{5250}^{\text{total}} - v_{5250}^{\text{total}}.$$  

(2)

Using equation (1), we may write:

$$v_{\lambda}^{\text{diff}} = \left( \frac{c_{\lambda}^{\text{mag}}}{c_{5250}^{\text{mag}}} - \frac{c_{\lambda}^{\text{mag}}}{c_{5250}^{\text{mag}}} \right) |B|.$$  

(3)

In order to eliminate $|B|$ from equation (3) we will need to use equation (1) for either $v_{5250}^{\text{total}}$ or $v_{5237}^{\text{total}}$. These equations also contain the oscillatory component which will mask the term we seek to evaluate. As long as the general features of the magnetic region remain slowly varying, we can average out the oscillatory component. Denoting the temporal average of any quantity $A$ by $\langle A \rangle$ we have:

$$\langle v_{\lambda}^{\text{total}} \rangle = c_{\lambda}^{\text{mag}} |B| + \langle v_{\lambda}^{\text{osc}} \rangle$$

(4)

which is:

$$\langle v_{\lambda}^{\text{total}} \rangle = c_{\lambda}^{\text{mag}} |B|$$

(5)

since $\langle v_{\lambda}^{\text{osc}} \rangle$ is zero and $|B|$ is roughly constant. Finally we obtain the formula:

$$\langle v_{\lambda}^{\text{total}} \rangle = c_{\lambda}^{\text{mag}} |B| \equiv v_{\lambda}^{\text{mag}} = \frac{c_{\lambda}^{\text{mag}}}{c_{5237}^{\text{mag}}} \frac{c_{\lambda}^{\text{mag}}}{c_{5250}^{\text{mag}}} v_{\lambda}^{\text{diff}}.$$  

(6)

The quantity $v_{\lambda}^{\text{cor}} = -v_{\lambda}^{\text{mag}}$ is the desired correction that should be added to the measured velocity in order to eliminate the magnetically induced line shift. According to the right side of equation (6), we do not need information about the magnetic field pattern including unknown filling factors in order to obtain this correction function. The individual correction coefficients $c_{\lambda}^{\text{mag}}$ need to be known and this requires a temporal average in order to eliminate the oscillation velocity. Longer period modes will continue to contribute to the total average velocity and variations in the magnetically induced velocity during the averaging process can introduce errors in the determination of the correction coefficients. The only approach available to us is to select exceptionally strong magnetic regions where the magnetically induced velocity dominates the errors. The next section explains how we have implemented this approach.

3. THE OBSERVATIONS.

The presence of the two exit slit assemblies at the 150-foot tower permits some flexibility in the observational program. We normally configure the system so that one exit slit assembly is always centered on $\lambda 5250$. The second has been centered on a variety of spectral features. Prior to this year, it was normally centered on an iodine absorption line at $\lambda 5248.7$ introduced by a vapor cell in the converging beam near the exit slit. This absorption line provides an absolute calibration of the system. Since January of 1988, the observing program at the 150-foot tower has been structured so that on alternating days the second slit is centered instead on the Cr II line at $\lambda 5237.3$. This observing plan maintains the availability of the calibration data for use in eliminating systematic effects while providing new information about the second spectral line on a regular basis.

As a departure from the regular plan for this study, a period of 9 days at the beginning of August 1988 was selected to temporarily suspend the alternating day pattern and concentrate on $\lambda 5250.2$ and $\lambda 5237.3$ pair. During this period, 101 doppler- and magneto-grams were obtained. Once each day a scan is made with a pixel size of 12ʺ. The remainder of the scans were made with a pixel size of 20ʺ. For all data during this period we have calculated the doppler $\lambda$-difference-gram from equation (2).

We have not used the raw data to form the $\lambda$-difference-gram because of the irregular way in which it is acquired. The position of the scanning aperture for each data point is not controlled but is known precisely. Rather than interpolate to a fixed grid which would introduce smoothing, for the purposes of various displays, we square up the image by shifting each point to the nearest point in a rectangular grid. This squaring up process depends on the location of the solar limb and that in turn changes according to which spectral line is in use. We now eliminate this effect by using the limb solution for $\lambda 5250$ for locating the correct pixel
MAGNETICALLY INDUCED SPECTRAL LINE REDSHIFTS

Figure 1. These figures show dopplergrams for \( \lambda 5237 \), (Fig. a), and \( \lambda 5250 \), (Fig. b), on August 7, 1988. The rectangles labeled a to d denote regions over which velocity differences were averaged. For this and subsequent velocitygrams, the scale is \( \pm 75 \text{m/s} \).

Figure 2. These figures show the doppler differences between \( \lambda 5237 \) and \( \lambda 5250 \), (Fig. a), and the magnetic field, (Fig. b), on August 3, 1988. The rectangles labeled a to d denote regions over which velocity differences were averaged.

for both \( \lambda 5250 \) and \( \lambda 5237 \). Also as was mentioned in the preceding section, the difference limb effect function for the two lines introduces a systematic dependence of the difference velocity on the limb distance.

Figure 1 shows in a the squared up doppler image in \( \lambda 5237 \) and in b the squared up doppler image in \( \lambda 5250 \). The grey scale for this and subsequent doppler images has been kept fixed so that full black and white correspond to \( \pm 75 \text{ m/s} \). Note that the full range of velocity is larger than the range allowed for in this figure. Marked out on the figure are 4 rectangles that were chosen for examination because of the presence of activity. The day, August 7, is shown in figure 1 because these two active regions were well placed on that day. The \( \lambda \)-difference-gram corresponding to figure 1 is found in figure 4. Note that almost all the velocity of the oscillations is absent from the \( \lambda \)-difference-gram. The active regions are also easily visible. These characteristics are typical of all the \( \lambda \)-difference-grams. We are now routinely making hard copies of the \( \lambda \)-difference-grams for the days when we take data in both spectral lines. Copies of these are available on request.

4. SELECTED AREAS.

In order to determine the effect the solar activity has on the spectral line, we need to compare the velocity differences within an active region to those outside the active region. Because we wish to study these differences over an extended period of time, we must allow the defined areas to follow the solar rotation. We have used the standard Carrington rotation rate to define the movement of the active region. This rate is too large for the higher latitudes but since our time period is only 9 days, the migration of the defined area relative to the solar matter is not significant. We have not attempted to define the region boundary in terms of the Carrington coordinates but instead have taken the boundary to be rectangular on the plane of the sky. The rectangle is defined by its SE and NW corners. These corners are fixed relative to the Carrington coordinates. Finally, no interpolation is done to determine velocities at precise Carrington coordinates. Instead, we have simply taken the area bound-
These figures show the doppler differences between $\lambda 5237$ and $\lambda 5250$, (Fig. a), and the magnetic field, (Fig. b), on August 3, 1988. The rectangles labeled a to d denote regions over which velocity differences were averaged.

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The studied areas are labeled with the letters a to d and marked on figures 1 to 5. We were fortunate enough to have our observing period coincide with the appearance at disk center of a new active region. This region appeared on about August 4 and grew rapidly until it was carried off the W limb after our special observing sequence ended. Prior to August 4, the selected rectangle in Box a showed little if any indication of the coming activity (see however the discussion of the $\lambda$-difference-grams in section 6 below). Box b is about the same size as Box a and at the same Carrington longitude but displaced slightly in latitude. This box was chosen to serve as a reference standard for box a. A similar pair of boxes was chosen as Box c, the active region box, and Box d, the comparison box, to study the more mature active region which dominates the northern hemisphere at the beginning of the observing period. These boxes are larger than Boxes a and b. Since they began near the center of the solar disk on August 1, by the end of the observing period, they had been carried off the W limb and could no longer be studied.

We show in Figures 2 to 5 the $\lambda$-difference-grams and the magnetograms for the slow scans that are taken each day. We show data from every other day in order to give the reader a sampling of the relationship between these two properties of the solar surface.

The Boxes a to d, which are shown in the figures, form our primary study set and are adequate for the purposes of evaluating the strength and variation of the magnetically induced line shift. However, the need to average out the oscillations and remove the rotation and limb shift based on our existing models of the behavior of these quantities as a function of position on the solar surface. The total velocity in each line $v_{\text{total}}$ is composed of many parts: rotation, limb shift, magnetic effects and oscillations. To study the magnetic effects we have to average out the oscillations and remove the rotation and limb shift based on our existing models of the behavior of these quantities as a function of position on the solar surface. The rotation and limb shift have been treated in this study according to the ap-
proach described by Howard et al (Ref. 17). This approach is adequate as long as both spectral lines are properly described by the expansion given in that paper. The rotation is treated adequately this way but the limb shift is not. We would like to determine the magnetic effect velocity by subtracting the total velocity in Box b from the total velocity in Box a. Because the center-to-limb angle for these two boxes is not the same, the difference of these two velocities will contain limb shift effects as well as the magnetic effects. In order to compensate for these effects, we have defined four boxes identical to Boxes a and b which are in pairs on each side of a and b. We used these to determine the difference in the limb shift function between Boxes a and b. The Carrington coordinates of the corners of all boxes used are given in Table 1.

Table 1
Coordinates of the SE and NW Corners of Selected Areas

<table>
<thead>
<tr>
<th>Box</th>
<th>bSE</th>
<th>tSE</th>
<th>bNW</th>
<th>tNW</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>-16.4</td>
<td>257.0</td>
<td>-11.3</td>
<td>264.8</td>
</tr>
<tr>
<td>b</td>
<td>-8.0</td>
<td>257.0</td>
<td>-3.0</td>
<td>264.8</td>
</tr>
<tr>
<td>c</td>
<td>28.0</td>
<td>290.3</td>
<td>32.0</td>
<td>304.0</td>
</tr>
<tr>
<td>d</td>
<td>17.0</td>
<td>290.3</td>
<td>21.0</td>
<td>304.0</td>
</tr>
<tr>
<td>e</td>
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<td>270.0</td>
<td>-11.3</td>
<td>277.0</td>
</tr>
<tr>
<td>f</td>
<td>-8.0</td>
<td>270.0</td>
<td>-3.0</td>
<td>277.0</td>
</tr>
<tr>
<td>g</td>
<td>-16.4</td>
<td>243.0</td>
<td>-11.3</td>
<td>250.0</td>
</tr>
<tr>
<td>h</td>
<td>-8.0</td>
<td>243.0</td>
<td>-3.0</td>
<td>250.0</td>
</tr>
</tbody>
</table>

5. AVERAGE λ-DIFFERENTIAL VELOCITIES.

The velocities in the Boxes a to d were computed for the 101 dopplergrams available. The λ-differential velocity, the difference between the velocity as measured using λ5237 and the velocity as measured using λ5250, are plotted as a function of time in Figures 6 and 7. The difference in the limb shift function for the two lines produces the overall up and down character of the curves. Noteworthy features include the close tracking of the two curves for Boxes a and b prior to the onset of the activity. The most important result from the comparison of Boxes a and b is the excess variation in the average of Box a after the activity begins. If we assume that roughly 5 independent active regions cover 5% of the solar surface this excess variation will cause full disk variations in the velocity on the level of 0.05 x (10m/s)/√5 ≈ 0.02m/s. The cycle time associated with this variation is between 20 to 60 minutes.

The results for Boxes e and d confirm those found from Boxes a and b. These regions have about 3 times the area of Boxes a and b and show about the same relative level of velocity variation. Of particular interest is the increase in the variation in Box c as it approaches the limb whereas Box d remains quiet. This effect is enhanced due to the reduced number of pixels from foreshortening but both boxes have about the same foreshortening so the difference between c and d is not due to that effect. The conclusion is that the total projected area of active regions is what will determine the increased level of low frequency noise introduced by the magnetic downdrafts and that the location of the active regions is of secondary importance.

6. AREA-DIFFERENTIAL VELOCITIES.

In order to study the effects of the activity on the average velocity as is required in equation (5), we need some standard of zero or unperturbed velocity. This reference velocity should come from a long time average function and should be a slowly varying or constant function of position on the solar disk. The function is mostly the limb shift function but could include such effects as the torsional oscillations and the meridional circulation. We have studied these effects for λ5250 (Ref. 18) and at the level of 10 m/s, we do not have a reliable understanding of all aspects of our system. We are in a worse position for λ5237. Consequently, we rely on adjacent quiet regions to provide the reference velocity. Coordinates for the box corners were given in Table 1.

Box a is the most interesting because it is a developing active region and because it is compact and there do not seem to be quiet areas inside the box. The primary reference box is b and the λ-differences were discussed above. The magnetic effect on Box a at λ5237 and at λ5250 should be the velocity difference

$$v_{\lambda}^{\text{mag}} = \delta v_{\lambda} = (v_{\lambda}^{a}) - (v_{\lambda}^{b}).$$

(7)

As discussed in §4, this definition of δv_λ is not reliable and we defined two additional pairs of boxes (Boxes e to h) which bracket Boxes a and b. We use the velocity difference $$\frac{1}{2} (v_{\lambda}^{a} - v_{\lambda}^{b} + v_{\lambda}^{e} - v_{\lambda}^{f})$$ as the quiet sun value for $v_{\lambda}^{a} - v_{\lambda}^{b}$. The
resulting values for $\delta v_A$ are given in Table 2. Table 3 gives the results for Boxes c and d. The numbers in Tables 2 and 3 for each day are the averages of all observations taken on that day. The number of observations in each day 1...9 were 2, 3, 7, 2, 16, 19, 17, 15 and 15.

The area differences in Tables 2 and 3 contain two noteworthy features:

First- Near the limb the active region velocities become negative rather than the typical positive. This may be due to the Evershed flow which consists of outflow from sunspots. At the limb the projection effects and the higher altitude cause this velocity pattern to become dominant. The trend in Table 2 for both velocities to decrease together as Box a approaches the limb suggests that this is a flow of matter rather than a line transfer process.

Second- Prior to the appearance of the activity, Box a has a definite negative velocity as if there is an upwelling flow caused by the rising magnetized matter.

In order to carry out our correction for the magnetic effect velocity using equation (6), we need the ratio: $c_{A5250}/c_{A5237}$. We can calculate that ratio from the data in Table 2. Unfortunately we cannot use all the data in this table for three reasons: Box a did not become active until after August 4, some of the daily averages contain too few points and the boxes are too close to the solar limb towards the end of the observing sequence. The magnetic effect in Box a is too weak until after August 5 and the effect of the limb on the boxes becomes dominant for August 7, 8 and 9. This leaves August 6 and 7 as the only days for which we can calculate the ratio. For Boxes c and d the situation is slightly better because the regions are fully developed throughout the run. We throw out August 1 and 4 which have only 2 observations and August 6 to 9 because Box c is too close to the limb. This leaves August 2, 3 and 5 as the only days for which the data is usable. For these 5 averages we obtain:

$$\frac{c_{A5250}}{c_{A5237}} = 2.1 \pm 0.1$$

which yields:

$$\delta v_{A5250} = 0.0 \pm 0.1 (v_{A5237} - v_{A5250})$$.  

This equation can be used to correct the observed velocities in A5250 for the active region effects. Unfortunately, before we can carry out that program, we must first improve our treatment of the limb shift so that the difference in this function is not incorrectly interpreted as an active region effect.
MAGNETICALLY INDUCED SPECTRAL LINE REDSHIFTS

Figure 8. This figure shows the dopplergrams for the Fe line at \( \lambda 5883 \) (Fig. a) and the NaD line at \( \lambda 5890 \), (Fig. b). Figure c shows the \( \lambda \) difference-gram for \( \lambda 5237 \) and \( \lambda 5250 \) in the same format as the previous figures. Also shown is the the magnetic field, (Fig. d). All these figures were measured on June 3, 1988 although the scan in the NaD wavelength band followed the regular \( \lambda 5250 \) and \( \lambda 5237 \) scans.

7. THE NaD LINE DOPPLERGRAM

The discussion throughout this paper has centered on the well studied lines near \( \lambda 5250 \). The synoptic program at the 150-foot tower of Mt. Wilson uses these lines almost exclusively. The interest in the metallic vapor resonance cells using either Sodium or Potassium makes a study of the appropriate lines desirable. The present system at the 150-foot tower does not permit regular observation of the NaD lines or any other line without disrupting the synoptic program and the unique character of the Mt. Wilson data set makes disruption of the observations very undesirable. However, in order to determine the feasibility of measuring full disk dopplergrams with the system, we have devoted part of one day on June 3, 1988 to a special setup. The dopplergram for the NaD line is shown in Figure 8b. Because of the dual line nature of the 150-foot tower system, we were able to obtain a dopplergram for another line in the spectral vicinity of the NaD line and we chose \( \lambda 5883 \), another Fe line. The dopplergram for this line is given in Figure 8a. We made the special setup the second observation of the day so we also have a comparable set of data to what we used in the other parts of this paper. We show in Figure 8c and 8d the \( \lambda \)-difference-gram for \( \lambda 5237 \) and \( \lambda 5250 \) along with the magnetogram from \( \lambda 5250 \).

We note two things from Figure 8 - First, the active region effects are weaker in the NaD lines than in the other lines we have studied. Second, we confirm the observation by Robillot et al (Réf. 4) that the sign of the magnetic effect for NaD is opposite from the non-resonance metallic lines such as the usual Fe lines. We cannot carry out an analysis of the NaD lines such as we did in the earlier parts of this paper for \( \lambda 5250 \) and \( \lambda 5237 \) because we have far less data and because the limb shift function for the NaD lines is too dissimilar to that for the Fe lines.

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8. REFERENCES


DYNAMO THEORY PREDICTION OF SOLAR ACTIVITY

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ABSTRACT

A "dynamo theory" technique appears to successfully predict decadal time scale solar activity variations. The technique was developed a decade ago, following some puzzling correlations involved with geomagnetic "precursors" of solar activity. Based upon this, a dynamo theory method was developed to predict solar activity. The method was used successfully in solar cycle 21 by Schatten, Scherrer, Svalgaard, and Wilcox, after testing with 8 prior solar cycles. Schatten and Sofia used the technique to predict "an exceptionally large" cycle, peaking early (in 1990) with a sunspot value near 170, likely the second largest on record. We now see sunspot numbers increasing at an alarming rate and realize 1) that a large cycle is developing and 2) that the cycle may even surpass the largest cycle-19. We use a "Sporer Butterfly" method to show that the cycle can now be expected to peak in the latter half of 1989, consistent with an amplitude comparable to our value predicted near the last solar minimum.

Key words: Solar Activity

1. INTRODUCTION

Within the past 20 years, the clear possibility of predicting solar activity on decadal timescales has developed. This work began on two fronts: a) the theoretical front, involving our deeper understanding of the origins of solar activity, together with b) the empirical front, wherein the relationships of solar dynamo theory to the actual level of solar activity in the real world was considered.

The theoretical understanding of dynamo theory has progressed in this century from the early models of Waldmeier (Refs. 1-2) to the later models of Babcock, Leighton, Parker, Howard, Yoshimura, Snodgrass, and Wilson (Refs. 3-10). Although there are many puzzles in solar dynamo theory which still exist (Ref. 6), it is clear that the earlier dynamo views of Babcock (Ref. 3), that were supported by Hale's laws of sunspot polarities, remain an underpinning that still serves the Solar Physics community in understanding the solar dynamo. That is, in common with planetary dynamos, the solar dynamo uses differential rotation to magnify a weak poloidal magnetic field within a conducting material to generate a stronger toroidal field. Subsequently, cyclonic convection and magnetic buoyancy appear to play significant roles in regenerating the poloidal field. This latter aspect is not well understood, and thus the present solar activity prediction methods are only based upon the former Babcock dynamo view which does have the observational support of Hale's sunspot polarity law.

Empirical methods for predicting solar activity that actually seems to have a physical basis to them began with the studies of Ohl (Ref. 11), Brown and Williams (Ref. 12) (hereafter BW), and Ohl and Ohl (Ref. 13). These authors found the startling, surprising result that terrestrial geomagnetic activity could serve as a precursor to future solar activity. Although the physical basis for the predictions was not elaborated then, the authors found that the size of the sunspot maximum was anticipated from geomagnetic activity, in particular, the BW work used the previous minimum's number of "abnormal quiet days" (AQD's) in the geomagnetic field. How this method worked was a mystery.

A physical basis for the empirical work was required, or the results would have to be regarded, like so many other results, as a statistical fluke. Thus what was needed was essentially to fuse the above (a) and (b) categories--namely find a good theoretical underpinning on which to hang the empirical studies. In addition, the correctness of the ideas needed to be tested in the laboratory of our solar system. If the method really worked, it should also have predictive capability, as opposed to being only a curious correlation.

To accomplish these tasks, Schatten, Scherrer, Svalgaard, and Wilcox (Ref. 14) (hereafter SSW) suggested that the BW and Ohl effect might be understood in the following manner. The Sun's poloidal field at solar minimum might actually be providing the real signal associated with the size of the next sunspot cycle, as suggested by the conventional views of the solar dynamo. Figure 1 shows the physical basis for these ideas. When the Sun's polar field is weak at a particular solar minimum (left), dynamo processes magnify this weak field into a weak toroidal field and relatively few sunspots erupt in the following sunspot cycle. Conversely, when the polar field is strong (right), a large cycle emerges. This follows our conventional understanding of the Babcock dynamo.
SOLAR DYNAMO

Supported by Hale's sunspot polarity laws, the dynamo method appeared to work reasonably well in providing the solar cycle amplitude (the cycle peaked within the error bars of the SSSW prediction), but the prediction provided no information about solar cycle timing. Heretofore, we have relied upon average characteristics, and the official timing of the sunspot minimum, etc. to set our timing, and have awarded a generous ± 1 year uncertainty to the predicted solar maximum timing. With solar cycle durations varying between 8 and 17 years, the timing uncertainty is a serious drawback. The present paper suggests using Sporer's (Ref. 15) law (the butterfly diagram) to ascertain the progression of the cycle, and thereby obtain a better peak timing. This can be done only after the cycle starts, however, it does appear to offer hope in obtaining a good value for the timing. We outline how the method works, and then utilize it, thereby providing a test of the method when a future comparison is made. We refer to the technique we outline, as the "Sporer Butterfly" method (hereafter SB method) after Sporer who was primarily responsible for developing Carrington's (Ref. 16) ideas of how sunspots drift equatorward in latitude as the cycle progresses.

Figure 1. Shown are two representations of solar fields, drawn in accordance with the Babcock dynamo view. The fields on the left represent the state of the solar dynamo when the polar fields are weak. A consequence of this is that the toroidal field, wrapped up by differential rotation, also remains weak, so that a small cycle results. Conversely, when the polar fields are strong (right), a strong cycle results.

The fields on the left represent the state of the solar dynamo when the polar fields are weak. A consequence of this is that the toroidal field, wrapped up by differential rotation, also remains weak, so that a small cycle results. Conversely, when the polar fields are strong (right), a strong cycle results.

To test these ideas SSSW decided to compare the solar field directly with the strength of the solar activity cycle, rather than the geomagnetic AQD index. Unfortunately the Sun's polar fields, oriented virtually at the right angles relative to Earth, are the most difficult of the Sun's polar field to compare with future levels of solar activity. The proxies were: a) polar faculae counts, b) the shape of the Sun's coronal field, c) the flattening of the heliospheric current sheet, and d) the Ludendorf isophote index observed at solar eclipses. The first three seemed to provide fairly high correlations for eight solar cycles, suggesting that the technique might not be a statistical fluke, but might have physical underpinnings.
2. SPORER BUTTERFLY CYCLE TIMING METHOD

Figure 2 shows the general equatorward drift of sunspots as a cycle progresses (Ref. 17). Note that the SB drift depends strongly upon the smoothed maximum sunspot number, shown on the individual curves. This complicates matters, since to predict the solar cycle timing, the amplitude of the cycle must be known. We express the curves in Figure 1 mathematically as:

\[ \Phi = \left[ 8 - 6 \left( \frac{x}{50} \right) + 4 \left( \frac{x}{50} \right)^2 \right] + \\
\left[ 7 - 2 \left( \frac{x}{50} \right) - \left( \frac{x}{50} \right)^2 \right] \cdot \frac{R_m}{100} \]  

(1)

where \( \Phi \) is average spotgroup latitude; \( R_m \), the maximum smoothed sunspot number; and \( x \), time (in solar rotation units, 27.275 days) from solar maximum, positive in the future. Now, given a \( \Phi \) and an \( R_m \), values of \( x \) can be obtained by finding the solutions to the above quadratic equation. Let us examine this for the present cycle.

Examining sunspot latitudes for July, 1988 (NOAA Solar Geophysical Data Bulletins), a mean latitude for active regions of 22.5° is found. Similar values exist for the prior 3 months. Setting \( \Phi = 22.5, R_m = 170 \) (Ref. 18), we obtain the two roots \( x/50 = 4.35 \) or -0.26 (solar rotations); or \( x = 217.5 \) or -13 (solar rotations). The smaller second root relates to the present cycle and indicates that July, 1988 is 13 solar rotations before the sunspot peak, placing the peak near July 1989. We must consider uncertainties in the relation. An uncertain value of \( R_m \) of 25 corresponds to a timing uncertainty of 8.3 solar rotations or 227 days. We can also examine the current sunspot values and observe that they have been running early (or equivalently above) the previous prediction, thus a consistency is acquired only if we say the sunspot peak will arrive earlier than previously predicted (1990), namely in late 1989. Consequently, we revise the timing of our prediction to that shown in Figure 3, for a peak in the second half of 1989. We choose October, 1989 nominally rather than July, 1989 since the slightly later value allows the amplitude to match our previous prediction. Thus our revised prediction for the amplitude and timing of solar cycle 22 are for a value of \( R_m = 170 \pm 25 \) peaking near October, 1989 ± 7 months.

3. SUMMARY

We revise our estimated peak for solar cycle 22 to occur near October, 1989 ± 7 months, rather than in 1990. We examined the latitude of current sunspots and found a fairly low average latitude of 22.5°. The Sporer Butterfly method of examining the equatorward drift of active regions suggested that the present sunspot cycle would peak in late 1989.
Figure 3. Predicted smoothed sunspot number and radio flux (F10.7) from 1988 to 1995 (solid curve). Estimated yearly average 2σ uncertainties are shown as dashed curves. The curves differ from our earlier (Ref. 18) prediction through the timing change described here.

4. REFERENCES
Session 5

Space projects and low frequency oscillations

Chairman: R. Bonnet
LONG PERIOD SOLAR OSCILLATIONS

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ABSTRACT

Several years of high quality optical resonant scattering data from full disk solar observations are analysed to demonstrate the essential stability of the apparatus necessary to extract signals of periods hours, days or possibly years. The variation of the gravitational red shift is examined, the 13 day signal occasioned by solar surface inhomogeneities is demonstrated and the 160 mm signal in these data is shown to be an artifact. Finally the g modes extracted from these data are shown to be stable in period over the interval 1981 to 1987. Interpreted in terms of the Tassoul asymptotic relation a value of $T_0 = 41.2$ min is found which would tend to favor a solar model with mixing.

1. INTRODUCTION

The technique of optical resonant scattering (Ref. 1) has been used to study the solar surface for many years. In essence the line of sight velocity is determined by a comparison of the doppler shifted wavelength of light absorbed in the solar photosphere with that of identical atoms in the laboratory, in this instance potassium. As the properties of identical atoms at the two sites are being compared, the technique is intrinsically highly stable. It is this stability that renders the optical resonant scattering method well suited to a search for long period oscillations.

A second factor, not to be overlooked, is the observation of the 'whole' solar disk. Thus the multitude of random surface motions are averaged out as are the high order oscillations, leaving a response to mainly $Q$ values $<4$, thereby greatly reducing the number of contributions to the measured signal in the laboratory.

Although the whole solar disk is observed, when using a high resolution instrument the correlation between wavelength and position on a rotating body must be taken into account (Ref. 2). The effect of this becomes apparent when comparing the phase of the 13 day oscillations with that produced by solar surface inhomogeneities.

2. BASIC DATA

The data available for analysis, obtained at Izana Tenerife using a K 769.9 nm resonant scattering cell are summarised in table 1.

<table>
<thead>
<tr>
<th>Year</th>
<th>From</th>
<th>To</th>
<th>Span(days)</th>
<th>Data(days)</th>
</tr>
</thead>
<tbody>
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<td>18/7</td>
<td>17/8</td>
<td>31</td>
<td>29</td>
</tr>
<tr>
<td>1981</td>
<td>29/5</td>
<td>25/8</td>
<td>88</td>
<td>80</td>
</tr>
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<td>17/4</td>
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<td>122</td>
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<td>210</td>
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<td>1986</td>
<td>27/3</td>
<td>31/12</td>
<td>365</td>
<td>240</td>
</tr>
<tr>
<td>1987</td>
<td>1/1</td>
<td>31/12</td>
<td>365</td>
<td>215</td>
</tr>
</tbody>
</table>

Table 1 Dates of observations used in this analysis

Each daily data set of ~ 900 velocity determinations over a 10 h observing period is corrected for instrumental response (Ref. 3) and analysed to yield a set of residuals. A typical daily result is illustrated in figure 1, where I represents the solar intensity, $V_m$ the measured line of sight velocity and $V_R$ are the residuals clearly showing the presence of oscillatory signals, the 5 minute $p$ modes of high order.

![Figure 1. Typical data recorded on 16 May 1985.](image)

The measured velocity $V_m$ may be written as

$$V_m = V_{orb} + V_{spin} + V_{gr} + V(I)$$

(1)
where \( V_{\text{orb}} \) represents the earth's orbital motion, \( V_{\text{spin}} \) the line of sight velocity associated with the earth's rotation, \( V_{\text{grs}} \) the gravitational red shift and \( V(t) \) any time dependant motions giving rise to the observed solar oscillations. In order to extract the residuals \( V_R \), the basic data are fitted by

\[
\gamma(t) = A + B \sin \frac{2\pi}{T} (t - t_0) + C \sin 2\frac{2\pi}{T} (t - t_0)
\]

where \( T \) is the earth's rotation period (24 h), \( t_0 \) is the time of local noon and the second order term is included to reduce distortions. Normally this term is \(< 1\% \) of \( B \), the known line of sight rotation velocity at the site and epoch considered.

### 3. GRAVITATIONAL RED SHIFT

The constant \( A \) may, neglecting such effects as differing isotopic abundances in the photosphere and vapour cell or possible saturation effects arising from the greatly differing temperatures in the source and the detector, be equated to the gravitational red shift \( V_{\text{grs}} \) plus the mean orbital velocity \( V_{\text{orb}} \). A small linear correction, not shown in equation (2), is applied to allow for the daily change in \( V_{\text{orb}} \).

A plot of \( V_{\text{orb}} \) against \( A \) for the 210 determinations made in 1985 is shown in figure 2.

**Figure 2** The orbital velocity verses \( A \), found from eqn. 2, for 1985 data.

It should be emphasised that these results are from basic observed data and no daily normalizations have been applied, thereby clearly illustrating the basic stability of the resonant scattering method.

Fitting a straight line to these data yields an absolute calibration of the velocity sensitivity of the apparatus plus an estimate of the gravitational red shift. Similar analyses of the other data listed in table 1 yield the results shown in figure 3; a value based on 37 days of data obtained in 1977 is included. The error bars indicated in figure 3 are based on statistical considerations only, the larger values for 1977 and 1980 reflecting the short length of these data spans.

**Figure 3** The observed gravitational red shift variation.

Analysis of parts of these data have been previously held to indicate a possible gravitational red shift variation linked to the solar cycle (Ref. 4,5), however no systematic errors are indicated in figure 3 and any possible correlation should, at this stage, be viewed with extreme caution. There are know instrumental effects; for instance the deviation of the 5 points corresponding to \( A = 0.05 \) in figure 2 from the fitted line were occasioned by a blemish on the coelostat mirror. Another instrumental factor, the temperature and or positioning of the interference filter, produces line pulling effects which have been observed to cause shifts comparable with those illustrated in figure 3. These matters are currently under investigation and until resolved no conclusion should be reached about the observed variations in the value of the gravitational red shift, nor its apparent consistantly low value compared with the predictions of general relatively (632m/s) although this may at least be partially explained by the measured K 769.6 line asymmetry (Ref 3).

### 4. THE 13 DAY OSCILLATION

The yearly plots of the daily mean line of sight velocity, corrected for the earth's orbital and rotational motion and the measured gravitational red shift, clearly exhibit a residual signal which decreases in amplitude from 1981 to 1985, as illustrated in figures 4 and 5.

**Figure 4** Variations in the residual daily mean line of sight velocity for 1981 (x observations, - calculations based on plages).
points on the solar hemisphere displaced symmetrically about the central meridian. The affect is most pronounced during April and least during late August/September when at certain times of the day exact symmetry is obtained due to a cancellation of the terms in equation 1.

The velocity deduced from equation 3 assumes that the solar disk is observed symmetrically about the central meridian. However due to the shift caused by the \( V_{orb} \) and \( V_{grs} \) terms the actual centroid of the line of observation is shifted towards the red limb (west). Hence a phase shift between the actual observed signal and that calculated from equation 3, dependant on the value of \( (V_{orb} + V_{grs}) \), must be allowed for before a direct comparison is made. Taking this factor into account and merely assuming that the area of each plage makes no contribution to the detected signal, results in the solid curve shown in figure 4. Clearly good agreement is obtained in both amplitude and phase.

Similar calculations for 1982 and 1983 show good agreement with the observations although the amplitude of the predicted signal, based on Big Bear plage estimates, is approximately twice that observed in 1982.

After 1983 no plage data are available, only sunspot data. A similar calculation based on purely sunspot data although showing the same general behaviour grossly under-estimates the amplitude of the observed signal.

Hence it may be concluded that the observed 13 day oscillation results from the motion of surface inhomogeneities across the visible solar disk. A further check is the amplitude variation of the 13 day signal over the solar cycle, although care should be taken since a solar disk uniformly covered by blemishes will produce no net signal. Each year's data is subjected to an iterative sine wave fitting procedure and the amplitude of the 13 day signal found. The results are shown in figure 6.

Figure 6. Amplitude of the 13 day oscillation over the years 1980–1987.

The variation in amplitude of the 13 day signal is clearly suggestive of a correlation with the solar cycle.

4.2 Phase coherence

In an earlier paper Isaak et al (Ref. 11) indicated that the observed 13 day signal had apparently maintained a constant phase over the period 1977 to 1982 if a period of 13.035 days was assumed. Considering the present data, 1980–87, the positions of the maxima in each year's data string is found with a probable ± 1 day error. After 1983 no plage data are available, only sunspot data. A similar calculation based on purely sunspot data although showing the same general behaviour grossly under-estimates the amplitude of the observed signal.
fractional part. Using a period of 13.1 days the results over a span of almost 3000 days, are shown in figure 7.

There is a clear grouping of the results which may be emphasised by projecting the data onto the 'interval' axis as in figure 8 where a summation over a 2 day interval, corresponding to the probable error on the determination of the position of the peaks, has been made. This clearly does not represent a random distribution of phases but illustrates that within ± 1 day or ∼ 30° the phase of the oscillation has been maintained over a period of ∼ 3000 days.

A similar analysis assuming a period of 13.5 days, half the solar rotation period, produces the histogram shown in figure 9. The relatively featureless distribution shows that a period of 13.5 days does not maintain a constant phase over the interval considered.

As it has been demonstrated that the direct cause of the observed 13 day signal is undoubtedly solar surface inhomogeneities and that there is a very long phase coherence it would seem to imply that the positional occurrence of the sunspots and plages may themselves be determined by some internal phenomena of 13.1 day periodicity.

5. THE 160 MIN OSCILLATION

5.1 Introduction

The start of experimental helioseismology can probably be traced to the simultaneous publication in Nature of two papers announcing the discovery of a 160 min solar oscillation of amplitude ∼ 2 m/s (Ref. 12, 13). It was realised that the authenticity of this signal was in some doubt, as 160 minutes is exactly 1/9 of a day. Later analyses of data obtained over the period 1974 to 1976 (Ref. 4) indicated that the experimental evidence of a stable 160 minute solar oscillation was far from conclusive. Although the signal was still detected, the amplitude appeared variable and generally lower than first reported.

The reaffirmation of the 160 min signal was obtained from an analysis of the combined data of Stanford and Crimea. Using a superposed epoch technique, it was found that the phase of the signal was not constant but indicated a steady drift thus implying that the correct period was not 160.00 min but 160.01 min (Ref. 15). This slight difference, if substantiated, would establish the authenticity of the solar origin of the signal. In addition it was found that the phases of the 160 min signal, detected in the Birmingham, Crimean and Stanford data agreed. These two facts were held to show that the observed signal was not merely the 9th harmonic of a day introduced into the data by some artifact but truly constituted a solar signal.

Clearly to differentiate between 160.00 min (104.166 μHz) and 160.01 min (104.160 μHz) requires long stretches of data. If these data were to indicate that the signal had a frequency of 104.166 μHz (9th harmonic of a day), then a simple model should reproduce this signal, if it were not still to be ascribed to a solar origin and also predict the correct phase.

The daily measured line of sight velocity \( V_m \) may be expressed as in equation 1. The magnitudes and dominant periods of these terms are listed in table 2.
The term $V_{\text{spin}}$ may be expressed as

$$V_{\text{spin}} = V_s \cos \alpha \sin \frac{2\pi}{24} (t - t_0)$$  \hspace{1cm} (3)$$

where $t_0$ is the time of local noon expressed in hours, $V_s$ is the earth's peripheral velocity at the point of observation and $\alpha$ is the apparent solar declination. It should be noted that during any one year $t_0$ varies by \pm 31 min thus causing phase modulation of the $V_{\text{spin}}$ term.

Observations of the line of sight velocity of the sun during a particular day consist of three components, a) the 9th harmonic of the distorted sine wave of amplitude $V_s$ (\pm 400 m/s), offset by $V_{\text{orb}} + V_{\text{grs}}$, with a far smaller amplitude signal $V(t)$ superimposed as illustrated in figure 1. Any distortion of the line of sight velocity signal will generate harmonics of the 24 hour period, the 9th possibly being the reported 160 min oscillation. One possible factor that has been discussed many times is differential atmospheric extinction (Ref. 16). As the sun is rotating with an equatorial velocity of \sim 2 km/s, any partial obscuration of one limb over the other would cause a velocity shift. This process occurs periodically with one limb being emphasized in the morning and the other in the evening. This results in a slightly skew symmetric sine wave giving rise to harmonics of the day. Non-linearities in instrumental response could produce similar effects.

Precise frequency determination of any components in the basic signal require long data stretches when low frequencies (long periods) are considered. This introduces the further complication that as data from a single station are only obtained during daylight hours, the signal is amplitude modulated with a 24 hour period (window function) which causes sidebands of any signal present to the generated at spacings of 11.57 \mu Hz.

Thus any signal detected at 160 min could consist of three components, a) the 9th harmonic of the distorted diurnal signal b) sidebands of the other harmonics caused by the window function and c) a possibly truly solar signal.

### 5.2 Data analysis

The present analysis uses data obtained at Izana Tenerife between 1980 and 1985 as listed in table 1. Each data point is corrected for instrumental response (Ref 3) where available 10 hours of data, placed symmetrically about local noon are fitted by equation 2. In order to maintain a low diurnal distortion only these 10 hours of data are considered for each day, thus minimizing the effects of atmospheric extinction. The fitted function is then subtracted from the basic data string yielding a set of residuals ($V_R$) as illustrated in figure 1. These residuals now form the data for further analysis.

Considering the 1985 data, the full data string of 210 days of actual data, spanning an interval of 279 days, is subjected to an iterative sine wave fitting procedure using a frequency increment of 0.05 \mu Hz. The resulting spectrum up to a frequency of 0.15 mHz is plotted in figure 10.

### Table 2 Amplitudes and periods of dominant velocity terms.

<table>
<thead>
<tr>
<th>Term</th>
<th>Amplitude</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{\text{orb}}$</td>
<td>500 m/s</td>
<td>year</td>
</tr>
<tr>
<td>$V_{\text{spin}}$</td>
<td>400 m/s</td>
<td>day</td>
</tr>
<tr>
<td>$V_{\text{grs}}$</td>
<td>600 m/s</td>
<td>-</td>
</tr>
<tr>
<td>$V(t)$</td>
<td>1 m/s</td>
<td>minutes</td>
</tr>
</tbody>
</table>

The several distinct peaks correspond to the harmonics of a day, including the 9th at \sim 0.104 mHz. Thus if the signal at 0.104 mHz were attributed to a solar origin a new problem would arise - why does the 9th harmonic of a day not manifest itself when the others do?

Using a frequency increment of 0.01 \mu Hz the results of similar analyses of the other years considered are compared in figure 11 over a limited frequency interval around that of the 9th harmonic.

The loss of resolution with shorter data sets is clearly evident as is the variability of the signal amplitude. Note that the amplitudes are \sim 30 m/s whereas the amplitudes of the diurnal sine wave is \sim 400 m/s, hence any slight distortion would yield harmonics of these amplitudes. Further if this distortion were caused by differential atmospheric extinction, then variation in the quality of the season would be reflected in the amplitudes of the harmonics. The 1981 observing season had particularly clear skies and data were gathered during the summer months, whereas those obtained in 1984 and 1985 extended into the winter months when on average the atmospheric conditions deteriorate and the mean solar declination is decreased. Although the relative amplitude of the daily harmonics depends not only on the absolute quality of the day but also on the precise form of the distortion introduced, a clear correlation in the amplitude variation of the 7th and 9th harmonics over the years considered was found. Subsets of the 1985 data showed a marked increase in the amplitude of the 160 min signal as the winter season approaches.

**Figure 10** Frequency spectrum for 1985 data

The loss of resolution with shorter data sets is clearly evident as is the variability of the signal amplitude. Note that the amplitudes are \sim 30 m/s whereas the amplitudes of the diurnal sine wave is \sim 400 m/s, hence any slight distortion would yield harmonics of these amplitudes. Further if this distortion were caused by differential atmospheric extinction, then variation in the quality of the season would be reflected in the amplitudes of the harmonics. The 1981 observing season had particularly clear skies and data were gathered during the summer months, whereas those obtained in 1984 and 1985 extended into the winter months when on average the atmospheric conditions deteriorate and the mean solar declination is decreased. Although the relative amplitude of the daily harmonics depends not only on the absolute quality of the day but also on the precise form of the distortion introduced, a clear correlation in the amplitude variation of the 7th and 9th harmonics over the years considered was found. Subsets of the 1985 data showed a marked increase in the amplitude of the 160 min signal as the winter season approaches.

**Figure 11** Comparison of frequency spectra for data from 1980 to 1985 in the region of the 9th harmonic (0.001 mHz/div).
The phase stability of the signal is investigated by considering successive 30 day intervals over the 279 day data span and subjecting these individual data stretches to an iterative sine wave fitting procedure. The fitted phase for each 30 day interval, compared to the same epoch, is plotted against time for several fitted frequencies in figure 12. Were the fitted frequency to exactly match that present in the data set, the phase would remain constant, if too low the phase will increase and if too high decrease. Clearly from figure 12 the frequency component present lies between 104.16 and 104.17 µHz.

Figure 12 Phase variation of the fitted frequencies (µHz) over the 1985 observing period – actual data. The precise frequency is determined from the variation of the slopes of the fitted lines to the phases for each frequency value. The deviations from the fitted lines illustrate the phase modulation occasioned by the variation in the time of local noon. The 1980 data could not be analysed in this way as only a 31 day data span is available, however the other years yielded the results listed in table 3.

<table>
<thead>
<tr>
<th>Year</th>
<th>Frequency µHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1981</td>
<td>104.156</td>
</tr>
<tr>
<td>1982</td>
<td>104.165</td>
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<tr>
<td>1983</td>
<td>104.206</td>
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<tr>
<td>1984</td>
<td>104.167</td>
</tr>
<tr>
<td>1985</td>
<td>104.165</td>
</tr>
</tbody>
</table>

Table 3 Mean frequencies determine for each year.

The high frequency value obtained in 1983 should be treated with caution as some instrumental problems arose during this observing season.

The results strongly suggest that the frequency present is the 9th harmonic of a day, 104.166 µHz.

The phase of each fitted frequency may also be determined for each full years data set. Taking as an origin 0.0 GMT July 20 1980 (J.D. 4440.5), the phase of the 104.166 µHz (160.00 min) fitted frequency is shown in figure 13. The lines indicate the calculated phase behaviour for frequencies of 104.166 µHz and 104.160 µHz (160.01 min).

The conclusion reached from all the experimental data considered is that the signal present in the data is of period 160.00 min and not 160.01 min.

Figure 13 The phase of the 160.00 min signal over the period 1980-85: (O) actual data, (X) simulated data. The lines indicate the expected phase behaviour of the 160.00 and 160.01 min signals.

5.3 Simple Model Simulation

Having established that the signal has a period identical with the 9th harmonic of a day, this still does not exclude the possibility of a solar origin. A complete analysis of the lower frequency region of the 1985 data clearly showed (figure 10) that not only is there a peak corresponding to the 9th harmonic but that significant peaks occur at most harmonics considered. In order to test whether these and the 9th harmonic could be artifacts a simple model is constructed.

It was indicated that the daily sine wave resulting from the earth’s rotation could, if distorted, yield harmonics of the day which might account for the observed signal at 160 min. Hence a data set is generated where the \( V_{spin} \) term in equation (1) is given by

\[
V_{spin} = V_s (1 - 0.0003 (t-t')^2) \sin \frac{2\pi}{24} (t-t')
\]

thus introducing a small skew asymmetry. The constant in equation 4 is chosen to produce harmonics of amplitude similar to those in the actual data set.

The spectrum for the 1985 simulated data is calculated and the phase variation for various fitted frequencies is illustrated in figure 14. These data are virtually identical to those obtained with the real data (figure 12). In particular the phase of the simulated data determined from the full data string is found to be 0.8 rad for the 104.166 µHz frequency which is to be compared with that of 0.8 rad found from an identical analysis of the real data. A further phase comparison of the simulated and real data is given in figure 13.
The phase of the simulated data is governed solely by the time of local moon ($t_0$) and hence the agreement in phase with the actual data is the strongest yet indication that the observed 160 min signal is an artifact. As was pointed out at the IAU colloquium 66 (Ref 17) the observations carried out by the Birmingham, Crimean and Stanford groups were at sites separated in longitude by multiples of 160 min and thus the phase agreement between these sets of observations is not inconsistent if all three were to be interpreted in the present way. However an actual analysis of the Crimean and Stanford basic data would have to be undertaken to confirm this.

5.4 Conclusions
Long stretches of data extending over the years 1980 to 1985 and totalling 699 actual days of data have been analysed to show that the period of the 160 min oscillation is indeed 160.00 min and that its phase remains constant over the interval of 6 years considered.

Using a simple sine wave of amplitude determined from the daily, spin velocity of the earth, slightly distorted by a skew symmetric term, and period of 24 hours, the diurnal velocity change together with the known orbital and gravitational red shift velocity components, the measured daily signals of line of sight velocity are generated at precisely the same points at which actual data are recorded, thus maintaining the same window function. Analysis of these simulated data mimic the spectrum, frequency and phase, of the actual recorded data. Hence it may be concluded that, for the Izana observations, the observed 160.00 min 'solar' oscillation is an artifact produced by a simple distortion of the diurnal signal due primarily to differential atmospheric extinction interacting with a rotating sun.

An independent analysis of the same data presented at the IAU Symposium 123 (Ref 18) although misinterpreted at the time, confirms the phase constancy of the 160.00 min signal. In this analysis a signal of frequency $104.160 \mu$Hz (160.01 min) was fitted to the data and the phase was found to increase at the rate of ~32 minutes per year, thus confirming that the actual signal is at frequency 104.166 (160.00 min). The reported phase jump of $\pi$ radians was occasioned by the use of a computer routine which only determined $-\pi < \phi < \pi$.

A recent reanalysis of the Stanford data (Ref 19) using more complex functions to remove daily drifts, has led to the conclusion that the previously found phase change of the 160 min oscillation can no longer be supported.

Solar intensity variation data obtained by ACRIM on board the SMM space craft, although confirming the five minute solar oscillations, do not show any evidence for a 160 min signal (Ref 20).

A recent private communication Kotov (Ref 21) indicates that further analysis of the Stanford–Crimean data is in agreement with that presented here.

Thus at the present time analyses of available data suggest that there is no experimental evidence to support the existence of a solar signal at 160.01 min over the period 1980–1985.

6. G MODES

6.1 Introduction
As opposed to p modes which apart from the lowest $\ell$ values only probe the outer most layers of the sun, the g modes penetrate to the very core and thus could convey vital information concerning the solar interior. This fact has been realised for some time and yet the progress on the identification or even detection of g mode candidates has lagged well behind that associated with the shorter period p modes. The main reasons for this are four fold. The general noise level has been shown to rise in the low frequency region (Ref 22), the amplitudes are expected to be less than those of the p modes and in order to resolve the closely spaced frequency structure of the higher order modes long data spans are required. The anticipated periods for low $\ell$ high n g modes are 200–400 min, thus making it imperative that a high degree of instrumental stability is achieved. It has been demonstrated in section 1, that an optical resonance scattering spectrometer has good long term stability and that by operating the spectrometer at a good site, long data stretches with a useful filling factor are attainable.

Early attempts (Ref. 23, 24) at the identification of signals in the low frequency range were based on statistical criteria (Ref. 25) to separate the signals from noise plus an exploitation of the constant spacing in period of a given $\ell$ mode as predicted by the Tassoul relation (Ref. 26).

An alternative approach is the generation of a g mode spectrum over a range of $\ell$ and n values compatible with the instrumental response taking into account rotational splitting and the effects of the observing window function. Then by undertaking a cross correlation of the generated spectra and that measured, for the various parameters assumed, a three dimensional plot may be generated indicating the most likely set of parameters (Ref. 26, 27).

The two parameters extracted by either method are the basic period $T_0$ related to the Brunt–Vaisala frequency and the rotational frequency $\Omega_r$.

The basic Tassoul asymptotic relation for $n > \ell$ is

$$T_{\ell,n} = T_0 (n+2/2-1/2)\left(\frac{2(\ell+1)}{2\ell+1}\right)^{-1}$$

and the rotational frequency splitting is given by (Ref 28).

$$\delta \nu = m \Omega_r \left[1 - \frac{1}{\ell(\ell+1)}\right]$$
where \( T_{n,q} \) is the period, \( n \) the order and \( \ell \) the degree of the mode. The internal rotation frequency is characterized by \( v_r \) and \( m \) is the azimuthal number. Each mode splits into \( \ell + 1 \) components (\( m+\ell \) even) equally spaced in frequency, whereas the members of a given degree (\( \ell \)) are equally spaced in period.

The values of \( T_{0,0} \) predicted by the WIMP, standard, and models with mixing are 20 to 31, 33 to 36 and 40 to 57 minutes respectively. Hence, even a first estimate of this fundamental parameter from experimental data could prove useful in establishing which model to use to refine the calculations.

Unfortunately, the current experimental determinations do not agree on a unique value. The 1981 Izana data (Ref. 24) gave a value consistent with the model with mixing whereas the Stanford data (Ref. 23) favoured the standard model. More recent analysis (Ref. 26) does not always produce a unique answer and either model could be possible whilst an analysis of irradiance data (Ref. 27) comes out in favour of the WIMP model.

### 6.2 Data analysis

As seen from table 1 long data stretches are now available over several years; these data have been shown to be of good long term stability (figure 2) and should therefore represent the best currently available experimental data for the detection of solar g modes. An unbiased identification of series of approximately equally spaced peaks in the time spectrum of the data would serve as an indicator for a search for other members of the series. Some amplitude lower limit has to be determined to discriminate against noise for if a low enough limit were selected the peak density in the spectrum would be so great that almost any predefined series could be fitted. Secondly a region of the frequency spectrum should be selected where the density of anticipated peaks is not so high as to make the intrinsic resolution defined by the length of the data string less than the mode spacing. Also, if the g modes are to be identified from the selected signals above noise level on the basis of their equal period spacing, as predicted by the Tassoul asymptotic relation, the condition that \( n > \ell \) must be maintained. Hence the frequency region 50-75 \( \mu \text{Hz} \) is chosen.

The residuals, found as previously discrribed are subjected to an iterative sine wave fitting with a frequency increment of 0.01 \( \mu \text{Hz} \), thus over sampling the data. The resulting spectrum for 1985 is shown in figure 15.

![Figure 15 Frequency spectrum for 1985 data.](image)

This spectrum is clearly dominated by the 5th and 6th daily harmonics, but in addition several peaks probably exist above the general background noise.

Full disk velocity observations are sensitive mainly to \( \ell_1 - \ell_2 \) g modes, then assuming a value of \( T_{0,0} \) between 25 and 50 this would produce 25-50 peaks in the frequency range considered, neglecting rotational splitting.

The presence of discrete peaks above the general background may be visually emphasized by plotting the square of the frequency spectrum as in figure 16.

![Figure 16 The square of the frequency spectrum emphasising the existence of discrete peaks.](image)

Table 4. Number of peaks in the frequency range 50-75 \( \mu \text{Hz} \) of amplitude > 7.0 cm/s excluding daily harmonics.

<table>
<thead>
<tr>
<th>Year</th>
<th>peaks &gt; 7.0 cm/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1984</td>
<td>18</td>
</tr>
<tr>
<td>1985</td>
<td>44</td>
</tr>
<tr>
<td>1986</td>
<td>31</td>
</tr>
<tr>
<td>1987</td>
<td>30</td>
</tr>
</tbody>
</table>

This fairly high value for the amplitude cutoff is chosen for two reasons. Firstly, each genuine signal present will have associated sidebands due to the window function one of which at least will be in the frequency interval chosen. The window function for the 1985 data string is shown in figure 17.

![Figure 17 Window function for the 1985 data string.](image)

It is seen that the sidebands are considerably lower than the main peak and thus by choosing a relatively high amplitude cutoff a number of the sidebands are excluded. Secondly, based on results obtained on p modes, the amplitudes of the rotationally split components are generally lower than the main unsplit peak.
Figure 17 Window function for 1985 data.

The number of peaks in the chosen frequency interval is of the same order as that predicted by the Tassoul relation for $\ell_g - \ell_u$ with possible $T_0$ values 25 - 50 min.

Assuming the identified peaks are due to solar $g$ modes, there should be four series of equally spaced values in the time series corresponding to the $\ell_g - \ell_u$ modes. Not all members of a particular series may be present due to the amplitude selection criterion and/or interference effects between various modes. Also, the four anticipated series are interlaced and the number in each is low thereby rendering correlation techniques ineffective.

An alternative method, that of exact fractions, applied successfully in demonstrating the phase coherence of the 13 day signal, is well suited to the present problem. The method is illustrated by generating three series of interlaced points with constant period spacings of 11, 16 and 19 min. The period of each generated point is then divided by a test spacing and the resulting fractional part is plotted against the period as illustrated for a spacing $\delta T = 11.0$ min in figure 18

Figure 18 Fractional part versus period for generated data.

The series generated with a constant spacing of 11.0 min is clearly displayed as a horizontal line of points in figure 18 at a fractional interval value of 2 min corresponding to an arbitrary phase shift between the generated series and the starting point of the test spacing. Projecting these points onto the "interval" axis produces the histogram shown in figure 19.

Figure 19 Distribution of fractional parts for test data.

6.3 Experimental Results

This signature is used to search for equally spaced peaks in the time series of actual data. The peaks in each year's time series of amplitude > 7.0 cm/s are scanned with the value of $\delta T$ incremented in steps of 0.1 min from 5 to 30 min to determine the maxima in the histograms. The results obtained for 1985 data are shown in figures 20 - 22.

Figure 20 Distribution of fractional parts indicating a period spacing of 16.9 min. Amplitude > 7.0 cm/s.

Figure 21 Distribution of fractional parts indicating a period spacing of 11.4 min. Amplitude > 7.0 cm/s.
These plots suggest that three separate series are found in the data with constant spacings of 9.9, 11.4 and 16.9 min. To check that these distributions have statistical significance the 1985 data are reselected for peaks with a lower cutoff of 6.0 cm/s. The newly selected data is similarly analysed and the peak corresponding to \( \Delta T = 9.9 \) min is again found as shown in figure 23.

The peak at interval 0.1 min is clearly maintained whilst the numerous extra peaks selected at this lower cutoff level have contributed randomly to the background.

The approximate peak spacing given by the histograms is used in conjunction with equation 5 to define \( \xi \) and \( n \) values for each series. The periods corresponding to the peaks in each histogram are then plotted against the \( n \) value as in figure 24 for the \( l_2 = \ell_0 \) modes.
LONG-PERIOD SOLAR OSCILLATIONS

Figure 27 Period versus order of g modes identified in 1987 data.

The value of $T_0$ determined from the linear fits to each mode are given in table 5 for each year. The previously found values for 1981 data are also included (Ref. 24).

6.4 Conclusion

The search for g modes in four independent years of data each spanning about a full year have yielded consistent results. The present analysis has been limited to the 50 - 75 $\mu$Hz region and only signals of amplitude $> 7.0$ cm/s have been considered. With these choices many of the problems associated with g mode identification have been avoided as evidenced by the consistent results obtained. A method based on exact fractions was used to determine the basic period spacings for the modes visible to a full disk instrument and then by identifying the particular peaks contributing to each spacing found, $\ell$ and $n$ values assigned in accordance with the Tassoul asymptotic formula. The high $n$ values used assured small departures from linearity and estimates of the fundamental period $T_0$ were made for each $\ell$ value for the four years considered. Excellent consistency was obtained leading to a value of $T_0 = 41.2$ min. This would indicate that a solar model with mixing is to be preferred and seems to definitely rule out the WIMP model.

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>$\ell_1$</td>
<td>41.18 ± 0.14</td>
<td>41.31 ± 0.73</td>
<td>41.54 ± 0.38</td>
<td>41.21 ± 0.82</td>
<td>40.73 ± 2.32</td>
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<tr>
<td>$\ell_2$</td>
<td>41.32 ± 0.12</td>
<td>41.25 ± 0.96</td>
<td>41.20 ± 0.32</td>
<td>41.32 ± 0.32</td>
<td>40.64 ± 0.53</td>
</tr>
<tr>
<td>$\ell_3$</td>
<td>41.17 ± 0.14</td>
<td>41.15 ± 0.38</td>
<td>41.71 ± 0.42</td>
<td>41.81 ± 0.14</td>
<td>41.60 ± 0.17</td>
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</tbody>
</table>

Table 5 Determinations of $T_0$ over the years 1981 to 1985.

These determinations of $T_0$ show excellent consistency not only between $\ell$ values for any one year but also for all the years considered. The errors quoted on the 1981 determinations reflect the fact that a frequency range of 25 - 130 $\mu$Hz was used as opposed to 50 - 75 $\mu$Hz in the present analysis.

Taking the 1981 data as a reference the mean period shift for each mode is found over the years considered and listed in table 6.

<table>
<thead>
<tr>
<th>Year</th>
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<th>1986</th>
<th>1987</th>
</tr>
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<tbody>
<tr>
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<td>-0.7</td>
<td>-0.7</td>
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<td>-2.7</td>
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<td>$\Delta T$ min</td>
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<td>3.1</td>
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<td>$\Delta T$ min</td>
<td>0.6</td>
<td>-0.7</td>
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</tbody>
</table>

Table 6 Mean Shift in period for each mode compared to 1981.

The actual mean periods for each mode were compared with those previously determined in an analysis of 1981 data and no systematic shift in periods with the solar cycle was found.

7. ACKNOWLEDGEMENTS

The assistance of all members of the Birmingham and I.A.C. solar oscillation groups both past and present is gratefully acknowledged, especially the support of the technical staff. This work was partially funded by the SERC (U.K.) and the CAICYT (Spain).
<table>
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<tr>
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<td>240.6</td>
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<table>
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<th>frequency μHz</th>
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<td>330.1</td>
<td>50.49</td>
<td></td>
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</tr>
</tbody>
</table>

Table 7 Mean periods/frequencies for g modes in the frequency range 50–75 μHz

8. REFERENCES


CAN SOLAR G-MODES BE IDENTIFIED FROM GROUND-BASED VELOCITY MEASUREMENTS?


Instituto de Astrofísica de Canarias. E-38200. La Laguna. Tenerife.

ABSTRACT

After ten years of helioseismology research, the question of whether or not solar g modes can be detected and identified from ground observations, is still unclear. The limitation imposed by the earth's atmosphere, the background solar noise spectrum at low frequencies and a poor theoretical knowledge of these modes, are some of the reasons.

Using the best uninterrupted full disk velocity measurements obtained over the period 1984-1987, signals with amplitudes less than 5 cm/s are found in the g modes spectral region. Cross-correlation and other techniques used to detect stable signals (g modes) show negative results which allow to put an upper limit to their amplitudes and/or lifetimes.

1. INTRODUCTION

Till now, a great deal of the information on the inner solar structure has been obtained using the whole observed range of acoustic modes (p modes). It is now possible to test the solar models over more than 60% of the outer solar radius. The lack of information, mainly on the inner 40%, is accurately covered by the knowledge of the solar g modes since their frequencies are very sensitive to conditions in the core of the Sun, to the chemical composition, amount of mixing and to rotation rate.

From an observational point of view, their detection is very difficult; they exist in a spectral region (ν < 0.2 mHz) where noise (instrumental, atmospheric-terrestrial or solar) is very high; their densities, per unit frequency interval, also increase at very low frequencies and, finally, their amplitudes are very small in the photosphere due to their evanescent nature in the convection zone.

First claims of detection of solar g modes in solar velocity measurements (Ref. 1, 2 and 3) show serious discrepancies, not only in the amplitudes of individual peaks, but also in the determination of the parameter P0. Further, ACRIM solar intensity measurements, have been analyzed to look for g modes. The parameter P0 and the rotational splitting ω were fitted to the data showing ambiguous results as a consequence of the difficulty in the analysis (Ref. 4 and 5). When similar techniques were used in the solar velocity data over 4 individual years of observation (Ref. 6) the results showed different values of P0 along the years (41.1 to 35 minutes) and similar ones for the splitting (1 to 1.3 μHz).

Although the existence of solar signal above the noise level at low frequency ranges seems well established, their identification is not easy because of the density of peaks and also because the asymptotic theory is not fully understood and developed.

2. OBSERVATIONS AND DATA REDUCTION

The observations were carried out without interruption at the Observatorio de Izaña (Tenerife) over the period 1984 to 1987, using a resonant scattering spectrophotometer (Ref. 7), which measures the solar line of sight velocity by looking at the shift of the Fraunhoffer line at KI 769.9 nm, relative to the one produced in the laboratory by means of a stable vapour of potassium. The period of observations, 16 April 1984 to 31 December 1987, was selected amongst others, because it is the longest stretch without large interruptions.

Since we are interested in very long period solar oscillations, it is important to select for the present analysis those days with very good
atmospheric conditions (Figure 1-b), otherwise transparency fluctuations would introduce a lot of noise in the g modes region. Table 1 presents a summary of the observations: the year of observation, possible, available and selected days, hours of data, standard deviation of the time series and duty cycle (possible days times 24, relative to the used hours of data). Figure 2 shows the corresponding power spectrum of this observational window function.

The recorded daily solar signal is a distorted sine wave (Figure 1-a) that must be properly detrended. Thus, taking into account the overall response of the instrument and the potassium line shape (Ref. 8 and 9), the basic signal is fitted and calibrated, yielding the residuals (Figure 1-c) with data points spaced 40 seconds; they contain all the information on solar velocity fields with characteristic times shorter than 1 day.

Since the expected lifetime of the solar g modes is of the order of 10^6 years, a unique time series is formed with the same time reference, 1 April 1984 at 0 GMT. In addition, this series was resampled at intervals of 14 minutes, so that computing time decreases without losing any information on the g-modes region. The final time series covers a length of 32850 hours which provides a intrinsic resolution of 0.4 nHz.

<table>
<thead>
<tr>
<th>YEAR</th>
<th>Possi. Avail. Selec.</th>
<th>Hours</th>
<th>s.d.</th>
<th>Duty</th>
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<td>1227</td>
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<td>204</td>
<td>1612</td>
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<td>1987</td>
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<td>1475</td>
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<tr>
<td>TOTAL:</td>
<td>1355</td>
<td>1021</td>
<td>685</td>
<td>5594</td>
</tr>
</tbody>
</table>

Table 1. Summary of the data used in this analysis

Figure 2. Power spectrum of the observational window function. The first three side lobes at lower and higher frequencies are shown, each one in a frequency interval of 500 nHz.

3. ANALYSIS

3.1. Power Spectrum

To compute the power spectrum, an Iterative Sine Wave fitting procedure is used, producing an "amplitude square" spectrum. The calculation extends in the frequency range of 25 to 325 μHz, well above Nyquist frequency (~ 560 μHz) at steps of 4 nHz, which represents an oversampling factor of 2. The most important features in this power spectrum are the daily harmonics. Also, each signal present will have associated sidebands displaced at intervals of ±11.576 μHz due to the day-night data gaps. Therefore, at the daily harmonic frequencies there is a high degree of interference which produces spurious features that can mask solar information. Then the first stage of power spectrum analysis consists in cleaning in such effects but taking care not to introduce other artificial signals. Then a proper algorithm (Ref. 8) was developed to ensure the accomplishment of these two requirements. The result is shown in Figure 3, where signals, other than the fourth daily harmonic, have a maximum amplitude of less than 5 cm/s.

3.2. Noise level estimation

To calculate the probability distribution of any signal in the presence of noise, a technique proposed by Groth (Ref. 10) was used. As described elsewhere (Ref. 2), the analysis of the cumulative power distribution of the spectrum in a log diagram will show a departure of linearity beyond a certain power if any signal is present. From this graph, it is possible to calculate the noise variance under the assumption that it is normally distributed. This technique has been used on a clean spectrum over successive intervals of 5 μHz from 25 to 325 μHz. For each one, the noise variance (σ^2) is obtained, showing (Ref. 11), a very high correlation with the estimated solar continuum. This is so far the lowest noise level achieved at such low frequency region. Values vary from 0.11 cm/s at 50 μHz; 0.7 cm/s at 100 μHz and 0.4 cm/s at 300 μHz. It must be emphasized that these noise levels are lower by a factor of 20 to 40 than the ones reported in Ref. 2 and 3.

With these values of the noise level, the existence of peaks well above noise is clear; the problem is to know what they are.

4. GRAVITY MODES ANALYSIS

The asymptotic relation for the periods of low frequency gravity modes has been given by Tassoul (Ref. 12) up to the second order. For a solar model with radiative core and convective envelope, it can be written as:

\[ P_n = \frac{L}{(l+1)^2} \left( n + \frac{1}{2} - \frac{n}{4} \right)^2 \frac{L V_r^3 + \frac{1}{2} L^3}{P_n^3} \]

for \( n \gg 1 \)

where \( P_n \) is the frequency of the mode with order \( n \) and degree \( l \); \( L \) and \( l \) are related with the Brunt-Vaisala frequency (N). \( \Phi \) is a phase factor which varies with \( l \) from 0.18 to 0.9 for \( 10 < n < 27 \) and \( V_r \), \( V_0 \) are rather complicated constants very sensitive to the behaviour of \( N \) at the center of the sun, with values \( V_r \approx 0.4 \) and \( V_0 \approx 6 \) for the same range of \( n \) values (Ref. 13).

The main parameter \( P \) for standard, mixed and WIMP models is very different; 33 to 36, 40 to 57 and 29 to 31 minutes, respectively. Therefore the observation of g modes and the determination of
SOLAR G-MODES & GROUND-BASED VELOCITY MEASUREMENTS

P will place serious constraints on solar models.

From equation (1), it is clearly seen that the main spectral feature of g modes is that they are equally spaced in period for a given l value. This is no longer true if the rotational splitting is introduced; as the length of data is larger than the solar rotation this effect cannot be disregarded at all. Therefore, the period of any mode will split according to the expression

\[ P_{n,l,m} = P_{n,l} + \left[ m \nu \left( \frac{1}{2} \right) \right] \]

(2)

(Ref. 14), \( \nu \) being a measure of the rotation rate at the deep layers where the g modes propagate and \( m \) is the azimuthal number. Then each mode will split in \( 1+1 \) components (those with \( 1+m \) being even) equally spaced in frequency and not in period. Consequently, for degrees \( l<3 \), only the \( l=2 \) modes will have a \( m=0 \) component and therefore only this group of modes will be spaced in period.

To further complicate the situation, the existence of side-bands for any present signal, which are, again, peaks in the power spectrum spaced in frequency, should be remembered. Summarizing, previous analyses, based on the asymptotic behaviour of g modes and the property of period equidistances, must be regarded with caution and the need for other kinds of techniques becomes evident.

In the present work two different techniques have been tested; one based on individual and selected peaks and the other based on cross-correlation analysis.

4.1. Individual peak analysis.

Knowing the value of \( \nu \) for each 5 \( \mu \)Hz interval, peaks with amplitude higher than 3\( \sigma \) were selected in the spectral range 25 to 125 \( \mu \)Hz. The idea is to try to identify the selected peaks with the expected ones from the asymptotic relation (2) for a given value of the involved parameters. This is, of course, very difficult because of the number of unknown parameters \( (P, \nu, l, m, n, \Theta (1), \nu_1, \nu_2) \) and also because some of the parameters, usually neglected \( (\Theta (1), \nu_1, \nu_2) \), have uncertainties which produce frequency differences 10 to 20 times bigger than our intrinsic resolution (8 \( \mu \)Hz). In spite of these difficulties, we computed synthetic spectra of g modes for different combinations of the parameters and, in each case, try to fit the selected peaks. The obtained results are very poor and only a maximum of 20 to 30\% of the peaks can be identified for a given selection of the parameters.

This result can be more understood if we compare the density of expected peaks (splittered g modes and side bands) with the density of the experimental ones. This is shown in Figure 4 for a given set of parameters. The two distributions are different, suggesting that the selected peaks are not all g modes or that the noise is not normally distributed. As a conclusion, the single peak identification based on the asymptotic relation (1) and (2) is not, at present, a feasible way to detect g modes.
4.2. Cross-Correlation Analysis

Opposite to the previous technique, cross-correlation analysis makes use of all the information contained in the power spectrum. In the present analysis we will split the time series corresponding to 4 years of data in different subsets, then compute power spectra and cross-correlate them in all possible ways in order to obtain their degree of similarity. To check the consistency of the method as well as to eliminate peaks due to noise, different amplitude threshold levels can be used in the power spectra.

The first try was split the time series in two: one which contains data of even days and the other with the odd days. In such a way, we have the advantage that window functions are identical (differences being less than 4%) in both sets of data. Then the two corresponding power spectra were computed and noise level obtained, as explained above. Figure 5-a shows the autocorrelation of one of them, where the first side lobe is now at 5.8 $\mu$Hz (2 days). Then cross-correlation functions were calculated, via FFT, for different frequency intervals, of different length, from 25 to 125 $\mu$Hz and each time with different values of the threshold from 0 to 30. In all cases negative results were obtained, being all peaks at the same level ($\pm 10^2$). Figure 5-b shows one of the cross-correlation functions.

As a second step, we split the time series in two: first one contains the first half of the data and the other the second half, each one with equal

| Length of days (680). In this case window functions are slightly different, but if $g$ modes are present in both sets they will show up in the cross-correlation function. Again we repeat the same procedure with completely negative results. Figures 5-c and 5-d show, respectively, the autocorrelation and one particular case of the cross-correlation. Finally, we split the series in two different types of subsets. First one with subsets of 180 days each and the other with 90 days each. Shorter subsets were not created because 90 days provide just enough resolution to avoid interference between different $g$ modes with $\ell \geq 3$. As in previous cases, power spectra and cross-correlation functions were calculated using different thresholds; in any case cross-correlation values higher than 0.15 and well above noise were found.

As a conclusion, using the cross-correlation analysis indications of the existence of $g$ modes with amplitudes higher than 4 cm/s were not found.

To give more support to the previous result a simple test was performed. It consists in calculate, for a given frequency interval (in this case 50 to 100 $\mu$Hz) the mean power and the noise power for different data lengths. What it is expected if $g$ modes are present with amplitudes higher than 4 cm/s, is a decrease of the noise power as data length increases, and also a decrease of the mean power, both with much lower slope, until a point is reached where, increasing data length, mean power should be almost constant due to the presence of constant signal. Results of

![Figure 5. In a) and b) the auto and cross-correlation of power spectra corresponding to the even and odd time series. In c) and d) the same but for the first and second half series.](image)

Figure 5. In a) and b) the auto and cross-correlation of power spectra corresponding to the even and odd time series. In c) and d) the same but for the first and second half series.
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the test are shown in Figure 6 for data lengths from 80 days to 4 years, and it is clearly seen that both graphs decrease at the same rate with means that if any g modes are present, they are still at a level or below the achieved noise level. This test is confirmed if we look at, for different data lengths, the amplitude of the biggest peak; those are 15 cm/s for 80 days length, decreasing to 8 cm/s for 1 year and 4 cm/s for 4 years.

All these evidences, strongly suggest that the noise is the main feature present in the data and g modes, if they should have amplitudes smaller than 4 cm/s.

5. CONCLUSIONS

Analysis of high quality data on solar velocity measurements over a period of 4 consecutive years, has been carried out to search for g modes. The achieved noise levels (< 1 cm/s) at low frequencies (25 < \( f < 325 \) Hz), place an upper limit for the amplitudes of g modes of less than 4 cm/s.

Amongst the various techniques used in this analysis, the cross-correlation one give the most conclusive results showing the non existence of coherent solar signal, g modes, above 4 cm/s level. Therefore, more continuous observations are required to still improve the signal to noise ratio. An alternative explanation of the results can be made in terms of a short lifetime (< 90 days) and frequency changes which should also be frequency dependent, for solar g modes. In this latter case, even from the future space observations (GOLF and VIRGO experiments on board of SOHO) will be difficult their detection and identification.

6. ACKNOWLEDGEMENTS

We would like to thank all the members of the Seismology groups of the I.A.C. and Birmingham University, for support during all the years of observation. This work was partially funded by the CAICYT (under the grant PR84-0905) and SERC (U.K.).

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IPHIR: The Helioseismology Experiment on the PHOBOS Mission


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ABSTRACT

IPHIR (InterPlanetary Helioseismology by IRradiance measurements) is a solar irradiance experiment on the USSR planetary mission PHOBOS to Mars and its satellite Phobos. The experiment is a cooperative effort of PMOD/WRC, LPSP, SSD/ESA, KrAO and CRIP. The sensor is a three channel sunphotometer (SPM) which measures the solar spectral irradiance at 335, 500 and 665 nm with a precision of better than 1 ppm. A two axis solar sensor (TASS) is added to monitor the moderate solar pointing of the spacecraft. A microprocessor based data processing unit controls the sensor operation, acquires the data and performs the data compression for the transmission at a mean rate of 1 bit/s. The two spacecrafts have been launched on July 7th and 12th, 1988. The experiment on PHOBOS I gathered data during 45 days before the S/C was lost, the one on PHOBOS II is still operating. The data recovery is excellent with virtually 100 percent coverage. Although the signal is disturbed by the pointing of the spacecraft the results of a preliminary analysis in the range of the 5-minutes oscillations demonstrate the improvement achievable due to the fact that the time series is truly continuous and the instrumental and sampling noise is very low.

Keywords: Helioseismology, Solar Spectral Irradiance, PHOBOS Mission

1. INTRODUCTION

The opportunity for a helioseismology experiment on a planetary mission came up in 1985 when the final payload of the USSR mission to the martian satellite Phobos was discussed. The cruise phase of such a mission offers the unique chance to gather long uninterrupted time series - an important prerequisite for helioseismology. In the case of a Mars mission the time series can reach a length of more than 180 days. Due to the fact that the relative velocity between the spacecraft and the Sun during the transfer to Mars is varying and reaches quite high values oscillation measurements by Doppler observations of the global Sun would have been very difficult. Thus, irradiance observations were chosen and, more specifically, observations with a three channel sunphotometer (SPM) which was already developed for other investigations.

The PHOBOS Mission has a wide international participation and 27 experiments have been contributed by institutes from 13 different countries and international agencies. The helioseismology experiment, called IPHIR (InterPlanetary Helioseismology by IRradiance measurements), is a cooperative effort of the Physikalisch-Meteorologisches Observatorium, Davos, Switzerland (PMOD/WRC), the Laboratoire de Physique Stellaire et Planétaire, Verrières-le-Buisson, France, (LPSP), the Space Science Department of the European Space Agency, Noordwijk, the Netherlands, (SSD/ESA), the Crimean Astrophysical Observatory, Crimea, USSR (KrAO) and the Central Research Institute for Physics, Budapest, Hungary (CRIP).

The main scientific objectives of IPHIR can be summarized as follows:

- obtain continuous high quality measurements of spectral solar irradiance variation;
- determine the frequencies, amplitudes, phases and line profiles for low degree pressure modes in the frequency range from 2 to 8 mHz;
- if they exist, detect and classify low degree gravity modes in the frequency range from 10 to 500 mHz;
- study the phase and amplitude differences between oscillation observations in intensity and velocity by comparison with ground-based observations from e.g. the Crimean Observatory.

2. INSTRUMENTATION

The experiment IPHIR comprises a power supply, a spacecraft interface, a micro-processor based controller and an analog/digital data acquisition system and two sensors:

- a three channel sunphotometer (SPM),
- a two axis sun sensor (TASS).

The sensors with their analog circuits, the analog-to-digital converters, the control circuits for the instruments and the cover electronics are packaged...
into the sensor box. The microprocessor based controller, the data acquiring system, the S/C interface and the power supply are located in a separate box, the DPU. The experiment weighs about 7.2 kg (both boxes and the connecting cable) and consumes a mean power of about 5.5 W.

The sensor box has a cover which closes the entrance apertures of the SPM and TASS. During integration and testing on ground and during the first period in space it protects the optical surfaces from contamination. During flight it is closed every 27 hours for 40 s to check the performance of the SPM and TASS by internal electrical and optical tests.

### 2.1. Sunphotometers

The SPM has three independent channels at 335, 500 and 865 nm each consisting of a Si-diode interference-filter combination in a sealed housing. The filters have a half-power bandwidth of 5 nm. The three detectors are mounted in a common body, that is heated with electrical power to a constant temperature which is 1 to 5 K higher than the heat-sink and can be selected by tele-command. The view-limiting geometry is determined by two apertures at a distance of 10 mm, the front aperture having a diameter of 9 mm and the detector aperture one of 2 mm. To prevent heating of the detector aperture it is covered by a back-surface mirror. To avoid crosstalk by straylight between the channels three 10 mm diameter stainless steel tubes are used as baffles.

The detectors are used in unbiased mode in order to minimize 1/f noise. Ultra low bias current, low noise electrometer amplifiers are used to convert the detector current to voltage. The noise of the detector-amplifier circuit is below 1 ppm for an integration time of the order of 10 s. The three analog signals of the SPM are measured simultaneously by three dedicated channels of the data-acquisition system.

### 2.2. Two Axis Sun Sensor

The pointing capability of the spacecraft, which has to guarantee proper illumination of the solar arrays is specified to be within 1°. The TASS is needed to enable corrections of the data for the prevailing angle between the Sun and the optical axes of the sensor. The X- and Y-channels of the TASS have each a linear CCD array as detector with a slit in front, which is orientated perpendicular to the CCD axis and projects a diffraction image of the Sun onto the CCD (similar to a cylindrical lens). The CCD arrays have 1728 elements at 10 μm spacing. The distance between the detector and the slit ascertains a resolution of about 1 arcsec per pixel. The analog output of the CCDs is connected to the sequentially operated correlator via a multiplexer.

The image analyzer is a digital correlator, which determines the center of gravity of the image in a range of ±128 pixels (Figure 2). As the read-out of the CCD is driven by clock pulses at a constant rate interpolation between pixels is possible simply by analog integration of the step function arriving at the zero comparator. The present design interpolates to a quarter pixel, that is, the resolution corresponds to 0.26 arcsec. The output is a pulse train, which is stopped at the moment of the zero crossing. The number of pulses accumulated in the X- and Y-counters during a measurement phase correspond to the place of the center of gravity of the solar image. At the beginning of each 8.22 sec sampling period the position on both axis is read five times and summed up, which takes about 1 second. After read-out the TASS is switched off for the rest of the sampling period in order to diminish the overall power consumption.

### 2.3. Data Acquisition and Control System

The scientific objectives rely on long and uninterrupted time series. These time series are analyzed by calculating power spectra, the noise of which depend strongly on the way the sampling is done. Ideally one should integrate the signal during the whole sampling interval; otherwise high frequency noise leaks into the power spectrum. As the integration time relative to the sampling interval de-
creases, the power of the folded-in high frequency noise increases. It has to be noted that this noise is an inherent part of the time series and can not be reduced by a posteriori filtering of the time series. Simulation experiments show that in the case of solar observations with a sampling corresponding to a Nyquist frequency around 10 mHz, integration during at least 95% of the sampling interval is needed to reduce the folded-in noise to tolerable levels. Technically, this is best achieved with parallel voltage-to-frequency (V-f) converters which allow for true integration over a given time interval. Thus, the analog-to-digital conversion is performed by 4 V-f converters, 3 for the SPM channels, 1 for the various housekeeping signals. The basic sampling period is 8.22 s, out of which 64 ms are used to perform electrical calibrations (alternatively at 0 and ±5 volts).

Besides the V-f converters which are located in the sensor box, the data acquisition and control system comprises the S/C interface, the microprocessor, the timing module generating the internal timing and clock signals and the counter module, all located together with the power supply in the DPU box. Four counters of the counter module are used for the V-f converters and two for the TASS X and Y channels.

The data accumulated every 8.22 s (3 SPM and 1 HK, 4 calibrations, 2 TASS values) amounts to about 160 bits (=22 b/s). The allocation to the experiment, however, is about 1 b/s or roughly 10^6 bits per day. This low bit-rate is determined by the combination of limited amount of S/C storage capacity and the few hours contact every 4 days during the cruise phase. The organization of the data transfer is done in packets of 960 bits and four types of blocks are used: a science block (SB), a housekeeping block (HB), a test block (TB) and a processor/memory test block (DB). The data compression is organized around this packet structure and is done along the following lines:

- increasing the effective sampling time to 41 s (5 times 8.22 s);
- transmit in a SB the first SPM value out of 2⁰ with a resolution of 20 bits and the remaining 23 with as differences to the first value with 10 bits (the accumulation time for a SB amounts to ~ 990s);
- transmit in a SB the mean, standard deviation and drift over the time of a SB, of the TASS X and Y values;
- transmit in a HB means and standard deviations over a period of 12 SB of the 16 housekeeping parameters (a HB is sent every 3h20m);
- transmit the calibrations with full precision in the HB and means of differences in each SB;

A TB is completed every 26h20m and contains among other informations about the tests of SPM and TASS with closed cover and about the non-linearity of the V-f converters by performing three 8.22s measurements with 0, ±5s and ±5 calibration voltages. The SB, HB and TB organized in the above manner sum up to about 92000 bits per day.

3. Mission Profile and Operations

The following list summarizes the main events for the two PHOBOS spacecrafts (I and II, with the instruments IPHIR-2 and -3 respectively) during the mission:

PHOBOS I
- launch: 7.July 88, 18.38 UTC
- switch-off due to thermal problems of the S/C: 10.July
- switch-on: 12.July
- 3-axis stabilization: 16.July
- switch-off/on to reset CPU: 3.August
- loss of S/C: 28.August

PHOBOS II
- launch: 12.July 88, 18.01 UTC
- switch-on of IPHIR-3: 14.July
- 3-axis stabilization: 21.July
- planned switch-off of IPHIR-3 for Mars/Phobos operations: 6. January 89
- planned switch-on after Phobos encounter: 15.April 89
- planned end of mission (Mars-Earth opposition): end October 89

The data from the experiment reaches the team through the Space Research Institute (IKI) in Moscow, and the Centre National d'Etudes Spatiales (CNES) in Toulouse on a regular basis. The data coverage is excellent.

4. Performance of the Experiment

The electrical performance of IPHIR 2 and 3 are very satisfactory; the electrical calibrations are very stable and the SPM and TASS tests well within the expected limits. IPHIR 2 may have suffered during the thermal problem of the PHOBOS-1 at the beginning of the mission as the internal temperatures reached over 50°. Although there was no obvious malfunction, the transmitted HB were occasionally filled with zero, indicating that the memory and/or the CPU may have had some problems. The switch-off/on in August cured the problem, at least until the loss of the S/C. IPHIR-3 never showed similar problems, a further indication that the problems with IPHIR-2 may have been due to the high temperatures. The on-board software performs well, although some minor errors have been found during operations. These errors could be cured by adequate tele-commanding of the instrument.

The optical performance of the SPM channels is strongly wavelength dependent. The blue channel has a very important degradation with a 1/e time constant of about 30 days. This degradation is by one order of magnitude greater than the ones of the green and red channels with time constants of about 300 and 600 days respectively. As we are interested in relative changes on time scales of up to several days this gradual degradation is not really harming the data, as long as the sensitivity is still sufficient. The latter may eventually become a problem for the blue channel, but not for the other two.

A more important problem has been encountered which is related to the pointing of the S/C. The philosophy for the corrections of the signals due to offset pointing was based on fact that the sensitivity decreases with increasing offset angle. Laboratory tests confirmed this behavior of the SPM detectors, although the effect was slightly more
pronounced that one would expect from a cosine response. This is explained by the angular dependence of the characteristics of the interference filter. The data, however, showed spikes with amplitudes of a few tenths of percent, which seemed correlated with the pointing as recorded by the TASS. An example is shown in Figure 3, showing 24 hours of raw data from the red channel. The pointing of the S/C is operating in such a way, that in one direction it oscillates between about ±0.6° with more or less constant speed and fast turns at the extremes. In the other direction the movements are more erratic with seldom active control, but with indications of some crosstalk from the control of the other direction. The pointing oscillations are not strictly regular and have periods between 40 and 80 minutes. The fact that the movement is quite linear in time and that this seems to translate into a quadratic change of the signal with a minimum between the pointed apexes will help to develop cleaning algorithms. The reason for this effect was only found recently when tests with IPHIR-1, the spare model, were performed with the Sun as source. During these tests the IPHIR sensor was mounted to a solar tracker on a turn-table allowing accurate de-pointing and was compared to a SPM with fixed pointing. The behavior found in space could indeed be reproduced and it turned out that reason is straylight within the baffles produced by the light reflected from the back-surface mirror in front of the detector aperture. As long as the illumination is parallel to the optical axis, the baffle receives no light. With an angle the mirror reflects light onto the baffle which in turn reflects part of it back to the detector where the extra signal reaches about 0.5 percent at 1° deflection. As the SPM have up to now always been used on well pointed platforms and as the tests for IPHIR were done on the detector level, this kind of behavior was not detected.

Figure 3: 24 hours of raw data of the red channel. The numbers are in thousands of counts. The downward trend is due to the increasing distance between the S/C and the Sun.

5. First Results

The first test on the data is searching for the 5-minute p-mode oscillations and determine the achievable signal-to-noise ratio. Even without a sophisticated cleaning method a very reasonable spectrum results from a two week time series as shown in Figure 4. To obtain this spectrum the data points around the apexes have been suppressed and a filter was applied to the rest of the data. The signal-to-noise ratio of the power spectrum is about 10:1 compared to a power spectrum of 270 days of ACRIM data of 3:1 if standard techniques are used or 7:1 with improved methods (Woodard, 1984). The improvement is certainly due to the better signal-to-noise ratio of the instrument (inherent and by proper sampling). It has to be noted that ACRIM was never designed to be used for such kinds of studies. An other important point is the fact that the data coverage is much better, even with the missing points around the apexes producing gaps of about 10 percent, compared to the 30-35 percent gaps in ACRIM due to the orbit around the earth. The better sampling coverage (ACRIM has only about 10 percent) may also explain the much cleaner line-shapes observed in the IPHIR spectrum.

More efficient ways for cleaning are under development. A method based on fitting pieces of a parabola to both sides of the spikes looks promising. Tests on 24 hours data sets have shown a further signal-to-noise improvement.

6. Acknowledgements

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7. References

SOHO STATUS

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ABSTRACT

The scientific payload for Solar and Heliospheric Observatory (SoHo) space mission has been selected. SoHo will study the structure of the solar interior, the physics of the solar corona and the origin of the solar wind. The spacecraft definition phase will start in October 1989. The launch is expected for March 1995.

INTRODUCTION

The Solar and Helioseismology Observatory, SoHo together with the four Cluster spacecraft mission, is part of the joint ESA and NASA Solar Terrestrial Science Programme that was approved two years ago and is being developed to be in operation in July 1995. The Soho payload includes three instruments that will measure solar oscillations and need to be built in the next few years. The instruments design will be completed during next year, 1989. This meeting is an occasion for scientist not participating in their development to learn about the instruments, and thereafter comment and influence the kind of measurements that they are planning before the instrument design is closed. This paper describes the environment in which the instruments will be operated. The instruments are described in subsequent papers in this proceedings.

AIMS

Soho is a solar observatory with two main aims: one, is the study of the heating mechanisms and physical processes in general that take place in the solar atmosphere, from the chromosphere through the transition region to the corona, and give rise to the solar wind; the other main aim is the study of the solar interior by means of helioseismology, and the measurement of solar irradiance variations.

PAYLOAD

Twelve investigations have been selected to fulfill the aims of Soho (Table 1). Five of them (EIT, SUMER, COS, UVCS and LASCO) will study the solar atmosphere from the chromosphere out to the external corona ( ~ 30 solar radii). They use a combination of instruments that perform diagnostic (measurement of density, composition, velocity, temperature) in the plasma of the solar atmosphere by high resolution spectroscopy in the extreme ultraviolet (EUV) region of the spectrum, where many lines of the hot solar ions can be isolated, and by high to moderate resolution images (1.5 to several arc seconds) of the solar atmosphere at different levels both in EUV spectral lines and in white light.

One investigation (SWAN) determines the structure of the solar wind streams by mapping of the Lyman-alpha radiation scattered by the hydrogen in the inner heliosphere. Other three investigations (CELIAS, ERNE and COSTEP) determine "in situ" the composition (in charge or elemental, in mass or isotopic, ion charge state, and in energy) of the solar wind and of the suprathermal and energetic solar particles that propagate through it.

The part of the payload devoted to helioseismology consist of three investigations that are expected to complement the ground based observations in two main aspects: one, the study of long period, low order modes of oscillation, difficult to obtain from ground observatories, mainly due the Earth rotation; the other, the study of high order, high degree, modes of oscillation that require continued high spatial resolution, difficult to obtain from the ground due to the effects of atmospheric seeing. The three selected investigations are highly complementary among themselves: GOLF measures full solar disc Doppler shift and magnetic oscillations, VIRGO measures full disc and very low resolution - 12 pixels - imaging oscillations of the solar intensity in the visible part of the
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* MDI will transmit additional 160 kbits/s during the Soho high bit rate transmission mode.
THE STATUS OF SOHO

Figure 1. Soho insertion and halo orbit.

Figure 2. The expected Soho operation time in relation to the average solar cycle.

Figure 3. Time scale for the SOHO mission.

Spectrum, and MDI measures high resolution (up to 1.7 arc sec) velocity oscillations, thus being able to study the whole range of modes of oscillation. The three instruments are described elsewhere in this volume (Dame, 1988, Fröhlich, 1988, Scherrer, 1988).

SPACECRAFT AND ORBIT

Soho will be three-axes stabilized and will point to the Sun within an accuracy of 10 arc sec, with a pointing stability of 1 arc sec to be achieved over a 15-min interval. The spacecraft will consist of a payload module that accommodates the instruments and a service module to carry the spacecraft subsystems and the solar arrays. Its total mass will be about 1350 kg, and 750 W power will be provided by the solar panels. The payload will weigh about 650 kg and will consume 350 W in orbit.

Current plans call for Soho to be launched in March 1995. About four months later the spacecraft will be injected in a halo orbit around the L₁ Sun-Earth Lagrangian point, about 1.5 x 10⁸ km sunward from Earth (Fig. 1). The Halo orbit around the Lagrangian point with an orbital period of 180 days has been chosen because it offers a smooth Sun-spacecraft velocity change, appropriate for helioseismology. The Sun-spacecraft velocity will be measured to an accuracy better than 2 cm/s. Soho is being designed for a lifetime of 2 years, but it will be equipped with sufficient on-board consumables for an extra 4 years. Therefore SOHO will, in principle, be in operation during the next solar minimum and may well be operational during the rise to the following solar maximum (Fig. 2).

One important feature of Soho regarding helioseismology is that Soho will provide continuous data transmission during two years, hopefully six. Nevertheless, in the space missions there are some times data transmission interruptions due to diverse technical reasons, such as ground station failures, or servicing interruptions. In the case of Soho, ESA and NASA, and the experimenters building the helioseismology instruments will take special precautions to avoid the data interruptions, since one of the main reasons to go to space is precisely to avoid the data gaps produced in the ground observatories by the night and the clouds.

Regularly, Soho telemetry will be received by ground stations of NASA's Deep Space Network (DSN) during three short (1.3-hr) and one long (8-hr) periods each day, but during at least two consecutive months every year the telemetry will be received continuously (24 hours per day). Scientific data acquired outside these periods will be stored on magnetic tape aboard the spacecraft and transmitted to the ground during the short daily sessions. The SOHO payload will produce a continuous stream of 40 kb/s; however, the bit rate will be increased by 160 kb/s during the regular 8-hr direct telemetry periods and during the two months campaign of direct telemetry. The additional 160 kb/s are generated by the solar oscillations imaging instrument (MDI) in addition to its regular 5 kb/s bit rate.

The Experiment Operations Facility, located at NASA's Goddard Space Flight Center (Greenbelt, MD.), will be used to coordinate and plan the scientific operations of the payload. Its main task will be to organize the real-time operation of the payload and control of the solar remote sensing imaging and spectrometric instruments during the daily 8-hr ground linkage interval. ESA also intends to issue an Announcement of Opportunity to invite proposals for a second experiment operations facility in Europe.

A special data facility will be provided to handle and access the data of the helioseismology experiment MDI.

STATUS

The investigations for Soho and Cluster were selected by ESA and NASA in March 1988. ESA has issued an Invitation To Tender (ITT) to the European industry for the design and development
of Soho and Cluster in October 1988. The selected industrial consortium is expected to complete the definition of the spacecraft between October 1989 and November 1990. The development phase will start in December 1990 for a Soho launch on March 1995. As a consequence the scientific instruments must have their definition frozen for the summer 1989 and must be ready for delivery around the end of 1992.

SOHO IN RELATION TO OTHER HELIOSEISMOLOGY PROJECTS

Soho will come into operation after several ground network projects will be set for several years, therefore it is expected that the added features of the Soho mission will allow us to achieve a significant improvement upon what one will have learned at that time, particularly if the experience gained with these ground operations is used to interpret the data of Soho, let aside the benefits that will result of the comparison and correlation of the measurements taken simultaneously. In particular, it is expected that many of the facilities used by the Ground Oscillations Network Group, GONG, will be made available to the Solar Oscillations Imager (MDI) investigation in Soho.

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THE GOLF HELIOSEISMMOMETER ON BOARD SOHO

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ABSTRACT

We present the GOLF (Global Oscillations at Low Frequencies) investigation that was recently accepted on board the SOHO satellite of the ESA/NASA Solar Terrestrial Science Programme. The GOLF instrument is an improved version of ground based instruments using the resonant-scattering spectrometric technique to measure global velocities of the Sun and Stars. It has the unique capability of a 4 points measurement in the line profile for both the global velocity and magnetic field measurements.

1. INTRODUCTION

The GOLF instrument is specially designed for attaining the highest sensitivity (< 1 mm s\(^{-1}\) and 1 milligauss) in the observations of the global solar velocity and magnetic fields oscillations at very low frequencies.

The acoustic and gravity modes of long periods that GOLF will observe, free from the earth atmospheric noise, penetrate deeply in the solar core. The information they contain will permit a detailed assessment of the pressure and density stratification in those regions, where uncertainties in the equation of state, opacities and neutrino emission are large. Together with the measurement of the rotation rate of the inner regions, this investigation will allow for the first time the determination of key parameters of internal structure and evolution of the sun as a typical star. Simultaneously, the mode lifetimes as well as long term variations of their frequencies, will be studied from the long, uninterrupted time serie that the Lagrangian point orbit of the SOHO satellite will provide.

We will have access to short term effects on the mode energetics, together with secular changes related, for example, to solar activity. The latter will be monitored through the measurement of the global magnetic field to a precision that has not yet been achieved. This magnetic field correlates with the internal structure and rotation of the sun via the not yet understood dynamo mechanism, and also with the global structure of the corona and heliosphere that will be observed on board SOHO.

The instrument is an improved version of the ground based instruments using the resonant-scattering spectrometric technique. It features a 4 points velocity measurement in the line profile together with the global magnetic field measurement. This provides two powerful means to calibrate the instantaneous sensitivity of the instrument to activity/magnetic effects, which could allow to estimate part of the solar noise spectrum in the longer period range. This should help to disentangle the magnetic effects from the true velocity measurement but also, by correlating the two signals, to evidence possible magnetic influence on the velocities.

We shall review the scientific objectives, and the instrument principle and design. The short time left for the definition phase of GOLF will probably not allow to include in the development plan the necessary investigation of the performances of an instrument breadboard; this is a compromise which should not prevent us to built an as much as possible thoroughly understood and calibrated instrument.

2. SCIENTIFIC OBJECTIVES

GOLF is presumably the instrument with the highest sensitivity for the detection of gravity modes, one of the major scientific reason for having this type of instrument flown in Space on a remote orbit providing continuous coverage.

A detailed description of the Science Objectives is given in the proposal that we submitted to ESA/NASA in answer to the SOHO Announcement of Opportunity (ref. 1).

Long periods pressure modes will likely be studied on ground providing the several network facilities that will soon be available (refs. 2, 3). GOLF, with its ultimate sensitivity to velocity (< 1 mm/s), will bring confirmation of the results and evidence low amplitude modes, hardly possible to detect from noisy ground data.

The main breakthrough of GOLF will probably be to give confidence on gravity modes detection and identification. Up to now, attempts to detect gravity modes, with amplitudes inferior to 4 cm/s in a noise spectrum, are contradictory (refs. 4, 5). Given the importance of these modes in the comprehension of the solar interior, a clearly dedicated instrument aimed at this detection is desirable.

The duration of the continuous observations and the improvement of the signal/noise ratio that GOLF will provide,
will permit a clean measurement of those spectral features. In particular, the absence of aliasing that severely hinder ground based observations is essential in this frequency domain where the mode separations are of the order of the daily gap-induced window functions.

Aside those "normal" objectives for an Helioseismology instrument, GOLF has a broad scientific coverage by the access to the global magnetic field measurement. Being generated by dynamo processes in the solar interior, the magnetic field is responsible for the structuring, heating, and dynamical processes, not only in the solar atmosphere but throughout the heliosphere. The magnetic field of the Sun is the fundamental physical parameter that is at the heart of the SOHO overall objectives and unifies the science of the various experiments, not only of SOHO, but of the whole STSP.

The solar corona acts as a spatial filter for the various harmonic modes. The large-scale magnetic regions where the magnetic field opens up into the heliosphere appear near the sun as coronal holes, further out in the heliosphere frequently as fast-speed streams in the solar wind. Therefore there is a close correlation between the mean photospheric magnetic field and the interplanetary magnetic field. The mean solar magnetic field has not the past proven to be a very valuable parameter in the general study of solar-terrestrial relations, which is a prime objective of the STSP.

A somewhat speculative but quite exciting possibility is that the mean magnetic signal may vary on a time scale as short as the p mode oscillations and in fact correlated with them. Such fluctuations may arise due to a coupling between the interior properties of the photospheric kGauss fluxtubes and the p mode global oscillations. These oscillations cause a coherent perturbation of the atmosphere external to the fluxtubes, and thereby also perturb the equilibrium configuration of the fluxtubes. The amplitude and phase lag of these fluctuations with respect to the global velocity oscillations will depend on the details of the non-linear interaction between the magnetic fluxtubes and the dynamics of the surrounding medium, which is not well understood. GOLF will allow this qualitatively new physics to be explored.

3. GOLF INSTRUMENT PRINCIPLE

The GOLF instrument is based on the resonant-scattering spectrometric technique. The heart of the experiment is a resonance cell filled with sodium vapor which, by use of the Zeeman effect, enables to absorb light selectively in the two wings of the solar D lines (Na D1, 589.6 nm and Na D2, 589 nm). The beam is alternatively right and left circularly polarized (+, -) which allows to selectively analyse the left and right slope of the integrated line profile. Coupled to this normal approach used on ground instruments, we use a small modulation of the permanent magnetic field applied to the cell (fig. 1). This allows to get two measurement points for each polarization, i.e. to get an indication of the slope and width of the line profile.

In practice, since we want a velocity measurement, we deduce the velocity from a Bσ+ Bσ- combination or a Bσ+ Bσ+ combination. Using different combination of those quantities, one has also access to the profile width variations and slope variations, all indications of a given solar activity state (active regions...). The noise from the active regions can be really harmful at the precision level that GOLF is aiming for. R. Ulrich et al. (ref. 6) evaluated in line comparable to the sodium line, a noise level in the 0.2 m/s range, with periods in the 30 min. - 1 hour meter. This clearly has to be addressed to get the ultimate performances out of the GOLF measurements; a comparison of our slope and width measurements on the global integrated line profile with the more detailed results of resolved sun velocities in the sodium line, has to be undertaken.

By allowing access to the circular polarization of the solar emission (doing the polarization analysis after a first quarter wave plate), GOLF has access to the global magnetic field by measuring the two Stokes components, V and V.-.

In practice, has illustrated on fig. 2, depending of the solar polarization, the profile can be more or less broader or deeper, a measurement that, again, the 4 points approaches will make much more precise.

This approach, which implies a eight points measurement sequence which last 40 seconds, is easier to carry in Space since no rapid change can interfere during this rather long elementary registration time.

4. GOLF OPTICAL DESIGN AND p-HOTOMETRY

The basic optical design principle of GOLF is pupil re-imaging to limit the sensitivity to pointing errors and solar structures.

The light collection and re-emission are not homogeneous in the cell (ref. 7); each part of the cell should then be equally irradiated by each part of the sun surface. This will minimize the sensitivity to pointing offsets and jitter, particularly accentuated by the solar rotation. At first order, this design guarantees that the spatial energetic reparation in the beam entering the cell, is independant from the spatial energetic and velocity reparation on the Sun.

In practice, and even if the pupil is imaged somewhere in the cell, the beam has an opening angle (divergence) which leads to some dependance with pointing offsets and jitter, particularly accentuated by the solar rotation. At first order, this design guarantees that the spatial energetic reparation in the beam entering the cell, is independant from the spatial energetic and velocity reparation on the Sun.

A schematic drawing (fig. 3) illustrates the main features of the proposed optical layout. After having been spectrally selected, the incident beam is focussed by L1 on the solar image position S1. L2, which has its focus on S1 position, re-images the Sun at the infinite, while doing an image of the pupil P (L1 position in practice) in the center of the cell (P'). An output lens (not shown) re-images the sun on the back detector (flux monitor).

The first quarter wave plate is placed just after L1. The

Figure 1. The magnetic field applied to the alkali vapor in the cell is slightly modulated by an amount ΔB (in between 25 and 100 gauss) in order to get 2 measurement points in each slope of the line profile (the absorption window is 25 mA for the sodium ).
Figure 2. Each orientation of the entrance linear polarizer will lead to 4 measurements in the profile by a subsequent analysis of the right and left circular polarization and the small AB modulation. This linear polarizer is placed after a first quarter wave plate which gives access to the intrinsic solar components of the Stokes vector, i.e. V and V- that allow the determination of B. Alike the velocity case, different combinations can be used to derive the magnetic field.

longitudinal positions of the linear polarizer and second quarter wave plate are not critical since their localization in the section between L1 and S1 makes that they see the same beam divergence.

The first lens is 20 mm in diameter and 200 mm in focal length (beam divergence F/10 : ± 3°). The second lens has a focal of 50 mm. The distance between L2 and the center of the cell is 62.5 mm. The cell characteristic are given in ref. 7.

The use of 6 photomultipliers tubes (PMTs) for the detection is based on two considerations. First, we use PMTs because of the type of noise limitation that they provide, i.e. photon noise which has no dependence with frequency (contrary to solid state detectors like diodes). Second, we use six PMTs because of a compromise between an acceptable noise level and photometry consideration.

We have an incoming solar flux in D1 and D2, in the 25 mA bandpass of the analysis window, of ~ 0.44 \(10^{-2}\) W/m². At the end of the detection chain the residual flux should be such that the counting rate is \(N = 1.2 \times 10^{7}\) counts/s, from noise consideration in the 5 min. range (instantaneous sensitivity of 1 m²s⁻²Hz⁻¹ in order to be below the solar noise continuum). In order to avoid further problems such as counts piling-up, linearity, photocathode degradation, we have chosen to share this flux over six individual counting chains (2 \(10^{6}\) counts/s/PMT). This figure is fully compatible with space qualified devices. In addition, use of six independent counting chains increases significantly the overall reliability of the detector unit. Photometry is further detailed in ref. 7, when considering the uncertainties on the cell re-emission.

5. GOLF TECHNOLOGY AND TRADE-OFF

Years ago, instruments using sodium vapor where know for their short life time due to the blackening of the cell, deteriorated by the hot sodium vapor. In the course of the GOLF investigation we have identified two type of glasses able to resist sodium vapor on much longer time than silica or classical pyrex. The first one is the Schott Glass 8636 which still present some significant sensitivity to sodium contamination but after periods of 2 to 4 years at normal working conditions.

The other glass, that is under selection for GOLF, is the Gehleniet glass (made by Phillips) which is remarkably insensitive to sodium so that it could stand more than 400 years without noticeable degradation at a nominal temperature of 180 °C. Due to a delay in the GOLF development plan, it has not yet been fully tested for the GOLF cell purpose (optical binding, polarization, mechanical stress, vibrations...) but the preliminary verifications were encouraging.

The magnetic modulation is another new technology that GOLF is addressing. Preliminary realisations with different coil wrapping geometries on a bare magnet where done last spring to show that level of 100 gauss can be achieved without a large thermal load or unreasonable current. However, no firm commitment is established on the right amount of the required amplitude (25 gauss ? 100 gauss ?) and no test have yet been made of the precision, repeatability and velocity induced signal, and on influence of the permanent magnet life time. The working point in the line profile, i.e. the magnet proper permanent field, is also to be determined on a global integrated solar profile rather than a sun center profile which is only a first order approximation.

In the GOLF design two other points were addressed that are specific of this Space experiment : the entrance filter (since in Space we can use the two sodium lines D1 and D2 - D2 has no more line blanketing caused by the atmosphere) and the polarization scheme (mechanical using incident circular light). It has proven to be feasible to realize a narrow band filter of 21 Å bandpass with a flat transmission (2 \(10^{-4}\) locally; 1% change in the slope) at top transmission on a 12 Å bandpass. Concerning the polarization scheme, the circular light in entrance allow to have only well polarized linear light re-emitted with an efficiency twice the approach where a linear light would be used in input. Precision on the mechanism are perfectly acceptable (±
Figure 3. Schematic optical principle of the GOLF instrument. A first lens L1 focalizes the incoming light (spectrally selected by the entrance filter) on the solar image SI. The second lens L2 re-images the pupil P (L1 in practice) inside the cell (P) while sending the solar image at the infinite. The polarization scheme first involve a quarter wave plate after L1 (to access the solar Stokes components V, V-), followed by a movable linear polarizer (calcite cube) and a movable quarter wave plate which finally alternativelly deliver a right or left circularly polarized light. The light is absorbed and re-emitted by the cell (in linearly polarized light at 90° of the optical axis) for a final detection on six photo multipliers tubes.

6 arcmin: which leads to a ratio the incident right circular polarization on the undesirable one of 7\(10^6\) and do not need any development compare to other approaches (PEM or LCC). The baffling of the rejected light in the linear (and rotating) polarizer is however to be adressed particular attention.

6. GOLF DEVELOPMENT PLAN

GOLF development plan is not firmed yet, and has been delayed nearly one year due to a lack of resources agreed in between the CNES (Centre National d'Études Spatiales: the French financing Agency) and the LPSP Direction, that hoped for a delay in the ESA/NASA SOHO programme.

The GOLF instrument is rather simple and nothing unreasonable has to be developed for this investigation. However the instrument is asking high stability and repeatability and requires a thorough calibration of all its physical parameters in order to achieve the required precision expected.

How harmful for the GOLF programme will be this lack of a careful definition of the instrument required performances and suitable technologies (the definition phase ends normally in march 1989), is not known but will not help in providing the community with the high quality data that are expected from this unique Space opportunity: a satellite at the Lagrangian point.

7. CONCLUSION

The GOLF instrument, free of the earth atmosphere fast changes, has an improved operational measurement sequence compared to ground based instrument. It will allow, with precision never reached before, comprehensive study of the global velocity and magnetic fields of the Sun, using a unique 4 points measurement sequence in the line profile.

Acknowledgments. We are grateful to the GOLF Co-Investigators who did a marvelous work outlining the unique science objectives and carefully assisting in the design of the GOLF instrument. In particular we shall address thanks to J. Stenflo for the magnetic field approach, R. Ulrich for the solar activity influence, M. Decaudin for the early instrument definition and T. Roca Cortés for his help in getting through a comprehensive set of science specifications for the instrument. We are embedded to R.M. Sonnet who developed, initially with the DISCO programme, the interest of Global Oscillations measurements at LPSP.

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The VIRGO Experiment (Variability of solar Irradiance and Gravity Oscillations) contains two types of active cavity radiometers for monitoring of the solar "constant", two three channel sunphotometers (SPM) for the measurement of spectral irradiance at 335, 500 and 865 nm and a low resolution imager (LOI) with 12 pixels. The main scientific objective is probing the solar interior by helioseismology with $p$- and $g$-mode solar oscillations determined from spectral irradiance (SPM) and radiance (LOI) variations on time scales of minutes to the mission time. Moreover, the measurements of the variability of the solar "constant" and spectral irradiance over periods of days to the mission time will yield information about the convection zone, as well as the comparison of the amplitudes and phases of the oscillations as manifested in irradiance and radiance (from VIRGO) and velocity (from GOLF and SOI).

Keywords: Helioseismology, Solar Constant, Solar Spectral Irradiance, Low Resolution Solar Radiance, SOHO Mission

1. INTRODUCTION

The VIRGO investigation (Variability of solar Irradiance and Gravity Oscillations) will observe and study the irradiance and radiance of the Sun with high precision, high stability and high accuracy. The total irradiance will be observed with active cavity radiometers (PM06 and CROM) and the spectral irradiance measurement will be carried out by two three channel sunphotometers (SPM). The radiance will be measured with 12 resolution elements on the solar disc using the luminosity oscillations imager (LOI). For the spectral irradiance and the radiance data output the signal to noise ratio will be at least one order of magnitude higher than any previous space measurements.

The main data output of the VIRGO package will consist of time series of the irradiance and radiance data. As a secondary output the internal guider of the LOI component will produce time series of data proportional to the polar and equatorial solar diameters. The time series of the total and spectral irradiances will resemble that of the ACRIM instrument on SMM. However the data will have higher precision and due to the location of the SOHO spacecraft the observations will be continuous. In addition the spatial resolution and the spectral data will enable to deconvolve the simultaneous effects of different photospheric structures on the solar irradiance. The variations seen in these time series are a superposition of all the random and periodic phenomena that cause as a whole the variability of the solar irradiance.
two types of absolute radiometers (CROM & PM06).

the channel sunphotometer (SPM).

- luminosity oscillation image (LOI).

The sensors are packaged in a unit as shown in Figure 1. This box contains also the controller and the analog-to-digital acquisition system. The power supply and control interface are located in a separate box.

2.1. Absolute Radiometers

Absolute radiometers are based on the measurement of a heat flux by using an electrically calibrated heat flux transducer. The radiation is absorbed in a cavity which ensures a high absorptivity over the spectral range of interest for solar radiometry. During practical operation of the instrument, an electronic circuit maintains the heat flux constant by accordingly controlling the power fed to the cavity heater. This is called the active mode of operation, hence also the name "active cavity radiometer". The irradiance can be calculated from the shaded and irradiated electrical powers $P_i$ and $P_a$ by,

$$S = C (P_a - P_i)$$

with $C$ being a constant, which is the inverse of the aperture area times a correction factor for the deviations from ideal behavior.

The correction factor accounts for different effects such as the reflectivity of the cavity, the losses due to diffraction at the apertures, stray-light in the view limiting muffler, heating of leads and the nonequivalence of electrical and radiative heating. The corresponding correction factors are determined experimentally and their uncertainty determines the absolute accuracy of the radiometer. By present state-of-the-art absolute radiometry an accuracy of the order of ±0.15% is achieved.

Although the design of both radiometers are based on the same principle, the physical realization is different and this difference is the main reason for having both. In Figure 2, the detector arrangements of the CROM and PM06 type radiometers are shown for comparison. Major differences are the arrangement of the compensating cavity and the form and coating of the cavities. The main advantage of the CROM instrument is that the compensating cavity can also be used for radiation measurements, whereas the back-cavity of PM06 cannot be exposed to the sun. From the experience with the ACRIM radiometer on the SMM spacecraft it is known that continuously exposed radiometers degrade relatively to only occasionally exposed sensors: in this way a drift of about 10 μg pm per year was measured for the continuously used ACRIM sensor. This means that one needs at least one spare sensor for each radiometer: in the case of CROM it is the compensating cavity; for PM06 a complete second instrument is needed, which in turn also increases redundancy. Moreover, the different geometries and coatings (diffuse for CROM and specular for PM06) of the cavities will most probably lead to different and thus distinguishable degradation. A further concern about degradation of the cavity absorptance is due to prolonged exposure to vacuum which is taken into account with a reflectometer built into PM06, which allows for an independent in orbit determination of the cavity absorptance. With these measures - two different types of instruments and the reflectometer - the detection of even small long-term trends of the solar "constant" will be possible. The back-up sensor will be exposed at most once every 100 hours during the mission.

2.2. Sunphotometers

The SPM has three independent channels at 335, 500 and 665 nm each consisting of a Si-diode interference-filter combination in a sealed housing. The three detectors are mounted in a common body, that is heated with constant power and remains always a few degrees above the temperature of the heat sink. This prevents degradation of the optical element due to condensation of gaseous contaminants. The actual detector temperature is monitored by two thermometers and used to correct the sensitivity during evaluation of the data.

Although the design of both radiometers are based on the same principle, the physical realization is different and this difference is the main reason for having both. In Figure 2, the detector arrangements of the CROM and PM06 type radiometers are shown for comparison. Major differences are the arrangement of the compensating cavity and the form and coating of the cavities. The main advantage of the CROM instrument is that the compensating cavity can also be used for radiation measurements, whereas the back-cavity of PM06 cannot be exposed to the sun. From the experience with the ACRIM radiometer on the SMM spacecraft it is known that continuously exposed radiometers degrade relatively to only occasionally exposed sensors: in this way a drift of about 10 μg pm per year was measured for the continuously used ACRIM sensor. This means that one needs at least one spare sensor for each radiometer: in the case of CROM it is the compensating cavity; for PM06 a complete second instrument is needed, which in turn also increases redundancy. Moreover, the different geometries and coatings (diffuse for CROM and specular for PM06) of the cavities will most probably lead to different and thus distinguishable degradation. A further concern about degradation of the cavity absorptance is due to prolonged exposure to vacuum which is taken into account with a reflectometer built into PM06, which allows for an independent in orbit determination of the cavity absorptance. With these measures - two different types of instruments and the reflectometer - the detection of even small long-term trends of the solar "constant" will be possible. The back-up sensor will be exposed at most once every 100 hours during the mission.
in order to minimize 1/f noise. Ultra low bias current, low noise electrometer amplifiers are used to convert the detector current to voltage. The noise of the detector-amplifier circuit is below 1 ppm for an integration time of 10 s. The three analog signals of the SPM are measured simultaneously by three dedicated channels of the common data-acquisition system.

2.3. Luminosity Oscillation Imager

The LOI instrument is a very high stability solar photometer able to resolve the solar disc into 12 spatial elements. The precision for each pixel is about 1 ppm for an integration time of 10 s. The 12.3 mm diameter image is provided by a 50 mm diameter, 1300 mm focal length Ritchey-Chretien telescope. The spectral bandwidth is defined by a 5 nm bandwidth interference filter with center wavelength at 500 nm (placed in front of the telescope). The position of the secondary mirror of the telescope is controlled by two orthogonally placed piezoelectric stacks allowing the image to be stabilized on a custom Si-diode array. In order to allow for the possible misalignment of 10 arcmin between the LOI and the spacecraft sun pointer a coarse offset mechanism is included. It is planned to use this mechanism only at the startup of operations, but in case of failure of the fine guider the coarse system may stabilize the image to ± 5 arcsec.

The detector is a neutron doped silicon single substrate multi-element detector custom made to our specifications with 12 separate diode elements. Four of these elements constitute an outer circular boundary of the detector and are used for stabilizing the image. The other 12 elements are shaped specifically to allow detection of spatial scale corresponding to spherical harmonics up to degree 10. As in the SPMs, the detectors are used in unbiased mode in order to minimize 1/f noise. Ultra low bias current, low noise electrometer amplifiers are used to convert the detector current to voltage.

The analog-to-digital conversion is performed in parallel for all channels by 12 separate voltage-to-frequency converters (VFC) within LOI. The output of each VFC is counted in separate 24-bit counters in the common data acquisition system. The electrical calibration of the VFC is performed in a similar way as in the common data acquisition, but also within LOI.

2.4. Data Acquisition and Control System

The data acquisition and control system comprises the S/C interface, the microprocessor, the timing module which generates the internal timing and clock signals from the high frequency clock and reset pulses of the S/C, the counter module with 24 bit counters directly controlled by the processor and the voltage-to-frequency converter (VFC) module with 8 VFC and the corresponding multiplexers.

The scientific objectives rely on long and uninterrupted time series. These time series are analyzed by calculating power spectra. The noise of which depend strongly on the way the sampling is done. Ideally one should integrate the signal during the whole sampling interval; otherwise high frequency noise leaks into the power spectrum. As the integration time relative to the sampling interval decreases, the power of the folded-in high frequency noise increases. It has to be noted that this noise is an inherent part of the time series and can not be reduced by a posteriori filtering of the time series. Simulation experiments show that in the case of solar observations with a sampling corresponding to a Nyquist frequency around 10 mHz, integration during at least 90% of the sampling interval is needed to reduce the folded-in noise to tolerable levels. Technically, this is best achieved with parallel voltage-to-frequency converters (VFC) which allow for true integration over a given time interval.
3. SCIENTIFIC OBJECTIVES

The scientific content of the gathered time series may be studied in different ways. Solar variability in itself is very interesting, but it can also be used to study other physical phenomena, such as solar oscillations, phenomena related to solar activity, etc. We may be interested in the phenomena causing the variation, e.g. sunspots, or we may be mainly interested in the underlying physical causes for the existence of sunspots, e.g. dynamo theory, rotation and convection. The causes of irradiance changes are crucially important for the understanding of solar and stellar evolution. Independent of the cause, the knowledge of the possible medium and long term variations of the solar irradiance are equally important for the understanding of terrestrial climatic changes. These non periodic or quasiperiodic phenomena are best studied in the time domain where these variations may be directly compared with the signatures of solar surface intensity structures.

Although irradiance variations and solar oscillations are for simplicity - treated as independent, it is highly likely that these domains overlap physically. The long period solar oscillations may be coupled to the seemingly nonperiodic or quasiperiodic variations in irradiance, either directly or by influencing the causes of surface intensity configurations.

The precision of the oscillation measurements are determined by the instrumental noise sources and the non-oscillatory solar background signals. Although the solar background signal has the effect of a noise signal on the oscillation measurements this "noise" contains valuable information about the originating solar phenomena, granulation, mesogranulation, supergranulation and active regions.

The stellar analogy to Helioseismology, Astroseismology will be able to reach a much larger number of stars with irradiance measurements than with velocity measurements. This is due to the photon noise limitation and the large variation in the absolute line of sight velocity due to stellar proper motions. Information about the solar "noise" in intensity observations will directly be applicable to the stellar observations. In addition no star other than the Sun can currently be spatially resolved to observe time/spatial variation as function of disc position, thus making it necessary to make geometrical assumptions of the causes of the observations. When the Sun is observed as a star simultaneously with resolved observations we can calibrate the effects that given physical phenomena have on unresolved observations.

The following list summarizes the scientific objectives of the VIRGO investigation:

- determine with high accuracy the amount of spectral redistribution of the solar output.
- determine the frequencies, amplitudes and phases for oscillation modes in the frequency range of 1 mHz to 10 mHz.
- if they exist, detect and classify low degree modes of solar oscillations.
- study the phase and amplitude differences between intensity and velocity measurements in correlative studies with the other helioseismology investigations (GOLF and SOI) on SOHO.
- use the determined oscillation mode parameters as input to model calculations and inversions to determine values for sound speed, density stratification, rotation in the solar interior.
- search for the long periodicities or quasiperiodicities found in other solar parameters.
- complement the other helioseismology investigations on SOHO by overlapping in spatial resolution (degrees 0 to 7).
- study the influence of active regions and other large scale solar intensity structures on the solar irradiance and radiance.
- investigate the energy storage in the convection zone in connection with the energy blocking by active regions.
- study the solar energy budget.
- provide accurate total and spectral irradiance data for input in terrestrial climate modeling.

4. THE VIRGO TEAM

The VIRGO Team consists of the authors as Investigators supplemented by the following associated scientists: D.Gough, University Cambridge, T.Hoeksema, Stanford University, J.Provost, Observatoire de Nice, R.C.Willson, Jet Propulsion Laboratory. The following Institutes contribute with hardware: PMOD-WRC, SSD-ESTEC, IRMB and IAC. The participation in the team guarantees the experimental and theoretical expertise needed to plan, design and build such an experiment and also the scientific liaison to the other helioseismology experiments on SOHO, GOLF and SC1.
THE SOLAR OSCILLATIONS IMAGER FOR SOHO

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ABSTRACT

The Solar Oscillations Imager (SOI) program for SOHO will consist of a Michelson Doppler Imager (MDI) instrument, a data reduction and analysis capability, and a coordinated set of investigations designed to address a set of science objectives. The program is a collaboration of 15 Investigators (P&Co), and 21 Associate Investigators. The overall investigation is organized to address a set of science objectives. These include primary objectives which can be investigated using the techniques of helioseismology and associated objectives which can be addressed by other analysis techniques. The MDI is designed to take advantage of the anticipated SOHO telemetry by organizing the observations into 4 observing programs: structure (at all times), dynamics (2 months per year), campaign (8 hours per day, 10 months per year), and magnetic fields (few minutes per day). The MDI will be able to measure line-of-sight velocity by Doppler shift, transverse velocity by local correlation tracking, line and continuum intensity, and line-of-sight magnetic fields with both 4 and 1.4 arc-second resolution (2 and 0.7 arc-sec pixels respectively). The flight instrument will be built by the Lockheed Palo Alto Research Laboratory.

Keywords: Helioseismology, Solar Oscillations

1. OVERVIEW

The Solar Oscillations Imager (SOI) program for the Solar and Heliospheric Observatory (SOHO) is designed to allow a coordinated set of investigations which will address a broad set of science objectives. SOHO is a joint NASA - ESA mission which will place the SOHO spacecraft in a halo orbit around the Sun-Earth L1 Lagrangian point in early 1995. In this location SOHO will provide an unprecedented opportunity to probe the solar interior with the techniques of helioseismology.

The flight instrument for the SOI program will be the Michelson Doppler Imager (MDI). The MDI is a modification of the Fourier Tachometer and can measure line-of-sight velocity by Doppler shift, transverse velocity by local correlation tracking, line and continuum intensity, and line-of-sight magnetic field by the Zeeman effect.

The MDI will be built by the Solar Physics Group at the Lockheed Palo Alto Research Laboratory under the direction of Alan Title.

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Data management, including preliminary data reduction and production and distribution of datasets for analysis will involve a distributed system, with significant segments at Stanford and at the National Solar Observatory in an extension of the GONG project.

The science analysis will involve the entire team with organizational support from the group at Stanford.

1.2. Science Objectives

The overall investigation is organized to address a set of science objectives, which are divided into two general categories of primary and associated science objectives. The primary scientific objectives of the SOI program are to measure the internal stratification and dynamics of the sun by the standard techniques of helioseismology. They will be based upon precision measurements of the line-of-sight surface velocity field and determination of spherical harmonic coefficients.
The primary objectives of the MDI are to investigate the solar interior using the tools of helioseismology. To fulfill the potential of helioseismology observations free of the distortions from the Earth’s atmosphere and free of diurnal data gaps must be obtained. Figure 1 shows the familiar \( l-v \) or \( k-\omega \) diagram. The entire region of Figure 1 will be accessible to the MDI. Note that regions of the spectrum which are essential to developing a full understanding of the solar interior are accessible to MDI but not to previous experiments. (However the sensitivity to modes of degree 0-2 and modes with periods in excess of several hours has been reduced to an as yet undetermined extent in order to reduce cost).

The continuous curves are the frequencies versus degree \( \ell \) of \( g_1 \), \( p_1 \), plus the even-order \( p \) and \( g \) modes for a standard solar model. [Odd order (except \( n=1 \) were omitted to avoid cluttering the diagram.] There are no lines in the upper left-hand corner of the diagram because the frequencies are above the Lamb cutoff \( \nu_c \), so the modes are not trapped. At high \( l \) frequencies can exceed \( \nu_c \) because the modes are reflected by the corona. These modes provide information about the structure of the corona. The \( g \) modes are indicated only for \( \ell \leq 10 \), because interior \( g \) modes of higher degree are unlikely to be detectable. The heavy shaded region indicates where ground-based observations have detected modes. The light shaded region is the region of greatest power in Alan Title et al.’s SOUP observations. The vertical dashed line is at \( \ell > 200 \); GONG will detect modes only to the left of this line. The four thin oblique straight lines marked a, b, c, d indicate the \( p \) modes whose lower turning points are respectively at the base of the convection zone, the middle of the HeII and H ionization zones, and the peak in the superadiabatic temperature gradient in the upper convective boundary layer. These lines are extended (dashed) beneath \( \ell_1 \) for clarity.

The primary objectives identified for the SOI investigation are:

<table>
<thead>
<tr>
<th>Science Objective</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radial Stratification</td>
<td>Spherically symmetric component of mean structure in and below convective envelope: ( \bar{\rho}, \bar{p}, \xi, \zeta, \rho ) and ( g ) modes.</td>
</tr>
<tr>
<td>Internal Rotation</td>
<td>Rotation as function of radius and latitude, ( \Omega (r, \theta) ), ( g ) modes.</td>
</tr>
<tr>
<td>Large-Scale Convection</td>
<td>Large-scale convection cells and associated thermal structures: Giant Cell Tomography, sampling radius, latitude, and longitude.</td>
</tr>
<tr>
<td>Large-Scale Asphericity</td>
<td>Non-spherically symmetric component of mean structure in convective envelope, ( \rho, \rho, \xi, \zeta ).</td>
</tr>
<tr>
<td>Upper Convective Boundary Layer</td>
<td>Detailed sampling of sub-surface regions: upper reflection point for most ( p ) modes. Highly turbulent convection, fibril magnetic fields.</td>
</tr>
</tbody>
</table>

---

1.3. Observing Programs

The MDI is designed to take advantage of the anticipated SOHO telemetry availability by organizing the observations required to support the investigations into 4 observing programs:

- Primary structure program,
- Primary dynamics program,
- Campaign programs,
- Magnetic fields program.

The observed data will be returned to the earth through a virtual continuous low bandwidth (5k-baud) channel for the primary structure program. To allow full recovery of the million pixels of data a high bandwidth channel will also be available some of the time. This 160 kbps channel will be needed continuously for two months per year for the primary dynamics program. In other months it will be available for about 8 hours each day. The magnetic field data will be contained in the first few minutes of these times. The remaining high bandwidth time is required to complete the campaign programs. The magnetic program is designed primarily to support the interpretation of the velocity data but the field data may be available for collaborative studies.

1.4. The MDI Flight Instrument

The primary observable of the MDI is line-of-sight velocity. The MDI will make velocity maps by sampling the solar Ni I 6768 Å line profile at four points and finding the Doppler shift by determining the phase of the first Fourier coefficient of the line shape with respect to a reference wavelength. The MDI achieves a full width 100 mÅ bandwidth with a sequence of filter elements. These include a 50 Ångstrom front window, an 8 Ångstrom blocking filter, an 400 mA Lyot filter and two tunable Michelson interferometer used as analogs of birefringent elements. The Michelson elements will be tuned by rotating half-wave plates. The filters will be calibrated with an onboard laser.

The filter chain is preceded by a f/15.7 Cassegrain telescope with a piezo-electric driven secondary to achieve a stable image. The primary image is reimaged through the filters in a telecentric imaging relay. The final relay image is then imaged onto a 1024x1024 pixel CCD camera with a magnification of 1 or 3 selected by the shutter. By selecting one of the light paths, the MDI can operate in either normal or high resolution modes with either 4 or 1.4 arc-second resolution (2 or 0.7 arc-sec pixels respectively). Each exposure and readout cycle will take about 2 seconds.

An important design principle of the MDI is to allow high resolution observations at all times. In order to accomplish the desired observing sequences with the limited telemetry to be available on SOHO, the MDI will contain an on-board computing capability sufficient to extract a subset of the observed solar oscillation modes. A complete full-disk Dopplergram will be obtained during the first half of each minute. This Dopplergram will be Fourier transformed and sampled to extract a set of oscillation modes. This sampled data will be for the primary structure program and must be obtained continuously for many months. Interleaved in the other half of each minute, further observations will be made for the primary dynamics and campaign programs, depending on telemetry availability.

The MDI conceptual design is described in more detail in an accompanying paper (Ref. 1) in this volume.

2. SCIENCE OBJECTIVES

2.1. Primary Objectives

The primary objectives of the MDI are to investigate the solar interior using the tools of helioseismology. To fulfill the potential of
Chromospheric Structure: Magnetic and thermal structures within chromospheric cavity.
Excitation and Damping: Coupling of waves to turbulent convection. Mode lifetimes, driving mechanisms.
Active Region Seismology: Scattering and absorption of waves by active regions; wakes behind sunspots; search for magnetic fields yet to erupt.
Internal Global Scale Magnetic Fields: Toroidal fields at base of convection zone; Solar cycle variations of large-scale magnetic fields. Solar core fields.
Seismic Response to Flares: Wave excitation and absorption by major flares.

2.2. Associated Objectives

In addition to the primary objectives, the SOI program will allow investigation of several associated science objectives. They are called associated objectives because they involve the study of observable solar phenomena other than oscillations but have the same goals of understanding the structure and variability of the sun. These investigations are no less important to the understanding of the solar structure and dynamics than the primary objectives.

<table>
<thead>
<tr>
<th>Science Objective</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large-Scale Surface Flows</td>
<td>Global surface flows: meridional circulation and giant cells; temperature field associated with flows.</td>
</tr>
<tr>
<td>Magnetic Fields</td>
<td>Synoptic magnetograms, repeat as needed for validation of velocity data.</td>
</tr>
<tr>
<td>Turbulent Convection</td>
<td>Study granulation and mesogranulation, evolution and advection of smallest-scale intensity features; study physics of intense turbulence.</td>
</tr>
<tr>
<td>Magnetohydrodynamics</td>
<td>Study MHD processes by simultaneous sampling of velocities, temperatures, and magnetic fields.</td>
</tr>
<tr>
<td>Supergranulation</td>
<td>Study evolution of supergranule convection cells and magnetic network reorganization.</td>
</tr>
<tr>
<td>Figure of the Limb</td>
<td>Study long-term variations in the figure of the limb.</td>
</tr>
<tr>
<td>Flux Budget</td>
<td>Spot and plage contribution to radiant flux.</td>
</tr>
</tbody>
</table>

3. OBSERVING PROGRAMS

The science objectives will be addressed by a set of observing programs. The programs are essentially the sets of observations to be accomplished during each of the kinds of telemetry opportunities.

In order to observe the p-mode oscillations, we must make a velocity map about once per minute. With reasonable compression, it takes about 160 kbps for one minute to transmit an observation. SOHO will provide a 160 kbps channel for as much of the time as is practical considering ground station availability and cost. This high bandwidth time will most likely be limited to 8 hours per day for ten months with one interval of two months each year of 24-hour per day coverage. In addition, SOHO will provide a continuous channel of 5 kbps at all times.

3.1. Primary Structure Program

The primary structure objectives relate to determining the solar internal structure require continuous or nearly continuous observations for many months to years, so the needed data must be acquired in the low bandwidth channel. Since we can telemeter only about 1/32 of the observed data, we must extract the required components of the data onboard the spacecraft. The MDI includes an image processor that will allow these calculations. Basically we will select a few thousand of the several million oscillation modes observed. We plan to acquire all the low-degree modes up to about l=24, limb position data up to about l=100, and several thousand selected high degree modes. The observations will be made in the first half of each minute at all times.

This data will allow us to study the radial stratification, internal rotation, large-scale asphericity, internal global fields, large scale surface flows, and figure of the limb science objectives.

3.2. Primary Dynamics Program

While we can use partial data recovery to achieve some of the science objectives, the detailed study of the solar convection zone requires all observable modes for a long interval. During high-bandwidth telemetry intervals exceeding a day, we will directly recover full disk Dopplergrams and intensity maps each minute for the primary dynamics program. These will be the identical data used by the on-board processor for the Primary Structure Program reductions, which will continue uninterrupted. We will also make line-of-sight magnetograms as required to interpret the velocity data.

3.3. Campaign Programs

During the daily 8-hour intervals of high bandwidth coverage outside the primary dynamics program we will carry out our campaign programs. These are a collection of observing sequences designed to measure internal and photospheric dynamics using both helioseismology and traditional analysis techniques.

The science objectives studied in the campaign programs include the upper convective boundary layer, excitation and damping, chromospheric structure, response to flares, turbulent convection, and magnetohydrodynamics. The campaign programs will also allow some opportunity to make coordinated observations with other SOHO experiments.

3.4. Magnetic Fields

We will make full disk magnetograms at the start and possibly also at the end of each 8-hour high data rate interval. These are primarily for the purpose of interpreting the velocity maps, since even with the Ni line we expect some contamination of the velocity measurements. We will also make partial data recovery to achieve some of the science objectives, the detailed study of the solar convection zone requires all observable modes for a long interval. During high-bandwidth telemetry intervals exceeding a day, we will directly recover full disk Dopplergrams and intensity maps each minute for the primary dynamics program. These will be the identical data used by the on-board processor for the Primary Structure Program reductions, which will continue uninterrupted. We will also make line-of-sight magnetograms as required to interpret the velocity data.

3.5. Modes of Observation

In order to accomplish the observing programs the MDI will have several modes of observation. We can select among six observable quantities shown in Table 2.

In addition to various observables, we can also observe with either normal resolution or in high resolution mode. In either resolution we can select a partial data grid to either allow more quantities to be measured or to track features through our field-of-view.

4. DATA ANALYSIS

4.1. SOI Data Processing Requirements

SOI will produce about 300 Gigabytes of raw telemetry data per year for at least 2 and perhaps 6 years. After decompression and production of over 20 sets of reduced and calibrated data required
for analysis, this will expand by a factor of 10. Access to this unique dataset of 6 to 18 Terabytes must be available to members of the helioseismology community worldwide. The large quantity and variety of data will require special efforts and significant capabilities in data management. As part of the SOHO definition phase we are examining ways to implement the required processing capability, making optimal use of the efforts and facilities of the team. In particular, we intend to develop a data processing plan which takes the GONG experience into account, with possible overlap in both software design and hardware. While we are still at a preliminary stage in this study, it is clear that there are significant benefits to be gained by combining the efforts in some manner, and also significant problems in distributed data management to be solved.

4.2. Data Products

Five classes of SOI data are distinguished by their scientific purpose, frequency of collection, and size:

4.2.1. Primary Dynamics Data. The helioseismology data produced and transmitted with minimal on-board processing during the two months a year of continuous 160 kbps telemetry. 150 GB/yr of raw (compressed) data, but requiring processing at a rate of 600 GB/yr for near-real-time analysis.

4.2.2. Campaign Data. Data supporting a variety of science objectives and programmable during the mission, produced during the 8 hours per day of 160 kbps telemetry in the remaining 10 months per year. 170 GB/yr of raw data.

4.2.3. Primary Structure Data. Helioseismology data produced and transmitted continuously in the 5 kbps channel after substantial on-board processing. Requires essentially no processing beyond decoding and sorting except for possible calibrations and corrections based on experience and comparisons with the Dynamics data.

4.2.4. Magnetic Data. Magnetograms with supporting continuum maps and Dopplergrams transmitted twice a day, at beginning and end of 160 kbps telemetry, or at specified times during the 2 months of continuous high-rate telemetry. 3 - 5 GB/yr of raw data.

4.2.5. Housekeeping Data. Data transmitted continuously from the Dedicated Experiment Processor in support of instrument monitoring and control functions. About 1.5 GB/yr.

The widely varying size scales and access requirements for these data streams and their associated individual datasets, such as time-sorted Dopplergrams and mode-sorted amplitudes, suggest the use of a variety of media for storage and distribution, including magnetic tape, magnetic disk, magnetic tape cassette, optical disc, and optical tape. Probably all of these will be in use at different points in the data management program.

## Table 2. SOI - MDI Observables

<table>
<thead>
<tr>
<th>Observable</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_d$</td>
<td>Doppler Velocity</td>
</tr>
<tr>
<td>$V_L$</td>
<td>Transverse Velocity</td>
</tr>
<tr>
<td>$I_L$</td>
<td>Line intensity</td>
</tr>
<tr>
<td>$I_r$</td>
<td>Continuum intensity</td>
</tr>
<tr>
<td>$B_{II}$</td>
<td>Line-of-sight Field</td>
</tr>
<tr>
<td>$R$</td>
<td>Solar limb position</td>
</tr>
</tbody>
</table>

4.3. Data Flow

A useful way to understand the SOI data plan is to examine the data paths from the instrument through to science analysis. The following scenario, based on several stages of data preparation, illustrates the present direction of our planning.

4.3.1. On-board Processing. The data processing to select the modes available to the Primary Structure Program must be performed by an onboard Image Processing Unit. Its output will require only minor additional processing before being available for direct analysis in the form of tables of mode amplitudes. Design of this critical processor will be the responsibility of the Lockheed instrument design team, working together with the Stanford group, but its usefulness rests on the proper selection of modes and analysis techniques. These must be provided by members of the team based on both theoretical modeling and observational analysis of prototype data.

4.3.2. Telemetry and Data Capture. Telemetry from the spacecraft will be provided by ESA as an integral part of the SOHO mission, with NASA’s Deep Space Network and the GSFC mission support operation providing capture, decommutation, and transmission to the appropriate facility. This will provide raw datasets in separate streams according to the nature of the telemetry: low-bandwidth, high-bandwidth, and housekeeping data. It is expected that telemetry tagging will allow the magnetograms to be extracted for quick look.

4.3.3. Operations and Quick Look. Housekeeping data (about 0.5% of the raw data) must be available in near real time to the operations team. In addition, it is highly desirable to have some of the data, particularly the magnetograms, available for quick look both by investigators and by other SOHO experiments at the Experiment Operations Facility at GSFC. Coordination of this aspect of the data management will be in the hands of the joint Stanford/Lockheed operations team.

4.3.4. Decompression, Validation, Calibration, and Routing. All raw data must undergo a certain low level of processing, beginning with decoding, and involving various kinds of validation and instrumental calibration depending on the nature of the data. This will be done at Stanford. Some of the data can then be routed directly to individual investigators for analysis, while others will require very substantial additional processing. At this point the data stream breaks up into several branches, depending on the programs.

4.3.5. Data Product Pipelines. The data branch requiring the most additional processing is that for the Primary Dynamics Program. When calibrated, it will closely resemble merged GONG data at a factor of 4 higher resolution. Production of the final datasets at NSO could take advantage of tools already in place for the comparable GONG reduction. Production of the Primary Structure Program and the Magnetic Program datasets on the other hand should involve no more than sorting. The various data branches appropriate to the Campaign Program will have many processing requirements, and will not generally produce standard datasets in pipeline fashion. To the extent that long-term or repeated campaigns have been identified we anticipate additional data reduction programs at Stanford, Lockheed, NSO, HAO, the University of Colorado, and probably other Co-I Institutions. An essential feature of these efforts is that the data products from all branches will be available to the entire team.

4.3.6. Data Access, Distribution, and Archiving. In order to avoid costly duplication of hardware while providing for maximum rapid access to the data, it would be ideal to configure a distributed archive with primary servers at Stanford and NSO, additional servers as appropriate at other sites with significant Campaign Program data archives, and servers at one or more supercomputing sites. This will require a more complex data management scheme than would a centralized database, but by distributing both the processing tasks and data storage requirements over several smaller
facilities it may well result in net hardware and operational cost savings. Most importantly, the data will be available where the investigations are. Furthermore, we anticipate that data media now being developed will make multiple copies of large portions of the final data products practical. Thus we plan to have copies of each data product available at each processing site whenever practical. The essential requirement will be a distributed data cataloging system. In this data management scheme Stanford will be the clearing house for tracking data through to the distributed data products for investigations by members of the SOI team, while NSO and NSSDC will provide broader community access.

We plan to have the long-term data archive maintained at NSO, at least insofar as the helioseismology data is concerned, to take advantage of its prior location there, proximity to GONG data, and continuing support resources available. The remaining data will be archived at either at NSO or at NSSDC.

4.3.7. Science Analysis. Scientific analysis of the data will be a team effort. We expect that subgroups of the team will take responsibility for the first analyses related to each of the identified science objectives. To support the science analysis a common workstation architecture will be defined and provided as possible. Network access to the Stanford and NSO systems will allow data searches, browsing, requests, updates, and some analysis. It is not expected that the SOI program will have the computing resources to fully investigate all of the science objectives. Access to supercomputers will be required to fully achieve some of the scientific objectives. We are exploring joint efforts with both the facilities at GSFC and with the National Supercomputing Centers to facilitate access and availability of data.

4.3.8. Theory Support. To aid in the design of observing programs and of data reduction and analysis procedures, the SOI program requires parallel development of helioseismology theory in areas such as inversion and modeling. Some of the computing requirements of this effort will come from the SOI systems and workstations, some will come from SOI-supported use of institutional computing resources, and some must come from the supercomputer facilities. We hope to include support for theoretical work in joint agreements with those facilities.

To support these efforts we will need a software management system, a sophisticated distributed database management system designed to work over a network with multiple media and multiple servers, and an analysis management system consisting of a science analysis library and a reduction and analysis system. These will provide the tools with which our widely dispersed team of investigators can work together conveniently and efficiently, exploring the huge amount of data that we hope will yield a much better understanding of the interior of the nearest star.

5. REFERENCES

ABSTRACT

The VIRGO (Variability in Irradiance and Gravity Oscillations) investigation has been selected to fly on the SOHO mission. One of the components of VIRGO is a small imaging instrument with limited spatial resolution. Calculations have been carried out in an attempt to find an optimal geometrical detector configuration for the observation of low degree g-modes in the solar luminosity. The constraints are given as a wish of having the largest possible sensitivity and redundancy with the smallest number of detectors. In addition the crosstalk for different modes at same frequency should be minimized.

Keywords: Solar g-modes, Radiance observations, Detector configurations.

1. INTRODUCTION

The VIRGO (Variability in Irradiance and Gravity Oscillations) investigation has been selected to fly on the SOHO mission. One of the components of VIRGO is a small imaging instrument with limited spatial resolution. This instrument, Luminosity Oscillations Imager (LOI), will observe the solar radiance with 12 pixel resolution. A detailed description of a prototype of the LOI is given elsewhere (Ref. 1).

The properties of the g-mode oscillations of low degree depend mainly upon the physical conditions in the central regions of the sun. Thus, they are more suited as probes to the solar core than the p-mode oscillations. However the g-modes are damped in the solar convection zone and the amplitudes of the modes are strongly reduced before they reach the observable surface. No single modes of g-mode oscillations have been identified unquestionable in the luminosity or Doppler measurements. Since the modes are difficult to detect the observations should be optimized with the greatest sensitivity for the most probable modes.

Calculations have been carried out to estimate the damping of low degree g-modes through the solar convection zone (Ref. 2). These calculations clearly show that if the low degree modes are exited with equipartition in energy per mode then it is very unlikely to observe global modes at the surface with degree higher than about \( l = 6 \).

Some work has been carried out in calculation spatial response functions for different detector configurations used in observations of solar oscillations (Ref. 3). They develop response functions for velocity observations with low or no spatial resolution on the solar surface. The relative variation of sensitivity with frequency and degrees \( l = 1 - 3 \) for radiance observations have also been done (Ref. 2). In this paper we want to look at the response functions for g-mode luminosity oscillations with specialized detector configurations with no more than 12-14 pixel elements over the solar disc.

2. DETECTOR SENSITIVITY

We assume that the radiance variation over the solar surface due to low degree g-modes can be described with spherical harmonics in the same way as the p-mode oscillations. The surface distribution of the radiance signal \( Y_l^m(\theta, \phi) \) may be described as

\[
Y_l^m(\theta, \phi) = I(\mu)P_l^m(\cos\theta)e^{im\phi}
\]

where \( I(\mu) \) describes the limb darkening function where \( \mu \) is the cosine of the heliocentric angle, and \( P_l^m \) is the associated Legendre function. The angles \( \theta \) and \( \phi \) are the latitudinal and the azimuthal angles, respectively. In the calculations the value of \( \phi \) may be looked upon as a phase angle. If only a single value of \( \phi \) is used then the calculated mode sensitivity for the low \( l \) values may be strongly influenced. In order to circumvent this problem we calculate the sensitivity as an average value of the sensitivity for 11 values of \( \phi \) ranging from -90 to +90 degrees.

The associated Legendre functions are calculated up to a value of \( l, m = 10 \) with the exact analytical formulas. The
limb darkening is calculated for a wavelength of 550 nm.

As reference we calculate the sensitivity for the different modes for a theoretical detector with 256×256 square detectors in the same manner as described by Ref. 4. This configuration is assumed to have full sensitivity for the modes with \( l \leq 10 \).

We have calculated the theoretical sensitivity for more than 90 different detector configurations, the results for three of them are given in Figs. 1-4. For all the multielement configurations the shapes of the detectors and thus the boundaries between them have been chosen to fit where the node lines of different modes fall on the Sun under the assumption that the rotation axis falls in the plane of the sky. This is done in an attempt to isolate different modes from each other in order to ease the mode identification. If some modes have larger amplitudes than others this selection process may randomly reduce the detectability of solar g-modes.

The choice of detector configuration depend very strongly upon the combination of the modes we want to observe. For a given number of detectors the total number of modes with \( l, m \) values we can observe with high sensitivity is roughly constant.

For these low degree modes the contamination between different modes caused by the apparent forward and backward tilt of the solar axis during one year is small. The SOHO spacecraft will be rotated such that the meridian of the detector is pointing along the solar rotation axis with an uncertainty of 15 arc minutes. This error is thus negligible for these calculations.

3. CONCLUSION

We have mainly investigated detector configurations with less than 12-14 detectors in order to get a realistic picture of how the different possible configurations may detect low degree g-modes. The exact configuration depends on which modes we are most interested in observing. For the measurement of frequency splittings caused by the rotation of
the solar core it is most interesting to resolve the modes with \( m \approx l \), however this is under the assumption that the core is not rotating obliquely. For a large number of possible g-mode frequency there is near degeneracy both with and without the inclusion of rotational splitting.

The argument may also be turned around, since the mode power spectrum will be most crowded by modes that may be split in frequency the best possibility to identify modes may be to look for \( m = 0 \) modes. This would indicate that we should choose a configuration which accentuates these modes and attenuates the modes with higher values of \( m \).

In reality we will have to make tradeoffs in the choice of mode sensitivity, this should be done on basis of the inclusion of mode combinations which will give the largest scientific output.

Figure 4. Same as Fig. 1, but for the configuration shown in the figure.

The current calculations have assumed that the potentially observable g-modes on the Sun have a surface distribution that may be described with simple spherical harmonics. If there is non-linear interaction between large amplitude modes below the convection zone this assumption may not be valid. It may then be more effective to chose a symmetrical configuration, but this non spherical harmonics representation will for a given number of detectors necessarily have sensitivity for lower values of \( l \). This may be counterproductive since the non-linear effects may decrease the spatial scale of the oscillatory disturbance.

More work has to be done at two levels before the optimum detector configuration for the flighty model of LOI can be chosen. Firstly it has to be decided which modes are the most interesting physically, and which tradeoffs must be made in this choice. Secondly the best configuration to meet the physical contents requirements. The latter is a relatively simple task using the methods described above.

4. REFERENCES

The VIRGO (Variability in IRradiance and Gravity Oscillations) investigation has been selected to fly on ESA's SOHO mission. One of the components of VIRGO is a small imaging solar photometer. This instrument, the Luminosity Oscillations Imager (LOI), will observe the solar radiance with 12 pixels resolution.

A prototype of the LOI has been developed at ESTEC. This prototype was built mainly to test possible detector configurations, the data acquisition system and the internal guider. The prototype has been operating at Izana, Tenerife since April this year.

Introduction

The VIRGO investigation is formed through a collaboration of five Helioseismology groups in Europe under the leadership of C. Fröhlich, Davos. As is indicated from the acronym the scientific goals may be divided into two related areas, first the measurement of the total and spectral irradiance of the Sun, and secondly the search for and identification of low degree gravity mode oscillations. A detailed description of the whole investigation is given by Fröhlich (Ref. 1).

The package to fly on SOHO comprises two units, the sensor unit which contains six sensors of four different types, and a power supply unit.

The instruments contained in the sensor unit are two types of absolute radiometers, two f-3 disc Sun photometers, and the LOI. The LOI consists of a small telescope which projects the solar image on a specially configured solid state multielement detector. The geometrical configuration of the detector is chosen to optimise the sensitivity of g-modes with degree in the range l=1-7 (Ref. 2).

In the frequency region of 10-100 mHz the density of g-modes is so high that the identification of single modes constitutes a major problem. In the lower frequency region, with a two year observation time string only 60% of the l=1-3 modes will be resolved in the frequency domain. It is therefore necessary to have spatial resolution in order to resolve the different modes. The other helioseismology instrument with spatial resolution onboard SOHO may not have the spatial and time stability to accurately measure the lowest degree long period g-modes.

The prototype LOI was built at ESTEC. It comprises a 70mm diameter 1300 mm focal length Ritchey-Chrétien telescope forming an image of the Sun of approximately 12.7 mm in diameter. The secondary mirror is mounted with two orthogonally placed piezoelectric pushers. The action of these pushers are used for fine guiding of the image.

The input optical bandwidth of the instrument is limited to~10nm centered at ~500 nm by a filter in front of the telescope. The filter comprises an edge interference filter combined with a GG495 glass filter, and an RG3 infra-red blocking filter.

The whole instrument with the telescope is mounted in a Spindler and Hoyer macrobench.

The LOI prototype is mounted on the same mount as the SLOT (Solar Luminosity Oscillations Telescope) at Izana on Tenerife. A detailed description of SLOT is given in Ref 3.

The Detector

For the prototype LOI a 10X10 element Si detector from CENTRONIC was used. These elements cover an area of 15 mm\(^2\) and the effective pixel size is a square with side of 1.4 mm. The various elements of the detector are paralleled to give the pattern shown in the figure. This detector configuration can easily (relatively) be changed by changing the strapping of the pixels. The detector is used non-biased.

The current configuration gives 10 surface elements over the visible solar hemisphere, in addition 4 are elements placed orthogonally along the solar limb. The signals from surface elements are sent through amplifiers to the data acquisition system. The output of the limb elements is used in a hardwired analog servoloop that drive the piezoelectric actuators on the secondary mirror.

The choice of the prototype detector configuration was done with two reasons in mind, firstly the configuration should be able to detect low degree g-modes from the ground and secondly the configuration should not be to different from the flight model.
Data Acquisition System

The output of the 10 surface elements of the detector are feed into 10 separate low power, low noise operational amplifiers (AD549). The amplified signal (amplification of approximately $10^4$) is transformed into frequency with 10 AD651 voltage to frequency converters. These are synchronous VFCs, this means that a reference frequency from an external oscillator is feed into the VFC and the output is a series of pulses of integer fractions of the input reference. The advantage of this is that when the same oscillator is used for the reference and for the filtering of the readout then small changes in the gaiting time due to oscillator variations are completely compensated for. For normal VFCs such small variations in the gaiting would directly be transferred into errors in the output signal.

In the prototype the gaiting signal taken from the SLOT instrument (Ref. 3) and is currently set at 13 seconds. The reference oscillator is taken from the same base oscillator and is currently at 1.25 MHz this implies a maximum count rate at full scale of half this value. The pulses from the VFCs is counted and entered into shift registers before they are loaded into a HP1986E microcomputer via a serial link. The data are stored on a floppy disc, writing one disc per day. The control computer is able to manipulate the data in real time giving plots of both time strings and power spectra of single channels or ratios of channel pairs.

Preliminary Results

The operation of the LOI prototype was commenced at Izafla in April 1988 and has continued intermittently to the present. Very strong interference from a nearby radio transmitter caused initial problems. With appropriate shielding and grounding these problems were sorted out. However the further results were hampered by problems with the internal guiding system. The construction turned out to have too small dynamic range to be able to compensated the angular offset errors caused by the SLOT mount on which the LOI is placed.

Only for short periods the guider managed to lock the solar image. When the offset was within the dynamic range of the secondary mirror tilting the guider locked the image to better than 1 arc second. The limit seems to be caused by the differential movement of the opposite limbs caused by seeing noise.

For some of the better periods the observations show power in the 5 minute band of the solar p-modes, but these initial data have not been of the quality to do quantitative analysis.

The data acquisition system is functioning as expected with noise levels at the level to be expected from the sampling. This is promising for the development of the flight model.

FLIGHT MODEL

The design of the flight model is of course not complete yet, though a number of parameters have had to be defined in the proposal. The instrument will measure $263\times90\times90$ mm, will weigh about 2950g, and consume about 2.875W. The telescope will be reduced in diameter to a 55mm, primary but the focal length will be maintained at 1300mm. The detector will be custom built with the required pixel shapes, and at the moment it is planned to produce this so that the front window can be used for ground testing, and then removed for the flight.

The design of the guider will have to be radically altered as this is the system causing the most problems at present. At the moment the plan is to use a two stage guiding system. The first stage will be a coarse guider, capable of providing offsets of up to 10 arc minutes, and will be operated at the beginning of the mission to first center the optics, and will only be operated again if there are any gross pointing errors. The second system will be the fine guider, which will be a closed servo loop using piezo-electric actuators to wobble the secondary mirror to take up any motion of the platform.

It is also planned to use the signal from the guiding segments of the detector to provide a monitor of the solar diameter, and maybe provide another means of measuring the solar oscillations.

The electronic design for the LOI is probably almost in it final form. The analog part of the data acquisition system will use discrete low noise/low power operational amplifiers in transconductance mode. It is hoped that the amplifiers can be obtained in die form, and they can then be assembled along with their resistors in the form of a hybrid using chip and wire technology. This gives a reduction in size, increase in reliability, and thermal matching all the components. Further, we intend to match the temperature coefficients of the resistors to minimise the temperature coefficient of the overall data chain.

The VFCs will again be of the synchronous variety, and hopefully also, formed from chip and wire hybrids with their associated passive components.

The counters and shift registers of the prototype LOI are a breadboard form of an Application-Specific Integrated Circuit (ASIC) being developed at SSD (Ref 4) for both this experiment and also particle physics experiments. It is hoped that the first ASICs will be delivered in the first quarter of next year (1989) for testing and that fully space qualified components will be available by the end of that year.

CONCLUSIONS

The experience gained from the prototype has been of great significance for the further development. In positive aspects we have proved that our design of the data collection chain is sufficiently accurate to meet the design specifications. In negative sense we have learned that our guider mechanics and possibly the electronics have to be redesigned, both these items are currently being investigated.

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INTEGRATED LIGHT AND VELOCITY OF SOLAR G MODES OSCILLATIONS

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ABSTRACT

The properties of the integrated intensity and velocity variations of the solar gravity modes for different spatial filters are studied in the adiabatic and nonadiabatic cases in the frequency range 40-120 MHz. Predicted amplitudes are given assuming equipartition of energy between the modes. As far as the intensity is concerned, a lack of amplitude has been pointed out around 6l/2, which should have implication on the whole disc Virgo and ground based observations.

Keywords: oscillations of the sun

1. INTEGRATED NON ADIABATIC INTENSITY PERTURBATION AND VELOCITY

In order to help the detection and identification of low frequency gravity modes in the context of future helioseismological experiments (Soho, Iris...), it is necessary to have an idea of which modes could be preferentially observed. The observed velocity and intensity perturbation are obtained by the integration of the respective quantities on the whole disk or on a portion of the solar surface according to the experience. Following the analysis developed in Reference (1-2):

\[ V = \omega a_{n, l, m}(x) A_{n, l, m} P_l^m(\cos \theta_0) \cos(\omega t + m\varphi_0) / (u_1 + W_{nad}(\omega, l) u_1) \]  (1)

\[ \delta F / F_0 = a_{n, l, m}(x) A_{n, l, m} P_l^m(\cos \theta_0) \cos(\omega t + m\varphi_0 + \psi_f) \]

\[ \left| \frac{\delta F}{F_0} \right|^2 = \frac{2^l G_{nad}(\omega, l)^2 + (2l - c_1)^2}{2l + 2G_{nad}(\omega, l)^2} \left( \frac{2l}{2l + c_1 \cos \theta_0} \right)^{1/2} \]  (2)

- \omega is the non dimensionned pulsation related to the frequency \( \nu \) by \( \omega = 2\pi \nu / \Omega_p \) with \( \Omega_p = \sqrt{GM / R^3} \).
- \varphi_0 and \( \theta_0 \) are the spherical coordinates of the observer.

The rotation axis of the Sun is taken as z axis. \( \theta_0 \sim 83^\circ \) is the mean angle the x axis and the ecliptic plane.

- The coefficients \( u_1, v_1, \theta_1, c_1 \) depend on the geometry of the portion of the Sun which is observed and of the limb darkening function.
- \( A_{n, l, m} \) is a normalizing coefficient which depends on the excitation mechanism. \( a_{n, l, m}(x) \) is the radial displacement at the level of observation \( x \), assuming \( a_{n, l, m} \) \( \sim \) 1 at the surface.
- \( W_{nad}, G_{nad} \) and \( \psi_0 \) are respectively the ratio between the horizontal and radial displacements, the ratio of the modulus and the phase lag between non adiabatic mean intensity fluctuation and radial displacement in the observed layers. They depend on the non adiabatic properties of the waves, on the structure of the atmosphere and of the level of observation. The rotation is assumed to be weak and its influence on \( \omega \) and the eigenfunctions is neglected.

Approximated expressions of \( W_{nad} \) and \( G_{nad} \) can be obtained in the adiabatic case, assuming an outer boundary condition which is a fit with an isothermal atmosphere. For frequencies of the 40 to 100 MHz range (\( \omega \) of order 1), a development relatively to the large parameter \( \Omega_c^2 (\Omega_c - (\Gamma g/2c))/\Omega_p \) gives:

\[ G(\omega, l) \approx 4 \Omega_{nad} \left( \frac{I - 1}{\omega^2} \right) 4 \omega^2 \left( 1 + \frac{\beta(\omega, l)}{4\omega^2} \right) \]  (3)

with

\[ \beta(\omega, l) \frac{l(l + 1)}{\omega^2} (1 - 2\Gamma) + 4(\Gamma - 1) \cdot \omega^2, \quad \Gamma = \left( \frac{d \log p}{d \log \rho} \right)_{ad} \]  (4)

\[ W(\omega, l) \approx \frac{1}{\omega^2} \left( 1 + G(\omega, l) \cdot \frac{\Gamma}{16 \Omega_{nad}^2} \right) \]  (5)

In the considered range of \( \omega \), \( G(\omega, l) \) has only one zero which corresponds for large \( l \) to frequencies of surface divergence free modes (\( \omega = \kappa g \)) for which the surface intensity of the wave is very small. For \( l = 1, \omega \approx 67 \mu Hz, l = 2, \omega \approx 10 \mu Hz, l = 3, \omega \approx 140 \mu Hz \ldots \) They will correspond to a minimum of the predicted intensity flux.
Figure 1: Variation of $\psi_0$ and $\psi_D$ (in degrees) relatively to the frequency for $l = 1$ and $l = 2$, at different levels near $\tau = 1$: $\log P = 5.13 (1), 5.08 (2), 5.01 (3), 4.95 (4), 4.83 (5), 4.73 (6), 4.63 (7), 4.53 (8)$.

The nonadiabatic numerical computation of the frequencies and eigenfunctions of the $g$ modes for a standard solar model assuming Eddington approximation has been performed in the frequency range $40-120 \mu Hz$ and for degrees $l = 1, ..., 7$. The convection is described by the classical mixing length theory. The perturbations of the nuclear energy and of the convective heat flux have been neglected. The results show that the radial displacement near the surface is not much modified while the intensity fluctuation is greater, so that $W_{naj}$ and $G_{naj}$ has the same behavior as $G$ but is enhanced by a factor $\sim 4$.

Due to the nonadiabatic process, there exists a phase lag $\psi_D$ between the integrated flux and the velocity perturbation, which is related to the phase lag $\psi_0$ by:

$$\cotg(\psi_D) = \cotg(\psi_0) + \frac{2b_0 - c_1}{b_1 G_{naj} \cos \psi_0} \quad (6)$$

The values of $\psi_0$ and $\psi_D$ are given in Figure 1 at different heights in the atmosphere around the level of observation $\tau = 1$, for $l = 1, 2$. For the deeper layers, $\psi_0$ is almost constant with a jump of about $180^\circ$ around $60 \mu Hz$ for $l = 1$ and around $100 \mu Hz$ for $l = 2$. Correspondingly the amplitude of $\psi_D$ is almost constant relatively to the frequency and varies continuously with increasing height from $-90^\circ$ to $0^\circ$. For layers nearer the surface, the behavior is more complex.

The estimation of the normalizing coefficient $A_{naj,m}$ requires the knowledge of the amplitude of the modes as a function of the degree and of the frequency. Up to now, different excitation mechanisms have been considered: excitation by convection, nonlinear coupling of modes, overstability due to nuclear burning effect, excitation by magnetic torque, but no convincing conclusions are achieved (Ref. 3).

Figure 2: Variation of the logarithm of the non dimensional energy as a function of the logarithm of the frequency, for different degrees $l, m = 0$, and assuming a radial displacement of the mode equal to 1 at the surface.
We have thus assumed an equipartition of the energy between the modes. The numerical values of the energy,
\[ E_{i,m}(\omega) = \frac{1}{2} \int \rho(\delta r, \delta r') \, dv \]
are given in Figure 2 for \( m \leq 0 \). Except for \( l = 1 \), the curves exhibit a nearly linear behavior corresponding to a power law \( E_{i,m} \sim \omega^{3.3} \), which can be compared to asymptotic expression (Ref. 2). If all the modes have an energy \( E_0 \) chosen arbitrarily equal to \( 10^{37} \text{erg} \), the normalizing coefficient is given by
\[ A_{n,i,m} = \sqrt{E_0 / E_{i,m}(\omega)} / [(l + m)! / (l - m)!] \]

2) VISIBILITY IN VELOCITY AND INTENSITY FOR DIFFERENT FILTERS

2.1) Predicted velocity amplitudes

Three different filters have been considered: whole disc, the difference between an external ring and a central disc corresponding to Stanford experiment (Ref. 4) and the difference between whole disc and external annulus corresponding to the Crimea experiment (Ref. 5).

As expected, for the whole disc observation the modes with low degree \( l = 1, 2 \) have the largest amplitudes (Figure 3) of the order of 0.5 to 1 \( \text{m} \text{s}^{-1} \) for modes with an energy of \( 10^{37} \text{erg} \), while modes with higher degrees may dominate in Stanford and Crimea observations. The lower value of the equidistance period of these modes \( (P_0 / \sqrt{l(l + 1)}) \) and the rotational splitting lead to a very dense and intricate spectrum, which will make the identification of the modes very complicated.

Figure 4: Predicted velocity for Stanford observations assuming an equipartition of energy \( E_0 = 10^{37} \text{erg} \).

Figure 5: Predicted velocity for Crimea observations assuming an equipartition of energy \( E_0 = 10^{37} \text{erg} \).

The deep minima for \( l = 1, 2, 3 \) which occur in the range 90 to 120 \( \mu \text{Hz} \) are due to pure geometrical effects through the combination of spatial filters.

2.2) Predicted intensity amplitudes

The whole disc observation favours the modes with degree \( l = 1 \) and \( l = 2 \) over almost the spectrum but with the peculiarity that there appear a deep minimum for \( l = 1 \) around 60 \( \mu \text{Hz} \) and for \( l = 2 \) around 100 \( \mu \text{Hz} \) (Figure 6). These frequencies are close to the zero of \( G_{\text{rad}} \) and they correspond to modes which are almost non divergent near the surface and have about ten radial nodes. The numerical computation of the eigenfunctions shows that these modes have a node near the bottom of the convection zone. The modes \( l > 3 \) have much lower amplitude. This behavior would render more difficult the detection of modes around 60 \( \mu \text{Hz} \) in experiment like Virgo. With the same assumption as before for the energy, the amplitude will be about \( 2 \times 10^{-5} \).

Another way to visualize these results is given in Figure 7, where an artificial spectrum of \( g \)-modes is plotted for \( l = 1, 2 \)
assuming a uniform rotation of the order of twice a mean solar surface rotation of $\Omega_R \sim 0.810^{-2}\Omega_\odot$ and taking into account the mean declination of the solar rotation axis relatively to the ecliptic. This induces the possibility to see with a small amplitude the modes with odd values of $l + m$.

If the intensity oscillations were observed with Stanford and Crimea filters (Figures 8 and 9), the largest amplitudes would be obtained for larger degrees respectively $l = 3, 4, 5$ and $l = 3, 4$ with $l = 2$ at low frequencies. These results are dependent on the assumption of energy equipartition and on the simplifying description of the non adiabatic effects.

2.3) Ratio between the amplitudes of the predicted magnitude fluctuation and velocity

This ratio is independent of the assumption made on the normalization. The result for whole disc observation is given in Figure 10. It has the same order of magnitude as the ratio given for radial $p$ modes (Ref. 7). However the behavior of the curves may be modified if both the perturbation of the convective flux by the pulsation is taken into account and the convection in the equilibrium model is treated in a more sophisticated way. In the case of $p$-modes for example, an extension of formulae (1) and (2) gives a monotonic increase of the ratio between the amplitudes of the predicted magnitude fluctuation and velocity relatively to the frequency (Ref. 8), while the introduction of the perturbation of the convective flux for radial $p$ modes leads to a minimum around $3mHz$ (Ref. 7-9).

A more complete discussion for the amplitude prediction concerning $g$ modes is given in Reference 2.
Figure 10: Variation of the ratio between the amplitudes of the predicted magnitude fluctuation and velocity relatively to the frequency for whole disc observations.

3. REFERENCES

The solar g-modes are extremely difficult to detect because of their very small amplitude and because of the absence of any regular pattern in their frequency distribution. A frequency dependent noise power spectrum and a g-mode power spectrum can look extremely similar. We propose to use cross-correlation and autocorrelation methods to test the presence of g-modes in a real power spectrum. This method is used on 3 years of ACRIM irradiance data with negative conclusion. It is also tested with synthetic g-modes computed into a standard and a non-standard model. Only the standard model seems to offer enough regularity in rotational splitting and period separation to make reliable the measurement of these two parameters.

Key words: Sun, Oscillations, g-modes. Simulation

1. INTRODUCTION

In the last ten years, helioseismology has developed at an increased speed mostly through the more and more precise measurements of pressure eigenmodes, with periods ranging between 3 and 8 minutes. These modes are trapped inside resonant cavities located between the solar surface and a turning which is deeper and deeper for lower and lower degree of the corresponding spherical harmonic function. Due to the temperature gradient inside the Sun, the major part of the travel time of a sound wave inside its resonant cavity is passed in the upper layers. As a direct consequence, the frequencies of such p-modes are more determined by the physical conditions in the outer part of the solar body. Only differential informations, such as small differences of frequencies of neighbour low degree modes, give access to the measurement of the sound speed inside the solar core. This measurement has from a major success and seems to exclude the influence of enough mixing in the core of the solar model to resolve the neutrino problem. However, the p-mode information is limited to one parameter, namely the sound speed, which itself depends on both temperature and the molecular mass. Therefore, different estimations of these two physical quantities can give the same sound speed, and a fixed range of different solar models can still be constructed and be compatible with p-mode helioseismological results.

This ambiguity would be ruled out by the detection and the identification of some gravity eigenmodes. Unlike p-modes, these g-modes have frequencies mainly determined by the physical conditions in the solar core. For this reason, the detection of just a few of them would be of great benefit for the seismology of this deep internal part of the Sun.

Although several results concerning the detection of such g-modes have been reported, they are conflicting and so, we can only conclude to-day that no convincing evidence for g-mode detection has been obtained so far. Indeed, the detection of the solar g-modes have proved to be extremely difficult for several converging reasons.

- They have certainly very small amplitudes in the convective envelope, where they are evanescent (although it is hopefully possible that their amplitudes increase in the upper layers of the solar atmosphere) and moreover, any noise spectrum displays increasing power with decreasing frequency and therefore, the noise level is much enhanced in the very low frequency range of the possibly excited g-modes.

- Their rotational splitting is of the same order as the separation between the frequencies of modes of consecutive radial orders, making the figure confused.

- In the standard solar model, the rotational splitting is almost constant in frequency, different for modes of different degrees. Modes of consecutive radial orders are almost constantly separated in period, with different separations for different degrees. The whole figure is then totally confused. In a model with a mixed core, splitting and period separation are not constant, making the situation even more complex.

The result is that a real g-mode power spectrum may very much look like a p-mode power spectrum. As an example, the figure 1 shows the power spectrum of 7 months of irradiance data obtained by the ACRIM instrument. The same figure could contain a gravity spectrum, or only a frequency-dependent noise spectrum of solar or instrumental origin.

Searching for Solar g-modes
Tests of a Statistical Method

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ABSTRACT

The solar g-modes are extremely difficult to detect because of their very small amplitude and because of the absence of any regular pattern in their frequency distribution. A frequency dependent noise power spectrum and a g-mode power spectrum can look extremely similar. We propose to use cross-correlation and autocorrelation methods to test the presence of g-modes in a real power spectrum. This method is used on 3 years of ACRIM irradiance data with negative conclusion. It is also tested with synthetic g-modes computed into a standard and a non-standard model. Only the standard model seems to offer enough regularity in rotational splitting and period separation to make reliable the measurement of these two parameters.

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1. INTRODUCTION

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2. METHODS FOR G-MODE DETECTION

With the space projects and the ground-based networks, the next few years will certainly provide lots of excellent data, in velocity and in photometry. We should be prepared, in this data, to look for g-modes with a very small signal-to-noise ratio, and consequently to develop adequate strategy. So far, the search for g-modes was made by means of spectrum by a window function makes necessary to approach the g-mode detection by a statistical method.

The basic idea that we wish to emphasize here is that if we consider two different noise power spectra, then their cross-correlation function will be only a cross-correlation of noise, with a flat behaviour around the zero frequency lag (Figure 2). If they contain also some g-mode pattern, then their cross-correlation function will also contain a term of autocorrelation with a typical central peak (Figure 3).

3. CROSS-CORRELATION AS A G-MODE SENSOR

This method can be applied to various circumstances, such as different observations obtained at different times by the same instrument, or obtained at the same time by different instruments. An illustration of the first case has been presented by Palle et al (These Proceedings), without positive conclusion. We present another illustration of this first case obtained with 4 time series of ACRIM data obtained in 1980, and between 1984 and 1986, of 6 to 9 months each. The Figure 4 shows the average of the 6 cross-correlation functions computed between the power spectra of these 4 time series, taken between 20 and 100 $\mu$Hz. It shows no evidence for g-modes with mean amplitude larger than 3x10^{-5} per mode. If only 10 radial orders of modes of degree 1 and 2 would be present, their mean amplitude would be less than 4x10^{-5}.

Figure 1. Low frequency range power spectrum of 7 months of ACRIM irradiance data.

Figure 2. The cross-correlation of two independent noise spectra should show no significant correlation near the zero frequency lag.

two different approaches. The first one consists in studying the statistical significance and/or the repeatability of the major peaks in a given frequency range (Ref.1). This method is well adapted if only one or a few g-modes are excited to an amplitude accessible to measurement. The second one makes use of an asymptotic approximation of the frequencies distribution, with two parameters, the period equidistance and the rotational splitting (Ref.2). Cross-correlations between real and artificial spectrum are used and are useful to disentangle the window effects. A two-dimensional map of the correlation coefficient can be drawn with the two parameters. It remains to study the statistical significance of the highest point of such a map. Unfortunately, the departure of the real frequencies from the asymptotic equation (Ref.3) makes this method not totally adequate because too constraining.

We think that, except in the case of the excitation of only a very few individual g-mode frequencies, the general confusion in the power spectrum, eventually made worse in the case of gaps in the time series implying the convolution of the

Figure 2. The cross-correlation of two independent noise spectra should show no significant correlation near the zero frequency lag.

Figure 3. If they include a solar g-mode pattern, the cross-correlation of two spectra will contain a central peak, signature of g-mode autocorrelation.

Figure 4. Average of the 6 cross-correlation functions of 4 ACRIM irradiance low frequency power spectra, from data time series of 6 to 9 months each.
For the next step of the proposed method of analysis, we shall assume that a positive result has been obtained by means of the cross-correlation, and that we will now deal with power spectra in which we know that there are peaks with measurable amplitude due to solar g-modes.

In this case, if the rotational splitting of the dipolar g-modes would not depend on frequency, and would be constant, assuming that only the modes of tesseral order \( m=1 \) can be observed, our power spectrum would contain two identical sets of peaks, just translated in frequency by twice the amount of rotational splitting. This situation is ideal for an autocorrelation analysis.

In order to test the reliability of this possibility, we have computed the frequencies of all g-modes of frequencies comprised between 20 and 100 \( \mu \text{Hz} \), in a standard model and in a model with a mixed core (mixed mass = 0.4). With the frequency, the rotational splitting has been computed for all modes of degrees 1 and 2. The Figure 5 shows this rotational splitting in the two models, for g-modes of degree 1 and 2, as a function of radial order, assuming four different depth dependences for the internal solar rotation. Assuming a frequency resolution of the order of 0.1 \( \mu \text{Hz} \) (4 months of data), it is clear that the rotational splitting can be regarded as constant in the standard model, within this resolution, for all modes of period longer than 2.5 hours. It is also clear that this is not true in the mixed model, where the splitting appears to be strongly dependent on the radial order. We have pushed the simulation one step forward by adding the \( l=1, m=1 \) and \( l=2, m=0, \pm 2 \) g-modes of radial order 7 to 25 of the standard model to the four ACRIM power spectra, with individual amplitudes of \( 7 \times 10^{-5} \). The Figure 6 shows the autocorrelation functions computed between 23 and 90 \( \mu \text{Hz} \). The mean splitting is clearly visible in the three cases.

This last analysis makes possible to measure the mean splitting inside the selected frequency range. The next step of our proposed analysis consists in eliminating the \( m \)-dependence, in order to represent the power spectrum as a function of the period \( T=1/\nu \), and to look for the period equidistance \( T_0 \). In the case of the \( l=1 \) g-modes, if we call \( s \) the mean frequency splitting, this is possible by replacing \( P(\nu) \) by \( P(\nu+s) + P(\nu-s) \). We have then a g-mode spectrum in which all peaks of degree 1 are added up on the \( m=0 \) positions. In the representation as a function of period, they will represent a set of almost equidistant peaks. A new auto-
Figure 6. All g-modes of degree 1, tesseral order ±1, and of degree 2, tesseral order 0, ±2, have been artificially added to the four ACRIM power spectra in the frequency range comprised between 23 and 90 $\mu$Hz, with the three slowest laws of rotation and individual amplitudes of $7 \times 10^{-5}$. The mean autocorrelation computed in this frequency range makes possible an estimation of the mean rotational splittings.

The period equidistance is limited in precision. The Figure 7 shows the distribution of periods of $l=1$, $l=2$, $m=0$ g-modes as a function of the radial order in the standard model and in the model with a mixed core. Here again, one can see that the period equidistance is consistent with the typical resolution of the observations in the same range than before, for periods longer than 2 or 3 hours. Clearly also, this is not true in the case of the non-standard model.

4. CONCLUSIONS.

If the real sun is well described by the standard model and if the cross-correlation analysis of two different power spectra has proved the existence of g-modes, then an autocorrelation analysis can be performed in order to find the mean rotational splitting.

As a next step, this mean splitting can be used to add together all modes of the same degree.
and to turn from frequency to period. A new autocorrelation analysis can be performed in order to find the mean period equidistance.

The following step, which consists in the identification of individual modes, is then made easier.

This method is a statistical one, which is inadequate if only a few g-modes are excited to amplitudes just above the surrounding noise level.

If the sun is not described by the standard model, the efficiency of this statistical method will be decreased because both rotational splitting and period spacing may not be constant any more. More tests remain to be performed in this case and will be dealt with in a forthcoming paper.

9. REFERENCES

OBSERVED ASYMPTOTIC PROPERTIES OF LOW-DEGREE SOLAR GRAVITY-MODE EIGENFREQUENCIES

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ABSTRACT

The asymptotic properties of the low-degree solar gravity modes classified by Hill and Gu (Ref. 1) are studied in the framework of first- and second-order asymptotic theory predictions (Ref. 2). The results of this analysis demonstrate the necessity of retaining the second-order term in asymptotic theory to describe the eigenfrequency spectrum. In this theory, there are two first-order parameters, $T_D$ and $\delta$, and two second-order parameters, $V_1$ and $V_2$. Values of the parameters obtained in this analysis are: $T_D = 36.31 \pm 0.12$ min, $\delta = -0.43 \pm 0.13$, $V_1 = 0.35$, and $V_2 = 4.76$. There remain differences of $\approx 0.3 \mu$Hz between the asymptotic theory eigenfrequencies and observed eigenfrequencies which are quasi-periodic functions of the radial order $n$ for a given value of the degree $l$.

Keywords: gravity modes, asymptotic parameters

1. INTRODUCTION

The theoretical predictions of asymptotic theory (Ref. 2) have been tested both with eigenfrequency spectra of standard solar models (Refs. 3, 4) and an eigenfrequency spectra based on observations (Refs. 5-7). The period $T_{nf}$ of a mode of radial order $n$ and degree $l$ (azimuthal order $m = 0$) in second-order asymptotic theory (Ref. 2) is given by

$$T_{nf} = T_C \left( n + \frac{l}{2} + \delta \right) \left[ \frac{1}{T_D + (l+1) V_1 + V_2} \right]^{1/2}.$$

where $T_D$, $\delta$, $V_1$, and $V_2$ are constants. The term $T_C$ is related to the Brunt-Väisälä frequency $N$ by the equation

$$T_C = 2\pi \left( \frac{T_C}{N} \right)^{1/2},$$

where $r_c$ is the radius of the convection zone. The constant $\delta$ depends on the structure just below the base of the convection zone. The expressions for $V_1$ and $V_2$ are much more complicated than that for $T_D$ and reference is made to Tassoul (Ref. 2) for these details. In first-order asymptotic theory, $V_1 = V_2 = 0$.

The extent to which the asymptotic theory predictions are realized depends upon the degree to which the properties of the solar interior meet the conditions required for the applicability of the theory. As Christensen-Dalsgaard et al. (Ref. 8) state, "The asymptotic analysis is valid if the scale of variation of the background is much greater than all oscillation wavelengths considered, and that condition is indeed satisfied throughout most of the interior of any traditional theoretical model of the Sun." As a corollary, any observed systematic deviations from asymptotic theory predictions may be the manifestation of conditions in the solar interior which do not meet the requirements for asymptotic theory to be applicable. Therefore, the study of the asymptotic properties of the observed eigenfrequency spectrum takes on a particular importance: in addition to possibly obtaining constraints on $T_D$, $\delta$, $V_1$, and $V_2$, it may offer a diagnostic probe to those regions of the solar interior where a significant fractional change occurs in the background state in a characteristic length of the eigenfunctions used in the study. This is suggestive of the null experiments encountered in physics and enjoys the same attractive feature. That feature in this case is that the deviations from the predictions of asymptotic theory allow us to study directly those presumably restricted regions where the conditions for the applicability of asymptotic theory are not met.

In the following sections we examine $(\nu_n^{s})_{ib} - (\nu_n^{s})_{ob}$ for the $l = 2, ..., 5$ gravity modes in the mode classifications of Hill and Gu (Ref. 1) for asymptotic properties in the framework of first- and second-order asymptotic theory predictions given by Eq. (1).

2. LOW-DEGREE GRAVITY-MODE CLASSIFICATIONS

A series of low-degree gravity-mode multiplets with $m = 0$ eigenfrequencies between 60 and 220 $\mu$Hz have been classified by Hill and Gu (Ref. 1). The work on gravity-mode classifications is part of an extensive mode classification program at SCLERA (Ref. 9) that was started in 1982. This program has been described in numerous works (Refs. 1C-14) and is based on the physical properties of the eigenfunctions, on the use of combinations of observations that pass different spatial filter functions, and on the theoretical eigenfrequency spectrum of a standard solar model for determining the radial order $n$.

The gravity-mode spectrum, in general, is much more complex than the low-order, low-degree, acoustic-mode spectrum. The complexity of the spectrum was first
reduced to a manageable level in the work of Hill (Ref. 12) by the successful combination of differential velocity observations from the Crimean Astrophysical Observatory (Ref. 15) and of the 1979 differential-radius observations made at SCLERA (Ref. 16). Because of the different spatial filter functions for these two sets of observations, it was possible to decouple I and m in the mode classification program. The result was the classification of 31 gravity-mode multiplets with I = 1, ..., 5. In a subsequent analysis, Gu and Hill (Ref. 17) and Hill and Gu (Ref. 1) extended the Hill (Ref. 12) work using the mode classification techniques developed at SCLERA. The result was 53 classified gravity-mode multiplets.

3. TESTS OF GRAVITY-MODE CLASSIFICATIONS

There have been a series of independent tests made of the Hill (Ref. 12), Gu and Hill (Ref. 17), and Hill and Gu (Ref. 1) gravity-mode classifications. The first series by Hill and Kroll (Ref. 18); Kroll, Huang, and Hill (Ref. 19), and Kroll, Hill, and Chen (Ref. 20) is based on an analysis of the total irradiance observations obtained from the Solar Maximum Mission. The second series is based on the power spectrum of the 1985 radiation intensity observations made by Oglesby (Refs. 21, 22). The third series by Hill (Ref. 23) is based on an analysis of the 1978 solar diameter observations made by Caudell et al. (Ref. 24). These series of tests fall into several categories. Some test for detection of gravity modes, some test for identification of multiplets, and others test I and m assignments that have been made. Highly statistically significant positive results have been obtained in each of the series of tests. It is the strength of these independent tests that motivate the determination of the asymptotic properties of the classified spectrum by Hill and Gu (Ref. 1).

4. RESULTS

The values of \( v_{nl} \) for the gravity modes classified by Hill and Gu (Ref. 1) were first examined in the framework of first-order asymptotic theory. In this case, the theoretical \( v_{nl} \) are given by Eq. (1) with \( V_1 = V_2 = 0 \). Results similar to those reported by Rosenwald and Hill (Ref. 6) were found. In the work of Rosenwald and Hill (Ref. 6), the values obtained for \( T_0 \) were given by the second-order \( V_1 \) and \( V_2 \) terms retained in Eq. (1). In this analysis, the observed \( v_{nl} \) for each given I were fitted to the theoretical \( v_{nl} \) given by Eq. (1) for the best values of \( T_0, V_1, \) and \( V_2 \). The results of the analysis are:

\[
T_0 = 36.31 \pm 0.12 \text{ min} \\
\delta = -0.43 \pm 0.13 \\
V_1 = 0.35 \\
V_2 = 4.76 \\
\]

The least-squares analysis was done separately for each set of multiplets with \( I = 2, ..., 5 \). The \( I = 1, m = 0 \) classified eigenfrequencies were not used in this analysis because there are only a few data points available from this set of multiplets.

Using the values of the constants in Eq. (1), the proficiency of the second-order theory to describe the observed \( v_{nl} \) is tested by examining the difference \( (v_{nl})_{ba} - (v_{nl})_{ab} \) as a function of \( n \) for each of the values of \( I \). The results for \( I = 4 \) are shown in Fig. 1, which clearly show deviations that are quasi-periodic functions of the radial order \( n \). Similar results are found for \( I = 2, 3 \) and 5. The values of \( n_{max} \) and \( n_{min} \) are listed in Table 1, where \( n_{max} (n_{min}) \) denotes values of \( n \) for which \( (v_{nl})_{ba} - (v_{nl})_{ab} \) obtains a local maximum (minimum). A typical amplitude for these deviations is 0.3 \( \mu \text{Hz} \).

5. SUMMARY

It is found that the g-mode spectrum classified by Hill and Gu (Ref. 1) can be represented quite well by second-order asymptotic theory accompanied by a quasi-periodic term. The quasi-periodic term has an amplitude of approximately 0.3 \( \mu \text{Hz} \).
The set of second-order asymptotic theory parameters obtained in the fit to the classified spectrum can give us important information on the internal structure of the Sun and, in particular, can be used to reduce the number of viable solar models. The value of \( T_2 \) can be computed for different solar models. On comparing these theoretical predictions with the observational-based value of \( T_2 = 36.31 \) min given by Eq. (3), it is found that the present result is in general agreement with standard solar models, but inconsistent with WIMP models and mixed models (see Ref. 25 and references therein). The later two models were suggested as possible solutions of the solar neutrino paradox. The value of \( \delta \) is predicted to have a value between \( -\frac{1}{4} \) and \( -\frac{1}{10} \), depending on the structure of the temperature gradient at the base of the convection zone. The observed value of \( \delta = -0.425 \) is close to \(-\frac{1}{4}\). The value obtained by standard solar models computed with local mixing length theory (Ref. 26). This limiting value is predicted if the subadiabatic temperature gradient falls linearly to zero as the lower boundary of the convection zone is approached from below and being essentially zero in the convection zone (Ref. 26).

This work was supported by the U.S. Department of Energy and the U.S. Air Force Office of Scientific Research.

6. REFERENCES

9. SCLERA is an acronym for the Santa Catalina Laboratory for Experimental Relativity by Astrometry, operated by the University of Arizona.
INVERSION OF QUASI-PERIODIC DEVIATIONS BETWEEN LOW-DEGREE SOLAR GRAVITY MODE EIGENFREQUENCIES AND ASYMPTOTIC THEORY EIGENFREQUENCIES

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ABSTRACT

The fine structure found by Gu, Hill and Rosenwald (Ref. 1) between asymptotic theory eigenfrequencies and the observed eigenfrequencies reported by Hill and Gu (Ref. 2) is interpreted as the result of conditions not being met for the applicability of asymptotic theory at one or more radii in the solar interior. From an inversion of the observed fine structure, reasonably good agreement is obtained between observation and theory for either a localized perturbation in internal structure at $r/R = 0.06$ or at $r/R \sim 0.23$. The latter solution is, however, the better one. The amplitude of the perturbation in the mean molecular weight required to produce the fine structure is also inferred.

Keywords: Gravity-mode, Internal structure, Inversion

1. INTRODUCTION

A fine structure may be present in an eigenfrequency spectrum for a number of different reasons. The more widely known example from solar seismology is the fine structure generated by rotation of the Sun. For a given set of normal modes, a fine structure may also be caused by localized changes in the background state of the Sun when those changes occur over a distance shorter than the local radial wavelengths of the respective eigenfunctions. Examples of this are found in the works of Hill and Rosenwald (Ref. 3), Kidman and Cox (Ref. 4), Gavryusev and Gavryuseva (Ref. 5), and Berthomieu and Provost (Ref. 6). Analogous effects have received much attention in the study of the normal modes of the Earth. For example, Lapwood and Usami (Ref. 7) have called this fine structure effect the "solotone" effect.

The first example of such structurally induced fine structure on solar eigenfrequencies was obtained by Hill and Rosenwald (Ref. 3) for low-degree five min oscillations. This fine structure has been identified with structural changes that occur over a relatively short distance at the base of the convection zone. A second example of observed fine structure which is presumably structurally induced has been found in the study of low-degree solar gravity modes by Rosenwald, Hill and Gu (Ref. 8) and Gu, Hill and Rosenwald (Ref. 1). This fine structure was detected as quasi-periodic deviations of the observed eigenfrequency spectrum from asymptotic theory predictions.

It was observed in (Refs. 1, 8) that a perturbation in the Brunt-Väisälä frequency localized at 0.2 solar radii would give rise to the type of fine structure observed. An inversion of the observed fine structure is made in the following sections to better determine the properties of the localized structural changes that could account for the fine structure.

2. OBSERVED FINE STRUCTURE

A series of low-degree gravity mode multiplets have been classified in the SCLERA (Ref. 9) mode classification program (Ref. 2). The $m = 0$ eigenfrequencies $\nu_n$ of the above modes lie between 60 and 220 $\mu$Hz with degrees $\ell = 1$, $\ldots$, 5. Subscript $n$ is the radial order and $m$ is the azimuthal order. A total of 53 multiplets are included in this set. The eigenfrequencies of these multiplets were observed to have a fine structure in addition to that due to rotation. A quantitative study of the newly discovered fine structure has been made by Gu, Hill and Rosenwald (Ref. 1) by comparing the observed $\nu_n$ with those predicted by asymptotic theory.

The observed fine structure effects, which are of interest here, are characterized as quasi-periodic deviations of the classified spectrum from the spectrum predicted by second order asymptotic theory. The deviations are periodic in radial order $n$ for a fixed $\ell$ and not dependent on $m$. For more specifics concerning these deviations, reference is made to Gu, Hill and Rosenwald (Ref. 1). Both the amplitudes and the periods of the deviations will be used in the inversion project.
A particularly simple inversion technique may be employed to infer the internal properties of the Sun which give rise to quasi-periodic deviations of \( \nu_n^2 \) from the values predicted by asymptotic theory. As noted in the Introduction, quasi-periodic deviations from asymptotic theory are expected for a given set of normal modes when a significant fractional change in the background state of the Sun occurs within a characteristic length shorter than the distance characterizing changes in the respective eigenfunctions. The location in radius of a significant fractional change in the background state determines the period in \( n \) of the periodic deviation; the magnitude of the fractional change in the background state determines the amplitude of the periodic deviation. It should thus be possible, by using observational results and sensitivity analysis results for a given standard solar model, to determine both the location and magnitude of the change in the background primarily responsible for the observed quasi-periodic deviations.

The sensitivity analysis required for the inversion project was performed using the standard solar model of Saio (Ref. 10). In this analysis, the derivatives \( \Delta \nu_n^2 / \Delta A^* \) were determined as a function of \( n, \ell \), and \( i \), where \( \Delta \nu_n^2 \) is the perturbation of linear eigenfrequency \( \nu_n^2 \) due to a perturbation \( \Delta A^* \) in \( A^* \). The parameter \( A^* \) is related to the Brunt-Väisälä frequency \( N \) by

\[
A^* = \frac{\mathbf{g}^2}{\mathbf{g}}
\]

where \( g \) is the value of the acceleration of gravity at radius \( r \). For the determination of the derivatives \( \Delta \nu_n^2 / \Delta A^* \), the \( \nu_n^2 \) for the \( \ell = 2, 3 \), and \( 5 \) gravity mode multiplets classified by Hill and Gu (Ref. 2) were calculated both for the unperturbed Saio model and the Saio model perturbed by \( \Delta A^* \). In these calculations, standard inner and outer boundary conditions (Ref. 11) were used. It is noted that the Saio (Ref. 10) standard solar model may furnish an adequate starting point for the inversion project because the \( \nu_n^2 \) of this model are in reasonably good agreement with the classified eigenfrequencies of Hill and Gu (Ref. 2).

Two independent parameters appearing in the differential equations which uniquely describe linear adiabatic oscillations of the Sun are \( A^* \) and \( \nu_n^2 \), where

\[
\nu_n^2 = \frac{\mathbf{g} \epsilon}{c^2}
\]

and \( c \) is the speed of sound. It is only necessary in the present analysis, however, to consider perturbations in \( A^* \). This is because, first, in the solar interior below the convection zone, the magnitude of the terms \( \Delta \nu_n^2 / \Delta A^* \) are typically smaller than \( \Delta \nu_n^2 / \Delta A^* \) by a factor of 100. Second, \( A^* \) can exhibit rather large peaks due to the relatively high sensitivity of \( \nu_n^2 \) to gradients in the mean molecular weight \( \mu \). The \( \Delta \nu_n^2 / \Delta A^* \) are found for a given \( \ell \) and \( i \) to be a quasi-periodic function of \( n \). The results for \( \ell = 4 \) are shown in Figure 1 for \( \Delta A^* / A^* = 0.06 \) in a zone with a mean fractional radius of \( x = 0.249 \) and a zone width \( \Delta x = 0.0089 \), where \( x = r/R \) and \( R \) is the solar radius. Similar results are obtained for \( \ell = 2, 3 \), and \( 5 \). The location of the minima \( n_{\min}(\ell, x) \) in the set of figures are determined as functions of \( \ell \) and \( x \). For each value of \( \ell \), a diagnostic diagram is generated by plotting the fractional zone radii \( x_i \) as a function of the location of the minima \( n_{\min}(\ell, x) \). The diagnostic diagram obtained for \( \ell = 4 \) is shown in Figure 2. The minima are used in this analysis because a positive perturbation in \( A^* \) gives rise to a negative value of \( \Delta \nu_n^2 \) (see Figure 1).

The second phase of the inversion technique consisted of placing vertical lines on the diagnostic diagrams at the locations of the observed minima and testing for a solution. The vertical lines appropriate for the \( \ell = 4 \) mode are also shown in Figure 2.
The family of curves which gives the locations of the minimum $n_{\text{min}}$ for $\ell = 4$ multiplets in the derivatives $\nu_{nl}^2 / \Delta A^2$ as a function of the $x$ location of $\Delta A^2$.

A solution is considered to exist if there is a set of intersections of the two families of lines in the diagnostic diagrams that lie on a horizontal line. It is apparent from inspection of Figure 2 that for $\ell = 4$, there are solutions for $x = 0.23$ and for $x = 0.06$. Results consistent with the $\ell = 4$ solutions at $x = 0.23$ and $x = 0.06$ are also found for $\ell = 2$, 3 and 5 modes. However, the relative scatter of the locations of the intersections in the diagnostic diagrams about the $x = 0.06$ solution is about a factor of 5 larger than the relative scatter about the other possible solution at $x = 0.23$. Also, the better $x = 0.23$ solution is obtained when comparing the observed $n_{\text{min}}$ with the model determined $n_{\text{min}}$ not the observed $n_{\text{max}}$ with the model determined $n_{\text{max}}$. It is thus concluded that the primary source of the observed periodic deviations is a positive perturbation in $A$ located at a radius near $x = 0.23$. The weighted average $\bar{x}$ of the solutions for $\ell = 2, 3, 4$ and 5 yields the result

$$\bar{x} = 0.235 \pm 0.005$$

The error in Equation (3) is based on the standard deviation of the $x$ coordinates of the intersections found in the diagnostic diagrams for the $x = 0.23$ solution and is a measure of the internal consistency of the findings. Certainly, the value of the absolute error depends on the correctness of the Saio (Ref. 10) solar model and no estimate of this type of error has been made.

The strength of this secondary perturbation in $A^*$ would be about 1/3 of the primary perturbation in $A$ located at $x = 0.23$. It is assumed in the following sections that the sign of the secondary perturbation in $A$ is also positive. However, no information about the sign has been inferred from the observations.

4. MAGNITUDE OF $\int \left( \frac{\Delta A^*}{A^*} \right) \, \text{dr}$

The magnitude of $\int \Delta A^*/A^* \, \text{dr}$ can be estimated from the observed amplitudes of the periodic deviations and the computed derivatives $\nu_{nl}^2 / \Delta A^2$. The results are

$$\int \Delta A^*/A^* \, \text{dr} = \begin{cases} 0.0045 \text{R} ; \bar{x} = 0.235 \\ 0.0015 \text{R} ; \bar{x} = 0.19 \end{cases}$$

It is assumed in obtaining the results expressed in Equation (5) that the effective radial widths of $\Delta A^*$ are less than approximately one quarter of the radial wavelength of the eigenfunctions used in the observations. This corresponds to $\Delta x \leq 0.015$.

5. INFERRED RADIAL PROPERTIES OF $\mu$

One physical parameter that could be responsible for the localized perturbations in $A$ is the mean molecular weight $\mu$. For a sharp transition of $\mu$ from one value to a second value, the change $\Delta \mu = \mu(r_2) - \mu(r_1)$ can be expressed to a good approximation in terms of $\Delta A^*$ as

$$\Delta \mu = \frac{R c_1}{M} \cdot \frac{R^2}{r_1^2} \int_{r_1}^{r_2} \left( \frac{\Delta A^*}{A^*} \right) \, \text{dr}$$

where

$$\frac{1}{c_1} = \frac{M_r}{R^2} \cdot \frac{3}{M}$$

$r_2 > r_1$, $M_r$ is the solar mass interior to radius $r$ and $M$ is the total solar mass.

Using results for $\int (\Delta A^*/A^*) \, \text{dr}$ given by Equation (5), we find

$$\Delta \mu = \begin{cases} -0.009 ; \bar{x} = 0.235 \\ -0.003 ; \bar{x} = 0.19 \end{cases}$$

5. DISCUSSION

The basic features of the fine structure found in the classified low-degree gravity mode spectrum...
change in the mean molecular weight occurring in
the effective radial width of $\Delta x \leq 0.015$. The
magnitude of the fractional change in $\mu$ is $|\Delta \mu/\mu| =
0.009$ at $x = 0.235$.

Does this inferred change in $\mu$ imply a mixed core
and if so, is a mixed core consistent with other
observational results? If the core of the Saio
model (Ref. 10) were completely and instantaneously
mixed out to $x = 0.235$ (but was unmixed prior to this),
there would exist at $x = 0.235$ a decrease in $\mu$ with the fractional change of

$$\left(\frac{\Delta \mu}{\mu}\right)_{\text{mix}} = -0.060 \quad (9)$$

Therefore, the inferred $\Delta \mu/\mu$ at $x = 0.235$ given in
Equation (8) is only a fraction of $(\Delta \mu/\mu)_{\text{mix}}$.
Quantitatively,

$$\left(\frac{\Delta \mu}{\mu}\right)_{\text{mix}} = 0.15 \quad (10)$$

It is concluded from the result in Equation (10)
that the small mixing required to yield the inferred
$\Delta \mu/\mu$ given by Equation (8) is sufficiently
small so as not to be very important in resolving the
solar neutrino paradox. The fractional change is
also sufficiently small so as not to be inconsis-
tent with the observational based value of $Y_0$
obtained by Gu, Hill and Rosenwald (Ref. 1) and the
five min oscillations which indicate that the core
is not mixed.

Clearly, the inferred $\Delta \mu/\mu$ could be the result of a
partial mixing of the core in the past. One inter-
esting question that might be raised is what would
be the time of the following hypothetical complete
mixing event: The core of the Sun becomes com-
pletely mixed at an early age out to $x = 0.235$, then
the interior becomes permanently stable
against mixing while the discontinuity in $\mu$ at
$x = 0.235$ remains as a "fossil". The answer to
this question is an age of the order of 0.7 Gyr.
Actually, the restriction on stability can be
relaxed to the situation where subsequent mixing
occurs at $x$ values significantly less than 0.235.
In fact, the second inferred $\Delta \mu/\mu$ at $x = 0.19$
might be a location of subsequent mixing.

In light of this discussion, the observed fine
structure in the eigenfrequencies may be considered
"fossil" records or "footprints" in the Sun. This
is a particularly intriguing possibility.

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of Energy and the Air Force Office of Scientific
Research.

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9. SCLERA is an acronym for the Santa Catalina Laboratory for Experimental Relativity by Astrometry, operated by the University of Arizona.


THE MICHELSON DOPPLER IMAGER FOR THE SOLAR OSCILLATIONS IMAGER PROGRAM ON SOHO

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ABSTRACT

The Michelson Doppler Imager (MDI) will be the instrument used in the Solar Oscillations Imager Program on SOHO. MDI will make a line-of-sight velocity map of the full solar disk with 2 arc-second pixels each minute. The instrument will be a modification of the Fourier Tachometer and will operate by using narrow bandpass solar images at four wavelengths to measure the line profile of the Ni I line at 6768 Å. This method is relatively insensitive to line profile changes and has a linear response to velocity. The instrument is also capable of making partial maps with 0.7 arc-sec pixels. All data will be transmitted to the ground for two continuous months each year and 8 hours each day (160 kilobits/sec). At all times the on-board computer will compute and transmit a selection of modes (5 kilobits/sec) to take full advantage of the advantages of a space based telescope. Line-of-sight magnetic fields will also be measured regularly. The flight instrument will be built by the Lockheed Palo Alto Research Laboratory.

Keywords: Helioseismology, Spacecraft Instrumentation, Solar Oscillations

1. INTRODUCTION

The Michelson Doppler Imager (MDI) instrument on the Solar and Heliospheric Observatory (SOHO) will greatly enhance our ability to measure solar oscillations, particularly those of high degree. Going to space provides the required conditions for continued progress in helioseismology: high resolution images undistorted by atmospheric seeing and freedom from a host of problems caused by Earth’s rotation, particularly diurnal gaps and varying atmospheric transparency. The MDI design has been optimized to take full advantage of going to space. Global observing networks can eliminate gaps, but will not completely eliminate these effects. The SOI program will complement these networks and the other helioseismology instruments on SOHO.

A description of the scientific objectives of the SOI program, the team approach to conducting the science investigation, the observing program, and the plans for data reduction and analysis are described elsewhere (Ref. 1). The primary scientific objectives are to measure the internal stratification and dynamics of the Sun, providing stringent tests of stellar models and leading to understanding of solar activity and the solar cycle. The primary objectives will be addressed using the methods of helioseismology whose primary observable is the line-of-sight velocity. The set of associated objectives, necessary for understanding solar dynamics, require measurements of additional quantities including transverse velocity, line-of-sight magnetic field, and intensity. The list of MDI observables, methods of observation, noise, and sensitivity are given in Table 1.

2. OBSERVATIONAL METHOD

Correct measurements of the line-of-sight velocity are crucial to the success of the SOI program. Our method is an adaptation of the Fourier Tachometer and has several advantages: it is stable, provides a linear signal in velocity, and is insensitive to line profile variations and to CCD irregularities.

2.1 Description of the MDI Velocity Measurement Technique

The line position is determined by making four intensity measurements at wavelengths evenly spaced across the Ni I 6768 absorption line. The wavelength is selected by a fixed Lyot filter and tunable Michelson interferometers. Figure 1 shows the theoretical spectral profiles of the solar line and various components of the instrument.

Figure 1: The light solid curve is the assumed line profile; the dotted curve is the Lyot transmission profile, the heavy solid curve is the product of the line profile and the Lyot transmission; and the dashed curve is the transmission profile of the Michelsons tuned to zero displacement.

Two linear combinations of the four intensities give the fundamental sine and cosine components of a four-point discrete Fourier transform in wavelength. If we describe the line intensity by \[ I(t) = I_0 - A e^{-j2\pi ft} \] and consider the the spectral range 2FWHM around line center, then over 99% of the power is contained in the fundamental Fourier coefficient.
Table 1. SOI - MDI Observables

Direct and derived observables. The time shown is the minimum time needed for one measurement. The noise describes the formal statistical uncertainty in the measurement process for that time. The sensitivity quoted is the estimate of residual instrumental systematic effects. The ability to measure a given physical phenomena is usually limited by solar noise sources rather than the quoted sensitivity or noise.

<table>
<thead>
<tr>
<th>Observable</th>
<th>Method</th>
<th>Time</th>
<th>Noise</th>
<th>Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{d}$</td>
<td>Doppler Velocity</td>
<td>MDI adaptation of Fourier Tachometer</td>
<td>20 sec</td>
<td>12 m/s</td>
</tr>
<tr>
<td>$V_{l}$</td>
<td>Transverse Velocity</td>
<td>Correlation tracking of granulation intensity</td>
<td>8 hr</td>
<td>15 m/s</td>
</tr>
<tr>
<td>$I_{l}$</td>
<td>Line intensity</td>
<td>Derived from $V_{l}$ measurement</td>
<td>20 sec</td>
<td>0.1%</td>
</tr>
<tr>
<td>$I_{r}$</td>
<td>Continuum intensity</td>
<td>Direct measurement</td>
<td>5 sec</td>
<td>0.2%</td>
</tr>
<tr>
<td>$B_{r}$</td>
<td>Line-of-sight Field</td>
<td>Derived from $V_{l}$ in both circular polarizations</td>
<td>40 sec</td>
<td>6 gauss</td>
</tr>
<tr>
<td>$R$</td>
<td>Solar limb position</td>
<td>Derived from $I_{l}$ measurement</td>
<td>5 sec</td>
<td>0.001</td>
</tr>
</tbody>
</table>

$U/l = \sum a_{i} \cos \frac{2\pi \lambda}{2 \text{FWHM}} + \sum b_{i} \sin \frac{2\pi \lambda}{2 \text{FWHM}}$ and $P_{a} = a_{2}^{2} + b_{2}^{2}$ we find that $P_{a} \leq \sum P_{a} > 0.99$. The phase measures the Doppler shift of the spectral line.

This technique retains the advantages of the Fourier Tachometer without the disadvantage of sampling a broad spectral range and requires fewer computations. Because one spectral bandpass is measured for all pixels simultaneously, the dynamic range must allow a linear measurement of velocity from any pixel. Over the disk the following contribute to the velocity signal: rotation, ±2000 m/s; limb shift, 700 m/s; meso- and supergranulation, ±600 m/s; granulation, 1000 m/s; p-mode oscillations, ±500 m/s; observer motion, several hundred m/s; and various smaller effects. Thus the dynamic range must be about ±4000 m/s.

Figure 2 compares the theoretical response using the Ni I 6768 line and the transmission curves above for the four point method and the traditional two-point dopplergram method. The figures show the velocity determined over a ±5000 m/s range. The two point method is highly non-linear and much more sensitive to variations in the line profile.

2.2 Description of the Instrument

The measurement technique was described above; we present the details of the implementation in this section. It should be emphasized here that the project is still in the definition phase and so the details presented here are preliminary and do not necessarily present the final configuration of the instrument. The instrument will be described roughly in the order that light will pass through it. Diagrams of the optical and mechanical layouts are presented in Figures 3 and 4. The instrument is 145 cm by 22.5 cm by 30 cm. We estimate that the instrument will have a mass of slightly over 55 kg and consume about 45 watts.

Light passes through front window when the front door is opened. The front window reflects all the light outside of a 50 angstrom band centered on the Ni line. The primary mirror of 12 cm aperture together with an active secondary form a classical Cassegrain telescope. To match the size of the sun to the size of the detector requires a focal length of 189 cm and yields an $f/15.7$ telescope. Piezoelectric transducers on the secondary are controlled by a limb detector assembly that centers the image by measuring intensity at four points around the limb. The same system flown on SOUP demonstrated image stability of 0.003 arc-sec RMS during flight. Light passes from the telescope through a polarization analyzer wheel having four positions: a pair of quarter wave plates at ±45°; a transparent section, and a thin film mirror facing back into the instrument for calibration purposes. It will take about 1 second to move from one position to another. After the analyzer a polarizing beam splitter allows p-waves to pass through and reflects the s-waves out to the limb detectors. Light traps will catch stray light.

Figure 2: The upper panel shows the simulated velocity measurement using the MDI 4-point method versus the true velocity. The new different lines correspond to varying the width and depth of the line parameters by 10% about their mean values. The velocity signal is quite linear and fairly insensitive to line profile variations. The lower panel shows the response of the traditional two-point dopplergram method to the same input parameters.
Next will be an 8 Å FWHM four-period all-dielectric blocking filter to select only one bandpass of the Lyot. The filter will be constructed such that light leaving the filter will have the same polarization at that going in. Reflected light will be trapped.

A three lens telecentric relay ensures that all ray bundles to image points have the same angular distribution in the filter section. Mounted in the center of the second lens is an optical fiber which can hold laser light into the filter section for calibration. It does not occult the image because is it mounted at the image of the secondary. The inner fully illuminates the Lyot and Michelson with collimated light. This system will allow measurement of both the transmission and Doppler offset of the entire filter system (except the blocking filter) on a pixel by pixel basis.

The Lyot filter will operate in two lines, Ni 6768 for solar measurements and Nb 6328 for calibrations. The filter will have a bandpass of 0.4 Å and a free spectral range of 8 Å.

The Michelson interferometers are the primary wavelength analyzers of the instrument. The have FWHM of 0.2 and 0.1 Å and are tuned by rotating half waveplates. For the required accuracy of 1 m/s the position of the waveplates must be known to 1 minute of arc. The waveplates rotate simultaneously in a 2:1 ratio and move in less than a half second.

The reimaging optics form an image on the CCD camera. A rotating shutter determines which of two light paths falls on the camera. Normally a full disk image with 4 arc-sec resolution (2 arc-sec pixels) will be made. An alternate path with a magnifying lens forms a 1.4 arc-sec resolution image 717 arc-sec square. The beam splitter divides the light such that the both beams have the same exposure time. The final reimaging lens in the standard beam introduces aberrations so that there is no image structures below 2 arc seconds. The high resolution image is diffraction limited. Exposure times are on the order of 0.2 seconds and is precisely controlled to within about 3 microseconds.

The 1024 by 1024 CCD camera will be exposed on average to half the well full depth of 200,000 electrons and will be photon limited. The camera will be read out with an 11-bit A/D converter in about 2 seconds. The selection of the CCD chip and the camera design are not final.

Normally about 30 seconds of observation will be required for each velocity map. Two measurements will be taken at each of the four wavelengths to reduce the effects of acceleration. In 30 seconds we can measure velocity at each point to well beyond the theoretical solar noise limit of 12 m/s; giving the instrument a sensitivity to solar modes of less than 1 m/s. During the other half of the time other observations can be made.

The thermal design of the instrument has been optimized for constant temperature. Temperature controls will be provided for the critical components: the Michelsons, the Lyot filter and the blocking filter. Because we can no longer make absolute calibrations of the instrument, the temperature constraints are somewhat relaxed. The critical parameters are the thermal time constants and the linearity of the temperature shifts. By reflecting most of the incoming light at the entrance window, thermal gradients should be kept to a minimum.

3. DATA PROCESSING

MDI requires a large amount of data to accomplish the scientific objectives of the mission. To communicate a compressed frame of oscillations data each minute requires 160 kilohits/second of telemetry. We expect that this data rate will be available 8 hours each day and for 2 continuous months each year. In addition a 5 kbyte stream will be available at all times. Over 300 gigabytes of raw compressed data will be received each year. We have organized our observing strategy into four categories, each of which takes advantage of a particular telemetry mode. These are more fully described in Ref. 1. We intend to use the flexibility of the instrument to constantly review the observing programs as new scientific questions arise between now and the end of the mission.

The on-board computer system is an essential feature of the MDI experiment and consists of two units. The Dedicated Experiment Processor (DEP) controls the operation of the various electro-mechanical devices (e.g. the Michelson waveplates), processes commands sent from the spacecraft, returns housekeeping data to the spacecraft, and controls the operation of the Image Processing Unit (IPU). The DEP is an 80C86 based CMOS processor designed for minimum power consumption.

The IPU is a special-purpose image processor. It contains direct interfaces to the CCD camera, the SIC/low and low rate data channels, and to the DEP. The IPU is slaved to the DEP; however, the software interface between the two processors typically operates at a very high level. For example, a single command causes the CCD camera data to be read and added to the proper accumulating memory pages. This allows the system to realize the full potential of the specialized IP architecture while maintaining a familiar, straightforward high-level interface to the user. The IPU will execute 10 million operations per second, contain 24 megabytes of RAM and consume about 15 watts. The IPU will perform the complete set of onboard reductions necessary while data from two interleaved observing programs is accumulated separately in parallel buffers. Data compression is also performed by the IPU as appropriate for each type of data.

3.1 Low Data Rate Data Processing

The primary structure program will operate continuously and is designed to investigate the static or slowly varying structures of the solar interior. It requires uninterrupted data and will therefore use the 5 kb/s telemetry channel. Each velocity image will be reduced to retain all modes of degree 24 or less and a selection of several thousand higher degree modes. The higher order modes will be computed using a novel technique described in Ref. 1. This will provide a virtually gapless stream of data providing a significant fraction of the solar interior over a several year interval. This program will run continually, even during the high data rate times for continuity, comparison, and calibration.

3.2 High Data Rate Processing

The computing task during the high data rate times is conceptually simpler, since the data must only be compressed and passed on to the spacecraft. However the various kinds of data: doppler, intensity, and magnetic field, must be compressed in different ways.

Magnetic data is required for understanding the oscillations and other velocity data. A full disk magnetic image will be transmitted regularly, probably two or three times per day. We intend to make that data available on a quick-look basis on the ground if possible.

The primary dynamics program, run during the two continuous high rate data months each year, will essentially transmit full disk velocity and intensity measurements once each minute. Campaign programs are those utilizing the daily 8 hours of high rate data. Many of the primary and associated science objectives require different measurements from those described above. The primary structure program will run at all times using the photons during half of each minute. The campaign programs will run during the other half of those minutes. These campaigns might include intensive monitoring of evolving active regions at high or low resolution, intensity measurements for determining transverse velocity fields, etc. In each of these cases special commands will be sent to the spacecraft and the data will be partially reduced and/or compressed before transmission to the ground.
Figure 3: MDI Optical Layout
Figure 4: MDI Mechanical/Structural Layout
3.3 Ground Based Processing

The ground based processing is described elsewhere (Ref. 1) and will be a major part of the effort in this program. The requirements for handling this volume of data in a useful way are demanding. We have proposed to establish a Data Reduction and Analysis Center (DRAC) dedicated to the reduction, analysis, archiving, and distribution of MDI data. The facility will serve to convert the raw data into a set of useful data products. It will also serve as a primary analysis facility for the members of the science team, as a network communications and data server for remote workstations, and as a data archive with the capability to reduce and extract data in forms that can be transmitted to and used by Co-Investigators, Associate-Investigators and other computing facilities. Our goals for the facility are to have sufficient on-site capability to allow the basic investigations to be carried out to completion, and to provide sufficient support to allow effective access to the data archive by all members of the community. We are currently working closely with our co-investigators at GONG to find the optimum manner in which to proceed so as to take advantage of the expertise and facilities there and exploit the benefits of using similar techniques for complementary analyses.

4. SUMMARY

SOHO offers an unprecedented opportunity to probe the interior of a star with the techniques of helioseismology. Continuous distortion-free observations of the Sun will provide unique measurements of its internal structure and dynamics. The MDI instrument provides the capability to obtain observations with very high spatial resolution with high precision at all times. The instrument is designed so that as helioseismology matures it will be able to collect novel data sets that exploit the discoveries which are certain to arise.

5. REFERENCES

A SPACE QUALIFIED APPLICATION-SPECIFIC INTEGRATED CIRCUIT (ASIC) CONTAINING 8 X 24 BIT COUNTERS AND SHIFT REGISTERS

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Abstract

A single chip counter has been developed in the Space Science Department (SSD) of ESTEC. This Application-Specific Integrated Circuit (ASIC) is to be produced as a space qualified device, and realised in Silicon on Sapphire (SOS) technology. This technology has the twin advantages of very high radiation tolerance (100 krad) and is latch up free.

Introduction

Many types of experiment need some form of counter, whether it is to count particles, or pulses that are themselves the main quantity of interest, or else in the form of an analog to digital converter (ADC) used in conjunction with a voltage to frequency converter.

The chip described here has been developed with these needs in mind, and will be constructed in a space qualified technology. A single chip will contain 8 X 24 bit counters, and 8 X 24 bit shift registers arranged as shown in figure 1. To construct such a system using conventional technology would require at least 40 individual chips, and would be less reliable as the number of interconnections would be many times higher.

The Chip

The block diagram of the chip is shown in figure 1. Functionally it is grouped into two sections, this reduces the number of external connections, whilst still providing some degree of flexibility. Each counter has an individual Schmitt-trigger input. The inputs are enabled by the \(C_{en}\) which when [HI] will allow pulses to pass to the counter, and when [LO] will not. The counters are rising edge triggered, and the maximum input frequency is 10 MHz. A pulse on the \(P_{i}\) pin will asynchronously load the shift registers associated with each counter with the current counter value. This can then be read out serially from the shift registers (S\(_{out}\)) by strobing the \(R_{clk}\). The counters can be reset with a pulse to \(M_r\). A suggested timing diagram is shown in figure 2. The units can be "daisy chained" by connecting \(S_{out}\) of one to \(S_i\) of the next and paralleling the \(R_{clk}\) pins. Each of the control lines is buffered to look like a single CMOS input.

The device is designed to operate at standard TTL supply voltage, and the \(S_{out}\) provide TTL compatible outputs in both signal levels and fan-out capacity.

Current Status

A CMOS prototype has been constructed in the form of hybrids. These units have been successfully tested and are now incorporated in the data acquisition chain of a prototype Luminosity Oscillations Imager (LOI), a small resolved Sun photometer to be flown on the ESA/NASA SOHO Satellite as part of the VIRGO investigation.

Status of the ASIC is that the final design is complete, and prototypes will be produced early next year. Space qualified devices should be available by the end of the first quarter of 1990.

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![Figure 1 Block diagram of chip](image1)

![Figure 2 Possible timing diagram](image2)
PROPERTIES OF SOLAR GRAVITY MODE SIGNALS IN TOTAL IRRADIANCE OBSERVATIONS

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ABSTRACT

Further evidence has been found that a significant fraction of the gravity mode power density in the total irradiance observations appears in sidebands of classified eigenfrequencies. These sidebands whose amplitudes vary from year to year are interpreted as harmonics of the rotational frequencies of the nonuniform solar surface. These findings are for non axisymmetric modes and corroborate the findings of Kroll, Hill and Chen (Ref. 1) for axisymmetric modes. It is demonstrated that the generation of the sidebands lifts the usual restriction on the parity of the eigenfunctions for modes detectable in total irradiance observations.

Keywords: gravity modes, total irradiance

1. INTRODUCTION

It has been reported by Kroll, Hill and Chen (Ref. 1) that for the $l = 2, 3, 4$ and $5, m = 0$ classified gravity modes of Hill and Gu (Ref. 2), a significant fraction of the gravity mode power density in the total irradiance observations appears in sidebands of the eigenfrequencies where $l$ and $m$ refer to the degree and angular order of the eigenfunction, respectively. The total irradiance observations were obtained with the Active Cavity Radiometer Irradiance Monitor [ACRIM] (Ref. 3) on the solar Maximum Mission. They also conclude that these sidebands are produced by the rotation of a nonuniform solar surface. Other examples of the manifestations of a rotating nonuniform surface are well documented. (Refs. 4-8) Such findings as those reported by Kroll, Hill and Chen (Ref. 1) may impact in a rather dramatic way on how total irradiance observations are used in a solar seismology program. The presence of these surface-rotation associated sidebands adds considerable complexity to the power spectra and significantly reduces the signal-to-noise ratio from what is normally expected for the symmetry allowed gravity modes signals ($\pm m$ even). Although the additional complexity in the power spectra reduces the effectiveness of total irradiance observations in the study of even ($\pm m$) gravity modes, the generation of the sidebands lifts the usual restriction of the parity of the eigenfunction ($\pm m$ must be even) that can be detected in whole disk type observations. As a result, it should be possible to study gravity modes with even and odd values of ($\pm m$) in solar total irradiance. It may also be possible with a refined analysis to test the sign of the assigned values of $m$ and obtain information on the values of $l$.

The lifting of the usual restriction on the eigenfunction parity for the detectable modes in total irradiance observations has already been demonstrated (Ref. 1) in the case of $l = 3$ and $5$, $m = 0$ gravity modes classified by Hill and Gu (Ref. 2). We further examine the total irradiance observations in the following sections for additional evidence of deviations of the observed eigenfunction from that expected for an axisymmetric system for the gravity modes classified by Hill and Gu (Ref. 2) with $|m| \geq 1$.

In general, the rotating nonuniform surface of the Sun will lead to gravity mode power density at sidebands of the eigenfrequencies $\nu$ of the normal modes where $n$ specifies the radial order of the eigenfunction. We assume that the nonuniform rotating surface effects on the observed radiation intensity $I'$ can be described to a good approximation by

$$I' = A Y_l^m(\theta, \phi) f(\theta, \phi, \Omega) e^{i 2\pi n \Omega t},$$

where $A$ is a complex amplitude, $\Omega$ is the angular rotational speed of the surface which is a function of $\theta$; $\theta$ and $\phi$ are heliocentric spherical coordinates, and $t$ is the time variable. For a uniform surface, $f \equiv 1$. It is implicitly assumed that $f$ changes slowly over the period of months, at least that part of $f$ relevant to these observations.

The frequency locations of the sidebands of the $\nu_m$ where gravity mode associated power density is predicted are determined by the $\theta$ and $\phi$ dependence of $Y_l^m$ and $\Omega$. The maximum value of $\Omega$ is 428Hz observed at the equator and the minimum value is 295Hz observed at the poles of the Sun (Ref. 9).

2. TOTAL IRRADIANCE OBSERVATIONS

Observations of solar total irradiance made during 1980 and 1984 with ACRIM are used for this analysis. Obvious spikes in the data are removed and the data effectively high-pass filtered with a cutoff frequency of 4 µHz. The time series were also low-pass filtered (triangular filter) with the first zero at 1 mHz.

The time series for each year are then divided approximately in half resulting in two 135.40 d time series from 1980 and 118.14 d and 116.85 d time series from 1984. Four power spectra are then computed from the Fast Fourier Transform of each of the data sets.

3. POWER SPECTRUM ANALYSIS PROGRAM AND RESULTS

The properties of the total irradiance power spectra are studied by examining the mean power density and mean square deviations of the power density in the 1980 and 1984 power spectra separately. The two years are treated separately because a higher noise level is observed in 1980 relative to 1984. The mean power density and the mean square deviations of the power density are independent quantities and both are examined for properties of the gravity mode signals. Differences between the mean power density at frequencies where gravity mode associated sideband power density is predicted and the mean power density at frequencies where only background power density is expected are calculated. Similar differences are calculated for the mean square deviations of the power density.

The frequencies at which the power spectra were examined are given by $|\Delta\nu| = 0.35, 0.42, 0.64, 0.71, 0.78, 0.85, 0.98, 1.05, 1.12, 1.19, 1.26, 1.33, 1.46, 1.53, 1.60, 1.67 \mu Hz$ and $|\Delta\nu| = 0.086 \mu Hz$. The mean power density, $P_{bg}$, was observed to be approximately proportional to $1/(1+0.5|\rho|)$.

The predictions of gravity mode signals obtained with the 1979 differentials are consistent with that which would be predicted if the mode classifications are correct and if there are gravity mode associated sideband signals in the power spectra of total irradiance observations. Based on the observed amplitudes of the g-mode signals obtained with the 1979 differentials, we first look for each value of $|m|$, which is the orbital radius observations (Ref. 10) and the $m$ dependence of the spatial filter function appropriate to these latter observations (Refs. 11,12). It is predicted that the gravity mode power density in the total irradiance observations should be approximately proportional to $1/(1+0.5\rho)$. Using the same values published for the spatial filter function, the value of $|m|=1$ in Equation (4), and taking into account the number of classified modes used for each value of $|m|$, we would predict that these corresponding signals for $|m|=2, 3, 4$ and 5 would be $(8.3 \pm 2.9, 6.5 \pm 1.6, 1.6 \pm 0.6$, and 0.8) for $8.3 \pm 2.9, 6.5 \pm 1.6, 1.6 \pm 0.6$, and 0.8 $\mu Hz$.

The distribution of the $\Delta P$ in Equation (4) is consistent with that which would be predicted if the mode classifications are correct and if there are gravity mode associated sideband signals in the power spectra of total irradiance observations. Based on the observed amplitudes of the g-mode signals obtained with the 1979 differentials, we first look for each value of $|m|$, which is the orbital radius observations (Ref. 10) and the $m$ dependence of the spatial filter function appropriate to these latter observations (Refs. 11,12). It is predicted that the gravity mode power density in the total irradiance observations should be approximately proportional to $1/(1+0.5\rho)$. Using the same values published for the spatial filter function, the value of $|m|=1$ in Equation (4), and taking into account the number of classified modes used for each value of $|m|$, we would predict that these corresponding signals for $|m|=2, 3, 4$ and 5 would be $(8.3 \pm 2.9, 6.5 \pm 1.6, 1.6 \pm 0.6$, and 0.8) for $8.3 \pm 2.9, 6.5 \pm 1.6, 1.6 \pm 0.6$, and 0.8 $\mu Hz$. Similar values would result if the predictions were based on the m=0 results obtained by Kroll, Hill and Chen (Ref. 1) for
It is apparent from the predicted distribution of gravity mode associated signals and the standard deviations quoted in Equation (4) that it is only meaningful to test at this time for $|m|-1$ gravity mode related signals. For this reason the analysis in the following sections will be confined to consideration of the $|m|-1$ classified modes except when performing tests with the set of unclassified modes.

The probability $p$ for obtaining the result for $|m|-1$ in Equation (4) assuming that the null hypothesis is correct and assuming a normal distribution for the sum is

$$p = 1.7 \times 10^{-3}$$

This finding indicates that it is unlikely that the null hypothesis is correct for the $|m|-1$ modes. More specifically, for the hypothesis that gravity mode associated sidebands are not present, this finding indicates that it is equally unlikely that this hypothesis is correct.

The preceding test was repeated for the modes which were not classified in the work of Hill and Gu (Ref. 2) with $i = 1, 2, 3, 4$ and $5$, $|m| = 0$ and with $75 \leq \nu \leq 135\text{Hz}$. The results are

$$\frac{\Delta P_{0\nu}}{\sigma_1} + \frac{\Delta(P-P)_{0\nu}}{\sigma_2} = \begin{cases} 3.50 \pm 4.47; & |m| = 0 \\ 3.42 \pm 5.66; & |m| = 1 \\ 8.50 \pm 5.66; & |m| = 2 \\ 4.19 \pm 4.90; & |m| = 3 \\ 3.90 \pm 4.47; & |m| = 4, 5 \\ \end{cases} \tag{6}$$

These results are consistent with the predicted null hypothesis that no gravity mode associated signals are present. In particular, the results in Equation (6) are consistent with zero for the $|m| = 0$ and $|m|-1$ unclassified modes.

We now return to the $|m|-1$ modes for a series of significance tests because of the statistical significance of the $|m|-1$ results given in Equations (3) and (6). First for the $|m|-1$ modes, the combined sums of $\Sigma (\Delta P_{0\nu}/\sigma_1)$ and $\Sigma (\Delta(P-P)_{0\nu}/\sigma_2)$ over $i = 1, 2, 3, 4$ and $5$ and the $32 \Delta \nu$ are examined for 1980 and 1984 separately. Under the null hypothesis that no gravity mode signals are present and/or the gravity mode classifications are incorrect on the whole, the sums for 1980 and 1984 should both be zero. The results are:

$$\frac{\Delta P_{0\nu}}{\sigma_1} + \frac{\Delta(P-P)_{0\nu}}{\sigma_2} = \begin{cases} 9.76 \pm 4.47; & 1980 \\ 8.74 \pm 4.47; & 1984 \end{cases} \tag{7}$$

The probability $p$ for obtaining these results assuming that the null hypothesis is correct and assuming a normal distribution for the respective sums is

$$p = \begin{cases} 0.015; & 1980 \\ 0.025; & 1984 \end{cases} \tag{8}$$

These findings indicate that it is unlikely that the null hypothesis is correct for both 1980 and 1984.

It is interesting to note that the strength of the sidebands for the $i = 1, 3$ and $5$ classified modes is similar to that observed for the sidebands of the $i = 2$ and $4$ classified modes. For a relative measure of these strengths, the combined sums of $\Sigma (\Delta P_{0\nu}/\sigma_1)$ and $\Sigma (\Delta(P-P)_{0\nu}/\sigma_2)$ are calculated separately for the even $i = 2$ and $4$ modes and the odd $i = 1, 3$ and $5$ modes. In this case, the sums are extended over 1980 and 1984, the $32$ values of $\Delta \nu$ and the respective even and odd values of $i$. The results are:

$$\frac{\Delta P_{0\nu}}{\sigma_1} + \frac{\Delta(P-P)_{0\nu}}{\sigma_2} = \begin{cases} 10.64 \pm 4.00; & 1 = 2, 4 \\ 7.86 \pm 4.90; & 1 = 1, 3, 5 \end{cases} \tag{9}$$

The comparable strengths indicated by these findings for what is usually considered the symmetry allowed even ($\ell=m$) signals and the symmetry forbidden odd ($\ell=m$) signals is consistent with the sidebands being generated by a rotating nonuniform surface. A similar violation was reported by Kroll, Hill and Chen (Ref. 1) for the $m=0$ modes.

The strength of the sidebands in 1984 relative to 1980 are also different by a factor of $0.50 \pm 0.31$ as measured by $\Sigma (\Delta P_{0\nu})$ and $\Sigma (\Delta(P-P)_{0\nu})$. This result is in good agreement with the corresponding factor of $0.54 \pm 0.33$ obtained by Kroll, Hill and Chen (Ref. 1) for $m=0$ modes. The mean background power density for 1980 and 1984 differ by a factor of $0.50 \pm 0.05$. This observed correlation between strength of sidebands and power background density is also consistent with the sideband power density being generated by a rotating nonuniform surface whose characteristics vary from one year to another.

It was further found that the total gravity mode associated power density in the sidebands is an order of magnitude larger than the gravity mode power density located at the $\nu_{nm}$. This finding may be important in understanding the difficulty experienced in detecting gravity mode signals in total irradiance observations (Ref. 13, 14), a conclusion similar to that made by Kroll, Hill and Chen (Ref. 1) based on work with $m=0$ modes.

In summary, statistically significant evidence of $|m|-1$ gravity mode associated sideband power density has been found in both the 1980 and 1984 total irradiance power spectra. Thus, combined with the findings of Kroll, Hill and Chen (Ref. 1), we can now conclude that $|m|-0$ and $|m|-1$ gravity mode associated sideband power density is present in the total irradiance observations. Furthermore, the sideband structure is interpreted as the effects of rotation of a nonuniform surface and not the signature of rotational splitting, since the strength of the sidebands for 1980 and 1984 are different by a factor of $2$, the sideband strength is proportional to the mean background power density, and the strength of the even ($\ell=m$) and odd ($\ell=m$) sidebands are comparable.
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9. REFERENCES


THE SEARCH FOR SOLAR GRAVITY MODES

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ABSTRACT

We have maintained a solar oscillations observing program for more than 13 years at Stanford. The observations are most sensitive to low degree solar modes and have been used for the study of long period p-mode and g-mode oscillations. At the start of the 1987 observing season (summer) some long-standing problems with the instrument were corrected which (along with good weather) allowed the cleanest set of data to date. This paper reports the current state of the search for evidence of g-modes in this data. Analysis of this data shows good evidence for g-modes. Various methods were used for mode identification with a statistical search for a simple pattern of even spacing in period selected as the most robust. Using this method, a possible g-mode identification was made with an asymptotic period separation \( T_0 = 37.1 \) minutes. This identification was consistent with a rotation splitting of 1.6 microHz. Tests with randomly generated spectral peaks find as significant a possible set of modes in only 2 out of 100 cases.

Keywords: solar oscillations, g-modes.

The search for solar oscillation gravity modes, called g-modes for short, has produced several results, but no agreement between the identified modes or even the value of the Brunt Vaisala period \( T_0 \) has been reached. In this analysis we identify possible g-modes in the Stanford differential observations of the 1987 season based on a systematic statistical search which provides a high level of confidence in the solar origin and g-mode identity of the observed peaks in the power spectrum. The resulting \( T_0 \) has a value of 37.1 minutes, somewhat higher than in current solar models. The rotational splitting \( v_\theta \) is determined to be 1.6 \( \mu \)Hz.

The difficulty of g-mode detection arises from a combination of problems. Firstly, the g-modes lie at very low frequencies, below 300 \( \mu \)Hz, with the vast majority actually below 100 \( \mu \)Hz. At such frequencies the amount of power deriving from non-solar systematic errors in the data starts to get large. Below 50 \( \mu \)Hz the solar noise contribution also becomes large, making detection in that region even less likely. Moreover, the amplitude of the g-modes is theoretically expected to be low at the photosphere, perhaps so small as to be undetectable with current data. Thus the first problem is to reduce the level of noise in the data to allow g-modes to be seen in the first place.

The approach developed for this analysis involves first filtering out the frequencies below 50 \( \mu \)Hz to avoid the majority of the error signal in the Stanford data. Then the average remaining daily signal is subtracted from each filtered day separately before combining the days. This method works well for removing the large, dominating peaks in the spectrum located at frequencies of 1/day times an integer without destroying the information which is not synchronized to a 24 hour period. This is a marked improvement over the parabolic fit used in the previous analysis of the Stanford data. Combined with the much better quality and quantity of 1987 data compared to the 1979 season, this analysis thus works with a greatly improved power spectrum, which is shown in Figure 1.

The confusion of peaks with overlapping window sidelobes in this spectrum makes it necessary to approach the g-mode detection with a statistical method. The efficacy of different statistical methods has been examined by constructing artificial power spectra with model modes provided by J. Christensen-Dalsgaard. It was found that the method used by Frohlich to search for g-modes in the ACRIM data could not possibly work with the Stanford data. The data has a filling factor of only 24% after the filtering necessary for the first steps of the data reduction mentioned above. This data window, combined with the level of noise in the data, results in a power spectrum from which Frohlich's method cannot retrieve the input model g-modes. Moreover, even for noise-free data with a 100% filling factor this method was found to only barely identify the model g-modes.

The method developed in this study builds on the fact that the previous method failed because it was too constraining. The model g-modes of different degrees \( l \) cannot be well described by Tassoul's asymptotic equation in the region of interest between 50 \( \mu \)Hz and 100 \( \mu \)Hz. Since Frohlich's method depends on this asymptotic equation, it has trouble extracting the modes. The improved method, hereafter referred to as the linear search method, relaxes the asymptotic requirement by searching only for a set of peaks separated by a constant period splitting. It does not require that all such sets of peaks fit into one equation. Examination of the model peaks shows that a reasonable linearity in peak separation might be expected above a period of 200 minutes, corresponding to 83.3 \( \mu \)Hz. The linear search thus runs between 83.3 \( \mu \)Hz and 50 \( \mu \)Hz.

The peaks which are subjected to this statistical search are obtained by the peak extraction procedure used on the 1979 data. Basically this treatment selects the largest peak in the spectrum and removes it and all its window aliases from the spectrum by directly subtracting a sine wave of the appropriate amplitude and phase from the data. Recursive application produces a set of peaks which is then examined with the linear search method.

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419

Figure 1. *Reduced Low Frequency Power Spectrum.* The 1987 power spectrum, cleaned of the peaks at 1/day times an integer, in the region of interest.

Figure 2. *Linear Search of Extracted 1987 Peak List.* The double peak corresponds to the $l=1$ g-modes detected in the data. The contour levels are measured in units of the standard deviation $\sigma$ of the whole contour map.
Figure 2 shows the result of the linear search on the 1987 extracted peaks. The abscissa of this parameter map corresponds to the splitting between the peaks. The ordinate corresponds to the location of the first peak of a set of peaks. Each point on the map thus corresponds to one possible set of peaks in the spectrum. The linear search adds up the power in this possible set of peaks and records it as one value of the map. A high value in this map would correspond to the detection of an actual set of peaks in the spectrum.

In the 1987 linear search map one notices a dominant double peak. The location and separation of the two map peaks is exactly right to correspond to two sets of peaks in the spectrum with pairs separated by a fixed splitting in frequency. The Stanford measurement is most sensitive to the lowest degree g-modes, $l=1$ or 2, but only those with $lm$ even. Furthermore, the amplitude of g-modes at the photosphere depends on the degree $l$ such that the higher degree modes have a much lower amplitude. One would thus expect to see mostly $l=1$ modes, with perhaps some $l=2$. With these considerations, the double peak is consistent with a detection of $l=1$ g-modes with $m = -1, 1$.

To attach a statistical significance to this possible detection requires some knowledge of how the method acts on a random set of peaks. To this end a Monte Carlo simulation was devised which generates a set of peaks distributed randomly in the spectrum. The number of random peaks in the detection region between 50 nHz and 83.3 nHz is on the average kept the same as the actual number of peaks extracted from the data in that region - 27. The linear search is then performed on the random spectrum. If the two highest map peaks in the linear search are located with respect to each other so that they correspond to two sets of spectrum peaks separated by a constant splitting in frequency then they could be misinterpreted as a g-mode detection. In 100 iterations of this procedure there were only 5 occurrences of that condition. One can conservatively conclude that the double peak in the 1987 data is consistent with the signature of g-modes with a confidence level of over 95%. However, 3 of the 5 cases would have an unreasonable frequency splitting of over 4 nHz. The other 2 cases had barely feasible frequency splittings of 2.5 nHz and 3.3 nHz. Thus assuming a reasonable limit on the range of the rotational splitting further increases the confidence level to over 98%.

An important instrumental improvement was made before the 1987 season. In prior years there were significant variations in instrumental sensitivity which had seriously degraded the amplitude information. Thus the 1987 data is unique in its superior quality. However, other years of Stanford data have been reexamined using the new identification technique. The other good seasons of data - 1981, 1984, and 1986 - all to some extent show power in the same area of the search map as the 1987 double peak. Thus these other years are consistent with the detection of g-modes in the 1987 data. However, the level in any one of those three years would not be significant by itself.

The value of the separation between successive $l=1$ modes of $m = 0$, obtained by taking the separation at the middle between the two peaks, is measured to be 26.25 minutes. Multiplying by $\pi$, one obtains a value of 37.1 minutes for $T_x$. Taking the position of the two peaks, which corresponds to the position of the first peak of each set in the power spectrum, one measures 219.4 minutes and 224.1 minutes. These correspond to 74.4 nHz and 76.0 nHz, which gives a value of 1.6 nHz for the rotational splitting $v_x$. This implies a rotation about 3 times the surface rate in the core of the sun. This agrees with the strong increase suggested by measurements of rotational splitting of p-modes.

If one assumes that the $T_x$ also applies for $l=2$ modes, which is asymptotically true, but probably not exact in this range, one can calculate the expected period spacing for those modes. The resulting number is 15.3 minutes. It is interesting to note that at double that separation (30.6 minutes) the 1987 linear search map displays its only other large group of power, at a position of 207.5 minutes. This is similar to the pattern seen in the artificial data, where the $l=2$ modes were more visible at a separation of twice their expected separation. Thus virtually all the peaks in the 1987 map are consistent with a g-mode detection. The measured values of the peak positions allows one to calculate the periods of the peaks. The results are summarized in Table 1.

<table>
<thead>
<tr>
<th>Period (min)</th>
<th>Frequency (nHz)</th>
<th>Degree (l)</th>
<th>Azimuthal order (m)</th>
<th>Radial order (n)</th>
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<tbody>
<tr>
<td>219.4</td>
<td>76.0</td>
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<td>1</td>
<td>8</td>
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<tr>
<td>224.1</td>
<td>74.4</td>
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<td>-1</td>
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</tr>
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<td>245.1</td>
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<td>1</td>
<td>1</td>
<td>9</td>
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<td>277.7</td>
<td>60.0</td>
<td>1</td>
<td>-1</td>
<td>10</td>
</tr>
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<td>296.5</td>
<td>56.2</td>
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<td>11</td>
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<td>304.5</td>
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<td>322.2</td>
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<td>222.7</td>
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<tr>
<td>268.3</td>
<td>62.1</td>
<td>2</td>
<td>0</td>
<td>17</td>
</tr>
</tbody>
</table>

Table 1. This table lists the frequencies of the g-modes detected in the 1987 data, as calculated from the values corresponding to the location of the $l=1$, $m = -1, 1$ double peak in the linear search map. The list is only extended up to the limits of the linear search region, between 50 nHz and 83.3 nHz. The $l=2$ modes are stopped at $n=17$ since there is no evidence for higher order modes in the extracted peak list (see figure 3).

In Figure 3 the actual peaks which were extracted from the 1987 data are plotted together with the location of the modes as calculated from the measured values of the separation, and the position of the first peak. The dashed arrows correspond to modes which lie outside the range of the linear search (above 83.3 nHz), and are thus only an extrapolation using the linear calculation. The lower row of arrows corresponds to both $m$ values of the $l=1$ modes, and is extended throughout the whole frequency range. One can see that below 80 nHz the locations match up quite well and completely with extracted peaks. The upper row of arrows corresponds to the position of the $l=2, m=0$ mode. The locations again match up well with groups of extracted peaks, especially below 85 nHz. One can say that all extracted peaks below about 82 nHz can be accounted for as g-modes.
Returning to the original power spectrum, one finds that the highest of the peaks has an amplitude of about 40 cm/s. The g-modes detected by $S$ lie at levels from a few centimeters to about 10 cm, indicating that the ratio in sensitivities between the differential and full-disk observation methods may be as large or even larger for the g-modes than for the p-modes.

The peaks previously identified in the 1979 Stanford data must be compared to this new identification. In the range below 83.3 μHz three of the previously published peaks lie right at the frequency of a g-mode identified in this study, another three lie very close or could be $l=2$ peaks with $m=-2$, while the final three lie relatively far from an identified g-mode. At the natural width of these peaks, one can statistically expect in this case one chance superposition. That there are three, with three more close ones, is suggestive, especially in light of the fact that the 1979 data from which the peaks are derived is of significantly lower quality than the data analyzed in this study. The filling factor is only 8%, which has been shown to have severe effects on the spectrum. The parabolic fit used in that analysis introduces power in a broad region of the spectrum, which has further potential for resulting in erroneous peaks. The data used in this analysis, as well as the reduction method, are a significant improvement over these first studies.

The value of $T_0$ as measured is accurate to within ±0.2 minutes. However, it is possible that this value has a systematic error in it which gives a value lower than its actual one. Examining the model peaks, one sees that the actual $l_m$ sets of peaks lie on lines which curve in such a way as to have a larger separation at higher periods. A linear fit to such a slightly curved line would give a lesser slope than the curve is asymptotically approaching. Therefore the measured $T_0$ may actually be too small. The amount of systematic error is hard to estimate. In the model peaks the $l=1$ and $l=2$ modes have a separation which results in two values for $T_0$ which differ by 1.3 minutes, with the $l=2$ modes having the higher value. If this is taken as an indication of the possible systematic error, the measured value for $T_0$ is then 37.1 ±0.2/±1.3 minutes. Another number which comes out of these calculations is the value of the constant $e$ in the Tassoul equation, which is generally taken to be $\frac{1}{4}$. The calculations give a value of $e = 0.05$ for the $l=1$ modes, and $e = 3$ for the $l=2$ modes. However, these values are extremely sensitive to the exact value $T_0$, meaning that those values for $e$ are as consistent with each other as they are with $\frac{1}{4}$ or 0.

The measured value of $T_0$ is higher than the value calculated with the best current solar models, which lies around 34.5 minutes. To raise the value in the models would require a reduction in the buoyancy frequency. This could be achieved either by mixing the interior, or by reducing the initial Helium abundance $Y_0$. Either of these changes would have the tendency to decrease the temperature in the interior. However, these changes would also increase the discrepancy between the models and the well-established frequencies of p-modes in the 5 minute region.
We had hoped to confirm these observations in the just completed 1988 observation season. Unfortunately the sky was not cooperative and a long sequence of -10-hour observations was not possible. Further continuation at Stanford must await the 1989 season.

ACKNOWLEDGEMENT

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REFERENCES

GOLF : SIMULATIONS OF THE RE-EMISSION OF THE RESONANCE CELL

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ABSTRACT

Applied to the GOLF (Global Oscillations at Low Frequencies) helioseismometric instrument to be flown on SOHO, we analyse the re-emission process in the resonance cell, heart of the instrument. The cell is filled with a sodium vapor and absorbs selectively in the two sodium lines D1 and D2. We simulated the different influence of the physical parameters (cell geometry, temperature) and report herein the temperature analysis which allow to constrain the nominal temperature working point and the required stability, as a function of the operational conditions (orbital velocity variation).

1. INTRODUCTION

To evaluate the photon flux on each photomultiplier tube (PMT), and then to estimate its sensitivity to different parameters of the instrument, we can simulate the physical processes of the radiative transfer in the sodium vapor. Firstly, the absorption profiles of the vapor must be known. A $\sigma$ component contains one group of four hyperfine lines in the NaD1 line, and two groups of four each in the NaD2 line. With a magnetic field greater than about 2000 Gauss, an asymptotic range is reached (GOLF design is based on a permanent magnetic field of 4750 Gauss, cf. ref. 1). This means that the transition probability of the hyperfine lines and the distance (in wavelength unit) between two of them are constant in one group. At first sight, the hyperfine structure acts like three 25 mA gaussian filters. Thus, knowing the relative transition probability and the Lande factor of each group, we can sample both sodium solar lines around the working points, and finally estimate the photon flux absorbed and reemitted towards the collecting optics. This flux will depend on time via the radial orbital velocity evolution of the satellite, which will result in different positions in the lines.

2. SIMULATIONS

According to the satellite orbital conditions, at the Lagrangian point L1 in between the earth and the sun, one can evaluate the change in the orbital velocity of the satellite along the year. The derivation is straightforward and leads to the velocity expression:

$$v_{\text{orb}} = n a e \left(1 - e^2\right)^{1/2} \sin \theta$$

with:
- $n = 2\pi/T$ (T is the orbital period)
- $a =$ half major axis
- $e =$ orbital eccentricity
- $\theta =$ angle between the major axis and the sun-L1 vector

This variation is illustrated, starting with the winter solstice, on fig. 1. The uncertainty shown is taking into account the maximum amplitude of the Halo orbit (80 m/s) and the 1 % maximum uncertainty on the velocity (~ 6 m/s) due to the approximation in the calculation of the Lagrangian point orbit (the Lagrangian point is placed at the earth position).

![Figure 1. Orbital velocity along the year displaying the variation of the associated uncertainty level.](image)

Figure 1. Orbital velocity along the year displaying the variation of the associated uncertainty level.

Then, for a given time, but also for a given instrumental configuration (left or right solar wings), we can obtain a curve showing the counting rate on a PMT versus one parameter of the instrument, for example the temperature of the cell (the head or the stem). Such a curve is presented in figure 2, including the uncertainties of the optimal values due to those of the solar profiles (c-shape of the bisectors, cf. ref.3) and the orbital velocity. Firstly, it should be noticed that the absolute values are less significant than the slope and curvature parameters.
The optimal values depend on the position of the working points essentially because the hyperfine groups have different transition probabilities. The values obtained for the different configurations are summarized in Table 1, the slope of the corresponding previous curves being the same. We can verify in this table that, in April, the wavelength shifts from the Einstein redshift and from the orbital velocity of the satellite, add up, leading to a larger influence on the optimal temperature than solely the effect of the orbital velocity variation. In October, the two effects are in opposition and nearly cancel themselves.

<table>
<thead>
<tr>
<th></th>
<th>December</th>
<th>April</th>
<th>October</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left wing</td>
<td>T_s (K)</td>
<td>419.0</td>
<td>419.0</td>
</tr>
<tr>
<td>Flux (10^6/s)</td>
<td>15</td>
<td>16</td>
<td>13</td>
</tr>
<tr>
<td>Right wing</td>
<td>T_s (K)</td>
<td>419.2</td>
<td>419.3</td>
</tr>
<tr>
<td>Flux (10^6/s)</td>
<td>11</td>
<td>10</td>
<td>13</td>
</tr>
<tr>
<td>Difference</td>
<td>DTs (K)</td>
<td>0.2</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table 1: Optimal values of the temperature of the stem (T_s) and of the corresponding counting rate (F_x) on one PMT. The difference of the temperatures in the two wings (DTs) is also given (the uncertainties mentioned in the previous paragraph give a maximum precision of 0.3 K on the temperature, and 10% on the counting rate).

If we compare the results obtained on the counting rate for an optimized temperature adjustment with the GOLF design (see ref. 1 & 2), we remark that the estimated counting rate is in between 3 and 10 times greater than the desired one. Thus, if these estimations are confirmed, we would be able to improve the performances of the instrument either by reducing for example the entrance pupil size, or the collecting angle in the cell (less scattered light; better polarized re-emission).

For the fixed temperatures of the two parts of the cell (head and stem), the criterion used was to optimize the total flux on the left and right solar wings for a given time. This approach leads, effectively, to associated optimal temperatures that remain practically constant over the year (change < 0.1 K). Also, for most of the measurements, the flux will not be a maximum but the sum "left wing + right wing" will be. Then, we may evaluate the corresponding counting rate and the new positions on the previous curves. For the worst orbital situation (April), the so-determined optimal points are at maximum at 0.2 K from the top. A zoom around those points is shown in Fig. 3, corresponding to the winter solstice. Note that those curves are a direct indication of the re-emission curve (efficiency) of the cell since the other parameters that bear on the final counting rate (flux) are only multiplicative factors.

With such curves we can evaluate the stability requirements, necessary to reach a given precision in the measurements, on the parameters which influence the system. We shall study the influence of the temperature itself.

We may suppose that the noise induced by temperature fluctuations is firstly due to random fluctuations, and secondly to impulsions more or less regularly imposed by a feedback process. The instrument is photon noise limited, so we might impose the first fluctuations to generate at most a noise, being, for example, ten times in power below the photon noise. The variance of the velocity noise due to the photon statistics is:

$$\sigma_v^2 = 4 k^2 R / (R+1)^2 (k+1),$$

where $k$ is the sensitivity coefficient ($k \approx 4000$ m/s), $R = h / h$ with $h$ and $h$ the measured intensities. In the worst case (April), $R=1.6$, and with a counting rate of $2 \times 10^5$ per photo multiplier tube which corresponds at present to the GOLF design, we estimate the white noise level to be of the order of $3.8 \text{m/s}^2/\text{Hz}$. Then, after 4 seconds of integration on 6 PMTs, we obtain a photon noise of about 40 cm/s. So, with the criterion chosen above, we should have at maximum a noise of 12.5 cm/s produced by random fluctuations of the cell temperature. The variance of the velocity noise is directly related to that of the corresponding intensity noise by the relation:

$$\sigma_v^2 = 4 k^2 (\sigma_i^2 + \Delta_i^2) / (\Delta_i^2).$$
Moreover, we may fit our previous curves by second order polynomials to obtain the variance of the corresponding temperature noise. In other words, we may write:

\[ l_1 = a_1 T^2 + b_1 T + c_1 \]
\[ l_2 = a_2 T^2 + b_2 T + c_2 \]

This gives us the relations:

\[ \sigma_T^2 = (2a_1 T + b_1)^2 \sigma_l^2 \]
\[ \sigma_T^2 = (2a_2 T + b_2)^2 \sigma_l^2 \]

and finally:

\[ \sigma_T^2 = \frac{(a_1 T + b_1)^2 + (a_2 T + b_2)^2}{4k^2} \]

In the worst case, this formula gives us: \( \sigma_T = 0.1K \) for \( \sigma_v = 12.5 \text{cm/s} \). This standard deviation must be observed during the 10 seconds necessary for an elementary velocity measurement, and is quite feasible.

Let's consider now the noise induced by pulses. In the worst case, for which the temperature follows a sinusoidal variation or for which all the energy of the induced spectrum is in one bin of frequency resolution, we shall have in the power spectrum one mode with an amplitude of \( \Delta T/4 \) where \( \Delta T \) corresponds to a temperature jump. Also, for any frequency of a sinusoidal temperature signal, we may evaluate here the lower limit for the greatest jump allowed, to induce at this frequency a noise lower than a given value. Thus, considering a maximum noise of 1mm/s, and a signal to noise ratio of 200, we have estimated the lower limit being \( \Delta T = 5 \times 10^{-3} K \).

This is a first evaluation which will have to be verified in particular by the knowledge of the typical spectrum of the temperature regulation signal.

3. CONCLUSION

We have presented here preliminary results of our simulations of the re-emission of the resonance cell, heart of the GOLF experiment. Firstly, the efficiency of the cell appears better than the corresponding ones estimated for the ground-based instruments, even though none of them has been studied specifically on the cell re-emission. This gives us the eventual opportunity to achieve better performances with the GOLF instrument. Secondly, because of the combined wavelength shift, from the Einstein redshift and from the orbital velocity of the satellite, the optimal temperature of the cell varies from a configuration to another (left or right wings configuration at the same time, or different time in the same wings). The extreme optimal values are separated only by few tenths of a degree, but we have to choose fixed temperatures for the cell head and stem. We propose to optimize the total flux in both wings, the associated temperature remaining practically constant over the year. Finally, these simulations confirm to us that measuring solar oscillation frequencies with a 1 mm/s precision needs very careful analysis and experimental tests on each part of the instrument. This stresses the need for a breadboard model of the instrument, with particular attention to the cell environment.

Acknowledgements. This analysis was partly performed with the computing facilities of the I.A.C (Instituto de Astrofisica de Canarias). We wish to thank particularly its department group of Helioseismology for its welcome and helpful discussions.

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Session 6
Solar modelling techniques
Chairman: M. Gabriel
STUDY OF SOLAR STRUCTURE BASED ON P-MODE HELIOSEISMOLOGY

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ABSTRACT
A principal goal of helioseismology is to test the physics, assumptions and procedures used in computing models of solar and stellar structure and their oscillation frequencies. Indeed, the connection between the physics and the assumptions on the one hand, and the observable frequencies on the other, may be regarded as the principal forward (and inverse) problem, with the model serving as a necessary intermediate step.

Here I consider how the model and its frequencies react to changes in the physics and the assumptions. I also briefly discuss tests of numerical precision, and compare computed frequencies with the observations.

Keywords: Helioseismology, solar structure, solar oscillations, p modes, asymptotic theory, physics of the solar interior.

1. INTRODUCTION
At the most naive level the goal of the study of solar structure by means of helioseismology might seem to be to match frequencies computed for solar models to those observed. This, however, is too narrow a perspective. Even if one were able by some procedure (such as trial and error, or even pure luck) to achieve a fit that fell within the observational errors, such a fit would in itself provide relatively little information. It is conceivable that the fit might have resulted by accident from effects of numerical errors, or that the model is only one among several, possibly quite dissimilar, models that fit the data equally well. Resolution of these questions requires tests of the sensitivity of the frequencies to the numerical procedures and the physical assumptions used in their calculation.

An alternative attitude is to regard the principal role of helioseismology, insofar as solar structure is concerned, to be to test the physics and assumptions used when computing models of solar and stellar evolution, the precise details of solar structure being largely irrelevant. From this viewpoint the computation of solar models, or the determination of solar structure through application of inverse techniques, are to be seen mainly as intermediate steps in the connection between the observed quantities, namely the frequencies, and the principal objects of study, namely the physics and assumptions. This may be a rather extreme position. However, among the aims of helioseismology are to provide a more secure basis for computing models of other stars, and to obtain an understanding of the properties of matter under solar conditions. In these contexts the precise temperature, say, at a given point in the present Sun is of limited interest.

The possibility of establishing the connection between the physics and the oscillation frequencies depends on the generally assumed simplicity of conditions in the solar interior. This allows ab initio calculations of solar structure, based on simple assumptions of hydrostatic and thermal equilibrium, on a description of the chemical evolution of the Sun, and on microscopic properties of matter which (in principle, at least) can be determined from laboratory measurements or theoretical work. This situation could be contrasted with that pertaining to geoseismology, where such calculations are impossible, and where the determination of the structure of the Earth is therefore a primary goal; an analogous situation applies in helioseismic investigations of solar rotation or magnetic fields (Ref. 1), where the construction of reliable theoretical models from first principles is also not possible. If conditions in the Sun were to be less simple than normally assumed, there would no longer be a direct and simple link between physics and frequencies. Thus, helioseismology must also test the validity of the assumptions used in the calculations, and possibly establish how they should be modified. An important example, to which I return below, is the assumption that no mixing takes place in the solar core.

An essential requirement on calculations of solar models and frequencies is that errors introduced by the numerical solution of the equations of solar structure and the oscillation equations, for given physics and assumptions, are sufficiently small. In particular, the numerical errors should be much smaller than the effects of likely changes to the physics and assumptions. Also, it is evidently desirable to match the accuracy of the observed frequencies. From calculations where, say, the number of mesh points is varied it is straightforward to estimate the intrinsic truncation errors in a given calculation. Much more difficult to eliminate are "trivial" programming errors. This probably requires detailed comparisons between results of independent calculations, based on precisely

the same physics and assumptions. Such comparisons are now under way as part of the GONG project (Ref. 2). Calculations of "standard" solar models were recently analyzed by Turek-Chlêze et al. (Ref. 3).

To evaluate the sensitivity of solar models and oscillation frequencies to various aspects of the physics and the assumptions one may study the changes in the models and frequencies induced by a given change in the physics or the assumptions. If the changes are sufficiently small the relevant equations can be linearized around a reference model. In this case one can determine kernels connecting the assumed change, which for simplicity I take to be a function $\delta \omega (r)$ of the distance $r$ to the centre of the Sun, to the frequency change $\delta \omega_i$:

$$\delta \omega_i = \int_0^R K_i^2(r) \delta \omega(r) \, dr \quad (1.1)$$

(e.g. Ref. 4, 5, 6; see also Ref. 7). Here $R$ is the surface radius, and $n$ and $\ell$ are the order and degree of the model; for simplicity I assumed spherical symmetry, so that the oscillation frequencies are independent of the azimuthal order of the mode.

The power of equation (1.1) is evident. By inspecting the kernels one can estimate the effect on the frequencies of any change $\delta \omega$ to the quantity being considered. Furthermore relations of this sort form the basis for inverse analyses of differences between the observed frequencies and those computed for a solar model, to determine corrections to the model (Ref. 5).

In this paper I take a simpler and less general approach to the sensitivity analysis, by comparing models and their frequencies where just one aspect of the physics or the assumptions is varied. This evidently limits the number and types of changes considered. On the other hand linear relations between the change and its effects are not assumed, and so it is possible to test the accuracy of linearization. Furthermore, an analysis of the results of these experiments provide some understanding of the connection between the physics, the structure and the frequencies, and so may aid the interpretation of the results of more elaborate treatments.

Calculations of this nature have been made by a number of authors. Much of this effort has been motivated by the solar neutrino problem (Ref. 8), which has led to extensive investigations of the effects on the computed models of uncertainties in the input to the calculation (e.g. Refs. 9, 10). Other calculations (e.g. Ref. 11) have been aimed at explaining the discrepancy between observed and computed oscillation frequencies. However, rather than reviewing these calculations, I present results that are to a large extent based on my own computations. A justification for this limitation is that it provides a unified basis for discussing effects of different modifications to the calculation. Furthermore, the results of other investigations are often so briefly described in the literature that a detailed analysis of the changes considered is not possible.

From the discussion in these proceedings (Refs. 12, 13, 14) it is evident that the detection and identification of solar g modes is still in considerable doubt. For this reason I only consider the information that may be obtained from p modes. However, it should be noted that measurement of frequencies of identified g modes, or even just determination of their mean period spacing, would result in far stronger constraints on conditions in the solar core than provided by p modes alone.

There exist several reviews (e.g. Ref. 15, 16, 17) on the general properties of solar oscillations, and so these are only discussed briefly. Neither is the subject of inversion discussed, except in passing. Reference may be made, for example, to the reviews by Gough (Ref. 5, 18) on this topic.

2. PHYSICS OF THE SOLAR INTERIOR

2.1 "Standard" solar models.

Most of the calculations reported here are made within the framework of the "standard" theory of solar structure and oscillations. Here the model is assumed to be spherically symmetric, and in hydrostatic and (very nearly) in thermal equilibrium; there is no mixing except in the outer convective envelope; the convective flux is calculated using some version of the mixing length prescription; dynamical effects of convection (i.e. "turbulent pressure") are neglected in the calculation of both model and oscillations; the oscillations are adiabatic; and effects of magnetic fields are neglected.

Under these assumptions the structure of the model, and its oscillation frequencies, are determined by the physical properties of matter in the solar interior, as given by

- The equation of state
- The opacity
- The rates of energy generation and change of composition.

In addition we need to specify certain parameters, namely

- The mass of the Sun
- The present age of the Sun
- The present radius of the Sun
- The present luminosity of the Sun
- The abundance $Z$ of heavy elements.

Here $Z$ should in principle be regarded as a vector, consisting of the abundances of each element heavier than helium. In practice the relative contributions of such elements are often assumed to be given, and only the combined abundance, or its ratio to the abundance $X$ of hydrogen, is regarded as a parameter of the calculation. The abundance $Y$ of helium is determined, together with a parameter in the calculation of the convective flux, by requiring the model of the present Sun to have the correct luminosity and radius.

Among the assumptions made in the standard calculation of solar models and frequencies, the most questionable are those relating to the structure of the model and the behaviour of the oscillations near the surface. Thus standard calculations neglect

- The details of convective energy transport
- Dynamical effects of convection
- The nonadiabaticity of the oscillations
- Other effects connected with mode excitation and amplitude limitation
- Effects of atmospheric structure
- Effects of radiative transfer
- Possible effects of surface magnetic fields.

Of these, the most significant effects are probably those associated with convection, and with mode excitation and damping. As discussed by Llibrechts
(Ref. 19) and Coldreikh & Kumar (Ref. 20) in these proceedings, different calculations of the stability of solar oscillations yield rather different results, largely due to the uncertain treatment of convection. Nonadiabaticity apparently decreases the computed frequencies by several kHz (e.g. Ref. 21, 22); however a more precise estimate of the frequency change has to await a better understanding of these processes. Similarly, although there have been investigations of the effects of surface magnetic fields on the oscillations (e.g. Ref. 23, 24, 25), the current information about the field geometry is inadequate for an accurate calculation of the associated frequency change. Given such uncertainties it is clear that all current calculations of solar oscillation frequencies suffer from errors coming from the treatment of the surface layers.

These errors have potentially serious consequences for the study of the solar interior by means of helioseismology. On the other hand the errors are very likely confined to the superficial layers of the Sun. Regardless of the detailed treatment of convection there is little doubt that the stratification in the convection zone is essentially adiabatic, except in a thin region near the top. Furthermore the convective velocities are almost certainly very small, compared with the sound speed, except in the significantly superadiabatic region; the same is therefore true of the dynamical effects of convection. The effects of nonadiabaticity on the dynamics of the oscillations, and hence on their frequencies, are confined to a narrow region near the solar surface where the thermal timescale is comparable with the oscillation period. Also, unless the magnetic field strength increases rapidly with depth, the effects of the magnetic fields associated with the visible flux tubes are likely to be confined near the surface (e.g. Ref. 23). As discussed in the next section, the likely confinement to the surface layers of the known errors in the physics of the model and the oscillations leads to considerable simplifications in the analysis of the oscillation frequencies.

A related question concerns the proper boundary conditions to be imposed on the oscillations at the surface of the model, particularly in the light of the very inhomogeneous nature of the upper solar atmosphere. Fortunately the computed frequencies are insensitive to the precise boundary condition, provided that it is applied sufficiently high in the atmosphere (Ref. 11, 26), because the modes are evanescent in the atmosphere. Thus the oscillations are in general insensitive to the details of the atmosphere above the temperature minimum. Exceptions may be modes of frequency exceeding 5 mHz, where there is no reflection beneath the photosphere, or possibly chromospheric modes trapped in the acoustic cavity near the temperature minimum (e.g. Refs. 21, 27 - 30).

The fact that the convection zone is essentially adiabatic, apart from a thin boundary layer, considerably simplifies analysis of its structure. The density stratification satisfies

\[ \frac{1}{\Gamma_1} \approx \frac{d \ln \rho}{d \ln \eta} \approx \frac{1}{\Gamma_1}, \tag{2.1} \]

where \( \Gamma_1 \approx (\frac{d \ln \eta}{d \eta \rho}) \) (the derivative being at constant specific entropy \( s \)). To the extent that this relation is valid, the variation of pressure \( p(r) \) and density \( \rho(r) \), and hence of the adiabatic sound speed \( c(r) \) given by

\[ c^2 = \frac{\Gamma_1 p}{\rho} \tag{2.2} \]

is determined by the equation of state, the composition and the (constant) value of \( s \). The latter is fixed (e.g. by adjusting the mixing length parameter) by requiring that the model have the correct radius. In particular it should be noticed that the opacity in the convection zone has essentially no direct effect on its structure. It is true that the opacity affects the structure of the upper, superadiabatic part of the convection zone but this can be regarded as part of the general uncertainty about the structure of that region. After calibration the atmospheric opacity has very little effect on the structure of the bulk of the convection zone, and of the radiative interior. On the other hand, as discussed below, the value of the opacity below the convection zone has a substantial effect on \( s \) and hence on the structure of the convection zone.

![Figure 1. Schematic representation of solar structure. The thin hashed area near the surface indicates the region where the physics is uncertain, because of effects of convection, nonadiabaticity, etc. The adiabatic part of the convection zone is specified by the equation of state (EOS), and the constant values of specific entropy \( s \), and composition (given by the abundances \( X \) and \( Z \) of hydrogen and heavy elements). Beneath the convection zone the structure also depends on opacity \( \kappa \) and the energy generation rate \( \varepsilon \). From the preceding discussion follows the schematic picture of solar structure, and its dependence on the physics, illustrated in Figure 1. The insensitivity of the convection zone structure to opacity makes oscillations trapped in this region particularly attractive for obtaining a measurement of the helium abundance of the Sun that is independent of the calibration of the luminosity of the model, as well as for testing the equation of state. The former aspect is discussed in Refs. 31 and 32, whereas the testing of the equation of state is considered in Ref. 33, and in Section 4.3. Below the convection zone the situation is evidently more complicated. However, here the major chemical elements constituting the gas are probably essentially completely ionized, and hence the uncertainties in the equation of state associated with the ionization balance have little effect on the structure. The principal uncertainty in the physics affecting this region, apart from failure of the standard assumptions, is probably associated with the opacity, and there might therefore be some hope of using information about that region to test opacity calculations (Ref. 34; see also section 4.2).}
2.2 Departures from the standard assumptions.

Apart from the problems near the surface, there is no compelling evidence for the failure of the assumptions of standard solar model and oscillation calculations. However, various difficulties with the standard solar models, notably the neutrino problem, have led to consideration of models where some of these assumptions have been relaxed. Among the effects that have been considered are

- Full or partial core mixing
- Effects of magnetic fields in the solar core
- Substantial departures from thermal equilibrium
- Heavy element enrichment of the convection zone due to accretion of interstellar material
- Effects of diffusion or settling of heavy elements
- Energy transport by means of Weakly Interacting Massive Particles (WIMPs)

Substantial mass loss during the evolution of the Sun.

In this paper I mainly stay within the limits of the standard theory; however effects of partial mixing are discussed in Section 4.7 below. Further discussion of mixing, possibly due to weak turbulence, can be found, for example, in Refs. 35 and 36. The possibility of heavy element enrichment of the convection zone, with the solar interior having a substantially lower Z (Ref. 37), can be excluded on the basis of existing frequency measurements (Ref. 38). Effects of diffusion were considered, for example, in Refs. 39 and 40 (see also Ref. 41); they appear to be possibly of some, although still rather uncertain, significance. Approximate results on the effects on oscillation frequencies of a small core in the Sun enriched in heavy elements were obtained in Ref. 42. Mass loss is discussed in these proceedings (Ref. 43). A temporary departure from thermal equilibrium, in connection with a mixing episode, was proposed by Dille & Gough (Ref. 44); Ref. 45 reports new results on this mechanism. It could also be connected with a possible weak magnetic field in the solar core (Ref. 46), and related to the curious behaviour of the equatorial rotation in the core inferred in certain inversions for rotational splitting (Ref. 47). WIMPs (e.g., Ref. 48, 49) might reduce the temperature gradient in the solar core and hence the neutrino flux; the effects on the oscillation frequencies are at present only marginally detectable.

2.3 Relations between changes in the model.

To aid the interpretation of the numerical results presented below, it is useful to derive approximate relations between changes in quantities characterizing the model. The relations are particularly simple in the convection zone, where the adiabatic relation (2.1) is satisfied. Also I assume that the changes are so small that the linear approximation is applicable. Here and in the following "s" is used to denote differences between two equilibrium models at fixed radius r.

Of particular interest is the change in the sound speed, since this to a large extent determines the change in the frequencies of the p modes (cf. equation (2.6) below). From equation (2.2) follows that

\[ \delta v = \sqrt{\frac{2}{\rho}} \left[ \delta \ln r - \delta \ln \rho \right] \]  

(2.3)

where ln is the natural logarithm. Since only about 2 per cent of the solar mass is contained in the convection zone, it is an excellent approximation to assume that the mass is constant in this region. From the

equation of hydrostatic support, and the definition of \( \Gamma \) in equation (2.1), it is then straightforward to show that

\[ \delta \ln \frac{\rho}{\rho_i} = \left( \frac{T}{T_i} \right) \left[ \delta \ln r + \frac{2}{\rho_i} \int_{r_i}^{r} \left( \frac{T}{T_i} \right)^2 \delta \ln \rho \right] \]  

(2.4)

where \( \delta \ln r \) is the intrinsic change, at fixed \( \rho, \rho_i \) and \( \Gamma \), brought about by a possible change in the equation of state; the remaining terms arise from the changes in \( \rho, \rho_i \) and \( \Gamma \), brought about by a possible change in the equation of state; the remaining terms arise from the changes in \( \rho, \rho_i \) and \( \Gamma \), brought about by a possible change in the equation of state. The change in \( \Gamma \) can be written as

\[ \delta \ln r = \left( \delta \ln \Gamma \right) \]  

(2.5)

where \( \delta \ln \rho \) is the intrinsic change, at fixed \( \rho, \rho_i \) and \( \Gamma \), brought about by a possible change in the equation of state; the remaining terms arise from the changes in \( \rho, \rho_i \) and \( \Gamma \), brought about by a possible change in the equation of state. The change in \( \Gamma \) can be written as

\[ \delta \ln r = \left( \delta \ln \Gamma \right) \]  

(2.6)

The change in \( \Gamma \) is significant size only in the ionization zones of hydrogen and helium. The same is therefore true of \( \delta \Gamma \). Since \( H_i \) decreases roughly as \( T^1 \) with increasing depth (cf. equation (2.5)) it follows from equations (2.3) and (2.4) that within the convection zone, and outside this ionization zones, \( \delta \Gamma \) likewise decreases with increasing depth. In particular, if the changes in \( \Gamma \), and the superadiabatic gradient can be neglected, there is no change in the sound speed in the convection zone (cf. Ref. 38).

From the equation of hydrostatic support, again neglecting the variation of mass, also follows that the change in \( \rho \) is related to the change in \( p/\rho \) by

\[ \delta \ln \rho = \left( \delta \ln \rho \right) - \int_{r_i}^{r} \delta \ln \rho \frac{\rho_i}{\rho} dr' \]  

(2.7)

Since pressure varies very rapidly with depth in the outer part of the convection zone (between depths of 0.001R and 0.01R p increases by roughly a factor 1000) even a fairly small change in \( p/\rho \), of predominantly the same sign, can be associated with a considerable change in \( \rho \). Examples of this will be considered below.

Obviously the changes in \( p/\rho \) and \( \rho \) are coupled through the change in \( \Gamma_i \), as is evident from equation (2.6). Thus in general the changes to the model satisfy a set of coupled, linear differential equations, the solution of which effectively leads to kernels such as indicated schematically in equation (1.1) (see also Ref. 4). Nevertheless the integral relations (2.4) and (2.7) provide some indication of the behaviour of the changes. Also, since the derivatives of \( \Gamma_i \) in equation (2.6) are quite small except in the hydrogen ionization zone, there may be circumstances where the intrinsic
3. PROPERTIES OF THE OSCILLATIONS.

3.1 Asymptotic behaviour of p modes.

The properties of the p modes are largely determined by the adiabatic sound speed $c(r)$. From asymptotic analysis (Refs. 15, 50, 51, 52) one finds that the angular frequency $\omega$ is related to the sound speed through the Duvall law (Ref. 53)

$$\frac{\pi}{\omega} = F\left[\frac{\omega}{L}\right]$$  \hspace{1cm} (3.1)

where

$$F(\omega) = \int_{r_t}^{r_s} \left[1 - \frac{c^2}{r^2\omega^2}\right]^{1/2} \frac{dr}{c}$$  \hspace{1cm} (3.2)

and the integration extends from the inner turning point $r = r_t$, determined by $c(r_t)/r_t = \omega$, to the surface, $r = R$. Here $L = \ell + \frac{1}{2} \alpha$, which is essentially a function of frequency but not of $\ell$, depends on the properties of the model very near the surface.

From equation (3.2) follows that the frequency depends solely on the structure of that part of the model which is outside $r_t$. The dependence of $r_t$ on frequency and degree is illustrated in Figure 2. The most important aspect is evidently the variation with $\ell$ over the range of modes that are observed the region to which the frequency is sensitive varies from the entire Sun to a layer whose thickness is less than 0.5 per cent of the solar radius. This variation enables inversion of the frequencies to obtain $c(r)$ (e.g. Ref. 51), and it greatly simplifies the interpretation of the differences between frequencies of different models, or between observed and computed frequencies (Ref. 54).

The asymptotic behaviour of the frequencies is particularly simple for modes of low degree (e.g. Ref. 55). Here one finds

$$\nu_0 = (n + \ell + \frac{1}{4} + \alpha)\Delta \nu + \epsilon_n$$  \hspace{1cm} (3.3)

where $\epsilon_n$ is a small correction term; also

$$\Delta \nu = \left[2 \int_{r_t}^{r_s} \frac{dr}{c} \right]^{-1}$$  \hspace{1cm} (3.4)

This behaviour is reflected in the well-known approximately uniform spacing of the peaks in spectra of low-degree oscillations. It follows from the properties of the correction term that

$$\nu_{n,0} - \nu_{n-1,4} = (4\ell + 6)\Delta \nu$$  \hspace{1cm} (3.5)

where the quantity $\Delta \nu$ is determined by equation (3.6). This is evidently a very special dependence on $\ell$ and degree. The use of this relation for inversion to determine $\Delta \nu$ is discussed in Refs. 57 and 58.

It should be noticed that $c/r$ decreases quite rapidly with increasing $r$. Thus $L^2\omega^2/r^2c^2 \ll 1$ except near the turning point $r_t$, and $1 - L^2\omega^2/r^2c^2$ may be approximated by 1 in the integrals in equations (3.6) and (3.7). If furthermore the term in $\Delta \alpha$ can be neglected, the result is the very simple relation between the changes in sound speed and frequency:

$$S = \frac{\omega}{L}$$

where the two functions $H_1$ and $H_2$ are determined by equation (3.6). This is evidently a very special dependence on $\omega$ and $L$. The use of this relation for inversion to determine $\Delta \alpha$ is discussed in Refs. 57 and 58.

Figure 2. The turning point radius $r_t$ (a) and the penetration depth $R - r_t$ (b), in units of the solar radius $R$, as a function of degree $\ell$ for three values of the frequency $\omega$. The curves have been calculated for a normal model of the present Sun.
Physically this shows that the change in sound speed in a region of the Sun affects the frequency with a weight determined by the time spent by the mode, regarded as a superposition of traveling waves, in that region. Thus changes near the surface, where the sound speed is low, have relatively large effects on the frequencies.

3.2 General perturbation analysis.

The preceding discussion of the effects on the frequencies of changes to the model was based exclusively on the asymptotic properties of the oscillations. However it is possible to derive more general expressions for the frequency changes. The equations of stellar oscillations can formally be written as an eigenvalue problem (Ref. 59, 60)

\[ \mathcal{L} \psi = \omega^2 \psi \]

where the eigenvector \( \psi = (\psi_r, \psi_t, \psi_z) \) consists of the amplitudes of the radial and tangential components of displacement, and the operator \( \mathcal{L} \) describes the physics of the oscillations. For adiabatic oscillations \( \mathcal{L} \) is Hermitian in an appropriately defined vector space; this is equivalent to the well-known variational property of the eigenfrequencies of adiabatic oscillations (Ref. 61). Any small change to the model, or to the physics of the oscillations, can then formally be described as a perturbation \( \delta \mathcal{L} \) to the operator \( \mathcal{L} \), and its effects on the frequencies may be obtained from first-order perturbation theory (Ref. 62); the result may be written

\[ \delta \omega = \omega \int_0^R \frac{\delta \mathcal{L}}{\mathcal{L}} \psi^2 \rho^2 dr \]

In equation (3.12) the expression for the inner product in the vector space considered has been explicitly written out. For the purposes of the following discussion the eigenfunctions have been normalized with the surface value of \( \psi_{z,0} \).

It should be noted that equations (3.11) and (3.12) are closely analogous to the asymptotic equations (3.6) and (3.7). In fact \( \delta \) is, apart from a factor, the asymptotic approximation to \( \mathcal{L} \). In both cases the factor multiplying \( \delta \omega \) measures the inertia in the mode (or the mode mass, cf. Ref. 19); the dependence of \( \delta \omega \) on \( \delta S \) or \( \delta \mathcal{L} \) will result from the fact that modes with larger inertia are more difficult to perturb. The effect on the frequencies of the change in the model is described by the right hand sides of equations (3.6) or (3.11), and depends on the overlap between the change in the model and the eigenfunction. This is explicit in equation (3.11), where the eigenfunction appears directly; in equation (3.6) the function multiplying \( \delta \mathcal{L}/\mathcal{L} \), restricted to the region outside the turning point of the mode, is the asymptotic representation of the eigenfunction.

In analysis of frequency differences between models, or between observations and theory, it is common practice to regard the results near the surface of displacement, and the operator \( \mathcal{L} \) describes the

\[ Q_{\delta \omega} = \frac{\delta \omega}{\omega} \]

and \( \delta \omega(\omega) \) is the change in frequency \( \omega \). Thus, roughly speaking, the scaled frequency difference measures the effect on a radial mode of the same frequency of that part of the perturbation \( \delta \mathcal{L} \), which is localized to the region where the actual mode is trapped. \( Q_{\delta \omega} \) is illustrated in Figure 3.

Near the surface the vertical wavenumber becomes much larger than the horizontal wavenumber; as a consequence the behaviour of the oscillations, and in particular the eigenfunctions, depend on frequency but not on \( \ell \) (see for example Ref. 16). Thus, if \( \delta \mathcal{L} \) is confined close to the surface the integral on the right hand side of equation (3.11), and hence the scaled frequency difference \( Q_{\delta \omega} \), is a function of frequency alone. The condition for this to be true is that the extent of the region over which \( \delta \mathcal{L} \) is significant is much smaller than the depth of penetration of the modes considered. It follows that if \( Q_{\delta \omega} \) does depend on \( \ell \) for a set of modes, the change in the model extends at least to the lower turning point of those modes.

The upper reflection of the modes occurs at a point where their frequencies are equal to the acoustical cut-off frequency (Ref. 62)

\[ \omega_{ce} = \frac{2\pi}{2c} \]

Thus \( \omega_{ce} \) decreases with increasing depth; consequently, with decreasing frequency the modes are reflected more and more deeply, and their amplitude very near the surface, relative to the amplitude in the interior, decreases (Refs. 19, 64). From equation (3.11) then follows that low-frequency modes are insensitive to perturbations \( \delta \mathcal{L} \) that are confined to the superficial layers of the model. An example of this is considered in section 4.1 below.

This property of the oscillations is particularly important in the light of the discussion in Section 2.1 of the currently unavoidable errors near the surface.
of the model. Given that these errors are probably
collided to the outer 0.2 per cent of the radius, they
may be expected to lead to scaled frequency errors
that are essentially independent of \( \xi \) and small at low
frequency. Frequency errors that do not have these
properties are therefore indicative of errors in the
bulk of the model.

4. RESULTS ON SOLAR MODELS AND FREQUENCIES.

Here I present results on a number of calculations of
solar models, and adiabatic oscillation frequencies of
these models, and interpret the results in the light of
the discussion in the preceding sections.

The computational techniques were discussed in
Ref. 65. All models are calibrated to have a radius
(at the point where the temperature equals the effec-
tive temperature) of \( 6.9599 \times 10^6 \) cm; except where
otherwise noted the luminosity of the model is
3.826 \times 10^{32} \text{ erg s}^{-1} \) and the age of the present Sun is
taken to be \( 4.75 \times 10^6 \) years. The accuracy of the
calibration in radius and luminosity is at least \( 10^{-4} \) in
all cases.

In most cases the reference model is Model 1 of
Ref. 65, which resulted from a full evolution calcula-
tion. In some cases, however, I consider instead
static models of the present Sun, with hydrogen abun-
dance profiles \( \lambda (q) \) \( (q = m/M \) being the mass frac-
tion) derived from that of Model 1 through scaling
with a constant factor. The factor, together with the
mixing length parameter, is determined such that the
model has the correct luminosity and radius. This
technique is evidently much faster than a full evolu-
tion calculation, and has been shown in several cases
to give very similar results for changes in the model
caused by changes in the physics.

Great care has been taken to use the same
parameters in the calculation of reference and modi-
fied models, to ensure that the effects of for example
numerical errors are the same in both cases. Only in
this way can one be sure to obtain reliable estimates
of the occasionally rather subtle effects of the
changes to the physics.

4.1 Treatment of convection.

As a first example I consider a change in the calcu-
lation of the superadiabatic gradient near the top of
the convection zone; this is discussed in more detail in
Ref. 66. The effect of the change is to decrease
the maximum value of the superadiabatic gradient,
extending the region where it is significant, in such a
way that the specific entropy \( s \) in the adiabatic part
of the convection zone is roughly unchanged. The
analysis was carried out by means of calibrated static
models.

Figure 4 shows the resulting changes in sound
speed and \( \Gamma_1 \) in the outermost part of the model.
Here and in the following the changes in the model
are evaluated at fixed radius \( r \). The changes in the
bulk of the model are entirely negligible (cf. Ref.
66). The computed frequency differences are shown in
Figure 5. To illustrate the effect of the scaling with
\( Q_{\xi \xi} \) both unsealed and scaled differences are presented.
It is evident that although there is a sub-
stantial variation in \( \Delta \nu_{\xi \xi} \) with \( \xi \), this is almost
type caused by the variation in the mode inertia.
The scaled differences are essentially independent of
degree for \( \xi \leq 500 \). From Figure 2, we see that the
lower turning point for \( \xi = 500 \) is at a depth of
about 0.012\( R \). At higher degrees the modes extend over
only part of the region of substantially positive \( \Delta \nu_{\xi} \), and hence the region of negative \( \Delta \nu_{\xi} \) becomes
increasingly important with increasing degree, causing
the decrease in \( Q_{\xi \xi} \Delta \nu_{\xi \xi} \). Thus this example demon-
strates the ability of resolving, at least qualitatively,
sound-speed differences in the outer 1 per cent of the
Sun by means of observations of high-degree modes.

The frequency difference is essentially zero for
\( \nu \leq 2000 \text{ mHz} \). This appears to be consistent with the
arguments presented in the previous section for the
insensitivity of low-frequency modes to changes near
the surface.

It might be noted that effects on the structure
and the oscillations of adopting a non-local form of
mixing length theory were studied in Refs. 67 and 68.

4.2 The opacity.

The next example concerns the effect of modifications
to the opacity. I have arbitrarily determined the
opacity \( \kappa \), as a function of \( \rho \) and temperature \( T \), as
\[
\log \kappa = A(f(\log T)) \cdot \log \kappa_T. \tag{4.1}
\]
Here "log" is logarithm to base 10, and \( \kappa_T \) is the
opacity obtained by interpolation in the tables of Ref.
69, which were used for computing the reference
model, the function \( f \), which is shown in Figure 6,
is non-zero in a restricted temperature range around the
base of the convection zone (the location of the
lower limit of \( f \), which is within the convection zone,
is of course of no importance), and the constant \( A \)
may be adjusted to test the extent to which the
response of the model is linear in the change in the
opacity.
Figure 5. Frequency differences between the model computed with modified superadiabatic gradient (2), and the reference model (1), for selected values of \( \xi \). Points corresponding to a given value of \( \xi \) have been connected, according to the following convention: \( \xi = 0, 5, 10, 20, 30 \) (-----); \( \xi = 40, 60, 70, 100 \) (--------); \( \xi = 150, 200, 300, 400 \) (------); and \( \xi = 500, 600, 700, 800, 900, 1000 \) (-------). In addition a few values of \( \xi \) have been indicated in the figure. In a) are shown the original differences; b) shows the differences after scaling by the normalized mode Interta (cf. equation (3.13)).

Figure 6. The function \( f(\log T) \) used in the modification of the opacity (cf. equation (4.1)).

I have computed models with \( A = 0.1 \), corresponding to a maximum change in opacity at fixed \( T \) and \( \rho \) of 26 per cent, \( A = 0.2 \) and \( A = 0.4 \). These calculations also used static models. As shown in Table 1, the changes in opacity required fairly substantial changes in the composition and mixing length parameter to get the correct the luminosity and radius, and led to a significant change in the depth of the convection zone. Note that \( A = 0 \) corresponds to the unperturbed static model; this is barely distinguishable from Model 1. In Figure 7 the heavy lines show the changes in \( p, \rho, T, \sigma \) and \( \Gamma_1 \) resulting from an opacity change with \( A = 0.1 \). As might have been anticipated from equations (2.4) and (2.6), given that there is no intrinsic change in \( \Gamma_1 \), the change in the convection zone in \( p/\rho \) and hence in \( \sigma \) is mainly concentrated to the ionization zones. It then follows from equation (2.7) that the changes in \( p, \rho \) and therefore in \( \sigma \), are essentially constant in the bulk of the convection zone. Since the convection zone extends more deeply in the modified model, the temperature gradient is higher (being adiabatic) in this model than in the reference model just beneath the bottom of the convection zone of the latter. This is the reason for the increase in \( \delta \ln T \) and \( \delta \ln (p/\rho) \) with increasing depth just beneath the convection zone. The changes in the deep interior of the model are more difficult to describe in simple terms. However it should be noticed that, due to the high sensitivity of the energy generation rate on temperature, the temperature change in the core has to be small.
Figure 7. Differences between the model with modified opacity and the reference model, in the sense (modified model) - (reference model). Illustrated are $\delta \ln p$ (--- --- ---), $\delta \ln \rho$ (----- -----), $\delta \ln T$ (----- -----), and, in (b), $\delta \ln \alpha$ (----- -----). The heavy lines show the differences for an opacity modification with $A = 0.1$ (cf. equation (4.1)). To illustrate the linearity of the response of the model, the thin lines show the differences for $A = 0.2$, multiplied by $\frac{1}{k}$.

To test the linearity of the response of the model the thin lines in Figure 7 show the changes for $A = 0.2$, but multiplied by the factor $\frac{1}{k}$. Had the response been precisely linear, these curves would have coincided with the heavy curves. In fact it is obvious that the non-linearity is relatively modest, even for the substantial changes in $p$ and $\rho$ in the convection zone. Similarly, the variation in the properties shown in Table 1 also depends nearly linearly on $A$. Given the size of $A$ this success of the linear approximation is remarkable.

Table 1

<table>
<thead>
<tr>
<th>$A$</th>
<th>$\chi$</th>
<th>$\alpha$</th>
<th>$D_{CZ}/R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.9264</td>
<td>1.8984</td>
<td>0.2825</td>
</tr>
<tr>
<td>0.1</td>
<td>0.7225</td>
<td>1.6941</td>
<td>0.3013</td>
</tr>
<tr>
<td>0.2</td>
<td>0.7126</td>
<td>1.7473</td>
<td>0.3180</td>
</tr>
<tr>
<td>0.4</td>
<td>0.6962</td>
<td>1.8330</td>
<td>0.3501</td>
</tr>
</tbody>
</table>

Effects of changes to the opacity. $A$ is the amplitude of the opacity modification. $\chi$ is the surface hydrogen abundance, $\alpha$ is the mixing length parameter and $D_{CZ}$ is the depth of the convection zone.

Scaled frequency differences between the model with $A = 0.1$ and the reference model are shown in Figure 8. These differences can be understood relatively simply in terms of the differences in sound speed shown in Figure 7, by using equation (3.6) or (3.9), and the behaviour of the turning point illustrated in Figure 2. At very low degree the modes penetrate essentially to the centre, and the frequency change is given by the weighted average in equation (3.9), which is dominated by the region of positive $\delta c$ beneath the convection zone. As $\ell$ increases to 10 the turning point moves out through the region of

Figure 8. Scaled frequency differences between the model computed with modified opacity, with $A = 0.1$, (2), and the reference model (1), for selected values of $\ell$. Points corresponding to a given value of $\ell$ have been connected, according to the following convention: $\ell = 0, 5, 10, 20, 30$ (--- --- ---); $\ell = 40, 50, 70, 100$ (----- -----); $\ell = 150, 200, 300, 400$ (----- -----); and $\ell = 500, 600, 700, 800, 900, 1000$ (----- -----). In addition a few values of $\ell$ have been indicated in the figure.
slightly negative $\delta c$ near the core, and $Q_{0} \delta v_{e} / v_{a}$ increases. At higher degrees, beginning at low frequency for $E = 20$, and at increasing $\nu$ when $E$ increases to 50, the modes become largely confined within the convection zone. Here $\delta c$ is negative and significant only near the surface; thus the frequency differences are negative, depend little on $E$, and are furthermore small at low frequency. It should be noticed that the negative $\delta c$ near the surface has a substantial effect on the frequencies, despite its insignificant appearance in Figure 7a. The reason is the weighting with $e^1$ (cf. equation (3.9)) which makes the frequencies very sensitive to changes in the model near the surface.

This description of the relation between the sound speed and the frequency changes is evidently in accordance with equation (3.8). Thus $Q_{0} \delta v_{e} / v_{a}$ may be expected to be predominantly a function of the turning point position or, equivalently, of $\nu_{a} / L$ (Ref. 65). This is confirmed by Figure 9, where the scaled relative frequency differences are plotted in this form. Particularly striking is the transition near $\nu_{a} / L \approx 100 \mu Hz$, corresponding to modes with turning points in the vicinity of the base of the convection zone. It should also be noticed that for the modes confined to the convection zone the term in $H_2$, corresponding to the change in $\alpha$ between the models, is responsible for a substantial part of the frequency change.

A much more extensive study of the effects of modifications to the opacity was carried out by Korzennik & Ulrich (Ref. 34). By modifying the opacity locally at different temperatures they obtained kernels relating the opacity change to the change in oscillation frequencies; they then attempted to invert the differences between observed and computed frequencies to estimate the modifications required to the opacity. It might also be mentioned that studies have been made of the effects of opacity changes on double mode Cepheids and $\delta$ Scuti stars (e.g. Ref. 70).

4.3 The equation of state.

Most of the calculations reported here used a very simple description of the equation of state, proposed by Eggleton, Faulkner & Flannery (EFF; Ref. 71); among other simplifications, it assumes that all atoms and ions are in the ground state. To test the sensitivity of the models to the equation of state I consider results of a calculation using a much more sophisticated treatment of the thermodynamics of the gas, developed by B. and D. Mihalas, Hummer & Dappen (MHD; Refs. 72, 73, 74; see also Ref. 75). Here the partition functions of the atoms are computed taking into account the interaction with the other particles in the gas. The effects of using the MHD equation of state on the models and the frequencies are discussed in Ref. 33, but are summarized here.

Figure 10 shows differences between the MHD and the EFF models. Furthermore, in Figure 10b the thin line shows $\delta v_{e}$, calculated using equations (2.3) and (2.4), with $\delta \Delta P$ replaced by $\delta \Delta P^r$. The close agreement with the actual sound-speed difference indicates that the difference between the superadiabatic gradients has a minor effect in this case. It should be noticed that in contrast to the modification of the opacity, $\delta \Delta P$ is non-negligible in much of the convection zone. As a result $\delta \Delta P$, which is again given by equation (2.7) varies throughout the convection zone. The large positive peak in $\delta \Delta P$ near the surface is
Figure 10. Differences between the models using the MHD and the EFF equations of state, in the sense MHD - EFF. In a) are illustrated \( \delta \ln \rho \) (---), \( \delta \ln T \) (-----), and \( \delta \ln n \) (-----). In b) are shown in addition \( \delta \ln n \) (---), and, as the thin continuous line, the approximation to \( \delta \ln n \) obtained from \( \delta \ln n \) by using equation (2.4), with \( \delta \ln n \) replaced by \( \delta \ln n \), and equation (2.3).

Figure 11. Scaled frequency differences \( \nu_{P}^{\text{MHD}} - \nu_{P}^{\text{EFF}} \) between the MHD and the EFF models, for selected values of \( \ell \). Points corresponding to a given value of \( \ell \) have been connected, according to the following convention: \( \ell = 0, 5, 10, 20, 30 \) (--); \( \ell = 40, 50, 70, 100 \) (-----); \( \ell = 150, 200, 300, 400 \) (-----); and \( \ell = 500, 600, 700, 800, 900, 1000 \) (-----). In addition a few values of \( \ell \) have been indicated in the figure.
Table 2.

<table>
<thead>
<tr>
<th>Case</th>
<th>$X$</th>
<th>$\alpha$</th>
<th>$D_{cz}/R$</th>
<th>$D_{\nu}$ (µHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>0.73344</td>
<td>1.6357</td>
<td>0.2827</td>
<td>1.540</td>
</tr>
<tr>
<td>10 per cent increase in $S_{pe}$</td>
<td>0.73107</td>
<td>1.6376</td>
<td>0.2870</td>
<td>1.517</td>
</tr>
<tr>
<td>Age 5.25 $\times 10^9$ years</td>
<td>0.73852</td>
<td>1.6870</td>
<td>0.2875</td>
<td>1.476</td>
</tr>
<tr>
<td>$L_0 = 3.846 \times 10^{33}$ erg s$^{-1}$</td>
<td>0.73287</td>
<td>1.6457</td>
<td>0.2827</td>
<td>1.537</td>
</tr>
<tr>
<td>Partially mixed core</td>
<td>0.7286</td>
<td>1.5563</td>
<td>0.2727</td>
<td>2.002</td>
</tr>
</tbody>
</table>

Effects of changes to reaction rates, solar age, and solar luminosity, and effects of partial core mixing. $X$ is the surface hydrogen abundance, $\alpha$ is the mixing length parameter, $D_{cz}$ is the depth of the convection zone, and $D_{\nu}$ is a parameter characterizing the separation of frequencies of low-degree modes (cf. equation (3.5)).

Effects on frequencies of high-degree modes of modifying the equation of state were also studied in Refs. 67 and 76.

4.4 The rate of energy generation.

To test the sensitivity of the models to the parameters determining the rate of nuclear reactions, I have
artificially increased the cross-section factor $S_{pp}$ of the basic p-p reaction by 10 per cent. In this case full evolution calculations were used. Similar calculations were carried out by Lebreton et al. (Ref. 77), motivated by the uncertainty in the axial-vector weak-interaction coupling constant. The changes in the model are illustrated in Figure 12; other properties of the normal and the modified models are given in Table 2. The changes in the convection zone follow the same general pattern as was found when the opacity was modified (cf. Figure 7). However a more careful comparison between Figures 7b and 12b reveals significant differences in the detailed behaviour of $\delta ln\alpha$. This is probably related to differences in the relative importance of the change in composition and specific entropy. It follows from the discussion in Section 2 that, given the equation of state, the structure of the adiabatic part of the convection zone is determined by the composition and the specific entropy $s$, which in turn are determined by the hydrogen abundance $X$ and the mixing length parameter $\alpha$. Thus the change in the properties of the convection zone should essentially be given as a linear combination of the changes resulting from "unit" modifications in either $X$ or $\alpha$, weighted by the actual changes in $X$ and $\alpha$. In fact a comparison between Tables 1 and 2 shows that for the modification in opacity the change in $X$ is substantially larger, relative to the change in $\alpha$, than for the modification in the reaction rates; in the latter case the change in $\alpha$, and hence in specific entropy, dominates the changes in the outer part of the convection zone.

As with the change in opacity $\delta ln\rho$ is essentially constant in much of the convection zone; also the small increase in the depth of the convection zone causes the positive $\delta ln\tau$ and $\delta \ln c$ just below the average of the frequency spacing $\nu_{n,0} - \nu_{n-1,2}^2$.  

4.5 The solar age.

This example considers the sensitivity of the solar model to the assumed age. Similar model calculations were reported by Guenther & Sarajedini (Ref. 82). Although there are indications that the age of the Sun should be smaller than the value of $4.75 \times 10^9$
years assumed for the reference model (Ref. 83), I here consider a model of age $5.25 \times 10^9$ years. Separate tests have shown that for age changes of this magnitude, the response of the model is very nearly linear in the change in age.

Parameters for the model are presented in Table 2, and Figure 14 shows changes in the structure. The overall features of the changes in the convection zone are very similar to those found when the opacity or the reaction rate was changed, but the details (cf. Figure 14b) differ from both cases. In fact Table 2 shows that the change in the frequency in $X$ and $\alpha$ is of similar magnitude to the ratio for the opacity change, but the change in $X$ and $\alpha$ are now of the same, rather than opposite, sign. There is a fairly substantial change in the sound speed in the core, resulting from the lower central hydrogen abundance which is caused by the higher age of the model.

The change in the core sound speed has a significant effect on the frequencies, as shown in Figure 15. The change with $\kappa$ in $Q_{\kappa \nu \nu}$ among the lowest-degree modes is more pronounced than in the case of the modified $S_{\nu \nu}$. This causes a significant decrease in $D_0$ (cf. Table 2). A similar change with increasing age occurs in evolution sequences of models (Refs. 84, 85, 81).

The relations between age, hydrogen abundance, and frequencies of low-degree modes are similar to those obtained in Ref. 82. From these results the authors conclude that a determination of the solar helium abundance $Y$ from $p$ mode frequencies will be very difficult. This conclusion may be premature. It was pointed out in Refs. 31 and 32 that a more direct determination of $Y$ may be possible from analysis of the frequencies of high-degree modes which, being trapped in the convection zone, are insensitive to the uncertainties of the physics of the solar interior. In contrast, the analysis presented by Guenther & Sarajedini depends on the calibration of complete solar models. Furthermore, within the framework of such a calibration, it is important to consider all the information available. Thus it is clear from Figure 15 that modes of 10 - 20 are considerably more sensitive to changes in age, or equivalently helium abundance, than the low-degree modes discussed in Ref. 82.

4.6 The solar luminosity.

The surface luminosity $L_s$ used in the model calculation should really be the value corresponding to thermal equilibrium. Given that the solar irradiance varies through the solar cycle (Ref. 86) it is not obvious how to choose the equilibrium value. To this uncertainty must be added the intrinsic errors in the measurement of the luminosity. Thus there is some interest in investigating the sensitivity of the model to the assumed value of $L_s$.

To do so I have computed a model with $L_s = 3.846 \times 10^{33}$ erg s$^{-1}$. This differs from the reference value of $3.826 \times 10^{33}$ erg s$^{-1}$ by considerably more than the observed range of variation (Ref. 86), or the likely measurement error. It is evident from the values in Table 2, and the differences shown in Figure 16, that such a modification of $L_s$ has a very small effect on the model. Similarly, as shown in Figure 17, the changes in the frequencies are insignificant, at the present level of computational and measurement accuracy.

It is important to emphasize that this result is not immediately relevant to the possible frequency changes that might be associated with the solar cycle. The models considered here both reach thermal equilibrium and have the same radius. In contrast, the variations during the solar cycle are presumably associated with energy storage in the convection zone, and hence involve departures from thermal equilibrium. Thus to study these variations, a proper time-dependent calculation is required (see also Refs. 4, 6).

4.7 Partial core mixing.

To study the effect of partial mixing in the core, I have computed a model whose hydrogen profile $X(q)$ was obtained by scaling with a constant factor from that of a model of Schatzman et al. (Ref. 35). Here mixing was modelled through weak turbulent diffusion; I used the model for which the so-called turbulent Reynolds number $Re^t$ was 100, leading to a neutrino flux which is consistent with the observed value. This example is discussed in more detail in Ref. 66.
Figure 17. Scaled frequency differences between the model with increased surface luminosity (2) and the reference model (1), for selected values of \( \varepsilon \). Points corresponding to a given value of \( \varepsilon \) have been connected, according to the following convention: \( \varepsilon = 0, 1, 2, 3, 4, 5, 10, 20, 30 \) (-----); \( \varepsilon = 40, 50, 70, 100 \) (-----); \( \varepsilon = 150, 200, 300, 400 \) (-----); and \( \varepsilon = 500, 600, 700, 800, 900, 1000 \) (-----).

Figure 18. Differences between the model with a partially mixed core and the reference model, in the sense (modified model) - (reference model). Illustrated are \( \delta \ln p \) (-----), \( \delta \ln \rho \) (-----), \( \delta \text{ln} \) (-----) and, in (b), \( \delta \text{ln} r \) (-----).

Figure 19. Scaled frequency differences between the model with partially mixed core (2) and the reference model (1), for selected values of \( \varepsilon \). Points corresponding to a given value of \( \varepsilon \) have been connected, according to the following convention: \( \varepsilon = 0, 1, 2, 3, 4, 5, 10, 20, 30 \) (-----); \( \varepsilon = 40, 50, 70, 100 \) (-----); \( \varepsilon = 150, 200, 300, 400 \) (-----); and \( \varepsilon = 500, 600, 700, 800, 900, 1000 \) (-----). In addition a few values of \( \varepsilon \) have been indicated in the figure.
Parameters of the model are given in Table 2, and differences in structure are shown in Figure 18. The most striking effect in this case is the very large increase in sound speed in the core, caused by the decrease in mean molecular weight resulting from the mixing. The pattern of change in the outer part of the model is by now familiar. It might be noticed that in this case both \( \alpha \) and \( \chi \) are reduced, and hence \( \delta \chi \) is similar, though of opposite sign, to the change (cf. Figure 1d) resulting from an increase in model age.

The rapidly varying change in sound speed in the core results in a rapid variation with \( \ell \) in the frequency change, shown in Figure 19. This also leads to the substantial increase in \( D_0 \) shown in Table 2. The dependence of the frequency change on \( \ell \) clearly reflects the variation of \( \delta \chi \) with \( r \). Indeed it is possible to carry out an inversion of the frequency differences to recover the difference in sound speed (Ref. 57). At higher degrees there is yet again a transition to modes trapped in the convection zone, and dominated by the positive \( \delta \chi \) in the ionization zones.

Effects on \( p \) mode frequencies of core mixing were also calculated by Ulrich & Rhodes (Ref. 11). A more detailed analysis of the results was given by Ulrich et al. (Ref. 87).

5. COMPARISON WITH OBSERVED FREQUENCIES.

When comparing with the observed frequencies great care is required to ensure that the numerical precision of the computed frequencies, given the assumed physics, is adequate. The Sun does not make numerical errors!

I have carried out a series of tests, to optimize the distribution of mesh points used in both model and oscillation calculations, and to estimate the residual error in the frequencies. The computations reported in this section all used 600 spatial mesh points and 26 time steps in the model calculation, with a distribution somewhat different from that used in Ref. 65; the resulting models were interpolated by means of cubic interpolation to new meshes, with 1200 points, optimized for the computation of \( p \) modes. The accuracy of the resulting models and frequencies were estimated by comparing with calculations using fewer mesh points in either the model or the oscillation calculation, or fewer time steps in the model calculation. The maximum relative error, at fixed \( r \), in \( p \), \( \rho \) and \( T \) in the model with 600 points is below 0.1 per cent, the maximum error in \( \rho \) being probably less than 0.03 per cent. Also the contribution from the model error to the \( Q_0 \)-scaled error in the frequencies is less than about 1 \( \mu \text{Hz} \) over the entire range of modes considered. The errors introduced in the frequency computation for a given model are considerably smaller. Thus it can be assumed that the computed frequencies are accurate to about 1 \( \mu \text{Hz} \), for given physics. This is comparable to, although possibly somewhat worse than, the accuracy of most observed frequencies at low and moderate degree (e.g. Ref. 89).

Figure 20 shows unscaled differences between observed frequencies and frequencies for a model computed with the EFF equation of state, for selected values of \( \ell \). Points corresponding to a given value of \( \ell \) have been connected, according to the following convention: \( \ell = 0, 5, 10, 20, 30 \) (--- model---); \( \ell = 40, 50, 70, 100 \) (-----); \( \ell = 150, 200, 300, 400 \) (-----); and \( \ell = 500, 600, 700, 800, 900, 1000 \) (-----). In addition a few values of \( \ell \) have been indicated in the figure.

Figure 20 shows unscaled differences between observed frequencies and frequencies for a model computed with the EFF equation of state, for selected values of \( \ell \). There is evidently a substantial general increase in the errors in the computed frequencies with \( \ell \). Much of this variation, however, is caused by the variation in mode inertia, and is absent in the corresponding scaled differences, shown in Figure 21a. There remains some variation in the error with \( \ell \) between degrees of about 20 and 40. In Ref. 54, where a similar analysis based on unscaled frequency differences was carried out, this variation was attributed to an error in the model near the base of the convection zone. Indeed the effect is superficially quite similar to that observed in the model comparisons in Section 4, and caused by the change in the depth of the convection zone between the models. It was pointed out in Ref. 54 that apart from this effect, the frequency errors depend predominantly on frequency, indicating, as discussed in Section 3, that most of the remaining error in the model is located near the surface. It is therefore tempting to associate this error with the known uncertainties in the treatment of the superficial layers of the Sun.
Figure 21. Differences between observed and computed frequencies, scaled with the normalized mode inertia (cf. equation (3.13)), for selected values of $\tau$. The results in a) are the same, apart from the scaling, as those shown in Figure 20. In b) were used frequencies for a model computed with the MHD equation of state. Points corresponding to a given value of $\tau$ have been connected, according to the following convention: $\tau = 0, 5, 10, 20, 30$ (------------); $\tau = 40, 50, 70, 100$ (-------); $\tau = 150, 200, 300, 400$ (--------); and $\tau = 500, 600, 700, 800, 900, 1000$ (---------). In addition a few values of $\tau$ have been indicated in the figure.

With the improved data now available, and the use of scaled frequency differences, this conclusion needs to be reconsidered. Thus it is clear from Figure 21a that there remains some variation in the error with $\tau$, for $\tau > 200$. Furthermore, the errors are quite substantial at low frequencies. As argued in Section 3 this suggests that the errors in the model are not solely confined to the uncertain layers near the surface.

When the observations are compared with the results based on the MHD equation of state, a strikingly different picture emerges, as shown in Figure 21b. Now the error at low frequencies is very much reduced, and, except for $\tau = 900$, the systematic dependence of the error on $\tau$ is quite small. The variation of the error with $\tau$ associated with the base of the convection zone remaining; but apart from this it appears that the use of the improved equation of state has removed most of the errors in the interior of the model.

Inversions of the observed frequencies (Refs. 51, 58, 90, 91) have shown that the variation at $\tau = 20 - 40$ is associated with an error in the sound speed at $r/R = 0.3 - 0.7$. This is absent in models that employ newer and higher opacities (e.g. Ref. 92). Thus this part of the frequency error is probably caused by the use of inaccurate opacities. This is consistent with the effect discussed in Section 4.2 of opacity modifications on the frequencies.

The dominant component of the frequency error which is a function of frequency alone must be associated with the errors in the calculation that are surely present due to the insufficient treatment of the region of substantially superadiabatic convection and nonadiabatic oscillations. Indeed, given the approximations made in the present calculations, absence of such errors in the frequencies would have been cause for some concern. We are currently unable to make precise and realistic calculations that take all effects in this region into account. However, it is very interesting that detailed numerical models of granulation (Ref. 93) show five minute oscillations; the modes are shifted in frequency relative to those of an equivalent simple models by an amount that, if properly scaled by the mode inertia, is not unlike the remaining frequency differences in Figure 21b (Ref. 94).

6. CONCLUSIONS.

Within the standard theory of solar structure and oscillations the frequencies are determined by the assumed physics of the solar interior, and the parameters used in the model calculation. The results presented in Section 4 give some indications of the sensitivity of the frequencies, by exploring the effects of changes to the physics and the parameters. In each case the modifications were probably representa-
tive of current uncertainties. The most significant effects were caused by the changes in the opacity and the equation of state, as well as a (largely artificial) change in the calculation of the superadiabatic gradient in the upper part of the convection zone. Not surprisingly, the frequencies were relatively insensitive to a change in the nuclear energy generation rate, and almost unaffected by a change in the assumed surface luminosity. An increase in the assumed age of the Sun had a relatively modest effect; nevertheless, because the change in sound speed was largest in the core of the model, it resulted in a significant decrease in the frequency separation $\nu_{1,2}$ which is now observationally very well determined.

It is evident that observations with finite precision of a finite number of modes can never provide complete information about the essentially infinitely complex properties of the solar interior. The task of interpreting the results is considerably simplified, however, by the properties of the oscillations, which make certain modes, or combinations of modes, sensitive predominantly to specific aspects of the model. In its most general form this principle underlies inverse analyses of the frequencies. Here I have used it in qualitative discussions of the relations between model and frequency changes. A specific, and very useful, example is that properly scaled frequency differences, relative to frequency differences in the model computed near the surface, are functions of frequency but not of degree. Also, since the structure of the convection does not depend directly on the opacity, frequencies of modes trapped in the convection zone are determined by the equation of state, the composition and the value of the specific entropy $s$, which is constant in the adiabatic part of the convection zone; the uncertainty in the opacity only enters indirectly, through its effect on $s$.

Except for the region near the solar surface, there is no compelling evidence in the observed oscillation frequencies for departures from the standard assumptions of stellar structure calculations; however other difficulties with the standard models have lead to suggestions of such departures. Thus it is of obvious interest to investigate their effects on the oscillation frequencies; in this way the observed frequencies may put limits on the possible magnitude of the deviation from standard theory. As an example, I have considered briefly the effects of partial mixing of the solar core; mixing of this magnitude seems firmly ruled out by the observed value of the frequency separation at low degree. However the list of possible non-standard effects is long, and requires further extensive investigations. It is likely that these investigations will also lead to improved insight into the relations between structure and frequencies for normal models.

The comparison of theory and observations in Section 5 suggests that, if the best possible physics of the solar interior is used, the dominant part of the error in the computed frequencies may be associated with the superficial layers of the Sun. These layers are extremely difficult to model, due to the effects of convection and radiative transfer; a measure of the difficulty is the lack of agreement between different calculations of the stability of the solar modes. Thus it is very encouraging that information about this region is now becoming available from detailed hydrodynamical calculations (Ref. 94). Although the results are preliminary, they appear to account roughly for the remaining frequency errors in the calculations. Thus we may be getting close to reproducing the gross features of the observed frequencies.

This is evidently just the starting point for the investigations of solar structure by means of helioseismology. On this basis we can begin to probe the details of the solar interior; for example the sensitivity of the equation of state found in Section 4.3 suggests that it may become possible to study the thermodynamic properties of matter in the solar convection zone. In this process we should keep an open mind for possible indications of more fundamental failings of the model, which may show up in what could initially appear as insignificant features in the observations. On the other hand one should evidently be wary of adopting striking departures from the standard theory before the possibilities of more mundane explanations have been thoroughly explored.

The analysis of the processes near the solar surface will undoubtedly be greatly aided by the measurements of line widths (Ref. 19) and of the phase and amplitude relations between intensity and velocity oscillations (e.g. Ref. 95). Also progress is being made in calculating the effects of convection and radiation on the stability of the modes (e.g. Ref. 96), and in the modelling of non-adiabatic effects between convection modes and (Ref. 97) and among the modes (Ref. 98). By combining the results of the calculations, both detailed and simplified, and the observed properties of the oscillations, including their frequencies, we may hope eventually to get an understanding of this complex region.

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STUDY OF SOLAR STRUCTURE BASED ON P-MODE HELIOSEISMOLOGY

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CURRENT OPTIONS FOR THE EQUATION OF STATE OF THE SOLAR INTERIOR

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ABSTRACT

The principal open problem of the equation of state is the number of excited states of hydrogen and helium in the zones of partial ionization. The number of excited states is affected in two ways: first by destruction of the levels due to the (neutral and charged) surrounding species, and second by statistical mechanics which governs the population of the available levels. I discuss recent progress in this field and explain the two principal current "philosophies". One is the chemist's view, in which bound systems are interpreted as autonomous species with reactions between each other, the other is the physicist's view, in which a virial expansion of pressure is obtained starting out from fundamental species only (i.e. electrons and nuclei), and in which the so-called Planck-Larkin partition function appears.

Keywords: solar and stellar structure, equation of state, Planck-Larkin partition function

1. INTRODUCTION

For many astrophysical applications crude recipes for the equation of state are quite adequate (e.g. Saha's equation with ground-state partition functions, together with an imposed total ionization above a certain temperature). For finer, for instance helioseismological, purposes a better description is needed. Firstly, because in the usual derivation of the stellar pulsation equations one makes use of thermodynamic identities. Therefore, the equation of state has at least to be formally consistent. Imposing \textit{ad-hoc} total ionization in the central layers of stars, however, violates thermodynamic consistency. Secondly, because helioseismology has attained such a level of precision that better descriptions of the physics of stellar interiors will be needed. In particular, the determination of the helium abundance of the solar convection zone by a thermodynamic method, based on the signature of the second helium ionization zone on local sound speed and thus on acoustic-mode frequencies, will require an \textit{accurate}, and not just formally correct equation of state (Refs. 1-4). Further examples of the relevance of the equation of state for stellar structure can be found elsewhere in these proceedings (Ref. 5).

In the following, I discuss the requirements demanded from an equation of state, then I discuss some typical equations of state (for more detailed compilations of the literature see Refs. 6 and 7). The discussion starts with a simple but consistent equation of state, goes then to a recently finished formalism that incorporates a large number of atomic data, and ends with a somewhat alternative approach to the equation of state that is currently being developed at Livermore.

Finally, I demonstrate the importance of the equation of state at the example of solar high-degree \textit{p} modes, where a recent study (Ref. 8) has shown that better agreement between theory and observation can be obtained by using a more sophisticated equation of state.

2. REQUIREMENTS ON AN EQUATION OF STATE

I'd like to emphasize the importance of a \textit{consistent} equation of state for stellar applications. Without exaggeration one can say that consistency is even more important than accuracy of the physical description. The reason is that in many conventional applications formal consistency of thermodynamic quantities is used several times during the manipulations of the hydrodynamical equations. Consider, as an example, how the adiabatic temperature gradient enters the derivation of the adiabatic pulsation equations after the substitution of pressure perturbations by density perturbations (which are further transformed using mass conservation). The adiabatic gradient thus inserted must be consistent with the equation of state used in the equation of hydrostatic support.

From these considerations it is clear that the degree of formal accuracy required of an equation of state strongly depends on the chosen purpose. Equilibrium models and adiabatic pulsations need second-order thermodynamic quantities (the terminology refers to derivatives of the free energy or any other equivalent thermodynamic potential). Nonadiabatic pulsation calculations have to go one level deeper: third-order quantities such as derivatives of the adiabatic gradient or specific heat are also required. In a complicated equation of state that comprises many nonideal effects it is a highly nontrivial matter to achieve accurate third-order quantities.
Formal accuracy is of course not enough, and perhaps the most transparent solar physics application that demonstrates the need for real accuracy of the physical description is the thermodynamic method to determine the helium abundance of the solar convection zone (Refs. 1, 2, 4). Third order quantities are needed to interpret the so-called "helium hump" in the derivative of the sound speed (inferred from inversions of solar p-mode frequencies). The height of the helium hump depends on the internal partition functions (Ref. 3).

3. SIMPLE EQUATIONS OF STATE

Disregarding the even simpler polytropic relations, the simplest prescriptions available consist in mixtures of ideal gases with ionization- and sometimes molecular dissociation- reactions. In these formalisms, the statistical weight of the bound species contains the ground-state contribution only. Some of the simple equations of state can treat partially degenerate electrons according to Fermi-Dirac statistics. Although time-honoured and highly successful in stellar envelopes, such simple equations of state all suffer from their inability to describe pressure (or synonymously density) ionization. From physical arguments it is clear that atoms must be ionized at very high densities, but the Saha equation predicts just the contrary, i.e. an unphysical recombination of atoms. In the solar center, for instance, as much as 30% neutral hydrogen would be allowed. This is clearly at variance with elementary volume considerations, from which one concludes that at densities of 150 g cm\(^{-3}\) (of the solar center) there is no room for neutral hydrogen atoms, which have (in tightly packed configurations) densities of the order of 1 g cm\(^{-3}\). Since the aforementioned simple equations of state know nothing about the radius of the hydrogen atom, their predicted recombination is, in this approximation, a legitimate quantum-mechanical effect, reflecting the fact that at higher densities (despite simultaneously higher temperatures) the continuum states of the electrons become less accessible. This is due to a smaller density of electronic states per energy interval at higher densities (think of the problem of electrons in a box, and note how the discrete energy spectrum gets wider-spaced the smaller the box is). The test is done by the Pauli principle, which causes, at high densities, a piling up to such high continuum energies that the system finally responds with favouring atomic recombination as the lesser evil. Only more realistic equations of state can get rid of this unphysical recombination.

Eggleton, Faulkner and Flannery (Ref. 9) developed a simple equation of state (EFF) that is formally consistent and that describes pressure ionization at least qualitatively. But their pressure ionization device is not achieved by a physical model (e.g. by a description of an atom and its surrounding particles), but is imposed in an ad-hoc way by forcing the anticipated result, i.e. full ionization at high densities. Parameters fix the actual location (in temperature and density) of the pressure ionization zone. Varying these parameters might give us some indications about the sensitivity of stellar models on the precise nature of pressure ionization, as has been suggested by Bahcall and Ulrich (Ref. 10). Despite the lack of physical foundation, EFF is a useful equation of state because of its thermodynamic consistency. Compared with simple prescriptions, like imposing full ionization beyond an empirically determined adiabate (such devices can indeed be found in some of the current programmes of stellar evolution), EFF is better suited for stellar pulsation applications, and has been, e.g., successfully employed in the solar models of Christensen-Dalsgaard (Ref. 11). In addition, the relative simplicity of the EFF formalism allows accurate numerical realizations that can be put directly in stellar programmes.

On the negative side, I mention the principal limitations of EFF, given by the absence of: 1) a physical mechanism for pressure ionization, 2) excited states in the bound systems, 3) a treatment of hydrogen molecules (important for low-mass stars, see Ref. 5), and 4) the Coulomb-pressure correction.

4. TOWARDS REALISTIC EQUATIONS OF STATE

4.1 The free-energy-minimization method

Most realistic equations of state that have appeared in the last 30 years are based on the free-energy minimization method. The free-energy minimization method uses approximate statistical mechanical models (for example the nonrelativistic electron gas, Debye-Hückel theory for ionic species, hard-core atoms to simulate pressure ionization via configurational terms, quantum mechanical models of atoms in perturbed fields, etc.). From these models a macroscopic free energy is constructed as a function of temperature \(T\), volume \(V\), and the concentrations \(N_1,...,N_k\) of the \(k\) components of the plasma.

Usually, the chemist's view is taken, in which bound species (atoms, ions, molecules) are treated in the same way as fundamental particles (electrons and nuclei). There is an intuitive simplicity contained in this view (we usually take the existence of atoms in plasmas for granted, at least at not too high densities), but this simplicity has to be paid for by additional minimization procedures in the multidimensional space of abundances of each species, restricted by the appropriate stoichiometrical relations and by mass and charge conservation. The physical idea behind this minimization is simple: the "internal" degrees of freedom (like ionization degrees) are not adjustable by the experimenter, who can only control "external" parameters (like temperature, density, and mass fractions of each chemical element). The thermodynamic equilibrium is then determined as the one configuration (compared to those having different internal parameters) that minimizes the free energy, or equivalently, maximizes entropy. Once this minimum is found, the model free energy delivers all thermodynamic quantities in a straightforward way by differentiation.

4.2 Tables and approximative analytic versions

Fontaine, Graboske and van Horn (Ref. 12, hereinafter FGH) published extensive tabular material based on the free-energy-minimization method at lower and intermediate densities, and on Thomas-Fermi theory at high densities. Between the two zones a thermodynamically consistent interpolation procedure was employed. However, the relative coarseness of the tables rendered them difficult for use in stellar models. Firstly, only second-order quantities are tabulated, and the spacing in temperature and density chosen does not allow numerical differentiation within the tables. Therefore, it is not possible to obtain third-order quantities. Secondly, practical applications of
the tables are restricted by the lack of choice in the chemical composition: only pure-hydrogen and pure-helium mixtures were presented, together with a prescription for a volume-weighted interpolation for stellar H-He mixtures.

However, interpolations of that kind do violate thermodynamic consistency, because each thermodynamic quantity has a different nonlinear dependence on the chemical composition. Therefore I proposed a simplified model (Ref. 13), based on the confined atom model (hereinafter CAM), which can produce the results of these tables approximately, and which is simple enough to be programmed within stellar evolution codes. In the confined-atom model, the Coulomb potential outside a sphere of radius $R$ is replaced by an infinitely high potential wall (this is equivalent to a zero boundary condition of the wave function at $R$). The value of $R$ is chosen as a function of the volume available for a given bound species. For $R < \infty$, all bound-state energies are lifted from their unperturbed values; with decreasing $R$ the higher states are gradually spilled over into the continuum. While physically this is certainly not a very realistic procedure, it has some formal advantages (like being able to ionize "from the cold", i.e. without "seed electrons" or other starters (usually assumed available from alkali-type metals).

Compared with the simple equations of state discussed in section 3., both FGH and CAM represent major physical improvements: realistic pressure ionization devices, Coulomb pressure, and polynomial fits for excited states.

4.3 The MHD equation of state

Mihalas, Hummer and Däppen (Refs. 6 and 14-15, hereinafter MHD) have recently developed a new treatment of the equation of state, which is part of an ongoing opacity project (Ref. 16). While the basic concept of the MHD equation of state is conventional (it is based on the free-energy-minimization method), it is characterized by detailed internal partition functions of a large number of atomic, ionic, and molecular species. Full thermodynamic consistency is assured by analytical expressions of the free energy and its first- and second-order derivatives. This not only allows an efficient Newton-Raphson minimization, but, in addition, the ensuing thermodynamic quantities are of analytical precision and can therefore be differentiated once more, this time numerically. Reliable third-order thermodynamic quantities are thus calculated.

Apart from formal thermodynamic consistency, the MHD equation of state also achieves some degree of statistical mechanical consistency (Ref. 6). Statistical mechanical consistency refers to the more subtle requirement that each time a bound configuration (like an atom) is modified by its surroundings in the plasma, then the relevant force has to be described in the physical description of the surroundings as well. To be more specific consider as an example the EFF equation of state (Ref. 9), which is thermodynamically consistent, but not statistical mechanically consistent, because atoms are pressure ionized by an ad-hoc mechanism which does not have its counterpart in the free particles.

In the MHD equation of state, perturbations by charged and neutral species are taken into account. These perturbations are described by an occupation probability for each energy level. These occupation probabilities are reduced by excluded-volume effects for neutral perturbers, and by the Stark effect for charged perturbers (see Ref. 6). A first comparison of these occupation probabilities with experiment has been made: Däppen, Anderson and Mihalas (Ref. 7) have used them to simulate the radiation from a precision plasma experiment (Ref. 17), and the agreement is good.

The MHD equation of state is a computational heavy weight. Unless drastically simplified to a small number of atomic and ionic species, it has to be used in the form of tables. A tape with first results is already available (Ref. 18). Furthermore, Yveline Lebreton and I have developed programmes (Ref. 5 and 19) that can automatically create tables that are centered around the temperatures and densities of stellar interiors. These table-creating programmes can also handle the changes in chemical composition during the main-sequence evolution of stars. As I have mentioned above, the smoothness of the MHD formalism allows tabulation of third-order thermodynamic quantities. Furthermore, by tabulating the results from EFF in the same way as those of MHD, and by comparing the models that use the interpolated EFF results with the models that call EFF directly (Refs. 5, 8, 19), we can control the accuracy of the interpolation process and adjust the necessary fineness of the tables.

Before leaving the MHD equation of state I'd like to add just one general remark. Though the MHD formalism was developed for stellar envelopes, there is no problem in applying it to interiors. Though one is leaving the domain for which MHD was originally conceived, the physical ingredients are, even at these high densities, quite the same as the ones conventionally used in models of stellar interiors. Only theories that treat higher order correlations in plasmas seriously would distinctly go beyond our assumptions. Such theories still await to be applied in the context of stellar interiors, and in order to extend them to the low density regime, special care will have to be given to overall consistency.

As an illustration for the successful extrapolation of the MHD formalism to high densities, I would like to cite an extreme example: Pesnell (private communication) has succeeded in constructing models for structure and pulsation of white dwarfs using the MHD equation of state.

4.4 An alternative approach developed at Livermore

All equation-of-state formalisms mentioned so far have been formulated in the so-called chemical picture, where bound systems (atoms, ions and molecules) are treated on the same level as the fundamental ones (electrons and nuclei). Reactions between the different species are assumed. As we have seen in subsection 4.1, the price to pay for this intuitive simplicity is the necessity to perform a minimization of the free-energy. Since realistic free-energies are highly nonlinear, and since the minimum of the free energy has to be found in a multidimensional space this is indeed a high price to pay. Understandably therefore, alternative approaches have been tried. They are essentially based on many-body expansions of the grand-canonical partition function. Only fundamental particles (electrons and nuclei) enter these formalisms explicitly. This has of course the advantage that minimization procedures like those in the free-energy method are not necessary.

However, this physical picture is difficult to realize, too. The reason is that the grand-canonical partition function is an infinite sum comprising terms involving subspaces of one, two, etc.
particles. Since the quantum-mechanical three-body problem is not solvable, quasi-classical approximations must be used for the higher terms. The summation technique of the grand-canonical partition function goes back to Mayer (Ref. 20). Reviews of more recent developments can be found in Refs. 21-22 and 7. The natural thermodynamic potential of the grand-canonical partition function is pressure as a function of temperature, density, and the chemical potential. The sum in the grand-canonical partition function has its analogue in the virial expansion of pressure, which is essentially a power series of pressure with respect to density. While the lowest order term correspond to the noninteracting gas, the higher terms represent the correlation effects of two and more particles. Several authors have observed independently (see e.g. Ref. 7) that in the twobody part of pressure (and also in the higher terms) cancellations of (weak) bound state and continuum contributions occur. Note that these cancellations affect pressure, and not the internal partition functions of the chemical picture, which would have only positive terms. They are therefore perfectly legal.

However, the net result, i.e. the remaining bound-state contribution (of the second and higher virial coefficients) does look like internal partition function (which it is not), and it is called Planck-Larkin partition function (PLPF). If the PLPF were identified as an internal partition function, it would act like a (convergent and differentiable) truncation of the usual internal partition function, cutting off at an energy equal to the equipartition value \( kT \). I stress, however, that the PLPF must not be used as an internal partition function (see Ref. 7), but be left at its place in virial expansions. This remark should be sufficient to settle the recent controversy about the PLPF (Refs. 23 and 24).

The Livermore equation of state follows these lines. It is the basis for the Livermore opacity calculations. In principle the bound state contributions to the opacity could be computed coherently within the same formalism, but such a theory is not yet developed (Rogers, private communication). Therefore, the perturbation expansion is, for the opacity, regrouped to pull back some of the weakly-bound state contribution for opacity purposes, though in the equation-of-state part their place would be in the continuum (Rogers, private communication and Ref. 25).

5. IMPLICATIONS FOR SOLAR PHYSICS

The so far most convincing demonstration of the sensitivity of an observable astrophysical quantity on details of the equation of state is the influence of hydrogen and helium partition functions on high-degree p-mode frequencies. Observed oscillation frequencies have been compared with theoretical results, based in one case on the EFF and in the other case on the MHD equation of state. The results of this comparison are discussed in Ref. 19 and also in these proceedings (Ref. 26). Here, I will give the physical explanation of the different behaviour of EFF and MHD. Before doing this, I would like to summarize the situation.

In conventional standard models, represented by the EFF calculation, the frequency differences \( \delta v \) between observation and theory depend both on the frequency of the modes and on their angular degree \( l \). The frequency dependence (which is similar for all degrees \( l \)) is thought to be due to an inadequate (because adiabatic) treatment of the pulsations in the uppermost part of the convection zone and the solar atmosphere (see Ref. 27). But there is also a distinct dependence on the angular degree \( l \) which suggests inadequacies of the model in deeper layers, coming from the position of the lower turning point. Apart from these two features one notes a substantial frequency error at low frequencies, where one would expect no problems arising from the usual assumptions (like adiabatic oscillations). This low-frequency discrepancy also suggests inadequacies of the model in the interior.

The same comparison between observations and a model using the MHD equation of state shows: 1) little systematic variation with angular degree \( l \), 2) a significant reduction of the low frequency discrepancy, and 3) no change in the overall frequency dependence of the frequency differences. Since the first two effects are related to the interior, it appears that much of the inadequacy of the theory of the interior has been eliminated by using the MHD equation of state. Furthermore, the fact that the overall frequency dependence is unaffected by changing the equation of state supports the interpretation that it is caused by errors in the treatment of the surface layers.

Let me now come to the physical explanation. Firstly, we note that the effect of the MHD equation of state on oscillation frequencies can be understood in terms of the local sound speed. As shown in Ref. 8 and also in these proceedings (Ref. 26) the relative sound-speed difference \( \delta c/c \) alternates between \( \pm 1-2\% \) in the sub-photospheric layers (with \( 0.95<r/R<1.0 \)). Secondly, it can be shown that the sound speed difference \( c^2=\rho T/p \) is qualitatively dominated by the influence of \( \Gamma_1 \), thus roughly \( \delta c/c = 0.5 \delta \Gamma_1/\Gamma_1 \). This relatively simple relation is by no means evident, since changing the equation of state also causes changing the equilibrium structure. Nevertheless, Ref. 8 shows that the influence of \( \Gamma_1 \) is the most important one. Thirdly, \( \Gamma_1 \) is connected to the internal partition functions. In this part of the Sun \( 0.95<r/R<1.0 \), the principal effect of the MHD equation of state is caused by the larger than usual internal partition functions of \( H \), \( He \) and \( He^+ \), which increase the statistical weight of these species.

Why are the internal partition functions of the MHD equation of state significantly larger? It is true that normally the weight of the ground state (with [negative] energy \( E_0 \)) dominates the one of each excited state (with [negative] energy \( E_1 \)) by a factor of \( \exp(-E_1/E_0)/kT \). For the temperatures encountered in the \( H \) and \( He \) ionization zones, this domination can easily be by a factor of \( 10^{2-4} \). However, the sheer number of excited states explicitly dealt with in MHD can lead to a sizeable correction of the total statistical weight of the \( H \) and \( He \) atoms or the \( He^+ \) ion. More specifically, this increase of the statistical weight can get as high as \( 10-30\% \). This increase is then translated into a decrease of the \( H \), \( He \), and \( He^+ \) ionization fractions. This decrease is of the same order as the increase of the statistical weights, which is obvious from the Saha equation. MHD thus pushes deeper down the location of the ionization zones. All this, however, does not yet explain the change of sign that appears in \( \delta c/c \) as a function of depth. The change of sign is explained by considering \( \Gamma_1 \). We know that in a partially ionized plasma \( \Gamma_1 \) is lowered from its ideal value of \( 3/2 \) (at full ionization or recombination); the minimum values are about 1.20 and 1.55 in the \( H \)-ionization and the second \( He^- \)ionization zone.
respectively. Consequently, the MHD equation of state also pushes down the zones where this lowering of \( \Gamma_1 \) takes place. The \( \delta \Gamma_1 \) that results from this translation of the \( \Gamma_1 \) "dip" has the opposite sign on the upper flank than on the lower flank of the \( \Gamma_1 \) dip, thus explaining the alternation of positive and negative \( \delta \epsilon_c \).

6. CONCLUSIONS

There has been substantial progress on the equation of state of the solar interior. The example of the high-degree p modes indicates that the recent efforts (with the MHD equation of state) have gone in the right direction. However, further studies are needed to isolate the effects of the equation of state from other possible effects. Furthermore, in the light of the principal conceptual differences encountered in the equation of state (regarding the realizations of the chemical and physical picture), Forrest Rogers and I envisage to prepare the Livermore equation of state for the the same confrontation with helioseismology to which the MHD equation of state has been subjected (Ref. 8).

Given the importance of the equation of state for opacities and the helium abundance determination (Refs. 2 and 4), such comparisons from solar physics and also laboratory tests (Ref. 7) will help us to distinguish between different equations of state.

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Scattering And Multiple Scattering Of Acoustic Waves In a Stratified Medium.


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Abstract

Bogdan and Zweibel have studied the effect on wave propagation of scattering from an ensemble of vertical magnetic flux tubes in a stratified atmosphere, using the WKB approximation. We analyse the effect of an ensemble of general axisymmetric scatterers on acoustic waves in a stratified atmosphere, without using the WKB approximation, and obtain a dispersion relation valid for small filling factors. The properties of this dispersion relation are studied for scattering from density inhomogeneities in a two-layer model atmosphere. It is found that increased stratification tends to decrease the frequency perturbation induced by a population of cylindrical inhomogeneities.

1. Introduction

It is known (ref. 1) that the magnetic field in the solar atmosphere is concentrated into slender (1-200 km), intense (2-3kG) flux tubes. The effect of these fibrils on the frequency of solar p-mode oscillations is greater than that of a diffuse magnetic field of the same total flux by a factor of order 1/\(f\), where \(f\) is the fractional volume filled by the flux tubes. This difference occurs because the gas pressure in the flux tubes must be lower than in the surrounding medium, whereas, for example, a uniform magnetic field has constant magnetic pressure and so does not affect the stratification of the medium.

Bogdan and Zweibel (refs.2-4) have analysed the effect of fibril magnetic fields on acoustic modes using the theory of multiple scattering. They consider the effect of a population of parallel, axisymmetric scatterers, oriented parallel to the direction of stratification. In Section 3, a simple 2-layer "toy" atmospheric model is used to test some of the properties of this dispersion relation. In section 4, we present our conclusions.

2. Multiple Scattering In a Stratified Atmosphere

We wish to consider the effect of scattering from a population of axisymmetric scatterers embedded in a stratified medium, on the dispersion relation for acoustic modes. If the medium has an acoustic cutoff, so that the modes form a discrete set then, taking the z-axis as parallel to the direction of stratification, a scalar wavefunction (e.g. the pressure perturbation) can be written as the sum of plane waves of the form

\[
\phi = A e^{ik \cdot z} f(z) e^{-i \omega t},
\]

where \(k\) is a unit vector perpendicular to the direction of stratification (i.e. a "horizontal" unit vector), \(f(z)\) is an eigenfunction corresponding to an allowed wave number, \(k_n\), at frequency \(\omega\) and \(A\) is the amplitude of the mode.

Since scattering is assumed to be from axisymmetric objects, it is convenient to rewrite eqn.(1) as a Fourier-Bessel expansion in cylindrical polar coordinates \((s, \theta, z)\):

\[
\phi = A f(z) e^{-i \omega t} \sum_{n=0}^{\infty} c_n^m J_n(k_n s) \cos m \theta,
\]

where

\[
c_n^m = \begin{cases} 
2 & m = 0 \\
1 & \text{otherwise},
\end{cases}
\]

and \(J_n\) is a Bessel function of the first kind of order \(n\). From this point on, we shall use eqn.(2) to allow us to switch freely from plane-wave to Fourier-Bessel representations of the oscillation eigenmodes.

We now consider what happens when a wave of form (1) is incident on a single axisymmetric scatterer at \(s = 0\). The wave scattered from (2) can be expressed in the form:

\[
\psi_s = A \sum_{nm} c_n^m T_{nm}^m H_n^{(1)}(k_m s) \cos m \theta f_m(z) e^{-i \omega t},
\]

where \(H_n^{(1)}\) is a Hankel function of the first kind and the scattering coefficients, \(T_{nm}^m\), are considered, at this stage, to be...
known. It may be noted that where scattering is into an evanescient mode for which $k_n$ is imaginary, the Hankel function becomes proportional to the modified Bessel function $K_n(k_n|\alpha)|$. In what follows, the $e^{-i\omega t}$ time dependence will not be explicitly included.

To analyse the effect of multiple scattering by a population of scatterers, the $i$th scatterer situated at $r = r_i$ and distributed with number density $n(r)$, we consider a general incident wave

$$\phi = \int \sum_i A_i(\vec{k}) e^{i\vec{k}\cdot \vec{r}} f_i(z) d\vec{k},$$

where $A_i(\vec{k})$ is the amplitude of a wave travelling in the direction $\vec{k}$.

We now consider the scattered wave averaged over the distribution $n(r)$ of the flux tubes, given by

$$\langle \phi \rangle = \int \frac{\sum_i A_i(\vec{k}) e^{i\vec{k}\cdot \vec{r}} r_i H_i^{(1)}(k_i|r - r_i|) f_i(z) n(r)}{\sum_i A_i(\vec{k}) e^{i\vec{k}\cdot \vec{r}} r_i H_i^{(1)}(k_i|r - r_i|) n(r)} d\vec{k} d\tau.$$  

Equation (6) represents an approximation, the Foldy-Lax-Twersky relation, to the full scattering equation, in which tertiary and higher orders of multiple scattering are neglected. For present purposes, we simply note that this approximation is believed to be valid where the filling factor, $f_i$, is less than about $10^{-3}$, a value similar to that found in the quiet solar photosphere. To obtain a dispersion relation for the medium, we now use a self-consistency argument to claim that the total wave at any point is the sum of all the scattered waves at that point: i.e. $\phi = \langle \phi \rangle$. The resulting equation requires some manipulation, but since this is essentially the same as that performed by Bogdan and Zweibel (ref. 2) we shall use the full equation (8).

3. A Simple Test Model

As has been mentioned, the principal difference between diffuse magnetic fields and fibril magnetic fields, is that the latter inevitably give rise to pressure and density fluctuations in a model atmosphere. This suggests that it may be instructive to consider oscillations in an atmosphere containing non-magnetic, axisymmetric fluctuations. Indeed the results of Bogdan and Zweibel (ref. 2) suggest that such non-magnetic inhomogeneities on oscillation eigenfrequencies is qualitatively similar to that from magnetic fibrils.

3.1. The Model Atmosphere

To simplify calculations, we shall use a model atmosphere without gravity (and therefore, necessarily, isobaric), consisting of two layers each of constant temperature. We consider the atmosphere to be bounded by hot uniform tenuous gas at $z = 0$ and $z = d$ with the boundary between the layers being at $z = \lambda d$. The sound speed is

$$c_{\text{ref}} = \begin{cases} c_1 & \text{if } z < \lambda d \\ c_2 & \text{if } z > \lambda d \end{cases}$$

and we define $\alpha = c_1/c_2 = \rho_2/\rho_1$. We measure all lengths and wave numbers in units of $d$, the height of the layer, and $d^{-1}$ respectively, and we use the dimensionless frequency $\tau = \omega d/c_1$.

Adiabatic pressure fluctuations of the external medium satisfy the scalar Helmholtz equation with variable sound speed,

$$\nabla^2 p' + \frac{\omega^2}{c_{\text{ref}}^2} p' = 0,$$

which has solutions

$$p' = J_m(k_1 \alpha)^e^{i\omega t} f_1(z) e^{-i\tau z},$$

where $J_m$ is any solution of Bessel's equation of order $m$. The vertical eigenfunctions must satisfy $f(z) = 0$ at the boundaries and be continuous at $z = \lambda d$ so

$$f_1(z) = \begin{cases} \sin[\gamma_1 z]\sin[\gamma_1 (z - \lambda)] & z < \lambda \\ \sin[\gamma_1 z]\sin[\gamma_2 z] & z > \lambda, \end{cases}$$

where $\gamma_1^2 = \alpha^2 - k_1^2$ and $\gamma_2^2 = \alpha^2 k_1^2 - k_2^2$.

Continuity of normal velocity at $z = \lambda$ leads to the dispersion relation

$$\alpha^2 \tan[\gamma_1 z]\sin[\gamma_1 (z - \lambda)] + \gamma_2 \tan[\gamma_2 z] = 0$$

and the index $f_1$ on $f_1(z)$ labels the solutions to eqn.(15). The solutions of this dispersion relation for $\alpha = 0.2$ are shown in fig.(1).

It is useful to have a qualitative understanding of the properties of this model. The first point to note is that solutions of eqn.(15) can exist with $\gamma_2^2 < 0$, although all solutions have $\gamma_1^2 > 0$. These solutions correspond to modes which are propagating in region 1, but evanescent in region 2. It is intended that such behaviour may be used to mimic an important aspect of solar acoustic modes, which have lower turning point depths which vary with horizontal wave number.
A complication to this picture arises because of the discontinuity in the unperturbed atmosphere at \( z = \lambda \). For illustrative purposes, consider the case \( \alpha \ll 1 \), so that the discontinuity is large. Then acoustic waves will tend to be almost totally reflected at the discontinuity, so waves will be almost totally trapped in one region or the other. Plots of the solutions \((k^2, \omega^2)\) to the dispersion relation thus consist of essentially two sets of parallel lines, one set with slope \( c_2^2 \) and one with slope \( c_1^2 \), but exhibiting avoided crossings, as seen in fig. (1).

3.2 Scattering

To calculate the scattering coefficients for a single scatterer at \( \delta = 0 \), we write the incident wave, \( p' \), the scattered wave, \( p'_s \), and the internal (transmitted) wave, \( p'_t \), as Fourier-Bessel series thus

\[
p' = f_n(z) e^{i(k_n - \tau - \tau')} = f_n(z) \sum_{m} c_m z^m J_m(k_m z) \cos m \theta e^{-i \tau}
\]

\[
p'_s = \sum_{m} c_m z^m T_m(r) J_m(k_m z) \cos m \theta e^{-i \tau}
\]

\[
p'_t = \sum_{m} C_m \sin(j \pi z) J_m(\sigma_m z) \cos m \theta e^{-i \tau}
\]

where \( \sigma_m^2 = \delta^2 \tau^2 - j^2 \pi^2 \).

To obtain an equation for the scattering coefficients we apply conditions of continuity of total pressure and normal velocity at \( z = \epsilon \). The resultant equations are multiplied by \( \sin \pi z \) and integrated over the depth of the layer, to obtain a matrix equation for \( T_{mn} \).

We do not here present detailed discussion of the form of the resulting scattering coefficients, but we note that, as mentioned above, the scattering coefficients depend strongly on our, as yet arbitrary, normalisation of the eigenfunctions. To obtain physically meaningful results, one would prefer to normalise the incident wave to unit energy flux (in some sense), whereas for comparison with observations, it might be better to normalise the eigenfunctions to something like unit average vertical velocity in some region near the upper surface of the layer. It should also be noted that, for comparison with observations, the quantity \( \left| T \right|^2 / (1 + \left| T \right|^2) \) (assuming unit incident amplitude) is better behaved than \( \left| T \right|^2 \) itself. Thus Braun, Duvali, and LaBonte (Ref. 5) calculate \((\text{scattered power}) / (\text{total power})\) from observations.

3.3 Choice of Parameters

One would naturally like to choose parameters for this model which are in some way representative of solar values, despite the model's essential limitations. The principal parameter of interest is the ratio of the flux-tube radius to the density scale height. In the upper photospheric regions, this has a typical value of about \( 0.25 \), an order of magnitude smaller than \( \alpha \), for \( \alpha \ll 1 \). We take values of \( \alpha \) of 0.5 and 1.0. The sound speed ratio, \( \delta \), may be greater or less than 1. One anticipates that solar fibrils will be colder than their surroundings; however their dominant mode of oscillation will be fast magneto-acoustic waves whose group velocity is higher than the local external sound speed. Indeed, Bogdan and Zweibel (see ref. (3) Figure (1)) find that over a wide range of parameter values, magnetic flux tubes in a solar model increase the frequency of acoustic waves of a given wavelength. We therefore present results for \( \delta = 0.5 \) and \( \delta = 2.0 \). The real part of the resultant frequency shifts are shown in figs. 2(a) - 2(e) and the damping rates in figs. 3(a) - 3(e). The corresponding parameter values are listed in Table 1. Since, in the region of validity of equation (9), the frequency shifts are linear in \( f \), we present results only for \( f = 10^{-5} \), a typical solar value.

![Figure 1](image)

**Table 1.**

<table>
<thead>
<tr>
<th>Figure</th>
<th>( \alpha )</th>
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<tbody>
<tr>
<td>2.3(a)</td>
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<tr>
<td>2.3(b)</td>
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<tr>
<td>2.3(c)</td>
<td>1.0</td>
<td>0.5</td>
<td>0.25</td>
</tr>
<tr>
<td>2.3(d)</td>
<td>0.5</td>
<td>2.0</td>
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Key to figures (2) and (3).

4. Results and Conclusions

Regrettably, it is not possible to make a direct comparison between our results and the numerical results presented by Bogdan and Zweibel. In Ref. (1), they consider only long wavelength waves travelling transverse to the flux tube axis whereas, in the long wavelength limit \( (\alpha a = 0) \), the waves in our model are travelling parallel to the cylinder. In Ref. (2), results are only presented for non-zero magnetic fields, so no direct comparison with our model can be made.

The important point to notice from figures (2a-d) is that as \( \alpha \) increases towards unity, the frequency shift increases. This is to be expected since for small \( \alpha \), the mismatch between the eigenfunctions in the external medium and in the tube is large, and so the overlap integrals between the modes are small. This reduces the magnitude of the scattering coefficients and hence the perturbation.
It is also noticeable that the frequency shift tends to increase with decreasing tube radius, at constant filling factor. This result is to be expected from the form of equation (8). The perturbation to the wave number squared is proportional to the product of the number density of scatterers and the matrix of scattering coefficients. The value of a scattering coefficient depends principally on the product of the horizontal wave number with the tube radius, and only weakly on the vertical structure of the wavefunction, while the number density varies inversely as the square of the flux tube radius if the fractional volume filled is constant, hence the perturbation increases with decreasing radius.

The damping rates shown in fig. (3) go to zero as the transverse wave number becomes small, in agreement with the results of Bogdan and Zweibel, and contrasting with the behaviour of the real component of the frequency shift which is of order $f$ in the long wavelength limit. However, at shorter wavelengths, the damping rate is comparable in magnitude to the real shift.

We have introduced a formalism for multiple scattering in a stratified atmosphere with which it is possible to calculate the frequency shifts due to a population of axisymmetric scatterers embedded in the medium. Provided that the scatterers are sufficiently well separated that the approximations used hold, this method can be used with any model of a scatterer for which the single scatterer scattering coefficients can be found. We have used this formalism to study the eigenfrequency perturbations in a simple model atmosphere, and found that the mismatch in the eigenfunctions inside and outside our model flux tube leads to a reduction in the magnitude of the scattering coefficients, and hence to a reduction in the size of the frequency perturbation.

References
Towards an Independent Calibration of the Mixing-Length Theory

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ABSTRACT

The standard astrophysical mixing-length theory of turbulent convection, and modifications thereof, are calibrated with laboratory experimental data over a range of Rayleigh numbers and Prandtl numbers. Both local and nonlocal theories are considered, and an analysis made of the effects on the calibration of different degrees of small-scale turbulence, eddy decay-probability and presumed nonlocalization (both symmetrical and asymmetrical). It is found that while the local theory does adequately describe some of the features of laboratory convection (though it would predict a mixing-length parameter of at least 2.4), its qualitative agreement is poor. Nonlocal theories with this parameter around 1.8-2.0 give a much better agreement with experiment over the whole range of experimental Rayleigh and Prandtl numbers. We conclude that both local and nonlocal mixing-length theories can be adjusted to give apparently accurate descriptions of turbulent laboratory convection in the ranges $10^8 \leq \text{Rayleigh number} \leq 10^9$ and $10^{-3} \leq \text{Prandtl number} \leq 10^3$, but that the nonlocal theory provides a more physically realistic description of the flow.

Keywords: convection, mixing-length, turbulence, nonlocal

1. INTRODUCTION

Most stars are thought to contain at least one region in which convection plays a dominant role in the transport of energy. The development of a reliable theoretical description of these convection zones is thus of great importance for the modelling of stars. However, relatively little attention is being paid to this branch of astrophysics today, partly because of a feeling that since conditions within a star are so different from anything likely to be seen in terrestrial convection one will never be able to tell whether one theory is better than any other; and indeed, it does seem that the bulk of convection zone can apparently be quite well treated using an extremely simple procedure. The details of the theory influence only the thin boundary layers, and for the Sun a gross calibration of the jump-conditions across the upper boundary layer appears to be all that is required for an adequate description of the solar interior (Ref. 1).

The most commonly used procedures for constructing stellar models use a formulation known as local mixing-length theory, based largely on the formalism of Viensen (Ref. 2, 3). In its derivation it must be assumed that the typical scale of convective motion is much less than the scales over which fluid properties vary (Ref. 4, 5). This is not the case within the body of most convection zones, and it is certainly not so in the outer layers of convective envelopes of late-type stars. This inconsistency has rarely worried stellar physicists in the past, and will not be addressed in this paper either.

Recently, however, improving observations of the Sun, and the promise that very high-order solar p-modes of oscillation (which are concentrated in just these layers) offer to probe the upper convective boundary layer, encourages one to search for an independent calibration of the theory, in the hope of bringing us closer to testing the consistency of the formalism. One ought to be able to make some attempt to do this by comparison with results from laboratory experiments with turbulent convection. The premise of this paper is that, regardless of the dissimilarities between the convective regimes of the laboratory and of a star, if a suitably modified form of mixing-length theory describes turbulent laboratory convection only poorly, then one cannot expect it to be adequate in the stellar case; if, on the other hand, it does give a good description (in some sense), then one might have more confidence in its applicability to stars.

It should be pointed out that the use of mixing-length theory to describe terrestrial turbulence is by no means novel: in fact, it was with this idea in mind that Taylor (Ref. 6) and Prandtl (Ref. 7) originally formulated the ideas behind it.

2. LOCAL MIXING-LENGTH THEORY

2.1 The Linear Growth-Rate

The following derivation is based on that of Gough (Ref. 8). Throughout this paper the term eddy will be used to describe the fundamental convective entity, briefly coherent in space and time. Our theoretical system will be a plane-parallel layer of fluid of infinite horizontal extent, with rigid, perfectly conducting boundaries at fixed temperatures: that is, Rayleigh-Bénard convection. The boundaries are separated by a distance $d$, the lower being hotter than the upper by $\Delta T$.

Following standard procedure, we take the Navier-Stokes Equation, retaining the viscous term $\nabla^2 u$, the heat equation (in which we make the diffusion approximation for the radiative flux) and the continuity equation, and decompose variables into mean (averaged over horizontal coordinates and time) and fluctuating (with zero horizontal and time average) parts; apply the Boussinesq approximation (set density constant except in the product $g\Delta$) and take the (vertical) $z$-component of the double curl of the Navier-Stokes Equation. The procedure is tantamount to linearizing this and the heat equation (by dropping terms involving $u \nabla$) and replacing $\partial / \partial t$ and $-\nabla^2$ by $q$ and $k^2$, the eddy linear growth-rate and wave number squared.

respectively. We also define an eddy shape-factor

$$\Phi = 1 + \frac{k^2}{k^2_h},$$

(1)

where \(k_v\) and \(k_h\) are vertical and horizontal wave numbers respectively, and \(k^2 = k^2_v + k^2_h\). Our equations are thus

$$q_u = \frac{\omega^2}{\Phi} - \nu k^2 w,$$

(2)

$$q_\theta = \beta w - k^2 \theta,$$

(3)

which can be solved for the growth-rate:

$$q = \frac{k^2}{2} \left[ (\nu - \kappa)^2 + 4 \frac{\omega^2}{\Phi} \right] - (\nu + \kappa).$$

(4)

In the above, \(w\) and \(\theta\) are the fluctuating parts of vertical velocity and temperature respectively, \(g\) is the acceleration due to gravity (assumed constant), \(\kappa\) is the coefficient of thermal expansion, \(\nu\) the kinematic viscosity and thermal diffusivity respectively, and \(\beta = -\frac{\kappa}{\Delta T} - \beta_{sd}\) is the superadiabatic lapse rate. \(\Delta T\) is the mean part of temperature and we assume that \(\beta_{sd} = g/c_p\) for gases and is zero otherwise. The coefficient \(c_p\) is the specific heat at constant pressure.

The retention of the viscous term in the Navier-Stokes Equation, not usual in stellar mixing-length theory, is necessary owing to the important role played by the viscosity in laboratory Rayleigh-Bénard convection, as will be evident later. In fact, one ought even to consider the temperature variation of the quantities \(w\) and \(\theta\) (yielding the so-called weak Boussinesq approximation) because of the substantial variation of these coefficients in many of the experimental situations (eg Ref. 20).

Luckily, it turns out that we can make progress without introducing this dependence into the original equations, but taking cognizance of it only later in the analysis.

2.2 Turbulent Energy Transport

The efficacy of turbulent convection appears to be reduced by the generation of locally almost isotropic small-scale turbulence in which velocity and temperature fluctuations are only poorly correlated. Indeed, our preliminary work showed that if we ignored this turbulence our theoretical heat flux was too high at large Rayleigh numbers (defined below). We included \(\lambda\) by replacing \(\nu\) and \(\kappa\) in Eq. 2.3 by \(\nu = \nu + \nu_t\) and \(\kappa = \kappa + \kappa_t\), where \(\nu_t\) and \(\kappa_t\) are turbulent viscosity and thermal diffusivity respectively. Following Ref. 8, 9 we write

$$\nu_t = \kappa^{-1} (u.w)^\frac{1}{2},$$

(5)

where \(\kappa\) is a constant of order unity relating the large-scale and small-scale velocity and length scales to each other. The diffusivity \(\kappa_t\) can then be related to \(\nu_t\) in terms of a turbulent Prandtl number \(\sigma_t = \nu_t/\kappa_t\): experimental evidence indicates that \(\sigma_t\) is remarkably constant under a great variety of conditions (Ref. 10), taking a value of around 0.7, which increases to about 0.9 in boundary layers. There does seem to be a small inverse dependence on \((molecular)\ Prandtl number, \(\sigma = \nu/\kappa, with \(\sigma_0 \approx 2 \sigma = 0.055.\) Astrophysically, Massaguer & Zhao (Ref. 11) and Nakano et al. (Ref. 12) estimate \(\sigma_0 \approx 0.4,\) respectively, while Ariša et al. (Ref. 13) estimate \(\sigma_0 \approx \frac{1}{4}\) by using it to calibrate sizes of surface features of the Sun. We adopt \(\sigma_0 = 0.7\) in this paper.

We now scale all quantities using \(d\) (length), \(\Delta T\) (temperature), \(d^2/\kappa\) (time) and \(d^4/\rho\) (mass). Thus the layer extends from \(z = 0\) to \(z = 1\) with \(T(0) - T(1) = 1.\) Eq. 4 becomes

$$Q = \frac{2\sigma}{k^2} = \left[ (\sigma_0 - \sigma(1 - 1/\sigma_t)) \right]^2 + \eta^2 R_\sigma \beta^2 t,$$

(6)

in which \(R = g\Delta T d^5/\kappa\nu\) is the Rayleigh number of the layer, \(\sigma = \sigma_t/\sigma_0,\ \eta = (\sigma - 1)/\sigma_0 \eta_0,\) and \(\ell = \pi/k \nu d\) is the mixing-length, defining the vertical extent of an eddy and hence the typical convective length scale.

2.3 Breakdown of Eddies

Since we discarded the nonlinear terms in deriving Eqs. 2, 3 we need to provide a mechanism by which the eddies' growth is halted. Following Gough (Ref. 8, 14) we use the idea of an eddy disruption probability given by

$$p dt = (c \omega u) \frac{1}{2} dt,$$

(7)

where \(p dt\) is the probability that an eddy is disrupted between \(t\) and \(t + dt,\) and \(\omega\) is the eddy's vorticity, double overbars denoting an average over the volume of the eddy. The constant \(c\) is of order unity. Such a concept is consistent with turbulence theory (Ref. 13).

Thus, we envisage eddies which form, grow according to the linearized theory, and, after some time determined by Eq. 7, are instantaneously disrupted. Thus they create the uncorrelated small-scale turbulence which we parameterize with \(\nu_t\) and \(\kappa_t.\)

2.4 Convective Flux and Reynolds Stress

Dimensionless convective flux and Reynolds stress (scaled by \(d^2/c_p\Delta T\) and \(d^2/\rho^2\) respectively) take the forms

$$F_c = u \omega,\quad p_\theta = \omega^2,$$

(8)

(9)

respectively. Following the procedure discussed in Ref. 8, 14 this leads to

$$F_c = \frac{4}{C} \frac{Q^2 (Q + 2\Delta T)}{\eta^2 R_\sigma \beta^2 t},$$

(10)

$$p_\theta = \frac{2}{C} \left( \frac{Q}{C - 1} \right)^3,$$

(11)

where \(C\) is a factor proportional to \(c^2\) and will be treated as a constant. Aside from the nondimensionalization, differences between Eq. 10, 11, and the corresponding results in Ref. 8 result from the presence here of (molecular and turbulent) viscous terms, and also from our use of a different form (Eq. 7) for the disruption probability, used also in Ref. 14.

It has been explicitly assumed in the derivation of Eq. 10, 11 that horizontal variations in \(u\) have a purely sinusoidal spatial structure. From this assumption it follows that

$$\langle u.w \rangle = 0,$$

(12)

which can be substituted into (the dimensionless version of) Eq. 5 and then combined with Eq. 9, 11 to yield

$$\sigma = Q E,\quad E = c(2/C)^\frac{1}{2}.$$

(13)

Substituting this into Eq. 6 and rearranging gives

$$G = \frac{Q}{H} \left[ \frac{Q}{J} - J^2 + \frac{\eta^2 R_\sigma \beta^2 t}{\ell} \right] - \frac{(Q/j)}{J},$$

(14)

in which \(H = (2E + 1)(2E/\sigma_t + 1)\) and \(j = \sqrt{(2E + 1)/(2E + 1)}.\) Thus Eq. 10 & 11 become

$$F_c = \frac{4}{C H \Phi} \frac{G^2 (G + 2\Delta T)}{\eta^2 R_\sigma \beta^2 t},$$

(15)

$$p_\theta = \frac{2}{C H \Phi} \left( \frac{\eta^2}{\ell} \right)^2,$$

(16)
If we assume that $f = 1$ and $R > \sigma$ (the usual astrophysical assumptions, the first being equivalent to $\varepsilon = 0$) then these relations reduce to

$$F_1 = \frac{4\eta}{C\Phi} (Re)^\frac{2}{3} \beta^3 E^2,$$

(17)

$$p_1 = \frac{2\pi^2 \eta^2}{C(\Phi - 1)} Re \beta \rho^4,$$

(18)

which are equivalent to the usual formulæ used for computing stellar models (Ref. 8).

The sum of the (horizontally averaged) radiative and convective fluxes is independent of $z$ and equal to the Nusselt number of the layer, $N$:

$$N = N_0(z) + N_1,$$

(19)

where $N_0$ is given by $\Phi = 3/2$ and $\Phi = 5/3$ respectively. In this paper we assume $\Phi = 5/3$.

Eq. 19 is a differential equation for $\Phi$ with eigenvalue $N$, which is to be solved subject to the condition $T(0) = T(1)$. From Eq. 14 it can be seen that $G \leq 0$ if $\eta^2 \beta R^2 \leq 4$: this defines a boundary layer in which no convection occurs, and in which we must set $G = 0$ and $\beta = N - \beta_d$.

We intend to compare with experiment. To do so we plot theoretical $N$ against $R$ and $\sigma$ at constant $\alpha, C, e$, the relations between which we choose to make the theoretical value of $N$ agree with the experimental determination (namely, $N = 8.41$ (Ref. 21, 24) for water $(\sigma = 0.65)$ at $R = 10^3$).

In the weak Boussinesq approximation, the formulæ for $R$ and $\sigma$ vary with $z$, their nominal values, the Rayleigh number and Prandtl number, being those at $z = 0.5$. Accordingly, the real temperature dependences of $\nu, \kappa$ and $\Phi$ (Ref. 16, 17) were approximated for each of the fluids mentioned in Eq. 21 by quadratic functions of temperature in the experimental range $20^\circ C \leq T \leq 40^\circ C$, the fictitious fluid with $\sigma = 0.001$ being given the properties of Mercury. Typical experimental values of $\nu$ (Ref. 18-24, usually a few cm), and the values of $\Delta T$ calculated from these and the assumed value of $R$, were used to find $R$ and $\sigma$ as functions of (scaled) temperature, and hence of $x$, at every step of each Newton-Raphson iteration, whence they could be incorporated into the solution process.

2.5 Experimental Data

Fig. 1 shows a plot of experimentally determined $\log N$ against $\log R$ for various fluids of different $\sigma$. In all the experiments considered here (Ref. 18-25) the coefficients $\nu, \kappa$ and $\Phi$ are assumed constant; as much as given experimental conditions allowed, their data were modified to allow for such variation by replacing quoted values of $R$ (the midpoint values) with a mean value of $R$ integrated over the layer, which was usually somewhat lower. This, incidentally, increased the slopes of some of the curves, particularly at high $R$ where the mean and midpoint estimates of $R$ tended to differ most. Several features, some difficult to discern on so small a graph, deserve comment:

- **Figure 1**: Experimentally determined Nusselt numbers as a function of Rayleigh number. Many different results are shown, at several different Prandtl numbers, the tendency being for the lower curves to represent lower Prandtl numbers. The two lowest are determined for mercury (Ref. 23, 24). The dashed line, of slope 1/3, is purely for comparison.

- The slopes of the curves seem to increase with $R$, towards a limiting slope which could not reasonably be $\frac{1}{3}$; this is consistent with our theory (and many others), for which it can be shown (from Eq. 19) that the asymptotic limit at large $R$ is of the form $N \sim R^2$. In the past, it has been the subject of some discussion that the experimental slopes of such curves tend to be in the range 0.28-0.30; we conjecture that much of this apparent discrepancy is due to overestimation of the experimental $R$ at large $R$ through not allowing for the temperature dependence of transport coefficients; and because experimental $R$ are still not asymptotically large.

- The data for mercury from Ref. 23, 24 are consistent if one ignores the two lowest-$R$ data points of Ref. 23.

- Several small jump discontinuities occur, as predicted by Malkus (Ref. 26).

- Although there seems to be a tendency for $N$ to decrease at fixed $R$ with decreasing $\sigma$, there are some discrepancies: for example, the data of Ref. 25, and between the high-$\sigma$ curves of Ref. 24 and those of other references.

2.6 Theoretical Results

Figs. 2-5 show examples of the results for $\alpha = 2.5$ and $\alpha = 4.0$. For each, a plot is shown of $\log(N - 1)$ against $\log R$ for three different values of $R$, and of $N$ against $\log R$ for five values of $\sigma$. We note the following:

- Agreement with experiment is quite good in Fig. 2 but very poor in Fig. 4.

- Theory does badly in Fig. 3, with slopes much too large; it does somewhat better in Fig. 5 but still not well at large $R$.

- As one might have expected, $C = C(\alpha, e)$ is a monotonic function of $\alpha$: for fixed flux we can have either large, short-lived eddies or small, long-lived ones. The reason we have no results for $\alpha = 2.5$ is because for such $\alpha$ (and the assumed $R, \sigma$ and $N$) there is no solution of Eq. 19 which satisfies the boundary conditions for any $C$ or $e$. This is consistent with an asymptotic analysis of Eq. 19 for $C \to 0$: we find that (for
The smallest $\alpha$ for which a solution exists is given by

$$\alpha_{\text{min}} = \frac{8}{3} \left( \frac{4N^2}{\eta R} \right)^{1/2}$$

$\simeq 2.35$ at $R = 10^6$, $N = 8.41$. (22)

Physically, one can see that if $\alpha$ is small then eddies are small, and thus the efficacy of convection is low. Although one can compensate for this reduced efficacy by increasing the eddies’ lifetimes (decreasing $C$), $\alpha$ approaches a finite limit as $C \to 0$: even infinitely long-lived small eddies cannot transport enough heat to generate a unit temperature difference across the layer. As one would have expected, $\alpha_{\text{min}}$ was found to increase with increasing $\epsilon$. Against expectations, $\epsilon$, for which we considered only the values $\epsilon = 0, 0.1$ and $0.3$, did only a little to reduce the theoretical $N$ at high $R$.

Thus, while local mixing-length theory seems able to describe turbulent laboratory convection in a general qualitative way, it fails in important details. The size of $\alpha$ is a serious problem, for, taking our physical description of the motion literally, $\alpha > 2$ implies that the eddies extend beyond the boundaries, which is hardly possible.

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Figure 4: See caption for Figure 2.

Figure 5: See caption for Figure 3.

### 3. NONLOCAL MIXING-LENGTH THEORY

#### 3.1 The Theory

In real convection, $\alpha$ does appear to be quite large, for example, theoretical models of the Sun are best calibrated with $\alpha \gtrsim 1.7$, while in the laboratory eddies often extend over much of a convecting layer.

The idea of a nonlocal mixing-length theory is that if $\alpha$ is of order unity, implying that fluid properties vary over the extent of an eddy, then the representative value of a variable that a convective eddy centred at a height $z_0$ experiences should be calculated by averaging that variable throughout the eddy. Moreover, fluxes of momentum and heat at $z = z_0$ must be computed by averaging contributions from all eddies that intersect the plane $z = z_0$.

The averaging has been discussed by Gough (Ref. 6) based in part on the earlier work of Spiegel (Ref. 27). The average lapse rate experienced by an eddy was estimated by Spiegel as

$$<\beta> = \frac{2}{\ell} \int f_{-\ell/2}^{+\ell/2} \beta(z') \cos \left[ \frac{\pi(z'-z)}{\ell} \right] dz'$$

where henceforth angle brackets denote that a quantity is calculated nonlocally. Similar expressions for $< F_t >$ and $< p_t >$...
can be obtained in which the term $\beta(z')$ in Eq. 23 is replaced by $F_X(z')$ and $p_{th}(z')$ respectively, these two functions being defined as the right-hand sides of Eqs. 15 & 16 in which $\beta$ is replaced by $<\beta>$.

Spiegel, using the concept of an eddy phase-space derives a similar expression for $<F_X>$, in which the kernel $\cos^3$ is replaced by $E_X$, the second exponential integral. These kernels are greatest near the centre of the eddy, as one would expect. Using the Eddington approximation from the theory of radiative transfer (Refs. 8, 29), this phase-space formalism leads naturally to the much more tractable equation

$$\frac{1}{\alpha^2} \frac{d^2 <F_X>}{ds^2} - <F_X> = -<F_X>,$$

(24)

in which $\alpha^2 = 3$, with an analogous equation for $<p_{th}>$. The new variable $s$ is defined by $ds = dz/\ell$ and $\alpha^2 = 61$. At this stage we note that an exact solution of Eq. 24 is

$$<F_X>(s) = \int_{-\infty}^{\infty} F_X(z') K(s'-s)dz',$$

(25)

where

$$K(s'-s) = \frac{1}{2} a \exp(-|a|s'-s).$$

(26)

A suitable coefficient $a$ can be determined by demanding that terms in the Taylor expansions about $s$ of $X$ and $E_X$ differ only at fourth order (cf Ref. 30). This yields $a^2 = 2$. To achieve the same agreement between the Taylor expansions of Eq. 24 and the kernel of Eq. 25 (which may be rewritten as $2 \cos^2 (x'-a)$) one needs to set $a^2 = 4$. A further problem arises through our treatment of turbulent viscosity, as Eq. 12 is valid only in the local theory and we cannot use it to simplify Eq. 5. Instead, we define a horizontal viscosity, as Eq. 12 is valid only in the local theory and we can-
When $\mu^2 = 0.5a^2$, $N$ was very often anomalously large around $R = 10^3$, as shown in Fig. 9. When $\mu^2 = 0.2a^2$, this discrepancy vanishes, so we prefer the latter value.

The second region is characterized by excellent agreement when $R$ and $\sigma$ are moderately large. In particular, the theoretical slope of the log $N$–log $R$ relation can be made to agree exactly with the experimentally determined slope up to the largest experimental $R$. This is illustrated in Fig. 8, 9. In this region, $a^2$ is relatively small (4-10) and $C$ is moderately large (0.1-8): eddies are short-lived and convective properties are highly non-local. In almost all cases, $\epsilon$ was best set at 0.1: when $\epsilon = 0$, the theoretical temperature profiles were dominated by regions of negative temperature gradient, which is inconsistent with experiment; while when $\epsilon = 0.3$ no solutions could be obtained. For the reason given above, $\mu^2 = 0.2a^2$ is to be preferred here, though as an illustration Fig. 8, 9 have $\mu^2 = 0.5a^2$.

4. CONCLUSIONS

The local formulation of the mixing-length theory, while able to reproduce the general dependence of the Nusselt number, $N$, on the Rayleigh number, $R$, and the Prandtl number, $\sigma$, quite well, is unable to provide the correct experimental slope in the log $N$–log $R$ relation: the theoretical slope is always too high. Moreover, the local theory cannot give solutions at all unless the mixing-length parameter, $\alpha$, is larger than about 2.35, which seems not to be possible on physical grounds.

The nonlocal formulation does allow solutions with $\alpha \leq 2.0$, but it would seem that no solutions of interest exist if $\alpha \leq 1.8$. If the nonlocalization parameter, $\alpha^2$, is relatively small (4-10), the disruption parameter, $C$, relatively large (near unity), the turbulent transport parameter, $\mu$, is around $\sqrt{0.2a}$, then for moderately large $\sigma > 0.1$ the theoretical and experimental Nusselt numbers can agree exactly for $R > 10^3$. This region of parameter-space is to be preferred over the other region mentioned in Section 3.2 because values of $C$ of order unity imply that eddies exist for about a turnover time, whereas if $C \sim 10^{-2}$, as in the other region, eddies would exist for many turnover times, which does not seem to be the case experimentally. Also, low Prandtl number convection is poorly understood theoretically and so one should be less concerned about disagreement in this regime than in the high Rayleigh number limit.

It should be stressed that the fact that low-$\sigma$ convection is poorly understood does not affect the relevance of this paper to stellar convection, in which $\sigma$ is nevertheless extremely small. What we have investigated here is mixing-length theory itself, in a regime, the laboratory, where one has as much right to expect the theory to apply as in a star. We find that the local theory is unable to provide a physically consistent description of turbulent convection in the laboratory, whereas the nonlocal theory, with $1.8 \leq \alpha \leq 2.0$ and physically plausible values of the other parameters, appears to be adequate if the asym-
motry parameter is positive but not large. Consequently, we have a good reason for preferring the use of nonlocal over local mixing-length theory in stellar models, with a top-biassed nonlocalization.

It is interesting that we can model bumps and temperature-gradient sign-changes, as these have been reported in the literature (e.g. Ref. 18, 21). Unfortunately, in the parameter regime where these effects can be seen the theoretical Nusselt numbers do not agree so well with experiment.

5. REFERENCES
ABSTRACT

We solve the inverse problem of the solar p-mode spectrum based on a recently developed asymptotic inversion method, and infer the sound velocity distribution with the accuracy better than a few percent. In this method, the equation governing high order p-mode oscillations has been reduced to a form of the Schrödinger equation in quantum mechanics. By using the quantization rule based on the WKBJ method, we formulated an integral equation to infer the form of the "acoustic potential". The acoustic potential itself consists of two parts. One of them is dependent on the degree $l$ and the sound velocity and is in the dominant term in the deep interior while the other is independent of $l$ and dominates in the outer envelope. By examining the $l$-dependence of the acoustic potential thus inferred, we separate these two terms and then we infer the sound velocity distribution in the Sun. We apply this method to the real observational data of Duvall et al. (Ref. 1) and Libbrecht and Kaufman (Ref. 2) and obtain the sound velocity distribution in the Sun. We compare the result with our model and find that there is a small discrepancy between the model and the inferred result in the range of $0.3 < r/R < 0.4$.

Key words: Inverse problem; Solar oscillations; Solar structure.

1. INTRODUCTION

The most important and unique aspect of solar oscillations is the possibility of a seismological diagnosis of the solar internal structure. There are various ways to extract information on the solar structure from the oscillation data, but they are divided into two major categories, the forward problem and the inverse problem. In the former, the observed eigenfrequencies of the Sun are compared with the theoretically calculated eigenfrequencies of various solar models constructed with some parameters and the best fit model is searched. On the other hand, in the inverse problem, an integral equation whose known function is provided from the observed eigenfrequencies of the Sun is introduced and functional forms of certain physical quantities are directly determined by solving the integral equation.

There are also various ways to attack the inverse problem and inversion methods based on asymptotic formulism are some of them. Gough (Ref. 3) is the first who developed an asymptotic inversion method to use in helioseismology. Starting from the quantization rule for eigenmodes, he theoretically derived Duvall's (Ref. 4) relation between $(n+n)/4$ and $/m+1)/4$, where $n$ and $l$ denote the radial order and the degree of the mode, respectively, $\omega$ is the eigenfrequency, and $\alpha \approx 1.5$. He then derived from this relation Abel's integral equation and obtained its analytic solution yielding the sound velocity distribution in the Sun. His method was applied to the observational data of Duvall (Ref. 4) and Harvey and Duvall (Ref. 5) by Christensen-Dalsgaard et al. (Ref. 6), and it was well provided us information about the sound velocity distribution in the outer 60 percent of the Sun. Recently, the present authors developed another asymptotic inversion method of inferring the sound velocity distribution in the Sun (Refs. 7-9), which gives a better resolution in the deep interior. As for observational data, progress of the observational technique and much effort of many observers have made both the data amount and the data quality quite better. Hence a combination of an efficient inversion method and the updated observational data will provide us more information on the sound velocity distribution in the Sun.

In this paper, we first perform numerical simulations to examine the validity of the inversion method and then apply the method to Duvall et al.'s (Ref. 1) and Libbrecht and Kaufman's (Ref. 2) observational data.

2. SUMMARY OF THE INVERSION METHOD

Detailed mathematical description of the present inversion method has already been given in Refs. 7-9, but we outline here the method again for later discussions in the following sections.

The basic equations governing linear, adiabatic, nonradial oscillations of stars are reduced to a second order differential equation with respect to distance from the stellar center $r$:

$$\frac{d^2v_r}{dr^2} + \frac{1}{r(\frac{l+1}{4})^2} \left( L^2_{l+1} - \Phi(r) \right) v_r = 0$$

where $v_r(r)$ is the radial part of an eigenfunction $v$ of a mode with radial order $n$ and degree $l$, that is,

$$v(r, \theta, \phi, t) = v_r(r)Y_{l,m}(\theta, \phi) \exp(i\omega_{nl}t)$$

(2)

$\omega_{nl}$ is the corresponding eigenfrequency, $v(r)$ denotes the sound velocity, and $\Phi$ is the "acoustic potential" defined by

$$\Phi(r) = \frac{l(l+1)(\frac{L^2_{l+1}}{r^2} + \Phi(r))}{l(l+1)}$$

(3)

Here, $l$ denotes the degree of spherical harmonic, by which the angular dependence of the eigenfunction is described, and $\Phi(r)$ represents the $l$-independent part of the acoustic potential. The exact forms of the acoustic potential depend on the approximations adopted to derive equation (1), but the first term dominates over the $l$-independent term in the deep interior, while the $l$-independent term dominates near the surface. An example of the acoustic potential can be seen in Fig. 1 of Ref. 9. The asymptotic WKBJ approximation leads the quantization rule holding between the radial order $n$ and the eigenfrequency, which is written as

$$n = \frac{4}{\pi} \int_{r_1}^{r_2} \left( \frac{L^2_{l+1}}{r^2} - \Phi(r) \right)^{1/2} dr$$

(4)

Here $r_1$ and $r_2$ are the turning points at which the integrand of equation (4) vanishes, and $\epsilon$ represents a phase correction due to the reflection of the wave at the turning points. We extend this relation which holds only for discrete eigenvalues to a relation between continuous variables.
\[ \rho u = 1. \] Then by transforming variables, we rewrite equation (1) as

\[ (u + r x) = \int_{\Phi_{\text{max}}}^{x} \left( \omega^2 - \Phi' \right) \frac{db}{d\Phi} \, d\Phi. \tag{5} \]

where \( \Phi_{\text{max}} \) denotes the potential minimum which is estimated by extrapolation from low order modes, and

\[ \omega^2 \equiv \int_{r_{\text{in}}}^{r_{\text{out}}} \frac{dr}{\rho(r)} \tag{6} \]

means the traveling time by the sound wave between the two turning points and is called the "acoustic length". Differentiation of equation (5) with respect to \( \omega^2 \) leads to

\[ 2\pi \frac{d\rho}{d\omega^2} = \int_{\Phi_{\text{max}}}^{x} \left( \omega^2 - \Phi' \right)^{-1/2} \, db \, d\Phi. \tag{7} \]

where the left-hand side and squared frequencies \( \omega^2 \) are obtained from observational data. Hence equation (7) can be regarded as Abel's integral equation, of which the known function is the left-hand side, the kernel is \( (\omega^2 - \Phi')^{-1/2} \) and the unknown to be solved is \( db/d\Phi \) (Ref. 9). The solution is analytically obtained and it is given by

\[ s(\Phi) = \int_{\Phi_{\text{max}}}^{\Phi} \left[ \frac{dh}{d\omega^2} \right]^{-1} \, db \, d\Phi. \tag{8} \]

The acoustic length \( s \) is uniquely determined as a function of the level in the potential, but the potential itself is dependent on the degree \( l \). Therefore we hereafter regard \( s \) given by equation (8) as a function of both \( \omega^2 \) and \( l \) and rewrite it as

\[ s(\omega^2, l) = \int_{\Phi_{\text{max}}}^{\Phi} \left[ \frac{dh}{d\omega^2} \right]^{-1} \, db \, d\Phi. \tag{9} \]

We should note here that the term \( (l + 1)(r/n)^2 / r^3 \) dominates over the \( L \)-independent term \( \Psi(r) \) of the acoustic potential in the deep interior of the Sun, while the dominant term near the surface is \( \Psi(r) \). Therefore, the inner and the outer turning points \( r_1 \) and \( r_2 \) satisfy

\[ r_1^{(l + 1)}(r/n)^2 = \omega^2 \tag{10} \]

and

\[ \Psi(r_1) = \omega^2 \tag{11} \]

respectively. With the help of equations (10) and (11), differentiation with respect to \( (l + 1) \)-reduces equation (9) given as the solution of the integral equation (7) to

\[ \frac{dh(r)}{dr} \bigg|_{r_{\text{in}}}^{r_{\text{out}}} = 2(l + 1) \left[ \frac{dh}{d(l + 1)} \right] \tag{12} \]

where

\[ o(r) \equiv c(r)/r \tag{13} \]

The right-hand side of equation (12) is evaluated from the solutions \( s(\omega^2, l) \) of the integral equation (7) for various values of \( l \), and then equation (13) should be regarded as a differential equation from which we can determine the sound velocity distribution in the Sun together with an approximate boundary condition. The boundary condition should be given by

\[ r = R \quad \text{at} \quad u = c(R)/R. \tag{14} \]

but it is practically given at the outermost point among the turning points in hand, where \( c \equiv c(r) \) can be inferred from a theoretical model with little uncertainty.

3. NUMERICAL SIMULATIONS

3.1. Numerical Simulation using Complete Frequency Spectrum

In order to examine the validity of the present method, we first apply it to an idealistic case in which eigenfrequencies of all the modes in the range of \( 1 \leq l \leq 600 \) are supposed to be obtained. The eigenfrequencies were theoretically calculated by using Shibahashi et al. (Ref. 10), and we performed the inversion of their frequency spectrum. We used larger number of \( \rho \)-modes than actually detected modes and assumed the data were error-free, since our purpose was to examine the theoretically limited best resolution of the method in an idealistic case. The detail was described in Ref. 9. As for the boundary condition of (14), we have set the boundary at \( \log_{10}(r) = -4.3 \) and adopted the boundary condition as \( r/H = 0.99 \).

Figure 1 shows \( c^2 = c^2(r) \) of the final result. The monotonic thin curve represents the true value of \( c^2(r) \) of the model, and the solid curve shows the square of the sound velocity that solved by means of the present inversion method. (The thick curve is explained in the next subsection.) The thin curve in Fig. 2 shows the relative difference between the true value of \( c^2(r) \) and the deduced value from the inversion: \( (c^2_{\text{true}} - c^2_{\text{inversion}}) / c^2_{\text{true}} \). The absolute error incurred from the present method is estimated at most at smaller than 4%. Inaccuracy in the deep region is caused by smallness of number of modes which penetrate to there. On the other hand, inaccuracy near the surface comes from the fact that the equilibrium structure changes so steeply that the asymptotic analysis itself is less valid there. Besides that, since the modes whose inner turning points are distributed there are only low order modes with high degree, the numerical differentiation in the right-hand-side of equation (9) leading to the acoustic length is less accurate, and this fact also makes the final result of the inversion less accurate near the surface.

3.2. Numerical Simulation using Incomplete Frequency Spectrum

Low order \( \rho \)-modes with low degree have not been detected. To examine the validity of the present inversion method in practical cases, we pick up 1971 \( \rho \)-modes in the range of \( 1 \leq l \leq 600 \) from the set of theoretically calculated frequencies used in the previous subsection. These modes are a part of the modes reported by Duvall et al.'s (Ref. 1) and Libbrecht and Kaufman (Ref. 2). Most high degree modes have been observed, but the number of modes belonging to such a high degree \( l \) is limited to only a few and the inversion method outlined in the previous section is not appropriate to be applied to such high degree modes. Therefore we limit ourselves to deal with \( \rho \)-modes of only \( 1 \leq l \leq 600 \). Some alternative methods should be applied for the higher degree modes (Ref. 9). In the simulation, we regard the theoretical eigenfrequencies of these modes as error-free observational data.

The lack of low order modes with low degree affects severely the estimation of \( \Phi_{\text{max}} \), which in turn responsible for the accuracy in the acoustic lengths as the solution of the integral equation (9). Since it is
such low degree modes that penetrate into the deep interior of the Sun and extract information from there, the absence of low degree, low order modes will make the accuracy in the inferred sound velocity distribution in the deep interior of the Sun worse. As for high degree modes, to observationally distinguish a degree \(l\) from its neighbors becomes more difficult with increasing \(l\). In the previous subsection and Ref. 9, we used eigenmodes of various degrees \(l\), \(l = 200\) to 600 with a step of 10, but the resolution of \(l\) in the actual observational data is worse than 10. Since the high degree \(p\)-modes are responsible for diagnosis of the outer envelope below the photosphere, the coarse resolution of \(l\) will make the accuracy in the sound velocity distribution in the outer envelope inferred from the \(p\)-mode oscillations spectrum.

The thin curve in Fig. 1 shows the squared sound velocity thus obtained from the \(1971\) \(p\)-modes. As a reference, we reproduce the sound velocity inferred from the complete spectrum as well as the true sound velocity of the model in the same figure. The thick curve in Fig. 2 shows the relative difference between the true value of \(v(r)^2\) and the deduced value from the inversion of the \(1971\) \(p\)-modes, \(\frac{[v(r)^2 - \langle v(r)^2 \rangle]/\langle v(r)^2 \rangle}{\langle v(r)^2 \rangle}\). The thin curve shows that from the complete \(p\)-mode spectrum. As seen in these figures, the inverted results based on the \(1971\) \(p\)-modes reproduces less accurately the true sound velocity than that of the complete \(p\)-modes spectrum. But the relative difference is as small as 1 percent at most, and hence we feel the present inversion method is still useful to the incomplete \(p\)-mode spectrum.

### 3.3. Numerical Simulation with Error-Free Data

As the next step, we take account of observational error and examine the sensitivity of the present inversion method to the observational error in determination of the eigenfrequencies. We use the same \(1971\) \(p\)-modes as those used in the previous subsection, but we add some fluctuations to the eigenfrequencies of the modes. In doing so, we first generate a set of \(1971\) standard normal random numbers by a computer. Then, we multiply these random numbers by the standard deviations of observed eigenfrequencies of each mode. We regard the values thus obtained as errors in determination of eigenfrequencies. Adding these estimated errors to the theoretically calculated eigenfrequencies, we make an artificially simulated observed frequencies. Also, by subtracting the random error from the calculated frequencies, we make another set of simulation. We repeated this procedure for five times and made eventually ten sets of simulate observational frequencies. We suppose the true eigenfrequency is in the range of these simulate observational sets and treat these sets as statistically equivalent sets to estimate the standard deviation of the inversion result. We carried out the inversion procedure and obtained the sound velocity distribution as the solution for each of the sets of observations spectrum. Figure 3 shows these six solutions together with the solution in the case of error-free observational data obtained in the previous subsection and the true sound velocity of the model. The ten sets of solutions are statistically...
distributed due to the statistical distribution of the frequency data itself around the solution in the case of error-free data. The thin curves in this figure indicate the envelopes of the solutions and we can regard it as the most reliable range for true sound velocity distribution. Figure 4 shows the relative difference between the true sound velocity distribution and the solutions deduced from the set of p-modes spectrum. The thick curve is a result of the error-free data, while the thin curves show the envelopes of a set of erroneous data. We see the inversion of the erroneous data still reproduces the true sound velocity in the range of 0.29 ≤ \( c/R \) ≤ 0.30 within a few percent error.

4. INVERSION OF REAL OBSERVATIONAL DATA

Since we have verified usefulness of the present inversion method by performing numerical simulations, we now apply the method to real observational data of Duvall et al. (Ref. 1) and Libbrecht and Kaufman (Ref. 2). Since the inversion method adopted here cannot be applied to high degree modes such that the function of \( n(\zeta^2, l) \) is limited to only a narrow range of the radial order \( n \), we pick up only modes with \( 1 ≤ n ≤ 900 \). The total number of modes is then 1971. The practical procedure is the same as that used in the previous section. To estimate the error in the result of the deduced sound velocity distribution caused by the observational error in frequencies of modes, we construct 72 sets of frequency spectrum by adding some amounts within the standard deviation to the reported frequencies without taking account of standard deviation in the same figure, we plot the sound velocity distribution of the model 1 of Shibahashi et al. (Ref. 10). Figure 6 shows the relative difference between the sound velocity distribution deduced from the observational data and the theoretical model: \( (c_n^\text{obs} - c_n^\text{theor})/c_n^\text{theor} \). Since we have used 72 statistical sets of frequency spectrum, the confident level shown by these envelopes is estimated as \( \pm 1 \sigma \). We find the sound velocity \( c_n \) in the range of 0.29 ≤ \( c/R \) ≤ 0.30 deduced from the p-modes spectrum is slower than that of the model.

5. DISCUSSION

Christensen-Dalsgaard et al. (Ref. 1) concluded that the solar internal sound velocity in the range of 0.3 ≤ \( c/R \) ≤ 0.4 appears to exceed that of the model by a few percent. The present inversion result seems inconsistent with their conclusion. There are three possible causes for this apparent contradiction: 1) the difference in the theoretical models of the sun, 2) the difference in the observational data sets, and 3) the difference in the inversion methods. As for (ii), comparing the results performed in section 3.1 by means of these two methods, the difference seems small. Also, as for (i), the frequency difference between the observational data sets seems small. Hence, the difference between the referenced theoretical models seems to be the most likely cause of the apparent contradiction. Indeed, it is known that there are small differences among the standard solar models constructed by various researchers.

It should be noted here, however, that the sound velocity inferred by means of the asymptotic inversion method is independent of the choice of reference models.

6 REFERENCES

HELIOSEISMOLOGICAL INVERSE PROBLEM:
SOUND SPEED IN THE SOLAR INTERIOR
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ABSTRACT
The asymptotic technique for the reconstruction of
the sound speed distribution in solar interior from
the observational frequencies of the five-minute
oscillations is described. The significant
improvement of the efficiency of reconstruction of
the structure of the central solar regions is
achieved due to the inclusion of the effects of
self-gravitation into the asymptotic theory and due
to the general improvement of the inversion
technique. The results of the inversion of the
experimental data are discussed.

Keywords: Five-Minute Oscillations, Inverse Problem,
Solar Interior Structure

1. INTRODUCTION
Recent observations of the solar five-minute oscil­
lations have led to the significant improvement of
the volume and quality of the experimental data now
available. The frequencies of the five-minute modes
have been measured with high accuracy over a wide
range of degree. These observational data represent
the firm basis for the study of the solar interior
structure within the framework of the helioseismo­
logical inverse problem.

At the initial stage of the solution of the inverse
problem, in order to reveal the main sources of
discrepancies between the experimental and theore­
tical frequencies, the use of the asymptotic theory
is of the particular interest. It permits to
construct an efficient technique of nonlinear
inversion, which is based solely on the experimental
data and do not require the use of any solar model
as a zero-order approximation (Refs. 6, 7, 9, 1, 2,
12, 14 and references therein).

When the frequencies of the acoustic modes are used
for the study of the solar interior, the major
difficulties appear to be connected with the
structure of the central solar regions. At the same
time, the study of these regions is also the most
interesting, particularly in relation with the
solar neutrino problem. The difficulties in the
sounding of the structure of the solar core are due
largely to the fact that corresponding effects in the
frequencies of the acoustic modes are very small
(of the order of $10^{-5}$). To obtain the information
about the structure of the central regions from the
frequencies of the acoustic modes, the effects of
the buoyancy forces and gravity perturbations must
be taken into account (Refs. 2, 14). The general
improvement of the inversion technique also appears
to be necessary.

2. ASYMPTOTIC THEORY OF THE ACOUSTIC OSCILLATIONS:
EFFECTS OF GRAVITY PERTURBATION
The asymptotic theory of linear adiabatic stellar
oscillations was first constructed by Vandakurov
(Ref. 13) and further developed by a number of
authors (see e. g. review by Vorontsov and Zharkov
(Ref. 14) and references therein). The asymptotic
theory assumes the Cowling approximation, in which
the effects of gravity perturbation
are
neglected; the problem of linear adiabatic oscillations can be
described in this approximation by a second-order
system of differential equations.

The influence of the effects of gravity perturbation
on the oscillation frequencies can be determined
with a good accuracy by using the variational
principle and the eigenfunctions obtained in Cowling
approximation (see e. g. Refs. 5, 14):

$$
\delta (\omega_{0}^{2}) = \frac{\int \rho_{0} \psi^{*} \psi \, dv}{\int \rho_{0} \psi_{1} \psi \, dv}.
$$

Here $\omega_{0}$ is the angular frequency, $\psi$ is vector
displacement field, $\rho_{0}$ is unperturbed density, its
Eulerian perturbation $\rho_{1}$ is related with the displacement field by the
Poisson equation $\nabla \psi_{1} = 4\pi G \rho_{1}$, which has an
analytical solution in this problem. The asterisk
denotes complex conjugate, the time dependence of
the perturbed quantities is separated by the common
factor $\exp (i\omega t)$, the integration is performed
over the volume of the star.

Now let us estimate the right-hand side of the Eq. 1
by substituting the asymptotic eigenfunctions. For
the high-frequency acoustic modes which may be
considered as a superposition of the trapped

acoustic waves we replace the Laplace operator \( \nabla^2 \) in the Poisson equation by \(-1/4\), where the local wavenumber \( k = \omega/c ; c \) is adiabatic sound speed. We use also the fact that the mean potential energy of an acoustic mode (and hence the mean kinetic energy) is close to the mean potential energy of the elastic deformations. The final result for the correction to the squared eigenfrequency is

\[
\delta (\omega^2) \approx \frac{1}{4} k^2 (\frac{\omega^2}{L^2} - \frac{L^2}{r^2})^{-1/2} dr,
\]

where \( L = 1 + 1/2 \). The limits of radial integrals are the turning point \( r_1 \) for which \( c^2 - \psi^2 = \omega / L \), and the solar surface \( R \) which is taken to be at the region of the temperature minimum.

The comparison of the corrections to the eigenfrequencies obtained by using this asymptotic estimation with the results of the direct numerical computations (based on the variational principle) is shown in Figure 1. It is seen that the asymptotic approximation may be used as a good first approximation; the accuracy increases rapidly with the increase of the degree 1.

![Figure 1](image)

**Figure 1.** The corrections to the eigenfrequencies of the five-minute oscillations obtained by using the asymptotic estimation (Eq. 2) (solid curves) and by the direct numerical computations. The degree 1 is shown near each curve.

The use of the correction given by the Eq. 2 in the asymptotic equation for the eigenfrequencies (Ref. 14) leads to the more accurate equation

\[
F(\omega) + \frac{1}{\omega^2} \left[ F(\omega) - \psi(\omega) \right] - \pi \frac{\delta(\omega)}{\omega} \approx \pi \frac{n}{\omega_0},
\]

where \( \omega = \omega / L \), \( r \) is the radial order of the acoustic mode,

\[
F(\omega) = \frac{R}{r_1} \int_{r_1}^{R} \left( \frac{c^2 - \psi^2}{w^2} \right)^{1/2} dr,
\]

\[
\psi(\omega) = \frac{1}{2} \int_{r_1}^{R} N^2 \left( \frac{c^2 - \psi^2}{w^2} \right)^{1/2} dr.
\]

3. NUMERICAL INVERSION TECHNIQUE

The experimental information determined by the right-hand side of the Eq. 3 may be considered as a function of two variables \((w, \omega)\), known at the nodes of some grid. The three terms in the left-hand side of the Eq. 3 have different functional dependences on \( w \) and \( \omega \), which makes it possible to split the experimental information and to determine these terms separately from the experimental data. If such a splitting has been performed, the sound speed distribution is determined from the Eq. 4, which admits the analytical inversion using the Abel transform. Note that the effects of the buoyancy forces and gravity perturbations lead to the terms with similar functional dependence on \( w, \omega \) (both are determined by some unknown function of \( w \), divided by \( \omega^2 \)), so that only the difference \( \Phi(\omega) - \psi(\omega) \) is reconstructed from the experimental data.

The splitting of the experimental information is performed in two stages. At the first stage the frequency dependence of the phase shift \( \Phi(\omega) \) is reconstructed. It is convenient to use the nodes of sufficiently high degree for this purpose. These nodes penetrate not too deep into the solar interior and the influence of the buoyancy forces and gravity perturbations on their frequencies can be neglected; the splitting of the experimental information can then be performed using only the two main terms in the left-hand side of the Eq. 3 by the method described by Brodsky and Vorontsov (Ref. 1). The results described below have been obtained by using the nodes in the degree range \( 30 \leq \ell \leq 80 \) for this purpose (for higher degree the accuracy of the experimental frequencies becomes lower). The result of the reconstruction of the frequency dependence of the phase shift and the information about the structure of the outer solar layers which is contained in this frequency dependence are discussed in the paper by Brodsky and Vorontsov (Ref. 3). For the convenient use in the computations the phase shift \( \Phi_2 \) has then been approximated as a function of \( \omega_2 \) by a 15-th degree polynomial in the frequency range from 1.0 to 3.8 MHz; the degree of the polynomial and the frequency range are determined by the quality of the experimental data. The reconstruction of \( \Phi(\omega) \) is performed only with the accuracy of an arbitrary additive constant; this constant is not significant in further computations because only the derivative of \( \Phi(\omega) \) is used for the reconstruction of the sound speed.

It is important to note that if the influence of the non-adiabatic effects is restricted by the outer solar layers, the corresponding effects in the oscillation frequencies are contained in the frequency dependence of the phase shift and filtered out from the experimental data together with \( \Phi(\omega) \) when the sound speed is reconstructed.
The second stage consists in the further splitting of the "cleared" experimental information into two terms: \( F(\omega) \) and \( \frac{dF(\omega)}{d(1/\omega)} \). Each of the two functions \( F(\omega) \) and \( \frac{dF(\omega)}{d(1/\omega)} \) is found as its approximation by the cubic B-splines (it is convenient to use \( 1/\omega \) as an independent variable). Such a problem of the approximation of a function of two variables which can be described by the two functions of the single variable leads to the solution of the system of matrix equations of the type

\[ A_{ki} \rightarrow \phi_i \rightarrow v_k, \]

where the two-component vectors \( \phi_i \) correspond to the coefficients of the approximation of the two unknown functions by B-splines, each element of the matrix \( \{ A_{ki} \} \) is the matrix of the dimension \( 2 \times 2 \), the right-hand side is determined by the experimental data. The matrix \( \{ A_{ki} \} \) has a banded 7-diagonal structure; it is symmetric in the sense: \( A_{ki} = A_{ik} \).

The Eqs. 7 are solved by reducing the matrix \( \{ A_{ki} \} \) to the matrix product

\[ \{ A_{ki} \} = U W U^T, \]

where \( U \) is lower triangular and \( W \) is diagonal matrix. The method of numerical solution is similar to that of the standard spline-approximation problem; the only difference is that the arithmetic operations with numbers are replaced by the operations with matrices. The study of this method of parallel approximation on the model problem has shown its very high efficiency and stability.

The useful boundary conditions may be introduced into the approximation problem. It may be shown using the Eqs. 4-6 that for a solar model of arbitrary stratification

\[ \lim_{1/\omega \to 0} \frac{dF}{d(1/\omega)} = -\frac{R}{2}, \quad \lim_{1/\omega \to 0} \frac{d(\phi-Y)}{d(1/\omega)} = 0. \]

The introduction of the boundary conditions is facilitated by the convenience of the differentiation of B-splines. The result is reduced to the modification of the first equations in the system of Eqs. B; the replacement of these equations by their corresponding linear combinations returns the symmetric form to the matrix \( \{ A_{ki} \} \). The use of such an a priori information (Eqs. 9) leads to the significant improvement of the stability of the results in the central solar regions.

The distribution of the breakpoints for the spline approximation is chosen by establishing the correspondence between the variable \( 1/\omega \) which determines the position of the turning point \( r^1 \) and radius \( r \) in the standard solar model. The breakpoints are chosen to be equidistant in \( dr/c \); such a choice leads to the homogeneous correspondence between the resolving capability of the spline approximation and the resolving power of the acoustic modes which is limited by the radial wavelength. The interval of the approximation is taken to be from \( 1/\omega = 0 \) (corresponding to \( r = 0 \)) to some maximum value which is determined by the maximum degree of the modes which are used. In the computations described below the modes of degree up to \( l \geq 200 \) have been used and the upper point of the approximation interval corresponds to \( r = 0.98 R_\odot \). The mesh with 20 intervals in \( 1/\omega \) has been used, with the real noise level in the experimental data such a restriction of the resolving capability of the inversion technique has the regularization properties which are nearly optimal.

Because the influence of buoyancy forces and gravity perturbation on the high-degree modes with penetration depth limited by the outer convection zone are negligible, it is reasonable to perform a smooth transition from the parallel spline-approximation to the ordinary spline-approximation of function \( F(1/\omega) \) at some value of \( 1/\omega \). This transition is also facilitated by the convenient properties of B-spline representation: it is performed simply by setting the corresponding elements in matrices \( A_{ki} \) to zero. This modification is reasonable because of the lower accuracy of frequency measurements of high-degree modes; it permits to improve to some extent the stability of the results in the outer solar regions.

4. NUMERICAL RESULTS

The observational oscillation frequencies published by Jimenez et al (Ref. 10) (0 < \( l \leq 3 \)), Duval et al (Ref. B) (4 \( l \leq 9 \)) and Libbrecht and Kaufman (Ref. 11) (100 \( l \leq 200 \)) have been used for the inversion. The similar inversion have been performed for the theoretical frequencies of the standard solar model, using the same set of the oscillation modes (about 1100 modes in the frequency range from 1.8 to 3.8 Hz) and using the same numerical technique. The solar model 1 of Christensen-Dalsgaard (Ref. 4) have been used for the comparative computations. The eigenfrequencies have been computed in Cowling approximation with further improvement using the variational principle (Refs. 5, 14). The second-order difference scheme with 2000 mesh points have been used. The relative error of the eigenfrequency computations is estimated to be within \( 5 \times 10^{-4} \). The error is due largely to the variational treatment of the effects of gravity perturbation; it has the maximum value for low-degree modes.

Figure 2. The squared sound speed distribution reconstructed from the experimental frequencies (solid curve) and from the theoretical frequencies (dashed curve). The dotted curve shows the sound speed distribution in the model.

The results of the reconstruction of the sound speed distribution are shown in Figure 2. The inversion of the theoretical frequencies appears to be in much better agreement with the sound speed in the model comparing with the results of the previous
asymptotic inversions. The remaining discrepancies for \( r < 0.15 R_\odot \) are within few percent and probably due to the inaccuracy of the asymptotic description. Because the influence of this inaccuracy on the inversion of the theoretical and experimental frequencies should be of the similar character, the conclusions about the deviation of the solar structure from the standard model predictions may be drawn using the relative analysis of the sound speed inferred from the experimental and from the theoretical frequencies.

The relative difference between the squared sound speed inferred from the experimental and from the model frequencies is shown by the solid curve in Figure 3. We turn to the discussion of the discrepancies beginning with the outer layers.

Figure 3. The relative difference between the squared sound speed reconstructed from the experimental and from the theoretical frequencies (solid curve). The dashed curve shows the similar result obtained when the theoretical eigenfrequencies have been computed with the number of mesh points reduced twicely; this procedure adds the systematic error to the eigenfrequencies of the order of \( 10^{-3} \) in all the degree range.

The fluctuation of the curve for \( r > 0.9 R_\odot \) is connected with inaccuracy of the inversion technique in the outer layers and due largely to the restriction of the data set from higher-degree modes (see e.g., Refs. 9, 14; for \( w < w_\odot \approx 8 \times 10^5 \text{ s}^{-1} \) which corresponds to \( r \approx 0.97 R_\odot \), the derivative \( dF/dw \) becomes very small; this error rapidly decreases with depth.

The first significant discrepancy between the inferred sound speed distributions is the small hump in the curve in Figure 3 at \( r \approx 0.6 R_\odot \). Note that the similar effect can be seen in the results discussed in Refs. (6, 14), also not so clearly; those results have been obtained using the different inversion technique and different sets of the experimental data.

Figure 4 shows the radial dependence of the variable which is determined by the derivative of the squared sound speed. This variable has approximately constant value \(-2/3\) in the adiabatic regions of the convection zone (Refs. 9, 14). The deviation from this value at \( r \approx 0.8 R_\odot \) in the solar model is due to the ionization of carbon and oxygen; this deviation is reproduced rather well by the reconstruction of the sound speed from the theoretical eigenfrequencies. The significantly different result is seen in the curve obtained from the experimental frequencies. It makes the impression that the corresponding ionization zone is more deep, probably near the base of the convection zone. Just this difference between the gradients of the sound speed is responsible for the hump in the curve in Figure 3 at \( r \approx 0.8 R_\odot \). It may be expected that the detailed study of this region makes it possible to improve significantly the equation of state and to determine the absolute abundance of the corresponding heavy elements.

Near the base of the convection zone (\( r \approx 0.714 R_\odot \)) the oscillation frequencies have a somewhat smoothed character when compared with the model curve; some oscillations are also seen, particularly at \( r \approx 0.6 R_\odot \). It is due to the wave nature of the information contained in the oscillation frequencies. The convection zone may be somewhat more deep than in the model. The rough estimation indicates that the base of the solar convection zone is located at \( r = 0.70 \pm 0.01 R_\odot \).

The second region of significant discrepancies in the sound speed corresponds to \( 0.5 R_\odot < r < 0.7 R_\odot \). In this region the results of the inversions are in complete agreement with the similar results obtained previously (Refs. 6, 14). The main source of these discrepancies is related probably with inadequate description of the opacity in the radiative interior.

We turn now to the discussion of the region for which the reliable and stable results are the most difficult to obtain - to the region of the solar core. The results shown in Figures 2, 3 reveal the drastic discrepancies between the inferred sound speed distributions. Because the sensitivity of the acoustic mode frequencies to the structure of the solar core is very small, the detailed analysis of the possible sources of errors in both the experimental and the theoretical frequencies is needed.

The errors in the theoretical eigenfrequencies have a systematic character and may be related with
insufficient accuracy of the finite difference scheme and with the treatment of the effects of gravity perturbation. The dashed curve in Figure 3 shows the result of using the theoretical frequencies computed with the number of mesh point reduced twicely. It is seen that the accuracy of the difference scheme can not be the source of discrepancies which are revealed. The errors in the eigenfrequencies which are due to the treatment of the effects of gravity perturbation by using the variational principle with eigenfunctions computed in Cowling approximation have been studied in detail by Christensen-Dalsgaard (Ref. 5). The analysis of these errors shows that they can be significant but their magnitude is insufficient to explain the discrepancies.

Figure 6 shows directly the difference between the functions $F(l/w)$ reconstructed from the experimental and from the theoretical frequencies. The values of $\Delta F$ shown by dots for the individual modes have been obtained after the initial information $K_r \omega$ have been cleared from the effects of the frequency-dependent phase shift (using the corresponding polynomials) and from the effects of buoyancy forces and gravity perturbations (using the corresponding splines). The reduction of the experimental and theoretical quantities to the same values of $1/\omega$ have been performed by the interpolation in the theoretical data set for a fixed degree. The solid curve shows the difference between the spline approximations of the functions $F(l/w)$. The vertical scale, as is the function $F(l/w)$, is determined with an accuracy of an order of magnitude in the model. Because the possible differences in the stratification are reflected in the density profile, the differences between the experimental frequencies by $1 \times 10^{-3}$ leads to the decrease of $\Delta F$ by 3.5 s. The information about the difference in the sound speed in central regions is thus based upon the small difference in the structure of the frequency distributions at the level of $10^{-3}$.

Figure 5 shows that the signal to noise ratio in this region only slightly exceeds unity. In this situation the quantitative conclusions about the differences in sound speed in the core are impossible. The only conclusion is that in the region $0.2 R_s \leq r \leq 0.3 R_s$ the sound speed in the Sun increases towards the center more slowly than in the standard model. The reliability of this conclusion is almost beyond doubt. In the region of thermonuclear reactions the effect seems to have the different sign; this conclusion, however, needs to be confirmed using more accurate experimental data.

It is interesting to compare these conclusions with the results of the reconstruction of the function $\Phi(l/w) - \Psi(l/w)$, which describes the influence of buoyancy forces and gravity perturbations. These results are shown in Figure 6. Because we are dealing here with the reconstruction of the small effects estimated in the first approximation, the accuracy of these results is much lower than in the reconstruction of the sound speed. The certain information, however, can be obtained by the intercomparison of the curves obtained from the two frequency sets.

The significant scatter of the points in Figure 5 is difficult to explain by somewhat else than by the errors in the experimental data. The particular example is the mode with $1 = 2$, $n = 11$; its frequency appears to be $2 - 3 \, \text{mHz}$ too high. The magnitude of the scatter indicates that the accuracy of the experimental data is probably somewhat overestimated.
value of the adiabatic exponent $P_2$ in the deep solar regions is almost constant. In the small deviation of the stratification the increase of $N^2$ corresponds to the increase of $\frac{dc}{dr}$, that is to the more slow increase of the sound speed towards the center. The conclusions about the discrepancies in the distributions of the Brunt-Väisälä frequency are thus in the complete agreement with the conclusions about the discrepancies in the sound speed.

The discrepancies which are revealed may be interpreted the most naturally as a result of some sort of mixing in the solar core. The decrease of the sound speed towards the solar center in the energy generating core of the standard model is due to the gradient of the mean molecular weight which arises when hydrogen burning leads to the increase of the helium content. The lower values of $\frac{dc}{dr}$ or, which is the same, the lower values of $N^2$ obtained from the experimental frequencies indicate to the more uniform chemical composition of the core. The higher values of $\frac{dc}{dr}$ and $N^2$ in the region $0.2 R < r < 0.3 R$ may be explained either by the lower value of $N^2$ by the formation of the chemical composition gradient due to the transfer of the products of nuclear reactions into this region.

Note that the models with the mixed core represent one of the most natural solutions of the solar neutrino problem.

5. CONCLUSIONS

The information contained in the observational five-minute oscillation frequencies gives some evidence for the nonstandard structure of the solar core. The mixing in solar interior seems to be the most natural interpretation.

The information about the nonstandard structure of the core has been inferred from the oscillation frequencies at the level close to the noise level. The definitive solution of this problem will probably be obtained by the application of the inversion technique which has been developed to the more accurate experimental data which will be available in the near future. The use of the more accurate theoretical frequencies is also important.

Beyond the solar core the results confirm those obtained previously. The radius of the base of the convection zone is estimated to be $0.70 \pm 0.01 R_\odot$.

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6. REFERENCES


OSCILLATION FREQUENCIES OF SOLAR MODELS

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ABSTRACT

Two solar models have been constructed, one with no diffusion of the atomic nuclei, and another including diffusive element separation. The opacity at the bottom of the convection zone was increased 15-20 percent (within its theoretical uncertainty) to obtain a few microhertz agreement with observed p-mode frequencies. Original helium mass fractions were 0.291 and 0.289 for the no-diffusion and diffusion models, respectively. The diffusion model evolved to a surface Y=0.255 at the solar age, and the original Z-value of 0.0300 decreased to 0.0179. Agreement of l=0 and 2 p-mode frequency separations with those observed is good. The g-mode nonadiabatic solutions do not have equal period spacings until high radial order. The lowest order modes are more visible if they all have the same kinetic energy. High central temperatures, produce over 9 SNU's from the B and 1.5 SNU's from the Be reactions. Models with iron condensed-out below the convection zone, and with WIMPs cooling the central regions to reduce the SNU's, agree less well with p-mode frequency separations.

Keywords: Solar Oscillations, Solar Models

1. BACKGROUND

Global oscillations of the sun have received considerable observational interest in the last 20 years. Frequencies for low degree p-modes (Pallé et al., 1986) and both low and high degree p-modes (Duvall et al., 1986 and Libbrecht and Kaufman, 1988) are now typically measured to an accuracy of better than about 1 microhertz in 3000. Further observations are now being made to see if these frequencies vary with the solar activity cycle as proposed by van der Raay (1984), Woodard and Noyes (1985), Henning and Scherrer (1986), and Fossati et al. (1987). Also the splitting of these mode frequencies for a given order (n) and degree (l) is being actively studied to give the internal rotation structure of the sun. See for example Brown (1987a). However, theoretical models of the sun are not able to predict global oscillation frequencies with anywhere near the observational accuracy. Some successful work on this problem is reported in this paper.

In this work we report the results of the evolution of a solar model using our best available material property data and including the effects of diffusive processes that lead to a partial sorting of the elements with depth. Our two models are without and with the effects of diffusion. The diffusion processes of gravitational settling, composition gradient diffusion, and thermal diffusion are taken from and added to the method described by Iben and MacDonald (1985). While the surface abundances of helium and the heavier elements are decreased at the solar age relative to the case of no diffusion, the pulsation frequencies are not much changed. Important results of our work are that a high original helium content (Y=0.29) is required to match the observed global oscillation data, and the total neutrino flux detectable by the chlorine detector accordingly is increased to over 10 SNU's.

2. MATERIAL PROPERTIES

The accuracy of the calculated frequencies from solar models depends strongly on the accuracy of the material properties used to develop the model. These properties are in three categories: equation of state, opacity, and nuclear energy production. The energy production seems to have little uncertainty even though we have used only the older Fowler, Caughlin, and Zimmerman (1975) reaction rates for this study.

The importance of the equation of state is for the p-modes that strongly sample the solar convection zone. This convection zone structure depends both on the pressure and energy from the equation of state, because the amount of convection depends not only on the pressure the gas can produce by its nuclei and free electrons, but also on how superadiabatic the temperature gradient is.

Convection zone conditions for the diffusion model are given in Table 1, which gives the temperature, density, T₁, and the gas pressure constant β = PV₂/T from detailed calculations and those values from the Iben fit used in our calculations. The calculated data come from Dimitri Mihalas (Mihalas, Hummer, and Däppen, 1987) who has specially computed a precision equation of state for the solar mixture. Given beside the T₁ and β values are the values obtained from the Iben fit procedure that computes the ionization of only the hydrogen electron, the two helium electrons, and a single electron from a representative heavy element with an ionization potential of 7.5 electron volts. The justification for this simple procedure is that the heavy elements of a solar mixture contribute less than 1 percent of the free particles that cause pressure. The Iben (1963) procedure makes the further approximation that the ratio of the internal partition functions needed for the ionization equilibrium calculation is equal to the ratio of the ground state statistical weights. The agreement between the T₁ values can be seen to be usually better than 0.01, with somewhat larger differences at temperatures lower than 90,000K in the hydrogen and helium ionization zones that are only 1.3 percent of the radius and 4.5x10⁻⁶ of the mass into the sun. The maximum difference for the T₁ is at 50,000K (over 0.04), apparently because the Iben procedure does not precisely calculate the second helium ionization. Only a small part of these differences come from the interpolation necessary in the Mihalas data at the second ionization temperature of He, where values change rapidly.

Table 1

Convection Zone Equation of State

<table>
<thead>
<tr>
<th>T (10^4 K)</th>
<th>ρ (g/cm³)</th>
<th>Γ₁</th>
<th>Iben b (10^6)</th>
<th>Iben (10^6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4818</td>
<td>9.5392</td>
<td>1.664</td>
<td>1.667</td>
<td>1.328</td>
</tr>
<tr>
<td>1.0178</td>
<td>5.4200</td>
<td>1.664</td>
<td>1.668</td>
<td>1.325</td>
</tr>
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</tr>
</tbody>
</table>

At 0.000K, the b from the Iben fit, modified to include coulomb pressure reducing effects, is 3 percent too low, but the pressure is generally accurate to better than 1 percent. Shibahashi, Noels, and Gabriel (1983) and Stix and Knoelker (1986) have discussed the effects on models and p-mode oscillation frequencies when the coulomb effects are included.

It is important to realize that the oscillation frequency squared in simple adiabatic radial oscillations involves a term with the factor 3Γ₁/4 and another term with Γ₁ as a factor. Thus with uncertainties of almost one percent in Γ₁ averaged over the ionization zone region where, say, a p-mode feels the structure of the star, the frequency can be uncertain by some fraction of a percent. Thus, unless the Γ₁ can be made considerably more accurate than what we currently have, agreement with observed p-mode frequencies to better than a microhertz can be sought, that is, to 0.1 percent, does not seem possible with present solar composition equations of state. While the Iben procedure for the ionization could be improved, it is probably never going to be possible to calculate the gammas and the pressure more accurately than 1 percent everywhere.

In the outer layers, where evolution program Γ₁ values compare poorly with those calculated more accurately, the modes with l greater than about 100 have great weight for their period determination. These are layers where the temperature is less than 60,000K. Nevertheless theoretical frequencies appear to agree reasonably well with observations, as we see later. Only for l=200 or more, where only the hydrogen ionization zone is sampled, does there seem to be a significant additional effect of the Γ₁ errors on the comparison between theoretical and observed p-mode frequencies.

The small amount of extra pressure in the Iben procedure is compensated for by our being forced to reduce hydrogen (by maybe 0.01 in X) throughout the solar model in order to obtain the observed luminosity and radius of the sun at its age of 4.6 billion years. One can see that an error of ΔP/P relates to a ΔY change by the following reasonably accurate formula based on complete ionization, appropriate for all but about the outer one percent of the solar mass:

ΔP/P = -5ΔY/4(2X + 3Y/4 + Z/2)

where X, Y, and Z are the mass fractions of the solar composition for, respectively, hydrogen, helium, and the assumed fixed mass fraction for all the heavier elements. For this equation we assume that all the pressure is from the gas with no corrections for its very slight electron degeneracy, coulomb effects, or for the very small amount of radiation pressure. Evaluating this expression for the composition X = 0.70, Y = 0.28, and Z = 0.02, we get 5ΔY = -1.36P/ΔP.

Complete ionization occurs at a temperature less than 2x10^6K for the gammas. However, we have switched from the ionization calculation procedure to complete ionization in the Iben program at 4.5x10^6K to be sure of a reasonably smooth transition, especially for the Brunt-Väisälä frequency. At higher temperatures where a slight amount of electron degeneracy occurs, the Eggleton et al. (1973) procedure is used in the Iben fit.

The Iben (1975) fit for the opacity that we use deeper than the convection zone has been adjusted to both fit the latest Opacity Library values, and also to better fit the p-mode frequencies. In the expression for the Z effect (Iben, 1975) a factor of 1.30 has been added. Also the "esk" electron scattering factor has been doubled to better match the more modern central opacity. Detailed comparisons show that between 2 and 7 million kelvin the Iben fit opacities are 15-20 percent larger than those given by the latest Opacity Library values, and this increase has been made deliberately to improve agreement with the observed p-mode frequencies. According to Merts (1988, private communication) such an increase is "wet" in the theoretical uncertainty of the opacity at these temperatures and densities. At temperatures above 10 million kelvin the agreement with the Opacity Library is within 5 percent with the central opacity from the Iben fit being too small. The main effects, as will be seen later, are that the convection zone is deeper than any previous model based on evolution, and the central temperature is just slightly too small. These higher opacities also require higher helium, just as the pressure equation of state systematic errors do, because the higher opacity is compensated for by needing more of the lower opacity helium.

3. MODELS

Evolution calculations were made using the Iben evolution program (originally described in 1965) with modifications for the equation of state, for the opacity, and for the element diffusion. Details of the results of the two evolution calculations are in Table 2. Standard mixing length theory (Cox and Giuli, 1968) is used. Note that the evolution runs were done with over 300 time steps in contrast to the typical case of 10 steps as most recently used by Guenther and Sarajedini (1986). The required mixing lengths in units of the pressure scale height are just slightly higher than conventional. The radius for the models at the solar age matches the solar radius to allow accuracy to better than one microhertz in the p-mode frequencies.

We have assumed that the solar mass is 1.989x10^32 g, the solar radius is 6.9599x10^10 cm, and the solar luminosity is 3.826x10^33 erg/s. This solar luminosity was suggested by Christensen-Dalsgaard in a private communication, but it seems small, and a check case at 3.862x10^33 erg/s was made to bracket the true solar luminosity. With all other parameters held fixed, except the required small increase in Y (by 0.001) and the mixing length (by 0.7 percent) to keep the radius fixed, the p-mode frequencies are only slightly changed as we see later.

Results for the present sun are in Tables 3 and 4. The small effect of heavy element settling can be seen by the central Y and Z values. The higher central helium than conventional gives a higher central temperature and a higher neutrino flux at the earth. The largest effect on composition needed to be changed throughout the entire model by +0.00016 in the no-diffusion and -.00032 in the diffusion program for the pulsation analysis, the helium mass fraction to 0.2315 for an 11 step diffusion calculation. Details of the results of the two evolution program (originally described in 1965) with modifications for the equation of state, for the opacity, and for the element diffusion. Details of the results of the two evolution calculations are in Table 2. Standard mixing length theory (Cox and Giuli, 1968) is used. Note that the evolution runs were done with over 300 time steps in contrast to the typical case of 10 steps as most recently used by Guenther and Sarajedini (1986). The required mixing lengths in units of the pressure scale height are just slightly higher than conventional. The radius for the models at the solar age matches the solar radius to allow accuracy to better than one microhertz in the p-mode frequencies.

The composition structure is taken from the Iben evolution models and put into another program to construct a model with mass zoning suitable for nonradial pulsation analyses. Due to small differences in the treatment of the Z part of the composition when calculating the opacity between the evolution program and the model builder program for the pulsation analysis, the helium mass fraction needed to be changed throughout the entire model by +0.00016 in the no-diffusion and -0.00032 in the diffusion models to account exactly for all the mass, radius, and luminosity.
4. P-MODE FREQUENCIES

The 1700 zone model is introduced into our nonradial pulsation programs to calculate the adiabatic and nonadiabatic frequencies. Two radial and nonradial linear theory pulsation programs have been developed by Pesnell (1988) using a Lagrangian mesh and using a solution method described by Castor (1971) for the radial nonadiabatic case. The second smaller program gives only adiabatic solutions for nonradial cases, but it has more elaborate inner boundary conditions. This second program has also been used for the radial adiabatic and nonadiabatic modes.

The outer mechanical boundary condition has been tested to see its effect on p-mode frequencies and decay rates. A change in the inertial mass from 7.7 to 1.0 (a relic factor frequently used in Los Alamos pulsation calculations) to approximate the overlying chromosphere and corona mass.

The influence of the luminosity on the p-mode frequencies has been studied by using two luminosities differing by 0.94 percent. The l=2 and l=200 p-modes differ by less than one microhertz with the higher luminosity giving lower frequencies. The differences grow with mode order, reaching one microhertz just over 4000 microhertz for l=200. They are smaller for l=2, as expected because these lower degree modes feel less the surface conditions that are affected by the change in luminosity. Thus we would predict that as the solar luminosity increases going into the next solar cycle, the p-mode frequencies might decrease by a fraction of a microhertz. However, the observations in the last few years where the luminosity was decreasing, may indicate the opposite effect. But, Brown (1987b) and Jefferies et al. (1988) suggest that the p-mode frequency changes may be much smaller than other authors have suggested.

The comparison between our theoretical adiabatic and nonadiabatic frequencies for the no diffusion model and those observed is given in Figure 1 for low l modes. Figure 2 shows this comparison for the same modes for the model where diffusion is allowed to separate the elements. One can see that the nonadiabatic effects reduce the slope of the O-C curves to nearly zero, but there remain a few microhertz differences from observed frequencies. The effects of the diffusion make the agreement poorer, but that is not entirely unfortunate because we feel that our diffusion calculation has indeed overestimated the element separations. Figure 3 gives the same comparisons for higher l values in the no- diffusion case, showing that our equation of state problems in the convection zone do really affect the l=200 modes.
The tendency for the O-C to get larger at higher frequency may be due to the overestimate of the nonadiabatic effects there. We get very rapid mode decay rates using only radiation luminosity modulation, but if there is occasional mode excitation by coupling to convection, the effective nonadiabatic effects and the period increments will be somewhat smaller.

The agreement between theory and observations for these p-modes is as good as or better than the most relevant recent models by Christensen-Dalsgaard (1982) (see Christensen-Dalsgaard and Gough 1984), by Shibahashi, Noels, and Gabriel (1983), by Scuflaire, Gabriel, and Noels (1984), or by Bahcall and Ulrich (1988). The first of these has the observed minus calculated (O-C) positive at low frequency and negative at high frequency. The second has agreement to often better than 5 microhertz, but use of nonadiabatic frequencies would make the agreement worse. The third has O-C negative over the whole 5 minute band, while the last has O-C positive over the whole band. In these last three cases, only low l values were used. Better agreement of our frequencies could maybe have been produced if we had used an even higher opacity in the few million kelvin temperature region requiring an even larger initial helium abundance to match the observed luminosity.

An earlier version of this paper presented O-C plots of the p-mode frequencies calculated from models without the coulomb effects in the gas pressure and energy. Then the O-C values were 10 to 20 microhertz larger, a similar finding to those of Shibahashi, Noels and Gabriel (1983) and Stix and Knoelker (1986).

A widely used quantity for measuring the central solar structure is:

\[ \delta(n) = \nu_{n,0} - \nu_{n-1,2} \]

with n the radial order of the mode and the second subscript being the l value of the spherical harmonic function. Since the radial and quadrupole modes penetrate deeply into the sun, but are similar to each other in the outer regions, their frequency difference (or ratio) is indeed sensitive to the solar center as pointed out by many before (Faulkner, Gough, and Vahia, 1986 and Däppen, Gilliland, and Christensen-Dalsgaard, 1986). Figure 4 gives \( \delta(n) \) versus mode order for the two models using nonadiabatic frequencies. These nonadiabatic frequencies are typically a few microhertz smaller than the adiabatic ones (see Kidman and Cox, 1984, for quantitative nonadiabatic period increments), but understandably the nonadiabatic frequency increment, determined right at the top of the convection zone, is almost the same for the two modes being subtracted. Thus the differences between the adiabatic frequencies is the same as those plotted to within 0.1 microhertz. Also plotted in the figure are the observed differences of the mode frequencies (Pallé, et al. 1986), and an asymptotic theory line (for the no-diffusion case) using the formula:

\[ \delta(n) = 6A_0/(n + 1) \]

where

\[ A_0 = (c(R_0)/R_0 - \int_0^{R_0} \frac{d\rho}{dr} \frac{dr}{dr})/4\pi^2 \]

and

\[ \nu_0 = 1/(2\int_0^{R_0} (dr/c)). \]

Here \( c \) is the sound velocity. The integrals \( A_0 \) for the no-diffusion and diffusion models are 38.8 and 39.3 microhertz. The corresponding \( \nu_0 \) values are 139.0 and 138.9 microhertz.

The agreement with asymptotic theory (Vandakurov, 1967 and Tassoul, 1980) gets better for higher order as it should. However, most workers have found closer agreement with asymptotic theory than we have at these orders. Recent discussions of the asymptotic theory by Provost and Berthonkieu (1986) and by Smeyers and Tassoul (1987) do not change the above comparison. We suspect that 1) the decrease of the sonic velocity with radius at the solar model center (due to the high temperature gradient set by the high opacity), followed by 2) a frequently seen increase further out, and finally 3) a decrease way out to the surface, may produce gradients that are too severe for the asymptotic theory. An indication that the theory is not exactly valid for our models is that the \( \nu_0 \) also is calculated to be too large at 139 microhertz, whereas the separation between the actually calculated frequencies is nearly 135 microhertz.

The agreement with observed frequency differences, at least at low order, suggests that we do not need to invoke any special cooling mechanism, such as WIMPs, for the solar center (see Faulkner and Gilliland 1985). In two papers (Faulkner, Gough, and Vahia, 1986 and Däppen, Gilliland, and Christensen-Dalsgaard, 1986) it was pointed out that a cooler central temperature and a denser core, with not much change in the central composition, would give a lower central temperature and therefore a lower neutrino production rate. The lower temperature also would result in a lower sonic velocity. This then gives a lower \( A_0 \) and a lower \( \delta(n) \) which seemed to agree better with observations.

This conclusion is in direct conflict with the recent result of Gilliland and Däppen (1988) who find support for WIMPs in the p-mode differences. A possible explana-
tion is the opacities (Cox and Stewart, 1965) used by these authors are lower than those from the latest Los Alamos calculations. They are certainly lower than our arbitrarily increased value between 1 and 7 million Kelvin. There may also be equation of state differences between their calculations and ours.

The agreement between observations and theory for \( \delta(n) \) at the highest frequency is not satisfactory. Perhaps it is due to observational errors. It does not seem to be possible that our larger than corrected helium abundance could be the problem, because a decrease of 0.02 in \( Y \) would produce only a one percent increase in the sonic velocity everywhere and a one percent increase in \( \delta(n) \).

One can speculate that the larger \( \delta(n) \) for the deepest penetrating nonradial modes reflects an even higher central sonic velocity. Such a high sonic velocity might be the result of a larger central temperature (our central opacity needs to be slightly increased) or more central hydrogen (and therefore a higher sonic velocity) than we have in our models produced by a mixing event billions of years ago. Of course, too much hydrogen would upset the luminosity that has been matched to the present solar value. If the observations are correct, the discrepancy points to a fascinating probe of the central few percent of the solar mass.

Modulation of the radiation flow produces some \( \kappa \) and \( \gamma \) effect driving just at 10,00K at the top of the convection zone, but exterior to that point, there is radiation damping. The relative amount of driving and damping depends greatly on the temperature and density derivatives of the opacity, and with our use of the Stellwarg fit adjusted by a factor of three (to allow for molecules), some net pulsation drive is found for this model. Use of the King IVa table of Cox and Tabor or the unadjusted Stellwarg fit (1975ab) produces damping of this mode, as reported by Kidman and Cox (1984). Even though their damping seems to accord well with the measured lifetime of modes from their line widths in the Fourier spectrum, there must be damping or the modes would never be observed. We suggest that there are brief episodes of convection coupled driving to excite the modes, but the damping we calculate each cycle occurs continuously over the life of each manifestation of a mode.

We get pulsational instability for the p-modes in the sun depending greatly on the opacity derivatives with respect to temperature and density. The fact that Ando and Osaki (1976) and Goldreich and Keeley (1977) get pulsational instability in nonadiabatic calculations from the \( \kappa \) effect depends greatly on several things including the accuracy of their opacities. The former authors used the Paczynski (1969) opacity formulation, whereas the later ones have used the Christy (1966) fit. We find a great sensitivity to opacities, and have found that the King IVa opacity table seems to have the opacities that best fit the solar p-modes in the sun. Such modes would have lifetimes continuously over the life of each manifestation of a mode.

An interesting question concerns the maximum \( l \) value of p-modes in the sun. Such modes would have lifetimes equal to their periods. We do not have a definite answer to this question, but at \( l=1400 \), the 5 minute p-modes have decay rates approaching 10 percent per period. We guess that any \( l=2000 \) p-mode would be difficult to detect.

5. G-MODE FREQUENCIES

The spacing between the g-mode periods should be a value \( P_0/\sqrt{l(l+1)} \) for high order, where

\[
P_0 = 2\pi^2 \int r^6 N(r) dr.
\]

Here \( N \) is the Brunt Väisälä frequency that is positive only out to \( r_c \), the radius of the convection zone bottom. The integrals for the no-diffusion and diffusion models are 34.6 and 34.5 minutes, respectively, with possible integration uncertainties of less than a few percent. Because of our high central helium and central temperature, this value is smaller than that obtained for many recent solar models. Our gradient at the solar center is more adiabatic than usually calculated. This effect, and the composition gradient just below the convection zone, make the Brunt-Väisälä frequency higher than obtained previously both at the center and near the bottom of the convection zone of our diffusion model.

Figure 5. The period spacing between mode periods multiplied by \( \sqrt{l(l+1)} \) is plotted versus g-mode order for \( l=1 \) to 5. The fluctuations at the low order are caused by mode "bumping" as discussed in the text.

The most recent determination of \( P_0 \) is by Barry et al. (1987) where the value fit to asymptotic theory is 30.25 minutes. This value is based on 53 g-mode multiplets classified by \( n, l, \) and \( n \) values. The value of 29.85 by Fröhlich (1986) seems incorrect because he assumed that \( P_0 \) was constant for all g-mode orders.

6. DISCUSSION AND CONCLUSIONS

The present solar structure is determined in this work by fitting the solar mass, radius, luminosity, age, and observed p-mode oscillation frequencies. The effects of diffusion reduce the surface helium and Z to where the Z seems to match the present observed value of about 0.018. Apart from a problem of our pressure equation of state that artificially requires a small excess of helium, we feel that matching the observed solar parameters is now about the best that one can do.

Our models require high helium, but also high central temperatures. Thus instead of improving the solar neutrino problem, we have made it worse, regardless of whether we have diffusive element separation. It appears that to save our models from the criterion of also producing low neutrino fluxes, neutrino oscillations as discussed by Bethe (1986) and Rosen and Gelb (1986) are needed. A way to reduce this central temperature is to assume the presence of weakly interacting massive particles (WIMPs) that can conduct heat outward. However, the \( \delta(n) \) differences suggest that pulsation frequencies do not require WIMPs.
We have calculated models with low central temperatures such as one with all the iron condensed-out below the convection zone and one with WIMPs. Figure 6 shows the δ(n) plot for the WIMP model, which produces only two SNU. The agreement with observations is not as good as for figure 4, leading us to conclude that this is not a good solar neutrino problem solution.

Measurement of the period spacing of the g-modes has been expected to reveal details of the central solar structure, but this work shows that the spacing is not as simple as asymptotic theory (Tassoul, 1980) predicts. The variation of the spacing at low mode order and at low ℓ needs to be considered.

![Graph showing δ(n) vs. Order n (ℓ)](image)

Figure 6. The δ(n) nonadiabatic frequency difference between the radial and quadrupole modes for radial orders 14 to 30 is plotted versus radial mode order for the WIMP model.

7. REFERENCES


HELIOSEISMOLOGICAL INVERSE PROBLEM:
DIAGNOSTICS OF THE STRUCTURE OF THE LOWER SOLAR ATMOSPHERE

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ABSTRACT

The five-minute solar oscillation frequencies contain information about the phase shift corresponding to the reflection of the internal acoustic waves from the outer solar layers. The results of the reconstruction of the frequency dependence of the phase shift from the experimental data are described. This information provides the efficient way of seismic diagnostic of the structure of the outer regions of the convection zone and lower solar atmosphere. The method of direct computation of frequency dependence of the phase shift for a solar envelope model is described.

Keywords: Five-Minute Oscillations, Inverse Problem, Solar Atmosphere

1. INTRODUCTION

The solar five-minute oscillations may be considered as the result of the interference of the trapped internal acoustic waves. The information contained in the oscillation frequencies may be regarded as consisting of two parts; these parts may be separated by the splitting of the experimental information. The first part is determined by the seismic structure of the solar interior - by the distributions of sound speed, density and Brunt-Vaisala frequency; this information may be used for the reconstruction of the seismic structure within the framework of the asymptotic inverse problem (Refs. 6,7 and references therein). The second part of the information has the non-asymptotic character; it is determined by the reflecting properties of the outer solar layers, in which the reflection of the trapped acoustic waves occurs. This part of the information may be separated from the experimental data (Refs. 1, 2) in the form of the frequency dependence of the corresponding phase shift.

The equation for the eigenfrequencies is constructed by the matching of the asymptotic solutions in the internal solar regions with nonasymptotic solutions in the outer reflecting layers (Ref. 7). When the effects of gravity perturbation are taken into account (Ref. 6), this equation has the form

\[ F(w) + \frac{1}{\Omega^2} \left[ \Phi(w) - \Psi(w) \right] - \frac{\Omega'(w)}{\Omega} \approx \Pi \frac{\Omega}{\Omega}, \]

where \( \Omega \) is the angular frequency, \( n \) is the radial order of the acoustic mode, \( w = \Omega/(1 + 1/2) \), \( l \) is the degree of the mode. The functions \( F(w), \Phi(w), \Psi(w) \) are determined by the distributions of the sound speed, density and Brunt-Vaisala frequency in solar interior, \( \Omega'(w) \) is the frequency dependence of the phase shift.

The frequency dependence of the phase shift may be determined from the experimental data in the form of the function

\[ \Phi(w) = -\omega^2 \frac{\partial}{\partial w} \frac{\partial \Omega}{\partial w}. \]

This function may also be calculated from a model of the solar envelope; their comparison represents the efficient way of the diagnostics of the structure of the reflecting regions.

2. RECONSTRUCTION OF \( \Phi(w) \) FROM THE ACOUSTIC OSCILLATION FREQUENCIES

For the reconstruction of the phase shift from the oscillation frequencies it is convenient to use the modes of sufficiently high degree. These modes penetrate not too deep into the solar interior and the influence of the buoyant forces and gravity perturbation on their frequencies can be neglected, i.e. the functions \( \Phi(w) \) and \( \Psi(w) \) in the Eq. 1 can be neglected when the splitting of the experimental information is performed. The experimental frequencies have been taken from the paper by Duval et al (Ref. 4) for the degree range 30 \( \leq l \leq 80 \); for higher degree the accuracy of the experimental data decreases. For the reconstruction of \( \Phi(w) \) the expression obtained by Brodsky and Vorontsov (Ref. 1) has been used:

\[ \Phi(w) = \frac{\omega - \frac{\partial \omega}{\partial n} \frac{\partial \Omega}{\partial \omega}\left(1 + \frac{1}{2}\frac{\omega}{\Omega}\right)^2}{\frac{\partial \omega}{\partial n}}. \]

The derivatives were computed by the differentiation of the frequency table using the approximating formulas.

M A BRODSKY & S V VORONTSOV

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The r e s u l t i s shown i n F i g u r e 1 . The s i m i l a r c a l c u l a t i o n s have been c a r r i e d o u t w i t h t h e t l i e o r e t i c a l
e i g e n f r e q u e n c i e s c o m p u t e d f o r t h e s o l a r model 1 o f
C h r i s t e n s e n - D a l s g a a r d (Ref. 3) w i t h t h e accuracy o f
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were V is frequency in mHz. For the experimental
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shown i n t h e u p p e r p a r t Df t h e f i g u r e . The h o r i z o n t a l l i n e s i n d i c a t e t h e f r e q u e n c y band f r o m 1 . 5
t o 4 mHz.


Because the function $\beta(\omega)$, rather than $\alpha(\omega)$, is reconstructed from the experimental frequencies, it is also more convenient to calculate directly the function $\beta(\omega)$ as the asymptote of the solution of the corresponding initial value problem. It is not difficult to show that for this purpose the Eq. 8 should be solved simultaneously with the equation

$$
\frac{d}{d\tau_a} \frac{\pi}{\omega} \left( 1 + \cos \theta(\tau_a) + \left[ \frac{1}{4} \theta(\tau_a) + \frac{\pi}{4} \beta(\tau_a) \right] \sin \theta(\tau_a) \right) = 2 \left( \omega \tau_a - \frac{\pi}{4} - \alpha(\tau_a) \right).
$$

The boundary conditions in the region of temperature minimum, corresponding to the "reflection condition" usually used in the adiabatic oscillation problem, are

$$
-\frac{\pi}{4} - \alpha(0) = \arctg A(\omega),
$$

$$
\alpha(0) = \alpha(0) + \frac{-A(\omega) + 2\omega(\tau_a)}{1 + A(\omega)} r = R.
$$

Instead of the Eqs. 12, the alternative boundary conditions may be used, which are obtained directly from the Schrodinger equation in the assumption of the constant value of $V^2$ in the vicinity of the temperature minimum:

$$
-\frac{\pi}{4} - \alpha(0) = \arctg \left[ \sqrt{\frac{\omega_0^2}{\omega^2} - 1} \right],
$$

$$
\alpha(0) = \alpha(0) + \frac{1}{\sqrt{\omega_0^2 - 1}}.
$$

The results are very weakly dependent on the choice of the boundary conditions: this property corresponds to the similar situation in the eigenfrequency computations.

The value of $\beta(\omega)$ is calculated for each $\omega$ as the asymptote of the solution of the Eq. 11 for sufficiently high values of $\tau_a$. The convergence of $\beta(\tau_a)$ for the standard model is illustrated by the dotted curve in Figure 3.

The convergence of the numerical solution $\beta(\tau_a)$ to the constant value for the standard solar model at the frequency of $3 \text{ mHz}$ is illustrated by the dotted line in Figure 3. The solid curve is obtained when the correction $\delta \beta$ is used (see text).

The result of the calculation of $\beta(\omega)$ for the standard envelope model is shown by the solid curve in Figure 4.
The result agree completely with the similar curve reconstructed from the eigenfrequencies of this model (lower curve in Figure 1). The small difference between the two curves (within some percent) reflects the accuracy of the numerical computations; it is connected with double differentiation of the sound speed profile in the model with insufficient number of mesh points when the reflecting potential has been computed.

The certain periodic component is clearly seen in the frequency dependence of the phase shift. Its origin is due to the local disturbance in the profile of the reflecting potential at $r = 0.98 \, R$, connected with the second helium ionization zone. The smoothing of this disturbance in the numerical experiment by a straight line in the coordinates $(T_\text{c}, \ln V^2)$ leads to the disappearance of the periodic component in $B(\omega)$, which is illustrated by the dashed line in Figure 4. The similar periodic component is revealed when $B(\omega)$ is reconstructed from the observational frequencies of the oscillations (Figure 1). The amplitude of this periodic component may be used for the determination of the absolute abundance of helium in the convective zone (see also Ref. 7).

4. CONCLUSIONS

The reconstruction of the frequency dependence of the phase shift from the observational frequencies of the solar five-minute oscillations together with the simple method of the calculation of this dependence for a solar envelope model open the new efficient way of the seismic diagnostics of the structure of lower solar atmosphere and outer layers of the convective zone. The significant feature of the method is that it permits to verify the different models of the solar envelope avoiding the construction of the full evolutionary solar model.

The difference between the experimental and theoretical frequency dependences of the phase shift is very large and determines the main source of the discrepancy between the observational and theoretical frequencies of the five-minute oscillations. This difference is directly related with the disagreement of frequencies in the echelle diagrams usually used for the comparison of the experimental and theoretical frequencies of low degree oscillations (Ref. 7).

The study of the different models of the solar envelope should make it possible (to some extent) to verify and calibrate the different variants of turbulent convection theory, the equation of state and the models of the solar atmosphere.

All the considerations described above assume the adiabatic approximation. The study of the influence of the nonadiabatic effects on the frequency dependence of the phase shift will be very important.

A few words about the new measurements of the line widths connected with the damping of the five-minute oscillations (Libbrecht, Ref. 5). If we assume as a working hypothesis that the nonadiabatic effects are restricted by the outer solar layers, then the damping of the five-minute modes is equivalent to the imperfect (partial) reflection of the acoustic waves from the outer layers. In this case the line width may be converted to the absolute value of the reflection coefficient, which complements the data on the phase shift to the full (complex) reflection coefficient as a function of frequency. The nonadiabatic problem of the five-minute oscillations is then significantly simplified from the mathematical point of view, leading to the introduction of a complex reflecting potential.

The relation between the line width $\Delta$ and the absolute value of the reflection coefficient $|R|$ may be determined in a following way. Denoting by $E$ the total energy of an oscillation mode, we have

$$\Delta = -\frac{1}{E} \frac{dE}{dt} = \frac{E}{E} (1 - |R|)$$

where $q$ is acoustic energy flux incident on the solar surface. This flux and the total energy of the mode are easily estimated using the asymptotic eigenfunctions. The result is

$$\frac{q}{E} \approx \left\{ \frac{E}{T} \left\{ -\frac{E}{2} \frac{1}{(1+2)^2} \right\}^{1/2} \right\}^{-1} \text{dr}^{-1} = \int_1^a \frac{dr}{r}$$

where $r_1$ denotes the lower turning point. Due to the use of the asymptotic description, this expression has a clear physical meaning; the total energy $E$ of the mode is equal to energy flux $q$ multiplied by the time between two consecutive reflections from the surface. For low-degree modes the value of $q/E$ corresponds to the value of the "large splitting" $\Delta V$.

The reduction of the linewidths measured by Libbrecht (Ref. 5) to the reflection coefficient $|R|$ leads to the following results. The value of $|R|$ is 0.99 at the frequency of 1.6 mHz, decreases when the frequency increases, nearly constant value of 0.95 between 2.5 mHz and 3 mHz and then fall to 0.6 at the frequency of 4 mHz. It is interesting to note that at more higher frequencies, where the experimental data is less reliable, the reflection coefficient becomes negative. If it is really so, it would mean that the linewidths are too large to be explained only by the energy dissipation near the surface and energy leakage through the potential barrier to the upper atmosphere, so that some volume dissipation processes should be significant. More accurate and reliable measurements of the linewidths of the high-frequency modes can shed light on the solution of this interesting question.

We are grateful to K. G. Libbrecht for providing experimental data and J. Christensen-Dalsgaard for his solar model I. We thank V. N. Zharkov and V. N. Strakhov for useful discussions.
5. REFERENCES


DETERMINATION OF THE SOLAR INTERNAL SOUND SPEED BY MEANS OF A DIFFERENTIAL ASYMPTOTIC INVERSION.

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ABSTRACT

We present a simple asymptotic inversion method for obtaining an estimate of the difference in internal sound speed from differences between five-minute p-mode oscillation frequencies. Thus, given a known model and a set of frequencies of a model of unknown structure, or of the Sun, the internal sound speed of the latter can be estimated. Numerical experiments with error-free frequencies indicate that this method can provide an estimate of the sound speed, from the energy-generating core to the helium ionization zone, with smaller fractional error than any previously published results. We have applied the method to real solar data, and find that the solar sound speed is indeed broadly as determined by an earlier investigation at those depths for which the earlier inversion should be reliable.

Keywords: Solar internal sound speed, helioseismic inversion techniques, asymptotic behaviour.

1. INTRODUCTION

Most of the solar five minute oscillations are p modes, and their frequencies are determined mainly by the adiabatic sound speed $c(r)$ in the solar interior, as well as by the properties of a thin layer near the solar surface where the modes are reflected. The latter properties are not well understood; this complicates the task of interpreting and inverting the observed frequencies.

The p modes can be described in terms of asymptotic theory, according to which the frequencies $\omega$ satisfy the Duvall law (Ref. 1):

$$\frac{(n+\alpha)n}{\omega} = F(\omega/L),$$  (1)

where

$$F(\omega) = \int_1^R \left[ 1 - \frac{c^2}{r^2} \right]^{1/2} c^4 \, dr,$$  (2)

and $\omega = \omega/L$. Here $n$ is radial order, $\ell$ is degree, $L = \ell + 1/2$ and $\alpha$, which is essentially a function of frequency but not of $L$, depends on the uncertain properties of the upper reflection layer. The lower limit of the integral is at the inner turning point $r_1$, defined by $c(r_1)/r_1 = \omega$. Christensen-Dalsgaard et al. (Ref. 2) showed that the observed frequencies could be fitted to a relation of this form, resulting in a determination of $\alpha$ (which they assumed to be constant) and $F(\omega)$. They inverted the relation for $F$, to obtain an estimate of the sound speed in the solar interior. Subsequently the technique has been improved by allowing for the frequency dependence of $\alpha$ (Refs. 3, 4).

2. DIFFERENTIAL INVERSION

The asymptotic relation is not exact. Hence the inversion of equation (2) suffers from systematic errors. These can be partly eliminated by considering differences between the results of inversions based on frequencies of different models, or on observed and computed frequencies (Ref. 2). In Ref. 5 we have shown that the systematic errors in the asymptotic relation are eliminated more efficiently by inverting frequency differences to obtain directly the difference in sound speed. By linearizing equations (1) and (2) in terms of small perturbations $\delta c$ and $\delta \alpha$ in the model, and the corresponding frequency perturbations $\delta \omega$, we obtain, approximately (Ref. 6),

$$S(\omega) \frac{\delta \omega}{\omega} = \int_1^R \left[ 1 - \frac{c^2}{r^2} \right]^{1/2} \frac{\delta c}{c} \, dr + n \frac{\delta \alpha}{\omega}$$  (3)

where

$$S(\omega) = \int_1^R \left[ 1 - \frac{c^2}{r^2} \right]^{1/2} \, dr.$$  (4)

Hence $S(\omega)\delta \omega/\omega = H_1(\omega) + H_2(\omega)$ in this approximation, the functions $H_1$ and $H_2$ being defined by equation (3). If a relation of this form is fitted to the differences between frequencies computed from two models, or between the observed and computed frequencies, the sound-speed difference can be obtained by inverting $H_1$. As with the inversion of $F(\omega)$, the equation to be inverted can be transformed into an integral equation of the Abel type, and hence the solution can be written down analytically.

To analyse the frequency differences we fit the expression

$$\tilde{H}_1(\omega) + \tilde{H}_2(\omega)$$  (5)

to $S\delta \omega/\omega$ by a least-squares method. Here $\tilde{H}_1$ and $\tilde{H}_2$ are splines; for the cases considered here the splines are defined on 20 knots approximately uniformly spaced in log$\omega$, and uniformly spaced in $\omega$. The sound-speed difference is then estimated by inverting $\tilde{H}_1(\omega)$. The procedure is discussed in more detail in Ref. 5.
Figure 1. Relative sound-speed differences between a chemically homogeneous model and a normal model of the Sun. The dashed line shows the exact difference. The solid line was obtained by inverting \( \tilde{H}_1 \), determined by fitting the expression (5) to the scaled frequency differences shown in Figure 2.

3. RESULTS

We have applied this method to the compilation of observed solar frequencies in Refs. 7 and 8 and, to test the technique, to frequencies for the same set of modes in a chemically homogeneous model, calibrated to have the solar radius and luminosity. The sets included modes in the five-minute band of all degrees up to 100, of degrees between 110 and 400 in steps of 10, and of degrees between 420 and 1320 in steps of 20. To ensure adequate accuracy of the simple asymptotic representation (1) we excluded the following relatively low-frequency modes (Ref. 5):

- \( \ell = 0, \nu < 4480 \mu Hz; \)
- \( \ell = 1, \nu < 3203 \mu Hz; \)
- \( \ell = 2, \nu < 2430 \mu Hz; \)
- \( \ell = 3, \nu < 2311 \mu Hz; \)
- \( \ell = 4, \nu < 2111 \mu Hz. \)

Also the \( f \) modes clearly had to be excluded.

The reference model used was Model 1 of Ref. 9. The difference in sound speed between the homogeneous model and Model 1 is illustrated in Figure 1. Scaled frequency differences between the models are illustrated in Figure 1, as functions of \( \nu/L \) (\( \nu = \omega/2\pi \) being the cyclic frequency). The original scaled differences (Figure 2a) clearly display the expected dependence on \( \nu/L \) and \( \nu \) (cf. Ref. 6). Figure 2b shows the scaled data after subtraction of the \( \tilde{H}_2(\omega) \) obtained by fitting the expression (5), as well as the \( \tilde{H}_1(\omega/L) \) obtained from the fit; evidently the fit successfully eliminates most of that part of the frequency difference that is not a function of \( \omega/L \). By inverting \( \tilde{H}_1 \), one obtains the estimate for \( \delta \omega/\omega \) shown as a solid line in Figure 1. The agreement between the exact \( \delta \omega/\omega \) and the result of the inversion is relatively good, demonstrating the ability of the method to recover even a fairly large sound-speed difference. Further tests are discussed in Ref. 5. They give some confidence in the application of the method to the observed data.

Scaled differences between the observed frequencies and the frequencies of Model 1 are illustrated in Figure 3. In contrast to Figure 2, most of the variation in this case is associated with \( H_2(\omega) \), and hence with the properties of the surface layers. Figure 3b shows the fitted \( \tilde{H}_1 \), and the residuals of the scaled frequency differences after subtracting the fitted spline \( \tilde{H}_2 \). It is evident that much of the variation in \( \delta \omega/\omega \) is indeed contained in \( H_1 \) and \( H_2 \), as was the case with the homogeneous model. However there appears to be a systematic shift in the residuals in the vicinity of \( \nu/L = 25 \mu Hz \). This may be related to the transition between two data sets, from observations of modes of moderate (Ref. 7) and high (Ref. 8) degree, and could be caused by a residual scale error in the latter (cf. Ref. 2).

The result of inverting \( \tilde{H}_1 \) for the observed frequency differences is shown in Figure 4. The error in the inversion was estimated by noting that the inversion determines coefficients \( d_{\nu L} \) such that the resulting \( \delta \omega/\omega \) can be written in the form:

\[
\frac{\delta \omega}{\omega} = \sum_{\nu L} d_{\nu L}(r) \frac{\delta \omega L}{\nu L},
\]

(Ref. 10). If the errors in \( \delta \omega/\omega \) are assumed to be random and Gaussian, one can evaluate the error in \( \delta \omega/\omega \) from this expression. In fact we have used as an estimate of the error in each individual frequency difference the magnitude of its deviation from the fit (5), which may be estimated from Figure 3b. The corresponding departures from the fit to the frequency differences between the chemically homogeneous model and Model 1 are much smaller (cf. Figure 2b); thus although the effect of the systematic error resulting from the inaccuracy of the asymptotic representation is partially included in our error estimate, it is probably insignificant compared with the effect of the random errors in the data. Except for modes of degree greater than about 400, the individual mode frequency errors we infer are in general substantially larger than those quoted in Refs. 7 and 8; so unless the actual function \( H_1(\omega) \) for the Sun is much less
Figure 2. Scaled differences between the frequencies of the chemically homogeneous and the normal model. Here \( \tau_0 = \int_0^\infty \frac{dr}{c} \) is the value of \( S(w) \) in the limit of large \( w \).

In a) are shown the original differences, each curve corresponding to a fixed value of \( \xi \). In b), the solid lines show the result of subtracting from the scaled differences in a) the frequency-dependent term \( H_i(w) \) obtained by fitting the expression (5). The dashed line shows \( \tau_0^* H_i(\nu/L) \), where \( H_i \) was obtained from the same fit. The upper axis is labelled in terms of the turning point position \( r_i/R \), related to \( \nu/L \) through \( c(r_i)/r_i = 2\pi v/L \). The tick marks are at \( r_i/R = 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.7, 0.8, 0.9, 0.95, 0.99, 0.995 \) and \( 0.999 \).

The mapping connecting the residuals from the fit with the errors in the inversion can be estimated by comparing Figures 3b and 4. Since the residuals shown in Figure 2b are only about a third of those in Figure 3b, it follows that 1-\( \sigma \) errors in the inversion illustrated in Figure 1, estimated in this manner, would be only about a third of those indicated in Figure 4. Throughout the entire region this is substantially less than the differences between the two curves illustrated in Figure 1, which provide an estimate of the combined systematic errors in the inversion procedure. Hence those errors, which are below about 0.3 per cent almost everywhere, are not predominantly due to the failure of fitting the relation (5) to the scaled frequency differences, but rather indicate that the resulting \( H_i \) does not accurately approximate the first term on the right hand side of equation (3).
4. DISCUSSION

Broadly speaking, the results shown in Figure 4 are consistent with those obtained in Ref. 2. This is true in particular of the dominant positive hump beneath the convection zone; as was pointed out in Ref. 2 this might possibly be caused by the opacity used to compute the model being too small by a relatively modest amount, of order 20 per cent, at the temperatures corresponding to this region. Also it appears that in most of the convection zone \( \delta c/c \) is essentially zero; its behaviour for \( r/R > 0.9 \) is apparently associated with the shift in the frequency residuals mentioned above, and hence could be a result of systematic errors in the observations. The results in the deep interior are uncertain, largely because they are based on relatively few modes. The negative sign of \( \delta c/c \) at \( r/R = 0.3 \) appears to be significant, but the detailed behaviour probably cannot be trusted. We note that although this behaviour is absent from the homogeneous model inversion shown in Figure 1, it might still conceivably result from systematic errors in the inversion procedure, since such errors may depend on the details of the underlying exact solution to the inverse problem (e.g. Ref. 10). Nevertheless it is interesting that the previous inversion (Ref. 2) showed a similar feature.

Results from asymptotic sound-speed inversions have also been presented by Vorontsov (Ref. 11), Kosovichev (Ref. 12) and Shibahashi & Sekii (Ref. 13) at this meeting. Both Kosovichev and Vorontsov also used Model 1 as a reference, and obtain positive humps in \( \delta c/c \) similar to that found here. In contrast Shibahashi & Sekii used a different reference model, based on more recent (and higher) opacities; in this case the hump is absent, confirming its interpre-
Figure 4. Sound-speed difference between the Sun and a normal solar model (Model 1 of Ref. 9), as obtained by inverting the fit $\tilde{H}$, shown in Figure 3b. The thin dashed lines indicate 1-$\sigma$ error limits, estimated from the scatter around the fit.


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5. REFERENCES
ON THE RELATION OF THE DUVALL PHASE FUNCTION TO THE UPPER LAYERS OF THE CONVECTION ZONE

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ABSTRACT

A method to obtain the DuVall phase function from the structure of a stellar model is presented. This can be a good tool for understanding the dependence of p-mode frequencies on the structure of the upper layers of the convection zone. In particular we have obtained a linear relation between phase function differences of two models and their equilibrium structure differences. To test the validity of this expression we have used several solar models. These models are also useful for estimating what kind of information can be obtained from observational DuVall phase function determinations.

1. INTRODUCTION

Because of the rapid decrease of the solar sound speed close to the surface, the p-mode energy contribution increases rapidly and therefore their frequencies depend to a large extent on the upper layers of the Sun. Thus, in principle, helioseismology is a powerful tool for providing information about this zone. In fact the comparison between the observed frequencies and those of solar models suggests that most of the differences arise in these layers (e.g. Ref. 1, 2). They also introduce problems in asymptotic inversions to determine the sound speed in the solar interior, because asymptotic theory is not strictly valid near the surface, where the scale heights of pressure and density are small compared with the wavelength of the modes, and rapidly varying. Further problems are caused by the insufficient understanding of the physics of the oscillations in the superficial layers (Ref. 3).

In the context of the asymptotic theory and in particular in the DuVall relation the influence of the upper layers is included in the so-called effective phase function in order to absorb the errors introduced by the failure of the asymptotic description in these layers. We have computed effective phases, isolating the contribution of the upper layers and investigating possible errors in the asymptotic description in these layers. We have computed DuVall phase function determinations.

2. THE EFFECTIVE PHASE FUNCTION

2.1 Definition

If we require the DuVall relation (e.g. Ref. 4) to be exactly satisfied, we obtain the effective phase, from computed eigenfrequencies, as:

\[ \alpha_{eff}(\omega, l) \equiv F \left( \frac{\omega}{L} \right) \left( \frac{\omega}{\pi} - n \right) \]  

where:

\[ F(x) = \int_{r_1}^{R} \left( 1 - \frac{c^2}{r^2} \right)^{1/2} \frac{dr}{r}. \]  

Here \( l \) is the degree, \( n \) the radial order, \( \omega \) the frequency of the mode, and \( L = l + 1/2 \). \( c(r) \) is the local sound speed, \( r_1 \) is the inner turning point given by \( c(r_1)/r_1 = \omega \) and \( R \) the photospheric radius. For high frequency \( \omega \)-modes of sufficiently low degree \( \alpha_{eff}(\omega, l) \approx \alpha(\omega) \) is a function of frequency alone.

In order to find \( \alpha(\omega) \) we have integrated the wave equations in the adiabatic approximation taking boundary conditions at the surface: thus for a given frequency and degree we obtain the radial amplitude \( \xi(r) \) of the radial component of the oscillation displacement. On the other hand, according to asymptotic theory, for \( r \ll r_f \) we have:

\[ \xi(r) \approx A_{eff} \left( \frac{1}{r_f/r} \right)^{1/2} \left( 1 - \frac{L^2 \rho^2}{r^2 \omega^2} \right)^{1/4} \cos(\omega \tau_f + \phi_0), \]

where:

\[ r_s = \int_{r_1}^{R} \left( 1 - \frac{L^2 \rho^2}{r^2 \omega^2} \right)^{1/2} \frac{dr}{r}; \]

also we introduce \( \alpha_{eff}(\omega, l) \) by:

\[ \alpha_{eff}(\omega, l) = \left( 1 - \frac{A_{eff}}{A_{eff} + \alpha(\omega, l)} \right) \pi. \]

Here \( \rho \) is the density and \( A_{eff} \), \( \alpha_{eff}(\omega, l) \) are \( r \)-independent functions. We have determined \( A_{eff} \) - within a constant factor - and \( \alpha_{eff}(\omega, l) \) by fitting \( \xi(r) \) to equation (3) by least squares over a small interval in \( r \), at a point \( r_0 \) where the asymptotic relation may be expected to be valid. Tests have shown that the \( A_{eff} \) and \( \alpha_{eff}(\omega, l) \) so determined are insensitive to the precise location of the fitting point \( r_0 \) over a wide range of \( r_0 \).

If in equation (4) we replace the limit \( r_f \) by \( r_1 \) and the limit \( R \) by \( r_f \) and integrate the wave equations from the interior to the surface we can find in the same way an effective phase \( \alpha(\omega, l) \). It can be shown that:

\[ \alpha_{eff}(\omega, l) = \alpha(\omega, l) + \alpha(\omega, l) - 1/2. \]

Because \( \xi(r) \) is \( l \)-independent close to the surface for sufficiently small \( l \), one expects that \( \alpha_{eff}(\omega, l) \approx \alpha(\omega) \), where \( \alpha(\omega) \) is computed for radial oscillations (numerically it can be shown that this is highly accurate provided the fit is made far from the inner turning point) and, if asymptotic theory is correct in the interior, from JWKB
Figure 1. The dot-dashed line is \( a_{\text{df}}(\nu, l) \) for the degrees \( l \) indicated. The continuous line is \( \bar{a}_s(\nu) \).

Figure 2. The dashed line is the dimensionless quantity \( \frac{S_{\text{nl}}}{\omega_{\text{nl}}} \) from equation (7).

Figure 3. The dotted line shows the effective phase function for the model with modified convection using in both cases Model 1 of Christensen-Dalsgaard (Ref. 5). Had we used the Cowling approximation to get \( a_{\text{df}}(\nu, l) \) most of the \( l \)-dependence for low degrees would have disappeared; this means that the main error in using asymptotic theory in the deep interior is the neglect of the perturbations in the gravitational potential. However, although \( a(\nu, l) \neq 0 \), \( \bar{a}_s(\nu) \) still depends only on the structure of the upper layers.

2.2 Differences in the phase function between two models

Our work is based on the comparison between models; thus it is interesting to consider how frequency differences between two models are related to the differences between the structure of their upper layers. We show here a method to obtain the effective phase differences \( \delta a_s(\nu) = a_{\text{df}}(\nu, l) - a_{\text{df}}(\nu, l) \) from frequency differences and in the next section we consider the relation between this and the equilibrium structure.

Frequency differences between \( \nu \)-modes of two models calibrated to the same radius satisfy the relation (Ref. 6):

\[
S_{\text{nl}} \cdot \frac{\delta \omega_{\text{nl}}}{\omega_{\text{nl}}} \approx H_1 \left( \frac{\omega}{L} \right) + H_2(\omega),
\]

where:

\[
S_{\text{nl}} = \int_{r_i}^{R} \left( 1 - \frac{L^2 c^2}{r^2 \omega^2} \right)^{1/2} \frac{d r}{c} - \frac{\pi}{\omega} \frac{\partial a_{\text{df}}(\nu, l)}{\partial \omega}.
\]

\[
H_1(\omega) = \int_{r_i}^{R} \left( 1 - \frac{c^2}{r^2 \omega^2} \right)^{1/2} \frac{d c}{c} \frac{d r}{c}
\]

and:

\[
H_2(\omega) = \frac{\pi}{\omega} \delta a_\omega(\omega).
\]

Here \( \delta a_\omega = a_{\nu, 2}^{\nu} - a_{\nu, 1}^{\nu} \), \( \delta c(r) = c^{(2)}(r) - c^{(1)}(r) \), \( \delta a_s(\nu) = a_s^{(2)}(\nu) - a_s^{(1)}(\nu) \), where the superscripts (1) and (2) refer to the two models and the differences between the models are assumed to be sufficiently small. Some properties of this equation are discussed in Ref. 6 and 7; in particular Ref. 7 shows a method for estimating \( H_1 \) and \( H_2 \) from frequency differences and to invert the \( H_1 \) so obtained to determine the sound speed difference (see also Ref. 8).

Since \( a_\omega(\nu) \) is not precisely defined in the Duvall relation, the same is true of \( \delta a_\omega(\nu) \) as defined here; thus we need to prove that \( \delta a_\omega(\nu) \approx \delta a_\omega(\nu) \) where \( \delta a_\omega(\nu) \) is obtained from frequency differences, but, noticing that the determination of \( H_2 \) from eigenfrequencies can be done only within a constant (Ref. 7). The method employed here uses the fact that in the case of the Sun most of the convection zone is nearly adiabatically stratified and so \( \delta c(r) \approx 0 \) for \( r_i \), \( r < r_i \), \( r_i \), where \( r_i \) is the upper base of the convection zones and \( r_i \) the deepest point of the second He-ionization region (e.g. Ref. 9). Therefore, for modes with

\[
\frac{\delta c(r_i)}{r_i} > \frac{\omega}{\omega_{\text{nl}}} \approx \frac{\delta c(\nu)}{r_i}
\]

one has from equation (7):

\[
\frac{d}{d\omega} \left( S_{\text{nl}} \frac{\delta \omega_{\text{nl}}}{\omega_{\text{nl}}} \right) \approx \frac{d}{d\omega} H_2(\omega).
\]

Hence, we want to test the accuracy of the expression:

\[
\frac{d}{d\omega} \left( \frac{S_{\text{nl}} \delta \omega_{\text{nl}}}{\omega_{\text{nl}}} \right) \approx \frac{d}{d\omega} \left( \frac{\delta a_\omega(\nu)}{\omega} \right)
\]

where \( < \) is an average over a small frequency range (typically \( \approx 100 \mu \text{Hz} \)).

To test this relation, we compare two models with Model 1, both with the solar radius and luminosity:

a): a model with a different treatment of the convection, but otherwise with the same properties as Model 1 (Ref. 3, 9)

b): a model with hydrogen abundance \( X(r) \) constant (the same as used in Ref. 6).

In case a) the differences are predominantly concentrated near the surface, whereas in case b) there are substantial differences in the sound speed throughout the Sun.

Figures 2 and 3 show the left hand side of equation (13) using a set of modes with degree 50 \( \leq l \leq 100 \) and 15 \( \mu \text{Hz} \leq \nu / L \leq 50 \mu \text{Hz} \) (\( \nu \) is the cyclic frequency) for the model with the modified convection (Figure 2) and the chemically homogeneous model (Figure 3). Also shown is the right hand side of equation (13) for each case. It can be seen that at least for these two cases \( d/\omega (\delta a_\omega(\nu)/\omega) \) can be computed from \( \nu \)-mode frequency differences with a good accuracy.
3. EFFECTIVE PHASE DIFFERENCES AS FUNCTIONS OF EQUILIBRIUM MODEL DIFFERENCES

3.1 The equation

As it is well known (see also Ref. 10), frequency differences can be expressed as functions of only two variables, e.g., sound speed $c$ and first adiabatic exponent $\Gamma_1$; therefore $\delta n_\omega(\omega) \equiv \delta n(\omega)$ that is related through equation (7) with $\delta n_{\text{int}}$ can be expressed as $\delta n_\omega(\omega) = f(\omega; \beta_c, \beta_\Gamma)$.

Because we are going to integrate the wave equations in the convection zone only we use the Cowling approximation; furthermore, as has been pointed out in the previous section, it is sufficient to consider radial oscillations. We have also used the adiabatic approximation. Given a small change in the equilibrium model we have, at fixed $\omega$, small changes $\delta n_c$ and $\delta p'$ ($p'$ being the Eulerian pressure perturbation). Under the assumption of linearity $\delta n_c$ and $\delta p'$ can be expressed as integrals of kernels multiplying the changes in the model. By considering the perturbation of equation (3) it is then possible to show that $\delta n_\omega(\omega)$ satisfies:

$$\delta n_\omega(\omega) \approx \delta n_{c_1} + \delta n_c$$  \hspace{1cm} (14)

where:

$$\delta n_{c_1} = \int_{r_0}^{R} K^{(c_1)}(r) \frac{\delta n_c}{n_c}(r) \, dr$$ \hspace{1cm} (15)

and

$$\delta n_c = \int_{r_0}^{R} K^{(c)}(r) \frac{\delta n_c}{n_c}(r) \, dr.$$ \hspace{1cm} (16)

Here $K^{(c_1)}(r)$, $K^{(c)}(r)$ are obtained by solving numerically the appropriate differential equations, derived by perturbing the oscillation equations, and using a reference model; $r_0$ is a point where equation (3) is valid and $R$ a point in the atmosphere of the model where the energy density of the mode is small. In writing down equation (14) we have neglected a small term coming from differences at $r_0$ and $R$.

Figure 4a shows the kernels $K^{(c_1)}(r)$ and Figure 4b the kernels $K^{(c)}(r)$ for several frequencies. Model 1 has been used. Where the inverse of the density and pressure scale heights can be neglected, and the eigenfunctions satisfy the asymptotic relation (3), one has:

$$K^{(c)}_\omega \approx -K^{(c_1)}_\omega \approx \frac{2\omega}{\pi c} \cos \left[ 2(\omega R + \beta_\omega) \right].$$ \hspace{1cm} (17)

This behaviour of the kernels in the lower part of the convection zone can be seen in Figure 4. Notice also that the kernels decrease outside the external turning points.

3.2 Results

To test these expressions, particularly the accuracy of the linear hypothesis, we have used the two previous models. Figures 5 and 6 show $\delta n_\omega(\omega)$ as obtained both using the direct computation (see section 2.1) and from equation (14); results are given for the model with modified convection (Figure 5) and the chemically homogeneous model (Figure 6). These figures show that at least for these examples the results from equation (14) are quite accurate. Given the large differences between these two models and Model 1, one can expect equation (14) to be valid in many other cases if the physics of the oscillations is known. The differences between the observed frequencies and those computed for Model 1 are no larger than the typical frequency differences considered here and the same can be expected for them. However if the oscillation equations are wrong, e.g., because non-adiabatic effects must be included, equation (14) requires modifications.

Figures 5 and 6 also show each term of equation (14); in fact this equation allows us to look for the contribution of each part of the model to $\delta n_\omega(\omega)$. This can be helpful for understanding the dependence of $H_\omega(\omega)$ on equilibrium structure differences. As an example, Figure 7a shows for the model with modified convection zone the quantity:
Figure 5. The heavy continuous line is $\delta a_1(\omega)$ using the direct computation, the heavy dashed line $\delta a_1(\omega)$ using eq. (14). The thin continuous line is $\delta a_1(\omega)$ and the thin dashed line $\delta a_1(\omega)$. The model with modified convection zone has been used.

Figure 6. The same as Figure 5 but for the model with constant hydrogen abundance.

$$C_\omega(r) = \left( K^{\mu(r)}(r) \delta c(r) + K^{\mu(r)}(r) \delta c(r) \right) \frac{\pi}{\nu}$$

for several frequencies and Figure 7a shows for the same model the function

$$H_\omega(r) = \int_r^R C_\omega(r')dr'.$$

We notice that $H_\omega(r \simeq 0.97R) \simeq H_\omega(r_N) = H_2(\omega)$. From both figures it can be seen that almost all the contribution to $\delta a_2(\omega)$ for this model comes from the region where convection is significantly superadiabatic. This is partially due to the behaviour of $\delta c(r)$ and $\delta c(r)$ in this case (see Ref. 9) but it is enhanced because the kernels increase rapidly with $r$ in this region.

Figure 8a shows $C_\omega(r)$ and Figure 8b $H_\omega(r)$ for the chemically homogeneous model. In this case there is a significant contribution to $H_2(\omega)$ from a larger region, though, as it can be seen, the rather large negative value of $\delta a_2(\omega)$ (see Figure 6) is due mainly to the differences

Figure 7a. Dimensionless quantity $C(\omega) = (GM/R)^{1/2}$ for the frequencies indicated. The model with modified convection has been considered.

Figure 7b. Dimensionless quantity $H(\omega) = (GM/R)^{1/2}$ for the frequencies indicated. The model with modified convection has been considered.

Figure 8a. The same as figure 7a but for the model with constant hydrogen abundance.
in the uppermost layers. However, as Figure 8b shows, while for \( \nu = 3 \) and \( 4 \) mHz the integrated effect of the layers below \( r/R \sim 0.992 \) is small, for \( \nu = 2 \) mHz the region around \( r/R \sim 0.98 \), i.e. the second helium ionization zone, has an appreciable effect on \( H_2(\omega) \). By considering \( H_2(r) \) at more frequencies it can be seen that the contribution from this region to a large extent causes the oscillatory behaviour of \( \delta v_\mu(\omega) \) for low frequencies.

It is not easy to extract general conclusions from these two examples. However, it may be noticed that the differences in the sound speed and adiabatic exponent relative to Model 1 behave rather differently in the two cases. This is clearly reflected in the different behaviour of \( H_2(\omega) \). In particular it is likely that the oscillation \( \delta v_\mu \) at low frequency in Figure 6 is related to the substantial difference in helium abundance between the homogeneous model and Model 1.

4. REFERENCES

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FURTHER PROGRESS ON THE HELIUM ABUNDANCE DETERMINATION

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ABSTRACT

We report on further progress in attempting to determine the helium abundance in the solar convection zone by analysing the position and shape of the 'helium hump' in a thermodynamic quantity Θ which can be inferred from the sound speed in the vicinity of the He II ionization zone. At present we are estimating the sound speed from frequencies of high-degree solar oscillations by a differential asymptotic technique. The helium abundance Y is then determined by fitting the hump inferred to one obtained by interpolation in a grid of theoretical model envelopes. We have tested the procedure by carrying out a double-blind experiment on artificial data, and have found that accurate knowledge of the equation of state is essential for a useful determination. We have also carried out the procedure on real solar data, but we judge that the frequencies are at present too poorly determined to enable us to obtain a reliable estimate of Y in the sun.

1. INTRODUCTION

It has been argued (Gough, 1984; Däppen and Gough, 1984, 1986) that the helium abundance in the solar convection zone can be determined directly from frequencies of high-degree solar oscillations by measuring the depression in the bulk modulus of the gas brought about by the second ionization of helium. In the adiabatically stratified region of the convection zone (which contains the He II ionization zone) the first adiabatic exponent γ = (∂lnp/∂lnρ), and its thermodynamic derivatives are related to the sound speed c according to

Θ = \frac{1 - γ_ρ - γ}{1 - γ} = \frac{r^2 \, dc^2}{Gm \, dr} \equiv W \tag{1.1}

where r is a radial coordinate, γ_ρ and γ_ρ are the partial logarithmic derivatives of γ with respect to ρ and C^2 at constant C^2 and ρ respectively, m is the mass in the sphere r = constant, and G is the gravitational constant. The quantity Θ is a function solely of thermodynamic state variables; it takes the value -2/3 for a perfect monatomic gas with γ = 5/3, and it exceeds that value when the gas is in a state of partial ionization. The quantity W is determined entirely by a knowledge of c^2(r), the equations of hydrostatic support and a boundary condition relating ρ and ρ in the photosphere. Thus, a seismological determination of c^2(r) permits one to infer Θ(r), and in particular its deviation from -2/3 in the He II ionization zone. Evidently the magnitude of that deviation depends on the amount of helium present.

An attempt to develop a procedure for estimating Y was discussed by Däppen and Gough (1986). Sound speed was estimated from oscillation frequencies using an asymptotic procedure discussed by Gough (1986). Unfortunately, by the time c^2 had been differentiated to obtain W, the result was so noisy that we were chary of proceeding further. In this paper, we replace the absolute inversion for sound speed used previously, by the differential method discussed by Christensen-Dalsgaard et al. (1988), whose application to observed solar data is discussed in these proceedings by Christensen-Dalsgaard, Gough and Thompson. We find, using artificial data, that the method is potentially useful. However, we have failed to obtain a reliable estimate of the helium abundance of the sun, because the real solar data are apparently too noisy.

2. CALIBRATION PROCEDURE

The method is a calibration of solar envelope models. A rectangular grid of 9 envelopes with the solar mass, luminosity and radius and different helium abundance Y and mixing-length parameter α is constructed, and for each member of the grid the adiabatic oscillation eigenfrequencies of a set of five-minute p modes is computed. The differences between the sound speeds of the 8 models on the periphery of the grid from that of the central model is estimated from the oscillation frequency differences by the differential asymptotic inversion procedure of Christensen-Dalsgaard et al. (1988). From those differences and a knowledge of the sound speed in the central model, the sound speeds in the other 8 models can be inferred. From that the quantity W can be estimated for each model of the grid, and by interpolation (or extrapolation) it can also be estimated for any other values of Y and α. By matching...
W(τ; Y, α) with the function W inferred for the sun (or another model of unknown composition), the solar values of Y and α can thus be estimated. That estimate could then be improved further by computing a new grid model at the estimated values of Y and α, and iterating. (However, we have not yet carried out such an iteration.)

Since we are interested only in the values of W in the vicinity of the He II ionization zone, where M differs from the mass M of the sun by only about 10⁻²M, it is unnecessary to compute M explicitly. Instead, we work simply with

\[ W(\tau; Y, \alpha) = \frac{r^2}{GM} \frac{d^2}{dr^2} \]

which is obtainable immediately from \( e^2 \) without recourse to solving any differential equations.

We emphasize that the procedure outlined above is no more than a sophisticated calibration of envelope models, carried out by matching oscillation frequencies. Therefore to some degree it rests upon the validity of the theory used to calculate those models and their eigenfrequencies. Nevertheless, by comparing only that combination of frequencies that we believe is dependent predominantly on the region in which helium is undergoing second ionization, we hope to be able to eliminate the possibly larger discrepancies between the absolute frequencies of any of our theoretical models and those of the sun that arise from errors in the theory that have little or no bearing on the helium abundance. Thus our calibration should be more robust than the early naive comparison of k - ω diagrams (Gough, 1982). Moreover, it is certainly independent of all of the uncertain assumptions of the theory of stellar evolution that are required for determining the structure of the radiative interior, and upon which the calibration with low-degree modes (Christensen-Dalsgaard et al., 1980; 1981; Guenther et al., 1982; Sarajedini, 1988) depends.

In carrying out the calibration it is prudent always to use for the grid the same modes as are available for the sun. By so doing, it can be hoped that systematic errors in the inversion will largely cancel. Thus one might therefore wonder whether our use here of the differential inversion is really an improvement over the direct inversion used previously, because both inversions start from the same asymptotic formula. By inverting directly on frequency differences, most of the systematic error in the asymptotic formula and the procedure for inverting it is concealed, and the apparent advantage of the differential method might merely be cosmetic. In the absence of errors in the data, should not the two methods be equivalent? If one were calibrating another theoretical model computed in precisely the same way as the grid, the answer is surely yes, provided the iterations converge. But reality is different: because in practice there will never be a grid model that reproduces the data arbitrarily well (by virtue both of errors in the data and of the necessary approximations in formulating the theory for computing the grid), the explicit linearization of the asymptotic formula that is performed at the outset in the differential method does not commute with the operation of inversion. The tests performed by Christensen-Dalsgaard et al. (1988) suggest that at least for solar-type stars the differential method is more reliable.

It is important to realize that the role of the asymptotic inversion is only in defining the procedure whereby the frequencies of the sun are compared with the grid, and not in estimating the frequencies themselves. The eigenfrequencies of the grid models are computed numerically, to a precision at least as great as that set by the errors in the data. The importance of having a reliable inversion is in its ability to determine a combination of frequencies (whether it be linear, as it is in the procedure on which we are reporting here, or nonlinear, as it was previously) that measures the He II ionization zone yet which is insensitive to errors elsewhere in the models. Thus, for example, we were content with computing adiabatic eigenfrequencies on the grounds that nonadiabatic processes are important only in the superficial layers of the sun. We have not yet carefully assessed the errors that might have been introduced by so doing, though we are encouraged by the success of the direct asymptotic evaluation of W in measuring the depths of the conversion zones of three unknown theoretical models with different, and unspecified, superficial layers (Gough 1986).

3. TESTING THE METHOD

The procedure has been tested by calibrating theoretical models. In its final form the calibration has been carried out as a double-blind exercise in the following way:

Author A (in country C) constructed equilibrium model envelopes with the solar mass, luminosity and radius by inward integration from an optical depth \( \tau \) of \( 10^{-4} \). The atmosphere was assumed to be in hydrostatic equilibrium, and to satisfy the \( T - \tau \) relation of the Harvard-Smithsonian Reference Atmosphere (Gingerich et al., 1971). Beneath the photosphere the diffusion approximation to radiative transfer was employed, and the convective heat flux and Reynolds stress was computed from local mixing-length theory in the manner adopted by Baker and Gough (1979). Opacities were computed by cubic Lagrangian interpolation in the tables of Cox and Stewart (1970), and for the latest grid the Mihalas, Hummer and Dappen (in the following MHD) equation of state was used (Hummer and Mihalas, 1988; Mihalas et al., 1988; Dappen et al., 1988). The integration extended through about half the radius of the sun. A second-order accurate difference scheme was used on 1500 mesh points distributed according to a criterion limiting the variation of all dependent independent variables between consecutive mesh points. The information required for calculating linearized oscillation frequencies was then sent to Author B, who knew the parameters (Y, α) of the grid but not of the models to be calibrated.

Author B (in country D) then computed linearized adiabatic oscillations, ignoring pulsationally induced Lagrangian perturbations to the Reynolds stress. Since none of the modes was of low degree, the envelope equilibrium model was quite adequate for the task. The upper boundary condition was determined by matching the eigenfunction onto the causal adiabatic oscillation of a plane-parallel isothermal corona under constant gravitational acceleration at a temperature of \( 1.5 \times 10^6 K \). The lower boundary condition, which in all cases was well beneath the lower turning point of the mode, was obtained by matching onto an
asymptotic representation of the evanescent eigenfunction. Such care at the lower boundary is actually not necessary for the modes considered here, and indeed it was necessary to confine the computation of the highest-degree modes to only an outer portion of the envelope to avoid exponent underflow in the computer. Second-order accurate centred differences were used to represent derivatives in the numerical integration. About 1000 modes of degree \( \ell \) in the range \([40, 1300]\) and frequencies \( \nu \) satisfying \( 1.5 \text{ mHz} < \nu < 4 \text{ mHz} \) were computed for each model; the mode set contained all the modes with \( 40 \leq \ell \leq 100 \), and the values of \( \ell \) in steps of 10 for \( 100 \leq \ell \leq 1300 \). The frequencies of the models of the grid and of the models (of unknown \( Y \) and \( \alpha \)) to be calibrated were sent to Author C for analysis.

So as not to prejudice the intended calibration of the solar data, the parameters \((Y, \alpha)\) defining the grid were not divulged to Author C. Instead they were presented as two other parameters, \((u, v)\), say, which are linearly related to \((Y, \alpha)\). Author C knew that the relation is linear, but of course he was not told what it is. He was also provided with the sound speed of the central model of the grid, which is required for the differential inversions.

Author C (in country \( \Xi \)) then estimated sound speeds from the frequency sets provided, and from them computed estimates \( \overline{W}_0 \) of \( W_0 \) according to Equation (2.1). The extent to which he was successful can be gauged by comparing the actual values of \( W_0 \), illustrated in Figure 1, with the values \( \overline{W}_0 \) inferred from the frequency inversions, which are illustrated in Figure 2. Recall, however, that the final accuracy of the calibration is not directly related in a simple way to the accuracy of the inferred functions \( \overline{W}_0 \), since the calibration is carried out simply by matching oscillation frequencies alone, using only the curves in Figure 2.

From the grid of models a function \( \overline{W}_0(r; u, v) \) was defined by linear interpolation between the values \( \overline{W}_0 \) illustrated in Figure 2. The parameters \((u, v)\) for any model \( M \) to be calibrated were then obtained by minimizing

\[
E(u, v) = (r_2 - r_1)^{-1} \int_{r_1}^{r_2} \left[ \overline{W}_0(r; u, v) - \overline{W}_0(r) \right]^2 dr,
\]

(3.1)

where \( \overline{W}_0 \) is the function \( W_0 \) inferred from the frequencies of model \( M \). Typically, \( r_1 = 0.965R \) and \( r_2 = 0.985R \), where \( R \) is the radius of the sun. As is evident from Figures 1 and 2, the interval of integration includes most of the hump in \( W_0 \) caused by He II ionization.

Finally, Author C sent Author B his findings, who transformed \((u, v)\) to \((Y, \alpha)\) for communicating the result to Author A.
4. RESULTS OF THE TESTS

The first grid of models (grid 1) was that used by Däppen and Gough (1984) in their preliminary assessment of the feasibility of the method. The grid is composed of models with \( Y = 0.17, 0.22, 0.27 \) and \( \alpha = 1.4, 1.9, 2.4 \) in all combinations. The heavy element abundance \( Z \) was held fixed at 0.02. These models had been computed somewhat differently from the description in the previous section: a simple Saha equation of state had been employed, assuming all species to be in their ground states, and Reynolds stresses had been ignored. Mystery model 1 to be calibrated was the model used for the fitting by Däppen and Gough (1984), aside from the values of \( Y \) and \( \alpha \), and was computed under the same assumptions as the grid. Models 2 and 3 were computed in the manner described in the previous section, aside from the equation of state. They have the same composition. Model 2 has the equation of state of Eggleton, Faulkner and Flannery (1973). The equation of state of model 3 is that used by Berthomieu et al. (1980). It takes into account electron degeneracy, Coulomb pressure and an excluded volume term (these three things are of little importance in the solar convection zone), and contains internal partition functions of hydrogen and helium computed with the static screened Coulomb potential (though the heavy elements are still treated, consistently, with the simple Saha approximation). The functions \( \overline{W} \) inferred from the frequencies of the three models are illustrated in Figure 3; included, for comparison, is \( \overline{W}_0 \) of the central model of the grid.

The results of the calibrations are recorded in Table 1. Interestingly, the calibration of model 1 is actually slightly better than Däppen and Gough (1984) achieved with the exact functions \( W_0 \). The improvement is not significant and presumably results partly from the different range \((r_1 - r_2)\) of integration in Equation (3.1), and partly (and fortuitously) from what one would expect to be an inferior interpolation scheme. The success with model 2 is not so great, as was realised by Author C from the minimum value of \( E \). Unlike model 1, this model was not computed in the same way as the grid. We do not know how much of the residual misfit results from the different physics, and how much from possible numerical inaccuracies in the old grid.

The calibration of model 3 is very poor indeed. Even though \((Y, \alpha)\) is within the confines of the grid, the calibration via \( \overline{W}_0 \) was achieved by extrapolation. This emphasizes the importance of knowing the equation of state, which had already been realised by Däppen and Gough (1984) and by Däppen (1987) from comparisons of the exact functions \( W_0 \). Indeed, that is exactly what one would expect: what we measure is a variation in the adiabatic exponent \( \gamma \), which is related to \( Y \) directly through (derivatives of) the equation of state.

The final entry in the table is a calibration using a grid of models (grid 2) computed with the MHD equation of state. The heavy element abundance \( Z \) was again held fixed at 0.02. The test model 4 was computed with the same equation of state. We believe that numerical errors in these models are substantially less than many of the other errors in the model. The grid is somewhat finer than grid 1, as Author C realised from the eigenfrequencies, but we do not quote the parameters because they have not yet been divulged to Author C. (We appreciate that Author C now knows a single linear relation between \( Y, \alpha, u \) and \( r \).) The quality of the calibration is comparable with that of model 1, which like model 4 forms a homogeneous set with the grid to which it was fitted.
Figure 3. Functions $\tilde{W}$ inferred from the frequencies of the mystery models 1, 2, 3 (see text); included, for comparison, is $W$ of the central model of grid 1.

Table 1. Results of calibration tests; $E_{\min}$ is the minimum value of $E$.

<table>
<thead>
<tr>
<th>Model</th>
<th>Grid</th>
<th>actual</th>
<th>inferred</th>
<th>$E_{\min}$</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>1</td>
<td>0.235</td>
<td>0.233</td>
<td>2.02</td>
</tr>
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<td></td>
<td></td>
<td>2.00</td>
<td>2.02</td>
<td>$1.2 \times 10^{-3}$</td>
</tr>
<tr>
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<td>1</td>
<td>0.254</td>
<td>0.265</td>
<td>1.79</td>
</tr>
<tr>
<td></td>
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<td>$1.5 \times 10^{-2}$</td>
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<tr>
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<td>0.366</td>
<td>3.22</td>
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<td>3.22</td>
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<td></td>
<td></td>
<td>2.25</td>
<td>2.21</td>
<td>$3.7 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

6. FUTURE WORK

The obvious next step is to perform further tests on the sensitivity of the procedure to possible errors in the models. We should also study our fitting procedure, and possibly use some function other than $\tilde{W}$, if one can be found that is less sensitive to the most uncertain features of the model envelopes (such as $a$ and all the complicated physics it hides, and the other uncertainties in both the equilibrium state and its pulsations that are associated particularly with the upper boundary layer of the convection zone).

In our calibration tests to date, we seem to have been able to generate errors of a greater magnitude than the spread in values one finds in the literature from the calibration of evolved solar models. That is not to say our method is less reliable. Indeed, its strength lies in the fact that it rests on fewer unconfirmed assumptions of the theory of stellar structure, and therefore is less susceptible to serious systematic flaws. Evidently the method requires accurate knowledge of the equation of state, and for this reason we have generated a grid of models with what seems to be the best we can find. Our hope is that we shall refine the calibration procedure to the point where the equation of state is the most uncertain step. Then at least we shall know
exactly on what that calibration depends. Moreover, since we generate a function (such as $W_0$), and $Y$ is but a number, the data contain much more information than we set out to seek. Therefore we can hope to reverse the process to make a serious seismic investigation of the equation of state. Of course we shall also need better solar data, but we are always optimistic that that will be available in the not too distant future.

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MODELLING THE VARIATION OF SOLAR P-MODE FREQUENCIES

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ABSTRACT

We present a Green's function technique to study long-term variations of the Sun (with characteristic time scales from one month to millions of years). The method is the combination of two previous analyses carried out independently by Gough and by Dappen. Our study has been motivated by the recent observational progress on solar p-mode frequency variations.

Keywords: Sun, solar cycle, solar oscillation-frequency variations, solar radius variations, solar luminosity variations

1. INTRODUCTION

Helioseismology has grown to an age that allows serious studies of long-term p-mode oscillation-frequency variations. Though the situation is far from conclusive, there is an accumulation of results, indicating that low-degree p-mode frequencies have decreased by about 0.4 p.Hz during the period between 1981 and 1988 (Refs. 1-3). The situation is much less clear for possible solar radius variations. Solar diameter measurements performed at High Altitude Observatory between 1982 and 1986 are consistent with no radius change at all (Brown, private communication). However, Laclare (Ref. 4) does report a change of $\delta R/R$ of the order of $10^{-4}$. Even less clear is the situation for solar luminosity variations, but relative amplitudes of a few $10^{-3}$ are possible. Ongoing and planned space experiments to detect luminosity variations have been presented at this symposium (Refs. 5, 6).

Here, we present a Green's function technique that will allow to examine the effect of time-dependent localized disturbances on observable quantities such as oscillation frequencies, radius, luminosity and neutrino flux. Such localized disturbances could be, for instance, magnetic fields at the base of the solar convection zone.

We define specific disturbances of the form $\exp(\text{i} \omega t) \delta(r-r_0)$, representing a source at location $r_0$ acting periodically with a frequency $\omega$ ($\delta$ is the Dirac delta function). In the spirit of linear response theory, these functions contain, in principle, all information to compute the linear response of any source within the Sun, since any general (spatial and time dependent) source can be decomposed into elementary perturbations, and the response function of the general source is constructed from the elementary Green's functions by using the amplitudes of the same decomposition.

For technical reasons we restrict $\omega$ to values corresponding to periods that are longer than 1 month because of the assumed mixing-length formalism for convection. No restriction holds at the low frequency end: periods as long as the Kelvin-Helmholtz time of the Sun (about $10^7$ years) or longer can be treated within the formalism.

We compute observable quantities (frequency-, radius-, luminosity-, or neutrino-flux changes) as functions of $\omega$ and $r_0$, and for each of these observable quantity we also compute the phase shift with respect to the source. These Green's functions will become a valuable tool in the interpretation of future observed changes of solar oscillation frequencies, radius, luminosity, and perhaps neutrino-fluxes. The relative amplitudes of each of these quantities and, importantly, their phase relations (among themselves, and with respect to the solar cycle) will give us constraints on the possible location and strength of the physical mechanism responsible for their variations.

2. THE TWO PREVIOUS METHODS

2.1 Gough's study

Gough (Ref. 7.) computed the reaction of the entire Sun to a sudden change $\delta \ell$ in the mixing-length parameter $\ell = l H_p$ ($l$ being the mixing length and $H_p$ the pressure scale height). Disregarding the (dynamical) relaxation to the new hydrostatic equilibrium (attained after convective relaxation, i.e. after about one month), he computed the change of radius and luminosity during the transition to this new hydrostatic equilibrium. In this
context one often also considers the ratio \( W = \frac{\delta \ln R}{\delta \ln L} \), and Gough obtained \( W = 5 \times 10^{-4} \).

Gough's method is based on a modified nonadiabatic pulsation code. (Ref. 8). The method allows, in principle, to place the perturbations at any place in the Sun.

2.2 Däppen's study

Däppen (Ref. 9) introduced the Green's function type perturbations \( \exp(i\omega t)\delta (r-r_0) \) discussed in the introduction and computed the (stationary) reaction of the entire Sun to these perturbations. However, his analysis did not allow perturbations below the convection zone. This limitation was due to a simplified treatment of the central regions of the Sun. The high-frequency limit of these Green's functions confirmed the above-mentioned result by Gough.

3. SYNTHESIS OF THE TWO METHODS

We found that the two methods can be combined easily, choosing Gough's approach to compute Green's functions according to Däppen's definition. Indeed, the boundary value problem of nonadiabatic pulsations can be simply modified by inserting an additional constraint in the form of a discontinuous first derivative, imposed at the location in the Sun where we want to have the source of the Green's function. Thus modified the resulting pulsation equations have a solution for all values of \( \omega \) (which is now a simple parameter in the equations), and the eigenvalue nature of the problem disappears.

We have tested the ability of the modified nonadiabatic pulsation code to compute Green's functions by successfully solving the 1-D Schrödinger equation for the \( \delta \)-function-potential atom, whose exact solution is well known.

4. CONCLUSIONS

The result of this study is the formulation of a method to compute the response of observable solar quantities \( (p\)-mode frequencies, radius, luminosity, neutrino fluxes) to a wide range of perturbations inside the Sun. Observations of the variation of these quantities \( (e.g. \) during the solar cycle) will give indications on the seat and strength of the solar cycle.

We have begun to compute such Green's functions. The equilibrium model used is the Saclay standard solar model (Refs. 10, 11), which uses up-to-date nuclear reactions and opacities. Detailed results will be published elsewhere.

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ABSTRACT

The ratios between the amplitudes of p-modes measured simultaneously in the Na and K resonance lines are compared with the expected ratios for adiabatic waves at two different levels in the atmosphere. The results agree within errors with the energy decaying solution for the waves at the photosphere.

1. INTRODUCTION

Although the determination of p-mode frequencies has been the main goal of observed solar 5-minute oscillations, recently some important results about their amplitudes and phases have been obtained. These observational determinations are of major interest in order to establish the excitation and damping mechanisms and also the behaviour in the atmosphere of the p-modes. We mention the comparison between observed velocity and luminosity oscillations (ref. 1). Velocity observations in spectral lines formed at different heights in the atmosphere can be a good complement of such measurements. In this sense our note is concerned with the ratios between p-mode amplitudes of simultaneous observations in the Na and K Fraunhofer lines.

Although a full explanation of these data perhaps requires an elaborate theory we have compared, in order to illustrate the implications of the results, our data with the crude simple prediction from adiabatic evanescent waves in an isothermal atmosphere. This simplification can be justified as a first approach in the optically thin high atmosphere around the temperature minimum where the greater part of both lines are formed.

2. OBSERVATIONAL ANALYSIS

2.1 Observations

Observations were carried out from August 12 to September 16 1985 at the Observatorio del Teide using two independent resonant scattering spectrometers, one measuring Doppler velocities in the K Fraunhofer line (769.9 nm) and the other in the Na lines (589.0 and 589.6 nm). The data corresponding to these observations are the same as those used in ref. 2 and 3 where the apparatus, typical results of original data and p-mode spectra are shown. A detail of interest here is the magnetic fields used in each spectrometer for this observing season as they define the points being sampled on the solar lines. They were set at 1.8 and 1.3 kGauss for K and Na spectrometers respectively. The calibration constants (factors to convert amplitudes to velocities) obtained for these observations were $K_K = 2870 \pm 14 \text{ ms}^{-1}$ and $K_{Na} = 6350 \pm 190 \text{ ms}^{-1}$. We have considered here for each instrument a 36 day spectrum with a frequency range from 1.8 mHz to 5.8 mHz.

Two methods have been used to obtain the amplitude ratios between the two spectra. These are now explained in some detail.

2.2 Individual p-mode amplitude ratios

The frequency of each mode $(\nu_m)$ for degrees $l < 3$ has been determined by computing its centroid for the potassium spectrum using the method described in ref. 4. Once the centroid has been determined we estimate the amplitude of the mode for each spectrum as:

$$ r_m(\nu) = \frac{\sum_{j=1}^{k+1} A^2_{m,j}(\nu)}{(2k + 1)S^2_m(\nu)}^{1/2}, $$

where:

$$ A^2_{m,j}(\nu) = A^2_{m,j}(\nu) - N^2_m(\nu), \quad m = 1, 2. $$

Here $r$ is the spectral channel corresponding to the centroid. $2k + 1$ the number of channels in a fixed interval $2\Delta \nu$ around the centroid (typically $\Delta \nu = 2 \mu \text{Hz}$), $\nu_j$ the cyclic frequency of channel $j$, $A_{m,j}(\nu)$ the corresponding amplitude and $N_{m,j}(\nu)$ an estimate of the noise amplitude (this has been computed as in ref. 4). $m=1$ refers to the potassium spectrum and $m=2$ to that of sodium. $S_m(\nu)$ is an estimate of the ratio between the observed velocity and the r.m.s. velocity over the Sun’s surface computed as in ref. 4. Both instruments have slightly different $S_m(\nu)$ because the spectral lines used have different limb-darkening functions, the ratios $S_2(l)/S_1(l)$ are 1.01, 1.02, 1.04 and 1.09 for degrees $l = 0, 1, 2$, and 3 respectively.

In order to reduce the noise we have considered an average of the amplitude ratios of all the modes in a frequency interval (typically 300 $\mu \text{Hz}$). The errors have been estimated as the gaussian errors of the mean value within a given interval. We have also considered an additional error contribution from constant calibrations, added using a standard method for the propagation of errors. According to the values given above the induced relative error in the velocity ratios between both spectra is 0.03.

2.3 Direct ratios

By using the method explained above one cannot obtain values of the amplitude ratio close to the cut-off frequency. In order to get ratios at these frequencies we have computed the function:

where $\Delta f$, $f$, and $f_0$ are parameters to be specified and $\nu$ is the mean frequency for the channels considered. Conditions (4) and (5) with appropriate values of $f$, $f_0$ allow us to use, predominantly, channels corresponding to $p$-modes in eq. (3). The errors have also been estimated as the Gaussian errors of the mean values, taking into account the constant error coming from calibration constant determinations.

3. SIMPLE THEORY

Assuming an isothermal atmosphere for a classical ideal gas with constant first adiabatic exponent $\Gamma$, adiabatic non-propagating waves satisfy the equation:

$$\xi(r) = \tilde{a}_r e^\lambda \xi(r/H) + a_r e^{-\lambda} \xi(H/r),$$

where $\xi$ is the radial component of the radial displacement. If the scale height, $h$ is the height over the photospheric radius and:

$$\lambda = \frac{1}{2} \pm \frac{1}{2} \left(1 - \frac{w^2}{w_0^2}\right)^{1/2},$$

where $w_0$ is the cut-off frequency for the isothermal atmosphere. Here we have used the fact that we are dealing only with low degrees, so their dependence with height in the atmosphere is as the radial ones. From eq. (6) we have for the ratio between the radial components of the velocity amplitudes at two heights $h_1$ and $h_2$:

$$\frac{v(h_2)}{v(h_1)} = \frac{a_r e^{\lambda} \xi(h_2/H) + a_r e^{-\lambda} \xi(H/h_2)}{a_r e^{\lambda} \xi(h_1/H) + a_r e^{-\lambda} \xi(H/h_1)}.$$  

If $h_1/H, h_2/H$ and $w_0$ were known one could estimate the ratio $a_r/a_{2r}$. However to know $h_1$ and $h_2$ is not simple because the measurements are made over two average heights corresponding to the formation of the wing regions of the solar lines where the resonance laboratory lines lie. We consider here the case $a_r/a_{2r} << 1$ corresponding to an infinite atmosphere, where $p$-modes are pure evanescent waves. Under this circumstance one has:

$$\ln \frac{v(h_2)}{v(h_1)} = \frac{\Delta h}{2H} \left[1 - \frac{1 - \omega^2}{\omega^2_0}\right]^{1/2},$$

where $\Delta h = h_2 - h_1$. Although $\omega_0$ and $\Delta h/H$ are not precisely known they must be within reasonable limits.

4. RESULTS AND DISCUSSION

We first compare the two methods explained in subsections 2.2 and 2.3. In figure 1 the function $ln(v_h/V_1)$ is shown for both cases. It is clear that the direct ratios are systematically larger than the averages over individual $p$-modes. Although in eq. (3) we have not included a small term coming from $S_1(t)/S_2(t)$ the differences cannot be explained by including it. Perhaps the different averages involved in eq. (1) and (3) produce this systematic difference. Keeping this in mind, it is still of interest to use the results from eq. (3) because it contains values for high frequencies.

In order to compare the observational results with eq. (9) we have considered several values for $\omega$. For each one we take the value of $\Delta h/H$ that gives the smallest dispersion after neglecting the lower frequencies whose behaviour is not as expected and, of course, frequencies greater than the corresponding cut-off frequency. In figure 2 the observational points were computed using eq. (3) and three curves of parameters:

- $a_\nu = 4.9 mH$, $\Delta h/H = 3.60$
- $b_\nu = 5.2 mH$, $\Delta h/H = 3.44$
- $c_\nu = 5.5 mH$, $\Delta h/H = 3.80$

are shown for comparison.

As can be seen the three theoretical curves have only appreciable differences close to their cut-off frequencies. The large $\Delta h/H$ values required are partially due to the reduction of the observational data used here, as it has been pointed out above. The cut-off frequency can be estimated experimentally: by using the technique described in ref. 5 it has been obtained a mean value for the $p$-mode spectrum considered there ($K$ was used in those observations) of $\nu_c = 5.46 \pm 0.05 mHz$.

It seems that at least in the central frequency region the agreement is good. This is, as it is known, the region where $p$-modes have the greatest amplitudes and thus where there is higher reliability. The disagreement at the lowest frequencies is also present if individual modes are used as figure 1 shows; thus, if observational data have been carefully interpreted (ref. 4) at these frequencies. A large enough $a_\nu/a_{2\nu}$ (at least for low frequencies) would explain it but, given the assumptions involved in eq.(9), other reasons can cause this behaviour. The highest frequencies also do not follow eq.(9); $p$-modes could exist at those frequencies at least if one interprets the observational cut-off frequencies in this sense.

5. CONCLUSIONS

Eq. (6) with $a_\nu/a_{2\nu} << 1$ and $\approx k(T)$ (temperature minimum) is sometimes used to derive a boundary condition for computing the frequencies of standing waves from solar models. Thus, our results give some observational support for this boundary condition at least in most of the frequency range. However we do not claim this behaviour to be valid throughout the atmosphere and, also, the simple hypothesis used here can be inappropriate when looking for other effects in $p$-modes as for example their phase differences between luminosity and velocity (ref. 1) or their own frequencies.

6. ACKNOWLEDGEMENTS

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P-MODES IN NA AND K RESONANCE LINES

1.2

Figure 1: $\ln\left(\frac{v_2}{v_1}\right)$ versus cyclic frequency $\nu$. The $\square$ (connected by a continuous line) correspond to the method of subsection 2.2 and the $\circ$ (connected by a dot-dashed line) to the method of subsection 2.3 (eq. 3).

1.2

Figure 2: circles with error bars correspond to the observational data and the three discontinuous lines to the theoretical curves indicated in text.
SOLAR VARIABILITY AND CLIMATOLOGY

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ABSTRACT

The modern space geodetic techniques currently lead now to a determination of the Earth orientation with an accuracy of about 5 cm. Changes in the Length of day (LOD), which are associated with observed variations in the Earth's rate rotation may run from 1 year to centuries with amplitudes of many milliseconds. By analyzing the UT 1 data it is possible to get 10 frequencies, at least for the time interval used 1880-1984 (it is certainly not implied that these frequencies are of physical reality; the model must be regarded as a result of curve fitting).

On another hand, because planetary surface temperature is determined by the balance between absorbed solar and outgoing infrared radiation, it is not surprising that climate modelers have theorized that climate changes are driven by changes in solar luminosity S. Direct measurements of this parameter from above the atmosphere, have been available since early 1980 from the ACRIM instrument onboard the SMM satellite. Based on these data, a cluster of 10 periodicities can be found (same remark as precedently; the same number of 10 is fortuitous).

From the two data sets, it is easy to deduce the relationship between UT 1 and S, a quadratic curve being obtained. As a conclusion, it is suggested that for long periods, climate is influenced by astronomical cycles, and mainly by variations of the Earth's orbit (Milankovitch variables); for short periods, fluctuations in Earth rotation is dominated by atmospheric effects (changes in the net atmospheric rotation rate). Between these two fields, the sun activity cycle may affect the Earth, through the variation of the solar luminosity.

Keywords: sun, solar luminosity, solar activity, earth rotation, climate.

The desire to predict the weather for the next days is certainly of central importance to the humanity. In the past, Egyptian people built their first solar calendar by carefully observing the rhythms of the Nile's river floods. This may be considered as the first meteorological model, and the first step to understand the very difficult phenomena which occur in the solar-terrestrial system. Today, assessing the impact of a climatic change on the life of mankind leads to several strategies for societal action (including research). But if climate is of interest, if we presently know that climate shows significant variations on periods of time of the order of several decades, we are still ignorant of the real causes of these changes. What fraction of climatic variance is due to external forcing, what fraction is due to stochastic internal variations? The subject is still controversial, but variety and contradiction are valuable resources.

The motivation of this paper, which at first sight may seem a little odd looking face to the goals of this symposium, follows the recent findings of the cyclic solar diameter variations (Ref. 1). If so, theory predicts variation of solar luminosity and then, meteorological modelers can calculate the implications for the transient response of global Earth's temperature.

On another hand, it seems likely that changes in the average level of solar activity will have affected the Earth's spin over long periods of time. This was first stated by Chaiinnor (Ref. 2) when he published the results of his research on the changing length of day (LOD) and its relation to astronomical phenomena. As unpredictable abrupt changes in the LOD seems mostly driven by exchanges of axial angular momentum between the atmosphere and the solid Earth (Ref. 3) it appeared to us useful to compare
solar luminosity and Earth's motions, within a general scope of climatic changes.

2. IRREGULARITY OF EARTH'S ROTATION

In 1900, the international Latitude Service was established for the purpose of observing the Earth's polar motion. Since this date a great amount of data was collected through improved techniques and it is now proved that the rotation axis moves with respect to the Earth and that this motion has three components: a nearly circular annual wobble, a 14-months wobble (Chandler wobble) and a secular drift (polar wander).

Many investigations have confirmed the importance of air and water redistribution as the major causes of the annual wobble; a lot of work still remain to be done to explain the Chandler wobble, which is certainly a resonant motion of the Earth, but of unknown source of excitation. As far as the secular drift is concerned, recent studies have shown that it might be caused by deformations in the Earth's crust, according to global geomagnetic variations, which are indicative of motions in the core (Ref. 4).

There is now a general agreement on the fact that any changes in the circulation of the Earth's atmosphere cause fluctuations in the length of day (LOD), by exchanging angular momentum with the surface of the Earth (Ref. 5). If the Earth atmosphere is considered as a closed system, conservation of angular momentum, which is one of the basic principle of the physics, shows that any increase (decrease) of the angular momentum of the atmosphere must be accompanied by the decrease (increase) of the angular momentum of the Earth. Based upon these considerations, a lot of work has been made these last couple of years, thanks to the joint efforts in two main directions:

a) the unprecedented accuracy achieved by the techniques of modern space geodesy, such as satellite laser ranging, lunar laser ranging and very long base interferometry, and

b) the availability of regular and accurate determinations of the angular momentum of the atmosphere.

The first set of data is intensively analysed by the new International Earth Rotation Service (IERS) set up in Paris since 1988. The determination of the Earth's orientation in space (this term as defined to include the effects of Earth rotation, polar motion, precession and nutation) is now currently made with an accuracy of about 5 cm, that is to say about 2 milliarcseconds, which is roughly a factor of 5 to 10 times more accurate than data from older techniques (see for instance the analysis of the MERIT campaign, ref. 6).

The second set of data is provided by the following four meteorological centers: European Center for Medium Range Weather Forecasts (ECMWF), Japanese Meteorological Agency (JMA), the U.S. National Meteorological Center (NMC) and the United Kingdom Meteorological Office. A variety of atmospheric variables, including the local wind velocity vector, are estimated at each model grid point. The total angular momentum is estimated at twelve or twenty-four intervals (depending on the service used) from the appropriate integral of the grid point wind velocity and surface pressure estimates from the current update (Ref. 7).

Comparison of the two sets of data, which is given in Fig. 1 shows a high correlation, containing a large seasonal cycle, dominated by the annual term, and superimposed on this, an irregular 50 days oscillation, which is also found in zonal winds and other meteorological quantities (Ref. 8).

On a broader time scale, a spectral analysis performed through the time kept by the Earth, the UT1 data set (related to the LOD via the angular velocity of the Earth which is 7.292115 E-5 rad/s, ref. 9) reveals several periodicities of 0.137, 2.19, 2.83, 3.75, 4.9, 7.03, 8.56, 10.2, 12.2 and 14.8 years (Ref. 10).

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Figure 1. Atmospheric angular momentum (AAM) as estimated by the NMC (solid line, see text) together with the Length of day, LOD (dotted line). (Dicke et al. Ref. 7).

3. IRREGULARITY OF THE SOLAR LUMINOSITY

The solar luminosity is one of the most basic concepts for meteorologists because its value gives the amount of the solar energy output which reaches the upper layers of the Earth's atmosphere. Considering that the solar luminosity is constant, variations of the insolation (the effective solar
radiation incident on the Earth) by orbital changes as defined by the Milankovitch theory are now generally accepted as the fundamental basis for exogenetic control of climate changes on the time scale of 10,000 to 1,000,000 years (Ref. 11). On a more restricted period of time, a great number of authors reject all possibility of solar activity influence on weather (Ref. 12 - 14), essentially because links are based on statistically correlations that are regarded either as of not enough sufficient rigorous analysis or as of pure circumstantial effects (no plausible theory), or at last, as a pure fortuitous coincidence. But, if there is still many questionable conclusions, it seems from several independent sources of data that there may be a relationship between solar activity, solar luminosity and the climate, which can be resumed in this lapidary sentence: "the more sunspots, the milder may be the climate" (through a smaller diameter of the Sun, and an increased luminosity, ref. 15).

In the last decade, measurements of the solar constant on board satellites indicated that this parameter was increasing approximately 0.02 % per year, following a declining of about 0.03 % per year during the 1964 through mid-1986 period (Ref. 16). From the data set given in Ref. 17, it is possible to modelise the solar constant over a period of nine years, with fundamental periodicities of 4.3 years, 13.9 and 1.3 days. If we notice that it is impossible to find longer periods in the sample under consideration (no more than 1/4 th of the total length), these results are in good agreement with those deduced from the "Atlas of variations in the solar constant caused by sunspot blocking and facular emissions from 1874 to 1981", which are 95.5, 51.8, 24.4, 15.2, 11.2, 9.8, 8.2, 5.3, 4.2 years and 12 days (Ref. 18 - 19).

4 COMPARISON OF SOLAR ACTIVITY & EARTH'S SPIN

The spectral bands occurring in the power spectrum of the Earth's spin (UT1) can be clearly related to the periods deduced from the solar luminosity (Fig.2).

Obviously, it may be argued that the relationship presented is merely accidental, and this objection might arise because no physical explanation can be given. However, if we try to understand our solar system as a complete whole, our Earth is something as an internal body and the oscillations reflects the space-time organization.

Since the activity of the Sun is far from uniform from one cycle to an other one, its seems likely that changes in the average level of solar activity will have affected the earth's rotation over a period of hundreds or thousands of years. Historical reconstruction of past solar cycle is at this stage of discussion very important to be included in the astronomical theory of our terrestrial climate. Prospects for the future can include these mechanisms by which the climate system responds to the astronomically forced changes in the pattern of incoming solar radiation. These mechanisms are presently absent in the modelling climate variations, and may explain, for instance, the larger Pleistocene insolation found by people dealing with the Milankovitch theory (Ref. 20).

But the interest of such research remains in climate prediction, by using new sensing devices and observations, mainly concerning the solar luminosity and the solar diameter. As it is pointed out by Namias (Ref. 21), "the optimum results in this field of solar-wheather will be probably achieved by a marriage of statistical and observational techniques."

Figure 2. Periodicities deduced from the spectral analysis of the solar luminosity variations versus periodicities deduced from the spectral analysis of the Earth's rotation rate (UT1). It seems exist a break around 11 years. Below this value, fluctuations in Earth's rotation are dominated by atmospheric effects. Beyond, climate fluctuations may be solar driven.

5 CONCLUSION

We have investigated the possibility of finding a relationship between the variations of the Earth's rotation and the solar activity, through a comparative spectral analysis of homogeneous data sets (see also Ref. 22 - 24). The quadratic curve which seems to exist between the two phenomena, emphasizes possible significant analogies. The discrepancies can be probably attributed to some combinations of errors in the solar luminosity data. As a conclusion, it can be suggested that the short lived variabilities (periods less than 10 years) are related more to the general circulation and the longer ones to external forces. More precisely:
a/ for long periods, climate is influenced by astronomical cycles, and mainly by variations in the elements of the Earth's orbit (Milankovitch variables);  
b/ for short periods, fluctuations in Earth rotation is dominated by atmospheric effects (changes in the net atmospheric rotation rate);  
c/ between these two fields, the sun activity cycle and multiples of this cycle that have a clear origin in the Sun may affect the Earth, through solar luminosity changes.

5. ACKNOWLEDGEMENT

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6. REFERENCES


AN INVERSION METHOD BASED ON THE MOORE-PENROSE GENERALIZED INVERSE MATRIX

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ABSTRACT

We present an inversion method based on solving the algebraic equations by means of the Moore-Penrose generalized inverse matrix of inferring the solar internal rotation, and evaluate its validity.

Key words: Inverse problem, Solar oscillations, Solar rotation

1. INTRODUCTION

One of the most important aspects of helioseismology is the possibility of probing the solar internal rotation. Though the rotation is regarded as a key source of stellar and solar activities, the theory of the stellar rotation and its evolution has not yet been well established and we had not had a tool to measure the internal rotation of the Sun. Helioseismology provides us information of the solar internal rotation as a form of the frequency splitting of eigenmodes induced by the rotation. The frequency splitting due to the rotation is caused by the fact that those waves traveling in the same direction as the rotation have higher frequencies than those traveling in the opposite direction and then the degeneracy of frequencies in the azimuthal order is resolved. Roughly speaking, the amount of this splitting of frequency is proportional to some average of the rotation rate weighted by the eigenfunction of the mode, and it is dependent on the azimuthal order m. Since the form of eigenfunction is different from mode to mode, we can, in principle, infer the global structure of the solar internal rotation by measuring the frequency splitting of many eigenmodes. Mathematically speaking, the procedure of inferring the solar internal rotation from the frequency perturbation due to the rotation is to solve a set of integral equations, whose unknown function is the internal rotation of the Sun and the known function is the set of the frequency perturbations for various modes. A problem is how to solve this set of integral equations.

In this paper, we present a recipe to solve the integral equation based on the Moore-Penrose generalized inverse matrix and some preliminary results of numerical simulations performed to examine the validity of the method.

2. DISCRETIZATION

In this paper, we suppose the rotation rate is dependent only on the distance from the center r and independent of the latitude. In this case, the frequency perturbation is proportional to the azimuthal order m.

Then, the integral equation concerning the frequency splitting due to the rotation is, symbolically, given by

$$\Delta \omega_{n,m} = \int_0^\pi K_n(r) R(r) dr$$

where \(m \times \omega_{n,m}\) gives the frequency shift of the mode with radial order n, the degree l, and the azimuthal order m due to the rotation, \(R(r)\) is the solar internal rotation, and \(K_n(r)\) means the kernel which is dependent on modes. We replace the integral in this equation with the summation by discretizing the distance from the center r with N mesh points.

$$\Delta \omega_{n,m} = \sum_{j=1}^N \frac{\Delta r_j}{N} K_n(r_j) R_j$$

Let \(\mathbf{u} = \{\omega_{n,m}\}\) be a vector composed of a set of rotational splitting frequencies \(\{\omega_{n,m}\}\) Also let \(\mathbf{K} = \{K_n\}\) be a matrix composed of a set of kernels for \(\{\omega_{n,m}\}\) modes times \(\Delta r_j\), where \(j\) denotes the ordering number of discrete mesh points of the fractional radius, that is, the suffix of \(r_j\) is \(r_j = r_j / \Delta r_j\). Equation (2) is then written as

$$\mathbf{u} = \mathbf{K} \mathbf{v}$$

where \(\mathbf{v}\) is a vector composed of the rotational frequency at \(r_j\). The left-hand-side of equations (1) and (2) are observationally given. For the kernel \(\mathbf{K}\), if we have a good equilibrium model of the Sun, it is theoretically calculated. At the moment, the so-called standard model is sufficiently good to evaluate the kernel \(\mathbf{K}\) and we regard the kernels of equations (1) and (2) are known. Then the problem turns to how to solve algebraic equation (2).

3. THE LEAST SQUARES MINIMAL NORM SOLUTION

Let the dimension of \(\mathbf{u}\) be \(M \times 1\). Then the dimensions of \(\mathbf{K}\) and \(\mathbf{v}\) are \(M \times N\) and \(N \times 1\), respectively. The existence and uniqueness of solution of the algebraic equation (2) depends on the rank \(r \leq \min(M,N)\) of the matrix \(\mathbf{K}\) and, in the case of \(M > r\), on the consistency of the data \(\mathbf{u}\). We classify the following four cases:

(a) \(r = N\) and \(\mathbf{u}\) consistent
(b) \(r = N\) and \(\mathbf{u}\) inconsistent
(c) \(r < N\) and \(\mathbf{u}\) consistent
(d) \(r < N\) and \(\mathbf{u}\) inconsistent.

In the case (a), the solution is uniquely determined. No solution is given in the case of (b), but the least squares solution is uniquely determined. In the case of (c), there are infinite number of \(\mathbf{u}\)-satisfying equation (3). In the case of (d), there is no solution, while even the least squares solution is not uniquely determined.

The case of (c) and (d), some prior information is required to select a solution (case (c)) and a least squares solution (case (d)) among possible solutions. A well known treatment to do this is to choose the solution of minimal norm among infinite number of solutions or least squares solutions, that is, to choose \(\mathbf{u}\) which minimizes ||\(\mathbf{K}\)\(\mathbf{u}\||^2.

The solution thus determined is called "the minimal norm solution"
the case of (c) and \"the least squares minimal norm solution\" in the case of (d). An example of the adoption of the least squares minimal norm solution in the inverse problem of frequency splitting to infer the solar internal rotation and evaluate the prospect of this solution. To do so, we have theoretically calculated the frequency perturbation by assuming a rotational box and using a solar model. It should be noted here that the kernels \( K_i(x) \) of high order \( m \) in the case of frequency splitting are quite similar with each other near the outer part of the sun.

Besides this, because of round-off error in the course of numerical calculation, the set of equation (2) becomes slightly inconsistent and then the present case corresponds to case (d) among the above four categories. The eigenmodes we adopted are \( 1836 \) modes with \( 1 \leq l \leq 150 \) in the five-minute range. We set \( N = 250 \), that is, we divide the solar model by \( 250 \) shells. In evaluating the rank of the matrix \( K \), we adopt a parameter \( \epsilon \), i.e. we regard numbers smaller than \( \epsilon \) as zero to avoid getting practically meaningless principal values of \( K \). The example shown below in the case of \( M = 1836 \), \( N = 250 \), and \( \epsilon = 100 \). We do not take account of the surface boundary condition \( \Omega(x) = 1 \) or smoothness of \( \Omega \). The rotation law which we assume is a uniform rotation \( \Omega(x) = 1 \). The procedure to obtain the least squares minimal norm solution is described in the next section.

3. THE MOORE-PENROSE GENERALIZED INVERSE MATRIX

When the matrix \( K \) is not a regular square matrix, the inverse matrix of \( K \) is not defined and then some alternative definition of the inverse matrix is required. In general, a matrix \( K \) may be uniquely expressed by means of the so-called the singular value decomposition, in which \( K \) is written as

\[ K = U \Sigma V^T, \]

where \( U \) and \( V \) are unitary matrices of \( M \times M \) and \( N \times N \), respectively, and \( \Sigma \) is a diagonal matrix of \( M \times N \) whose diagonal components are composed of the singular values of the matrix \( K \):

\[ \Sigma = \begin{cases} \sigma_i & \text{if } 1 \leq i \leq r, \\ 0 & \text{otherwise}, \end{cases} \]

with \( \sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r \geq 0 \).

Here \( \sigma_i \) is the \( i \)-th singular value of \( K \) and \( \sigma_i \)'s are the singular values of the matrix \( K(K^T K)^{-1/2} \) and they are given to the square root of eigenvalues of the squares matrix \( K^T K \), where \( K^T \) denotes the transpose matrix of \( K \).

By introducing this singular value decomposition, we rewrite equation (5) as

\[ \Sigma \Omega = \omega, \]

where

\[ \Omega \equiv V^T \Omega \] (8)

and

\[ \omega \equiv U^T \omega. \] (9)

Let \( \Sigma \) be a \( N \times M \) matrix composed of

\[ \Sigma_{ij} \equiv \begin{cases} \sigma_i \delta_{ij} & \text{if } 1 \leq i \leq r, \\ 0 & \text{otherwise}, \end{cases} \] (10)

Then it can be shown that

\[ \Omega_{A_{\text{MIN}}} \equiv \Sigma^T \omega. \] (11)

is the least squares minimal norm solution of equation (3). As a consequence, the least squares minimal norm solution for \( \Omega \) is given by

\[ \Omega_{A_{\text{MIN}}} \equiv \Sigma^T \omega. \] (12)

since unitary transformation keeps the norm unchanged. The matrix \( K^T \) is known as the Moore-Penrose generalized inverse matrix of \( K \), and it is obtained, in practice, through the singular value decomposition (Ref 2).

4. LEAST SQUARES MAXIMAL SMOOTHNESS SOLUTION

The least squares minimal norm solution is simple, but it is not always suitable in the context of helioseismology. The requirement of the \"minimal norm\" is not based on some physical arguments but an a priori assumption. As seen in Fig. 1, the solution shows some unrealistic fluctuation around the true value. To avoid such unrealistic fluctuation, Phillips (Ref 3 4) introduced another alternative procedure in which a criterion for smoothness is satisfied for the solution. In their method, however, the solution is no longer one of the possible least squares solutions, and the weighting factors for the degree of smoothness, and for the least squares of the solution is arbitrary. Their method has been applied to the inverse problem of helioseismology by Jeffrey (Ref 5).

Here we introduce a new concept of the \"least squares maximal smoothness solution\", which satisfies both the conditions of the least squares and of the most smooth solution, and a mathematical procedure to obtain it. We measure the smoothness of the solution by

\[ S_\nu \sim \int |\partial^2 \Phi| dx \]

and try to find out the solution which minimizes \( S_\nu \) rather than the norm. Practically, the quantity \( S_\nu \) is rewritten in terms of a vector \( \Phi \) and it is evaluated by

\[ S_\nu \sim \| \Phi \|_0 \]

where the parenthesis means the inner product and \( \Phi \) is an \( N \times N \) hermitian matrix giving the numerical difference of the second order. It should be noted here that, in the case of (d) in section 2, the least squares solution is generally given by

\[ \Phi_{A_{\text{MIN}}} = \Phi_{A_{\text{MIN}}} + u, \] (15)

where \( u \) is a \( N \times 1 \) vector composed of

\[ u = \begin{pmatrix} u_1 \\ \vdots \\ u_N \end{pmatrix}. \] (16)

It is some prior information that determines the vector \( u \), and in the case of the least squares minimal norm solution, \( u \) is set to be \( 0 \). Instead of requiring \( \| u \| = 0 \), we impose \( u \), so that the solution is the most smooth, that is,

\[ \delta S_\nu/\delta u = 0 \] (17)
By setting this requirement, the final solution is not only the least squares but also the most smooth. After some manipulation, the condition (17) leads to

$$u = -QQ'_i \tilde{\Omega}_M \tilde{\Omega}_M$$  \hspace{1cm} (18)

where

$$Q = \begin{pmatrix} Q_{x, x} & Q_{x, y} \\ Q_{y, x} & Q_{y, y} \end{pmatrix}$$  \hspace{1cm} (19)

and

$$Q_i = \begin{pmatrix} Q_{x, x} & Q_{x, y} \\ Q_{y, x} & Q_{y, y} \end{pmatrix}$$  \hspace{1cm} (20)

and

$$Q' = V^T (D + D') W$$  \hspace{1cm} (21)

The least squares maximal solution given by equations (15) and (18) turns to the corresponding solution for $\tilde{\Omega}$ given by

$$\tilde{\Omega}_M = \tilde{\Omega}_M + V u$$  \hspace{1cm} (22)

which minimizes $S_M$ while minimizing $[\omega - \tilde{\Omega}]$.

Figure 2 shows the least squares maximal smoothness solution of equation (3) thus obtained. As seen in this figure, this solution is much smoother than the least squares minimal norm solution shown in Fig. 1.

From this numerical calculation, we conclude that the inversion method utilizing the Moore-Penrose generalized inverse matrix and requiring the maximal smoothness is quite effective.

6. REFERENCES

Solar models are calculated with the non-local form of mixing length theory, as proposed by Shaviv and Salpeter (Ref. 11), applied to the lower part of the convection zone. The resulting overshoot layer has a depth of 10 M\(_\odot\), this is about 30\% of the local pressure scale height. The parameters \( \alpha = L/H_\rho \) (mixing length / pressure scale height) and the initial He-abundance \( Y_0 \) are obtained by calculating a full model sequence in time to \( \alpha = 1.505 \) and \( Y_0 = 0.25637 \). The total convection zone has the depth \( d = 197 \) M\(_\odot\) at a temperature \( T = 2.166 \) MK compared to \( d = 181 \) M\(_\odot\) and \( T = 1.927 \) MK in the local case. This fact may have implications on the observed low \( L \)-abundance.

The influence of the overshoot layer on low-\( L \) modes shows an increase in the frequencies of the order of about \( 1 \mu \text{Hz} \), thus lowering the difference to the observed frequencies. For intermediate-\( L \) modes where the reflection boundary is just below the overshoot region the effect is strongest, up to about \( 4 \mu \text{Hz} \). High-\( L \) modes are slightly lowered by the overshoot effect. Nevertheless in all frequency ranges a discrepancy of several \( \mu \text{Hz} \) remains.

**Keywords:** convective overshoot \hspace{1cm} 5-min oscillations

**1. INTRODUCTION**

Solar oscillations are known to an accuracy better than \( 1 \mu \text{Hz} \), at least for low-\( L \) and intermediate-\( L \) \( p \)-modes (Refs. 3, 4). Theoretical calculations of oscillation frequencies carried out in Freiburg show the same behavior as the calculations of other groups, namely that for low-\( L \) modes with radial index \( n \leq 22 \) frequencies are about \( 5 - 10 \mu \text{Hz} \) too low compared to the measured frequencies. For intermediate-\( L \) modes the differences are even greater, whereas for high-\( L \) modes theoretical frequencies are too high. The reason for this should be found in the equilibrium model of the Sun, so a number of essential improvements were made in the past, e.g. the Los Alamos Opacity Library is now included into the evolutionary code as a standard, or the electrostatic correction in the framework of the Debye-Hückel theory is considered (Ref. 5).

Several authors found that a deeper convection zone may have the desired effect on the frequencies. Berthomieu et al. (Ref. 1) found that \( \alpha = L/H_\rho \) has a strong influence on the frequencies, and that increasing \( \alpha \) gives better agreement with the observations. Larger \( \alpha \) means a deeper convection zone. Noels et al. (Ref. 8) also confirmed that models with a deeper convection zone show better frequencies. But it is not quite clear to what improvement of their models this result should be attributed. In an evolutionary sequence the parameter \( \alpha \), or in other words the depth of the convection zone, is fixed by the constraint that the evolution should lead to the observed radius \( R \), and luminosity \( L \), of the present Sun. We therefore follow the idea to consider the overshoot layer at the bottom of the convection zone in order to extend the nearly adiabatic stratification deeper into the Sun. To obtain such an overshoot layer we employ a non-local version of the mixing length theory. For the upper part of the convection zone, including the Sun’s atmosphere, this has already been done by Ulrich and Rhodes (Ref. 13) who were interested in the effect on high-\( L \) \( p \)-modes. In the present paper we concentrate on the lower part of the convection zone where we may expect some effect on low- and intermediate-\( L \) frequencies.

**2. EQUILIBRIUM MODELS**

For the calculation of the equilibrium models we use a program originally developed by Kippenhahn et al. (Ref. 6). The overshoot layer at the base of the convection zone is included in the form of the non-local mixing-length formalism described by Shaviv and Salpeter (Ref. 11). This method was also applied by Pidatella and Stix (Ref. 9) to an envelope model and showed quite similar results as the more dynamically involved models of van Ballegooijen (Ref. 14) or Schmidt et al. (Ref. 10), in the sense that the depth of the overshoot layer is a few tenths of the pressure scale height. For our purposes the approach of Shaviv and Salpeter is entirely appropriate because we are mainly interested in the mean stratification of the Sun. The main simplification is that the radiative flux is treated in the diffusion approximation, so we restrict the application of this theory to the interior of the Sun.

The non-local calculation is essentially treated as outlined in the paper of Pidatella and Stix. We mention here only the convective heat flux

$$F_{\text{conv}}(r) = -f \varepsilon(r) p(r) c_p(r) \Delta T(r, r + A)$$

where $c_p$ and $p$ are respectively the specific heat and density. The temperature excess of a bubble or convective element $\Delta T(r, r + A)$ is obtained by integration of the difference of the temperature gradients $\nabla T - \nabla_{\text{adi}}$ over a mixing length $A$. $v(r, r + A)$ is the mean velocity of the convective element moving from $r + A$ to $r$. The filling factor $f$ is determined by the condition that $F_{\text{conv}}$ calculated from Eq. 1 in the lower part of the convection zone must fit $F_{\text{conv}}$ calculated from the local mixing-length theory in the upper part at an arbitrary chosen fitting point $r_{fit}$. In our calculation $f$ turns out to be 0.17. The main numerical results are listed in Table 1. Model 1 which includes the overshoot layer is compared to model 2 which everywhere uses the local mixing-length theory. We notice that the mixing length parameter $\alpha$ and the initial He-abundance is only slightly changed by inclusion of an overshoot layer.

Both calculations were done with a sequence of 25 models in time, the last one having an age of $4.64 \times 10^9$ years with an effective temperature of $T_{eff} = 5777.7 K$. The abundance of heavy elements was chosen to $Z = 0.017$. For the atmosphere the HSSRA $T - r$-relation is used. The opacity is taken from the Los Alamos tables and in the upper layers from Kurucz et al. (Ref. 7). In the equation of state the electrostatic correction due to the Debye-Hückel theory is considered. By the formula $F_{\text{rad}}/F_{\text{total}} = \varepsilon r p (1.4 d/H_p)$ we find the numerical result listed in Table 1 confirmed within 1%-accuracy (Ref. 9). This may serve as a test of the numerical calculation. Figure 1 shows the run of some variables characterizing the overshoot region.

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<th>model 1</th>
<th>model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>1.505</td>
<td>1.439</td>
</tr>
<tr>
<td>$\chi_0$</td>
<td>0.25637</td>
<td>0.25628</td>
</tr>
<tr>
<td>$r_{at}$</td>
<td>539.781 Mm</td>
<td>541.631 Mm</td>
</tr>
<tr>
<td>$r_{st}$</td>
<td>499.071 Mm</td>
<td>514.631 Mm</td>
</tr>
<tr>
<td>$d$</td>
<td>15.65 Mm</td>
<td>15.65 Mm</td>
</tr>
<tr>
<td>$d/H_p$</td>
<td>0.29</td>
<td>0.29</td>
</tr>
<tr>
<td>$b$</td>
<td>150 km</td>
<td>150 km</td>
</tr>
<tr>
<td>$D$</td>
<td>197 Mm</td>
<td>197 Mm</td>
</tr>
<tr>
<td>$\Delta T$</td>
<td>2.166 MK</td>
<td>2.166 MK</td>
</tr>
</tbody>
</table>

Table 1. Parameters of model 1 calculated with non-local mixing-length theory compared to model 2 calculated with local mixing-length theory. $\alpha$ is the ratio of the mixing length to the pressure scale height. $\chi_0$ is the initial He-abundance. At radii $r_{at}$, $r_{st}$ and $r_{sf}$ respectively, the superadiabaticity $\nabla T - \nabla_{\text{adi}}$, the mean temperature excess $\Delta T$ and the velocity $v$ become zero. (In model 2 these three points are identical). The depth of the overshoot layer is $d = r_{at} - r_{st}$. $D$ is the total depth of the convection zone, $T$ the temperature at this depth, and $b$ is the thickness of the boundary layer, i.e. the distance $r(F_{\text{rad}}/F_{\text{total}}) - r_{st}$.

The influence of the overshoot on the equilibrium variables is shown in Figure 2 where the ratio of temperature, pressure and density and the speed of sound for the two models is plotted. In and below the overshoot region the temperature is higher in model 1 than in model 2 by up to 1.7%, whereas throughout the remaining convection zone there is no change. The higher temperature is not entirely compensated by the lower density and so causes a higher pressure in this region, which in turn shifts some mass to higher layers in the Sun. Thus by the hydrostatic equilibrium the pressure, and also the density, is higher by about 1% throughout the convection zone. The speed of sound which is proportional to $\sqrt{T}$ shows the same behavior as the temperature. Only in and below the overshoot region there is an appreciable increase of the speed of sound which should shift the frequencies of those oscillations extending to this region.

![Figure 1](image1.png)

**Figure 1.** Overshoot layer: real, adiabatic and radiative temperature gradients $\nabla T$, $\nabla_{\text{adi}}$ and $\nabla_{\text{rad}}$. Temperature excess $\Delta T$ and mean velocity $v$, and the ratio of the radiative to the total heat flux $F_{\text{rad}}/F_{\text{total}}$, plotted against radius $r$.

![Figure 2](image2.png)

**Figure 2.** Ratio of equilibrium variables of model 1 to variables of model 2: temperature $T$, pressure $p$, density $\rho$ and speed of sound $c$, taken at the same radius in both models.
3. OSCILLATIONS

We determined the frequencies of p-mode oscillations with spherical harmonic index \( l \) in the low-, intermediate- and high-degree range for both models for comparison. A Henney technique is used to solve the linearized, adiabatic equations numerically. Initial estimates are obtained by a matrix method (Ref. 12). For \( l > 100 \) the Cowling-approximation is applied. The upper boundary condition is placed at an optical depth of \( r = 10^{-4} \).

Figure 3 shows an échelle diagram for the frequencies of low-degree modes \( l = 0, \ldots, 3 \). The overall appearance is that the frequencies are increased due to the overshoot by about \( 1 \mu \text{Hz} \). For low-\( n \) modes, where theoretical frequencies are still too low, the shift is in the right direction, indicating that a deeper convection zone indeed shows better frequencies, though the effect of an overshoot can not explain the whole difference to the measured frequencies. The fact that high-\( n \) mode frequencies are too high is independent from this consideration, because this can be attributed to the arbitrary cut-off at \( r = 10^{-4} \).

We also calculated high-\( l \) modes and obtained a decrease of frequencies by \( 1 - 2 \mu \text{Hz} \) for the overshoot-model. Perhaps this shift occurs because our equilibrium models showed slightly different values even in the upper layers of the Sun where the high-\( l \) modes are confined. But we think that numerical uncertainties in these layers are still too large to make a definite statement about this result.

4. CONCLUSION

Although the effect of an overshoot layer at the base of the solar convection zone can not explain the main discrepancies of theoretical to observed frequencies, the shift in frequencies of about \( 1 \mu \text{Hz} \) in general and up to \( 4 \mu \text{Hz} \) for some modes in the intermediate-\( l \) range is not small compared to the accuracy by which frequencies can be determined. Thus the overshoot should not be neglected in future models of the Sun.

REFERENCES


CONVECTION and P-MODE OSCILLATIONS

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ABSTRACT

We have simulated the upper 2.5 Mm of the solar convection zone using a three-dimensional, compressible, hydrodynamic computer code. Preliminary results show that convection excites p-mode oscillations. The frequencies of the modes in the numerical simulation agree well with the eigenfrequencies of our computational box calculated for the time averaged mean atmosphere. The agreement is excellent at low frequencies, and diverges at higher frequencies in a manner similar to the difference between observed and theoretical frequencies for the sun.

keywords: sun, convection, oscillations, p-modes, numerical simulation

1. OBJECTIVES

We address the discrepancy between the observed oscillation frequencies and the eigenfrequencies calculated for a spherically symmetric solar model (Ref 1). This discrepancy is primarily a function of frequency and is nearly independent of degree, ℓ, for small ℓ, which suggests the cause is confined to the outer layers of the solar cavity. A possible cause is interaction with convection. We investigate the effect of dynamic, inhomogeneous convection on p-mode frequencies. Our tool is a 3-D compressible (magneto-) hydrodynamic code for simulating the outer portion of the solar convection zone. We compare the oscillatory modes found in our numerical simulation (which we take to represent the actual sun) with the eigenmodes calculated for the time averaged mean atmosphere of the simulation.

2. NUMERICAL METHODS

Convection is a 3-D phenomenon, so we solve the hydrodynamic equations in three dimensions. Radiative energy exchange is crucial in determining the structure of the solar surface, so we include radiative transfer. The treatment is sufficiently elaborate to describe the sudden release of radiation, by ascending hot gas, in a thin layer at the solar surface. The upper solar convection zone is strongly stratified. Pressure and density vary by five orders of magnitude across our computational box. We increase the smoothness of the derivatives by replacing the gradients of density and pressure by the gradients of their logarithms. Hence, we solve the fluid equations in non-conservative form. Horizontal boundaries are assumed to be periodic. Top and bottom boundaries are as transmitting as we can easily make them while still preserving stability. The vertical heat flux is kept from drifting by specifying the internal energy of inflowing material at the bottom boundary. For a fuller discussion of our numerical methods see Ref 2.

We simulate a region 6 by 6 Mm horizontally, extending vertically from the temperature minimum (-0.5 Mm) to a depth of 2.5 Mm, using a grid of 63*63*63 points. The results presented here were obtained on the University of Colorado Alliant FX/8.

3. CONVECTION

Granulation is a surface phenomenon. Beneath the surface convection is organized on a large (meso) scale (figure 1). Ascending flow is hot, slow, and diverging. It is broad in extent, and nearly structureless. Descending flow is cool, fast and converges into disconnected, finger-like, downdrafts. Granules grow more readily in regions of upflow, and move towards locations of downdrafts, where granule growth is suppressed.

Fluid parcel trace plots show the position of all fluid parcels ascending through the plane z=0 (visible surface) at time t=0. Most of these parcels were also ascending at t=9 solar min. They ascend slowly, and originate from a small source volume. Only a small subset of ascending fluid at depth reaches the surface. At t=+9 solar min, most parcels have descended a substantial distance, and outline the downdrafts (figure 2).

4. OSCILLATIONS

Velocity oscillations for the vertically propagating (k=0) mode are illustrated in figure 3. The oscillation amplitude and growth rate are much larger in the modes of our computational box than in the sun primarily because our box is much shallower (2.5 Mm) than the solar cavity (710 Mm), so the simulation modes have much less inertia. The amplitude appears to grow in time approximately as (time)½, which suggests a stochastic excitation mechanism.

Figure 1: Temperature, vertical velocity, pressure and helicity on horizontal slices through the simulation domain at four depths. Note the change in morphology with depth.

Figure 2: Locations of fluid parcels ascending through the plane $z=0$ (visible surface) at time $t=0$ are shown both 9 solar minutes earlier and later.
Figure 3: Vertical velocity for the vertically propagating \((k=0)\) mode.

Figure 4: Power spectra of \(\langle \rho u_0 \rangle \) for modes with horizontal wavevectors \(k=0\) and \(1\) \(\text{MHz}^{-1}\).
5. OSCILLATION SPECTRUM

Resonant modes of the simulation's computational box are clearly revealed in the power spectra of the vertical mass flux, $p_{zxx}$, divided by the square root of the mean density. We show the spectra for modes with horizontal wave number $k=0$ (vertically propagating) and $k=1\text{ Mm}^{-1}$ (one horizontal cycle). The spectra at all depth points are superimposed. The mode eigenfunctions (depth dependence of the power spectra) are shown in the surface plots. The frequency spacing between modes is much larger than for the sun. Again, this is because our computational box is shallow.

6. COMPARISON OF SIMULATION MODES WITH MEAN ATMOSPHERE EIGENMODES

6.1 $k$-$\omega$ Diagram

The $k$-$\omega$ diagram compares the mode frequencies of the convection simulation with the eigenfrequencies of the same computational box with a plane parallel, static, mean atmosphere (obtained by averaging the simulation atmosphere horizontally and over time). Low frequency, vertically propagating (low $\ell$) modes have nearly identical frequencies in the simulation and mean atmosphere. For higher frequencies and larger horizontal wavenumbers (high $\ell$) the modes diverge, with the simulation modes (representing the sun) having lower frequency than the modes of the mean atmosphere (representing the spherically symmetric models used to calculate the global oscillation eigenmodes).

Figure 5: $k$-$\omega$ diagram for modes of the convection simulation and corresponding mean atmosphere.

6.2 $\Delta \nu$ vs. $\nu$

Figure 6 plots $\Delta \nu = \nu_{\text{simulation}} - \nu_{\text{mean atmosphere}}$ vs. $\nu$, which shows this trend clearly. It has a similar appearance to that found by Christensen-Dalsgaard et al. (Ref 1) for the difference between the observed and theoretically calculated mode frequencies.

7. CONCLUSIONS

These results indicate that the increasing disparity, with increasing frequency, between the observed and theoretical mode frequencies may be due to the interaction of the $p$-modes with the inhomogeneous, dynamic convection. We intend to evaluate this possibility by performing a simulation with a 10 Mm deep box, which will have a much richer spectrum. If the effect persists, we will explore the possibility of modifying the boundary conditions used in the global eigenmode calculations to represent the effects of convection.

8. REFERENCES

DETERMINATION OF THE SOLAR SOUND SPEED
BY AN ASYMPTOTIC INVERSION TECHNIQUE

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ABSTRACT

A new asymptotic inversion technique is developed using a weighted least-squares bicubic spline fit to the observational data. The accuracy of the asymptotic approach is discussed, and it is shown that the asymptotic theory is adequate to determine the sound speed in the solar interior between 0.4 and 0.9 R. The inversion of the data of Duvall et al. (1988) shows that the sound speed between 0.4 and 0.7 R is about 1 per cent greater than it is in the standard solar model of Christensen-Dalsgaard (1982); this result is in agreement with the previous asymptotic inversion of Christensen-Dalsgaard et al. (1985) and also that of Christensen-Dalsgaard et al. (1985). It is also consistent with the inversions presented by Gongh and Kosovichev (1985) using a kernel function approach.

Keywords: Helioseismology; inverse problem, the Sun: internal structure.

1. INTRODUCTION

The asymptotic inversion of p-mode frequencies presented by Gough (Refs. 1-2) is the simplest method to estimate the sound speed in the solar interior. It is based on Duvall's dispersion relation (Ref. 3):

$$
\frac{n + \alpha}{\omega} = F(L/\omega)
$$

where \( \omega \) is the frequency of a solar p mode, \( L = l + 1/2 \), \( l \) and \( n \) are respectively the degree and radial order of the mode, \( \alpha \) is a "phase shift" - a quantity which is a function of \( \omega \) and which depends predominantly on conditions near the surface. Harvey and Duvall (Ref. 4) have shown that the observed frequencies satisfy the relation (1) with an accuracy of about 1 per cent. In the asymptotic approximation the function \( F \) is considered as a differentiable function of \( n/\omega \). The sound speed can be found from a solution of the Abel integral equation (Ref. 4):

$$
\ln(r/R) = \frac{2}{\pi} \int_{r_0}^{r_0} \frac{dF}{dy} (y^2 - c_s^2)^{1/2} dy
$$

where \( c_s \) is the sound speed at the solar surface \( r = R \).

Therefore the determination of the sound speed is requires the evaluation of the first derivative of the function \( F \), which has been obtained from observational data.

A determination of \( F \) was first carried out by Duvall (Ref. 3) for constant value of \( \alpha \). The function \( F \) has been applied by Christensen-Dalsgaard et al (Ref. 5) to determine \( c(r) \) using equation (2). The inversion has shown that the sound speed in the radiative zone outside the solar core is about 1 - 2 per cent greater than it is in the standard solar model.

The frequency dependence of the phase shift \( \alpha \) can be determined simultaneously with the function \( F \) by making use of the known different functional dependences of these functions on \( \omega \) and \( l \) (Ref. 2). The functions \( \alpha(\omega) \) and \( F(y) \) have been fitted by smoothed piecewise linear functions (Ref. 2).

Brodsky and Vorontsov (Ref. 6) have suggested evaluating \( df/df \) by considering \( \omega \) as a two-dimensional function of \( n \) and \( L \) and using the derivatives \( \partial \alpha / \partial n \) and \( \partial \alpha / \partial L \) computed directly from the table of the observed frequencies by centered differences. They applied their approach to the inversion of the frequencies of p modes with 10 < \( l \) < 200 and did not find any differences from a standard solar model for \( r > 0.227R \). However, their method of numerical evaluation of the derivatives from the observational data may be unstable to observational errors, in particular for frequencies of high-degree modes which are measured with relatively low precision (Ref. 7).

In this paper I present a different approach, based on a bicubic least-squares spline fit to the parameter \( n/\omega \) regarded as a function of the two independent variables \( L/\omega \) and \( \omega \). My purpose was to estimate the accuracy of the asymptotic approximation and the asymptotic inversion, and to determine the sound speed from a new set of helioseismic data (Ref. 7).

2. BASIC FORMULAE

An application of short-wave asymptotic theory to the nonradial adiabatic oscillation equations yields, in the first-order approximation, the dispersive relation (Ref. 2):
Figure 1. Phase shift of the frequencies of model 1.

Figure 2. Relative differences of the exact frequencies of model 1 from the asymptotic approximation as a function of the radius of the turning point.

Figure 3a. Relative differences of the exact frequencies of model 1 from the asymptotic approximation in the solar core.

Figure 3b. Frequency corrections due to the perturbation of the gravitational potential.

Figure 3c. Frequency corrections due to the buoyancy.

Figure 3d. Frequency corrections due to the high-order terms in the asymptotic expansion.
3. ACCURACY OF THE ASYMPTOTIC APPROXIMATION

dept on observational errors (Ref. 9).

3.3. The asymptotic approximation should be a function of i* - but not I:

where N is the buoyancy frequency and r is radius of the turning point of the mode, (τ/ετ(r)) = L/ω). For high-frequency p modes ω² ≫ N², and therefore equation (3) can be rewritten (Ref. 2):

\[ \frac{\pi(n + \alpha)}{\omega} = \int_0^{r} \left( \frac{r^2}{c^2} - \frac{y^2}{c^2} \right)^{1/2} \frac{1}{r} \frac{N^2}{\omega^2} \frac{dr}{r} \]

or

\[ \frac{f(y, \omega)}{\omega} = F(y) - \frac{1}{\omega^2} \Psi(y) - \beta(\omega) \]

where

\[ f(y, \omega) = \frac{\pi n}{\omega} \]

\[ F(y) = \int_0^{r} \left( \frac{r^2}{c^2} - \frac{y^2}{c^2} \right)^{1/2} \frac{1}{r} \frac{N^2}{\omega^2} \frac{dr}{r} \]

\[ \Psi(y) = \frac{1}{2} \int_0^{r} N^2 \left( \frac{r^2}{c^2} - \frac{y^2}{c^2} \right)^{1/2} \frac{dr}{r} \]

\[ \beta(\omega) = \frac{\pi \alpha}{\omega} \]

All terms on the RHS of equation (5) have different functional dependences on y and ω, and therefore derivatives of F, Ψ and β can be determined separately (Refs. 2, 6):

\[ \frac{dF}{dy} = \frac{\partial f}{\partial y} + \omega \frac{\partial^2 f}{\partial \omega \partial y} \]

\[ \frac{d\Psi}{dy} = \omega^3 \frac{\partial^2 f}{\partial \omega \partial y} \]

\[ \frac{d\beta}{d\omega} = -\frac{dF}{\beta \omega} + \frac{2}{\omega^3} \Psi(y) \]

The application of the natural parameters of the asymptotic theory (n/ω) and (L/ω) simplifies the calculations of the derivatives.

A numerically stable method of evaluating the partial derivatives of the function f(y, ω) is provided by least-squares fitting of a bicubic spline to the observational data (Refs. 8-9). The method employs products of B-splines to represent the bicubic splines. The positions of the B-spline knots are used to control the smoothing process, which is dependent on observational errors (Ref. 9).

3. ACCURACY OF THE ASYMPTOTIC APPROXIMATION

Before solving the inverse problem one has to estimate how well the p-mode eigenfrequencies may be represented by the asymptotic approximation. In particular, the phase shift α should be a function of ω but not I:

\[ \alpha(\omega) = \frac{\omega}{\pi} \left| F(y) - \frac{1}{\omega^2} \Psi(y) \right| - n. \]

The RHS of equation (6) was computed using ε(r) and N²(r) of the solar model 1 of Christensen-Dalsgaard (Ref. 10) and the frequencies were calculated from the exact adiabatic pulsation eigenvalue problem. The results are shown in Figure 1. The phase shift can be represented by a single curve only for the p modes of intermediate degrees 10 < l < 200. There are large deviations for the low (l < 10) and high (l > 200) modes. I calculated a least-squares spline fit to the phase shift for the intermediate-degree modes, and the relative difference of the exact frequencies from the asymptotic approximation.

\[ \phi = \frac{(\nu - \nu_{as,N=0})}{\nu}, \]

where \( \nu = \omega/2\pi \) is cyclic frequency and \( \nu_{as,N=0} \) is the eigenfrequency calculated from equation (1). Figure 2 shows \( \phi \) as a function of the radius of the turning point. The asymptotic approximation is not valid in the solar core \( r < 0.3R \), nor near the surface at \( r > 0.9R \). Moreover, a smaller deviation is seen at the lower boundary of the convection zone \( r \approx 0.7R \). The accuracy of the asymptotic approximation is better than 0.5 per cent between 0.4R and 0.9R. Therefore in this region one can hope to estimate the sound speed using the asymptotic inversion technique.

The deviations from the asymptotic theory near the top and near the bottom (in particular, in the helium and hydrogen ionization zones) are evidently caused by small-scale variations of the sound speed. Near the centre of the Sun the first-order asymptotic approximation is inaccurate, due to perturbations of the gravitational potential, buoyancy, and high-order terms of the asymptotic expansion. The relative contributions from these effects were computed separately and the results are shown in Figures 3a - 3d. The main deviation from the asymptotics comes from the perturbations of the gravitational potential. The corrections due to buoyancy have values of the same order of magnitude as the high order terms, but their signs are different. Therefore the influence of buoyancy is partially eliminated by the high-order terms. Consequently to determine the buoyancy frequency the high order corrections to the asymptotic approximation should be considered.

4. THE ESTIMATION OF THE SOUND SPEED

The asymptotic inversion (2) has been applied to determine the sound speed for 0.4 < r/R < 0.9. Figure 4 shows the results of the inversion of the adiabatic eigenfrequencies of model 1 (solid line), as compared with the original solar model (dashed line). The relative difference between the inverted solar model and the original one is presented in Figure 5. The accuracy of the asymptotic inversion is about 0.2 per cent for 0.4 < r/R < 0.95. The deviation of about 1 per cent is seen near the lower boundary of the convection zone. Similar properties of the asymptotic inversion were established by Christensen-Dalsgaard et al (Ref. 5, figure 3).

Figure 6 shows results of the inversion of the frequencies of the solar 5-min oscillations (Ref. 7). The relative difference between the squared sound speeds the inverted model and model 1 is presented in Figure 7. The vertical bars show the mean-square deviations when artificial gaussian noise with standard deviation equal to the estimated amplitude of the observational errors is added to the frequencies.
Figure 4. Results of the inversion of the frequencies of model 1.

Figure 5. The relative difference between the inverted model 1 and the original model 1.

Figure 6. Results of the inversion of the observed frequencies.

Figure 7. The relative difference between the inverted solar model and model 1.
5. CONCLUSION

The sound speed in the solar interior between 0.4 and 0.7 R is about 1 per cent greater than it is the model 1. The result is in good agreement with the previous inversion (Ref. 5) and with recent inversions of frequency differences in linear approximation (Refs. 11-12).

The first order asymptotic inversion technique provides reliable estimations of the sound speed in the outer part of the radiative zone. However, it appears that the higher-order terms of the asymptotic expansion need to be considered in the inversion procedure to determine the structure of the solar core, and stratification near the bottom of the convection zone and near the surface.

6. ACKNOWLEDGEMENTS

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The Effect of a Nonspherical Sound Speed on the Acoustic Frequency Spectrum of the Sun

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Abstract

We study the effect on the solar acoustic frequency spectrum of a localised deviation from spherical symmetry concentrated near the outer convective zone of the sun. The local asymmetry is modelled by an angular-dependent contribution to the sound speed (Eq. 2.3). Calculations of the acoustic frequencies in the range 1500 to 4000 µHz have been made for a nonspherical deformation of relative amplitude α varying from 0 to 0.1. For α > 0.02 all acoustic frequencies of this range are strongly and irregularly displaced (quantum chaos). For lower values of α, only a small fraction of the frequencies are irregularly shifted. The deviation of the frequency spectrum of the deformed solar model with respect to the spectrum of a spherically symmetric model is measured by a relative scatter σ(ω). The relative scatter of the current observed solar frequencies with respect to the frequencies of a standard spherically symmetric solar model is found to correspond to an amplitude of the disturbance α = 0.025.

Our results are indicative that even slight local asymmetries influence the precise positions of the frequencies. They suggest the possibility of a diagnosis of nonspherical effects in the structure of the sun.

1. Introduction

The observed solar acoustic frequencies in the 3.5 mHz range are reported to be precise to one part in 10³ while theoretical models can reproduce these frequencies to within one part in 10⁴ only. The discrepancies between the observed frequencies and the frequencies of the currently favoured theoretical (spherically symmetric) solar models can presumably be reduced by incorporating better physics. However, a possible difficulty spotted in a recent inversion may be indicative that not all of these discrepancies can be accounted for by radial correction effects (Gough and Kosovichev 1988). On the one hand the inversion of the observed frequencies shows that the central density of the sun should be higher by about 10%. On the other hand, if the chemical composition as derived from the inversion is used to reconstruct a solar model, then a preliminary trial has given a lower central density. This apparent inconsistency does not arise if one modifies the symmetry over small scales. This problem can be resolved by solving the eigenvalue problem for a large number of solar parameters, are required to generate frequency families which cannot be characterised intrinsically by a set of 3 quantum numbers. In particular the frequencies define what has been referred to as the quantum chaotic part of the spectrum.

So far these theoretical results have been tested numerically for a globally deformed stellar model only (Nejad and Perdang 1988), and for the lower part of the frequency spectrum where the geometric theory is not strictly applicable. It was found that in the low part of the spectrum — the 25 lowest modes — very strong deformations of the star, of the order of 1/10 in the dimensionless deformation parameters, are required to generate quantum chaotic effects. It is the purpose of the present paper to extend the analysis to localised deformations of the equilibrium model which leave the global structure spherically symmetric over large enough scales but which modify the symmetry over small scales. This problem can be formulated in a mathematically straightforward way which enables us to solve the eigenvalue problem for a large number of solar frequencies (about 500 modes) in the observationally relevant frequency interval of 1500-4000 µHz.

2. The Model

We consider an equilibrium configuration in which the sound speed, denoted by c(r,z), is nonspherically distorted according to the following law:

\[ c(r,z)^2 = \left[1 + \alpha r \right]^2 \left[1 + \alpha \left(\hat{r} \cdot \mathbf{e}_\theta \right) \right]. \]

\[ f(p,\theta,\epsilon) = \sum_n C_n(r) \left(\hat{r} \cdot \mathbf{e}_\theta \right)^n \]

where \( C_n(r) \) is the sound speed of a spherically symmetric reference model, and the profile of the deformation, \( f(p,\theta,\epsilon) \), is given as an expansion in spherical harmonics. We shall assume that the coefficient functions \( C_n(r) \) differ from zero over a small radial
Fig. 1. The frequencies of the range 2000-2050 μHz as functions of the strength $c$, for a deformation $L = 10$

Fig. 2. The frequencies of the range 3000-3050 μHz as functions of the strength $c$, for a deformation $L = 10$

Fig. 3. The frequencies in the interval 3043-3048 μHz over a small range of strengths $c$, for a deformation $L = 10$

Fig. 4. The frequencies of the range 4000-4050 μHz as functions of the strength $c$, for a deformation $L = 10$

Fig. 5. The dependence of the variance $\sigma^2$ on $c$ (natural logarithms). For $c < 0.02$ (first mark) $\sigma$ is proportional to $c$; for $c > 0.07$ (second mark) we can approximate the dependence by a power law $\sigma = a c^k$, $k = 3$

Fig. 6. The frequencies of the range 2100-2300 μHz as functions of the strength $c$, for a deformation of large degree $L = 100$
interval only, stay over the range $r_{\text{g}} < r < r_{\text{f}}$ ($r$ is the radius of the star).

For a fixed normalisation of the profile (2.2) the parameter $c$ is a measure of the strength of the nonradial deformation. Interpreting $r_g$ and $r_f$ as the radial positions of the bottom and the top of the solar outer convection zone, we can think of our representation (2.1) as simulating any effect on the dominant factor specifying the adiabatic frequencies of the stellar model due to local asymmetries in the convection zone. The physical nature of these asymmetries is irrelevant at this stage. For instance the asymmetry might be caused by a stationary circulation pattern in the zone. Or it may be related to magnetic effects.

In order to keep our algebraic treatment simple enough we shall consider the case of a single component in the sum (2.3) which we represent in the form

$$f_{\text{nl}'}(r) = \int \frac{1}{r^2} \frac{\partial}{\partial r} \left[ \frac{\partial}{\partial r} \left( r^2 \frac{\partial v}{\partial r} \right) \right] dr.$$  

In this expression $\sigma$ stands for the dimensionless radial position $x = r/R$. The radial dependence of the asymmetry is then modelled by a Gaussian of relative width $\Delta$, centred at position $r_g = x_g R$. The angular dependence is specified by the spin-normal harmonic $Y_{\ell m}(\theta, \phi)$. By choosing $\Delta$ small enough, $\sigma$ sufficiently large, $\Delta$ does not perfectly satisfy our condition of vanishing at the levels $x_g$ and $x_f$, but it can be made to obey these requirements to a high degree of approximation.

We have solved the equation of the asymmetric acousto-thermal modes in the form given in Gough (1958), which is appropriate for the observed sound acceleration in the range we are interested in. We further disregard the effect of the buoyancy force which does not significantly influence the effect we are investigating here.

In the expression $v$ is a scalar function related to the displacement $\sigma$ by

$$\nabla^2 v = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial v}{\partial r} \right).$$

Since our perturbation in the sound speed $s = \sigma < 0.09$ is not (positively) affected, the boundary conditions remain identical with those of the conventional radially symmetric asymptotic eigenvalue problem (cf. Gough 1956).

Accordingly the eigenfunctions $v_{\text{nl}}(r, \theta, \phi)$ of the radially symmetric reference eigenvalue problem $(\sigma = 0)$ define a basis for the representation of the eigenfunctions of our nonsymmetric problem. Any eigenfunction $v(\theta, \phi, s)$ of our new eigenvalue problem can then be expanded as

$$v(\theta, \phi, s) = \sum_{\ell m} \sum_{n} \sum_{m'} r_{\text{nl}} Y_{\ell m}(\theta, \phi) Y_{\ell m'}(\theta, \phi).$$

where the summations extend over all modes. Therefore the original partial differential eigenvalue problem is reduced to the following algebraic eigenvalue problem

$$\sum_{\ell m} \sum_{n} \sum_{m'} \left[ \left( \nabla^2 + \phi \right) r_{\text{nl}} Y_{\ell m}(\theta, \phi) Y_{\ell m'}(\theta, \phi) - c^2 \right] \lambda_{\text{nl}} r_{\text{nl}} = 0.$$  

By introducing a new eigenvalue parameter $\lambda = \lambda_0 c^2$

$$\lambda_0 = 10^2$$  

the agebroic eigenvalue problem transforms into a standard matrix eigenvalue problem

$$(A - \lambda E) \mathbf{x} = 0.$$

In the latter expression $E$ is the unit matrix, $A_{\text{nl}m}$ are the matrix elements of $\lambda_{\text{nl}} m \ell (r_{\text{nl}} Y_{\ell m}(\theta, \phi) Y_{\ell m'}(\theta, \phi)$, $A_{\text{nl}m}$ is a $3 \times 3$ matrix of the form

$$\lambda_{\text{nl}} m \ell (r_{\text{nl}} Y_{\ell m}(\theta, \phi) Y_{\ell m'}(\theta, \phi) = \int \frac{1}{r^2} \frac{\partial}{\partial r} \left[ \frac{\partial}{\partial r} \left( r^2 \frac{\partial v}{\partial r} \right) \right] dr.$$  

The first integral is a standard angular integral of a product of three spherical harmonics which is known to obey classical selection rules. The second integral over the radius of the star involves functions in the radial position alone. In the present problem the angular and radial parts are thus seen to be independent. This contrasts with the general asymmetric acoustic eigenvalue problem for nonspherically deformed stars, in which the angular and radial parts do not decouple, and therefore a time consuming double (or triple) integration must be carried out numerically (Kuhn 1988; Nejad and Pendrington 1988). The computation of the matrix (2.12) of the present problem is numerically straightforward. The eigenvalue equation (2.10) can finally be solved by a standard matrix eigenvalue routine if we approximate the actual infinite-dimensional matrix by a matrix of a reasonable size $N$. We have chosen a dimension $N$ of about 500 in all of our actual calculations.

We wish to stress that the method of solution of the eigenvalue problem (2.4) we have adopted is an exact solution procedure. The only approximation we have made consists in truncating the infinite matrix $A^{\text{nl}m}$ in Eq. 2.10. This truncation introduces maximum errors of order $c^2$ in the lowest eigenvalue, and in the highest eigenvalue $\lambda$ of the matrix $A^{\text{nl}m}$.

Kuhn (1986) has recently investigated a more specific nonradial eigenvalue problem which is related to ours. The assumption on which his treatment is based is that the temperature profile in the sun is latitude-dependent, thereby producing a latitude-dependence in the sound speed. Kuhn then addresses the problem of the inversion of the nonradial correction to the temperature profile using the observed splitting data of the frequencies. To this end he considers the first order perturbations for frequencies as induced by the nonradial effects. As transpires from our numerical results, a first order perturbation may not be sufficient for a proper analysis of this question.

3. Numerical Results

We have performed a systematic investigation of our eigenvalue problem for the following set of parameter values:

$$c = 0.0; \quad \lambda = 0.05; \quad L = 10 \text{ and } 100; \quad M = 0; \quad M = 0.$$

The role of the strength $c$ has been studied by letting this parameter change over the range $0 \leq c \leq 0.1$.

In most of our experiments we have set $M = 0$; for reasons of numerical convenience. In fact, for this specific perturbation the selection rules imply that the matrix elements of $A^{\text{nl}m}$ are independent of $M$. Accordingly the matrix eigenvalue problem is split up into an infinity of separate eigenvalue problems, each one corresponding to a different value of $m$. Therefore the azimuthal number now remains an exact quantum number of the spectrum of the perturbed model. This property reflects the existence of the integral of the ray equations related to the axial symmetry of the perturbed model (cf. Pendrington 1988). Since the modes whose frequency corrections we are concerned with are typically characterised by rather high values of $l \times n$, a perturbation $M \neq 0$ would require us to deal with prohibitively large matrices if we wanted to take account properly of the effect of the coupling of modes of different $l$ values. The decoupling of the modes leads to an enormous gain in the numerical labour. We are well aware however that the price we have to pay is that we are dealing with a less realistic perturbation profile.

A preliminary experiment has been performed for a perturbation $L = 10$, $M = 5$, keeping just the modes $(l,n) = (39,3)$ and $(16,5)$ of a standard solar model (model 1 of Christensen-Dalsgaard 1982) in the expansion (2.7). With these modes in the range $1504-1509 \mu$Hz in the nonperturbed asymmetric spectrum, our matrix $A$ acquires already a dimension 146. For a strength of the asymmetry $c = 0.05$ we find that the separation in $m$ is always at least an order of magnitude less than the frequency shift due to the interaction of frequencies of different $l$. These results seem to be indicative that the frequency shift can be separated from the splitting. We are well aware, however, that the present experiment involving just 2 frequencies is not a full guarantee that we can extrapolate its result to the case of a large number of interacting frequencies of different spherical degrees.

Our main numerical experiments refer to a deformation of spherical degree $L = 10$. The results are summarised in Figs. 1-4. All of our calculations for the perturbed configuration are performed for modes of azimuthal quantum number $m = 0$ only. The nonperturbed model is model 1 of Christensen-Dalsgaard (1982). Our expansion of the eigenfunctions (2.7) takes account of $512$ modes of degree $3$ to $35$ and of order $24$ lying in the frequency range 1512 to 4236 $\mu$Hz. Fig. 1 shows the dependence on the strength $c$ for the frequencies in the range 2000-2050 $\mu$Hz. For a strength less than $0.01$ these frequencies are seen to be approximately linearly shifted. However, in the higher frequency range 3000-3050 $\mu$Hz (Fig. 2) we observe a curvature in the plots of the frequency against strength, which is manifest already at low strengths $0.005 \sim 0.01$. More specifically this is seen from Fig. 3, an avoided crossing occurs around at the very low strength $c \approx 0.003$ for the pair of frequencies close to 3045 $\mu$Hz. Finally, Fig. 4 displays the upper end of the set of computed frequencies (4000-4200 $\mu$Hz), again the existence of a curvature, and the presence of avoided crossings is encountered at low strengths $c \approx 0.01$. 

\[ \]
The first conclusion we can draw from these experiments is that a linear perturbation theory for the calculation of the frequency shifts ceases to be valid beyond a strength of 0.01 for a localised disturbance of spherical degree 10. Even for the lowest strengths we have investigated there may be an essential contribution to the frequencies which are not dealt with by the linear perturbation theory. To account for the latter phenomenon at least a two-mode interaction scheme is required, of the type first considered by Wigner and von Neumann (1929).

In the second place, we notice from Figs. 1, 2 and 4 that for a strength exceeding 0.02 avoided crossings become a generalised phenomenon. All frequencies are found to participate in avoided crossings; for most of the frequencies avoidances occur several times and at virtually unpredictable values of the strength parameter. In other words, within the regime the frequencies are very sensitive to the precise value of the perturbation parameter. The latter property is a characteristic feature of what has been termed quantum chaos in the literature. Our experiments indicate that the spectrum of the currently observable solar frequencies would be quantum chaotic if local asymmetries of spherical degree around 10 and strength > 0.02 were to exist in the sun.

We characterise by a single parameter the change in the frequencies due to the local symmetry perturbation, measuring the scatter of the frequencies with respect to the frequencies of a spherically symmetric model. To this end we adopt the relative variance \(\sigma^2(\varepsilon)\) of the frequencies \(\omega_p(\varepsilon)\) of the disturbed model with respect to the corresponding frequencies \(\omega_p(0)\) of the spherically symmetric model (\(\varepsilon = 0\)):

\[
\sigma^2(\varepsilon) = \frac{1}{N} \sum \left( \omega_p(\varepsilon) - \omega_p(0) \right)^2 / \omega_p(0)^2 \quad 3.2
\]

where the summation extends over all the N (about 500) modes we have computed. As discussed elsewhere the identification of the order and degree \((n, l)\) of the modes of the nonspherically symmetric problem is nontrivial; the quantum numbers \((n, l)\) of spherical symmetry cease to be quantum numbers once the spherical symmetry is broken. These numbers can however be used as pseudo quantum numbers which are naturally defined in our case by continuity with respect to the strength \(\varepsilon\) of the general procedure of defining pseudo quantum numbers. Perdang (1988). Since any pair of our set of frequencies, when regarded as functions of the strength, cannot to intersect, we have a simple algorithm of assigning pseudo quantum numbers to the frequencies of the deformed sun: We order the set of frequencies of the non deformed model \((\varepsilon = 0)\) in a sequence of increasing values. We likewise order the frequencies of the deformed model \((\varepsilon > 0)\). Any two frequencies of same rank (1st, 2nd, ...., Nth) in the two sequences are then assigned the same pair of quantum numbers \((n, l)\).

The variance (3.2) includes the following different effects. On the one hand it measures a systematic shift in the frequencies, \(\omega_{sys}(\varepsilon)\), roughly accounted for by a shift in the first order perturbation theory. New effects include in principle the contribution due to the occasional avoided crossings; but since these shifts are rare, their effect is washed out in the averaging procedure, and can therefore be disregarded. Finally it includes the contributions of the generalised avoided crossings in the quantum chaotic regime. For reasons which will transpire below, we shall write the effect of the latter as modulating the systematic contribution, so that our variance takes the form:

\[
\sigma^2(\varepsilon) = \omega_{sys}(\varepsilon) + \omega_{mod}(\varepsilon) \quad 3.3
\]

Since Figs. 1, 2, 4 show that the quantum chaotic regime sets in for \(\varepsilon > 0.02\), the systematic effect alone enters in the lower strength range \(\varepsilon < 0.02\) where \(\omega_{sys}(\varepsilon)\) then reduces to 1. Fig. 5 illustrates this point. A frequency less than 0.02 (the dispersion \(\varepsilon(\varepsilon)\) is in fact proportional to \(\varepsilon^2\) is one would infer from strict linear perturbation theory. The average proportionality remains actually meaningful over a range much larger than the range of actual validity of linear perturbation theory. This behaviour just translates the fact that the second order correction effects \(\omega_{sys}(\varepsilon)\) to the frequencies \(\omega_{sys}(0)\) with both signs in an essentially random fashion; this is indeed borne out in our Figs. 1, 2, 4 showing curvatures of both signs in the low-strength range; the average contribution of order \(\varepsilon^2\) then cancels out, so that only correction terms in \(\varepsilon^4\) will affect the variance. Accordingly the chaotic correction is such as to reduce the systematic variance \(\omega_{sys}(\varepsilon)^2\) or \(\omega_{sys}(\varepsilon^2)\) 3.1. This is due to the property that avoided crossings lead to interchanges in the identification of the modes; the latter operate in a way as to bring corresponding frequencies closer together. The effect of this reduction is actually clearly visible in Fig. 5 (strength range indicated by marks).

A different and perhaps more satisfactory representation of the variance consists in separating out a trend \(\omega_{trend}(\varepsilon)\), which is described by a smooth linear or quadratic fit of the data:

\[
\sigma^2(\varepsilon) = \omega_{trend}(\varepsilon) + \omega_{chaos}(\varepsilon) \quad 3.4
\]

Departures from this trend are then interpreted as chaotic corrections \(\omega_{chaos}(\varepsilon)\). The total variance can then be represented by

\[
\sigma^2(\varepsilon) = \omega_{trend}(\varepsilon) + \omega_{chaos}(\varepsilon) \quad 3.5
\]

When applied to our numerical results this latter formulation indicates that chaotic correction terms dominate for \(\varepsilon > 0.05\).

As a next point we have attempted an estimate of the variance of the observed solar frequencies for the range considered in our numerical experiments. This variance has been computed by the following formula:

\[
\omega_{trend}(\varepsilon) = \frac{1}{N} \sum \left( \omega_p(\varepsilon) - \omega_p(\varepsilon^0) - \omega_p(0) \right)^2 / \omega_p(0)^2 \quad 3.6
\]

where \(\omega_p(\varepsilon^0)\) stands for the observed solar frequency identified by its order and degree, and \(\omega_p(\varepsilon^0)\) is the corresponding frequency of a good standard solar model (of spherical symmetry) (model 1 of Christensen-Dalsgaard 1992). With the observed frequencies given in Duerr et al. (1988) and Libbrecht and Kaufman (1988) we find \(\omega_{trend}(\varepsilon)\) to be 1.1 x 10^-5; incidentally, if we include the full list of observed frequencies (about 2100), we find a higher value (2.9 x 10^-5) which remains however of the same order of magnitude. It is clear that our estimate (3.6) does include a contribution of strictly nonradial effects \(\omega_{nonrad}\). But it necessarily includes also contributions \(\omega_{rad}\) to the frequencies which are due to purely radial physical phenomena and which so far are not properly treated in current models. Furthermore the published observed frequencies are dealing with data which have already been smoothed with respect to the pseudo quantum numbers. Unfortunately, any such smoothing procedure has the effect of partially or completely wiping out the effect of fluctuations in the frequencies. \(\omega_{trend}(\varepsilon)\) due to localised avoided crossings and, more important, to generalised avoided crossings (quantum chaos). Accordingly, the true solar variance, \(\omega_{solar}\), is made up of the following terms:

\[
\omega_{solar} = \omega_{trend} + \omega_{rad} + \omega_{nonrad} + \omega_{mod} \quad 3.7
\]

where the contribution between square brackets is to be compared with the theoretical variance (3.2). In other words, the actual solar value we are entitled to compare with the values \(\sigma^2(\varepsilon)\) pictured in Fig. 5 is 1.1 x 10^-5 + \(\omega_{mod}(\varepsilon)\). The fluctuation component and the radial correction component are so far unknown, so that an accurate estimate of \(\varepsilon\) is manifestly not possible.

We may however consider two presumably extreme cases. On the one hand, let us assume that the radial contribution of a localised avoided crossing is about the same order of magnitude as the fluctuation term, and the fluctuation term is less than 10^-6. Then there still remains a nonradial contribution of 10^-6, which corresponds to a strength \(\varepsilon > 0.05\). Even with this large strength avoided crossings are to be expected, so that significant nonradial effects should be observable in the frequency spectrum. On the other hand, if we assume that 50% of the observational value is due to the nonradial contribution and that the fluctuation term is likewise of the order of 10^-6, then the corresponding strength becomes 0.035. Such a value lies well within the nonlinear range of the \(\varepsilon\) against \(\varepsilon\) dependence and signals quantum chaos at least in the higher frequency section (3000 mHz).

The perhaps most significant quantitative test of our experiments is that local nonradial effects of a relative amplitude \(\varepsilon > 0.005\) in the sun produce local deviations of individual frequencies from the expected positions in the observed acoustic spectrum - in addition to a general regular shift of the different frequency families. For nonradial effects which are just one order of magnitude higher, quantum chaos manifests itself in the highest part of the spectrum.

The above series of experiments refers to a deformation of degree \(L = 10\) (Eq. 2.3). We have carried out a similar experiment for a perturbation of degree \(L = 100\). We show here only the behaviour of the frequency spectrum in the range 2100-2500 mHz as a function of the strength \(\varepsilon\) (Fig. 6). The novel feature we observe is that the collection of frequencies \(\leq 2255\) mHz are all smoothly modified by the effect of the perturbation. Such families of frequencies are quantum regular (cf. Perdang 1988). Their existence is related to the survival of several families of regular acoustic rays under smooth deformations of the spherical symmetry. Notice that within the regular family we observe two isolated families of avoided crossings. For frequencies \(\leq 2255\) mHz quantum chaos is again found for higher values of the
The strength of the deformation. The $a^2 c$ dependence is here due to the three orders of magnitude less than the $a^2 c^2$ dependence. This experiment illustrates that the degree of the deformation strongly influences the correction effect on the frequencies, as is indeed obvious from the structure of the perturbation matrix M (Eq. 2.12: of the role of the angular integral).

We feel that deformations of a degree not too different from $c$ are the most efficient in modifying the observable part of the solar spectrum. For low degrees, the selection rules imply that most angular integrals vanish, while for high degrees the integrals are always small.

4. Conclusion

The general result of the present numerical investigation is that the non-spherical localized deformation required to produce a quantum chaotic part in the observable spectrum of solar and stellar oscillations is lower than one would have expected. In our admittedly restricted sample of experiments adapted to the 5-minute oscillations of the sun the phenomenon is found to occur for a strength of the deformation, $c$ (Eq. 2.1), exceeding 0.02. Above this threshold we notice that typically large groups of modes are interacting among themselves generating a complex pattern of avoided crossings. The individual frequency shifts become comparable with the average spacing. Linear perturbation theory is found to be totally inadequate for the calculation of the corrections at the lower threshold in the strength of $c < 0.01$. Linear perturbation theory cannot be applied indiscriminately to all modes. Avoided crossings seemingly can occur at arbitrarily small levels of deformation.

Regarding the specific problem of solar oscillations, our analysis is relevant on two accounts. In the first place, we have shown that the relative variance of the observed frequencies in the range (1500 - 4000 $\mu$Hz), $\sigma^2 = 3 \times 10^{-4}$, is of the same order as the relative variance of the computed frequencies of a deformed model, $\sigma(c)$, if the strength of the deformation is taken as $c = 0.025$. We cannot draw a strong conclusion from this numerical result. On the one hand the observed frequencies have been processed by a smoothing technique, so that the true observational relative dispersion is certainly larger than $3 \times 10^{-4}$. Alternatively, part of the observational dispersion can be accounted for by radial adjustments in the models, so that the variance to be interpreted by nonradial deformations is less than the true observational variance (cf. Eq. 3.7). Therefore, we cannot draw a reliable qualitative estimate of $c$ from the currently available observational data. As discussed above, however, it seems likely that $c > 0.005$.

In the second place, we have demonstrated in this paper, that any slight local asymmetry may produce observable frequency shifts in the asymptotic part of the spectrum (say for frequencies $> 2000 \mu$Hz). These shifts may be non-smooth functions of the (pseudo-)quantum numbers $n$ and $l$ of the sphere, as a result of avoided crossings between groups of modes of different spherical degrees (cf. Perdang 1986, 1988). Accordingly, in order to secure the preservation of these possible physically relevant irregular spacings in the frequencies of the real sun, we strongly advise the observers to avoid any smoothing procedures of the observed frequencies with respect to the quantum numbers. In the light of the present results there is no theoretical basis for an a priori requirement of smoothness of the frequency distribution in $n$ and $l$. The latter is guaranteed under conditions of strict spherical symmetry only.

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Session 7

Observations and techniques for asteroseismology

Chairman: J.W. Harvey
ABSTRACT

Five rapidly oscillating Ap (roAp) stars have shown evidence of multiple p-mode pulsations in their light or radial velocity variations for which estimates of their fundamental frequency spacing $\Delta \nu_0$ can be made. These stars and the Sun have ratios of $\Delta \nu_0/\nu$ which fall near 0.025 and 0.04. Since $\Delta \nu_0/\nu$ should be roughly proportional to $1/(n + \ell/2)$, it is possible that there are common selected overtone regimes for the pulsations of the roAp stars and the Sun.

Keywords: rapidly oscillating Ap stars, p-mode oscillations, asteroseismology

1. INTRODUCTION

The techniques of asteroseismology hinge on the detection of p- (or g-) mode eigenfrequency spectra from which parameters like $\nu_0$ and $D_0$ [Ref. 1] can be measured. Searches of bright solar-type stars for Doppler and light oscillations like those of the Sun have yet to yield unambiguous detections [e.g., Ref. 2]. This is not surprising given expected oscillation amplitudes of order cm/s and micromag. At present, the only main sequence stars other than the Sun to show oscillations consistent with high-overtone acoustic modes are the rapidly oscillating Ap (roAp) stars [Ref. 4]. Although their photometric and velocity amplitudes are at least 3 orders of magnitude greater than those of the Sun's "five-min" oscillations [Ref. 10], they appear to conform to the same general theory of p-mode pulsation.

Interpretation of stellar p-mode eigenfrequency spectra has been guided by the asymptotic approximations calculated by Tassoul [Ref. 12]. These have led to expressions of the form [Ref. 1]:

$$\nu_{n,\ell} \approx \Delta \nu_0 (n + \ell/2 + \epsilon) - D_0 [\ell (\ell + 1) - \delta] + \cdots$$  \hspace{1cm} (1)

such that

$$\Delta \nu_0 = [2 \int_0^R \frac{dr}{c^2}]^{-1}$$  \hspace{1cm} (2)

$$D_0 \propto \frac{c(R)}{R} - \int_0^R \frac{dr}{c}\frac{dc}{dt}$$  \hspace{1cm} (3)

where $c$ = sound speed, $\epsilon$ and $\delta$ = constants dependent upon the stellar structure. For the Sun, standard solar models predict $\Delta \nu_0 \approx 135 \mu$Hz and $D_0 \approx 1.5 \mu$Hz [Ref. 3]. Therefore, to first order, solar p-modes of alternating $\ell$ should be approximately equally spaced in frequency, as is in fact observed.

2. p-MODE SPECTRA OF roAp STARS

Two of the roAp stars, HD 24712 [Ref. 7] and HD 60435 [Ref. 9], exhibit at least 6 oscillation frequencies with roughly equal spacing. Three others, HD 201601 [Ref. 8], HD 203932 [Ref. 6] and HD 166473 [Ref. 5], each show at least 3 frequencies which are consistent with a simple p-mode pattern. The amplitude spectra of these five stars are plotted schematically in Figure 1.

For comparison, the solar oscillation spectrum is also shown in the first panel of Figure 1. Because roAp stars are observed in integrated light, the modes represented in their amplitude spectra are of order cm/s and micromag. At present, the only main sequence stars other than the Sun to show oscillations consistent with high-overtone acoustic modes are the rapidly oscillating Ap (roAp) stars [Ref. 4]. Although their photometric and velocity amplitudes are at least 3 orders of magnitude greater than those of the Sun's "five-min" oscillations [Ref. 10], they appear to conform to the same general theory of p-mode pulsation.

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For comparison, the solar oscillation spectrum is also shown in the first panel of Figure 1. Because roAp stars are observed in integrated light, the modes represented in their amplitude spectra are of low $\ell$ (likely $\ell \leq 2$). Therefore, the solar oscillation spectrum used here is also based on measurements in integrated light (by the ACRIM radiometer aboard the SMM satellite [Ref. 13]), treating the Sun as a distant star.

From such spectra, one can both determine the (amplitude weighted) mean frequency $\langle \nu \rangle$ of the envelope of oscillation peaks, and estimate the fundamental frequency spacing $\Delta \nu_0$. These quantities are listed in Table 1. The pulsation spectra of most roAp stars are less extensive than their solar counterpart; as a result, $\langle \nu \rangle$ for the roAp stars in Table 1 can be measured to a precision of better than 3-5%. On the other hand, the Sun's broader spectrum and flatter distribution of power introduces larger uncertainty into $\langle \nu \rangle$. Even so, this is probably no greater than about 15%.

Also, in the case of the roAp stars, there can be an ambiguity in selecting the spacing between adjacent frequencies as $\nu_0$ or $\nu_0/2$. However, the first choice usually implies that the star is well evolved from the main sequence [Refs. 9, 11], conflicting with other evidence for the evolutionary status of Ap stars. The lower values of $\Delta \nu_0$ for the roAp stars relative to the Sun is consistent with the larger radii (and hence, longer sound travel times) of these stars.
FIGURE 1. Schematic amplitude spectra of the rapid oscillations observed in the Sun [Ref. 13] and the roAp stars HD 60435, HD 24712, HD 201601 (ι Equ), HD 203932 and HD 166473 [Refs. 9, 7, 8, 6 and 5 respectively]. All frequencies and amplitudes have been derived from photometric observations except those of HD 201601, for which radial velocity data were also used. (Notes: Horizontal scale varies from panel to panel. The amplitudes shown for HD 166473 are approximate.)
3. DISCUSSION

To first order, equation (1) predicts that

$$\Delta \nu_0/\nu_n \approx \left( n + \ell/2 + \epsilon \right)^{-1}$$

Since the Tassoul approximation is valid only in the regime where $n \gg \ell$, and $\epsilon$ is a small constant, this means that the ratio $\Delta \nu_0/\nu_n$ should be roughly proportional to $(n)^{-1}$.

Values of $\Delta \nu_0/\nu_n$ for the Sun and the five roAp stars are also given in Table 1. Three of the stars - including the Sun - have ratios of $0.040 \pm 0.002$ (implying $n \approx 25$), while the other three fall at $0.025 \pm 0.001$ ($n \approx 40$). The uncertainties in each individual ratio, based on the precisions of $\nu$ quoted in Section 2 and uncertainties in $\Delta \nu_0$ of about 1 $\mu$Hz or less, are typically about $\pm 0.002$.

With the exception of HD 60435, none of the roAp stars appear to have long-lived non-radial p-mode spectra with more than about 4 consecutive overtones. HD 60435 has shown evidence for as many as about 15 overtones, but most of the lower overtones are transient and of low amplitude [Ref. 9]; this star's oscillation spectrum is also dominated by modes with an overtone range of 4 or 5. The Sun's photometric oscillations encompass only 9 - 10 consecutive overtones. The two groupings of "mean" overtone implied by Table 1 are separated by much more than the overtone ranges of the individual stars, particularly for the roAp variables.

3.1 Caveats

(a) **Small-number statistics.** With only six stars in the available sample, the apparent dichotomy in $\Delta \nu_0/\nu$ could be a random result. A few additional variables may be added to the sample in the next few years, which may serve to support/reject this result, but it is unlikely that we will have a statistically large sample for some time to come.

(b) **Non-uniform period distribution of the roAp stars.** Among the 13 identified roAp variables, there is a tendency for periods [frequencies] to fall close to 6 and 12 min [2.8 and 1.4 $\mu$Hz]. This would produce a bimodal distribution of $\Delta \nu_0/\nu$ if $\Delta \nu_0$ is fairly constant from star to star. Four of the roAp stars discussed here have periods 6 and/or 12 min. However, one (HD 166473) has an intermediate period, and the Sun ($P \approx 5$ min) is grouped with the longer-period oscillators.

It is possible that the tendency for period clustering in the roAp stars is actually a consequence of overtone selection in these pulsators.

(c) **Ambiguity in $\Delta \nu_0$.** As mentioned in Section 2, there is an inherent ambiguity of a factor of 2 in the choice of $\Delta \nu_0$, depending on whether the oscillation spectrum has modes of only even(odd) degree or both even and odd $\ell$. Evolutionary arguments [Refs. 9, 11] and observations of mode splitting by rotation [e.g., Refs. 4, 7, 9] can often resolve the uncertainty. Also, for the stars with only three observed modes, the constraints on frequency spacing are certainly less rigid.

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4. REFERENCES


RAPID PHOTOMETRY OF THE $\delta$ SCUTI VARIABLE 63 HER

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ABSTRACT

The $\delta$ Scuti variable 63 Her has been observed during 21 nights at two observatories (Observatorio del Teide, Tenerife Island, Spain and Observatorio Astronomico Nacional de San Pedro Martir in Baja California, Mexico). Six frequencies of pulsation have been unambiguously detected, among which the fundamental radial mode, two non radial pressure-like modes and probably three gravity like modes.

Key words: Star, $\delta$ Scuti, asteroseismology

INTRODUCTION

Among known variable stars, the $\delta$ Scuti stars present a number of interesting features. The mechanism of excitation of the pulsations (the $\kappa$-mechanism, ref 1) is understood for a number of years. However their position near the main sequence goes together with a complexity of the observed spectrum of frequencies (ref 1).

Evolutionary effects have a strong influence on the distribution of the Brunt-Vaisala frequency with depth in the central regions of the stars and one expects that some of the observed frequencies carry information on the stellar core. On the other hand, non linear coupling between the excited modes of pulsation is not yet as severe as it is in earlier type variables. However only a few modes have been recognized in the best studied stars of this class (see for example ref 2 or ref 9).

We have observed the known $\delta$ Scuti variable 63 Her together with a F2V star HD 155543 considered here as a reference star. The basic characteristics of the stars are in table 1.

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<td>-</td>
<td>3.32±0.29</td>
<td>6700±200</td>
</tr>
</tbody>
</table>

This $\delta$ Scuti has been previously observed by Breger, in 1969, (ref. 3) who reported a period of 0.077 day with an amplitude of 0.025 magnitude and by Elliott, in 1974 (ref. 4) who reported a period of 0.0797 day.

THE OBSERVATIONS

The two stars were observed during 21 nights (18 may to 7 june 1987) from two sites of latitude close to 30° North; the "Observatorio del Teide" in Tenerife Island (Spain), hereafter designed by OT. and the Observatorio Astronomico Nacional de San Pedro Martinez in Baja California (Mexico), hereafter designed by OSPM. In both sites similar 1.5m telescopes and identical photometers were used. Each photometer has three independent channels observing simultaneously the two target stars and the sky background. A cube beam splitter is located at the input of each channel and divides the incoming light into one part which is sent to the detector - a photomultiplier (one for each channel) Hamamatsu 7647-04.-and another part which is used for centering the star in the diaphragm and telescope guiding. The photomultipliers are operated at room temperature and in the photocounting mode; filters centered at 546 nm with 25 nm bandwidth were also used for the two stellar channels; the sampling time is 6 second, i.e. each 6 second the number of counts in the three channels is recorded simultaneously. Due to standard observing problems (bad weather in one or the other site), we obtained 184.35 hours of data - i.e. 38% of the total duration of the 21 days observing period and 70% of the whole nightly period.

DATA REDUCTION

In what follows we shall consider only the ratio of the counting rates measured for the two stars.
The main argument for using this ratio is that, since the angular distance between the two stars is of the order of 3', they will be affected in an identical way by the variations in the seeing conditions; this allows indeed to maintain a good photometric precision even in relatively poor weather (clouds, dust...).

On the other hand this ratio will mix the variations intrinsic to each star. However, a detailed study has shown that HD155543 does not present any evidence of periodic pulsations with amplitudes comparable with those of the pulsation of 63Hr which we have detected.

Thus for the present purpose we have used HD155543 as a constant reference star; each 6 sec the ratio of the count rates of 63Hr relative to HD155543 was calculated; then for a given night and a given site a cubic spline was adjusted to the resulting data by a least square fitting; this spline was sampled every 5 minutes, which is sufficient for an A8V star. As shown in ref. 5 this corresponds to a filtering by a low band pass filter which eliminates all frequencies above 1.7 mHz. The rms deviation from the spline was calculated for each 5 min intervals, and is shown in figure 1 for a typical night in O.T., together with the similar quantity calculated from the signal of each individual star.

It is seen that in good seeing conditions the photometric precision was of the order of a few mmag near to the zenith and degrades when the hour angle is larger than 2h; the precision on the measurement of the ratio between the two stellar signals is not better than the worst precision on each stellar signal. However when the seeing degrades, as seen in the large fluctuations in each individual stellar signal, the precision on the ratio is not seriously affected.

The filtered time series (the ratio of the two stellar counts) exhibit systematic drifts (of 20%) during a night, due probably to the relative drifts of the detector response of the two stellar channels of the photometer; indeed the ratio of counts measured when the two stellar channels were observing the same sky background in the beginning and at the end of an observing night generally differed also by the similar amounts.

To eliminate these drifts we determined a linear trend for each night and each site and normalised the 5 min data points to this linear trend. After this normalisation the data from the two sites were merged together; during the common observing periods (this happened for 7 nights and lasted for about an hour each time) the merged signal was taken as one half of the sum of the signals obtained in the two sites. This is justified by the very similar behavior of the two signals in the common observing periods as shown in figure 2.

Figure 1. RMS magnitude fluctuations over 5 minute intervals for : 63 Her, x:HD155543 and *; ratio, for a typical night in Teide Observatory

Figure 2: Corrected ratio of counts from the two stars from the two observatories (O: Teide Observatory o San Pedro Martir) for the night 23-25 May 1985.

FREQUENCY DETERMINATION

Since the photometric time series is not continuous over the whole observation period we have used two different methods to detect the frequencies of pulsation of 63Hr: a) the iterative sine wave fitting - hereafter called ISWF (ref. 6) well adapted to unequally sampled data and b) the "clean" deconvolution method (Ref.7), which allows to estimate the effects of the observation window on the pulsation spectrum. Actually Scargle (ref. 8) has shown that the first step in the ISWF method is equivalent to the computation of a modified periodogram. Let s be the observed signal sampled at times t = nAt (it being the sampling time, n = 1,...,N), w the observing window (= 1 if a measurement is available, = 0 in the opposite case), and a, the amplitude and phase of the least square fit of a sine wave of frequency to the time series s, then
RAPID PHOTOMETRY OF THE DELTA SCUTI VARIABLE 63 HER

\[ a^2 = \frac{1}{2} \left( \sum_{n=1}^{N} s_n w_n \cos(\omega(n\Delta t - \tau)) \right)^2 + \sum_{n=1}^{N} w_n \cos^2(\omega(n\Delta t - \tau)) \]

\[ \sum_{n=1}^{N} w_n \sin^2(\omega(n\Delta t - \tau)) \]

where \( \tau \) is determined by the observing window

\[ \tan(2\omega_1) = \frac{\sum_{n=1}^{N} w_n \sin(2\omega_1 \Delta t)}{\sum_{n=1}^{N} w_n \cos(2\omega_1 \Delta t)} \]

The corresponding periodogram is shown in figure 3a.

Furthermore, Scargle, ref. 8, has shown that for a normal white noise of variance \( \sigma^2 \), \( a^2 \) is distributed as \( \chi^2 \), so that the 99% confidence level for finding a group of frequencies is \( a^2 = \sigma^2 (4.6 + \log(m)) \).

To apply this criterion, we have proceeded iteratively; in the first stage, we select the frequencies whose amplitudes are higher than the critical levels calculated for continuous groups of \( m \approx 170 \) frequencies over which the periodogram appears to be roughly constant; then the corresponding sine waves are removed from the signal and a new periodogram is calculated, corresponding confidence levels calculated and the procedure repeated, if some frequencies have amplitude above the confidence levels. If not the iteration is stopped. The resulting residual periodogram and confidence levels are shown in figure 3b.

The corresponding detected frequencies and amplitudes are given in table 2.

The "clean" algorithm (Ref. 7) is based on an iterative deconvolution of the spectrum of the time series \( \{S_n\} \) by that of the window \( \{w_n\} \).

Let \( \hat{S}(\omega) \) and \( \tilde{U}(\omega) \) the discrete Fourier transforms of the two time series and \( \omega \) the frequency of the most prominent peak in the power spectrum

\[ P(\omega) = |\hat{S}(\omega)|^2, \]

the so-called "dirty" spectrum. An estimate of the "clean" amplitude of the transform \( \hat{S}(\omega) \) is given by

\[ a_1 = \frac{\hat{S}(\omega_1) - \tilde{U}(\omega_1)}{1 - |\tilde{U}(2\omega_1)|^2} \]

which is subtracted from the "dirty transform":

\[ S_1(\omega) = \hat{S}(\omega) - g \left( a_1 \hat{S}(\omega - \omega_1) + a_1 \hat{S}(\omega + \omega_1) \right) \]

g being a gain factor which we have chosen \( g = 0.8 \).

Figure 3: a) ISVF periodogram, b) residuals and 99% confidence levels.
The procedure is repeated producing successive clean components \( \ldots, \lambda_n \). The iteration is stopped at the step \( n \) when the residual spectrum looks very much as one due to pure noise. Quantitatively we used a criterion very similar to the one used in ISWF; namely at step \( n \), no frequencies of the residual spectrum has an amplitude higher than the 99% confidence levels calculated as above. The original dirty, residual and clean spectra are shown in figure 4.

**Comparison with theoretical pulsation frequencies**

The first point to notice is the comparison between the frequency of maximum amplitude \( f \approx 131 \, \mu \text{Hz} \) in our present observations and that reported in ref. 3 and 4 which is, within the experimental errors, the next frequency in our table namely \( f \approx 149.8 \, \mu \text{Hz} \).

The second point is that the lowest frequency \( f \approx 46.3 \, \mu \text{Hz} \) is very close to the fourth harmonic of the day and is probably not of stellar origin. We have compared the other observed pulsation frequencies with the prediction of a theoretical model, based on a standard analysis of the eigenmodes of the pulsation periods of a star with mass and age:

\[
1.8 \, M_{\odot} < M \leq 1.85 \, M_{\odot}, \quad 0.937 \leq t \leq 1.055 \times 10^{9} \text{years},
\]

which had on the ZAMS a standard chemical composition \( x = 0.73, \ Y = 0.25 \). It appears that the frequency of the most prominent peak, \( f \approx 131.0 \, \mu \text{Hz} \) corresponds well to the fundamental radial mode of such a model, confirming that 63 Her has evolved significantly off from the main sequence.

The corresponding list of frequencies and amplitudes is given in table 2.

**Table 2**

Detected pulsation frequencies of 63 Her

<table>
<thead>
<tr>
<th>ISWF</th>
<th>CLEAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(\mu \text{Hz}) )</td>
<td>( \Delta(\text{mmag}) )</td>
</tr>
<tr>
<td>46.32</td>
<td>4.5</td>
</tr>
<tr>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>73.75</td>
<td>2.3</td>
</tr>
<tr>
<td>89.31</td>
<td>2.5</td>
</tr>
<tr>
<td>131.00</td>
<td>7.5</td>
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<tr>
<td>149.82</td>
<td>1.9</td>
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<tr>
<td>232.32</td>
<td>1.8</td>
</tr>
<tr>
<td>301.08</td>
<td>1.1</td>
</tr>
</tbody>
</table>

It is seen that both methods give almost identical results except for two frequencies, one low frequency \( f = 56.15 \, \mu \text{Hz} \) detected only by the clean method and an high frequency \( f = 301.08 \, \mu \text{Hz} \) detected only by the ISWF.
The two confidently determined high frequency modes cannot be radial overtones since they do not have the canonical frequency ratios. From our models

\[
\frac{f_{0.1}}{f_{0.2}} = 0.76, \quad \frac{f_{0.1}}{f_{0.3}} = 0.61
\]

(\(f_{0,n}\) denoting the mode with degree 1 and order \(n\)) while we find in our observations for these ratios 0.87 and 0.56, respectively. These frequencies are most probably non radial modes of degree \(1 > 0\), and low values of \(n\). The most striking result is the existence of three or four frequencies lower than that of the fundamental \(f_{0,1}\), which we suggest to be gravity modes.

A more detailed identification is underway and will be published in a later paper.

Note that 63 Her is a rapidly rotating star. However we did not detect any rotational splitting of the pulsation frequencies.

REFERENCES

2. Breger M., Huang Lin, Jiang Shi Yang, Guo Zitte, Antonello E., Mantegazza L. 1987, Multiple close frequencies of the Delta Scuti star \(\delta\) Tau, Astron. Astrophys. 175, 117
Light curves and power spectra of a given Delta Scuti model have been synthetically constructed in order to investigate the effects of different techniques of photometric data sampling on the final structure of the periodograms. The main purpose of this analysis is to provide useful suggestions for the interpretation of periodograms obtained from real light curves of multiperiodic variables. Very preliminary results point out that an unambiguous detection of the pulsation modes from the periodograms is difficult also in the case of very long runs of observation and that maybe only international observing campaigns could provide the most useful photometric data sampling technique.

Keywords: Multiperiodic Variables, Photometric data sampling, Pulsation modes.

1. INTRODUCTION

Recently a considerable interest has been devoted to the problem of Delta Scuti variability. The observed properties of these pulsating variables, located in the lower part of the Cepheid instability strip, are well known, but, at present, not understood as well.

There are strong evidences that some of them pulsate with large amplitudes (many tenths of mag) and in this case just in one or a few modes. However many Delta Scuti exhibiting very low amplitudes (few mmag) show light curves variable in phase, shape and amplitude that suggest the coexistence of several pulsational modes (probably both radial and non-radial).

On this basis, few attempts to find some correlations between the physical parameters of the Delta Scuti stars and their properties of oscillation have been made (Ref. 1). Although the pulsators having the largest amplitudes appear to show the clear tendency to be slow rotating giant stars, it is still very difficult to give a definite physical interpretation of the bimodal behaviour of the Delta Scuti pulsators. In particular it appears very urgent to discover the factors limiting and selecting the modes and their possible links.

In the last few years several suggestions on that have been provided (Refs. 2-3). Anyway, the problem of the multiperiodic features noticed in the light curves of the Delta Scuti pulsators is far to be completely solved. The modes of oscillation are not always identified and sometimes even whether they are radial or not, p- or g-modes, is not clear; in general, at present, very few low amplitude Delta Scuti variables have unambiguously solved frequency spectra.

A powerful possibility of improvement in the identification of multiple periodicities should be supplied through the comparison between the power spectra obtained from observed light curves and those theoretically computed from a suitable model of the star of interest.

The purpose of the present paper is therefore to give general advices, using artificial data, concerning the photometric data sampling techniques to be adopted in the analysis of multiperiodic variables. This should provide some answers about the actual possibility of identification of the spectral features in the periodograms. Really, this is the starting point for the asteroseismological studies whose crucial goal is to decode the physical information hidden in the structure, often very complex, of the periodograms (a review of the physical implications of the spectral features is in Ref. 4).

The criteria adopted to construct the data sets are described in section 2. Section 3 illustrates how the analysis of the synthetic light curves and periodograms were performed. In section 4 the main results are summarized, while a brief discussion and some ideas for further experiments are presented in section 5.
2. SYNTHETIC LIGHT CURVES CONSTRUCTION

Time series of artificial data were constructed for a ZAMS model of Delta Scuti star kindly supplied by W. Dziembowski. The "theoretical" variable star (hereafter TVS) has the following parameters: $M = 1.4 M_\odot$, $R = 1.19 R_\odot$, $L = 4.33 L_\odot$, log $T_{\text{eff}} = 3.833$.

Eigenfrequencies of acoustic modes in the range $300 \leq \nu \leq 600 \mu$Hz were calculated for the model using a numerical code for linear adiabatic pulsations (Ref. 5). Both radial and non-radial modes were included; however the range of the spherical harmonic degree $\ell$ was restricted to $\ell \leq 2$, because, as it is well known, stellar modes with higher $\ell$ cannot be detected due to the effect of averaging over the stellar disc.

The frequencies rotationally split were included assuming a uniform rotation of the TVS with angular velocity equal to $2\pi \times 21.1 \text{ rad/sec}$. The split frequencies approximated to the first order, have been obtained by:

$$\nu_{\text{split}} = \nu_{\text{unperturbed}} - m (1 - C) \Omega \quad (1)$$

where $\nu_{\text{unperturbed}}$ is the unperturbed mode frequency, $C$ is the Édouard constant and $\Omega$ is the rotation rate of TVS.

The modes included in the analysis are listed in Table 1 together with their frequencies.

The light curves of TVS were then constructed assuming that the amplitude of the variation of the bolometric luminosity is given by:

$$A = a \rho_\ell (\cos \theta_0) \quad (2)$$

where

$$a = n \sqrt{(\ell - m)! / (\ell + m)!} \quad (3)$$

with $n$ azimuthal order of the mode and $a_\rho$ a parameter that depends essentially on the degree of the mode and on the adopted limb-darkening law (Ref. 6). $\rho_\ell (\cos \theta_0)$ is an associated Legendre function and $\theta_0$ is the polar angle of the observer (in this context arbitrarily fixed at $30^\circ$) in the frame of reference used to describe the oscillations.

$Q$ is a random variable with a given probability distribution (Ref. 3). Each choice is consistent with the hypothesis that a resonant mode-coupling between $g$-modes of low-$\ell$ degree and high-$\ell$ degree $g$-modes can be responsible in limiting the amplitude of the multiperiodic Delta Scuti variables (Ref. 3).

Since, in principle, for each acoustic mode there is a large number of gravity mode pairs that are non-linearly coupled to it, the determination of the amplitude for each eigenmode may be done only in a probabilistic way. In Table 1 the amplitudes for all the acoustic modes considered, computed in the way just described, are also reported.

Finally, the synthetic photometric signal was obtained as:

$$S(t) = \sum_i A_i \cos(\omega_i t + \phi_i) + N(t) \quad (4)$$

were the $\phi_i$ quantities are time-independent random phases and $N(t)$ is random gaussian noise with variance varying from night to night so as to simulate different photometric conditions of observation.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$\ell$</th>
<th>$m$</th>
<th>$\nu$ ($\mu$Hz)</th>
<th>$A$ (mmag)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>318.6</td>
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Ultimately, in order to obtain realistic light curves it was necessary to fix some observational parameters such as gaps, duty cycle, total observing time. To this aim a double channel photometer output, in the final form of magnitude differences between the TVS and a comparison star was simulated, with an integration time of 20 sec, a sampling interval between two consecutive sky measurements of 30 min (with a gap of more or less 7 min). The total observing time for each night was chosen according to the different experiments which were carried out. In any case, however, irregular gaps (of two data or more) were introduced inside the strings to simulate some bad measurements rejected.

In particular the first group of experiments carried out (some other tests were suggested by
The output consists of periodograms covering the frequency range of \( \frac{\nu}{2 \, \text{Hz}} \) to \( \frac{\nu}{7 \, \text{Hz}} \) and a total number of data of 3706.

ii) five consecutive nights of observation, runs of \( 5.9 \, \text{h}, \sim 24 \, \text{hour gaps}, \) total time of observation of \( 27.15 \) and total number of data of 3662;

iii) an observing campaign at the two good photometric sites of Catania and Canary Islands with a total of ten nights (not consecutive) of observation, runs of \( 7.5 \, \text{h}, \) total observing time of \( 75^h\) and total number of data of 9956.

Although many other experiments of this kind could be thought out, it seemed very instructive to analyze the results obtained from the two first "games" in order to determine the effects of different values of the duty cycles on the final structure of the periodogram that, in this context is expected to be very crowded.

On the other hand, the experiment number 3 should tell us something about the importance of long uninterrupted observations in the framework of international co-operations.

3. DATA ANALYSIS: PERIODOGRAMS

The synthetic time series, generated as specified, were then analyzed in frequency in terms of discrete Fourier transform of unevenly spaced data.

The choice of the finite set of frequencies at which to evaluate the periodogram for all the experiments was made according to that suggested by Scargle (Ref. 7). In fact an insufficient frequency step can invalidate the results producing spurious peaks and distorting the amplitude ratio among the components of the signal.

In some cases oversampling was adopted to improve the frequency resolution.

The determination of amplitude and phase of the signal's component corresponding to the most significant frequency in the periodogram was carried out by a least-squares fit of a single sine function to the entire data set. This function was then used to remove the dominant frequency (and with that the corresponding daily aliases) from the data, after which the "prewhitened" data were searched for a second periodicity. This process can be iterated until all significant periodi­

cies have been identified.

This procedure, unfortunately not always suitable, was adopted in the present analysis for sake of simplicity; final frequencies and uncertainties should be obtained through a simultaneous multi­

periodic least-squares fit to the original data using the previously determined frequencies only as initial guesses. The application of this technique could be planned for future experiments.

The output consists of periodograms covering the frequency range of \( 200 \leq \nu \leq 600 \, \mu \text{Hz}, \) with a frequency resolution of the order of 0.1 \( \mu \text{Hz}. \)

In all the examined cases, the structure of the spectra is too complicate to enable us to identify unambiguously all the components.

4. RESULTS

The comparison of the theoretical power spectrum of TVS (Fig. 1) with the periodogram computed in the experiment No. 1 (Fig. 2) points out that this approach fails to produce clearly de­

tectable peaks, in particular producing a very confused structure. The only evidence component in that at \( \nu = 313.13 \, \mu \text{Hz}, \) corresponding to the mode \((0,2, + 1)\) which has the largest amplitude associated, together with the daily aliases at \( \nu = 324.70 \, \mu \text{Hz} \) and \( \nu = 301.56 \, \mu \text{Hz}. \) After the filtering process the result became even worse since a spurious peak at frequency \( \nu = 402.20 \, \mu \text{Hz} \) emerged from the noise, probably caused by a type of interaction between the spectral window and the signal.
data show, as in the previous case, the possibility of identification of some equally spaced frequencies.

<table>
<thead>
<tr>
<th>( \nu ) (( \mu \text{Hz} ))</th>
<th>( A ) (( \text{mmag} ))</th>
<th>( \phi ) (( \text{rad} ))</th>
</tr>
</thead>
<tbody>
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<td>309.8</td>
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<td>1.17</td>
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<td>32.5</td>
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<td>316.5</td>
<td>17.4</td>
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</tr>
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<td>318.7</td>
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<td>379.0</td>
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<td>543.7</td>
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<td>0.59</td>
</tr>
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</table>

5. DISCUSSION AND FUTURE EXPERIMENTS

To summarize, the most impressive indication given by such a kind of approach is that important effects of interference among very closely spaced modes can produce strong difficulties in their identification from the periodograms. This may particularly be true if the interference works more effectively than shown in the present exercise, destroying some important components of the signal perhaps below the level of the detectability.

As a consequence, even though it is easy to realize that long uninterrupted runs of observation in each night are a stressing need to obtain good results (see exercise No. 1), this is only a necessary but not a sufficient condition to solve the modes of oscillations of a multiperiodic variable star and
to determine correctly their amplitudes. To do this, in fact, as the comparison between the general results obtained points out, a considerable amount of very long observing nights are required.

Figure 4. As for Figure 3, but for the successive five nights of observation. Only the component at \( \nu = 313.20 \) \( \mu \text{Hz} \) is evident.

Another important issue of this analysis is that which concerns the possibility of resolving the equally spaced frequencies, as shown in the periodograms of Figs. 6 and 7 and in the Table 2. In a real case, in fact, the presence of this kind of spectral features would provide an excellent method to identify non-radial modes. In addition, the equal-frequency split ratio may tell us something about the rotation rate of the star.

A huge amount of other interesting experiments should be played in the future to elucidate the relationship between what the observers can deduce out from the data and what really is the physics of a multiperiodic variable star.

i) An identical analysis should be done on the same time series with oversampled periodograms to improve the frequency resolution.

ii) Attempts should be made to extend the international observing campaign to a larger number of participants in order to avoid aliasing effects.

iii) The filtering process would be iterated to test the possibility to detect very low amplitude components of the signal.

iv) A multiple least-squares fit would be applied to compute frequencies, amplitudes and phases with a larger precision.

v) An analysis concerning the confidence level for each significant peak could be carried out.

The author wishes to thank the Director and the staff of the N. Copernicus Astronomical Center of Warsaw, where part of this work has been performed, for the kind hospitality and their assistance in using the computing facilities.

Figure 5. Normalized periodograms concerning a total number of data of 7409. The uppermost panel refers to the entire data set, the next ones to the successively pre-whitened data. The frequencies by which the prewhitening was performed are indicated by arrows (see Table 2).
Figure 6. Normalized periodogram of the entire data set for the observing international campaign of ten nights. The frequencies $f_1 = 313.29$ MHz and $f_2 = 319.13$ MHz are indicated by arrows.

Figure 7. Normalized periodograms of the data concerning the observing international campaign of fourteen nights. The uppermost panel refers to the entire data set, the next one to the data successively filtered from the strongest component at $f = 313.22$ MHz.

6. REFERENCES

THE SEARCH FOR SOLAR-TYPE OSCILLATIONS IN OTHER STARS USING THE MICHELSON INTERFEROMETRIC STELLAR OSCILLATION SPECTROMETER (IOS)

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1. INTRODUCTION

In taking the Sun as our model we find that by observing light from the entire Solar disc the now-familiar 5-minute oscillations have velocity amplitudes ranging up to ~20 cms⁻¹. These disturbances are the surface manifestations of low-order deeply-penetrating sound waves or p-modes which are excited internally. The Sun acts like a resonant cavity for these oscillations trapping them below its surface. Higher order oscillations inhabit less deep resonant cavities, also below the surface, and interfere with each other on time-scales of ~30 minutes, producing velocity amplitudes of up to 400 ms⁻¹. Unfortunately it is only the low order, low amplitude modes that would be detectable in other stars; requiring measurements with a precision of one part in 10⁶. The techniques used at present are capable of achieving 3.6 ms⁻¹, even when using very large telescopes. The ability to use more absorption lines from the stellar spectrum to increase the signal-to-noise ratio becomes essential if the target precisions are to be reached. The detection of such p-mode oscillations in other stars would allow the stellar interior to be probed yielding information about the conditions in and around the core and also about the stellar mass, age and radius. A sufficiently good SNR would remove the need for complicated and possibly misleading data reduction techniques such as the procedure of repeated power spectral analysis used to search for p-mode spacings (see ref. 1 for details).

2. INCREASING THE SNR BY USING MANY LINES

The tiny motions imparted to the stellar surface as the 5-minute p-mode oscillations propagate are reflected in the Doppler shifts of the absorption lines formed in the disturbed region. Each line carries essentially the same information about the velocity of the surface. Increasing the number of lines used increases the signal and, in a stellar seismometry instrument limited by photon statistics, the increase in SNR will be proportional to the square root of the increase in signal or the total number of lines used. Methods using alkali metal vapour cells are limited to one or perhaps two spectral lines, whereas spectrograph systems using masks or multi-point detectors are usually limited by reference spectrum, detector or spectrograph stability. These methods cannot, in general, be greatly improved by using more lines. The Coudé-reticon method (ref. 2) which uses 8 reticon arrays spanning a 1500Å range of spectrum was expected to achieve 3.6 ms⁻¹ noise levels. The still untested absolute accelerometer of Connes (ref. 3) should be able to use the whole available spectrum while eliminating all undesirable instrumental effects totally. The Michelson IOS, briefly described below, used in conjunction with a dedicated echelle spectrograph has the capability of using many spectral lines to significantly increase the SNR.

3. THE SINGLE LINE MICHELSON INTERFEROMETER

The single line instrument has been described fully in ref. 1 and its main features are shown in figure 1. In outline, the design of the instrument was for optimal measurement of Doppler shifts of a single absorption line in the spectrum of a candidate star by measuring the changes in the phase of the fringes at one path difference. The beam splitting and recombining areas are separated in the solid cube beam splitter by the use of retro-reflectors instead of plane mirrors. This gives access to the complementary pair of Michelson outputs. Relative changes in intensity at these two outputs are caused by the surface velocity changes in the star and are measured directly from the observed fringe pattern to change phase. Light from a HeNe laser, with a wavelength stability ±λ/Δλ=4.10⁻¹¹, follows the same path as the stellar signal and is used to maintain the path difference at ~5mm. The interferometer has also been field-widened by the inclusion of an extra thickness of glass in one arm and air in the other (see ref. 4).
Instrumental drifts are compensated for by rapidly swapping the signal paths, both optical and electronic, through the instrument at a rate of 25 Hz. This has the added advantage of eliminating scintillation effects.

4. MULTI-LINING THE MICHELSON ISOS

Well isolated Fe I lines in a 400Å bandwidth centred at a wavelength of 5500Å are used to increase the SNR. The properties of the chosen line interferograms are governed by the shape of the line profile and the wavelength of each line. The width and depth of the line determine the contrast of the fringes and deeper lines are to be preferred. The wavelength of the line determines the phase of the \( \cos^2 \) fringes at a given path difference. At such a path difference the phase of any pair of interferograms will be different and if added together the contrast of the resulting fringes is as likely to worsen as it is to improve. In order to maximise the signal from all of the spectral line interferograms they must be added coherently at the same path difference i.e. the slope of the separate interferograms must be in the same sense and the points of inflexion must coincide. The resultant fringes will be at a maximum in the slowly varying beat pattern which is modulating the interferogram.

4.1. Instrumental Considerations.

The absorption lines are provided by a dedicated spectrograph using a large echelle grating mounted in Littrow, with a dispersion of \( \sim 4 \) Å mm\(^{-1}\). Stellar light from the telescope is fed to the spectrograph along an optical fibre of core diameter

![Figure 1. Schematic of the Michelson ISOS](image1)

![Figure 2. Spectral profile at Michelson input](image2)
SEARCHING FOR SOLAR-TYPE OSCILLATIONS IN OTHER STARS WITH THE MICHELSON ISOS

200 µm which becomes the input slit. Individual optical fibres are placed to coincide with the FeI line images in the focal plane of the spectrograph thus acting as narrow band spatial filters in isolating the lines. The spectral profile of this arrangement is shown in figure 2 where the image of an emission line at the focal plane of the spectrograph is scanned across the coupling fibre. When using an absorption line, the fibre must be positioned so that the line falls on the flat region of this profile. These fibres are then placed in the input aperture of the interferometer, which is sufficiently luminous (see ref. 5) to accept ~100 fibres. Normally the position of a fibre in the entrance aperture of an interferometer would determine the phase of the fringes of the light it carries at some path difference by defining the off-axis angle \( \theta \) in equation 1:

\[ m\lambda = 2n\cos\theta \]  

where \( m \) is the order of interference, \( \lambda \) the wavelength, \( n \) the refractive index, and \( 2\theta \) the path difference. The field-widening property of the Michelson ISOS reduces this effect almost entirely over the input aperture, however, the circular symmetry of the entrance aperture is broken and a nearly linear change in phase imposed across it by a slight mis-alignment of one of the retro-reflectors relative to the other. The effect is similar to tilting one (plane) mirror in a standard Michelson, producing wedge fringes, but only depends upon the off-axis angle in one direction. Field-widening persists in the orthogonal direction. A knowledge of the change in phase with angle and the relative phases of all the separate fringe patterns allows the optimal placement of the optical fibres at the input. Since the optical fibres have a finite size and require support at the entrance aperture, there are only discrete values of off-axis angle which are available. A hexagonal arrangement is formed by the matrix of steel tubes that support the fibres. The low finesse, \( \cos^2 \), fringes of a two beam interferometer can be added to give only a slight reduction in overall contrast even when they are up to ~60° out of phase making this limitation less serious than it may at first seem.

4.2. Achieving Multi-lining

The 400 Å region of spectrum centred on 5500 Å from a solar-type star contains approximately 50 FeI lines of which ~30 are suitable. (Cross dispersion will eventually add more lines to this list.) Light from each of these lines in the Sun was separately fed on-axis to the Michelson and the phase of the resulting fringes measured relative to the HeNe reference wavelength. The 5406 Å line was chosen to be placed on-axis; it has an average fringe depth and is approximately central in the range of relative phases. The optimum positions of the fibres carrying light from other lines were found by maximising the fringe depth of pairs of lines, the 5406 Å line being one of each pair. The fringes produced by all the lines had a depth equal to the average fringes at the chosen path difference, which diminished by ~40% after the path difference was changed by 3 orders. A schematic of the multi-line system is shown in figure 3.

5. METHOD OF OPERATION AND ACCURACY

Once the positions of the fibres has been fixed the operation is the same as for one line. A path difference is chosen, by scanning ~5 orders, where a point of inflexion in the stellar signal fringes coincides with a point of inflexion in the reference wavelength fringes. This path difference is then kept constant with respect to the reference wavelength. Changes in the phase of the stellar signal will then represent radial velocity changes of the stellar surface. A change in velocity of ~30 km s\(^{-1}\) is required to change the order of interference by 1 i.e. a complete fringe in ~10^4 order. However, the magnitude of expected changes will only alter the order fractionally. With a single line a change of 15 ms\(^{-1}\) will cause a relative change in intensities in the Michelson out-
put parts of 0.01%; this requires $10^9$ photons to be counted. By using many lines the flux will increase in proportion to the number of lines thus allowing $10^9$ photons to be counted much more quickly; a time reduced by a factor of $\sqrt{n}$ for $n$ lines. The advantage of using many lines may be slightly diminished if the combined fringes have a lower contrast, a loss amounting to no more than 10-15% in sensitivity to velocity changes. It can be expected that a noise level of 1 ms$^{-1}$ will be achieved after ~4 hours while observing a 0$m$ star on a 4.2m telescope when using 16 lines.

6. MULTI-LINE OBSERVATIONS OF THE SUN

In order to show that the multi-lining technique improves the SNR it is necessary to detect radial velocity changes having added many lines together, maintaining the fringe contrast. A fringe of the 5406Å line and a fringe of 16 lines added together are shown in figures 4 and 5 respectively; roughly the same path difference region was scanned. There is a reduction in contrast of ~20% between the two fringes. The flux increased 14-fold when 16 lines were used. The short fall was due to dead time effects in the photon-counting electronics at the high count rates obtained from the Sun. Dead time effects also account for an increase in the mean level of the 16 line fringes and part of the reduction in contrast. The count rates expected from a 0$m$ star using a 4.2m telescope would be a factor of ~10 lower at ~$10^6$ photons per sec.

A detection of the solar 5-minute oscillation has been made after an observation of 2.28 hours using 11 lines. Light from a fractional area of the solar disc with a diameter 2% of that of the Sun was used, giving a larger signal than would be obtained from whole disc observations. This was, unfortunately, a necessary step to take due to the relatively rare occurrence of completely sunny days in London long enough to detect the lower-order 20 cms$^{-1}$ oscillations. The velocity amplitude spectrum is shown in figure 6. The 5-minute envelope is clearly visible although the spectrum is dominated by low frequency drifts, which are due to a rapid increase in the laboratory temperature housing the instrument caused by heat from the Sun. Such drifts do not, of course, arise at the telescope, where the instrument is placed in a stable environment and fed light from the telescope via an optical fibre. The noise level in figure 6 is below 1 ms$^{-1}$ after 2 hours, which indicates that at the counting rates expected from stars (see section 5 earlier) using a 4.2m telescope, the estimate of 1 ms$^{-1}$ after ~4 hours using 16 lines is likely to be achieved.

7. FUTURE OBSERVATIONS AND PROSPECTS

The above results on the Sun show that a noise level sufficiently low to detect sub-metre per second solar-type oscillations in other stars is now possible with the multi-lined Michelson ISOS. In forthcoming observations this winter at the 4.2m WHT it is hoped to detect such oscillations directly in the power spectrum of a stellar velocity curve. New observations of eCyg which are planned for summer '89 should confirm or not the possible detection of a mode spacing found in this star using the single-lined Michelson (ref. 1).

Observations of this kind would greatly enhance the overall prospects for solar-type asteroseismology and the theory of stellar interiors. Given sufficient funding these observations could easily be extended to many Sun-like stars over several years to build up an asteroseismology database which could also be used to monitor these stars for the presence of planetary sized companions; an area in which there is great scientific interest since the recent announcement of possible planetary detection by Campbell et al at the IAU in Baltimore.
ACKNOWLEDGEMENTS

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HIGH PRECISION VELOCITY OBSERVATIONS OF ARCTURUS USING THE 7699 Å LINE OF POTASSIUM

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ABSTRACT
The K giant Arcturus (α Boo) was observed with the Birmingham double magneto-optical filter spectrometer using the GHR1L facility at the Nasmyth focus of the 4.2 m William Herschel Telescope of the Observatorio del Roque de los Muchachos in 1988 April-May. Approximately 80 hours of data were obtained over a 2 week interval. Our preliminary analysis of the data shows the presence of the large (~200 m s⁻¹) amplitude velocity variation reported earlier by other observers. However, our more extensive data set strongly suggests that this variation is not singly periodic, as was previously indicated. We present some speculative comments as to the nature of this variation, and the implications for stellar seismology.

1. INTRODUCTION
As part of the Birmingham stellar seismology program, and in collaboration with the JAC and ESA observers, the star Arcturus (K2 IIIp) was observed on 9 full nights (and some part nights) over a 13 day interval with the 4.2 m William Herschel Telescope at the Roque de los Muchachos Observatory in 1988 April-May. The observations were taken with the Birmingham stellar spectrometer, which is a magneto-optical filter device based on the instrument originally described by Cimino et al (Ref. 1) for solar studies. Brief descriptions of the Birmingham instrument and previous observations in the program are given in Isaak and Jones (Ref. 2), and Innis and Isaak (Ref. 3).

In outline, the spectrometer measures the blue (B) and red (R) intensities in two bandpasses (of order 50 mA each), separated by ~5 km s⁻¹ (in velocity units), symmetrically placed about the rest wavelength of the Potassium 7699Å line. When observing a star which has a velocity, relative to the observer, within ±2.5 km s⁻¹, the spectrometer bandpasses are thus on opposite sides of the stellar potassium line.

The intensities, B and R, are measured, and the ratio (B-R)/(B+R) is to first order a linear measurement of the star’s velocity.

A block of three weeks of telescope time was granted to the project by the Spanish Committee to Allocate Time (CAT), as a commissioning run on the William Herschel Telescope. This is of course a large allocation on a telescope of this size. We were fortunate too as observing conditions were generally very good during our run, with the result that we have an extensive and high quality set of velocity observations of Arcturus. Our "first look" at the data shows the presence of the ~200 m s⁻¹ amplitude variation first reported by Smith et al (Ref. 4) and confirmed by Cochran (Ref. 5). However, our more extensive data set strongly suggests that the variation is not singly periodic, although we have not as yet determined the number or the frequencies of the major components present. A more complete discussion will appear elsewhere. We conclude with some speculative remarks regarding the possible nature of the variability, and the wider implications for the field of stellar seismology.

2. RESULTS
2.1 General
The raw data consists of measured ratios obtained every 8 seconds during the nightly run. We have taken averages over 16 points (to make 128 second integrations), and plotted nine full nightly data strings on the same set of axes in Figure 1. We stress that other than for the averaging over 16 points these data are as collected at the telescope during our run. For any given night the diurnal motion of the Earth is clearly seen in the data. This is of order 800 m s⁻¹ for the ~8 hr observing night. The offset from one night to the next is due to the changing component of the Earth's orbital motion relative to Arcturus. This effect is also present in any given night's data of course, and is of order 200 m s⁻¹ over one observing night. The "compression" in the nightly separation of the traces at the start and end of the run comes from the reduced sensitivity of the spectrometer at these times (i.e. a smaller observed change in ratio for a given velocity change): as we move away from zero relative velocity between the star and spectrometer, the spectrometer bandpasses are no longer symmetrically placed on the stellar absorption line, reducing the observed effect. As well, the "skewness" of the nightly curves relative to each other is due to the presence of the large variation discovered by Smith et al (Ref. 4), briefly mentioned above, and discussed in more detail in the following section.

Quite clearly, this is a very extensive set of observations of Arcturus, and of high precision. We did lose some observing time due to poor weather, and on occasion due to instrumental or telescope faults as well. However, given that we were observing from one site only, we consider our coverage is very nearly as full as we could have hoped for.

2.2 Calibration

The observed topocentric velocity of a star can be written as

$$v_{\text{obs}} = v_{\text{Hel}} + v_{g(\text{orb})} + v_{g} + v(t)$$  \hspace{1cm} (1)

where $v_{\text{Hel}}$ is the star's heliocentric velocity, $v_{g(\text{orb})}$ and $v_{g}$ are components due to the Earth's orbital and spin motion, and $v(t)$ is a time dependent term due to possible oscillatory or other variations. (For the case of the sun, $v(t)$ is small and quickly varying compared with $v_{g(\text{orb})} + v_{g}$). From a series of observations such as in Figure 1 it is possible to form a calibration between the observed ratio and the calculated topocentric velocity, by calculating the velocity for each of the given times of observation. For our Arcturus data, this gives the plot shown in Figure 2.

The large variation noted above is evident in this calibration plot. Data from adjacent nights, at the same calculated velocity, show significantly different observed ratios (i.e. different observed velocities), giving the segmented appearance to Figure 2. This is not an instrumental effect due to poor stability. If the intrinsic velocity of Arcturus was not varying then data from different nights would overlap to within the errors of the observation, as was seen in the data taken with this instrument on Procyon (Ref. 3).

In passing we note that Figure 2 is actually a crude differential of the line profile as discussed for the observations of Procyon in Ref. 3. Fitting a fifth order polynomial to the data in Figure 2, and integrating gives the curve plotted in Figure 3 - a very rough approximation of the Potassium 7699Å line in Arcturus.

Figure 1: The raw ratio traces for the nine full nightly runs on Arcturus, plotted against U.T. The traces correspond to the following dates (from bottom to top): April 21, 22, 23, 25, 27, 28, 29, 30, May 01. The 'wild points' scattered around zero are from the cloud broken 29 and 30 April nights. These points were not used in the analysis.

Figure 2: The derived Ratio Versus Velocity calibration plot for the data in Figure 1. The presence of the large intrinsic velocity variation of Arcturus causes the segmented appearance of this plot (see the text).
The presence of the intrinsic variation of size comparable to the other terms in Eq. 1 complicates the calibration. In the case of the Sun, where the amplitude of variation is small (~ 1 m s⁻¹), and fast (~ 5 min), a run of a number of hours defines the mean velocity well enough to allow a satisfactory calibration between ratio and velocity to be obtained. This is not necessarily true for the data we have for Arcturus, but to first approximation, we have used the fifth order polynomial, fitted to the data in Figure 2, to represent the velocity of Arcturus that would have been observed had the intrinsic variation not been present. From this we can relate the observed ratio to a topocentric velocity, thus forming data strings similar to that in Figure 1, but now showing the measured velocity changes of the star during the night. Clearly, this approximation will only be valid where we have data from two or more nights taken at the same topocentric velocity, which will not be the case for data in the first and last night. Consequently data from these two nights will not be used in the following analysis.

Figure 4 shows the observed velocity change of Arcturus for the night of 1988 April 25, obtained from the ratio trace shown in Figure 1 by the method outlined above.

Velocity residuals were formed from the calibrated data by subtracting off the contributions due to the Earth’s orbital and rotational motion. These residuals then represent the intrinsic variation of Arcturus as observed during our run. The residuals for other than the first and last days are plotted against time in Figure 5. The variation in velocity is clearly visible, with amplitude ~ 200 m s⁻¹. It appears that it is not a simple sinusoidal or similar variation.

Smith et al (Ref. 4) derived a most likely period of 1.84 day for the velocity variation in Arcturus, which gave a saw tooth velocity curve. We have plotted our data with this period in Figure 6. We do not recover the saw tooth wave, but obtain a scattered plot, as we do when plotting our data with the 2.18 day period (the day alias noted in Ref. 4) in Figure 7. We conclude that our data are not compatible with the most likely periods reported earlier.

Although our data are sparse, we calculated the power spectrum of the residuals in Figure 4 in an attempt to derive possible periods. The only significant power is in the low frequency section (~ 0.05 mHz), consequently we show only the 0 to 0.1 mHz region of the power spectrum in Figure 8. The two strongest peaks at ~ 0.0038 mHz and 0.015 mHz (~ 3.0 and 0.75δ) are day aliases of each other, while the next strongest peak at ~ 0.0057 mHz is essentially 2.0 days. Hence it appears that the power spectrum is dominated by the windowing in the data train.

As a result we have not as yet obtained further estimates of the periodicities present in Arcturus. However inspection of Figure 5 seems to indicate that more than one period is present - i.e. that Arcturus is multiperiodic. The analysis is continuing.

We also searched for the presence of higher frequency, smaller amplitude, velocity variability in Arcturus. The nightly velocity residuals in Figure 5 were de-trended by subtracting a fitted third order polynomial. A power spectrum of these data gave a noise level, per frequency bin of ~ 0.95 μHz, of ~ 0.9 m² s⁻². No significant peaks were apparent in this spectrum, setting an upper limit of ~ 3 m s⁻¹ for any possible higher frequency (~ 1 hour) variation of Arcturus.

### 3. DISCUSSION

Smith et al (Ref. 4) derived a most likely period of 1.84 day for the velocity variation in Arcturus, which gave a saw tooth velocity curve. We have plotted our data with this period in Figure 6. We do not recover the saw tooth wave, but obtain a scattered plot, as we do when plotting our data with the 2.18 day period (the day alias noted in Ref. 4) in Figure 7. We conclude that our data are not compatible with the most likely periods reported earlier.

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### 4. • ECULATION

The nature of the large velocity variation will now be discussed. Clearly orbital motion is not a possibility (see Ref. 5). This leaves rotation or pulsation as possible causes. Spectrometers such as ours would be sensitive to surface features such as spots and plages. However the rotation period of Arcturus is likely to be a hundred or more times longer than the typical time scales of the variations seen (see Kemp et al., Ref. 6), so that rotation must be considered extremely unlikely.

Quite obviously, pulsation must appear as the most likely cause. The well known period-density relation for pulsating stars leads to an estimated period of around one week, plus or minus a few days, for the fundamental radial mode of Arcturus (e.g. see Ref. 6). We raise the possibility that the variations, seemingly multiperiodic, observed in Arcturus are in fact the higher harmonics of this fundamental normal mode, in the same way that the solar five minute oscillations are harmonics of the (as yet undetected) solar fundamental period, estimated to be ~ 3.5 hours. A very rough estimate of the acoustic cut off frequency for Arcturus can be made (see Isaak, Ref. 7). Assuming that the equations in Ref. 7 are valid in such a rarified atmosphere, we derive an approximate cut off frequency of tens of hours. We speculate that the large amplitude of the observed velocity variation may be due to the thin stellar atmosphere, and the possibility that the star may have a much smaller number of allowed acoustic modes of oscillation due to the lower cut off frequency compared to the Sun.

Such remarks are of course very preliminary, and are offered mainly for discussion. Clearly the identification of the major frequencies present in Arcturus must be a
Figure 5: Velocity residuals for Arcturus after subtraction of the Earth's spin and orbital components. These data represent the intrinsic velocity variations of the star.

Figure 6: The velocity residuals of Figure 5 plotted with the best fit period of 1.84 days as determined by Smith et al (Ref. 4). (see the text).

Figure 7: As in Figure 6, but with trial period 2.18d - the next most likely period of Ref. 4 (see the text).

Figure 8: Power spectrum of the data in Figure 5, from 0 to 0.1 mHz (see the text).

priority. We are continuing the analysis of the data presented here, and further observations are planned. Observations of other similar stars are also high desirable, to search for the presence of oscillations of this type. (It seems that not all K giants oscillate, or perhaps they do not oscillate all the time, as Ref. 4 reports no detectable variations in the K0 III Star Pollux at the ~ 6 m s⁻¹ level). However, if the amplitudes of such stars are around tens (or even hundreds) of metres per second, then the future for ground based velocity seismology of such objects appears promising.
5. ACKNOWLEDGEMENTS

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VARIABLE STAR STATISTICS IN OPEN CLUSTERS.
NGC 6192

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ABSTRACT

A search for δ-Scuti stars and other variables in open clusters is in progress, and we report results for the cluster NGC 6192.

At the end the statistics for a number of open clusters on the frequency of variables in the Instability strip is presented. We think that there is now increasing evidence for a change with age of the fraction of variables.

In Ref. 1 indications were found of a change with time of the fraction of variable stars in the instability strip. The threshold for detection of variability was not uniform and in some clusters too high to give a clear picture of the statistics.

Gilliland and Brown (Ref. 2) reached a 3σ detection level of 0.5 mmag, observing ±2 hours with a 0.8m telescope and a Tektronix CCD detector. An RCA CCD chip performed slightly poorer reaching 0.7 mmag in 2.5 hours.

3. DATA

Thus motivated we collected around 3000 CCD-frames with an RCA chip on the Danish 1.5m Telescope at La Silla over 11 nights with exposure times in the range 20-60s. The reduction of two timeseries, consisting of 372 and 350 frames in Johnson B on the cluster NGC 6192 corresponding to two ±6 hour runs, is well underway. Autoguiding was used to limit the drift of the field to less than one pixel during 6 hours. Figure 1 shows the CCD-field which measures 2'×5'.

2. MOTIVATION

δ-Scuti stars are often multiperiodic and therefore candidates for seismological studies. In order to determine the pulsation frequencies long time series are needed. Such data are obviously more efficiently acquired when a fairly large group of stars can be observed simultaneously. In a previous paper Frandsen et al (Ref. 1), analysing CCD-images of open clusters, found a dozen probable δ-Scuti stars in the cluster NGC 2660.

So far no roAp star has been located in a cluster. Due to the peculiar atmospheres of these stars the determination of the age, the mass and the chemical composition is difficult by traditional spectroscopic methods. Membership of a cluster would answer some of the questions.

In Ref. 1 indications were found of a change with time of the fraction of variable stars in the instability strip. The threshold for detection of variability was not uniform and in some clusters too high to give a clear picture of the statistics.

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1. INTRODUCTION

Presented here are the first results from a survey of open clusters with the four main objectives:

- Find a field with a dozen or more δ-Scuti stars
- Find a rapidly oscillating Ap-star (roAp) in a cluster
- Determine the fraction of variables in the instability strip with a very low detection threshold.
- Check the ultimate noise limit of relative photometry for bright stars with different detectors and telescopes.

Figure 1. Grey scale plot. North is up. West is right.
4. LIGHT CURVES

Light curves for individual stars are obtained the following way: After bias-subtraction and careful flat-fielding the DAOPHOT package is used to calculate magnitudes within a set of apertures centered on each object, and for each star a list of raw magnitudes is obtained for all frames during a night. A careful choice of stars is made to define an ensemble mean magnitude. The selected stars are not affected by crowding or defects, are not deviating in colour and are not too bright or too faint. The stellar magnitudes are then computed relative to this mean removing the effect of extinction. The extinction and the seeing vary by more than a factor two during 6 hours. Figure 2 consists of four panels, two panels with raw magnitudes, one with relative magnitudes and one with the variation of the seeing around the mean.

Figure 2. Upper panels: Changes in raw magnitude as function of time. Lower panels: Relative magnitude for the 1. night and seeing changes. The mean seeing was 3.31 pixel. The scale is ±2 pixel/arc.

Star number 88 is clearly not a variable. In the following figure 3 the light curves for the 10 brightest stars have been plotted. The number and the magnitude of each star are indicated. The two brightest stars are overexposed under good seeing conditions, which accounts for some additional scatter in the measurements appearing before the middle of the period. Star number 71 and 143 are very red stars, and at the end of the night the high extinction causes the light curve to increase. A colour correction has been implemented which removes the effect. One genuine variable (30) is found with a period similar to the length of the time string and a semi-amplitude of a few percent. The rest of the stars are constant.

Figure 3. Light curves. Time in unit of 1000s. Stars have been shifted vertically for clearness.

King (Ref. 3) has made CCD-observations of the cluster and presents a colour-magnitude (CM) diagram. According to this diagram the bright stars should fall within the instability strip. We have analysed all the brighter stars for variability, but except for the star mentioned already, none of the bright stars are variables, not even at very low amplitudes (Ap-stars). Roberts et al (Ref. 4) have published an implementa-
tion of the CLEAN algorithm, with which we have computed amplitude spectra. As is evident from figure 4, even very small amplitudes are detectable, but we have found no peaks, not even at millimagnitude levels, which persist from night to night in the bright stars in the period range from 5 minutes to 3 hours.

As the current estimates of the frequency of δ-Scuti stars among field stars in the instability strip give a ratio between variables and non-variables of 1:2, we began to doubt the reddening and distance used by King (Ref. 3).

5. PHOTOMETRY

We also made observations with the Johnson U, B, V and the Gunn r filters. The sensitivity of the CCD and the filters does not give colours easily transformed to standard colours. Especially the U colour is difficult to calibrate, because the sensitivity of the CCD cuts the blue transmission of the filter. Nevertheless we have a better material, than King had, to find the reddening. We find a considerably larger value as can be seen in Table I, where also the estimated distance and age of the cluster are given.

![C-M diagram NGC 6192](image)

Figure 5. Colour-magnitude diagram for NGC 6192. The instability strip is indicated with dashed lines.

As a result the instability strip moves towards the faint end of the diagram. Figure 5 is the CM-diagram with the instability strip indicated, based on our parameters and observations. Even for the faint stars in the instability strip, we can detect variability if the semi-amplitude exceeds 1.5 percent.

### Table I

<table>
<thead>
<tr>
<th>Basic data for the cluster</th>
<th>King (Ref. 3)</th>
<th>This project</th>
</tr>
</thead>
<tbody>
<tr>
<td>E(B-V)</td>
<td>0.26</td>
<td>0.85</td>
</tr>
<tr>
<td>Distance (Kpc)</td>
<td>1.0</td>
<td>2.9</td>
</tr>
<tr>
<td>Log age (yr)</td>
<td>9.1</td>
<td>6.9</td>
</tr>
</tbody>
</table>

6. VARIABILITY

To identify possible variables we plot all stars in a scatter diagram \( \sigma(m) \), where \( \sigma(m) \) is the r.m.s deviation from the mean. The constant stars define a very precise curve which is a function of the different noise sources, mainly photon noise from the star or the background. Stars above this curve possibly are variables, but crowding, colour differences or bad columns can also affect the accuracy. The stars above the curve have been checked and the only real variable is number 30 seen in figure 2. It is marked by a star in figure 5 and 6. Its position in the CM-diagram suggests it is a δCepheid type variable.

![Figure 6. Rms deviation for each star against magnitude.](image)

The noise levels we reach correspond closely to the levels obtained by Gilliland and Brown. For bright stars we get the same error per point (0.0025 mag) using an RCA detector. The larger telescope we use is compensated for by a smaller duty cycle (20s vs. 35s exposure per minute) and a narrower filter.

7. STATISTICS

At last we present a small Table II, which summarizes the results of the search for δ-Scuti stars in clusters.
A general trend is beginning to appear showing a higher fraction of variables in old clusters (age > 10^8 yr). The threshold for detection is still too different (and too high) to make the comparison between clusters easy.

There are at least two ways to prevent the \( \kappa \)-mechanism to excite stars in the instability strip. Diffusion can with time remove the Helium ionization region necessary to drive oscillations, which would tend to stabilize stars with age. Timescales of 10^6 yr are typical for diffusion processes.

Non-linear mode-coupling can be enhanced by rotation (Dziembowski et al, Ref. 5), distribute the energy over many modes and thus decrease the amplitude. As the rotation of A-stars probably will decrease with time (on some so far unknown timescale), older clusters will tend to show stars with larger amplitudes. An increase in amplitudes also occurs, as the stars move away from the main sequence.

8. CONCLUSION

We did not succeed with respect to the first two objectives: The search for \( \delta \)-Scutis and roAp-stars. But the accuracy of the results obtained for NGC 6192 clearly has the potential to unravel the statistics of excitation of variables in clusters, and it is possible with the noise level reached to detect both very low amplitude \( \delta \)-Scutis and roAp-stars, if these stars are among the brighter stars observed in a cluster.

For observations of short period variations (P < 20 min) CCD photometry at the accuracy of the photon noise seems possible. For longer periods seeing and extinction changes makes it difficult to remove all the drift.

9. REFERENCES

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SOLAR CALIBRATION OF ASTEROSEISMOLOGY
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ABSTRACT / RESUME

The first expected result of asteroseismology will consist in the measurement of two parameters, \( \Delta v \) and \( D_0 \), which in the Tassoul's asymptotic approximation are close to the sound speed integral across the solar radius, and to the sound speed gradient inside the nuclear burning core, respectively. The first depends mostly on the mass and evolves slowly with age, while the second is strongly age dependent through the increase of molecular mass in the stellar core. Theorists have started to build \( \Delta v - D_0 \) diagrams, as a tool for the determination of mass and age of stars. For a precise calibration of such a diagram we have made a precise measurement of those parameters in the case of the sun.

Keywords: solar oscillation, solar structure, stellar masses, stellar ages.

1. ASYMPTOTIC APPROXIMATION

For modes of degrees 0 to 3 the frequencies of the solar eigenmodes are reasonably well described by Tassoul's asymptotic equation (Ref. 1):

\[
\nu(n, l) = \nu_0(n + l/2 + \epsilon) - \epsilon l(l + 1) - \epsilon \nu_0^2/\nu
\]

The role played by the various coefficients in this equation is efficiently illustrated by a representation of the measured Fourier spectrum on an échelle diagramme (Ref. 2), using the quasi-equidistance \( \nu_0 \) of the frequencies of successive harmonics of a given degree (Fig. 1). The first term would generate two vertical lines on the échelle diagram for modes of odd (1 and 3) and even (0 and 2) degree. The second term describe both the curvature and the separation (from 2 to 4) of the vertical line on the échelle diagramme through the parameter \( \epsilon \). Following Tassoul's equation, the separations \( \epsilon = 0 \) to \( \epsilon = 2 \) and \( \epsilon = 1 \) to \( \epsilon = 3 \) should decrease with frequency, roughly as \( \nu^{-1} \). Figure 2 shows that indeed, the decrease in frequency is present, but that it does not follow the \( \nu^{-1} \) variation. This is where the asymptotic approximation reaches its limit of validity.

2. POLYNOMIAL DEVELOPMENT.

\( D_0 - \Delta v_0 \) DIAGRAM

To have more precision on the measurements of the parameters \( \nu_0 \) and \( A \), several authors (Ref. 2, 3, 4, 5) have proposed to use, instead of Tassoul's equation, a polynomial development of the four lines in the échelle diagramme around a given point in the most energetic part. This lead to the substitution of the parameters \( \Delta v_0 \) and \( D_0 \) for the parameters \( \nu_0 \) and \( A \) of Tassoul's equation. The gain in precision using this method allows us to make much more precise comparisons of the theoretical predictions with the measured values.

The parameters \( \Delta v_0 \) and \( D_0 \) are geometrical parameters of the échelle diagramme, close to the parameters \( \nu_0 \) and \( A \) of Tassoul's equation (Ref. 5). \( \Delta v_0 \) measures the integral of the sound speed between center and surface; due to the sound gradient, it is predominantly sensitive to conditions in the outer layers. It depends mostly on the stellar radius, which itself depends on the mass, and will evolve slowly with the main sequence life of the star. \( D_0 \) depends mostly on the gradient of the sound speed in the core. It evolves with age, through the change in central molecular mass due to hydrogen burning into helium.

3. ANALYSIS AND RESULT

In our analysis, we have used the data of the two South Pole campaigns, in 79/80 (Ref. 2, 6) and in 84/85 (Ref. 7, 8), and also the 1980 ACRIM data (Ref. 9). \( \Delta v_0 \) has been estimated, using a polynomial fitting on the \( \ell=0 \) line of each échelle diagramme (Ref. 16) and has been used to determine the slope at \( n=22 \). The result is:

\[
\Delta v_0 = 135.1 \pm 0.1 \muHz
\]

The determination of \( D_0 \) is more difficult because we wanted to avoid the weakness inherent to the methods using the definition of the frequencies of the peaks. Two steps were used (Ref. 16):
- a first order approximation by a linear fit.
- using this first value to generate modified pairs of
eigenmodes, we performed cross-correlations between all our sets of data. These correlations are then used to determine accurately $D_0$ (Fig 3.).

$$D_0 = 1.52 \pm 0.03 \mu \text{Hz} \quad (3)$$

4. DISCUSSION.

In all standard solar models, $\Delta \nu_0$ is about 1 $\mu$Hz in excess of our value, much beyond the error bar. Possibly the error is located mostly in the outer layers of the standard model, at the eigenmodes boundary layer or in the upper part of the convective zone (Ref. 14).

The value of $D_0$ is directly related to the structure of the solar core. It is then of special interest in the questions of the neutrino flux or of the possible core mixing. The most recent frequency tables of standard models give a mean value of 1.52 ± 0.01$\mu$Hz. This is impressively consistent with the result of this analysis. It seems to exclude definitely the presence of enough mixing in the core to satisfy the neutrino flux (Ref. 10, 11). It does not exclude the possibility of non standard models, using ad-hoc mechanisms for the neutrino problem and still potentially consistent with the present result, like the WIMP model (Ref. 12, 13).

We just wish to point out the small uncertainties of 2% on $D_0$, the extremely severe constraint that it implies on the solar core model, and the fact that present standard models are perfectly consistent with the measured $D_0$.

It is possible to plot $D_0 - \Delta \nu_0$ for evolutionary sequences of star of a given mass and for variable masses at a given main sequence age (Ref. 14). One point in this diagram corresponds then to a unique pair (mass, age). It may be expected that within the next few years, the two parameters $\Delta \nu_0$ and $D_0$ will be accessible for at least a few bright main sequence stars. It is then of crucial importance to have a precise solar calibration for the determination of stellar mass and age through the $D_0 - \Delta \nu_0$ diagram. Figure 4 shows our result in such a diagram. However, one must keep in mind that this precise calibration is independent of the accuracy of theoretical models, and that the uncertainty on such parameters as effective temperature, temperature, or heavy elements abundance imply errors on mass and age much larger than the estimated uncertainty on our two asteroseismic parameters (Ref. 15).

5. REFERENCES

Fig. 2. Variation of $v(0) - v(2)$ (lower part) and of $v(1) - v(3)$ (upper part) as a function of frequency. On the lower part are the measured values, taken from the same compilation of all available results by Divall et al. (1987). On the upper part are the theoretical values from 6 standard models. Within the visible precision, both are consistent with a linear decrease between, say, 2.5 and 3.2 mHz. The polynomial development in $x$ explained in the text and used to describe the schelle diagram defines $\Delta f_0$ as being, on the lower curve, the point of abscissa 3.17 mHz. Authors on the upper part are: (*) Shibahashi et al, 1981, (O) Hirschl et al. 1983, (©) Shibahashi et al. 1983, (O) Lebreton et al. 1987, (©) Berthomieu et al. 1987, (©) Berthomieu et al. 1987

Fig. 3. After selection of 6 frequency windows which contain each a pair of modes of degree 0 and 2, and after adequate frequency scale modification inside each of these windows, this is the average of three cross-correlations using power spectra from ACRIM 1980, South Pole 1980, and South Pole 1984/85. Before averaging, the three curves have been translated in order to have a good coincidence of the central peak. The value of the needed translation is a measure of the mean frequency shift between the epochs of the two data sets. Then the two lateral peaks visible here are located respectively at $-6\Delta f_0$ and $+6\Delta f_0$. 

Normalized correlation

Frequency (mHz)


HIGH PRECISION VELOCITY MEASUREMENTS OF PROCYON USING THE 7699 Å LINE OF POTASSIUM.

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ABSTRACT

The Star Procyon (a CM) was observed for 2 weeks using the Birmingham double magneto-optical filter spectrometer on the 1.9 m reflector of the South African Astronomical Observatory in 1988 January. Our analysis of these velocity data shows that the noise level (per frequency channel of ~ 1 μHz) in the power spectrum of the combined nightly runs is approximately 12 m s⁻². These data were searched for the presence of small scale oscillations, but none were obviously present. On one night the surface integrated magnetic field of Procyon was measured using a rotating quarter wave plate to select alternatively opposite senses of circularly polarised light. Our preliminary analysis indicates that |B| is 1.3±4.0 Gauss (standard error), consistent with a non detection at the few Gauss level.

Keywords: Procyon, Stellar Seismology, Magnetic fields

1. INTRODUCTION

The Birmingham Stellar Spectrometer is based on the double magneto optical filter device, originally developed for solar studies by Cimino et al (Ref. 1), and is briefly described in Isaak and Jones (Ref. 2), along with the results of the first use of the instrument on Procyon during 1986 January. In outline, the spectrometer measures the blue (B) and red (R) intensities of star light in two bandpasses (of order 50 mÅ each), of separation ~ 5 km s⁻¹ in velocity units, symmetrically placed about the rest wavelength of the Potassium 7699 Å line. When observing a star which has a geocentric velocity within ~ ± 2.5 km s⁻¹, the spectrometer bandpasses are thus on opposite sides of the stellar potassium line. The intensities, B and R are measured, and the ratio (B-R)/(B+R) is to first order a linear measurement of the star's geocentric velocity. The components B and R are not measured simultaneously, but are chopped at 50 kHz.

The instrument is limited to solar type stars (those with a strong Potassium line), and whose velocities, relative to the earth, pass through or approach zero. For Procyon (F5IV), the earth's orbital motion exactly cancels the star's heliocentric velocity in late January each year, so that for ~ two weeks the star is observable with our instrument. The observation of the time the geocentric velocity equals zero (the crossing point) yields a precise measurement of the star's heliocentric velocity. A long series of such observations could, in principle, detect a Jupiter-like planet orbiting a solar type star. (This in fact was the impetus behind the initial studies that commenced in Birmingham in 1969). Three crossing points of Procyon have now been observed, in 1986 (see Ref. 2), 1987, and 1988. These results, along with a fuller analysis of the data discussed below (incorporating a full calibration of the data, see Ref. 3), will be presented elsewhere.

The observations obtained also allow us to search for small scale normal mode oscillations - the stellar equivalent of the solar 5 minute oscillation. We present here the preliminary results of our analysis.

2. OBSERVATIONS

We used the 1.9m reflector of the Sutherland station of the South African Astronomical Observatory to observe Procyon on eleven nights between 1988 January 12-25. We often observed over 8 hours per night, extending well into twilight with no effect on the measured velocity. The observed counting rates for each of the B and R channels varied from ~ 1 to 3 KHz, depending on observing conditions and where on the stellar absorption line the measurement was obtained. The integration time was eight seconds. The instrument is most sensitive when the B and R components are symmetrically placed about the stellar line (i.e. at the crossing point). Hence the first few days of data are of low sensitivity. The data were converted from the observed ratio to velocity using the calculated orbital and rotational changes due to the earth's motion (Ref. 2). This of course would remove any low frequency signal in the data.

The crossing point was observed on 1988 January 22. The velocity of Procyon as observed for this night is plotted in Figure 1.

In addition to the observations of Procyon, during the day we fed sunlight into the spectrometer via a 200 μm diameter silica optical fibre, as well as observing white light and a Potassium lamp as a check on instrumental offsets and drifts. An initial analysis indicates the instrument does not suffer greatly from such effects.
3. RESULTS

3.1 Procyon Data

The velocity data were detrended, and combined into points of 32 second integrations. Figure 2 shows a Power Spectrum of the velocity residuals of these data for the nights of 1988 January 17, 18, 21, 22, 24 and 25. The frequency resolution is 0.95 mHz. The vertical ordinate is the root mean square amplitude squared. The standard deviation of the spectrum per frequency interval (~1 mHz) is ~12 m$^2$s$^{-2}$, which we take as an estimate of the noise level of the data.

A close inspection of this spectrum showed there were a number of lines, greater than 3σ in power, with sidelobes of the spacing (~11.6 mHz) expected for a real signal in the velocity data. Such lines are found over a range of frequencies, including the 5 minute region, as well as much lower frequencies. We cannot as yet be certain of their origin, and are continuing the analysis.

There is little obvious agreement between our Figure 2 and the power spectrum obtained for Procyon by Grec et al (Ref. 4). It is of interest to compare the relative sensitivities of the two investigations. Grec et al show in their Figure 6 the filtered power spectrum, plotted with a resolution of 80 mHz. If the line widths of any modes present in Procyon are of the same order as for the Sun (<1 mHz at 2 mHz, Refs. 5 and 6), then such a coarse resolution has the effect of smearing the signal over a large range. Hence, for the data in Ref. 4, a measure of the strength of the individual oscillations reported can be made from reference to their Figure 6: For the peak near 1.5 mHz, taking a mean power of ~10 m$^2$s$^{-2}$ mHz$^{-1}$, and the bin size of 80 mHz, gives a mean square velocity for the signal of 0.8 m$^2$s$^{-2}$. If this came from just one mode (of width 1 mHz), it implies its strength is of order 60 m$^2$s$^{-2}$. If several lines contributed the mean figure for the individual lines would of course be reduced. As mentioned above, the noise level in our power spectrum (Fig. 2) is ~12 m$^2$s$^{-2}$ per frequency bin (of ~1 mHz). This suggests that the two investigations are of roughly equal sensitivity. Obviously further data, at lower noise levels, are needed to continue the study of Procyon.

3.2 Solar Data

As mentioned, we also obtained observations of integrated sunlight, which was fed into the spectrometer mounted on the telescope via a 50 m optical fibre. Typical counting rates were of the order of 20 kHz for each of the B and R channels. Figure 4 shows a power spectrum of the solar data taken on 1988 January 17, 18, 22, 24 and 25 (and thus is of comparable extent to the Procyon spectrum in Figure 2). The spectrum is dominated by the signal at low frequencies, which we believe is related to the thermal stability of the spectrometer, although poor weather contributes too. The standard deviation per frequency channel in the spectrum is ~0.4 m$^2$s$^{-2}$ per 1 mHz bin.

Comparing these solar data with those for Procyon, obtained at ~0.05 to ~0.1 of the counting rate, would suggest that the noise level of the Procyon spectrum should be between ~4 to ~9 m$^2$s$^{-2}$ per frequency bin (of ~1 mHz), slightly less than observed. However, Procyon has a broader and shallower absorption line, so that the sensitivity (proportional to the slope of the line) is reduced for Procyon compared with the Sun.
3.4 Line Profile Reconstruction

Figure 5 is a plot of the observed ratio against the calculated velocity (from the earth's orbital and rotational motion). Each data point is a 128 second integration. In effect it is also a rough differential of the line profile - being the slope of the spectral line plotted against wavelength (as over the limited wavelength range a change in velocity is essentially directly proportional to a change in wavelength). As a "zeroth" approximation to the line profile, a 5th order polynomial was fitted to the data in Figure 5.

4. CONCLUSIONS

From the above results we have shown that our instrument in its present form is capable of measuring velocities of bright stars to around the 10 m s⁻¹ level with ~ 1 week of observing time on a 2 metre class telescope. We also believe that the potential exists for high precision measurements of the integrated magnetic field of bright stars, almost certainly to below the 1 Gauss level. As well as this, the mean line profile of the observed spectral line is encoded in the data, and should be recoverable. We are currently upgrading the spectrometer to improve both the throughput and the sensitivity, and are hopeful of significantly improving the quality of the data we obtain.

3.3 Magnetic Field Observations

On the night of 1988 January 25 we incorporated a quarter wave plate in the light path of the spectrometer. This was rotated through 90° into one of two positions every 72 seconds (144 seconds for a complete cycle). The instrument is therefore sensitive to opposite senses of circularly polarised light. If, integrating over the stellar surface a small residual longitudinal magnetic field is present (as on the sun), this will separate the magnetically sensitive absorption lines in the star's spectrum. For the Potassium 7699 Å line the effect is a separation of around 3 m/s for every 1 Gauss of resultant field. A preliminary analysis of this night of data yields a mean magnetic field of $\mu B = 1.3G$, with the standard error in the mean of 4.0G, consistent with a zero resultant field for Procyon.

We point out though, that as for velocity, our sensitivity for this measurement decreases as we move away from the crossing point, so that we may be underestimating the error in the above measurement. As well, a full analysis of possible systematic effects has not been completed. However, we find this first result encouraging, as it indicates that a longer run should reduce the standard error to below the 1 Gauss level.
Dr. A.R. Jones to the construction of the stellar spectrometer. We also thank the Birmingham HIROS group technical staff for electronic and mechanical support, and for valuable assistance in the preparation for the observing run. We thank Dr. R. New of Birmingham for discussions concerning the analysis. Financial support for the construction of the spectrometer was from the School of Physics and Space Research at Birmingham. Travel to South Africa was funded by SERC. J. Innis is supported by an SERC Research Fellowship.

9. REFERENCES


5. ACKNOWLEDGEMENT

It is a pleasure to thank Dr. M. Feast, Director of the South African Astronomical Observatory, for the allocation of time on the 1.9 m reflector, and SAAO staff, particularly Mr. G. Woodhouse, for excellent assistance during the observing run. We acknowledge the contribution of former Birmingham research student

Figure 6 A "zeroth order" approximation to the 7699 Å line profile of Procyon, found from the integration of the polynomial that was fit to the data in Figure 5.
Radial velocity measurements were taken of a group of Ap stars using a newly improved FP-ISOS. Observations were made using the 1.5m TCS on Tenerife in May 1987 and December 1987. The already known Rapidly oscillating Ap stars 33 Lib and HR1217 were observed in order to search for radial velocity variations corresponding to the photometric periods found in these stars. Simultaneous photometric measurements were made on HR1217 on the last two nights using the nearby 0.5m telescope. The radial velocity and photometric data sets have been reduced using a weighted sine wave fitting routine. Promising results have been obtained for HR1217, while results obtained for the other Ap stars observed do not allow to any conclusion. Data taken several years ago on GCor was re-examined and the results obtained are also discussed.

Keywords: Ap stars, RV variations.

1. INTRODUCTION

Oscillations with periods in the range from 1 to 4 mHz have been discovered in, at least, twelve Ap stars so far via high-speed photometry. These stars have been named (Ref. 5) "Rapidly oscillating Ap stars" (hereafter ROApS). The physical characteristics of almost all the class are: strong magnetic field, low temperature (b-y<0.075), negative ÒC index, and strong spectral lines of Sr, Cr, Eu, and other rare earths. A comprehensive review of the class and their parameters can be found in Kurtz (Ref. 6). However a lot of questions remain open about the true nature of the class since the oscillations of ROApS have been seen basically photometrically. Are these stars actually pulsating? As the light variations are modulated by the magnetic period (Ref. 5), are the radial velocity (RV) variations modulated also? What is the geometry and nature of the pulsation? Is it similar to the one in GScuti stars?

To our knowledge, only two previous reports (Refs. 2, 10) have claimed to have discovered RV variations in these stars at a level of a few hundredths m/s, but the lack of information does not permit one to resolve completely any of the stated enigmas. Thanks to a collaboration between the "Seismology" groups of the Imperial College and the Instituto de Astrofisica de Canarias, a project to observe, simultaneously in radial velocity and luminosity, a group of Ap stars (two of them rapid oscillators) has been carried out in 1987.

RV measurements were taken using the FP-ISOS (Ref. 11), newly improved by referencing the instrument to the Gd blue line (Ref. 12), attached through an optical fibre to the Cassegraln focus of the 1.5m Casinos Sanchez telescope (TCS) at the Observatorio del Teide (OT). Observations were obtained on three nights in May 1987 and during a three weeks observing run in November and December 1987. The IAC/UBV photometer with a B filter attached to the nearby 0.5m telescope was used to perform the rapid photometry observations when the atmospheric conditions were sufficiently good.

In table 1, the journal of the observations is presented. At the end we were only able to obtain about 80 hours of data in both telescopes mainly due to the weather conditions. The ROApS 33 Lib and HR1217 were observed in order to search for velocity variations corresponding to the photometric periods found in these stars. Also, the "non-oscillating" Ap stars GCor and 49 Cam project to observe, simultaneously in radial velocity and luminosity, a group of Ap stars (two of them rapid oscillators) has been carried out in 1987.

<table>
<thead>
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<th>STAR</th>
<th>DATES</th>
<th>n' of hours</th>
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<td>126898</td>
<td>GCor</td>
<td>24/25 Feb 83</td>
<td>2.77</td>
</tr>
<tr>
<td>137909</td>
<td>33Lib</td>
<td>5/6 May 87</td>
<td>1.60</td>
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<td>-</td>
<td>6/7 May 87</td>
<td>6.25</td>
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<tr>
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<td>GCor</td>
<td>7/8 May 87</td>
<td>6.25</td>
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<tr>
<td>62140</td>
<td>49Cam</td>
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<td>-</td>
<td>-</td>
<td>15/16 Dec 87</td>
<td>6.50 (5.87)</td>
</tr>
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</table>

Table 1: Journal of observations.

The HD number and proper name of each star are presented together with the dates and number of hours it was observed. All the observations, but the first, were done with the 1.5m TCS at Izaña. The numbers in brackets are two nights of simultaneous photometric measurements obtained on the nearby 0.5m MOKO.
were monitored looking for radial velocity variations. The data was submitted to a harmonic analysis via iterative sine wave fitting (ISWF) procedure for unequally spaced data, looking for periodicities in the range of a few mHz. The results obtained are presented in section 2.

Using this reduction technique, the radial velocity measurements obtained on the star GCr, using the 1.9m telescope at SAAO by Pietrzewicz in February 1983 were re-examined looking for the photometric period known for this star (Ref. 6).

2. RESULTS AND DISCUSSION

2.1. High-speed photometry

Unfortunately, only two nights of data of very good photometric quality were obtained at the same time as the RV observations. In these nights (see table 1), the ROAPS HR1217 was monitored. This star, one of the best known rapid oscillators, has a very complicated structure of oscillation frequencies, with at least 5 nearby periods centred at 6.1 minutes (2.7 mHz), and amplitudes of a few millimagnitudes (mmag) modulated with a period of 12.4564 days (Ref. 8).

The highest peak in the plot of the three series, only for the night of 12/13 December which was replaced by a higher one at 2.7 mHz the next day.

Both sets of data were put together and reduced via ISWF. Obviously the aliasing problem becomes very important due to the small sample of data. Successive cleaning of the highest peaks, starting with the clearest at 2.6395 mHz, yielded the results presented in table 2. They agree quite well, within the resolution, with the ones in literature (Ref. 8). Considering the poor resolution, the daily aliases of the two last frequencies could have been chosen as the correct ones.

Table 2: HR1217 photometric results.

<table>
<thead>
<tr>
<th>Frequency (mHz)</th>
<th>Period (min)</th>
<th>Amplitude (mmag)</th>
<th>Phase (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.6395</td>
<td>6.31</td>
<td>1.36</td>
<td>0.14</td>
</tr>
<tr>
<td>2.7204</td>
<td>6.13</td>
<td>1.21</td>
<td>-1.14</td>
</tr>
<tr>
<td>2.7520</td>
<td>5.05</td>
<td>0.90</td>
<td>2.84</td>
</tr>
</tbody>
</table>

The amplitude maximum was very late in the two spectra obtained, with an amplitude near 500 mmag, more than one period can be clearly seen in both plots. A peak at 2.64 mHz with an amplitude near 1.4 mmag is present in the two spectra, however the second main frequency peak changed completely from one night to the other; the original peak at 2.74 mHz was replaced by a higher one at 2.7 mHz the next day.

2.2. Spectrometry

2.2.1. HR1217. This star has been studied very deeply since it was the only one with simultaneous photometric data. HR1217 was monitored for 8 nights in November and December 1987, however, only for the night of 12/13 December which included the amplitude maximum, was the peak centred at about 2.72 mHz the highest in the power spectra obtained, with an amplitude near 500 ms⁻¹, in agreement with Matthews et al. (Ref. 10). They claimed to have discovered radial velocity variations in this star at the photometric frequency of about 2.7 mHz, in the amplitude of a few hundred meters per second. A study of the daily amplitudes is deferred to section 2.3.1.

In order to reduce the noise and to increase the resolution, series of more than one observing run were produced and then analyzed via ISWF. For these series, only the best data, with the highest S/N and the lowest noise levels (below 500 ms⁻¹) were used. Three series were made, one with the four best nights (11/14/15), another with the data nearest to the amplitude maximum (12/14), and the last with the data with simultaneous high-speed photometry (14/15).

The highest peak in the plot of the three power spectra was one centred near 2.72±0.01 mHz (see figures 2 and 3), however no evidence of the other photometric frequency at 2.64 mHz were found. Table 3 gives the frequencies, amplitudes and phases corresponding to the peaks as well as the first daily alias (nearer to the photometric frequency) when the highest peak was at 2.73 mHz. Given the "noisy" appearance of the spectra and in order to know how much confidence we could have in the results, False Alarm Probabilities (FAP) (Ref. 4) were calculated. The values obtained not only for HR1217, but also for GCr and 49 Cam, which we will discuss later, are
have been related, within the resolution, to the 4.5 mHz, with an amplitude of 228 m/s, could be performed simultaneously. The second peak, near which agrees quite well with one of the frequencies found in the photometric observations performed simultaneously. The second peak, near 4.5 mHz, with an amplitude of 228 m/s, could have been related, within the resolution, to the one discovered in the photometric data at 2.7204 mHz. The other two frequencies found in luminosity were not in the RV spectra at confidence levels higher than 50%. For the longest series, the highest peak and its first daily alias, nearer to the photometric frequency, are presented.

2.2.2. 49 Cam. Since some evidence that the cool magnetic Ap star HD62140 (49 Cam) is oscillating with periods near one hour has been found, by rapid photometry, see Ref. 9, and later confirmed (Ref. 3), we decided to include this star in our observing plan. However, due to the short length of the data series obtained, only 9.1 hours, we could not claim to confirm the photometric results for the peak found at 0.37±0.08 mHz (see figure 4), with an amplitude near 1.22±0.15 kms⁻¹. This is not only because the frequencies were not exactly the same, but also because the FAP (see table 4) for this peak was as high as 48%.

2.2.3. 33 Lib. The ROAPs HD137949 (33 Lib) oscillates with a period of 8.29 minutes (2.01 mHz) and a peak to peak amplitude near 2.5 mmag (Ref. 6). It was monitored for more than 6 hours in May 1987 in an attempt to find the previously quoted period in radial velocity measurements.

Although there is a peak in the power spectrum obtained at 2 mHz with an amplitude of 310 m/s, the noise level is too high and obviously we are unable to come to a conclusion.

2.2.4. CorBor. Despite the photometric characteristics of this star, quite similar to ROAPs, no traces of oscillation have been reported so far in photometric measurements, however Ando et al. (Ref. 2) found some evidence of radial velocity variations at a frequency of about 2.7 mHz and amplitudes near 0.3 m/s. Although we would have liked to observe this star carefully, our initial results do not seem to confirm Ando et al.'s ones. The very low amplitude found for the peaks between 2 and 3 mHz cause us to think that it is also radial velocity stable in this range, at least at a level of 200 m/s⁻¹.

<table>
<thead>
<tr>
<th>STAR</th>
<th>n</th>
<th>σ</th>
<th>Δν</th>
<th>N_f</th>
<th>A</th>
<th>FAP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clr</td>
<td>316</td>
<td>2.90</td>
<td>0.8</td>
<td>78</td>
<td>1015</td>
<td>0.009</td>
</tr>
<tr>
<td>49 Cam</td>
<td>674</td>
<td>7.10</td>
<td>0.8</td>
<td>122</td>
<td>0.48</td>
<td></td>
</tr>
<tr>
<td>HR1217</td>
<td>3903</td>
<td>2.69</td>
<td>0.8</td>
<td>2760</td>
<td>258</td>
<td>0.29</td>
</tr>
<tr>
<td></td>
<td>3903</td>
<td>2.69</td>
<td>0.8</td>
<td>2760</td>
<td>228</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>3903</td>
<td>2.69</td>
<td>2.6</td>
<td>70</td>
<td>258</td>
<td>0.009</td>
</tr>
</tbody>
</table>

Table 4: Confidence levels of RV results.

False alarm probabilities (FAP) (Ref. 4) have been calculated for different amplitudes obtained via ISWF. In this table we present the values worked out for every star and amplitude of the peak, including: number of data points, standard deviation per measurement, frequency range and the corresponding number of independent frequencies. The amplitude of 228 m/s for HR1217 corresponds to the second peak at near 4.5 mHz.
correspond with the periods near 1 hour that some reached, the highest peak at 0.3 ± 0.08 mHz could not be explained by photometric measurements (Ref. 3). This peak has an amplitude near 1.22 km/s and a confidence level of 52% which does not permit us to be conclusive.

2.2.5. **ACir.** The radial velocity measurements obtained on the ROApS ACir using the 1.9 m telescope at SAAO several years ago was also re-examined via ISWF looking for the photometric period reported for this star (Ref. 6). Figure 5 shows the power spectrum obtained. The highest peak is at 2.5 ± 0.1 mHz with an amplitude of 1015 ± 160 m/s. With a confidence level higher than 99% (see table 4), it is certainly significant and agrees, within the resolution, with the one in literature. The second peak, near 4.8 mHz might also be significant although this has never been observed photometrically.

![Figure 5: Power spectrum of the whole sample of data on 45 Cam.](image)

The first conclusion could be that the ROApS have been studied over a period of time, and as a result, some other peaks have been discovered. Nevertheless, some other photometric measurements have not been found. Nevertheless, some other questions remain open.

2.3.1. **Amplitude modulation.** According to the "oblique pulsator model" (Ref. 5), the light amplitude modulation as well as the RV amplitude of the pulsation should be modulated with the rotation period of the star, and both should even have the same phase.

The fact that the amplitude of the light variations changes with the rotation period had been tested several times in various ROApS, and specially in HR1217. However, no strong evidence of this fact had been found in RV measurements. In order to provide our small but significant contribution, the RV amplitudes at three different frequency channels: 2.63, 2.73 and 2.78 mHz were calculated for every observing run on HR1217. The results are presented in table 5, while figure 9 shows the corresponding plot of the values. The first two channels were, given the resolution (0.02 mHz), the nearest to the highest photometric frequencies. The third was for use as a reference.

At first glance, it seems difficult to conclude anything. Nevertheless, the peak at 2.63 mHz for the 11th is not considered and taking into account the very high error of the 13th, a slightly tendency to modulation could be found in the data, considering that the amplitude modulation may have happened early on the 11th and later on the 13th; however the evidence is not very strong and we need to be cautious before claiming to have discovered RV amplitude modulation.

<table>
<thead>
<tr>
<th>Date</th>
<th>2.63 mHz</th>
<th>2.73 mHz</th>
<th>2.78 mHz (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>29 Nov</td>
<td>110</td>
<td>262</td>
<td>249</td>
</tr>
<tr>
<td>30 Nov</td>
<td>504</td>
<td>76(*)</td>
<td>238</td>
</tr>
<tr>
<td>9 Dec</td>
<td>340</td>
<td>322</td>
<td>161</td>
</tr>
<tr>
<td>11 Dec</td>
<td>638</td>
<td>276</td>
<td>446</td>
</tr>
<tr>
<td>12 Dec</td>
<td>220</td>
<td>426</td>
<td>75</td>
</tr>
<tr>
<td>13 Dec</td>
<td>464</td>
<td>715</td>
<td>166</td>
</tr>
<tr>
<td>14 Dec</td>
<td>186</td>
<td>106</td>
<td>251</td>
</tr>
<tr>
<td>15 Dec</td>
<td>223</td>
<td>148</td>
<td>161</td>
</tr>
</tbody>
</table>

(*): 432 m s^{-1} for the channel at 2.71 mHz.

Table 5: Radial velocity modulation.

2.3.2. **RV-to-light amplitude ratio.** In previous reports (Ref. 10) a value for the RV-to-light amplitude ratio of 2K/Δm=59±12 km s^{-1} had been found for data obtained on HR1217. According to the results that we obtained on the same star, this amplitude should have a value of 2K/Δm=139±6 km s^{-1} for the peak at 2.72 mHz, which is the only one present in photometric and spectrometric data. Why this discrepancy between the two values? Our value of 2K could be somehow overestimated by the relatively high noise level, or perhaps, as Matthews has suggested (private communication), the RV-to-light ratio may not be a constant for these stars.

Unfortunately, no simultaneous photometric observations were performed for the other stars,
given the ambiguity we have exposed in the determination of the frequency found simultaneously in luminosity and RV variations, between the peak at 2.732 mHz and its first daily alias at 2.720 mHz, two values for the phase lag could be worked out. Those would be 1.8±0.4 rads (10°±22°) for the first one and 2.1±0.4 rads (11°±22°) for the one at 2.72 mHz. Are the ROApS pulsating in the same way as the ÔScuti stars or, on the contrary, they are non-radial pulsators similar to the MS stars? This question is very difficult to answer with the marginal results obtained for just one star, but our previous point suggests that the second asseveration may be the correct one. However, further simultaneous RV and luminosity observations of other ROApS will have to be done before obtaining a final answer.

The 1.5m TTT and the 0.5m telescope in the island of Tenerife are operated by the IAC in the Spanish Observatorio del Teide.

3. REFERENCES

OSCILLATIONS OF JUPITER AS A TOOL FOR PROBING ITS INTERNAL STRUCTURE

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ABSTRACT

The internal structure of Jupiter is still uncertain. According to current models the planet exhibits two discontinuities of sound speed in the interior. The first one located in the external part of the hydrogen - helium atmosphere corresponds to the transition of molecular to metallic hydrogen, while the second one to the transition between the H-He envelope and the high density core. We use method of ray tracing of acoustic modes to predict the behaviour of expected pressure modes of Jupiter. Due to the inner discontinuity, the regular spacing between frequencies of adjacent models is broken in a characteristic manner which provides a clear signature. Observations of low degree p-modes, achievable from the ground, would accurately provide the value of the radius of the planetary core. Observations of high degree modes would provide a signature of the external transition.

Keywords: Jupiter, Oscillations.

1. JOVIAN INTERNAL STRUCTURE

The most recent models of the jovian internal structure are supported by the gravitational data from the Pioneer and Voyager measurements (Ref.1). Due to the uncertainty concerning both the compositional layering and the equations of state, the value of the estimated mass of the core fluctuates in a range of more than 50% (Ref. 2).

![Figure 1. Square of the sound speed from the values of pressure and density given by Hubbard’s model (Ref 3.). Both discontinuities in internal structure appear in the variations of \(c^2\).]

The presence of this denser core causes a sound speed variation, and it can be thus revealed by acoustic modes propagating in the deep interior. The core radius \(r_{\text{core}}\) is about \(R/10\), where \(R\) is the planetary radius. Another discontinuity appears in the jovian structure because of the possible existence of metallic hydrogen over 3Mbar; its properties remain uncertain but will induce a different propagation of the high-degree p-modes.

2. INNER DISCONTINUITY

The optical geometry approximation is sufficient to study the influence of both discontinuities. The dispersion law given by Gough (Ref. 4) includes the frequency of Brunt-Väisälä and a cut-off frequency. But calculations supplied with an interior model proposed by Hubbard (Ref. 3) show that these frequencies can be considered as null in the jovian interior. Then the p-modes verify only a pure acoustic dispersion law \(\omega = \frac{k}{c}\). The local sound speed \(c = \sqrt{T (p^2 M)}\) is inferred from the model of Hubbard. Concerning the inner discontinuity, we can use the Tassoul’s asymptotic expression of the low degree p-modes (Ref. 5,3) in which some free parameters are considered as null:

\[
\omega_n, \ell = \left( n + \frac{\ell}{2} \right) \frac{L^2}{2 \pi \omega_n, \ell} \int_\ell^\infty \frac{d \ell}{r} \left( \frac{d p}{d \ell} \right) \omega_n
\]

This expression remains valid despite the strong variation in the sound-speed. This affirmation proceeds from the concordance between the results given by an analytical method and the direct application of equation (1) in a case where only the sound speed discontinuity is considered, without any other radial variations.

Equation (1) shows that modes with the same \(n + \ell/2\) value differ because of the 2nd term which is, as for the sun, negative when the ray does not pass through the core, but positive in the opposite case. Its sign is settled by the sign of the sound speed gradient, normally negative when the chemical composition is uniform; here the structure discontinuity implies a positive correction as the density increases in the core. Therefore, the structure discontinuity causes a discontinuity in the regular spacing of the spectrum of eigenmodes (see fig 2). Only low-degree modes are represented; they correspond to periods between 4 and 17 minutes. The asymptotical expression (1) is not valid for the first harmonics (low \(n\)) so we have here \(n \geq 6\). The upper limit is arbitrarily \(n = 23\) in order to include the discontinuity of the degree \(\ell = 3\). The order \(n(\ell)\) at which the jump appears indicates the value of the core radius:

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If there is only a mixing of dense materials and helium and hydrogen (Ref. 6), there will be an abrupt discontinuity, but this will appreciably affect the sound speed gradient and its sign, and therefore the departure from the $n \to f/2$ rule.

$$ r_\text{core} = \frac{2L}{\pi(n(f+1/2) + L) \sqrt{f(f+1)}} $$  \quad (2)$$

3. OUTER DISCONTINUITY

Near the surface and the external discontinuity, the square of the sound speed increases almost linearly with depth, with a different coefficient in each phase of the hydrogen. This difference causes a break in the Duvall representation (figure 4). The value $\omega/k$ where the break appears gives the radius of the phase transition:

$$ r_\text{transition} = \frac{R_0 \left( 1 - \frac{R_0}{(7-1)GM_j/M_j} (\omega/k)^2 \right)}{(7-1)GM_j/M_j} $$  \quad (3)$$

the adiabatic exponent is determined by the slope of the graph $2(7 - 1)/\pi \times GM_j/R_j^2$. Unfortunately, high degree $p$-modes which would provide the corresponding signature cannot be detected from present available ground-based observations.

4. CONCLUSION

- Observation of low-degree $p$-modes would allow an accurate determination of the core radius.
- The detection of the phase transition requires information from high-degree modes, which will probably have to be observed from space.
- Future calculations will include the planet rotation (work in progress).

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SOLAR OSCILLATIONS, INSTRUMENTATION AND MEASUREMENT THEORY

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ABSTRACT

Solar-oscillation instruments have many common characteristics, such as detecting solar radial velocities on Fraunhofer lines with a 2-point measuring technique, high spectral resolution and stability, etc. A review of these common effects is made which can be used for optimizing most instruments in helioseismology. The choice of the spectral line is addressed for getting a high signal to solar noise ratio. Velocity imaging of solar oscillations modes is detailed including spatial sampling and span, highest observable degree. Applications of these different analysis is then applied to existing or future helioseismology instruments.

Key words: Helioseismology, Instrumentation, Optimization

1. INTRODUCTION

Helioseismology is the science of probing the internal structure of the Sun by observing its surface oscillations. The measurement of mode amplitudes of waves trapped in the Sun is at the core of the observation. These waves are reflected in the solar atmosphere where they affect the line strength, the shape and the wavelength of Fraunhofer lines. This is mainly the last effect which is detected by helioseismology experiments measuring solar radial velocities.

The Doppler shift of solar absorption lines is frequently deduced by using the 2-point measurement technique which has been used by many observers for experiments with or without spatial resolution (Refs.1-4). I will describe a complete model of the 2-point technique which can be applied to different line and filter profiles including effect of the prefilter or blocker. Calibration procedures, photon noise and parameters optimization will be also discussed.

The choice of the best spectral line is also a very important issue (Ref.5). Optimum choice depends on many parameters like line bisectors, height of formation or strength. These criteria can be used for optimizing stellar oscillations detection.

The following analysis can be applied to instruments making high resolution images of the Sun with discrete array detectors. Velocities measurements have two components non-oscillatory and oscillatory velocities. Non-oscillatory velocities are mapped for calibration purposes either spatially (Ref.3) or temporally (Ref.2); for inferring any unknown effects or for detecting global convective flows (Ref.6). This mapping was used to verify a solar rotation compensation scheme with a Fabry-Pérot (Ref.7). Instruments observing oscillatory velocities require temporal and spatial sampling to infer the frequency and the amplitude of the modes as a function of l,m and n. Temporal sampling and span has been discussed by J. Harvey and Christensen-Dalsgaard et al (Ref.5,8). Spatial sampling and span was discussed by J. Harvey (Ref.9) using some simple consideration. I describe a more complete model addressing spatial sampling, spatial filtering, aliasing and number of samples with instruments using discrete array detectors.

2. MEASUREMENT THEORY

2.1 Spectral response and sensitivity

The 2-point technique is quite simple. The integrated intensity of a solar absorption line is measured in the wings of the line giving the intensities, \( V_r \) and \( N_b \). The difference between these two intensities divided by their sum is the measured signal or \( D/S \). Intensities are measured with high spectral resolution filters: Fabry-Pérot, Lyot or magneto-optical filter; unwanted orders are blocked with a prefilter having a passband roughly equal to one free spectral range of the primary filter. Figure 1 shows typical response curves for various filter and line profile combinations. It is clear that \( D/S \) is roughly proportional to the reduced line shift \( v_R \).

Non-linearities are a strong function of \( v_R \); they can be studied by computing the sensitivity:

\[
S(v_r) = \frac{\partial (D/S)}{\partial v_R}
\]

The derivatives of the response curves shown in figure 1, are plotted in figure 2. Non-linearities are quadratic for small \( v_R \), as the
Figure 1. Typical response curves for various line and filter profile combinations, with no prefilter, with a prefilter having a lorentzian profile and a band width 10 times larger than the line width. The reduced scanning range (y) and the reduced filter bandwidth (x) are as wide as the line width. The line depth is (P) is 0.8. D/S is given as a function of the reduced velocity of the line (v). Response curves are almost similar with such a prefilter bandwidth. The response is higher when both profile are gaussian.

2.1 Photon noise

The 2-point technique has some disadvantages such as non-linearities and reduced dynamic range. It is one of the reason why a Michelson instrument, which has very high linearity and useful range (= 8 km/s), was selected for GONG. But compared to this kind of measurement, the 2-point technique has a better photon noise figure, mainly because of its higher spectral finesse. Photon noise computation is required for optimizing to the first order the parameters x and y; this noise in equivalent radial velocity is given by:

$$V_{\text{photon}} = Q(x, y, P, \sigma_0) \Delta V / \sqrt{N}$$

where

\begin{align*}
\Delta V & \text{ is the line width in km/s, } \\
N & \text{ is the number of photons transmitted by an instrument with a filter as wide as the solar line, } \\
\sigma_0 & \text{ is the reduced bandwith of the prefilter, } \\
Q & \text{ is the quality factor defined by: }
\end{align*}

$$Q(x, y, P, \sigma_0) = \frac{\int \left[ \frac{\partial I}{\partial V} \right]^{-1} }{\Delta V}$$

The lower the Q factor is, the lower the equivalent velocity will be. Optimum x and y will thus be given by the location of the minimum of Q, if any. The quality factor is almost independent of $\sigma_0$, for large value ($\sigma_0 > 10$). Assuming a large passband for the prefilter, I have computed Q factors for different combinations of line and filter profiles.

In figure 3 the reduced velocity range is plotted as a function of x and y, the ra. je being defined as 2 times the maximum velocity giving a variation of sensitivity less than 10% (the factor 2 taking into account the symmetry of the sensitivity). In the upper left corner funny isocontours are revealed, this is the region where the sensitivity curve does not look like figure 3; in this region, the sensitivity increase with the reduced velocity, reaches a maximum then decrease as in figure 2. Thus very high dynamic range can be obtained at the frontier of this region.
1. must be found between photon noise and spurious number of transmitted orders increases and so Prefilter spurious velocity is roughly inversely 

A/°C. The prefilter bandwidth is 10 times larger velocity; this approach was used by T. Brown

2.3 Prefilter effect

Most prefilters have a high temperature sensitivity. Temperature variations of the prefilter will induce a spurious velocity, which is shown as function of line depth in figure 5.

2.4 Calibration

For wide prefilter passband (O,>10), filter passband shifts are equivalent to line shifts, so scans of the solar line will calibrate the spectral response of the instrument. This method is used for tunable filters (i.e. Fabry-Pérot (Refs.7,11) or could be used for Lyot filters (Ref.4) to calibrate the whole response curvo.

Non-tunable filters like resonance cells or magneto-optical filters use the earth's rotation and/or the solar rotation for calibration (Refs.1-4). This method is not an absolute calibration since it relies on velocities previously measured by other instrument. Furthermore, only a small part of the response curve can be calibrated this way, so non-linearities are unknown especially when granulation or supergranulation is observed with imaging instruments. Daily calibration of resonance cells induces spurious velocity shifts with a period of one year (Ref.12). Recently, a new method to calibrate a resonance cell using magnetic coils has been developed (Ref.13) this allows to measure only the slope of the solar line (Zeeman effect is symmetrical). Once one is able to calibrate the response curve within 1%, velocity shifts can be inferred from D/S measurements with acceptable precisi.on.

2.5 Spectral line choice

Most solar spectral lines are non-symmetrical, and thus characterized by their bisectors (the so called C-shape). It is better to choose a line with a small bisector amplitude in order to minimize spectral response non-linearities. Furthermore, bisectors amplitudes were shown to be different between quiet and plage regions (Ref.14), and to decrease as the center-limb position (Ref.15).

The height of formation needs to be addressed: the amplitude of oscillatory velocity increases with height, as shown by Stebbins et al and Canfield et al (Refs.16-17), granular velocities decrease with height (Refs.17-18), supergranular velocities are supposed to be constant, with height as stated by Keil and Canfield (Ref.18), finally chromospheric velocities increase with height (Ref.19). These effects allow us to determine the first criteria for line choice:

- Minimal bisector change between active and quiet regions.
- Minimal bisector amplitude on any point of the Sun (from center to limb).
- Line formation should take place at the temperature minimum region (H=400-500 kms)

Bisector characteristics were extensively studied for solar Fe I lines (Refs.20-21). From these two reviews, simple considerations on the behavior of bisector amplitudes as a functions of different line parameters are drawn:

- Bisector amplitudes decrease with line depth.
- C-shapes have no dependence upon excitation potential for 70<cp<90%.
- C-shapes are less pronounced for low excitation potential for cp<70%.
- C-shapes have no dependence with wavelength.
- Fe II and Fe I lines bisectors have similar trends, but bisector amplitudes are larger for Fe II lines.
- No dependence on the Landé g-factor (Fe I lines)

It is not clear whether or not the difference of bisectors amplitude between quiet and active regions depends upon the Landé g-factor (or the excitation potential). Studies showed that the difference is not negligible even for g=0 lines (Fe I) (Ref.14). Other measurements demonstrated that the difference might be even smaller for g≠0 lines (Fe I) (Ref.22). Finally Cavallini et al (Ref.23) stated that the difference should rather depends upon the excitation potential; furthermore it seems that the lower the excitation potential is, the lower the difference is, at all center-limb position. GONG faced this problem and seems to agree with this conclusion (Ref.24):

- Bisector change between active and quiet region is minimal for low excitation potential and does not depend upon the Landé g-factor.

A method to compute heights of formation of photospheric lines was described by R.C.Altrock et al (Ref.25) using LTE and non-LTE models; heights were given mainly for Fe I lines, for different location in the disk or on the limb and on the line profiles. Similar computations can be done for other lines to give more data for line selection. Finally using McMath Observatory data or other work (Ref.26) minimal bisector change with the center-limb position can be known for solar lines; bisector amplitude seems to decrease towards the limb as measured also by Kostik and Orlora (Ref.27). Resonance lines are not suitable for helioseismology because of the greater sensitivity to active regions as shown by Ref.28. As a last word solar lines must be also free of blends and of atmospheric contamination (consult solar atlases).

From the previous paragraphs, the optimum line choice should be a deep and wide line in order to reduce non-linearities and increase the dynamic range. The criteria given above change with the center-limb position can be deduced for the solar rotation axis and e to is the physical spacing between pixel in the direction X (or Y). This criteria teaches us an interesting conclusion: even in the infrared the f/D ratio has to be greater than 20 because pixel dimension are not smaller than 10 μm. Furthermore most solar oscillations experiments actually make full disk observations, so it means that the equivalent focal length can be written as:

$$ \frac{f_X}{\lambda} = \frac{N_X}{\theta_0} \left( \frac{2\pi}{\lambda} \right) \left( \frac{\lambda}{2\pi} \right) \frac{\lambda}{\theta_0} $$

where λ is the solar line wavelength, f is the equivalent focal length of the optical system and e is the physical spacing between pixel in the direction X (or Y). This criteria teaches us an interesting conclusion: even in the infrared the f/D ratio has to be greater than 100 because pixel dimension are not smaller than 10 μm. Furthermore most solar oscillations experiments actually make full disk observations, so it means that the equivalent focal length can be written as:

$$ \frac{f_X}{\lambda} = \frac{N_X}{\theta_0} \left( \frac{2\pi}{\lambda} \right) \left( \frac{\lambda}{2\pi} \right) \frac{\lambda}{\theta_0} $$

As stated above the pupil might be rectangular. For a small number of pixels (=100), the typical dimensions are of the order of few millimeters. This solution is usually not workable because of the photon noise, so alternate solutions should be used, such as defocusing or injecting random noise in the guiding system. Both have the same drawback: the cutoff will not be as well defined as in the entrance pupil case. It is always possible to define an efficient cutoff frequency according to different criteria: first zero, transmitted amplitude, etc...

Thus optimization is possible to correct undersampling by discrete array detectors by choosing the contrast function of the optical system carefully.

3.1 Oscillatory velocities

3.1.1 Spatial sampling

When observations of velocities are performed with discrete array detectors, the Fourier transform of the measured velocities is given by:

$$ \mathcal{V}(V_x',V_y') = \sum_{j_1} \sum_{j_2} \delta(V_x' - \frac{k_1}{\Delta X}) \delta(V_y' - \frac{j_2}{\Delta Y}) \mathcal{I}(V_x',V_y') $$

with (V_x', V_y') are the coordinate in the Fourier plane of a point conjugate of a point of coordinate (X, Y) on the Sun, (X, Y) are the reduced coordinates to the solar radius, the axis X defined the solar equator, the axis Y defined the solar rotation axis. ΔX and ΔY are the pixel spacing, where \( \mathcal{I}(V_x',V_y') \) is the contrast function of the optical system and \( \mathcal{I}(V_x',V_y') \) is the Fourier transform of the pixel function. The Fourier transform of the finite Dirac pattern was approximated by a Dirac pattern too, the number of pixels being high (>100).

Let us suppose that the contrast function is a constant (i.e. perfect impulse response of the optical response), then the impulse response of the whole system will have strong aliasing problems. If one assumes that the contrast function can be band limited by construction (i.e. finite dimension of the pupil) then simple criteria are deduced for anti-aliasing:

$$ K(V_x',V_y') = 0 \text{ for } \frac{1}{2\Delta X} > |V_x| \text{ and } \frac{1}{2\Delta Y} > |V_y| $$

(5)

For rectangular pixels, the adapted pupil should be rectangular, it means that the lens or mirror has to be rectangular too. Then according to Eq.5 the lower limit on the f/D ratio is:

$$ \frac{f_X}{\lambda} = \frac{N_X}{\theta_0} \frac{2\pi}{\lambda} \frac{\lambda}{\theta_0} $$

(6)

where λ is the solar line wavelength, f is the equivalent focal length of the optical system and e is the physical spacing between pixel in the direction X (or Y). This criteria teaches us an interesting conclusion: even in the infrared the f/D ratio has to be greater than 20 because pixel dimension are not smaller than 10 μm. Furthermore most solar oscillations experiments actually make full disk observations, so it means that the equivalent focal length can be written as:

$$ \frac{f_X}{\lambda} = \frac{N_X}{\theta_0} \frac{2\pi}{\lambda} \frac{\lambda}{\theta_0} $$

(7)

where N is the number of pixel in the direction X (or Y) and θ0 is the solar angular radius. Then the equivalent focal length of the entrance pupil might be different if the array is not a square. Using Eq.6 and 7 we can deduce an upper limit on the dimension of the instrument pupil:

$$ D_X = \frac{N X}{\theta_0} $$

(8)

As stated above the pupil might be rectangular. For a small number of pixels (=100), the typical dimensions are of the order of few millimeters. This solution is usually not workable because of the photon noise, so alternate solutions should be used, such as defocusing or injecting random noise in the guiding system. Both have the same drawback: the cutoff will not be as well defined as in the entrance pupil case. It is always possible to define an efficient cutoff frequency according to different criteria: first zero, transmitted amplitude, etc...

Thus optimization is possible to correct undersampling by discrete array detectors by choosing the contrast function of the optical system carefully.

3.1.2 Spatial span and highest detectable degree

Scientific objectives are related to the number of detected degrees. It is intuitively thought that the higher the number of pixels, the higher the detected degree (hereafter J max). J. Harvey (Ref.9) showed that, using a simple model, J max is given by:

$$ J_{\text{max}} = N \pi \sqrt{1-f^2} \frac{1}{2} $$

(9)

where N is the number of pixel and f is the fraction of the observed Sun. Eq.9 is derived assuming two things:

- Aliasing is a function of J max (application of the Nyquist theorem)
The signal to noise ratio is infinity. Very small variations in the quasi-periodic mode pattern can be easily detected. The first assumption is not really correct as shown in the previous paragraph: observations of solar oscillations require a telescope with finite dimensions to make solar images and aliasing is independent of $l_{\text{max}}$. The second assumption is unrealistic, because solar and instrumental noise decrease the $S/N$ ratio. I will describe a model taking into account signal to noise ratio, number of pixels, spatial span and aliasing problem to derive $l_{\text{max}}$.

Firstly, I assume that there is no aliasing and that the product of the two functions $R$ and $B$ is roughly a triangle function $A$ with a cutoff frequency corresponding to criteria of Eq.5. Secondly, I study filtering of spherical harmonics for which $l=m$. The spherical harmonic is projected on the line of sight, and foreshortening is taken into account. Thus if we are able to detect these modes, spherical harmonics for which $m<1$ will be detected as well, because they contain fewer high spatial frequencies. Thirdly, I use the criteria for detecting a mode of amplitude $S$ attenuated by a factor $R$ (due to the instrument) with a noise floor $B$ is given by:

$$S/B=1/R$$

(10)

Of course the amplitude of the mode is dependent upon the degree $l$ and its frequency, and $B$ as well; then a local $S/B$ can be defined as a function of the location on the $(l,V)$ diagram (or $(m,V)$ diagram). The attenuation factor of a spherical harmonic $l=m$, for the instrument with the triangle response function, is given by:

$$F(l)=\int_0^1 A(v_j)\int_0^v V(l,v_j)dv_jdv/A(l,v_j)$$

(11)

where $v_j$ is the reduced frequency to the cut-off frequency, $V(l,v_j)$ is the module of the Fourier transform of the mode $l=m$. Then the final criteria for having $l_{\text{max}}$ is given by:

$$F(l_{\text{max}})=R(l_{\text{max}},V)$$

(12)

with $12F(l)\geq F(l)$ for $l=1/\text{max}$

Hereafter I will assume that the function $R$ is not dependent upon either $l$ or $n$, this simple hypothesis will lead us towards very useful conclusions.

Eq.12 was solved numerically with different numbers of pixels, different constant values of $R$ and various fractions $f$, of the observed Sun. I found that $l_{\text{max}}$ is roughly proportional to the number of pixels $N$, but the slope $a$ is a strong function of the fraction $f$, and the signal to noise ratio (figure 6).

As expected the slope decreases when $f$ increase (i.e. $l_{\text{max}}$ decrease). I assume that the noise (8) is not a function of $f$, which is not realistic since photon noise will increase as $1/f$, but this assumption should be true for $f$, down to 0.1. For small $f$, the slope is a linear function of $B/S$: for very high signal to noise ratio, the slope is very close to $N/2$ which supports the conclusion about the dependence of $l_{\text{max}}$ upon $N$ for small $f$. (Ref. 9): for signal equal to the noise the slope is close to 0 (i.e. no modes detected).

In conclusion after having sampled optimally to avoid aliasing, a typical span of $f=0.9$ and a signal to noise ratio of 4 should give a highest detectable degree of about the number of pixels of the discrete array, $N$.

4 DISCUSSION

Previously, solar oscillations experiments were not really completely designed for helioseismology! Already designed high resolution filters did not fit the described requirements. Perhaps only the imaging of the Sun onto a discrete array detector fitted the needs of helioseismologists (mode imaging).

Resonance cells and magneto-optical filters are not the best choice for this field of science: only one line (or two) can be studied, no absolute calibration can be achieved, resonance lines are wide, there is no tunability capability and they are sensitive to high solar activity: some of these drawbacks are compensated by a high dynamic range and good stability (atomic wavelength reference).

The Lyot filter as used by Libbrecht and Zirin (Ref. 4) seems closer to optimization: small scans are possible, thinner solar lines, relatively wide prefilter; drawbacks are almost complementary to the previous case: non-linearities, high spectral temperature sensitivity, no reference wavelength.

The Michelson interferometer, to which some results of this study might be applied, was the only instrument designed for helioseismology: it features linear velocity measurement, relatively low photon noise, absolute calibration, easy access to any solar line (only a wide prefilter required), no field effect, utilization of stabilized laser, spectral temperature sensitivity could be very low with a careful design of the Michelson, negligible sensitivity of the prefilter; drawbacks are technological, it took a long time to build and use a complete instrument (Ref. 29-30). That is for all these good points mentioned that the BROWN's instrument was chosen by GONG to observe with a 80% duty cycle solar oscillations. The GONG network is currently planned to be installed in 1991 at six stations. This network of instrument will be very close to fulfill all the criteria given in this paper.

Other instruments such as the Absolute Astronomical Accelerometer designed by CONNES (Ref. 12) is most likely to be the instrument able to detect solar g-modes in integrated light, and to do, in the near future,
astroseismology. This instrument features also linear velocity measurement, relatively low photon noise, absolute calibration, easy access to any solar/stellar line, utilization of stabilized laser.

The next step will be to observe from a space probe like SOHO. The proposition, which was selected by ESA/NASA for the 1995 SOHO flight, optimizes also most of the criteria given above (Ref.31).

By the year 1995, two optimized experiments will run simultaneously, which will allow time for other experiments to be developed. A ground based instrument, the so-called Stable Solar Analyzer, using the concepts of this study is being developed at the Applied Physics Lab (Ref.7). This instrument, which will be installed in Europe, will do mapping of solar radial velocities with a lithium niobate Fabry-Perot, the detector is a 100 by 100 Reticon (1...).

The same characteristics as the GONG Michelson interferometer but the linear velocity measurement is based onto a 6-point scan of the solar line (+1 point in the continuum): it has also active spectral stabilization of the Fabry-Perot onto a diode laser itself stabilized onto an atomic wavelength.

5. ACKNOWLEDGEMENTS

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A PROTOTYPE STELLAR PHOTOMETER FOR MAGNETIC FIELD 
AND DOPPLER MEASUREMENTS

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ABSTRACT

A stellar photometer is in preparation as a joint project of the University of Rome and the Astrophysical Observatory of Catania. It is based on a suitable version of the Magneto-Optical Filter (MOF). The aim is to detect magnetic and velocity signals on stars. The basic idea is shown in the figure. The photometer is able to maximize the collection of photons. As far as the magnetic field measurement is concerned we consider only the transverse component of the magnetic field that should provide detectable U and Q Stokes parameters (linear polarization). Tests on the sun are part of this program.

Keywords: Stellar oscillations, MOF, Planetary systems.

1. INTRODUCTION

The MOF is a narrow bandwidth filter in the wing of the sodium D lines which can be used to convert velocity fluctuations into measured intensity fluctuations. This filter has been successfully used at Mt. Wilson Observatory (MWO) to make high spatial resolution doppler and magnetic images of the sun. Recently, a modified version of this instrument has been successfully used at JPL to detect velocity signals of less than 1 meter/sec from the integrated solar disk with a precision better than 10 cm/sec in a single measurement. We propose to examine the suitability of using a modified version of this filter for detecting velocity motions associated with stellar pressure mode oscillations and motions associated with planetary companions. The modified version will have a substantial increase in the transmitted photon flux and the capability to use stars with a heliocentric radial velocity as large as ±50 km/s.

Detection and measurement of stellar pressure mode oscillations offers the potential for direct determination of the mean stellar density and interior chemical gradient. Combined with direct determination of stellar radii and theoretical stellar evolution models, this data will provide fundamental determinations of stellar masses and stellar ages. The detection of long term velocity modulations will provide evidence for the presence of low-mass planetary companions.

Interest in stellar planetary companions has grown recently with the detection of circumstellar material around Vega and β Pictoris (Ref.1). Work on the detection of planetary systems has centered on astrometric detection of periodic motions of nearby stars. Results to date are inconclusive (Ref.2). An alternative approach is to look for a periodic doppler shift in the spectrum of a star revolving around the center of mass of the star-planetary system (Ref.3). For the Sun, Jupiter produces periodic velocity fluctuation (period= 12 years) with an amplitude of 12.5 m/sec while the Earth produces a fluctuation (period= 1 year) of 9 cm/sec. Periodic velocity modulation detection and measurement, coupled with determination of the orbital inclination with respect to the plane of the sky using the calibration provided by the on-going MWO HK program (Ref.4), would permit positive identification of planetary systems.

The acceleration induced by a planet upon its parent star as the star and planet orbit their common center of mass has long been considered to offer a potential means of detecting planets, particularly those of substantial mass approaching that of Jupiter. Such accelerations could be searched for either astrometrically (for later components), or spectroscopically (yielding the doppler shift corresponding to the component along the line of sight). The observational requirements for such highly sensitive doppler measurements are closely related to the requirements associated with the study of stellar p-mode oscillations, except that the timescale of interest, measured in minutes for oscillations, is measured in years and longer for planetary orbital accelerations. Consequently the planet detection problem imposes very stringent requirements for long-term stability of the velocity or wavelength calibration of the measuring apparatus.

For this reason, we intend to understand the calibration of the MOF at the level required for planet detection and over the longest possible timescales in order to assess the applicability of the MOF to the planet...
detection problem, and to compare the technique with other candidate methods.

2. MOF PERFORMANCE CHARACTERISTICS

The MOF can be used in a variety of configurations as will be discussed in a forthcoming paper. This ability will provide us the flexibility of choosing the optimum configuration to maximize the S/N for our particular need. As a first order estimate of the performance of the MOF, we compute the expected photon flux through a 1 cm² aperture followed by an MOF. We do this calculation for a worst case and a best case MOF configuration. We then estimate the white noise power density associated with the flux level (the photon flux is assumed the dominant noise source in our system). The performance parameters for these two cases are summarized in Table 1.

These predictions reflect our expectation that the estimated losses from the optics (e.g. reflection losses from lenses, mirrors, etc.) and atmospheric extinction will be compensated by the simultaneous use of both Na D lines.

3. ESTIMATE OF SIGNAL AND NOISE LEVELS

With these parameters, count rates are estimated for the Sun \(m_v=26.6\) for which p-mode oscillations are readily observed and for a star with a visual magnitude of \(m_v=3.4\) typical of the type of bright star we would observe to detect stellar p-mode oscillations. The 30 magnitude difference between the two stars translates into an intensity ratio of \(10^{12}\). Using the measured flux of 170 erg/cm²/s/A observed in the solar continuum at the Na D lines (Ref.5), an aperture of 1 cm², we obtain the following range count rates, \(N_c\):

<table>
<thead>
<tr>
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<th>Worst Case</th>
<th>Best Case</th>
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</thead>
<tbody>
<tr>
<td>Sun</td>
<td>3x10¹⁰</td>
<td>3.5x10¹²</td>
</tr>
<tr>
<td>Star</td>
<td>3x10⁻²</td>
<td>3.5</td>
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</table>

We have independent empirical verification of these estimates from observations of the sun obtained with existing equipment in place at JPL using a one cm² aperture and an integration time of 3 sec. This data was obtained without any particular effort. Data from this run are plotted elsewhere in these proceedings. The 5 minute p-mode oscillation, superimposed on the Earth rotation trend, is clearly visible with a peak amplitude of \(\sim 1\) meter/sec. The noise level is at least 1 to 2 orders of magnitude below the oscillatory signal, consistent with the estimates derived from the following numbers:

- Photon flux on the detector: \(N_c = 10^{12} \text{ph}\)
- with 30 sec integration time
- Signal / Noise \(\propto 2 \times 10^{-6} \sqrt{N_c} \nu (\text{cm/sec})\)
- S/N = 1 for \(\nu \approx 1 \text{cm/sec}\)

Based on our analysis we have for the variance in the frequency domain the following expression:

\[
\sigma^2 = \left(\frac{2K}{3}\right)^2 \frac{1}{N_o} (m^2 s^{-2} H z^{-1})
\]

where \(K\) is evaluated from:

\[
\nu = K \frac{\Delta f}{I_c}
\]

which is the relationship between velocity amplitude and photometric intensity. In our case:

\[
N_c \text{ (counts/sec)}
\]

<p>| | |</p>
<table>
<thead>
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</tr>
</thead>
<tbody>
<tr>
<td>Sun (1 cm² aperture with worst case parameters)</td>
<td>(2.7 \times 10^7)</td>
</tr>
<tr>
<td>Sun (4 cm telescope)</td>
<td>(\geq 3.0 \times 10^{10})</td>
</tr>
</tbody>
</table>

using a value of \(K = 1.25 \times 10^4 \text{ m/s}\).

We therefore derive the following white noise levels for the two cases of interest in this analysis:

\[
\sigma^2_{\text{Sun}} = 2.3 \times 10^{-3} (m/s)^2 Hz^{-1}
\]

\[
\sigma^2_{\alpha \text{Cen}} = 2.5 (m/s)^2 Hz^{-1}
\]
4. THE PHOTOMETER

A development program is currently underway to design an MOF based on a cell incorporating potassium and other elements as well as sodium. If this effort is successful, we expect the performance of the MOF to improve proportional with the number of resonance lines available.

The optical arrangement of the stellar photometer is sketched in fig.1. The peculiarity of this design is that the usual system of two crossed polarizers is replaced by a new combination calcite + mirror. The calcite splits the incoming beam in two parallel beams (ordinary and extraordinary) that are reflected back by the mirror along the same paths. They pass twice through the same Na vapour and any change in polarization due to magneto optical effects results in a second beam splitting of the reflected rays. Four beams are created inside the calcite; two of them converge into one (continuous light; beam #3) while the other two are the monochromatic transmitted bands of a conventional Magneto-Optical Filter (beam #1 and 2). The head of the photometer is designed in such a way to easily separate the three exit beams.

5. OBSERVATIONAL STRATEGY AND CONCLUSION

The observational strategy that will be employed in direct or indirect solar observations will be to use a controlled and calibrated reduction of the level of solar intensity signal to the level obtained from bright stars.

We will due this by using neutral density filters to reduce the solar intensity and modifying our data acquisition system to incorporate a photon-counting photometer for low intensity observations. Other modifications include: the use of transverse magnetic field inside the MOF and the ability to detect linear polarization of the incoming stellar beam.

We are aware that the proposed program is difficult and long lasting. However the MOF is at present the only technology able to give an answer to most of the stringent requirements imposed by the aimed goals: it is simple, compact, stable, efficient and, most importantly, susceptible of further improvements.

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Abstract:

Our instrument devoted to stellar seismology uses a Cacciani-type sodium cell as a very stable frequency reference for radial velocity measurements. It was successfully operated for the detection of oscillations on Procyon and Alpha Centauri in 1983 and 1984.

In a new version, its sensitivity has been increased and the sources of noise have been reduced. We will give a brief description of the principle and discuss the limits of sensitivity. Other possible applications are also suggested. Finally we will discuss the capabilities of this instrument in the context of the development of asteroseismology.

Keywords: stellar seismology - sodium cell - optical resonance.

1. Description

1.1. Optical scheme

The instrument uses the magneto-optical resonance of the sodium vapour. The cell is heated at about 200 °C and thermally controlled to ensure a constant sodium pressure. A permanent magnet gives a magnetic field of about 1500 Gauss to separate the Zeeman polarizations.

Fig. 2. shows the location of the sodium cell between two crossed polarizer in order to select the Zeeman resonant light which is measured by the photomultiplier tube number 1. The intensity of this narrow band, located in the wing of the stellar sodium line, strongly changes with the radial velocity of the stellar photosphere.

The PMT 2 is placed on the beam of the other polarization and measures all the light selected by the interference filter F1 in a 20 Å bandwidth. This measurement is used as a reference to reject atmospheric transparency fluctuations. The pupil is imaged by lenses 1 and 2 in the center of the cell and by lenses 3 and 4 on the cathodes of the PMTs in order to minimize the effects of seeing. A camera is placed on the reference beam and provides an accurate guiding. Optical scheme has been calculated to be versatile so that the instrument could be operated at Cassegrain focus as well as at coude focus of every telescope.

Fig. 1. shows the seismometer at the Cassegrain focus of the CFH 3.6 meter telescope.

1.2. Technical characteristics

As said above, the measured intensity is a function of the radial velocity. Then the sensitivity of the instrument changes along the year with the orbital motion of the earth. For a given star with a radial velocity of less than 20 km/s, it exists then two periods of about one month each during which the slope of the line is maximum. The bandwidth of our filter is therefore calculated to give a good compromise between sensitivity and photon-noise. It is presently of .17 Å which gives us a sensitivity ΔI/I of 1.5 × 10^-4 per meter per seconds for a slow rotating G2 star. On Procyon, we have measured a sensitivity of 0.8 lfr"* meter per seconds and a photon counting rate of about 40000 c/s with a 3.6 meter telescope, corresponding to a photon noise of 9 m/s for 1 mm of integration time.

2. Performances and instrumental limitations.

From the analysis of the data obtained on Procyon and on Jupiter and the calibration tests, we conclude that the instrument was not photon-noise limited. However, the final noise level was reduced by a factor 2 or 3 in comparison with the previous prototype instrument. This is mainly due to a better correction of the atmospheric transparency. Dividing the signal of PMT 1 (i.e. which is proportional to the radial velocity) by the signal of the PMT 2 (i.e. the intensity of the continuum between the two sodium lines) remove quite well the effect of

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The need for large telescopes could be regarded as an important limitation to the observing programs as it is somewhat surprising for a telescope allocation committee to give one week at a 4 meter telescope on a zero magnitude star. The precision of velocity we could get by observing one star during one hour a few times around the conjunction should be better than 1 m/s until the fourth magnitude.

The total counting rate has also been increased by a factor 1.5 by using a more sensitive photomultiplier tube, and the dark current has been reduced by a more efficient cooling. What was suspected in the previous model to be a strong source of noise was the sensitivity to the guiding. The simplified optical path and the use of a guiding camera directly on the beam have reduced this problem, but it still remains. The guiding sensitivity appears to be about 1 m/s/arcsec (mainly due to the insufficient optical quality of the windows of the cell), which is well below the photon-noise in the case of good seeing and with an autoguider. However, it is really dramatic in the case of telescopes without autoguiding and which generally oscillates at long periods (from a few minutes to about one hour). This problem will be decreased by using optical quality windows.

Another source of noise which was not suspected before was the thermal stability of the cell, which control the sodium pressure. The accuracy of the control was of about 0.1 °C, providing a stability better than 1 m/s, but the thermal system had a resonance frequency of a few minutes, depending on the external temperature. Hopefully, the temperature of the cell was recorded with the observations and we could correct for this effect in the data. The remaining contribution to the noise was then reduced to a level of about 10 cm/s, and in the future, we will adjust the parameters to have a thermal cycle shorter than one minute.

Finally, after correction of the drift due to earth rotation, the r.m.s. value of each night of data recorded on Procyon at CFHT and Zelentchouck in February 1988 was less than 20 m/s for one minute of integration, which has to be compared with the 30 m/s obtained in February 1984 on the same star. It is already an important improvement, but it still lies above the photon noise which should be of 9 m/s. Assuming that the oscillations give a r.m.s. value of 6 m/s, it remains a noise of about 15 m/s coming from the thermal control, the guiding sensitivity and the low frequency variations of the photomultipliers sensitivity.

The first two points should be partly fixed for the next campaign in February 1989. The last point will be discussed later. So we expect to reach the photon noise level at least for frequencies higher than a few hundred micro-Hz and then to be able to detect oscillations of 20 cm/s in 3 nights of observations.

3. Discussion

As said above, the main limitations of our instrument are the photon-noise which limits drastically the number of accessible stars and which implies the use of large telescopes, and the frequency range limited to values higher than 200 μHz, due to the two different photomultiplier tubes for velocity measurement and reference, which prohibits observations of giant stars.

One of the possible improvements is to measure alternatively the velocity beam and the reference beam with the same photomultiplier, in order to access the low frequencies. So we could observe giant stars such as Arcturus, for example, or try and look at external planetary systems, by measuring the absolute radial velocity of a few stars during several years. The precision of velocity we could get by observing one star during one hour a few times around the conjunction should be better than 1 m/s until the fourth magnitude.

For this possible program, and also for low frequency measurement, we will be limited to high quality photometric nights: variations of humidity along the line of sight will produce a signal which cannot be corrected because of the presence of water lines in the continuum between the D1 and D2 lines. It is however possible to avoid this effect by selecting only a few Angstroms around the lines by using a Lyot filter or a Fabry-Perot but controlling them could be something complicated regarding the simplicity of the instrument.

The need for large telescopes could be regarded as an important limitation to the observing programs as it is somewhat surprising for a telescope allocation committee to give one week at a 4 meter telescope on a zero magnitude star.

But, if we can measure the main parameters $v_0$ and $D_0$ and the amplitude of oscillations at a given frequency for a given type of stars, the scientific return will be highly rewarding. In any
case, the extension of oscillation measurements to fainter stars will also require, even with multi-lines instruments, the use of large telescope because of the limited lifetime of modes.

In conclusion, we should say that the instrument has some intrinsic limitations but it has demonstrated the feasibility of oscillation measurements and it is still useful for these observations and other possible programs such as look for planetary system, for example. So we conclude that it had perfectly reached the goal it was designed for.

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SEARCH FOR RAPID OSCILLATIONS IN THE NORTHERN Ap STARS HD62140, HD81009 & HD22374

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ABSTRACT

High-speed photometric observations have been performed of the Ap stars HD62140 (49 Cam), HD81009 and HD22374 for more than 60 hours in October and December 1986 using the 1m-JKT at the Observatorio del Roque de los Muchachos (ORM) and the 1.5m-TCS of the Observatorio del Teide (OT), Canary Islands (Spain). Photometric measurements in the B band were reduced and the residuals obtained were subjected to a harmonic analysis searching for periodicities in the range of a few minutes to 2 hours. There is some evidence that the star 49 Cam is oscillating. HD81009 and HD22374 were observed in order to confirm some traces of oscillation present in earlier observations however, these possible oscillations did not repeat in the new data.

1. OBSERVATIONS

Rapid oscillations have been discovered in, at least 12 magnetic Ap stars (Ref. 6) so far, all but one (γ Equ) have been found in the southern hemisphere of the sky.

The necessity and the importance of a search for oscillations in Ap stars in the northern hemisphere encouraged us to observe a group of these stars looking for variations in the range of a few mHz. This paper presents the results obtained for three of them: 49 Cam, HD81009 and HD22374. Rapid photometry was performed and more than 60 hours of data were obtained over 11 nights in October and December 1986. The IAC/UBV one-channel photometer attached to the Cassegrain focus of the Carlos Sanchez Telescope (TCS), and the two-channel "People's Photometer" attached to the Jacobus Kapteyn Telescope (JKT) have been used. All the observations were made through a Johnson B filter (436 nm). One integration was obtained every 20 s (31 s for JKT).

Sky background correction was done through the second channel in the JKT and through a sophisticated chopping system between sky and star+sky in the TCS. The observations were corrected for atmospheric extinction by the standard astronomical procedure. The extinction coefficients, instrumental magnitude residuals and the standard deviation of the airmass fit were obtained. Since we did not use a calibration star, no attempt was made to transform the observations to the standard system. In table 1, we present the journal of the observations for every observing run and star.

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<th>b</th>
<th>c</th>
<th>d</th>
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</table>

Table 1: Journal of observations.

For every star, the date, time length, and the number of 20 s integrations (31 s for JKT) for each observing run, the standard deviation of the airmass fit (for data obtained with the TCS, also this value before and after the meridian is given), and the observatory-telescope combination used, are presented.

2. RESULTS AND DISCUSSION

Data residuals for one single night or a bigger sample of data in the case of 49 Cam and HD81009 were reduced by iterative sine wave fitting (ISWF) procedure for unequally spaced data (Ref. 9). The results that have been obtained and conclusions reached can be summarized as follows.

2.1. HD62140 (49 Cam)

49 Cam is a magnetic Ap star which is classified as FOpSrEu in the Yale Bright Star Catalogue. The Strongen photometry coefficients for this star are as follows (Ref. 4): b-γ=0.143; m,=0.265; c,=0.735; β=2.854; and δc,=0.083, all them very similar to those for already known Rapid Oscillating Ap Stars (Ref. 6). This star also has a strong magnetic field that seems to vary with a period of 4.23 days (Ref. 3).

On the night of JD 2446725 in October 1986, 49 Cam was monitored for four hours on the 1m-JKT at the ORM. In figure 1a the corresponding residuals are shown. The harmonic analysis, figure 1b, showed two peaks at 0.27 mHz and 0.46 mHz, with amplitudes 2.3 and 3.2 mmag respectively, above the 0.4 mmag 3σ noise level. Neither of these two periods were present in the periodogram of another star observed the same night with the same instrumentation.

Figure 1a: Light curve of 6 hours of observation of 49 Cam, obtained at the ORM Im-KJT, during JD2446725. The fit of the two major periods can be clearly seen.

Figure 1b: Amplitude periodogram of the data presented in figure 1a. The straight horizontal line represents the 3σ noise level. Two main peaks are raised clearly over the noise.

HD62140 was observed later on in December 1986 with the TCS, during a total of 31 hours on five nights. Although the peak structure near 1 hour was found again, with the same phase every night, no significant evidence was found of the peak at 0.48 mHz in any of the nightly series observed (see figure 2), nor in the periodogram of all the observations treated together.

So, 49 Cam might oscillate with a period near one hour, and perhaps with more than one frequency. Figure 2 presents two spectra of this star showing that the largest peak is, in both, located at a frequency near 0.27 mHz.

Moreover, 49 Cam was observed for two nights in 1984 (Ref. 7) with a total span of 9.19 hours and strong evidence of a single oscillation of 61.62±0.03 minutes period, with a peak-to-peak amplitude of about 2 mmag was found.

According to Kurtz's Oblique Pulsator Model (Ref. 5), the amplitude of the oscillation might change with the rotation of the star. In table 2a we present the amplitude of the peak located at 0.27 mHz as well as the corresponding noise level. As can be seen in these results, considering that data for JD 2446778 is absent due to bad weather, the amplitude seems to vary with a period from 4 to 6 days. Bonsach et al (Ref. 3) found a magnetic period for 49 Cam (related almost certainly to rotation) of 4.23 days. Although it would be tempting to assume a relation between these two facts, it is necessary to be very cautious since this range of frequencies is quite sensitive to slow atmospheric transparency fluctuations.

To obtain more information, an attempt was made to find out the frequency of oscillation of 49 Cam with higher resolution. Global residual series were analyzed via ISWF with a resolution of 1.0 μHz and the resulting amplitude spectrum can be seen in figure 3. Two frequencies, presented in table 2b with their respective amplitude and phases, seem to be the dominant ones in the periodogram in between a very complicated structure of side bands due not only to the one day alias, but also to the 4.23 day rotation period. There are some peaks at very low frequency that are probably due to slow transparency changes in the night sky, although the structure near 0.10 mHz could be related to some sort of variation reported in literature (Ref. 8). The results for 49 Cam are not at all conclusive, and hence we have planned to observe it again for at least one

Figure 2: 3.7 and 7.5 hours of B photometry data amplitude periodograms, of 49 Cam, during JD2446779 and 2446781. The plot for the first data has been shifted upwards by 4 mmag.

Table 2: Amplitudes and frequencies of oscillation of 49 Cam.
(a): Approximate amplitude and 3σ noise level for each day of data of HD62140.
(b): Frequencies of oscillation of 49 Cam.
week in October 1988 by rapid differential
photometry (Ref. 2) which is obviously the most
appropriate way to study periods between a few
minutes and a few hours simultaneously.

2.2. HD81009 and HD22374

HD81009 and HD22374 are both cool magnetic
ApSrCrEu stars with photometric characteristics
similar to the ones already known for rapid
oscillators. Belmonte et al (Ref. 1) found some
evidence of oscillations in these stars. In order
to confirm these results, both stars were
monitored and so far we have not been able to
confirm the past results.

HD22374 was observed for 8 hours in October
1986 with the JKT. Despite the very good quality
data obtained, with a noise level below 0.15 mmag,
as seen in figure 4, no trace of oscillations
appears in the amplitude periodogram at a level
higher than 0.4 mmag; even the peaks located below
this amplitude could not be correlated at all with
those found before. It is known that it is
possible to find a rapid oscillator at minimum
when it is observed for just one night, but the
extremely low noise level and low signal obtained
make us think now that the previous results for
HD22374 were probably noise. It is necessary to
check whether or not this star oscillates with
periods in the range of interest, nevertheless,
even if so, the amplitude of the oscillation will
surely be well below 1 mmag.

Figure 4: Amplitude periodogram obtained for the
star HD22374 on JD2446726. Notice the extremely
low noise level (0.15 mmag) obtained for a single
night of observation.

For HD81009, four series of data (see table 1)
using the TCS have been obtained with a total span
of 18.5 hours. Belmonte et al (Ref. 1) expressed
some caution about their results for this star,
since the peaks were only just at the limit of
detectability. Figure 5 shows the amplitude
periodogram obtained via ISUF of the global 18.5
hours length series of HD81009. As can be seen, no
significant peaks from 1 to 5 mHz appear over the
30" noise level. So, from these results, it is
concluded that HD81009 does not actually
oscillate, at least at a level as low as 0.5 mmag.

3. ACKNOWLEDGEMENTS

The help of the ORM staff and the maintenance
service and night assistants of the IAC at Izaña
(OT) are greatly acknowledged. The JKT, on the
island of La Palma, is operated by the RGO in the
Spanish Observatorio del Roque de los Muchachos;
the TCS, on the island of Tenerife, is operated by
the IAC in the Spanish Observatorio del Teide,
both observatories of the Instituto de Astrofisica
de Canarias.

Figure 5: Periodogram of the 18.5 hours long
global series of HD81009. No frequencies in the
range of interest are higher than the noise level.
Same resolution as in figure 3.

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A FABRY-PEROT INTERFEROMETRIC STELLAR OSCILLATION SPECTROMETER

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ABSTRACT

A Queensgate Instruments' Fabry-Perot Interferometer has been modified to enable a cadmium blue (4799.991 Å) line to be used as an optical reference. Laboratory tests have shown it capable of giving a noise level of about 30 cm s⁻¹. Observations last December on the 1.5 m IRFC, Tenerife, gave photon shot noise limited levels of 44 ms⁻¹ on HR1217 and 230 ms⁻¹ on 49 Cam. (The results of these observations are discussed elsewhere in the proceedings (ref. 1))

2. THE INSTRUMENT

2.1 Optics

Because of the proximity of telescope time the instrument had to be based around a test bed for demonstrating multilining in Fabry-Perot interferometers similar to that used in the Michelson ISCS (ref. 5). This meant that the interferometer was not optimized for observing Ap stars using a single line, for example, a larger than necessary etalon was used. Figure 1 is a schematic of the interferometer.

A cadmium lamp was chosen as the reference source as it had given the required degree of accuracy with a Michelson interferometer (ref. 5). The blue line (4799.991 Å) was chosen because it could be easily separated out from the stellar line (5317 Å) by using a dichroic beam splitter and also it matched the available etalon's coatings.

The stellar and reference light were introduced into the interferometer via two optical fibres. Optical fibres were used to reduce errors introduced by the star wandering over the input aperture and for convenience of being able to have the interferometer as a bench instrument. 600/1 fibres were used.

The etalon had a usable diameter of about 75 mm with a gap of about 92 µm, was coated for about 5000 Å and gave a finesse of over thirty.

As two fibres had to be used a field lens was necessary to stop vignetting. To give a large enough diameter and the correct focal ratio, a f/2.9, 200 mm aerial photography lens had to be used as the collimator.

The reference and stellar lines were separated by a dichroic beam splitter. A 10 Å wide, tilt tuneable filter was used to isolate the 5317.4 Å stellar line and a 100 Å wide filter was used for the cadmium blue reference line.

The stellar light was imaged onto a cooled R.C.A. 3140A/02 photomultiplier which has a high quantum efficiency and a low dark count. For the reference, as there were plenty of photons, an E.M.I. side window tube was used.

To reduce environmental effects the etalon was in a sealed pressure pot and the stellar line filter was temperature controlled to about 0.1°C.

Keywords: Stellar Oscillations, Radial Velocity Spectrometer, Fabry-Perot

1. INTRODUCTION

The CS100 capacitance micrometry servo control system for Fabry-Perot etalons as now manufactured by Queensgate Instruments was originally developed at Imperial College (ref. 2). This was used in an accurate radial velocity spectrometer by Pietraszewski (ref. 3) for observing δ-Scuti type stars. It gave results of about 15 ms⁻¹ on laboratory sources and 30 ms⁻¹ on ω-Car. (Even though this is not a δ-Scuti variable it was used as a calibration star.)

A very stable version of the CS100 has been developed by Queensgate Instruments (ref. 4). By using temperature stabilization of all critical components it should be capable of giving 30 cm s⁻¹ with a ±100 µm gap etalon.

Rather than duplicating the above method we decided to develop a technique of correcting for drifts in the CS100 and/or etalon by using a reference line from a stable source, either a spectral lamp or laser. This is possible as the CS100 offset stability is approximately ten times better than its absolute stability, so by periodically measuring the position of a known wavelength any drifts can be corrected for.

References:

2.2 Modifications to the CS100

The CS100 can be controlled digitally, i.e. by giving the CS100 a 12-bit number an offset is produced in the position of the F.P. plates. However, in the standard CS100 with a 92 μm gap étalon at 5317 Å, the minimum step size is equivalent to about 1350 m s⁻¹. Hence the étalon could be drifting by ±675 m s⁻¹ before any correction would be made. As we were hoping for a noise level of less than 1 m s⁻¹ this was unsatisfactory.

Finer control was obviously necessary for the gap. A second stage was added to the gap control channel (called the Z-channel) of the CS100 in which one step equalled one two thousandth of the original Z-channel. This enabled the fine Z-channel to cover ±1 coarse Z steps with a resolution of about 0.7 m s⁻¹.

2.3 Controlling the Instrument

The interferometer measures changes in the radial velocity by chopping on the points of inflexion of the absorption line and measuring the ratio \((A - B)/(A + B)\) (where \(A\) and \(B\) are the intensities of the two sides of the line). To reduce the noise caused by fluctuations in the sky transparency the étalon is chopped at a rate of 10 Hz and the results summed from each channel for the required integration time.

Similarly, the error in the gap is measured by chopping on the points of inflexion of the cadmium line and calculating the ratio \((C - D)/(C + D)\). This enables the correction to the gap of the étalon to be found by multiplying the ratio by a term based on the slope of the line.

3. STABILITY TESTS

When the interferometer was first tested with a cadmium lamp the noise levels were much larger than expected (100's of metres per second per point). The noise level appeared to vary periodically. We suspected that this was due to the counting period beating with variations in the 50 Hz mains frequency. Consequently a control unit was built which enabled the A.C. frequency the lamp was being run on to be controlled by the computer controlling the instrument. It also acted as a constant current source. This reduced the noise enormously and has been used for all the data presented here.

Figure 2 shows the stability of the interferometer measured using a cadmium lamp without the optical reference system. The 'signal' is being caused by ≈ 2°C change in the room temperature (in the form of a damped square wave) with ≈ 15 hour period. The 'structure' in the 'signal' is caused by the differing temperature behaviour of the interferometer and control electronics. The uninsulated electronics respond more or less instantly whereas there is an approximately 30 hour time constant for the interferometer due to insulation.
Figure 3. Stability of the interferometer with the optical reference for a one order chop.

Figure 3 shows the stability of the same interferometer using the optical reference. (N.B. the change of scale.) The interferometer is measuring the position of a cadmium blue line in one order to represent the stellar line and the same line in the next order as the reference. This gives an offset of one order which is twice the maximum offset that would normally be used. The signal, caused by a similar 2°C change, is just visible. (N.B. the change of scale.)

Figure 4 is the Fourier transform of the data in figure 3. This shows that the offset stability (and hence the radial velocity) is approximately 30 cm s\(^{-1}\) and is better than 1 ms\(^{-1}\) out to periods of over one hour.

Figure 4. Radial velocity spectrum of figure 3.

Figure 4 is the Fourier transform of the data in figure 3. This shows that the offset stability (and hence the radial velocity) is approximately 30 cm s\(^{-1}\) and is better than 1 ms\(^{-1}\) out to periods of over one hour.

4. STELLAR OBSERVATIONS

HR 1217 and 49 Cam were observed in the middle of December 1987 on the 1.5 m IRFC, Tenerife. Simultaneously J.A. Belmonte (of the Instituto de Astrofisica de Canarias) was taking photometric observations on the 0.5 m Mons telescope. A more detailed discussion of the combined observations is being presented elsewhere (ref. 1). Unfortunately, due to size of telescope, stellar magnitude, bad weather etc. neither of these stars gave a good enough signal to noise ratio to provide a significant test of the stability of the interferometer.

Figure 5. Radial velocity spectrum of HR 1217.

Figure 5 is the Fourier transform of the data in figure 3. This shows that the offset stability (and hence the radial velocity) is approximately 30 cm s\(^{-1}\) and is better than 1 ms\(^{-1}\) out to periods of over one hour.

Figure 5 shows the radial velocity spectrum from the Ap star HR 1217. This was taken over four nights, the 11th, 12th, 14th and 15th, with only the 14th and 15th being suitable for photometry. The noise on the mean is about 44 m s\(^{-1}\).

Figure 6. Radial velocity spectrum of 49 Cam.

Figure 6 shows the radial velocity spectrum of 49 Cam. This was observed for 3.75 hours on the 15th. The noise is consequently much higher, being approximately 230 m s\(^{-1}\).
5. POSSIBLE FUTURE DEVELOPMENTS

The present étalon is unnecessarily large for a single line instrument. A 28 mm diameter étalon would give a sufficiently large input aperture for 600 $\mu$ fibres. This would have the advantage that a wider choice of optics could be used, for example photographic lenses. Also, a smaller housing could be used. Using one fibre and introducing the reference light at the telescope would be more satisfactory than using the present two fibres, as it would eliminate any errors caused by movements of the separated fibres. Introducing temperature control for some of the electronics and improving that for the interferometer would help to eliminate low frequency drifts.

The current instrument only uses a single line. Instead of using large telescopes or/and long observing runs a better signal to noise ratio could be obtained by using large parts of the spectrum. Similarly to the Michelson, by using optical fibres (ref. 5), it is possible to use many lines in the Fabry-Perot. However, because field widening in a normal Fabry-Perot is impossible, this is much more difficult due to the fall in usable input aperture (because of loss of resolution) with off-axis angles. This means that large diameter étalons or small fibres have to be used and fibres towards the edge of the field have to be positioned very accurately.

7. REFERENCES


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SOME TESTS OF THE PRECISION OF CCD DIFFERENTIAL PHOTOMETRY MEASUREMENTS OF BRIGHT DOUBLE STARS

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Observatorio Astronomico di Brera, Milano-Merate, Italy

ABSTRACT

Two bright double stars were observed during two nights with a CCD detector in order to verify the possible precision of the CCD differential photometry for asteroseismology applications. The results indicate that the precision of the measurements for each double star, 0.0013 and 0.0011 mag, is better than the precision obtainable with photomultipliers; the improvement, however, is very small.

Keywords: CCD photometry, Double stars

1. INTRODUCTION

The possibility of studying asteroseismology photometrically from the ground or from space is related to the capability of the instrumentation and technique to perform (differential) photometry with very high accuracy. As well known, the earth atmosphere prevents us to perform very high precision photometric measurements, and it is difficult to obtain \( \Delta \)-values better than a value between 0.001 and 0.002 mag with photomultipliers from the ground. In the present note we report on the results of a test made on two bright double stars observed with a CCD. The aim of this test was to check the precision of this technique, because it has two advantages, the possibility of collecting much more photons per sec from a bright star than the photomultiplier, and the presence of more than one star per image.

2. OBSERVATIONS AND REDUCTIONS

The double stars HR 6184-6185/6186 and HD 164329-164330 (see Table 1) were observed during two nights at the 1.5 m telescope of Loiano Observatory (Bologna) with a CCD detector and \( V \) filter. The nights were not photometric because of thin clouds. The star images were defocussed in order to spread them over many pixels and allow us to make fairly long integrations (typically 20 - 30 sec); in this respect, the present test is different from that by Walker (Ref. 1). The time resolution of the observation varied between one and three minutes. The total observing time was less than three hours for both double stars. The reductions were performed at Brera Observatory (Milano) using MIDAS. After bias subtraction and flat field correction, an integration of the pixels covered by a star image was performed, and then an equal integrated area of background sky was subtracted and the magnitude difference (in the instrumental system) between the stars was computed. As regards HD 164329-164330, it was possible to check also the magnitude difference for two faint stars close to the bright double star.

3. DISCUSSION

A slow trend of the data of both double stars was detected. That for HD 164329-164330 could be explained by the color difference between the components of the double star; however, that...
should not be the reason for the trend of HR 6184-6185/6186 data. The observations are not sufficient for discussing this effect. Since we were interested especially in the estimation of the accuracy of measurements for short term variability, we corrected for the slow trend by using a parabola fit and derived the residuals. The power spectrum of the residuals did not show significant peaks, hence we assumed that there were no significant light variations. In Table 2, the $\sigma$-values of the residuals for each couple of stars are reported and they can be compared with the expected values computed taking into account the amount of photoelectrons collected, the background sky and the readout. The observed values are much higher than the expected ones for the brightest stars. The probable explanation of the strong difference could be the scintillation; other effects related to the acquisition and reduction techniques, such as small displacements of an image with respect to another, flat field corrections, integrations for given apertures, play a secondary role in this case. These secondary effects will be important when the main source of noise, very probably the earth atmosphere, will be removed.

The present results, even if obtained during not photometric nights, show that CCD detectors at ground based telescopes give differential photometry measurements of bright stars as accurate as (or slightly more accurate than) photomultipliers, but there is not a strong improvement of such an accuracy. However, other tests made at sites with excellent sky conditions should be performed.

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4. REFERENCES

Session 8
Models of stellar evolution from seismology
Chairman: H. Shibahashi
ON THE THERMAL SURFACE BOUNDARY CONDITION FOR SOLAR PULSATIONS

M. GABRIEL

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ABSTRACT.

The numerical coefficient which appears in the thermal surface boundary condition of radial and nonradial oscillations is computed for the Sun.

Key words: stellar oscillations, stellar stability, solar oscillations.

1. INTRODUCTION.

Recently I showed (ref. 1) that the thermal surface boundary condition for stellar pulsations is

\[ k \frac{F_r}{P} = \frac{P_r}{P} \]  

(1)

where \( F_r \) is the radial component of the radiative flux, \( P \) is the radiation pressure and the symbol prime stands for eulerian perturbation.

The coefficient \( k \) is a function of the degree and of the order of the mode under consideration. I have shown that for radial oscillations \( k = 1 \). For low degree nonradial oscillations \( k \) is also equal to one with an accuracy of a few percent. For higher degree \( k \) becomes significantly smaller than one.

The coefficient \( k \) is also a function of the limb darkening law. In my previous paper I used a very simple limb darkening law. As high and intermediate degree modes will ever be observed on the sun only, I have made new computations of \( k \) using the solar limb darkening law. I also give values for low degrees because one might need an accurate thermal surface boundary condition in connection with the solar oscillation problem.

2. EQUATIONS OF THE PROBLEM.

I have shown that to obtain the value of \( k \), one has to solve the following system from the "surface" of the star to infinity.

\[
\begin{align*}
\frac{dy_1}{dr} + \frac{i \sigma}{c f} y_2 - \frac{(t+1)}{r^2} \frac{2}{3 f - 1 - c E} y_3 &= 0 \\
\frac{dy_2}{dr} + \frac{f - 1}{fr} y_3 + \frac{i \sigma}{c} y_1 - \frac{(t+1)}{r^2} y_2 &= 0 \\
\frac{dy_3}{dr} - \frac{f - 1}{2 f} y_2 + \frac{i \sigma}{c} \frac{2}{3 f - 1 - c E} y_3 &= 0
\end{align*}
\]

(2)

with

\[
y_1 = r^2 F_r / c \\
y_2 = r^2 f E \]

\[
y_3 = \frac{3 f - 1}{2} \frac{E F_r}{r} \]

(3)

where \( r \) is the distance to the center of the star, \( \sigma \) is the eigenvalue, \( c \) is the speed of light, \( t \) is the degree of the oscillation, \( F \) is the equilibrium radiation flux, \( E \) is the energy density of radiation, \( F_r \) is the eulerian perturbation of the horizontal component of the radiation flux and \( f \) is the variable Eddington factor defined by

\[ P = f E \]  

(4)

For radial oscillation \( y_3 \equiv 0 \) and we have only the first two equations of (2).

The solutions of (2) must fulfil 3 boundary conditions. At infinity the solution must be an outgoing wave. This implies that \( y_1 = y_2 \). At the surface \( r = R \), I have taken the normalization condition \( y_1 = 1 \). I have also taken several values for \( \gamma \).

The value of \( \gamma \) is given by the integration of the oscillation in the star and here I have to take it as a free parameter.

Because our problem is linear, it is easily verified that, with our boundary conditions, \( k \) is a simple function of \( R \)

\[ k = [\Delta_1 + \Delta_2 R]^{-1} \]  

(5)

3. THE VARIABLE EDDINGTONG FACTOR.

In order to integrate (2) we must know \( f \) and \( F_r / c \) of all \( r \geq R \). \( E, F \) and \( P \) can be obtained by numerical computation of the integrals which define these variables. Therefore we have to compute integrals of the form

\[ I_n = 2 \pi \int_0^{\phi_n} I(\phi) \cos^m \phi \sin \phi d \phi \]  

(6)

where \( I \) is the intensity and \( \phi \) the angle of the radius vector and the light ray. \( \phi_n \) is such that the ray is tangent to the star.

It is easy to obtain \( I_n \) in terms of variables defined at the surface of the star. Simple considerations lead to
There $x = r/R$ and $\delta$ is the angle between the radius vector and the light ray at $r = R$. $I(\beta)$ is the observed limb darkening law given by Aller (ref. 2).

4. NUMERICAL RESULTS.

I found previously that the values of $k$ are nearly independent of the period. Therefore we give results for a period of 6 minutes only.

Table I gives the values of $k$ for a few values of $C$ and $\ell$. The accuracy is of the order of $10^{-3}$. It shows that $k$ differs significantly from one even for small $C$ values when $C > 10$. Also, for a given $C$ value, $k$ becomes constant for large $\ell$.

The values of $\Delta_1$ and $\Delta_2$ are given in Table II. Their imaginary part is small and may be neglected with an accuracy generally better than one per cent. For $\ell \geq 20$ their real part can be fitted to simple analytic laws which are

$$\Delta_1 = 1.332 + 0.2807 \ell^{-1} - 39.14 \ell^{-2} + 364.3 \ell^{-3}$$

$$\Delta_2 = -0.328 + 104.8 \ell^{-1} - 1666. \ell^{-2} + 13982 \ell^{-3}$$

The thermal surface boundary condition (1) has to be applied very far out. However, it is possible, as shown in Gabriel (ref. 1), to replace it by a condition applicable at the bottom of the isothermal zone.

5. REFERENCES.


---

**TABLEAU I : Value of $k$ for a few selected values of $C$ and $R$**

<table>
<thead>
<tr>
<th>$C/R$</th>
<th>$-10^{-4}$</th>
<th>$10^{-1}$</th>
<th>$-10^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.985 - i 3.1 10^{-4}</td>
<td>0.975 + i 4.8 10^{-5}</td>
<td>0.985 + i 9.3 10^{-3}</td>
</tr>
<tr>
<td>4</td>
<td>0.943 + i 2.6 10^{-3}</td>
<td>0.972 + i 4.2 10^{-3}</td>
<td>0.939 + i 7.9 10^{-1}</td>
</tr>
<tr>
<td>10</td>
<td>0.897 + i 4.6 10^{-3}</td>
<td>0.989 + i 1.5 10^{-3}</td>
<td>0.791 + i 1.86 10^{-1}</td>
</tr>
<tr>
<td>20</td>
<td>0.790 - i 1.0 10^{-4}</td>
<td>0.919 - i 0.6 10^{-4}</td>
<td>0.722 + i 2.02 10^{-1}</td>
</tr>
<tr>
<td>50</td>
<td>0.761 - i 4.1 10^{-4}</td>
<td>0.558 - i 2.87 10^{-3}</td>
<td>0.669 + i 2.42 10^{-1}</td>
</tr>
<tr>
<td>100</td>
<td>0.755 - i 7.10^{-5}</td>
<td>0.535 - i 3.1 10^{-4}</td>
<td>0.646 + i 2.64 10^{-1}</td>
</tr>
<tr>
<td>300</td>
<td>0.751 - i 2.3 10^{-3}</td>
<td>0.516 - i 1.2 10^{-4}</td>
<td>0.625 + i 2.81 10^{-1}</td>
</tr>
</tbody>
</table>

**TABLEAU II : Values of $\Delta_1$ and $\Delta_2$ (see eq. 5) for several $\ell$ and a period of 6 min.**

<table>
<thead>
<tr>
<th>$\ell$</th>
<th>$\Delta_1$</th>
<th>$\Delta_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.100 + i 1.16 10^{-3}</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-0.286 + i 1.06 10^{-2}</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-0.550 + i 1.80 10^{-2}</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-0.871 + i 2.53 10^{-2}</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-1.587 + i 2.80 10^{-2}</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>-2.258 + i 7.05 10^{-3}</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>-2.750 - i 3.36 10^{-2}</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>-3.189 - i 1.04 10^{-1}</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>-3.506 - i 2.80 10^{-2}</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>-4.787 - i 8.59 10^{-2}</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>-5.433 - i 7.85 10^{-3}</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>-5.997 - i 3.15 10^{-3}</td>
<td></td>
</tr>
</tbody>
</table>
ASYMPTOTIC APPROXIMATIONS OF NON-RADIAL OSCILLATION MODES OF THE SUN

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ABSTRACT

An asymptotic approximation of higher-degree p-modes in the Sun with periods near 5 minutes is developed. As a starting point, the property is used that the inner boundaries of the acoustic resonant cavities in which these modes originate, are located at depths in the solar interior where the local acoustic frequency becomes equal to the angular frequencies considered. The treatment is based on Olver’s asymptotic theory for second-order differential equations containing a large parameter. The equation determining the eigenfrequencies agrees with the equation derived earlier by Gough by the use of ray theory. The transition to the equation determining eigenfrequencies of higher-order p-modes is also considered.

Keywords: Solar oscillations, Stellar oscillations, Asymptotic theory.

1. INTRODUCTION

The 5-minute oscillations observed on the solar surface are ascribed to global p-oscillations originating from propagating acoustic waves that are reflected to and fro in resonant cavities below the solar surface. The extents of these cavities can be derived from a simple dispersion relation for the local wave propagation (Ref. 1). Their inner boundaries are located at depths in the solar interior where the local acoustic frequency depending on the degree of the spherical harmonic and the speed of sound at a particular point is equal to the angular frequency under consideration. We use this property as a starting point for the development of an asymptotic approximation of higher-degree p-modes in the Sun based on Olver’s asymptotic theory for second-order differential equations containing a large parameter (Ref. 2).

We find that the equation determining the eigenfrequencies agrees with the equation derived earlier by Gough by the use of ray theory (Ref. 3). This equation provides a theoretical explanation for the relation found by Duvall in the solar data (Ref. 4) and has served as the basis for inversion procedures leading to determinations of the variation of the speed of sound with depth in the solar interior that do not rely on theoretical models of the Sun (Refs. 5 and 6).

Finally, we consider the transition to the equation determining eigenfrequencies of higher-order p-modes.

2. DISPERSION RELATION AND PROPAGATION DIAGRAM

Let \( f_r, f_\theta, \) and \( f_\varphi \) be the components of the Lagrangian displacement with respect to the local orthonormal basis in a system of spherical coordinates \( r, \theta, \varphi \) with origin at the Sun’s center. These components are related to a single spherical harmonic \( Y^l_m (\theta, \varphi) \) by

\[
\begin{align*}
\frac{f_r}{r} &= \frac{u(r)}{r} Y^m_l (\theta, \varphi), \\
\frac{f_\theta}{r} &= \frac{v(r)}{r} \frac{\partial}{\partial \theta} Y^m_l (\theta, \varphi), \\
\frac{f_\varphi}{r} &= \frac{v(r)}{r \sin \theta} \frac{\partial}{\partial \varphi} Y^m_l (\theta, \varphi).
\end{align*}
\]

If we consider isentropic displacement fields and neglect the Eulerian perturbation of the gravitational potential, the radial functions \( u(r) \) and \( v(r) \) are solutions of the second-order system of differential equations

\[
\frac{du}{dr} = \frac{\nu}{c^2} u + \frac{\nu}{c^2} \left( \frac{\partial^2}{\partial r^2} - \frac{\delta^2}{r^2} \right) v, \\
\frac{dv}{dr} = \left[ 1 - \frac{\delta^2}{r^2} \right] \frac{u}{r} + \frac{\delta^2}{n^2} v.
\]

Here \( \delta \) is the angular frequency in the time dependency taken proportional to \( \exp \left( i \omega t \right) \), \( \nu^2 \) the square of the Brunt-Väisälä frequency, and \( \delta^2 = ((l + 1)\nu^2 + \omega^2) \) the square of the acoustic frequency.
By looking for local solutions
\[ u = A \exp(-i k r), \quad v = B \exp(-i k r), \] (4)
with non-zero constants A and B, we derive the relation between the complex wave number \( k_r \) and the angular frequency \( \omega \):
\[ \left( \frac{k_r + \frac{1}{2} \frac{d \ln n}{d r} \right)^2 = \frac{1}{c^2} \left( \frac{\omega^2 + \frac{S_0^2 N^2}{\sigma} - S_0^2 - \sigma_0^2}{\sigma} \right) \]
\[ = \frac{1}{c^2} F (\omega^2, \ell, r), \] (5)
where \( \sigma_0^2 = c^2 (\frac{d \ln M}{d r})^2 / 4 \) is the square of the acoustic cutoff frequency.

Local propagation in the radial direction is possible for a real \( \omega \) if \( k_r \) has a real part or, equivalently, if
\[ F > 0. \] (6)

When this condition is fulfilled, the left-hand member of Relation (5) reduces to the square of the real part of \( k_r \) and the dispersion relation corresponds to the relation given by Whitaker (Refs. 7 and 8) and used recently by Bahcall and Ulrich (Ref. 9).

For a given value of \( \ell \), \( F \) is positive in the solar interior for sufficiently large or sufficiently small values of \( \omega^2 \). The roots of the algebraic equation
\[ F = 0 \] (7)
which is quadratic in \( \omega^2 \), determine the lower and upper limits of the admissible values of \( \omega^2 \) and delineate the resonant cavities of the acoustic waves and the internal waves in a \((r, \omega^2)\)-diagram or propagation diagram.

Fig. 1 presents the propagation diagram for a solar model computed by Noels and Gabriel, for the degree \( \ell = 15 \) of the spherical harmonic. The abscissa is the fraction of the total solar radius, and the ordinate is \( c^2 \omega^2 \), the square of the acoustic cutoff frequency.

Local propagation in the radial direction is possible for a real \( \omega \) if \( k_r \) has a real part or, equivalently, if
\[ F > 0. \] (6)
For a higher-degree \( p \)-mode, we consider \((\ell + 1)\) as a large parameter, and the two terms \( c_0^2/4c^2 \) and \( 1/c^2 \) involved in the function \( \varphi(r) \) are of the same order of magnitude near the inner boundary of the acoustic cavity. The point at \( r = r_c \) where \( \varphi(r) = 0 \) is, therefore, a turning point of Eq. (8). The function \( M_1(r) \) is regular at that point.

We construct asymptotic approximations of the solutions of Eq. (8) from the turning point towards the center and towards the surface in terms of Airy functions.

The asymptotic approximation of the solution for \( v \) from the turning point towards the center takes the form

\[
v^{(2)} = \frac{1}{r^{3/2}} \left[ 1 - \frac{1}{4} \varphi(r) \right] ^{1/2} \left[ 1 + \frac{1}{3} (2) \right] J_{-1/4} \left[ \frac{1}{3} (2) \right] J_{1/4} \left[ \frac{1}{3} (2) \right]
\]

and \( B_1 \) and \( B_2 \) are two undetermined constants.

Similarly, the asymptotic approximation of the solution for \( v \) from the turning point towards the surface takes the form

\[
v^{(3)} = \frac{1}{r^{3/2}} \left[ -\varphi(r) \right] ^{1/2} \left[ 1 + \frac{1}{3} (3) \right] J_{-1/4} \left[ \frac{1}{3} (3) \right] J_{1/4} \left[ \frac{1}{3} (3) \right]
\]

and \( C_1 \) and \( C_2 \) are two undetermined constants.

The point at \( r = 0 \) is a double pole of \( \varphi(r) \), and an analytical point of \( M_1(r) \). Therefore, we construct another asymptotic approximation of the solution for \( v \) from that point towards the turning point at \( r = r_c \). The asymptotic approximation yielding a finite Lagrangian displacement at \( r = 0 \) is

\[
v^{(4)} = A_1 \left[ \begin{array}{c} r^{3/2} \left[ -\varphi(r) \right] \frac{1}{2} \exp \left[ \left( \ell + \frac{1}{2} \right) t^{(4)} \right] \end{array} \right]
\]

where

\[
\phi^{(4)}(r) = \int_0^r \left[ -\varphi(r') \right] ^{1/2} \, dr',
\]

and \( A_1 \) is an undetermined constant.

In the three cases considered for the construction of asymptotic approximations for \( v \), we derive asymptotic approximations for \( u \) by means of Eq. (3).

In order to construct asymptotic approximations of the solutions from the surface, we use an equation of the following form derived from Eqs. (2) and (3):

\[
u^{(0)} = \frac{1}{r^{3/2}} \left[ 1 - \frac{1}{4} \varphi(r) \right] ^{1/2} \left[ 1 + \frac{1}{3} (2) \right] J_{1/4} \left[ \frac{1}{3} (2) \right] J_{1/4} \left[ \frac{1}{3} (2) \right]
\]

where

\[
u^{(0)} = \int_0^r \left[ -\varphi(r') \right] ^{1/2} \, dr',
\]

This equation is appropriate from the surface to a radial distance to the center where \( S^2/c^2 \) is sufficiently smaller than unity so that the terms of that order may be neglected (Refs. 10 and 11).

We assume that the mass density in the superficial layers can be expressed as

\[
u^{(0)}(R - r) \approx 
\]

where \( \rho_0 \) and \( m_0 \) are constants. By setting \( m(r) = M \) and integrating the equation of hydrostatic equilibrium, we derive

\[
u^{(0)} \sim \frac{M}{R} \left( \frac{R - r}{R} \right) ^{m_0 + 1}
\]

From the use of these expressions for \( \rho \) and \( P \), it follows that the point at \( r = R \) is a single pole of \( \varphi(r) \) and a double pole of \( M_2(r) \).

The asymptotic approximation of the solution for \( u \) yielding a Lagrangian perturbation of the pressure that is zero at the surface takes the form

\[
u^{(4)} = D_1 r^{3/2} \left[ -\varphi(r) \right] ^{1/4} \left[ t^{(4)} \right] ^{1/2}
\]

where \( D_1 \) is a Bessel function, \( D_1 \) an undetermined constant, and

\[
u^{(4)} = \int_0^r \left[ -\varphi(r') \right] ^{1/2} \, dr'.
\]
We derive the associated approximation for \( v \) by means of Eq. (2).

We require the asymptotic approximations for \( u \) and \( v \) to be continuous from the center to the surface. By reducing the Airy functions and the Bessel function to their first asymptotic approximations for large arguments, and using the approximation

\[
\ell (\ell + 1)^{1/2} \approx \ell + 1/2,
\]

we see that

\[
B_2 = 0, \quad C_2 = 0,
\]

and that non-zero values can be found for the remaining constants if the angular frequency \( \omega \) satisfies the following equation:

\[
\int_{r_1}^{R} \frac{1}{c(r')} \left[ 1 - \frac{S_2^2 (r')}{\rho^2} \right]^{1/2} dr' = \int_{r_1}^{R} \frac{1}{c(r')} \left[ 1 - \frac{\ell (\ell + 1) c_0^2}{\sigma^2 r'^2} \right]^{1/2} dr' + \int_{r_1}^{R} \frac{dr'}{c(r')}.
\]

Integrating the first term, we find

\[
\int_{r_1}^{R} \frac{1}{c(r')} \left[ 1 - \frac{\ell (\ell + 1) c_0^2}{\sigma^2 r'^2} \right]^{1/2} dr' = \int_{r_1}^{R} \frac{r'^{1/2}}{c(r')} \left[ 1 - \frac{\ell_0 c_0^2}{\sigma^2 r'^2} \right]^{1/2} dr' = \frac{1}{c_0} \left[ \ell (\ell + 1) \right]^{1/2} \frac{1}{\sigma} \sec^{-1} \frac{r'^{1/2}}{r_1}.
\]

If we let \( r_1 \) go to zero, it follows that

\[
\int_{0}^{R} \frac{dr'}{c(r')} = \left[ \frac{1}{2} \ell (\ell + 1/2) + n + \frac{n}{2} \right].
\]

Finally, by substituting into Eq. (29) and again using Approximation (27), we find

\[
\int_{0}^{R} \frac{dr'}{c(r')} = \left[ \frac{1}{2} \ell (\ell + 1/2) + n + \frac{n}{2} \right].
\]
Asymptotic approximations of non-radial oscillation modes of the Sun


Figure 1. The position of the inner boundary of the acoustic resonant cavity at the period of 5 minutes for l varying from 1 to 1000.

5. REFERENCES

assumption that is made in counting the nodes of the eigenfunction \( u \) from the center (Ref. 11).

Relative to the solar non-radial p-modes with periods near 5 minutes, it should be noted that the inner boundary of the acoustic resonant cavity associated with the degree \( l = 1 \) of the spherical harmonic is still situated at a fraction of the radius from the center \( r/R = 0.05 \) as is seen from Fig. 3, where the position of the inner boundary of the acoustic resonant cavity at the period of 5 minutes is plotted for \( l \) varying from 1 to 1000. Fig. 3 is similar to a figure given by Duvall and Harvey (Ref. 13).
Abstract

The consequences of a possible mass loss in the early main sequence stage of solar type stars are investigated. Special attention is given to the Sun, the constraints from surface abundances and p-mode observations are taken into account. Only a total mass loss smaller than 0.2 $M_\odot$ seems to be consistent with the observations. Though the ensuing modifications of the internal structure and the cosmological consequences are very small, the $^4$He, $^7$Li, $^9$Be surface abundances are significantly modified.

Keywords: Standard solar model, Mass loss, Solar oscillations.

1. Introduction

The effect of mass loss on stellar evolution has been studied mainly in the phases where the mass loss is important and rather easily observable (Chiosi and Stellari 1981, Maeder and Meynet 1987). This effect is essential for the evolution of red giants or Wolf Rayet stars.

We discuss here the less evident effect of a possible mass loss in the main sequence phase for solar-type stars. The motivation given by its proponents (Willson et al 1987, Guzik et al 1987) was multiple: reconciliation of cosmic time scale (10 Gyrs) with the age of oldest stars (15 Gyrs), interpretation of the 1 $M_\odot$ bump in the initial mass function, modification of the $^7$Li, $^9$Be, $^3$He surface abundances.

From an observational point of view, the mass loss rate during this part of stellar life is totally unknown (Dupree 1986), the Sun is the only exception, with a present value of $\dot{M} = 2 \times 10^{-14} M_\odot/\text{yr}$. The observational upper limit is about $10^{-9}$–$10^{-10} M_\odot$ but there is no information on the original physical process. Might there, for example, be some relation between the observed rotational velocity in young clusters and mass loss?

Our aim is to study the influence of possible mass loss (with parametrised rate and time dependence) on the internal and external structure of the present Sun, for which we have precise measurements of luminosity, radius and p-modes frequencies. We want to determine how observations can limit the parameter range of the assumed mass loss.

Such a quantitative comparison of theoretical results with observations requires the development of an up-to-date solar standard model which is chosen as reference for this study. This allows to disregard present inadequacies of standard models and thus to isolate the non-standard effects. We therefore first present the Saclay standard model. Then we discuss the different models with mass loss, examine their main features and acoustic modes. A detailed discussion on surface abundances will appear in another paper.

2. The standard solar model

This model has been extensively discussed in Turck-Chièze et al (1988). It was obtained with a Paczynski code developed first for binary systems (De Grève and de Loore 1976) and adapted to single star evolution with mass loss (Prantszos et al 1986)
It includes the Los Alamos (Huebner 1977) opacities and the recent nuclear reactions rates from Caughlan and Fowler (1988) except for the $^7$Be($p,\gamma)^8$B reaction. For this reaction, which is crucial for $^8$B neutrinos, we have adopted the potential-model calculation of Barker (1983). A grey atmosphere described by Paczynski (1969) is used. The precision (around $10^{-4}$) required by helioseismology in the solution of the structure equations, has led us to improve the calculation of the partially degenerated electrons in the equation of state. The previously tabulated results were replaced by the high-precision Fermi-Dirac package employed in the MHD equation of state (Hummer and Hihalas 1988, Mihalas, Dappen and Hummer 1988, Dappen et al 1988). This modification in the equation of state did not lead to significant changes of the Saclay standard model results. Our present precise reference model gives a solar helium content of 0.271±0.012 by mass fraction, a mixing length parameter of 1.61, a capture rate of 5.7±1.3 SNU on $^{37}$Cl and 124±5 SNU on $^{71}$Ga. These results are very close to those of Bahcall and Ulrich (1988).

We have also compared our results with models developed by Christensen-Dalsgaard (1982); though the input physics of the two models (opacity, nuclear reactions rates) slightly differs we can see in Fig. 1 that the sound speed in the interior agrees well.

3. Models with mass loss

Guzik et al (1987) have recently presented models with an extreme mass loss of 1 $M_\odot$. Though their results are quite incompatible with observational limits, we have, for didactic purposes, recomputed these models to enhance some aspects not discussed previously.

We have considered two values of total mass loss: 1 and 0.2 $M_\odot$. The mass loss rate is parameterized as: $M=1/\tau \exp(-t/\tau)\Delta M$ (see also Guzik et al 1987). We have tried two different decay time scales, $\tau=0.13$ and $\tau=0.33$ Gyr, corresponding to $\dot{M}=7.7 \times 10^{-9}$ and $3.03 \times 10^{-9}$ $M_\odot$ yr$^{-1}$ for the 2 $M_\odot$ initial mass and $1.5 \times 10^{-9}$ and $6.1 \times 10^{-10}$ $M_\odot$ yr$^{-1}$ for the 1.2 $M_\odot$ initial mass, respectively.

We emphasize the danger of modelling extreme mass loss without treating overshooting of the convective core. The present-day calculations of stellar evolution have demonstrated the necessity to include an overshooting distance $d_{\text{over}} = 0.3 H_p$ (where $H_p$ is the pressure scale height). This value is imposed by the observed location of the top of the main sequence and its width (Maeder and Meynet 1988). In a standard solar model there is no convective core left at the solar age, hence no overshooting needs to be considered. This is different for an initial 2 $M_\odot$. And thus we examine for the sake of completeness the effect of increasing the size of the convective core by 0.3 $H_p$.

Table 1 summarizes the six models considered.

The effects of mass loss are:

- i) The absolute value of mass loss determines the initial luminosity of the star, which would be for an initial 2 $M_\odot$ 13 times greater than the luminosity of the present Sun. Is it a severe problem for the nearby planets?...

- ii) The main-sequence lifetime is influenced by the absolute mass loss and the time dependence of the mass-loss rate. In fact, the hydrogen burning is stronger in mass-loss models because of the higher initial central temperature; as a consequence, the present central hydrogen abundance is smaller and the main sequence lifetime reduced. In all cases, this reduction is never more than 10% of the lifetime of the standard model, even in the presence of extreme mass loss. However, the effect itself is strongly affected by overshooting from the convective core. Overshooting can partly compensate the effect of the greater initial mass and even increase the lifetime (see models 2 and

![Figure 1: Relative discrepancy between the squared sound speed of the two standard models (T-C et al 1988 and C-D 1982).](image-url)
Table 1: Models generated

<table>
<thead>
<tr>
<th>Initial mass</th>
<th>Overshooting</th>
<th>Y</th>
<th>Main seq lifetime</th>
<th>( ^{3}C_{1} )</th>
<th>( SNU )</th>
</tr>
</thead>
<tbody>
<tr>
<td>M ( \times 10^3 )</td>
<td>( \tau ) (Gyr)</td>
<td>( \alpha )</td>
<td>yr</td>
<td>yr</td>
<td></td>
</tr>
<tr>
<td>Standard</td>
<td>1</td>
<td>no</td>
<td>0.273</td>
<td>1.61</td>
<td>8.9 ( 10^9 )</td>
</tr>
<tr>
<td>Model 1</td>
<td>2</td>
<td>0.13</td>
<td>no</td>
<td>0.261</td>
<td>1.70</td>
</tr>
<tr>
<td>Model 2</td>
<td>2</td>
<td>0.13</td>
<td>d=0.3</td>
<td>0.265</td>
<td>1.67</td>
</tr>
<tr>
<td>Model 3</td>
<td>2</td>
<td>0.33</td>
<td>no</td>
<td>0.243</td>
<td>1.82</td>
</tr>
<tr>
<td>Model 4</td>
<td>2</td>
<td>0.33</td>
<td>d=0.3</td>
<td>0.245</td>
<td>1.81</td>
</tr>
<tr>
<td>Model 5</td>
<td>1.2</td>
<td>0.33</td>
<td>no</td>
<td>0.269</td>
<td>1.63</td>
</tr>
</tbody>
</table>

4). For a mass loss of 0.2 \( M_{\odot} \), the reduction of the lifetime is negligible.

- iii) The protosolar helium abundance \( (Y) \) is modified by mass loss. \( Y \) varies as \( W^{7.5} \) (where \( W \) is the mean molecular weight) and since it is observationally constrained for the present Sun, the increase in the central molecular weight (see previous point) must be balanced by a reduction of the atmospheric one (which is the initial one). The protosolar helium abundance is thus reduced in all models considered. This effect is, in all cases, in agreement with the generally assumed helium enrichment in the Galaxy before the birth of the Sun. Here, nevertheless, that the allowed galactic helium enrichment from the big-bang value of Pagel (1988) \( (Y=0.235\pm0.004) \) is uncomfortably small for models 3 and 4 \( (Y=0.243, 0.245) \). The situation is even worse in the equivalent model 3 of Guzik et al (1987) for which the initial helium mass fraction reaches 0.20, but this last result seems to be the consequence of the use of an old reference model. In our case, a maximum 10 \% change in the initial helium abundance is obtained for \( \alpha=0.33 \) and the extreme mass loss of 1 \( M_{\odot} \); this effect is quite independent of the presence of overshooting. For a mass loss of 0.2 \( M_{\odot} \) the modification of the initial helium abundance is very small.

- iv) The absolute value of mass loss and the time dependence of the mass-loss rate affect the convective envelope. For high initial mass-loss, the envelope is very thin at age zero and increases with time; this is in contrast to the standard model. The mixing length parameter \( \alpha \) (used as Henyey, Vardy and Bodenheimer 1965) is higher than the standard model one and increases with the decay time scale, the temperature of the bottom of the convective zone reaches 2.4 \( 10^6 \) K only in the extreme cases of model 3 and 4. Nevertheless, for \( ^{7}Li \) and \( ^{9}Be \), the most important factor is the evolution of the temperature of the 1 \( M_{\odot} \) shell, if we exclude a regeneration of these elements by flares as Ryter et al (1970) and Reeves (1986). Without regeneration, a high mass loss of 1 \( M_{\odot} \) would lead to a complete destruction of these two elements.

- v) As the central temperature of all these models is higher with respect to the standard one (at the present age), the chlorine neutrino capture rate is increased, even by a factor 2 for models 3 and 4 for which the internal structure is the most affected. For model 5 (smaller mass loss) the modification is rather minor.

4. Confrontation with helioseismology

Thermodynamical quantities are clearly modified by high mass loss, mainly in the central region as shown in Table 2. Moreover the models with overshooting (2 and 4) present a small but persisting convective core (about 0.1 \( M_{\odot} \)) at the present age.
We have therefore submitted the standard model and these non-standard models to seismological tests. The first test concerns the modes that are most sensitive to central conditions. Of course, internal g modes would be suited best if data were available. In their absence, we use p modes of low degree (Duvall et al 1988).

The Saclay code treats the atmospheric layers and those beneath the photosphere coherently, using the same formalism. The equilibrium model thus calculated can therefore be directly put into the adiabatic pulsation code of Christensen-Dalsgaard (1981, 1982), no additional procedure to determine the radius variable above the photosphere is required.

The results for our standard model are presented in Table 3. We compare the computed absolute frequencies with the recent compilation of Duvall et al (1988) for selected orders and degrees. Using the polynomial development discussed by several authors (for example Grec et al 1983 ) we compare the values of the parameters $\Delta v_0$ (the large frequency separation, probing the surface layers) and $D_0$ (related to the small frequency separation, probing central conditions) with the observed values. Our results are in the range of other recent standard models (Bahcall and Ulrich 1988, Christensen Dalsgaard 1982).

Corresponding results for high mass loss are summarized in Table 4 where we distinguish two classes of models, those without overshooting which give a $D_0$ value smaller than in the standard model and those with overshooting which give high values. The interpretation of the results is easier if we consider asymptotic analysis (Tassoul 1980). We calculate the expression:

$$A = \frac{1}{4\pi^2} \left[ \frac{c(R)}{R} - \left( \int_{R}^{R} \frac{dc}{dr} dr \right) \right].$$

Models 1 and 3, without a present convective core are very similar to the standard model. Using the relation $dc/dr=1/T \;dT/dr - 1/\mu \;dp/dr$, we realize that the difference mainly comes from the increase in the central mean molecular weight of about 10% which is only partly compensated by a temperature increase of 5%.

In the case of models 2 and 4, with a small convective core, the asymptotic calculation must be done carefully because the sound speed derivative is everywhere negative even at small radii, in contrast to the case of the standard model. In such models the contribution to the integral coming from small radii is extremely important. If the inferior limit of the integral is not properly taken into account ($r_1$ around 0.1 $R_0$ for $l=2$), the value of the integral $\Delta v_{20}$ could be modified by a factor up to 2 (see the contribution by Smeyers et al, these proceedings).

We have not yet obtained the numerical results for the model o with 0.2 mass loss but judging from the asymptotic prescription, we predict a $D_0$ of 9.8 $\mu$Hz which is slightly smaller than that of the standard model.

**Table 3 : Frequencies of p-modes**

<table>
<thead>
<tr>
<th>Model</th>
<th>1-0 n=20</th>
<th>1-2 n=19</th>
<th>1-1 n=19</th>
<th>1-3 n=18</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta$</td>
<td>135.55</td>
<td>135.24</td>
<td>-1.7045</td>
<td>10.23</td>
</tr>
<tr>
<td>$D_0$</td>
<td>-1.7045</td>
<td>-1.5040</td>
<td>-1.5040</td>
<td>9.0493</td>
</tr>
<tr>
<td>$\Delta v_{20}$</td>
<td>10.23</td>
<td>9.0493</td>
<td>9.0493</td>
<td>9.0493</td>
</tr>
<tr>
<td>$\rho_C$ (g/cm$^3$)</td>
<td>146.65</td>
<td>146.65</td>
<td>146.65</td>
<td>146.65</td>
</tr>
<tr>
<td>$T_C$ (K)</td>
<td>15.49 $10^6$</td>
<td>15.49 $10^6$</td>
<td>15.49 $10^6$</td>
<td>15.49 $10^6$</td>
</tr>
</tbody>
</table>

**Table 4 : Frequencies for high mass loss models**

<table>
<thead>
<tr>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-0 n=20</td>
<td>2893.19</td>
<td>2888.23</td>
<td>2888.81</td>
</tr>
<tr>
<td>1-2 n=19</td>
<td>2882.62</td>
<td>2877.04</td>
<td>2879.63</td>
</tr>
<tr>
<td>1-1 n=19</td>
<td>2820.98</td>
<td>2817.52</td>
<td>2817.33</td>
</tr>
<tr>
<td>1-3 n=18</td>
<td>2804.81</td>
<td>2798.70</td>
<td>2802.82</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>135.67</td>
<td>135.72</td>
<td>135.32</td>
</tr>
<tr>
<td>$D_0$ (MHz)</td>
<td>-1.589</td>
<td>-1.823</td>
<td>-1.418</td>
</tr>
<tr>
<td>$\Delta v_{20}$ (MHz)</td>
<td>9.53</td>
<td>9.93</td>
<td>8.52</td>
</tr>
<tr>
<td>$\rho_C$ (g/cm$^3$)</td>
<td>157.15</td>
<td>115.39</td>
<td>177.66</td>
</tr>
<tr>
<td>$T_C$ (K)</td>
<td>15.7 $10^6$</td>
<td>15.83</td>
<td>16.22</td>
</tr>
</tbody>
</table>

conv core
5. Discussion of the helioseismological results

The study of mass loss has led us to compute non-standard models that can have quite different central conditions (temperature differences up to 5%, density up to 20%, this corresponds to 10% in mean molecular weight). In the seismological comparison the astrophysical interest is doubled by our need to understand the real Sun better. So, to broaden the comparison of our models to higher-degree node observations, we draw on Figure 2 the difference of the square of sound speed between non-standard models 1,3,4,5 and our reference model (for clarity we did not draw model 2 which seems to have a too high value of $D_0$). The sound speed of the real Sun, deduced from p modes by an inverse technique is also included down to $r$ around 0.16 $R_\odot$ (Christensen-Dalsgaard et al 1985). We have indeed learned from several groups during this symposium that one can deduce the sound speed for $r < 0.2 R_\odot$ taking into account only 1 < 5. The sound speed between 0.1 and 0.2 $R_\odot$ seems to be smaller by about 2% than the standard prediction (Vorontsov, these proceedings, Christensen-Dalsgaard, Gough and Thompson, these proceedings; see also Shibahashi and Sekii, these proceedings).

For models with overshooting (models 2,4) we are in a situation similar to models with turbulent mixing in the core (Schatzman and Maeder 1981) even if our temperature variation is opposite to theirs. The reason is that in the two cases the frequency variation is dominated by the reduction of the mean molecular weight and its gradient due to the homogeneisation. At small radii, we observe in models 2 and 4 a smaller value of $D_0$ than in a model with turbulent mixing (Christensen-Dalsgaard 1986) due perhaps to the behaviour of the sound speed between 0.1 and 0.2 $R_\odot$. However at larger radii, model 4 seems to be in disagreement with observation if we believe the precision of the inferred sound speed of the Sun.

For models without overshooting (models 1,3,5) we notice a partial compensation of the increase in $T$ and $\mu$, and therefore the sound speed $c$ (proportional to $(T/\mu)^{1/2}$) is decreased by less than 5%. As a result, we see at small radii an apparently better agreement with observation (see models 1,3 in table 4). But we notice between 0.4 and 0.6 $R_\odot$, a compensating change in $c$ which seems to be sufficiently important to signal a disagreement with observation for model 3. It is not so clear for model 1. Concerning the center, the better

![Figure 2](image)

**Figure 2**: Relative discrepancy between the squared sound speeds of non-standards models [model 2 (--), model 4 (---), model 5 (--.--), and model 6 (-.-)] and the Saclay standard model. The shaded region obtained by C-D (1985) has been recompared to our standard case.
agreement of $D_0$ compared to that of the standard model is similar to the one of models with VIMPs (Dappen, Gilliland and Christensen-Dalsgaard 1986, Paulkmer, Gough, and Vahia 1986). However, while in models with VIMPs the central temperature is decreased, our models achieve the same effect on sound speed by an increased mean molecular weight.

From our compilation of non-standard models two general remarks can be made:
- The confrontation of models with observed low-degrees $p$ modes gives an estimation of the uncertainty in the sound speed at the center of the Sun. This uncertainty is about 5% and translates to an uncertainty in temperature or molecular weight of about 10%, if we consider the extreme case where one of these two quantities alone were responsible for the sound speed modification. Low-degree $p$ modes do therefore not yet allow us to answer the neutrino problem.
- However, if there were such a large effect at the center, the boundary value nature of stellar structure could lead to a counterpart of the sound speed change somewhere in the solar interior, where higher-degree $p$ modes could be used to test the effect. Figure 2 is an illustration of this. If we believe the error bars proposed and are confident in the precision of the non-standard calculation, we can certainly reject models 3 and 4 from the behaviour of the sound speed in the range of $r$ from 0.4 to 0.6 $R_\odot$. For appropriate $l$ values ($l < 35$) the frequency difference with respect to the standard model goes, in absolute value, up to 10 $\mu$Hz. In the case of turbulent mixing in the core there is no similar effect (Christensen-Dalsgaard 1986) but the model is rejected by $l$ $v$ degree $p$ modes already. What can we say in the case of WIMPs?

6- Conclusion

We have shown in this study of mass loss that extreme mass loss of 1 $M_\odot$ has severe difficulties to match the observed surface abundances of the Sun, (such models predict the complete destruction of $^7$Li and $^9$Be), and justify the absence of observed mass loss. Seismological observations disagree with most models considered, but do not rule out a rapid decay of the mass loss (model 1). Moreover the reduction of the lifetime on the main sequence (one of the motivations to consider mass loss) is not so evident, even in the case of large mass loss.

In the more plausible case (mass loss < 0.2 $M_\odot$) no cosmological effect is observed, the internal structure is very similar to the standard one, calculation of $p$ modes gives a small reduction of the parameter $D_0$ which might be welcome due to the slightly too high $D_0$ value of the standard model. Moreover, in this case, the surface abundances for $^3$He, $^7$Li, $^9$Be will be largely affected: $^3$He will increase relatively to the standard model by a factor of about two, this seems to be in better agreement with observation (Geiss et al 1972 and Black 1972) even if the situation is not so clear. Only a small mass loss could lead to a partial destruction of $^7$Li and $^9$Be - the present observed values are a destruction of 99% of $^7$Li (Cayrel et al 1984) and 50% of $^9$Be. This study is in progress.

We are extremely grateful to H. Reeves who has called our attention on the hypothesis of mass loss and thus motivated us to penetrate deeper in the helioseismological field during the presence of Werner Dappen in Saclay.

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ABSTRACT

Astroseismological methods can be used to calibrate the white dwarf stage of stellar evolution. The internal structure of the white dwarfs can be investigated and the cooling rate determined. Comparing evolutionary models with the luminosity distribution of nearby white dwarfs make a local determination of the age of the galactic disk population possible.

The white dwarf pulsators have multiple and closely spaced periods which can only be determined from continuous photometry covering several nights of observing. A campaign involving observatories around the world has been initiated and some first results are presented.

Keywords: Stellar Evolution, White Dwarf, Pulsations, Photometric Campaign.

1. INTRODUCTION

A solar type star will inevitably end up as a white dwarf after passing through a red giant and a planetary nebula stage. Almost independent of initial masses up to 4 to 5 solar masses, the mass in the white dwarf stage will be close to 0.60 solar masses.

The physical structure of the white dwarf stars is very simple. They have a degenerate carbon/oxygen core with extremely thin, mostly nondegenerate layers of pure helium and possibly hydrogen on top. The structure is a result of gravitational settling. The energy released from the white dwarf stars is a result of cooling processes first by plasmon neutrinos for the hottest and then normal photon cooling.

During their cooling history the white dwarf stars display pulsations when passing through 4 areas in the H-R diagram (Figure 1). The first is in the planetary nebula nucleus (PNN) stage. The second is in the hot DO stage and finally we have a DB and a DA instability strip.

An excellent review of the physical properties and the theory of the compact pulsators was given by Winget (Ref. 1) in the IAU symposium 123. In this contribution I will just repeat how the compact pulsators can calibrate the final stages of stellar evolution.

- The exact location of the blue edge of the DB and DA instability strips are very sensitive to mixing length theory parameters since these pulsations are driven by the partial ionization zones.

Figure 1. The 4 groups of white dwarf pulsators in the H-R diagram.

A model selection mechanism is at work. This is sensitive to the thickness and composition of the surface layers.

The period spacing observed in the DOV star PG 1159-035 has led to a very precise mass determination of 0.56 ± 0.02 $M_\odot$ (Refs. 2 and 3). The same technique may apply to other white dwarf pulsators.

Finally, the rate of period change as a first approximation related to the cooling rate of the white dwarf interior. This again is dependent of the core composition - and direct observations have determined $P/P_0 = 5 \times 7.4 \times 10^8$ yrs for the DOV star PG 1159-035 and eliminated other core compositions than a C/O core (Ref. 4). In a few years we expect to have period change observations on more objects.

2. THE AGE OF THE UNIVERSE

The study of the white dwarf pulsators has made it possible to calibrate the theoretical white dwarf cooling sequence (Ref. 3) and from this made it possible to determine the age of the oldest white dwarf observed. This again may be related to the oldest galactic disk population. From systematic observations it is found that there is a very real shortfall in the white dwarf luminosity distribution between log(L/L$_\odot$) = -4.35 and -4.65 (Ref. 5) (Figure 2).

The white dwarf luminosity function that matches this shortfall has a typical cooling age for the oldest white dwarf population of 9.0 ± 1.8 Gyr, and this is in agreement with the period change of one star.

**Figure 2.** The white dwarf luminosity function. The circles represent the observed number of white dwarfs in each luminosity bin. The solid line shows the theoretical distribution. (From Winget et al - Ref. 5.)

The observations are demanding. In order to "solve" systems with multiple and closely spaced periods, long continuous time series of photometric observations are needed. The only way to achieve this is to have observers around the planet Earth observing the selected objects as continuous as weather and political considerations allow. An extended coverage program has been initiated by the high speed photometry group in Texas, and the first observing session took place earlier this year.

3. THE WHOLE EARTH TELESCOPE

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The remarkable with this number is that it is determined locally and totally independent of uncertain distance indicators to remote corners of the Universe. This conclusion is based on the observation of the period change of one star only. More observations are needed to confirm the result and to investigate details in the structure and evolution of the white dwarf stars.

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using this technique. New versions of this photometer are under development. The goal is to have 7 observatories spread in longitude (Figure 3) and close enough to the equator to overlap. They will all be equipped with identical photometers and use identical sampling techniques. If possible a night’s data will be transferred within hours to a coordinating facility for a “quick look” and decision-taking. Two campaigns are planned each year for the next few years.

The first full scale campaign on the selected interacting binaries took place in March 1988, with PG 1346+082 and V 803 Cen as prime targets. Photometry was performed at 7 observatories and the time coverage during 17 days was nearly 60% for PG 1346+082. In addition high speed spectroscopy was performed by the MMT and at the Sutherland observatories, and IUE made exposures at low and high state of its cycle combining European and U.S. shifts.

The harvest was rich - and it is yet far from digested and analyzed. One important result can be seen from Figures 5-8.

If we take the power spectra of the data from one observatory (Figures 5 and 6) we find that the window function has a 7 tree structure (Figure 5) - and the power spectra look like a dense forest (Figure 6). It is not possible to determine which frequencies are real in this case. Look at the arrow. When data from all observatories are combined (Figures 7 and 8) the many trees in the window function disappear and we have only a triple structure left. This is because we did not get any observations from India this time. The power spectrum (Figure 8) is now much clearer and 13 peaks can be identified - suggesting that many of them arose from g-mode pulsations of the white dwarf member of the binary. Tests show that maybe 6 of these were coherent during the whole run.

The analysis continues - and the photometry will be analyzed together with the spectroscopic and the IUE data. It is clear that a wealth of information has been obtained, and that we may gain new information on these complicated objects.

Figure 3. Favored observing sites for the extended coverage program.

Figure 4. Whole Earth Telescope Coverage March '88.
Figure 5. The power spectral window for Texas data in the March 1988 campaign of PG 1346+082. The tree structure is due to the long periods of daylight breaking the continuous light curve.

Figure 6. The power spectra of Texas data, sampled as the window function shows in the previous figure. The frequency marked with an arrow is a "real" frequency corresponding to the period of 1471 s which has the maximum amplitude when data for more observatories are combined (compare Figure 8).

Figure 7. The power spectral window for data from all participating observatories. The side bands are due to lack of data at the longitude of India.

Figure 8. Power spectra using all data for PG 1346+082 in the March run. When analyzed with the power spectral window in Figure 7 - 11 peaks can be identified (arrows).
The program will continue, studying pulsators of various classes, sometimes with many observatories on line at other times with only a few to follow up. Sometimes a 1 m network will be employed like in March 88. At other times, like in Nov. 88 fainter objects request a 2 m telescope's network to get ample signal to noise. In some years it is hoped to have pinned down the physical parameters and the rate of evolution in the final stages of stellar evolution - thanks to the methods of asteroseismology.

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SOLAR-LIKE OSCILLATIONS IN LATE SPECTRAL CLASS STARS

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ABSTRACT

We investigate the minimum mass star that can exhibit solar-like nonradial p-mode oscillations. Solar models have almost equal pulsational driving and damping due to standard radiative effects. Coupling with surface convective elements occasionally tips the balance by strongly driving for a few days with convection timescales at the top of the convection zone. Then both convective and radiative damping decrease the mode amplitude at an observed decay rate (which is close to that calculated by Kidman and Cox [1984]), until the next convective reexcitation. Population I stellar models at 0.30, 0.45, and 0.75 $M_\odot$ have been constructed to see if they have a near equality of radiative driving and damping also. If so, then stars down to late spectral class could possibly display solar-like oscillations. We find that the near equality of radiative driving and damping occurs down to about 0.5 $M_\odot$. Possibly 100 to 300 second oscillations can be observed through most of the K spectral class, if magnetic activity does not overwhelm these small amplitude nonradial pulsations.

Keywords: Solar-like Oscillations, Late Type Stars

1. INTRODUCTION

The success of probing the solar interior by use of its oscillations has raised hopes that such probing could be done for other solar-like stars. To date, however, observations have not been able to detect reliably any such low degree modes in stars such as Procyon, a Cen, or $\epsilon$ Eri (R. Gilliland, private communication). A review of some of these data is available from Christensen-Dalsgaard (1981, degree modes in stars such as Procyon, a Cen, or $\epsilon$ Eri (R. Gilliland, private communication). A review of some of these data is available from Christensen-Dalsgaard (1981, degree modes in stars such as Procyon, a Cen, or $\epsilon$ Eri (R. Gilliland, private communication).

We have compared the Alexander (1975) opacities used by VHDA with the Ross-Aller opacity table (X=0.70, Y=0.28) with molecular effects in both the opacities and the equation of state added by Norman Magee. Recently we have added the effects of Coulomb interactions, following Clayton (1968), to the Iben equation of state (Iben, 1963, 1965, and 1975). At temperatures above 5000K (15,000K for the 0.75 $M_\odot$ model), we use the Stellingwerf opacity (1975ab) fit using the above X and Y values, and above 1.5 million kelvin, we switch to the Iben (1975) equation of state and opacity procedures. For regions with partial electron degeneracy, the Iben procedure includes some of the features of the Eggleton, Faulkner, and Flannery (1975) method.

We have compared the Alexander (1975) opacities used by VHDA with the Ross-Aller opacity table that we are using. The range of densities from $3\times10^{-6}$ to $1\times10^{-5}$ g/cm$^3$ we find good agreement from 3000K to 4000K but then the Alexander opacities drop to 1/2 of the Ross-Aller out to 10,000K. We confirmed the higher temperature behavior of the Alexander opacities by comparing them to the King 1Va non-molecular opacities to which the Ross-Aller opacities converge at higher temperatures. Indeed, the same factor of 1/2 of the King 1Va opacities was found. Since our models are so strongly convecting, we would not expect major differences with the VHDA models, although any slight discrepancy may be due to this difference.

Our material properties are different from the VandenBerg et al. ones so that we needed different luminosities and surface effective temperatures for these three masses. The VHDA values and ours are given in Table 1. We calculated a photospheric pressure for the 0.30 $M_\odot$ model of $1.3\times10^6$ for log g=5.0, larger, but in reasonably close agreement with the Mould (1976) atmospheres given in Table 1 of VHDA. Our two cooler models do not lie on the Z=0.02 main sequence line shown in Fig. 1 of VHDA, but rather closer to the Z=0.01 line, at a somewhat higher temperature. We were unable to determine complete models for the lower temperature. The 0.75 $M_\odot$ model is slightly on the higher luminosity side of the Z=0.02 line of VHDA.

Somewhat greater discrepancies were found when comparing to the 0.30 $M_\odot$ models of Grossman, Hays, and Gra...
boske (GHG, 1974), where the composition Y=0.29 and a Z=0.03 is assumed. While the luminosities are similar, an effective temperature of 3170K was used by GHG compared to our 3550K. Central temperatures were comparable however, as shown in Table 1, for the VHDA models. We did not compare their opacities to ours, and it may be that the differences between the GHG model and that of this paper may be ascribed to them as well as the differences in chemical composition.

Table 1 gives other details for these three 400 zone models. The ratio of the mixing length to the pressure scale height is set to unity in the standard theory presented by Cox and Giuli (1968). The temperature and fraction of the surface radius is given for the envelope convection zone bottom. The 0.30 $M_\odot$ model has convection right to the center, as other have found before. We give the central temperatures, but our models, constructed for pulsation analyses, are not highly accurate at the center. The outermost shell mass is set to have an optical thickness at its center of just over $10^{-4}$, and the shell masses, going inward, are chosen to have a temperature increase between 2 and 6 percent per shell up to 15,000K. The inertial mass of the outermost shell is increased by a factor of 7.7 in the pressure balance equation to allow for a possible chromosphere and corona mass. One should note that Cox, Shaviv, and Hodson (1981) found a radiative core in the model at 0.27 $M_\odot$, but now with our improved equation of state, we have models at and less massive than 0.30$M_\odot$ completely convective.

We have also investigated the case where it is assumed that the pressure in the deeper parts of the model is so large that all iron is condensed-out to form a liquid or solid, which quickly settles to the center. In this case, the photon absorption due to iron in the stellar composition is absent, making its opacity smaller. Investigation of the opacity contribution from iron has shown that the original [Fe] opacity fit should be used for the opacity without iron. This is because the earlier Cox-Stewart (1970) opacities had an iron abundance ten times smaller than accepted today and the more modern opacities of Cox and Tabor (1976). The eik term in the [Fe] fit to the Cox-Stewart opacities has been multiplied by a factor of 2.0 to allow for the current iron abundance in the mixture and 1.0 for the no-iron case.

The no-iron model for the 0.30 $M_\odot$ case is, however, essentially the same as before, and its radial and nonradial periods and growth rates are also almost the same.

## 3. PULSATION MECHANISMS

These models are found to have pulsational driving due to the following four mechanisms: $\alpha$ effect, $\gamma$ effect, radius effect, and gradient effect. The first effect takes place because of the positive slope of opacity with both density and temperature in the pulsation driving region. Figure 1 shows, versus the zone number in the temperature range from 17,000K to the surface (3100K), the logarithmic derivative of the opacity with respect to the temperature in the 0.45 $M_\odot$ model. In deeper layers, this derivative is negative and approximately -3.5 as for the Kramers opacity. The derivative with respect to density is positive almost always in stellar models. Thus, when compression occurs, the temperature and density rise, giving an increase in the opacity, which traps radiation luminosity. On decompression, the cooling reduces the opacity, releasing radiation to flow into the overlying layers and to produce a small excess in pressure. This causes mechanical driving.

The $\gamma$ mechanism occurs when $\gamma - 1$ is low, characteristic of ionization and dissociation zones where any added energy is mostly used to increase the ionization or dissociation instead of being put into the kinetic energy of the particles. Thus energy can be hidden on the compressional increase of internal energy. Recombination on decompression then releases this hidden energy, and pulsation driving can occur. Figure 2 gives the $\gamma - 1$ in the outer 100 mass shells. This variable does not reach 0.66 until a point deeper than the helium ionization zone at about 50,000K.

In the "radius effect," first discussed by Baker (1966), the effect of compression on the mass zone under consideration is to reduce its radius and its radiating surface, with a consequent trapping of luminosity. Expansion then increases the emitted radiation at all internal radii to produce driving.

Lastly, the gradient effect, contrary to the above effects, is sometimes responsible for pulsational damping. Gradient effect driving is due to the temperature variations increasing with radius near the surface, so that, again on compression, the temperature gradient is smaller in the surface layers than when the model is at its mean configuration. During expansion the temperature gradient is greater, increasing the radiative luminosity flow and driving motions. Damping can occur if the temperature variations decrease toward the surface, as actually happens for some modes in our models. Figure 3 shows that in the 0.45 $M_\odot$ model, between zones 200 (70,000K) and 350 (7000K), the temperature variation (in absolute value) increases towards the surface, giving pulsation driving. Exterior to there, and where the total of all the other effects produces driving, the gradient effect produces mechanical damping, because of the opposite behavior of the temperature variations.

## 4. PERIODS AND GROWTH RATES

Table 2 lists the periods and kinetic energy growth rates per period for the radial fundamental and first two overtone modes. For the 0.30 $M_\odot$ model we get periods of 24.6, 10.7, and 7.3 minutes. Comparable adiabatic radial mode periods found by Gabriel (1968) were 29.6 13.8, and 9.5 minutes, respectively for his 0.27 $M_\odot$ model. For the two lower masses, only the fundamental and first overtone are pulsationally unstable, but all three modes grow.
the \( l = 2 \) modes for the 0.30 \( M_\odot \) and 0.75 \( M_\odot \) models. All the later model modes have very long periods compared to

### Table 2

<table>
<thead>
<tr>
<th>Mass (( M_\odot ))</th>
<th>0.30</th>
<th>0.45</th>
<th>0.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Pi_0 ) (min)</td>
<td>24.6</td>
<td>34.8</td>
<td>59.9</td>
</tr>
<tr>
<td>( \eta_0 (10^{-12}) )</td>
<td>1.6</td>
<td>1.8</td>
<td>1.62</td>
</tr>
<tr>
<td>( \Pi_1 ) (min)</td>
<td>10.7</td>
<td>15.9</td>
<td>34.6</td>
</tr>
<tr>
<td>( \eta_1 (10^{-12}) )</td>
<td>0.14</td>
<td>0.53</td>
<td>0.827</td>
</tr>
<tr>
<td>( \Pi_3 ) (min)</td>
<td>7.3</td>
<td>10.8</td>
<td>24.8</td>
</tr>
<tr>
<td>( \eta_2 (10^{-12}) )</td>
<td>-1.2</td>
<td>-1.7</td>
<td>2572</td>
</tr>
</tbody>
</table>

### Table 3

Nonradial Nonadiabatic Periods (s) \( l = 1 \)

<table>
<thead>
<tr>
<th>Mass (( M_\odot ))</th>
<th>0.30</th>
<th>0.45</th>
<th>0.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_{10} )</td>
<td>139</td>
<td>207</td>
<td>520</td>
</tr>
<tr>
<td>( p_{11} )</td>
<td>128</td>
<td>194</td>
<td>480</td>
</tr>
<tr>
<td>( p_{12} )</td>
<td>119</td>
<td>178</td>
<td>445</td>
</tr>
<tr>
<td>( p_{13} )</td>
<td>111</td>
<td>166</td>
<td>415</td>
</tr>
<tr>
<td>( p_{14} )</td>
<td>104</td>
<td>156</td>
<td>389</td>
</tr>
<tr>
<td>( p_{15} )</td>
<td>98</td>
<td>146</td>
<td>368</td>
</tr>
<tr>
<td>( p_{16} )</td>
<td>92</td>
<td>138</td>
<td>346</td>
</tr>
<tr>
<td>( p_{17} )</td>
<td>84</td>
<td>132</td>
<td>329</td>
</tr>
</tbody>
</table>

### 6. CONCLUSIONS

In agreement with GHG and VHDA, our models are completely convective down to the core, and should not be expected to hide \(^{3}\)He in the core, as was predicted by Cox, Shaviv, and Hodson (1981). However, further efforts in modeling may yet show that radiative cores can exist, especially when models are considered with lower \( Z \).

The results of our linear nonadiabatic pulsational analysis of radial and nonradial modes of red dwarf models is that no nonradial pulsation is to be expected and that the fundamental and first overtone radial modes are weakly excited. We are thus in agreement with Gabriel (1968) in predicting radial pulsations, although to date none have
Table 4
Nonradial Nonadiabatic Periods (s) $l=2$

<table>
<thead>
<tr>
<th>Mass ($M_\odot$)</th>
<th>0.30</th>
<th>0.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{10}$</td>
<td>136</td>
<td>501</td>
</tr>
<tr>
<td>$p_{11}$</td>
<td>126</td>
<td>464</td>
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<tr>
<td>$p_{12}$</td>
<td>118</td>
<td>431</td>
</tr>
<tr>
<td>$p_{13}$</td>
<td>110</td>
<td>403</td>
</tr>
<tr>
<td>$p_{14}$</td>
<td>104</td>
<td>379</td>
</tr>
<tr>
<td>$p_{15}$</td>
<td>97</td>
<td>357</td>
</tr>
<tr>
<td>$p_{16}$</td>
<td>92</td>
<td>338</td>
</tr>
</tbody>
</table>

Table 5
Nonradial Nonadiabatic Growth Rates ($10^{-8}$) $l=1$

<table>
<thead>
<tr>
<th>Mass ($M_\odot$)</th>
<th>0.30</th>
<th>0.45</th>
<th>0.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{10}$</td>
<td>-0.21</td>
<td>-0.73</td>
<td>-220</td>
</tr>
<tr>
<td>$p_{11}$</td>
<td>-0.39</td>
<td>-1.4</td>
<td>-720</td>
</tr>
<tr>
<td>$p_{12}$</td>
<td>-0.63</td>
<td>-2.4</td>
<td>-1400</td>
</tr>
<tr>
<td>$p_{13}$</td>
<td>-1.1</td>
<td>-3.9</td>
<td>-2700</td>
</tr>
<tr>
<td>$p_{14}$</td>
<td>-1.8</td>
<td>-6.6</td>
<td>-4800</td>
</tr>
<tr>
<td>$p_{15}$</td>
<td>-2.6</td>
<td>-10</td>
<td>-7600</td>
</tr>
<tr>
<td>$p_{16}$</td>
<td>-3.7</td>
<td>-15</td>
<td>-11000</td>
</tr>
<tr>
<td>$p_{17}$</td>
<td>-</td>
<td>-22</td>
<td>-16000</td>
</tr>
</tbody>
</table>

Table 6
Nonradial Nonadiabatic Growth Rates ($10^{-8}$) $l=2$

<table>
<thead>
<tr>
<th>Mass ($M_\odot$)</th>
<th>0.30</th>
<th>0.45</th>
<th>0.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{10}$</td>
<td>-0.22</td>
<td>-470</td>
<td></td>
</tr>
<tr>
<td>$p_{11}$</td>
<td>-0.42</td>
<td>-980</td>
<td></td>
</tr>
<tr>
<td>$p_{12}$</td>
<td>-0.67</td>
<td>-1900</td>
<td></td>
</tr>
<tr>
<td>$p_{13}$</td>
<td>-1.1</td>
<td>-3500</td>
<td></td>
</tr>
<tr>
<td>$p_{14}$</td>
<td>-1.8</td>
<td>-5900</td>
<td></td>
</tr>
<tr>
<td>$p_{15}$</td>
<td>-2.7</td>
<td>-9200</td>
<td></td>
</tr>
<tr>
<td>$p_{16}$</td>
<td>-3.8</td>
<td>-13000</td>
<td></td>
</tr>
<tr>
<td>$p_{17}$</td>
<td>-</td>
<td>-18000</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4. The work over a pulsation cycle to drive pulsations is plotted versus zone number for the 0.75 $M_\odot$ model. Radiative effects driving, which peaks near shell 378, is limited there because the convection becomes very strong. The photosphere is at zone 391, and much of the driving and damping is in the optically thick regions.

Figure 5. The work over a pulsation cycle to drive pulsations is plotted versus zone number for the 0.45 $M_\odot$ model. Driving, which peaks near shell 388, is limited there because the convection becomes very strong. The photosphere is at zone 386, and much of the driving and damping is in the optically thick atmosphere.

Figure 6. The work over a pulsation cycle to drive pulsations is plotted versus zone number for the 0.30 $M_\odot$ model. Driving, which peaks near shell 388, is limited there because the convection becomes very strong. The photosphere is at zone 386, and much of the driving and damping is in the optically thin atmosphere.

been reported. The effects of convection probably damps the radial modes, but possibly there is coupling to drive some of the nonradial modes at the convection timescale.

We find that the driving and damping for solar-like nonradial low degree modes are almost equal for at least the 0.75 $M_\odot$ model and possibly for the 0.45 $M_\odot$ model. Since the convection timescale for the surface layers is about 200 seconds for these models, and coupling of the eddies with global pulsation modes is likely, as it is for the sun, it is possible that these solar like modes should be visible in red dwarfs through spectral class K.

6. REFERENCES

THE MASS-LUMINOSITY RELATIONSHIP FOR SOLAR-TYPE STARS

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The Center for Solar and Space Research
P.O. Box 6666
New Haven, CT 06511 U.S.A.

ABSTRACT

The mass-luminosity relationship for solar-type stars has been reinvestigated using data from the new edition of the Yale Parallax Catalogue and recently published orbits and mass ratios for the relevant visual double stars. The results of this investigation confirm that to within the errors of the data, the sun lies essentially on the mean relationship for the nearby stars. If there is to be further progress in the definition of the mass-luminosity relationship, then the accuracy of the trigonometric parallaxes must be improved substantially.

Keywords: Stellar masses, Luminosities, Parallaxes, Double Stars

1. INTRODUCTION

The fundamental information on the masses and luminosities of stars is based on the brightness, distance and orbital parameters of the binary stars. Both the luminosity, and especially the mass, through the use of Kepler’s Harmonic Law, depend critically on the distance, or parallax, of the binary system. With the imminent completion of the new edition of the Yale Parallax Catalogue (YPC) we felt that it would be important to see whether the modification of the system of the catalogue, the incorporation of more accurate data and the use of new corrections to absolute parallax would alter the mass-luminosity relationship for the solar-type stars as given in Refs. 6, 7 & 5.

2. FUNDAMENTAL DATA

The system of the YPC differs from that of the earlier catalogues (Refs. 8 & 9) in that the “observatory corrections” applied to the parallaxes in the Jenkins’ catalogue have been eliminated, since they are likely be an artifact of various selection effects (see, for example, Refs. 10, 11 & 14). On the other hand, a systematic error in the bright star parallaxes determined at the Allegheny Observatory was found by Hanson (Refs. 2 & 4). This effect has been corrected in the YPC since it would otherwise result in the bright star absolute magnitudes being too faint. In addition, new corrections from relative to absolute parallax have recently been computed at Yale using an improved model of the galaxy. These new corrections are smaller than previously used values (Ref. 15), therefore the resulting masses and luminosities are somewhat higher than the older ones. Finally, an analysis of parallaxes in common with two or more observatories has yielded new estimates for the external accuracy of the weighted mean parallaxes.

The stars in this investigation have been drawn from the YPC in the color range 0.15 ≤ (B - V) ≤ 1.20, and spectral types F, G and K. In addition, any star with a luminosity classification indicating that it was not a main sequence star was eliminated from further consideration.

The data on the orbits of the visual double stars were obtained from Ref. 15 and several more recent publications in the literature, primarily by Heintz.

The bolometric corrections which are used to convert the absolute visual magnitudes to the bolometric magnitude, given the B-V color index, are those used in Ref. 1 for main sequence (log g = 4.5) stars with solar metallicity ([Fe/H] = 0.0).

3. ANALYSIS

Since we are interested in calibrating the mass-luminosity relation, we must be concerned about possible selection effects in our data sample. For these stars we have a variety of selection effects, including: a) the initial ability to discover them as double stars, i.e. is their separation visually detected; b) is the observed orbital motion sufficiently rapid to make them “interesting” enough to be observed; and b) did the parallax observers consider the pair worth observing.

The selection effects are obviously complex and it is unlikely that they can be adequately untangled. Fortunately, Hanson (Ref. 3) has found that the impact of the selection effect on the luminosity calibration can be assessed by examining the slope of the cumulative proper motion distribution.

The conventional Lutz-Kelker (Ref. 12) corrections to the absolute magnitude assume that the sample of stars is uniformly distributed in space. In Hanson’s approach, we determine the slope of the cumulative proper motion distribution \[ \log (N(p)) \] versus \[ \log (p) \] of the stars in question, and from that, infer the true spatial distribution of our stars. Having determined the true distribution of our stars in space, we can then calculate the appropriate Lutz-Kelker-Hanson (LKH) corrections. In our case, which is illustrated in Fig. 1, the slope of the relation is approximately -1.5 which reduces the LKH corrections almost by a factor of two over the value for a uniform distribution in space for which the slope is -3.0.
The log of the parallax error to parallax ratio, $r = \frac{s.e. (\pi)}{\pi}$, is shown in Figs. 2 - 4 for stars with $r \leq 0.10$ and stars with $r \leq 0.30$. The slope of the relation is approximately 1.5 and essentially independent of the value of the ratio, $r$.

The sum of the masses can be computed from Kepler's Harmonic Law, given the semi-major axis $a$, the period $P$, and the parallax $\pi$. Unfortunately, this eliminates many stars from our sample. For this "eliminated" group of stars we have calculated the masses from the mass-luminosity relationship given by Heintz (Ref. 6) using the observed magnitude difference of the pair and visual magnitude corrected to bolometric magnitudes as described above. The individual masses are then scaled by the ratio of the sum of the masses calculated from Kepler's Harmonic Law to that obtained from the Heintz mass-luminosity relation. The double star components in this second group are therefore forced to have their mass sum consistent with Kepler's Harmonic Law and a log mass versus bolometric magnitude slope equal to Heintz's relationship. Since we are dealing with a relatively small range in stellar mass and our mass-bolometric magnitude relation for stars with observed mass-ratios is consistent with that found by Heintz, we feel that it is reasonable to include the "eliminated" stars in our comparison.

4. CONCLUSIONS

From this discussion and an inspection of Figs. 2 - 4, it should be clear that much remains to be done in the determination of stellar masses. While the Heintz (Ref. 6) relation is approximately correct, the details of the mass-luminosity relation remain obscured by observational errors. We have performed an error analysis of the data, and propagated the observational errors, where known, through to the plotted data. The error bars are too depressing to plot! It is clear enough that the lower quality orbits result in greater scatter in the derived masses. On the other hand, the dominant contributor is the parallax error. Unfortunately, it is likely to remain that way for some time to come due to the difficulty in determining parallaxes of double stars, since the spatial proximity of the two stars on the detector results in inherently lower accuracy than for single stars.
5. ACKNOWLEDGEMENTS

We are pleased to acknowledge the use of the plotting software package PASSAGE. This research has been supported in part by grants from the National Science Foundation.

6. REFERENCES


ABSTRACT

In the light of the unsatisfactory theoretical explanation of the observed p-mode frequency separation of \( \varepsilon \) Eri of 172 \( \mu \)Hz, and of the fact that the seismological observation has not yet been confirmed, we have computed a new series of models. We have taken into account only the observational input available prior to the seismological determination. We have obtained predictions for the p-mode frequency separation that lie between 198 and 244 \( \mu \)Hz.

Keywords: individual stars, \( \varepsilon \) Eri, stellar interiors, stellar pulsations

1. INTRODUCTION

Noyes et al. (Ref. 1) have reported the tentative observation of global p-mode oscillations on the K2 star \( \varepsilon \) Eri, finding a frequency separation of 172 \( \mu \)Hz. This has stimulated two independent theoretical studies on this star, the first by Guenther and Demarque (Ref. 2) and Guenther (Ref. 3), the second by Soderblom and Däppen (Ref. 4). Both attempts to model \( \varepsilon \) Eri have run into considerable difficulties. Guenther and Demarque (Refs. 2,3) concluded that the model of an old star (10 Gyr) fitted the data best, but they were aware that this would be hardly reconcilable with the well observed magnetic activity of \( \varepsilon \) Eri. Indeed, the activity would rather suggest a nearly zero-age-main-sequence star. This consideration has motivated the study by Soderblom and Däppen (Ref. 4), in which the indication of a young age is treated as an additional observational input. This assumption, however, has led to another difficulty: the resulting model has a very low value of the mixing-length parameter \( \alpha \approx 0.5 \) (with \( l \) being the mixing length, and \( H_P \) the pressure scale height).

Since the p-mode observation of Ref.1 has not yet been confirmed, we have computed new models of \( \varepsilon \) Eri that are both young and free of distasteful low mixing-length parameters. We have considered the seismological constraint of Ref.1 as the weakest, and we propose a prediction of the p-mode frequency separation.

Table 1 lists four typical models that all predict a p-mode frequency separation larger than the 172 \( \mu \)Hz reported in Ref. 1.
3. DISCUSSION AND CONCLUSION

It is somewhat discouraging to note that the provisional detection of stellar oscillations of ε Eri and the ensuing modelling effort has not brought much insight. The only thing that we can say is that among the three following statements all combinations of two are acceptable, but that the three together are in contradiction. The three statements are: 1) ε Eri is young, 2) ε Eri has a solar-type mixing-length parameter, and 3) ε Eri has a frequency separation of 172 µHz. One should therefore try to confirm (or refute) the seismological observation of Ref. 1. Of course it would be nice also to measure the small, not just the large frequency separation (see e.g. Ref. 13). Unfortunately this small separation lies near 10 µHz for a star like ε Eri, too close to the diurnal frequency (11.6 µHz). Using ground-based networks or going to space could help. The Hubble space telescope could be used for stars near the poles of its orbit (Ref. 14); however, Gilliland (private communication) has estimated that about 100 hours of target time would be needed...

Nevertheless, ε Eri should not discourage us from pursuing asteroseismology. Determination of age and mass of a given star is difficult, as has been demonstrated by Gough (Refs. 15,16). On the other hand, our knowledge of the interior of stars is much less detailed than of the Sun. Asteroseismologists are therefore much less ambitious, and any improvement in our knowledge of a star's fundamental properties is most welcome. In addition it follows from Gough's analysis (Refs. 15,16) that the interpretation of seismological observations will be much more accurate if we also know $\mu \lesssim$ mass of the star. Therefore stars lying in nearby binaries with well determined orbits are prime candidates for future seismological observations. The masses of these stars are known, and we can profit from the additional constraint that both stars should have the same age and composition. Good candidate systems could be α Centauri, ζ Bootis, and η Cassiopeia.

4. REFERENCES


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ABSTRACT: A model of first-generation intermediate mass star of 5 $M_\odot$ with no metals has been considered. The vibrational instability of this model has been investigated. The model, in question, burns helium in the core. Calculations have been performed for the first and second harmonics as well as for the fundamental mode. The model has been found to be vibrationally stable toward radial pulsations.

Keywords: Vibrational instability, pulsations, H-He Star.

1. INTRODUCTION

Looking at the positions of various types of pulsating stars in the Hertzsprung-Russel (H-R) diagram it is understood that there is a correlation with evolution.iben (1971) and Christy (1972) pointed out the importance of the understanding of stellar evolution to have an insight into the pulsational instability in stars.

In the present work pulsational properties of a first-generation intermediate mass star of 5 $M_\odot$ are studied. The first generation stars which have no heavy elements in their initial composition, might have contributed significantly to the early enrichment of the pre-galactic medium with nucleosynthesis products. The structure and evolution of first-generation stars was first treated by Ezer and Cameron (1971). Several others (e.g. Cary, 1974) followed them in searching for the first-generation stars' possible connection with the early evolutionary stages of the Galaxy. It was indicated by Ereryt-Ezer and Kiziloglu (1985) that the evolutionary history of these stars must be known in order to determine the amount of heavy elements ejected as a result of various mechanisms into the medium at certain stages of their evolution. Had helium was formed primordially hydrogen-helium stars could be born prior to any other known stellar population in the Galaxy. In fact, after encountering the difficulty of explaining the cosmic abundance of helium in terms of the nucleosynthesis in stars (Truran et al., 1965, Cameron and Truran, 1971), and the discovery of 3 K (Penzias and Wilson, 1965) microwave background radiation the idea that helium was formed primordially gained more acceptance. These ideas led us to think that the investigation of the pulsational instability of first-generation stars deserved attention.

It has been known for a long time that there exists a critical mass for the vibrational stability of main sequence stars (Ledoux, 1941). Schwarzchild and Harm (1959) showed that the critical mass was 60 $M_\odot$ for normal stars with $Y=0.22$ and $Z=0.02$. This limit has been found to be about 280 $M_\odot$ for pure hydrogen stars (Bouy, 1963) and 9 $M_\odot$ for pure helium stars (Bouy and Ledoux, 1965; Noels, 1967) using models constructed with electron scattering as the sole opacity source. Stothers and Simon (1970) have further increased the limiting value of 9 $M_\odot$ for pure helium stars by considering models with different values of $Z$ using a better opacity law. Noels and Maseeel (1982) studied the vibrational stability and critical mass of helium stars. They found that for models with 2% of heavy elements in mass, the critical mass was 16 $M_\odot$, while for pure helium models the critical mass was found to be 11.5 $M_\odot$. In the computation of these models the "Astrophysical opacity library" (Huebner et al., 1977) was used. However, for pure helium stars with electron scattering as the sole opacity source the value they obtained for the critical mass was 9 $M_\odot$. Noels and Napain (1984), by replacing the outer layers of a pure helium star by hydrogen rich ones, calculated the effect of it on the critical mass. They showed that the critical mass was extremely sensitive to the presence of a small hydrogen rich envelope. Ibrahim et al. (1981) studied the vibrational stability towards radial oscillations of homogeneous main sequence stars initially composed of 80% of hydrogen and 20% of helium in mass. They found the critical mass for vibrational stability to be 173 $M_\odot$.

The basic properties of the model are given in section 2. In section 3 radial adiabatic oscillations and vibrational instability are discussed and results are given in section 4.
We studied the vibrational stability of radial oscillations in a model of $5 M_\odot$ star with initial composition $X=0.8$ and $Z=0$. The model whose basic properties are given in Table I has been computed by Eryurt-Ezer and Kızıloglu (1985). The model star is on the helium main sequence. 

### Table 1: The characteristics of the equilibrium model

<table>
<thead>
<tr>
<th>$N/M_\odot$</th>
<th>$Y_c$</th>
<th>$\log T_c$</th>
<th>$\log n_c$</th>
<th>$\sigma_{cc}$</th>
<th>$P_c$</th>
<th>$\rho_c/\dot\rho$</th>
<th>$X_s$</th>
<th>$Y_s$</th>
<th>$\log n_c/\log T_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.998</td>
<td>8.063</td>
<td>4.154</td>
<td>0.018</td>
<td>0.996</td>
<td>2.782(4)</td>
<td>0.8</td>
<td>0.2</td>
<td>3.366</td>
</tr>
</tbody>
</table>

Number in parenthesis indicate the power of 10 which multiplies the corresponding number.

### 2. Model

The vertical axis is the absolute value of the difference between the LHS and RHS of equation (4). The horizontal axis is the trial value of $\omega'$.

### 3. Radial Adiabatic Oscillations and Vibrational Instability

The radial adiabatic oscillations are described by the application of the classical linear theory (Ledoux and Walraven, 1958). Equations governing the radial adiabatic oscillations are, in their linear form, given as (Schwarzschild and Harm, 1959; Ledoux, 1969),

\[
\frac{d}{dr} \left( \frac{\delta \rho}{\rho} \right) = -\frac{1}{\rho} \frac{d \rho}{dr} \left( \frac{\delta \rho}{\rho} \right) + \left( \frac{a^2}{c^2} \omega_x^2 \right) r \frac{d r}{d \rho},
\]

\[
\frac{d}{d r} \frac{d \epsilon_x}{\epsilon_x} = \frac{d \epsilon_x}{\epsilon_x} \left( \frac{\omega_x^2}{2} - (3 \epsilon_x - 12 \epsilon_x - 11 - 8) \right)
\]

where $\omega_x$ is the oscillation frequency, $\rho$ is the mass density, $r$ is the radius, and $\epsilon_x$ is the energy density. The boundary conditions are used. The outer boundary condition is (Cox, 1980)

\[
\frac{d}{dr} \left( \frac{\delta \rho}{\rho} \right) = \frac{1}{\rho} \frac{d \rho}{dr} \left( \frac{\delta \rho}{\rho} \right) + \left( \frac{a^2}{c^2} \omega_x^2 \right) r \frac{d r}{d \rho},
\]

\[
\frac{d}{d r} \frac{d \epsilon_x}{\epsilon_x} = \frac{d \epsilon_x}{\epsilon_x} \left( \frac{\omega_x^2}{2} - (3 \epsilon_x - 12 \epsilon_x - 11 - 8) \right)
\]

where $\omega_x$ is the ratio of gas pressure to total pressure $\rho$. Subject to the above boundary condition equations from (1) to (3) are solved for $(\delta \rho/\rho)$, $(\delta \epsilon_x/\epsilon_x)$, and $\omega_x$. In integrating the first-order differential equations Runge-Kutta method is used. The driving mechanism of vibrational instability is the nuclear energizing of pulsations in deep stellar interiors. In the first-order theory nonconservative terms in equation of motion and energy balance equation introduce a damping factor of the form $\exp(-\sigma^2 t)$. With the dependence of the perturbations of the form $\exp(-\sigma^2 t)$, the vibrational stability coefficient, $\sigma^2$, is given (cf. Boury et al., 1975)
Solving equations from (1) to (3) we obtain the radial adiabatic solutions, and thus we have the stability coefficient by using equation (5).

4. RESULTS AND DISCUSSION

For the model in question the behavior of the eigen function $\delta r/r$, of fundamental mode is given in Figure 2; while for the first and second harmonics it is given in Figure 3. In these figures the ordinate $\delta r/r$ is in arbitrary units, while the horizontal axis is the fractional distance from the center of the star. In Figure 4 we have the variation of $\sigma^*$ with $r/R$. When $\sigma^*$ gets negative it means that the model is vibrationally unstable.

![Fig.2. Change of the eigenfunction $\delta r/r$ with stellar radius, in fundamental mode for the corresponding model.](image)

![Fig.3. Change of $(\delta r/r)$ for the first and second harmonics for the model in question.](image)

In the analysis of the stability of radial adiabatic oscillations only the fundamental mode has been considered. The results are given in Table II.

### TABLE II

The pulsational quantities associated with the fundamental mode, the first harmonics, and the second harmonics of radial oscillations. The damping coefficient, $\gamma$, is in s$^{-1}$. Results are for $5 M_\odot$ hydrogen-helium star.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$c_0/C$</th>
<th>$\omega^2$</th>
<th>$\sigma^2$</th>
<th>$P(s)$</th>
<th>$\gamma(s^{-1})$</th>
<th>$\delta r/r$</th>
<th>$(\delta r/r)_{\sigma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.782(4)</td>
<td>9.76</td>
<td>1.394(-6)</td>
<td>5.321(3)</td>
<td>9.560(-9)</td>
<td>3</td>
<td>5.845(2)</td>
</tr>
<tr>
<td>1H</td>
<td>18.83</td>
<td>2.690(-6)</td>
<td>3.851(3)</td>
<td>-</td>
<td>-</td>
<td>-2.520(3)</td>
<td>6.080(3)</td>
</tr>
<tr>
<td>2H</td>
<td>30.98</td>
<td>4.426(-6)</td>
<td>2.987(3)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

Numbers in parentheses denote power of ten.
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Fig. 5. Energy generation rate throughout the model star.

Ledoux, P.: 1969, XIe Course de Perfectionnement de l'Association Vaudoise des Chercheurs en Physique, Saas-Fee, 24-29, Mars.
THE INFLUENCE OF THE EQUATION OF STATE ON THE ZERO-AGE MAIN SEQUENCE AND THE SUN

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ABSTRACT

In the light of the present uncertainty in the equation of state of stellar interiors, we have calculated stellar and solar models using three different formalisms for the equation of state. We have considered (1) a mixture of ideal gases using an artificial pressure-ionization device, (2) an equation of state where pressure ionization is achieved by the confined-atom model, and (3) an equation of state containing a large number of atomic and ionic species, with detailed internal partition functions, containing weighted occupation probabilities. We have examined the effect of these equations of state on the position of a star in the H-R diagram: the result is either a displacement of the star on the zero-age main sequence (ZAMS) or, for masses lower than 0.7\(M_\odot\) a change in the slope of the ZAMS. Furthermore, we have carried out the usual calibration procedure in order to obtain the initial He abundance \(Y\) of the solar model.

Keywords: solar and stellar structure, equation of state, zero-age main sequence

1. INTRODUCTION

While for many astrophysical purposes simple equations of state are amply sufficient, it can be shown that for finer applications such as the determination of the solar helium abundance or the modelling of low-mass stars in view of the anticipated Hipparcos data an additional effort will be required. In simple equations of state the plasma of stellar interiors is treated as a mixture of perfect gases of all species (atoms, ions, nuclei and electrons), and the Saha equation is solved to yield the degrees of ionization or of molecular formation.

The simplest approximation for the (divergent) sum over the bound states of the hydrogen atom consists in considering only the ground state. However, this approximation leads to a spurious recombination of the atomic and ionic systems at high densities. The reason is that partially degenerate electrons prohibit easy access to the continuum. Many classical "programmes" of stellar structure have overcome this problem by imposing full ionization all the way to the center of the star starting at a depth where the Saha equation ionizes everything. This solution to the problem actually works quite well; at the high density of the solar center (150 g cm\(^{-3}\)), all hydrogen atoms must be destroyed, because there would be no space available (fully packed hydrogen atoms correspond to a density of order of 1 g cm\(^{-3}\)). Realistic modifications of the Saha equation must be able to describe this ionization by pressure or density.

Several more realistic equations of state have been presented in the last years. The more conventional equations of state are based on the free-energy minimization method, which implies thermodynamical consistency. Mihalas, Hummer and Däppen (Ref. 1-3, in the following MHD) have recently developed such a treatment of the equation of state as part of an ongoing opacity recomputation project (Ref. 4). Less conventional equations of state (which are often associated with the Planck-Larkin partition functions) are currently been developed (Ref. 5-6) and their predictions could well be different from those of MHD. In view of several important questions in the equation of state, detailed confrontation with observational data either from the laboratory (Ref. 7) or astrophysics (Ref. 8) are needed. The present study performs this confrontation at the example of lower-main sequence stars.

2. SPECIFICATIONS FOR THE EQUATIONS OF STATE

We have examined three formalisms for the equation of state, they are described in detail in Table 1. below. We have considered (1) a mixture of ideal gases using an artificial pressure-ionization device (Eggleton, Faulkner & Flannery, in the following EFF, Ref. 9), (2) an equation of state where pressure ionization is achieved by the confined atom model (in the following CAM, Ref. 10), (3) an equation of state containing a large number of atomic and ionic species, with detailed internal partition functions, containing weighted occupation probabilities (Mihalas, Hummer & Däppen, hereafter MHD, Ref. 1-3).

3. CHARACTERISTICS OF THE MODELS

All stellar and solar models presented here have been calculated with up-to-date input physics. In particular, the most recent nuclear reaction rates for the proton-proton chain are used (Ref.11). In addition, we have used the opacity tables of Huebner et al. (Ref.12), complemented by molecular absorption coefficients at low temperatures (Ref.13). These molecular opacities are of great importance in the study of low-mass stars. Further details regarding the physical ingredients and the numerical realization of the models are given in Refs. 14-15.
Table 1. Specifications of the three equations of state used to calculate the stellar models. EFF stands for Eggleton, Faulkner and Flannery (Ref. 9), CAM stands for confined-atom model (Ref. 10) and MHD stands for Mihalas, Hummer and Dappen (Ref. 1-3)

<table>
<thead>
<tr>
<th>equation of state</th>
<th>EFF</th>
<th>CAM</th>
<th>MHD</th>
</tr>
</thead>
<tbody>
<tr>
<td>free-energy-minimization</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>(implies thermodynamical</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>consistency)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mechanism to ionize at high</td>
<td>arti</td>
<td>conf</td>
<td>con</td>
</tr>
<tr>
<td>densities</td>
<td>fial</td>
<td>fided-atom</td>
<td>fided-atom</td>
</tr>
<tr>
<td>Coulomb correction</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>of pressure</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>configurational terms</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>(correction for size of</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>particles)</td>
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<tr>
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<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>electrons</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>internal partition functions</td>
<td>no</td>
<td>simple polynomial approximations</td>
<td>detailed partition functions</td>
</tr>
<tr>
<td>(beyond ground state)</td>
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<tr>
<td>chemical composition</td>
<td>X,Y</td>
<td>X,Y</td>
<td>X,Y</td>
</tr>
<tr>
<td>Z in fixed ratios</td>
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<tr>
<td>partially degenerated</td>
<td>yes</td>
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<td>high</td>
</tr>
<tr>
<td>quantities</td>
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</tr>
</tbody>
</table>

4. THE ZERO-AGE MAIN SEQUENCE

In order to examine the effects of the equation of state on the theoretical zero-age main sequence we have calculated models of 0.5, 0.6, 0.7, 0.8, 0.9, 1.0 and 1.1 M\(_\odot\) with the three equation of state formalisms described in §2. The calculation of models below about 0.45 M\(_\odot\) has not been possible because of the limited range of the opacity tables (Refs. 12-13).

While we have varied the equation of state, we kept all input physics and parameters of the models unchanged. In particular, all the models considered have a helium abundance of Y=0.29 in mass fraction and a metal content of Z=0.02. We have chosen a mixing-length parameter \(\alpha = l/H_p\); \(H_p\)=pressure-scale height) equal to 2.2. These values of \(\alpha\) and Y are very close to those of a solar model using the same updated input physics (cf. Ref. 15), they are also suitable for models of population I stars.

In Figure 1, the zero-age main sequence (ZAMS) obtained with the CAM equation of state is compared with the ZAMS of the EFF equation of state. In the range of mass considered, the change of the equation of state from EFF to CAM does not noticeably modify the location and the slope of the ZAMS in the Hertzsprung-Russell diagram. However, the mass associated with a given logL-logT\(_{\text{eff}}\) point on the ZAMS is about 3% smaller on the ZAMS calculated with the CAM equation of state than the corresponding mass of the EFF zams. This shift of mass occurs because for a given mass CAM models have a smaller central pressure than EFF models. This decrease in pressure is due to both the Coulomb effect and the residual presence of He\(^+\) ions down to the center of the models. The precise contribution of these two effects will be discussed in a forthcoming paper (Ref. 16).

In Figure 2, the ZAMS calculated with the MHD and the EFF equations of state are compared. We note that above about 0.7
M_\odot, the effect of the MHD formalism is similar to that of the CAM equation of state, though slightly smaller. The decrease of mass from EFF to MHD is only 2% because the MHD equation of state leads to complete ionization of He^+. Below 0.7 M_\odot, however, the two ZAMS bifurcate, the ZAMS being steeper in the MHD case. A detailed explanation of this behaviour will be also given in Ref. 16. We merely note that the difference between MHD and EFF results from two effects. First, as mentioned above Coulomb correction contained in MHD slightly shifts a point of a given mass on the ZAMS. Second, the detailed inclusion of hydrogen molecules and their partition functions results in the noticeable change of the slope of the ZAMS for masses lower than 0.7 M_\odot in the MHD case.

5. THE STANDARD SOLAR MODEL

We have calculated three evolutionary sequences of the Sun using the three equations of state discussed in §2. Again, all other input physics has been kept unchanged. Each sequence has been calibrated in Y and \alpha in order to have at solar age (t_s=4.6 \times 10^9 yr.) the observed solar luminosity and radius (L_\odot=3.86 \times 10^{33} \text{ erg s}^{-1}, R_\odot=6.9599 \times 10^{10} \text{ cm}).

In Table 2, we give some basic properties of these evolutionary models. All the models attain L_\odot and R_\odot within better than 10^{-4}. N_v(\text{37Cl}) and N_v(\text{71Ga}) are the expected values for the rate of solar neutrino events in the 37Cl apparatus and the Gallex experiment, respectively. To derive these fluxes, we have used the neutrino-absorption cross sections of the 37Cl and 71Ga detectors given by Bahcall and Ulrich (Ref. 17). T_C, \rho_C and X_C are central temperature, density and hydrogen content, respectively. T_b is the temperature at the base of the outer convective zone (OCZ) and d is the depth of the OCZ measured in units of one solar radius. \delta v_{0,2} is a typical, asymptotically computed separation of p-mode frequencies v_{n,l} (n: order, l: angular degree, \delta v_{0,2}=v_{2,0}-v_{0,1}) where we have chosen n=22.

The use of the MHD formalism leads to a decrease of Y of 3.8% relative to the EFF model: The central pressure decrease in the MHD model, due to the Coulomb corrections, is compensated for by the decrease of the mean molecular weight (and thus of Y). We observe a further decrease of Y in the CAM model, caused by the ionized central He^+, leading to a total decrease of Y of 5.7%.

The neutrino fluxes and central temperature vary relatively little with the equation of state because they are essentially bound by the luminosity constraint. Only changes in the energy transport could alter this situation.

On the other hand there is a slight increase of both the depth and the temperature at the bottom of the outer convective zone: this could have an impact on the understanding of the observed lithium abundance at the solar surface and might also have seismological consequences.

Figure 3 gives relative difference in the sound speed between a solar model calculated with the MHD (or CAM) equation of state and the EFF equation of state, against fractional mass M/M_\odot.
We also note a significant change of the sound speed in the external layers, corresponding to the external convective zone. These differences of the sound speed can be understood in terms of the modifications of the partition functions in the MHD case. It has been shown recently by Christensen-Dalsgaard et al. (Ref. 8) that the modifications of the sound speed due to the MHD equation of state influence the high degree p-mode frequencies with the net result of a better agreement between the theoretical and observed frequencies.

Acknowledgments. The authors are indebted to A. Baglin and E. Schatzman for very useful discussions. Computing was supported by the C2VR of the Ecole Polytechnique at Palaiseau (France).

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MUSICOS (Multi-Site Continuous Spectroscopy): Objectives and Prospects for Asteroseismology

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ABSTRACT

MUSICOS, a project for a multisite facility network of high resolution spectrometers around the world partly dedicated to continuous spectroscopy, is under study in France and with European collaborators. This network aims to serve the solar/stellar community specially for programs requiring spectroscopic coverage around the clock. A major scientific goal is to allow asteroseismological studies using the most complete continuity. This goal drives the highest constraints on the spectrometer (efficiency, wavelength range, stability).

The current instrument design is described; the expected performances for asteroseismology and the organisation of the multisite network will be discussed.

1. INTRODUCTION

MUSICOS is a newborn international project which aims at developing a multi-site network of high resolution spectrographs. This project has received support from many scientists, mainly from European countries so far. The goal of Musicos is to facilitate multi-site spectroscopic observations, first by setting up an organisation helping the coordination of observations from existing instruments at different sites, then by designing, developing and installing similar spectrometers in well-chosen sites around the world, for which part of the time would be devoted to multi-site observations.

We organised in June a national workshop in France about the scientific use of MUSICOS. The workshop was attended by 50 participants, from different fields of solar and stellar physics. The project was recognised for various topics for the study of asteroseismology, stellar rotational modulation, surface structures, variable winds, for stellar activity programs where short term phenomena occurs, or fast rotation modulation allows application of Doppler imaging method. Also the study of wind variability, flare patrol, or eclipse imaging was stressed. Additionally, a joint network of high resolution spectroscopy and photometry would give a simultaneous support to continuous satellite observations. The need for multi-site continuous coverage (taking as example networks of solar seismology) was especially recognised for the asteroseismology programs (e.g. of OB, Be, δ Scu, fast rotating B stars and solar type stars). Given the positive impact of this workshop and the interest shown by several other European teams (Scandinavia, Italy, UK,...), we have decided to get started on the practical and technical aspects of the project as soon as 1989. We intend to perform laboratory tests to study the wavelength stability of fiber-fed spectrographs, to start the design and development of an echelle fiber-fed spectrometer, and to organise preliminary multi-site campaigns using an already existing mono-order fiber-fed spectrometer (ISIS, developed at Meudon Observatory and used at the moment at the 1.93 m telescope at OHP.

3. Asteroseismology requirements

For asteroseismology, the need for a high S/N, and the possibility of observing a large number of photospheric lines, was stressed, together
with the specific requirement of a high spectral stability and possibility of very accurate velocity calibration. An observing continuity better than 60% in order to decrease the effect of parasite sidelobes and aliasing in the power spectrum, and to increase the signal to noise and resolution on the oscillation modes was requested, which argues for multi-site network. The need for prereduction of data at each site, in order to control in real time the instrumental parameters (quality of night, drifts) was made clear at the workshop.

The application domain would concern first the β C Ma stars, which oscillations can be observed in radial velocity, and for which Doppler imaging effect due to fast rotation allows to have access to information on modes with degree 1 higher than 4. Other technique can make use of line intensities oscillations.

For the δ Scuti stars, continuous multi-site photometry has permitted the detection of non-radial modes. The Doppler rotation image information and the effect of large scale inhomogeneities can be studied spectroscopically. The same apply to Bp and Ap stars.

For solar-type oscillations, the velocity signal is much reduced (of the order of 15 cm/s per stronger solar mode). Classical spectrograph using one line are not stable and efficient enough. Resonance cell technique can be stable but can apply only to 0 or 1st magnitude stars with 4m class telescopes, which are oversubscribed and make difficult continuous observing from multi-site, except some results from Zelentchuk-Hawaii as reported by Fossat. Eventually this technique will have provided the first detection of oscillations in Procyon, α Cen A, ε Eri. For having access to fainter stars from later-type closest stars or from other classes in the H-R diagram, the multi-line concept may be relevant in the future. For instance, the MUSICOS spectrometer at 40000 resolution only with a 2m telescope would give on a 3 magnitude star a S/N of 300., which in principle, would allow to reach oscillation detection. Tests on such cross spectrometer have been achieved using Th lamps for accurate calibration. The fact that solar oscillations have been detected with this instrument by measuring scattered or reflected light, but that the noise on stellar observations do not go below a rms velocity error of 40 m/s, suggests errors arising from point sources, with the beam geometry being sensitive to depointing of the stellar beam for these measurements.

4 Technical design of MUSICOS instrument

From the scientific requirements after the MUSICOS workshop it was realised that a large range of scientific programmes require a spectral resolution 10000, 40000, and 80000. The main requirements from asteroseismology is the availability of large spectral domain in order to measure a number of photospheric lines in solar-type stars, and the stringent constraint is the need for spectral stability down to the 10m/s range. We have selected a versatile concept of a high resolution spectrometer, with 2 cross-echelle disperser modes at 10000, 40000 resolution, allowing a maximum domain on a array of 4 CCD detectors of 400 X 600 pixels, and a mono-order mode at 66000 using a double linear diode array from Thomson. The spectrograph is fed by an optical fiber which makes the instrument transportable and adaptable on another telescope. Also, fiber reduces the effect of guiding errors.

For seismology velocity measurements, we consider the 40000 resolution cross dispersion mode (For a S/N of 300, and 500 lines relatively free of blending available on the spectral domain for accurate velocity measurements) Stability test will be achieved, and accurate procedures for wavelength calibration are under study.

5. Project group

There is in France a project group (composed of C. Catala (Meudon) and B.H.Foing (IAS) as PIs, P. Felenbok, A.M. Hubert, J. Czarny (Meudon), J.M. Le Contel and E. Fossat (Nice). Also contacts were established with European groups in Italy (in Triest and Catania), in Scandinavia (in Upssala, Helsinki, and Aarhus), in Spain, in United Kingdom (at Armagh). Associate countries such as US (in Hawaii, in Boulder, Goddard, and with the SYNOP group), in USSR (in Crimea) and in China (in XingLong) wish to participate to the multi-site project.

6. MUSICOS Planning

Since 1982, with our collaborators, we have participated to Multi-Site Multi-Wavelength Observing campaigns. Next MUSICOS campaign with existing instruments would include sites in Hawaii, ESO, OHP, Pic du Midi, canarias, Crimea, Kitt Peak, AAT, etc.

At the end of 1989, we plan campaigns including transport of ISIS instrument and
prototype in complementary sites (Xing Long, Hawaii).
1988-1989 will be devoted also to the design and development of the MUSICOS European prototype, while 1990 should see the duplication of this spectrometer by the participant countries. In 1990-1991, we plan the installation in remote sites, and the start of the full network operations.

The joint analysis will start from the preliminary of the previous campaigns with existing instruments. When the network is operational, a Multi-Site Guest Observer program should be offered to the community.

The philosophy of the MUSICOS project is to associate the groups interested also on the scientific return, and this is the reason why we start already collaborative campaigns to learn how making, reducing, analysing the results from multisite data. Also, some programs require special observational strategy and may be interleaved or require service observing. Asteroseismology will require continuous coverage and a good control on the stability and the quality of the measurements.

The MUSICOS will be open to any input and collaboration during the development phase, and through the observing proposals to the community during the operational phase.

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Late papers
EFFECTS OF CONVECTIVE VELOCITIES ON SOLAR PRESSURE MODE FREQUENCIES

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ABSTRACT

There are persistent discrepancies between observations and theoretical calculations of solar p-mode frequencies, \( \nu_{\text{obs}} - \nu_{\text{calc}} \), in spite of improvements in the theory (e.g. equation of state).

In this short communication, we propose to approximate roughly the difference (observation-calculation) versus frequency as:

\[
\delta \nu = \nu_{\text{obs}} - \nu_{\text{calc}} \approx -Kn^2,
\]

where the constant \( K \) is of the order of 1 second, and independent of degree \( \ell \).

It has been already suggested by T. Brown that "Solar p-mode eigenfrequencies are decreased by turbulent convection" (Ref. 1). We want to revisit this suggestion and reformulate it in the following manner: is this physical effect responsible for some of the difference \( \nu_{\text{obs}} - \nu_{\text{calc}} \)? After a brief discussion of the mechanism, we present an order of magnitude evaluation which shows that, indeed, it is worthwhile to pursue the idea further: this will be done in improving the description of the coupling between convection and oscillations.

Keywords: Oscillations of the Sun.

1. OUTLINE OF THE PHYSICAL EFFECT

Consider an acoustic wave crossing back and forth a convective cell having a typical velocity \( v_c \). Its travel time is:

\[
T_b = L \left( \frac{1}{c + v_c} + \frac{1}{c - v_c} \right) \approx \frac{2L}{c} (1 + M^2),
\]

where \( L \) is the geometrical path length, \( c \) is the sound speed and \( M = v_c / c \) is the Mach number.

This time \( T_b \) is then always made longer by the convection by an amount:

\[
\Delta T_b \approx \frac{2LM^2}{c}.
\]

In the ray tracing approximation, the travel time \( T_{n,\ell} \) of an acoustic wave between two successive reflections at the upper boundary is:

\[
T_{n,\ell} = \int_{\text{ray path}} \frac{ds}{c} \approx \frac{n}{\nu_{n,\ell}},
\]

where \( s \) is the curvilinear ordinate along the ray, and \( \nu_{n,\ell} \) is the frequency of the mode of degree \( \ell \) and order \( n \). By differentiation, we obtain

\[
\delta T_{n,\ell} \approx -\frac{\delta \nu_{n,\ell}}{\nu_{n,\ell}^2}.
\]

By comparison with equation (1), we deduce

\[
\delta T_{n,\ell} \approx -nK \approx n \text{ seconds}.
\]

If this retardation effect is to explain \( \nu_{\text{obs}} - \nu_{\text{calc}} \), then this \( \delta T_{n,\ell} \) must be independent of \( \ell \) and must be increased by about 1 second for an increase of \( n \) by 1:

\[
\delta T_{n+1,\ell} - \delta T_{n,\ell} \approx 1 \text{ sec}.
\]

2. THE UPPER BOUNDARY FOR ACOUSTIC WAVES

The correction due to retardation of sound waves will be the most important in those regions where the Mach number is the greatest, namely close to the top of the convective zone. As is well known, an up going wave becomes evanescent as it approaches the acoustic cut-off, and is reflected downwards. A correct calculation should indeed include a proper description of the properties of the wave equation near the reflecting layers. However, in the present approach, we shall limit ourselves to the approximation given by Gough (Ref. 2) for the levels \( z_k \) at which ray paths become horizontal, in a polytropic model of index \( \mu \):

\[
z_k = (1 + \lambda)ak_\lambda^{-1},
\]
where: \( \lambda = \left(1 - \frac{\nu^2}{\pi^2}\right)^{1/2} \), \( \alpha = \frac{\nu^2 \mu^2}{\pi^2 \kappa^2} \), \( \beta^2 = \frac{\mu}{2} \left(\frac{\mu}{2} + 1\right) \), \( g \) being the acceleration of gravity and \( k_h \) the horizontal wave number.

In the limit where \( b/a \) is small compared to unity, which is the case if \( \ell \) is not too large (\( \ell < 300 \)) we obtain an approximation of equation (7) for \( z_+^+ \):

\[
z_+^+ \approx \frac{g}{8\pi^2} \frac{\mu + 1}{\nu^2}.
\]

We see that when we go from order \( n \) to order \( n + 1 \), the level \( z_+^+ \) rises inside the solar atmosphere by the quantity:

\[
\Delta z_+^+ \approx \frac{g}{8\pi^2} \frac{\mu + 2}{\nu^3} (\nu_{n+1} - \nu_n).
\]

We notice that the higher frequency modes are reflected higher in the solar atmosphere, and that there is no dependence in the wavenumber \( k_h \) (or equivalently in the degree \( \ell \)). Typical values for \( \Delta z_+^+ \) given by equation (9) are of order 50 km, which is probably an overestimation.

If the extra time \( \Delta T \) \( (L = \Delta z_+^+ = 50 \text{ km}) \) were to be of the order of 1 second, equation (3) would give an estimate of the Mach number:

\[
M \approx 0.3,
\]

which is not inconsistent with eddy velocities in the upper convective zone.

3. CONCLUSION

As a result of this very crude estimate, we consider that it is worthwhile to look more carefully into the effect initially suggested by Brown. In principle, one could think of an inversion process which would yield an estimate of the depth variation of the Mach number. However, as numerous other causes may also be invoked for explaining the discrepancies \( (v_{\text{obs}} - v_{\text{calc}}) \), we are still probably far from being able to achieve any significant inversion of the available data.

In the mean time, a better appreciation of the amplitude of this retardation effect, free from the acoustic ray approximation, could be derived from the numerical simulation of acoustic oscillations and convection such as the one that R. Stein and A. Nordlund (Ref. 3) have recently performed.

We hope to be able to report soon on the quantitative results obtained on these simulations.

4. REFERENCES

ABSTRACT

If the individual solar oscillations were pure standing waves of constant amplitude inside a stationary Sun, there would be in principle no intrinsic limits to the accuracy with which their frequencies could be measured. Unfortunately, it is not so. In their analysis of 10 months of irradiance data from the SMM/ACRIM experiment, Woodard and Hudson (Ref. 1) have proposed to describe the individual modes as independent and chaotically excited oscillators, the linewidths being of the order of 1.2/μHz in the central frequency range, around 3 mHz.

This paper deals with artificial full disk data. The goal is to determine the parameters of a randomly excited oscillator by comparison with the best data available to us. Such artificial signal is then used to test the methods of analysis used for the precise determination of mode frequencies, amplitudes and linewidths. We present tests of the reliability of centroid measurements made by barycenter, Lorentzian fit, Gaussian fit, with and without background noise. Several methods are also tested for linewidth measurements.

Keywords: Data analysis, Oscillations of the Sun.

1. THE FOURIER SPECTRUM “PREJUDICE”.

Let us start with well known, simple remarks:

i) The so called “window function” is the power spectrum of the time series equal to 1 when observations are recorded, and to zero otherwise. The power spectrum of a time series sampling a standing sinusoidal signal consists in the window function centered at the frequency of the sinusoid. For the present work, we shall limit ourselves to uninterrupted time series of finite, but long duration: the wave signature is then a peak in the power spectrum of width δν₀ = (duration of observation)^{-1}.

ii) Two (or n) standing waves sufficiently separated in frequency produce two (or n) peaks.

Hence, a spectrum like the one in figure 1 could be interpreted as a main peak, plus four smaller ones separated by a few tenths of μHz, plus noise.

If there are good reasons to believe that the peaks are real (e.g. two identical spectra obtained from two different instruments), then one could be tempted to conclude from such observations:

- one sees a splitting of the main peak into several (5?) subpeaks,
- oscillations lifetimes are longer than the observing run, since the width of each individual peak is of the order of δν₀.

At this stage, we would like to stress that such conclusions rely upon the implicit assumption that what is observed is a superposition of standing waves.

Figure 1. A power spectrum fitted by a Lorentzian and a Gaussian.
2. ANOTHER (IMPROVED?) PREJUDICE: RANDOMLY EXCITED, DAMPED OSCILLATIONS.

Again, we list some well known classical results:

i) A single damped oscillation gives a Lorentzian peak in the power spectrum, hardly modified by the window function if the damping time is short as compared to the duration of observations.

ii) Two (or n) damped oscillators, having identical frequencies, but excited at successive, randomly distributed times will exhibit the same Lorentzian peak in their individual power spectra. However, when added in the complex Fourier space, their phases will vary with frequency quite differently, thus causing interferences.

We have produced spectra of a large number of different realizations of time series from a randomly excited harmonic oscillator (period: 5 min, damping time ~ 4.5 days, duration of observation: 150 days, average rate of excitation: 1 per day, Gaussian statistics for excitation amplitudes).

Figure 1 was in fact a power spectrum of one typical realization. It is important to notice that no "noise" has been added to it. Everything is "signal" corresponding to an harmonic oscillator with a stable central frequency (taken here as the zero of the abscissa).

We conclude then that if we relax the implicit assumption of pre-existing standing waves there is no "proof" that this completely significant spectrum exhibits neither splitting nor long lifetimes of oscillations. Furthermore, it is obvious that the frequency of the main peak is a poor determination of the central frequency of the oscillator.

3. HOW CAN WE ANALYZE THIS KIND OF DATA?

To the set of time series of our harmonic oscillator, we have added different amounts of white noise. We have then analyzed the power spectra in the following manner:

- we assume that we have some idea of the position of the peak, but not a very precise one (±0.3 μHz),
- we compute a "noise level" from averaging the power outside the central part, over 100 spectral estimates,
- we compute the frequency of the center of gravity of the power spectrum in the central region, above this noise level, and estimate the integrated power in this domain.
- we identify the highest spectral estimate, and measure its amplitude,
- from this amplitude and the estimated power, we infer a guess for the "line width",
- we take those four guessed quantities (constant offset level due to noise, position of the center of gravity, integrated power, line width) and give them as starting conditions for least square fittings of a Lorentzian and a Gaussian curve.

On figure 1 are also shown the two curves fitted to the spectrum according to this procedure.

Having performed those calculations, we produce histograms of the different fitted parameters, some of which are shown on figures 2 and 3.

The signal to noise ratio of data that have been used for figure 2 is equal to 2.5. In spite of this rather high noise level, one sees that the central frequency is much better found by the Lorentzian fit. It can be noticed, however that there are some extended "wings" in the histogram. They correspond to situations where the fitting process tends to adjust to an isolated fringe, or noise peak; this phenomenon will show up in the linewidth determinations as we are going to see.

Figures 3 a & b show histograms of linewidths obtained from Gaussian and Lorentzian fits. The abscissa have been normalized in such a way that fitting the spectrum of our harmonic oscillator excited only once would give a value exactly equal to 1. Notice the difference in the ordinate scales. Figure 3a shows that a Gaussian fit tends to give a much too small line width, whereas a Lorentzian one (figure 3b) has obviously a bi-modal distribution (this character is better seen when the signal to noise ratio is higher: here it has been set to 10).

The explanations are simple: a gaussian function is very close to zero in the wings. Thus, it cannot take into account properly the wings of the "signal"; the fitting procedure compensates in increasing the "constant". It leaves then a part of the central region of the power spectrum above the constant which is narrower. One can improve the situation by forcing the "constant" to be equal to the "noise" level, but this assumes that our knowledge of this level is secure, namely that the regions where we measure it is signal-free. We know that it is the case in our artificial data. It might be dangerous to assume it for real data.
The bi-modal distribution of the Lorentz fits comes from the fact that the process sometimes likes to stick on one of the fringes of the interference pattern. When it does not happen, we see that the linewidth is essentially correctly determined. This is due to a much better determination of the value of the "constant". Histograms of values of the "constants" confirm this assertion. Instead of showing them, we prefer to display on figure 4a and 4b four examples of Gauss and Lorentz fits obtained with a signal to noise ratio equal to 2.5. The reader will find there a good illustration of the previous remarks.

Figure 3 a. Histograms of the linewidths found by fitting Gaussians.

Figure 4 a. Four Gaussian fits.

Figure 3 b. Histograms of the linewidths found by fitting Lorentzians.

Figure 4 b. Four Lorentzian fits corresponding to the same four original spectra.
4. CONCLUSIONS.

We shall not here enter into all the fine details that a careful study of the various possible histograms could reveal. We wish only to underline the following points:

i) For the determination of the central frequency, taking the highest peak is not the best choice. Fitting a Lorentz curve is better.

ii) As far as the linewidth is concerned, the Gaussian fit seems to be better defined, but indeed the Lorentzian one should be preferred, since it finds a better value for the actual width. However, since it happens sometimes that the fitting procedure yields erroneously to a very small linewidth, some care should be taken, especially when the number of realizations is not high enough to show clearly the bi-modal distribution. Once this distribution has been obtained, various techniques could be applied in order to prevent the fitting procedure to reach the absolute minimum of residuals, but instead to find a local minimum constrained to more likely values of the width.

iii) More detailed conclusions are of course somewhat dependent of the signal to noise ratio and will be dealt with in a forthcoming paper.

Performing real data analysis along these lines may yield new insight into interpretations of solar oscillations time series. However, we should not forget that we have replaced the standing wave prejudice by another one.

Even if it is likely to be a better model of the solar oscillations phenomenology, this remains to be demonstrated. Other non standing wave models should probably also be studied. It should be noticed also that we have alluded to the complex Fourier space, but we have only made use of the amplitude information. The phase remains to be exploited.

5. REFERENCE

1. Woodard M & Hudson H S 1983, Frequencies, amplitudes and linewidths of solar oscillations from total irradiance observations, Nature Vol 305, 599-593
Closing remarks
ON TAKING OBSERVERS SERIOUSLY

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Abstract

The lastest measurements of the even component of degeneracy splitting of five-minute oscillations announced at this symposium add substantially to the evidence for temporal variations in the solar acoustic asphericity that are correlated with the solar cycle. The asphericity appears to extend from the photosphere, through the convection zone, into the radiative interior. It may exist even in the core of the sun, but there is yet no sound direct evidence for that. The next observations will be carried out well into the new cycle, and are therefore likely to clarify the situation significantly.

1. Introduction

When observers, like me, of observers' observations take what they see quite seriously, hints of interesting phenomena which can stimulate the imagination are sometimes perceived. At this meeting I have been particularly impressed by the mounting, though arguably meagre evidence that perhaps the entire sun partakes in the solar cycle. Taken alone, each piece of evidence can hardly be considered convincing, but when taken together (after carefully rejecting the most suspect data: i.e. those that are not in accord with the ideas being developed here) a picture is emerging of coherent 11-year variations of large-scale features of the solar structure that are not merely on the surface and in the convection zone, but perhaps in the very core of the sun where the thermonuclear reactions take place. Although it would certainly be premature to conclude that this picture is definitely correct, the evidence must surely force us to entertain quite seriously the possibility that the solar cycle is basically a global oscillation, and is not entirely the product of a dynamo confined essentially to the turbulent convection zone.

2. Variation in acoustic asphericity

Even though extensive accurate helioseismic data do not yet extend over an interval as long as 11 years, some correlations with the cycle of variations in certain seismologically derived quantities are already appearing. My interest was triggered by the report at this symposium by S.M. Jeffries, M. Pomerantz, T. Duvall Jr., J.W. Harvey and D. Jaksha of the new measurements of the coefficients \( a_2 \) and \( a_4 \) in an expansion of degeneracy splitting of cyclic frequencies \( \nu \) of five-minute acoustic oscillations (p modes):

\[
d(L, m, t) \equiv N_{n,n0} \equiv \sum_{j=1}^{s} \alpha_j(L, t)P_j(m/L),
\]

where \( P_j \) is the legendre polynomial of degree \( j \), \( m \) is the azimuthal order of a mode, and \( L^2 = \ell(\ell + 1) \approx (L + \frac{1}{2})^2 \), \( \ell \) being the degree of the mode. Also \( t \) is time. (Strictly speaking the splitting \( s \) should depend also on \( n \), but the available data I discuss here have been averaged over \( n \).) The coefficients \( \alpha_j \) of the even-degree terms measure a deviation from spherical symmetry in the speed of propagation of acoustic waves (c.f. Gough and Thompson, 1988). Values of both \( \alpha_2 \) and \( \alpha_4 \) dating from 1981 have been declining with the retreat of the sunspots (Duvall, Harvey and Pomerantz, 1986; Libbrecht, 1986; Brown and Morrow, 1987), indicating a diminution in the magnitude of the asphericity. Indeed, I have heard it argued that neither the data of Libbrecht nor those of Brown and Morrow demonstrate significant non-zero values of \( \alpha_2 \) and \( \alpha_4 \), and that perhaps doubt should therefore be cast on the earlier measurements of Duvall et al.: then to within about a part in \( 10^4 \) the sun is spherically symmetrical. I do not share this attitude, and indeed can see an interesting coherent pattern emerging (see Figures 1 and 2): the new measurements of Jeffries et al., the first since sunspot minimum, show an upturn in \( \alpha_2 \), although in contrast, \( \alpha_4 \) has yet shown no indication of reversing direction. If one were to take all those data seriously, that would mean that not only the magnitude but also the shape of the acoustical distortion is changing.

If the varying component \( \delta c \) of the acoustic propagation speed \( c \) is assumed to be axisymmetric about the rotation axis of the sun, it can be expanded in the form

\[
\frac{\delta c}{c} = \sum_{k \geq 0} \beta_{2k} f_{2k}(r) P_{2k}(\cos \theta),
\]
where \( \theta \) is colatitude. (Here I have included a possible time-dependent spherically symmetrical component with coefficent \( \beta_0(t) \), to which I shall return when I discuss variations in luminosity and the frequencies of low-degree modes.)

If in addition, \( |\delta \omega/c| \ll 1 \), one can show from perturbation theory (e.g. Gough, 1988; Gough and Thompson, 1988) that this induces a frequency splitting

\[
\delta \omega = \sum_k \beta_0 k Q_{2k, tm} f_{2k},
\]

where the overbar denotes an average with respect to a radial coordinate \( r \) over the range spanned by the cavity within which the waves are trapped:

\[
\bar{f} = \int_0^R K f dr.
\]

For five-minute oscillations the kernel \( K \) is well approximated by

\[
K(r) \approx \int_0^R \frac{r^2 \omega^2}{1 - \frac{c^2 L^2}{r^2 \omega^2} \frac{1}{r^2} dr}
\]

(e.g. Gough, 1988; Gough and Thompson, 1989), where \( \omega = 2 \pi \nu \). The integration is from the lower turning point \( r_l \) at which \( K \) diverges:

\[
\frac{r_l}{c(r_l)} = \frac{L}{\omega},
\]

to the radius \( R \) of the sun (strictly speaking, the radius of the upper turning point). Notice that because we are discussing coefficients that have been averaged over \( n \), \( \omega \) can be regarded as constant (\( \omega/2 \pi \approx \nu_0 \approx 3 \text{mHz} \)), and \( r_l \) depends only on \( L \). The result of averaging over latitude is finally dependent part of \( \delta \omega \) is given by

\[
\frac{\delta \omega}{c} \approx \nu_0 \left[ \left( \frac{\omega}{c} \right)^2 \frac{\sin^2 \theta}{\cos^2 \theta} \right].
\]

(10)

As was pointed out by Gough and Thompson (1988), the insensitivity of \( \alpha_4 \) to \( L \) indicates that the averages (4) are insensitive to the extent of the cavity within which they are taken, implying that \( \delta \omega/c \) varies only weakly with depth. The best power-law fit is \( \delta \omega/c \propto (1 - r/R)^{0.3} \), but a depth-independent perturbation is not inconsistent with the data.

Aside from oblateness measurements, whose interpretation has been the subject of much debate, this is the first direct dynamical evidence for a deep-seated temporally varying asphericity in the hydrostatic structure of the sun.

3. The seat of the asphericity

How deep-seated is the asphericity? Present indications are that it extends beneath the convection zone to at least a depth of about 0.45 of the solar radius. This is the base of the cavity confining the lowest-degree modes (\( \ell \approx 25 \)) for which splitting coefficients are available. Evidently one needs to study modes of yet lower degree to learn about the inner core of the sun. This is an extremely challenging task, partly because the number of modes available to measure the splitting is an approximately increasing function of \( \ell \), so at low \( \ell \) there is less scope for averaging the data to reduce the noise. Unfortunately, a coherent picture of low-degree splitting has not emerged from this symposium, so I defer consideration of this issue until I have discussed the other evidence.

Of course, the first physical question one would naturally ask is: what is the origin of the acoustical asphericity and how is it related to the magnetic cycle? The first suggestion was that the asphericity was due to a latitudinal variation in thermal stratification resulting from the inhibition of convection by magnetic fields in the surface layers of the sun (Gough and Thompson, 1988). Although this produced the observed \( L \)-dependence of the coefficients \( \alpha_j \), in order to
impressive correlation between zonal flow and magnetic polarity reported at this symposium by E. Ribes, J.C. Ribes, whose temporal variation might be the so-called torsional oscillations reported by La Bonte and Howard (1982). The temporal variation contributing to \( \delta I / I \) by an amount comparable with that given by Equation (14), which implies a luminosity modulation \( \delta L / L \) of amplitude approximately \( 2 \times 10^{-4} \) in phase with the sunspot cycle.

4. Correlation with solar irradiance

An interesting consequence of a time-dependent latitudinal temperature variation is that even if the total luminosity \( L \) of the sun were constant, the irradiance \( I \) in the plane of the ecliptic (at constant distance from the sun) would change with time. Thus, setting

\[
\frac{\delta T_e}{T_e} \approx 2\pi c \sigma_0 \left[ \nu_0 / \nu_0 - 2\alpha_2 P_2(\cos \theta) + \frac{3}{2} \alpha_4 P_4(\cos \theta) \right] \tag{11}
\]

(which, aside from the potentially different origin, determined by \( \beta_0 \), is equivalent to Equation (10), where \( T_e \) is the spatially averaged effective temperature and \( \delta T_e \), the variation, one obtains

\[
\frac{\delta L}{L} = 8\beta_0 \tag{12}
\]

and

\[
\frac{\delta I}{I} = 8 \frac{\pi}{4} \frac{\delta T_e}{T_e} \sin^2 \theta d\theta \tag{13}
\]

Thus, if \( \delta L = 0 \), \( \frac{\delta I}{I} = \frac{6\beta_0 - \alpha_4}{3\nu_0}. \tag{14} \)

This quantity is compared with observed irradiance variations in Figure 4. Though it appears to have a similar functional form, its magnitude is somewhat less than half that required to explain the irradiance variations. Therefore we should perhaps conclude that there is also a spherically symmetrical component of the effective temperature variation contributing to \( \delta I / I \) by an amount comparable with that given by Equation (14), which implies a luminosity modulation \( \delta L / L \) of amplitude approximately \( 2 \times 10^{-4} \) in phase with the sunspot cycle.

5. Low-degree oscillations

Is the variation of \( \beta_0(t) \) implied by this conclusion consistent with observation? This question cannot yet be answered with confidence. First, we do not know what causes the spherically symmetrical component of the change in brightness structure, and second, the observational situation is not clear. Only observers of low-degree modes have reported absolute frequency shifts, but their results are apparently

Whatever the cause of the acoustical anomaly, there is likely to be an associated meridional flow, which will advect magnetic fields and angular momentum, creating a zonal flow whose temporal variation might be the so-called torsional oscillations reported by La Bonte and Howard (1982). The impressive correlation between zonal flow and magnetic polarity reported at this symposium by E. Ribes, J.C. Ribes, I. Vince and P. Merfin is not inconsistent with these ideas.
of course the splitting occurs for modes of low degree too. Therefore it is certainly in accord with expectation that used to determine order differently. Indeed, it is just that splitting that was finally, let us recall that the acoustic asphericity influences by torsional oscillations that are driven in the sun's core. (e.g. Dicke, 1978; thus reminded of the recent discussions rotation profile deduced by Duvall (1984). One is and Ventura (1985) and Gough (1985) by the interesting and neutrino flux displayed by E. Gavryu-

apparent correlation between sunspot numbers, apparent cline is situated well beneath the photosphere, though at present little more than that can be inferred. However, the apparent correlation between sunspot numbers, apparent semi-diameter and neutrino flux displayed by E. Gavryu-

what are the theoretical implications? It has been estimated (Gough, 1982) that a thermal adjustment at the base of the convection zone, which might mimic Parker’s thermal shadowing) causes a relative frequency shift \( \delta \nu / \nu \approx 0.5 \delta C / \ell \). For the decline of about \( 2 \times 10^{-4} \) in \( \delta \nu \) from sunspot maxima to sunspot minima, it is suggested that the frequency shift is caused by the non-radial mode that dominates the response in the previous section, which implies a fre-

section 2 of (\( 9 \)) to deduce the values of \( \delta_0 \) from the observed values of \( \alpha_0 \) plotted in Figures 1 and 2, which is valid because \( \ell \gg 2k \) for these observations, the frequency shifts \( \delta \nu / \nu \) can then be estimated by evaluating expression (15) using Equations (16), (3) and (7), assuming \( \delta_0 \) to be the same for the low-degree and the high-degree cavities. The result is that from 1982 to 1986 the mean dipole frequencies should have declined by about 0.10Hz less than the monopole frequencies, and the quadrupole and hexadecapole frequencies should have both declined by about 0.09Hz less than the monopole frequencies. These differences are substantially less than the (disparate) values that have been quoted by the solar observers.

it must be borne in mind that the reported variations might be due entirely to beating between rotationally split modes. Nevertheless, if variations in \( \alpha_0 \) as high as those that have been reported are finally confirmed, we shall be bound to expect that \( \delta_0 \) is greater when the average is carried over lower-degree cavities, and that therefore the acoustic asphericity is even greater in the radiative interior than it is in the convective envelope.

6. Prospects

The coming year will add an important chapter to this story. New seismic observations from the South Pole by the NAOA/NSO group are planned for the coming Austral summer. As we are now well into the rising phase of the sunspot cycle, we should therefore anticipate a further in-

This contribution to the proceedings is not a scheduled paper, but a synthesis of remarks made from the floor.
References


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