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ATOMIC ENERGY OF CANADA LIMITED

L'ENERGIE ATOMIQUE DU CANADA, LIMITEE

SENSITIVITY ANALYSIS IN MULTI-PARAMETER PROBABILISTIC SYSTEMS: AN EXAMPLE USING THE MCROC ROCK MICROCRACKING MODEL

ANALYSE DE SENSIBILITE DES SYSTEMES PROBABILISTES A PLUSIEURS PARAMETRES EN PRENANT COMME EXEMPLE LE MODELE MCROC DE MICROFISSURATION DES ROCHES

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Etablissement de recherches nucléaires de Whiteshell

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RÉSUMÉ

On peut appliquer des méthodes probabilistes impliquant l'emploi de l'analyse par la Méthode de Monte-Carlo à une grande variété de systèmes techniques. La sortie de l'analyse par la méthode de Monte-Carlo est une évaluation probabiliste de la conséquence des systèmes qui peut varier dans l'espace et le temps.

Le but de l'analyse de sensibilité est d'examiner comment la conséquence (sortie) est influencée par les valeurs paramétriques d'entrée. L'analyse de sensibilité fournit les renseignements nécessaires permettant d'optimaliser les propriétés techniques des systèmes. Dans ce rapport, on décrit en détail un ensemble de techniques d'analyses de sensibilité qui, toutes ensemble, constituent une méthodologie intégrée d'analyse de sensibilité de systèmes probabilistes. Ces technique ont des limites de confiance connues et on peut les appliquer à une grande variété de problèmes techniques.

On illustre la méthodologie d'analyse de sensibilité en exécutant l'analyse de sensibilité de modèle MCROC de microfissuration des roches.

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SENSITIVITY ANALYSIS IN MULTI-PARAMETER PROBABILISTIC SYSTEMS : AN EXAMPLE USING THE MCROC ROCK MICROCRACKING MODEL

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ABSTRACT

Probabilistic methods involving the use of multi-parameter Monte Carlo analysis can be applied to a wide range of engineering systems. The output from the Monte Carlo analysis is a probabilistic estimate of the system consequence, which can vary spatially and temporally.

Sensitivity analysis aims to examine how the output consequence is influenced by the input parameter values. Sensitivity analysis provides the necessary information so that the engineering properties of the system can be optimized. This report details a package of sensitivity analysis techniques that together form an integrated methodology for the sensitivity analysis of probabilistic systems. The techniques have known confidence limits and can be applied to a wide range of engineering problems.

The sensitivity analysis methodology is illustrated by performing the sensitivity analysis of the MCROC rock microcracking model.

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1 INTRODUCTION

Probabilistic methods are of interest to Atomic Energy of Canada Limited, since they can be applied to a range of engineering problems, mathematical models and parametric studies. For example, in waste management, they have been applied to the assessment of the hazard associated with toxic and radioactive wastes [1,2]. As another example, probabilistic methods are being employed in the development of fitness-for-service criteria [3] of reactor components.

Many of the probabilistic studies employ multi-parameter Monte Carlo techniques. In the Monte Carlo technique, input parameters of the system model are assigned probability density functions (PDFs). These PDFs represent the frequency of occurrence of the input parameters in the system model. A variate is sampled from each of the parameter PDFs that comprise the system under consideration. This set of variates represents a possible realization of the system, which has a corresponding system consequence. This process is repeated to produce many realizations of the system. The ensemble of realizations provides a probabilistic estimate of the system consequence, which may vary spatially and temporally.

It is often desirable to identify the critical input parameters of the model and to establish procedures to control them. The role of ***sensitivity analysis*** is to examine how the output consequences are influenced by the input parameter values, so that the system properties can be optimized.

Multi-parameter probabilistic systems are often too complex to allow the sensitivity analysis to be performed analytically. If analytical methods cannot be used, then alternative techniques must be employed. This report details a package of three techniques that together form an integrated methodology for the sensitivity analysis of probabilistic systems. The techniques have been successfully used in the assessment of the radiological risk from the disposal of radioactive wastes [4,5,6], but are not specific to any individual system. Here, the sensitivity analysis methodology is illustrated by performing the sensitivity analysis of a model that determines the extent of rock microcracking in the vicinity of a nuclear-waste disposal vault.

2 THE MCROC ROCK MICROCRACKING MODEL

The MCROC model [7,8] was developed to estimate the changes in intact rock elastic modulus (E) and permeability (B), caused by the heating of plutonic rock in the vicinity of a nuclear-fuel waste disposal vault.

The model considers that a microcrack exists in each crystal facet, throughout the rock mass. These microcracks are thin and penny-shaped, with an initial diameter of $2L_0$. The ratio of microcrack diameter to facet diameter is $2L_0/W$. This ratio and the facet diameter are different for each facet.

As the rock heats up, elastic strains due to differential thermal expansion (DTE) are developed at each crystal facet. The strain magnitude, at a given facet, depends on the local temperature increase (ΔT) and the difference in thermal expansivity ($\Delta\alpha$) between the crystals sharing the facet.

The local elastic strain energy, due to DTE, is assumed to be transformed into new microcrack surface (energy) at each facet. The velocity (V) of microcrack extension is given by

$$V = \frac{dL}{dt} = AK_I^n \quad , \tag{1}$$

where A and n are constants for a given rock type and environment, K_I is the stress intensity factor in tensile-opening mode, L is the microcrack radius, and t is time. The values of A and n for Lac du Bonnet granite have been determined experimentally [9,10], and are 10^{-178} and 29.0, respectively. The stress intensity factor, which depends on L, W, and the local stress (σ), is given by

$$K_I = Y\sigma\sqrt{L} \quad , \tag{2}$$

where Y is a local geometry factor that is a function of L and W, only. Stress changes both with reduced local stiffness (S) due to microcrack extension, and with the local temperature (T). Hence, V changes because K_I depends on σ and L. All these events are considered to occur within a small element of rock encompassing a single cracked facet. The local T change as a function of time is related to stress by a local displacement (U) within the rock element. The following equations are to be solved simultaneously:

$$\frac{dU}{dt} = \frac{dT}{dt}\frac{W\Delta\alpha}{2} \quad , \tag{3}$$

$$\frac{dL}{dt} = AY^n\sigma^n\sqrt{L^n} \quad , \tag{4}$$

$$\text{and} \quad \sigma = \frac{US}{W} \quad . \tag{5}$$

The solution of these equations yields K_I, σ, V, and L, all as functions of time and of the thermal history of the facet.

The thermal expansion of water trapped in closed microcracks can cause additional microcracking. When microcrack extension due to water expansion exceeds that resulting from σ alone, Equation 4 is replaced by

$$\frac{dL}{dt} = \frac{dT}{dt}L\frac{\gamma(T)}{2}\sqrt{\frac{L_0^2\rho_0}{\rho(T)}} \quad , \tag{6}$$

where ρ and γ are, respectively, the density and thermal expansivity of water. The water density at time zero is ρ_0. Equation 6 is valid for all temperatures below the boiling point of water.

The Monte Carlo method is used to generate the facet-microcrack array, which represents the rock mass. The parameters L_0/W, W, and $\Delta\alpha$ are assumed to have triangular-shaped probability density functions. The PDFs are specified in terms of minimum, mode, and maximum values of the parameters. Values for L_0/W, W, and $\Delta\alpha$ that are representative of Lac du Bonnet granite are given in Table 1. Parameter values are selected from their respective PDFs using a proven uniform pseudo-random number generator [11].

TABLE 1: REPRESENTATIVE PDFS FOR LAC DU BONNET GRANITE

Parameter	Minimum	Mode	Maximum
L_0/W	0.00000	0.10000	0.30000
W (m)	5.0000×10^{-4}	1.2500×10^{-3}	5.0000×10^{-3}
$\Delta\alpha$ (K^{-1})	2.0000×10^{-6}	1.0000×10^{-5}	1.7000×10^{-5}

The model predicts how the microcrack array develops with time, at various points (different thermal histories) in the rock mass. Specifically, it predicts the distributions of L from which $< L^2 >$ and $< L^3 >$ are estimated, where $<>$ indicates average value. The changes in intact rock elastic modulus (E/E_0) and permeability (B/B_0) are then estimated from:

$$\frac{E}{E_0} = \frac{\left(1 + \frac{16N<L_0^3>}{9}\right)}{\left(1 + \frac{16N<L^3>}{9}\right)} \quad , \tag{7}$$

$$\text{and} \quad \frac{B}{B_0} = \frac{< L^2 >}{< L_0^2 >} \quad , \tag{8}$$

where N is the number of microcracks per unit volume, E_0 is the value of E for uncracked rock, and B_0 is the value of B at time zero.

The heating of the rock in the vicinity of the nuclear-fuel waste disposal vault is caused by the decay of radioactive elements in the waste. The HOTROK computer code [12] has been used in conjunction with the vault design parameters [13] to obtain the thermal gradients in the rock mass. Figure 1 shows a series of temperature histories for different vertical distances above the centre of the vault. Figures 2 and 3 show the corresponding changes in elastic modulus and permeability, respectively, driven by the temperature histories of Figure 1. It can be seen from Figures 2 and 3 that the extent of microcracking decreases rapidly with increasing distance from the vault.

The Atomic Energy Control Board requires [14] that a quantitative assessment be made of the disposal system up to a period of 10^4 years after closure. A further requirement is that reasoned arguments must be presented to show that the risk from the disposal system does not dramatically increase after 10^4 years. Figures 2 and 3 show that

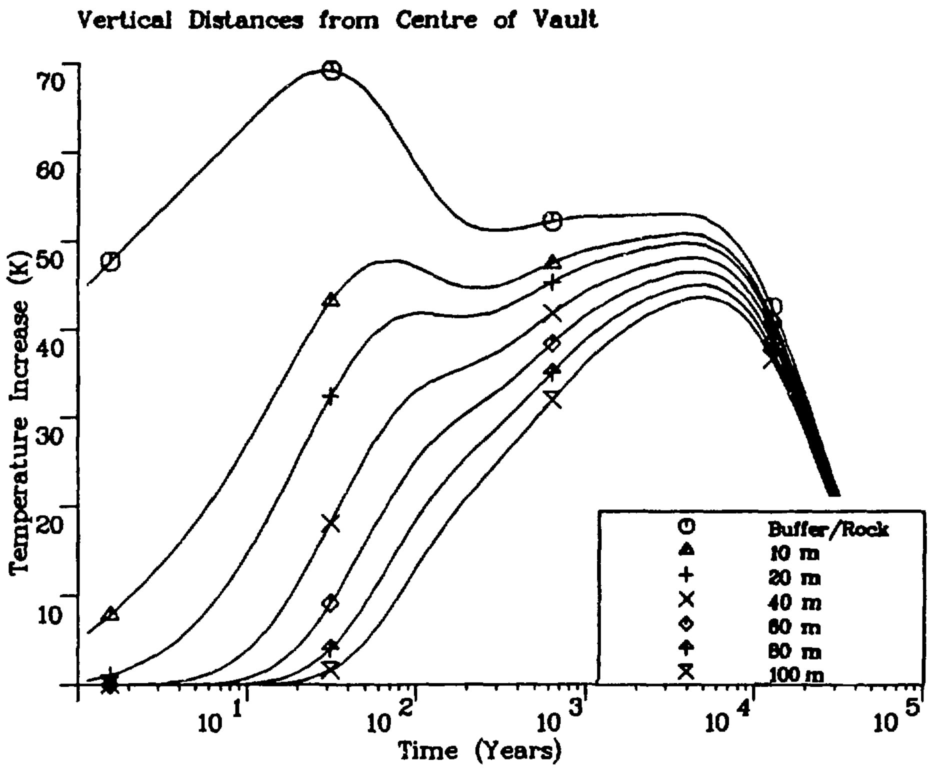

FIGURE 1: Temperature Profiles above Centre of the Vault

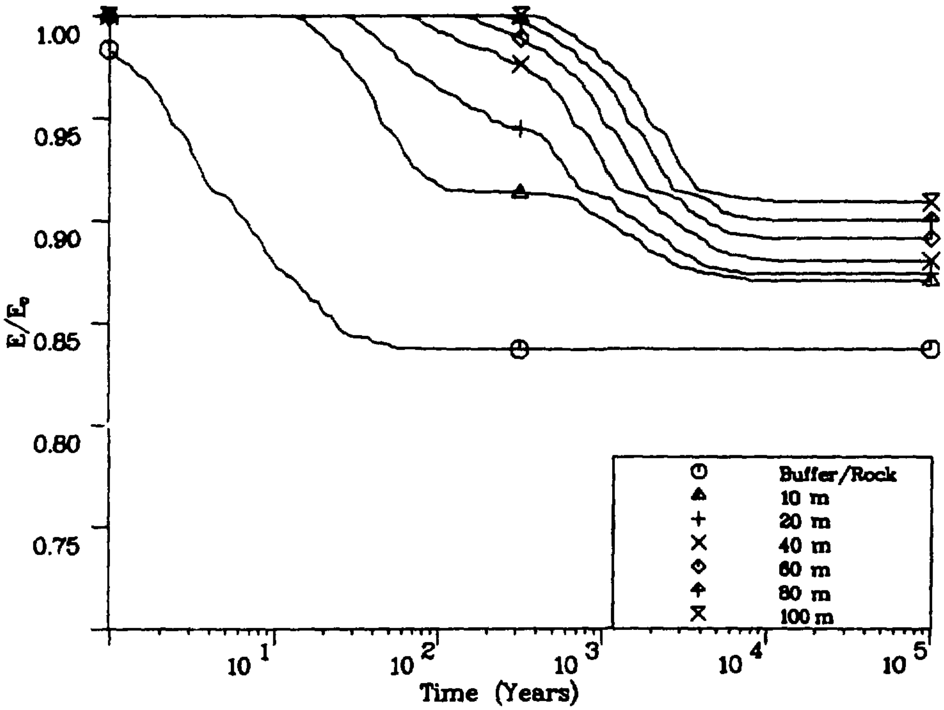

FIGURE 2: E/E_0 above Centre of the Vault

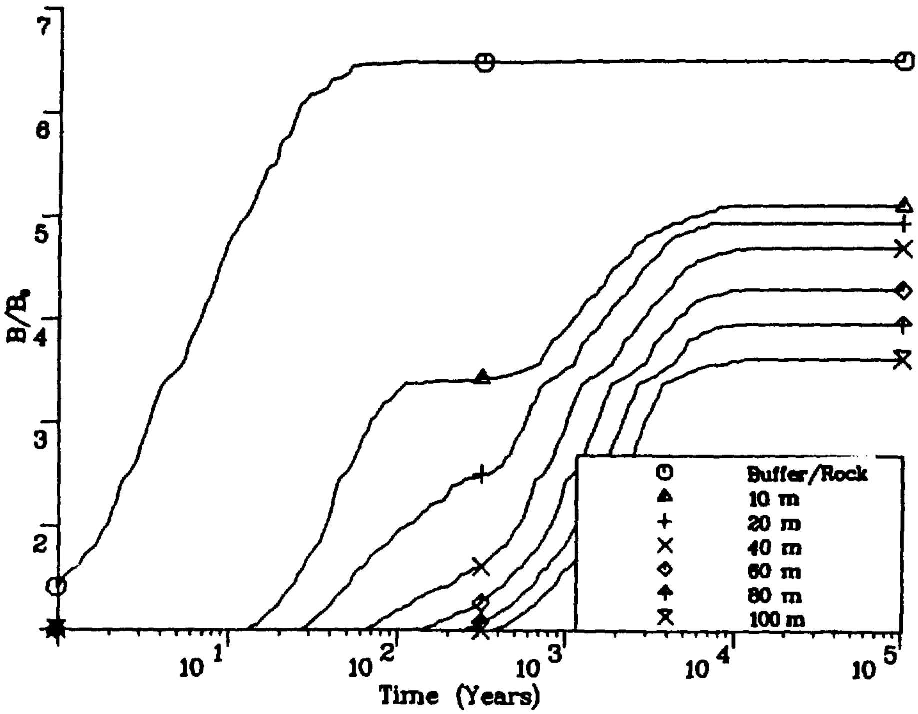

FIGURE 3: B/B_0 above Centre of the Vault

most of the microcracking occurs before 10^5 years after vault closure, and that maximum microcracking occurs at the buffer/rock interface. The reference point for the sensitivity analysis of the MCROC model was chosen to be at the buffer/rock interface at a time 10^5 years after vault closure. To facilitate analysis the spatial point with the maximum microcracking was chosen. The temporal point was chosen since no significant further microcracking is expected after 10^5 years, and it exceeds the minimum time period required for regulatory purposes.

3 SENSITIVITY ANALYSIS

Sensitivity analysis aims at determining how system output consequences are influenced by input parameter values. If Y is the consequence of interest, then it is convenient to think of the system model as a function

$$Y = f(X_1, \ldots, X_k) \quad , \tag{9}$$

where the variables $X_1, \ldots, X_k$ are the k input parameters in the system.

Previous approaches to the sensitivity analysis of multi-parameter Monte Carlo systems have concentrated on the production of a *response surface* for the system [15,16,17]. Since the true response surface ($Y = f(\boldsymbol{X})$) is *a priori* unknown, an approximate response surface must be constructed. A common approach is to use a second-order polynomial

$$\tilde{Y} = a_0 + \sum_{i=1}^{k} a_i X_i + \sum_{i=1}^{k} \sum_{j=1}^{k} a_{ij} X_i X_j \quad , \tag{10}$$

where $\tilde{Y}$ is the response surface estimate. The coefficients a_0, a_i, and a_{ij} are estimated by least squares fitting to

$$\sum_{i=1}^{k} (Y_i - \tilde{Y}_i)^2 = \min \quad , \tag{11}$$

or by some other interpolation scheme. Although, in principle, it is simple to produce an approximate response surface, in practice it is difficult to determine efficient functional forms for $\tilde{Y}$ that accurately represent the true response surface. Also, as the number of input parameters increases, the number of realizations necessary to cover the parameter hyperspace becomes very large. For example, to generate $\tilde{Y}$ from Equations 10 and 11 using only the minimum, mean, and maximum values of parameters in a 15-parameter system requires $3^k = 3^{15} = 14\ 348\ 907$ realizations. It is not possible to significantly increase the density of points in the parameter hyperspace, since impractically large numbers of realizations would be required. Fractional experimental designs [18,19] are sometimes used to reduce the number of realizations required, however, their use increases the sparseness of the points in the parameter hyperspace.

Sensitivity analysis techniques involving Green's functions [20] and the Adjoint technique [21] have recently been recommended. However, these techniques are effectively limited to relatively simple systems, since they require the iterative numerical solution of systems of partial differential equations. This is computationally expensive, and is subject to the compounding of rounding errors for even quite modest numbers of parameters.

From the perspective of the probabilistic methodology, the sensitivity analysis should provide information, to an acceptable confidence level, in three broad areas:

- What combination of parameters maximizes (and minimizes) the system consequence?
- How is the variability in system consequence influenced by the variability of individual input parameters?
- How do changes in the value of individual parameters influence the system consequence?

There is no single sensitivity analysis technique that can completely provide the required information in each of these areas. The following three techniques individually answer one of the three questions, as will be demonstrated in the case of the MCROC microcracking model. Together the three techniques provide an integrated methodology for the sensitivity analysis of probabilistic systems.

4 DIFFERENTIAL SENSITIVITY ENVELOPES

In first-order sensitivity analysis, the partial derivative of the consequence is determined with respect to the input parameters. The study of complex systems using first-order sensitivity analysis was first proposed by Tomović [22]. This technique is widely used when parameters deviate small amounts from their nominal values.

The MCROC model, in common with many probabilistic system models, has input parameters that vary simultaneously over wide ranges. Hence, first-order methods cannot be used to perform the sensitivity analysis of the MCROC model.

Although first-order sensitivity analysis techniques are inappropriate, a knowledge of the partial derivatives is essential in the understanding of how parameters combine to maximize (or minimize) the consequence. The procedure is as follows. Each of the k input parameters is sampled from its respective PDF. This parameter set ($X_1 = x_1, \ldots, X_k = x_k$) defines a particular point in the k-dimensional hyperspace, which has a corresponding consequence (Y). At that point in the hyperspace, the derivatives

(Y'_{x_i}) are calculated, where

$$Y'_{x_i} = \left.\frac{\partial Y}{\partial X_i}\right|_{X_j = x_j} \qquad 1 \leq i, j \leq k \ . \tag{12}$$

The input parameters are again sampled from their PDFs, which defines another point in the hyperspace, and the (Y'_{x_i}) are again calculated. This procedure is repeated many times to give an envelope (or histogram) of partial derivatives of Y with respect to each of the input parameters.

In the MCROC model both the change in elastic modulus and permeability are related to the ratio L/L_0. Thus, it is appropriate to set the consequence $Y = L/L_0$ at a time 10^5 years after closure of the vault. An example of the differential sensitivity envelope (DSE) for the parameter $\Delta\alpha$, without the presence of trapped water, is given in Figure 4.

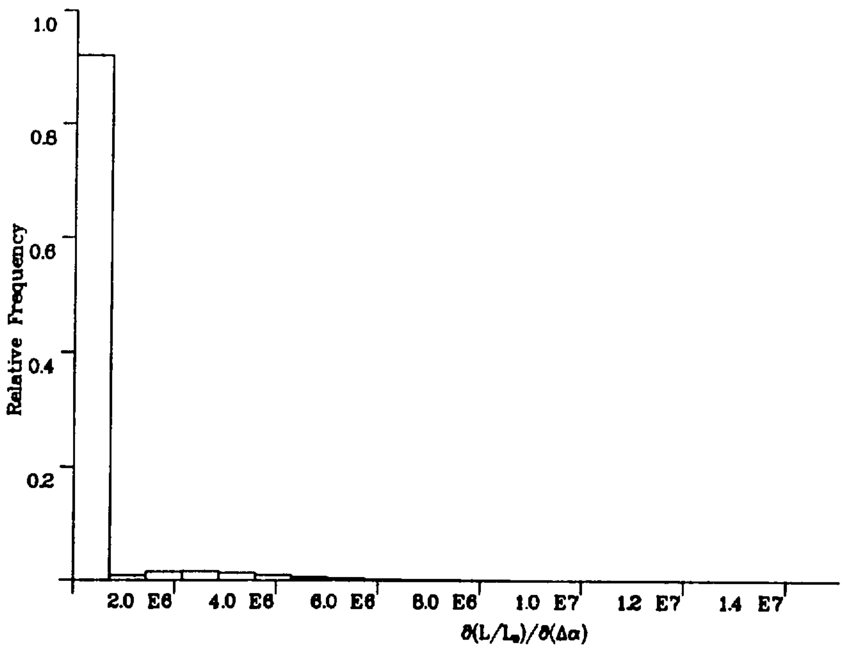

FIGURE 4: Sensitivity Envelope for $\Delta\alpha$ (No Trapped Water)

The utility of calculating the DSEs is in the understanding of how parameters individually, and in combination, effect the consequence (L/L_0). From Figure 4 it can be seen that, in the absence of trapped water, the derivative of L/L_0 with respect $\Delta\alpha$ is never negative. Thus, an increase in the value of $\Delta\alpha$ will always cause L/L_0 to either increase or remain the same, irrespective of the values of other parameters. In mathematical terms L/L_0 is monotonically non-decreasing with respect to $\Delta\alpha$. The extreme values of the DSEs of L/L_0 ,with respect to the distributed parameters of the MCROC model in the absence and presence of trapped water, are given in Tables 2 and 3, respectively.

TABLE 2: SENSITIVITY ENVELOPE EXTREMA FOR L/L_0 (NO TRAPPED WATER)

Differential	Minimum	Maximum
$\frac{\partial(L/L_0)}{\partial(L_0/W)}$	−407.36	3094.9
$\frac{\partial(L/L_0)}{\partial W}$	0.00000	4.8915×10^4
$\frac{\partial(L/L_0)}{\partial(\Delta\alpha)}$	0.00000	1.4343×10^7

TABLE 3: SENSITIVITY ENVELOPE EXTREMA FOR L/L_0 (TRAPPED WATER)

Differential	Minimum	Maximum
$\frac{\partial(L/L_0)}{\partial(L_0/W)}$	−407.36	3089.6
$\frac{\partial(L/L_0)}{\partial W}$	0.00000	4.8846×10^4
$\frac{\partial(L/L_0)}{\partial(\Delta\alpha)}$	0.00000	1.4335×10^7

As an example of the use of Tables 2 and 3, a unit increase in the value of W will cause an increase in L/L_0 of between 0.0 and 4.8915×10^4, in the absence of trapped water. In the analysis given in this report, the partial derivatives are calculated using 10 000 points in the hyperspace in both the absence and presence of trapped water. The binomial theorem can be used to calculate confidence bounds on the limits of the sensitivity envelopes. Since the sensitivity envelopes were calculated using 10 000 points in the parameter hyperspace, the probability of exceeding the limits given in Tables 2 and 3 is approximately 3.68×10^{-4}, at a 95% confidence level.

It should be noted that the ranges of the sensitivity envelopes are slightly smaller in the presence of trapped water, as compared to the case of the absence of trapped water. This is to be expected, since the presence of trapped water can cause additional microcracking. This additional microcracking is independent of the distributed parameters. Thus, the sensitivity of L/L_0 to the distributed parameters is slightly decreased by the presence of trapped water.

The behaviour of L/L_0, with respect to the distributed parameters, can be used to determine the combination of parameters that give rise to the extreme minimum and maximum values of changes in elastic modulus and permeability. In order to calculate the maximum possible value of L/L_0, the parameters that influence L/L_0 in a monotonically non-decreasing manner (i.e., W and $\Delta\alpha$)* are fixed at their maximum values. A linear search technique is then used to find the value of L_0/W that maximizes L/L_0. The calculation of the minimum possible value of L/L_0 is calculated in an analogous manner, by setting non-decreasing parameters at their minima, etc.. The value of L/L_0 obtained by this procedure is substituted into Equations 7 and 8 to obtain the extreme values of E/E_0 and B/B_0, respectively. The presence of trapped water does not effect the extreme values of E/E_0 and B/B_0, due to the properties of the MCROC model. The minimum and maximum possible values of E/E_0 and B/B_0 at 10^5 years after vault closure, in both the absence and presence of trapped water, are given in Table 4.

TABLE 4: MINIMUM AND MAXIMUM POSSIBLE VALUES OF E/E_0 AND B/B_0

Extremum	E/E_0	B/B_0
Minimum	0.72921	1.0000
Maximum	1.0000	826.56

This method of finding extreme values in probabilistic systems is widely applicable. A schematic of the general procedure to find the maximum possible value of a system consequence is given in Figure 5. The minimum possible consequence value is determined in an analogous manner.

On the basis of the maximum values of the partial derivatives given in Tables 2 and 3, the parameter importance ranking is $\Delta\alpha > W > L_0/W$. This parameter importance ranking should, however, be used with care since the distributions of the partial derivatives are not taken into account. Also, no allowance is made for the form of the PDFs assigned to each input parameter. Hence, the method will not detect differences caused by various types of PDF (triangular, normal, etc.). A sensitivity analysis method that does allow for the parameter distributions is given in the next section.

*Strictly speaking, it has not been proven that L/L_0 is non-decreasing with respect to W and $\Delta\alpha$. However, since the DSEs were calculated using 10 000 realizations, the probability of W and $\Delta\alpha$ influencing L/L_0 non-monotonically is less than 3.68×10^{-4} at a 95% confidence level. In order to unambiguously assign monotonicities, the DSE information is combined with the knowledge of the system under consideration. Further study of the constituent equations of MCROC (Equations 3–6), indicates that L/L_0 does indeed behave monotonically with respect to W and $\Delta\alpha$.

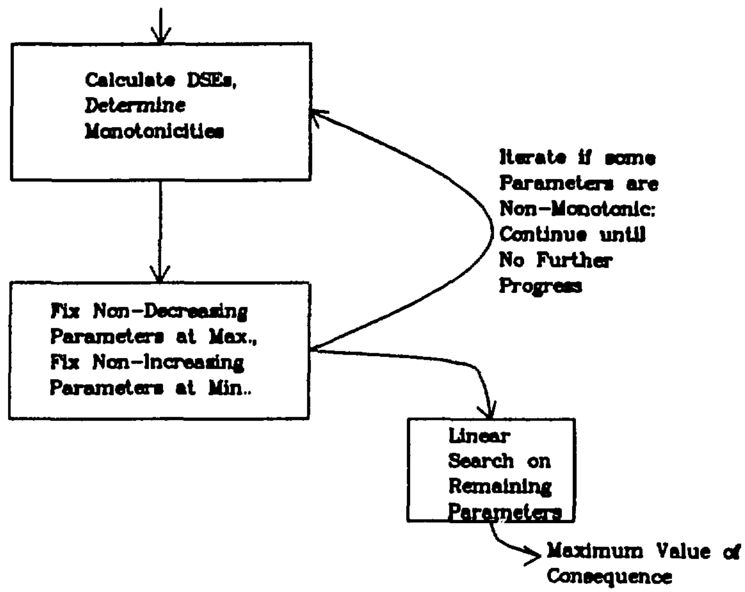

FIGURE 5: Procedure to Determine Maximum Value of Consequence

5 CORRELATION ANALYSIS

The aim of this correlation analysis is to determine how L/L_0 at 10^5 years after vault closure is related to the input parameters, and how the variability in L/L_0 is related to the variability of the input parameters.

The appropriate measure of correlation in multi-parameter probabilistic systems is the rank correlation coefficient of Spearman [23]. Spearman's rank correlation coefficient (ρ) measures the extent to which a consequence is determined by a monotonic relationship to each of the input parameters. The rank correlation coefficient can take values in the range $-1 \leq \rho \leq +1$. Positive values imply that the consequence is related in a monotonically increasing manner to the particular parameter. Negative values imply that the consequence is related in a monotonically decreasing manner. When $|\rho| = 1$, the consequence is entirely determined by the parameter in question. Lesser values of $|\rho|$ imply a lesser degree of correlation between the consequence and the parameter.

One of the advantages of the use of ρ as a statistical measure is that the statistical confidence in the measure is well understood [24]. The standard error in a measured rank correlation coefficient (σ_ρ) is given by $\sigma_\rho = 1/\sqrt{n-1}$, where n is the number of data points (in our case the number of Monte Carlo realizations). Since the errors in the statistical measure are known, a parameter importance ranking can be produced with a known level of confidence. This is unlike many other non-parametric measures, where the confidence bounds of a predicted parameter ranking may be unknown.

The analysis presented here is based on 100 000 realizations of the system model. Thus $\sigma_\rho = 1/\sqrt{99999} \approx 0.003$. Now, the 95% confidence interval corresponds approximately to ± 1.96 standard deviations. Therefore, the rank correlations given in Tables 5 and 6 are accurate to ± 0.006 at a 95% confidence level. The data ranking and calculation of ρ was performed with the CANAL correlation analysis program [25]. Tables 5 and 6 show that the parameter importance ranking is $\Delta\alpha > W > L_0/W$, with or without the presence of trapped water.

TABLE 5: RANK CORRELATION COEFFICIENT FOR L/L_0 (NO TRAPPED WATER)

Parameter	ρ
L_0/W	0.30467
W	0.42107
$\Delta\alpha$	0.67541

From the definition of the rank correlation coefficient it can be shown that

TABLE 6: RANK CORRELATION COEFFICIENT FOR L/L_0 (TRAPPED WATER)

Parameter	ρ
L_0/W	0.30126
W	0.42060
$\Delta\alpha$	0.67581

the square of the correlation coefficient (ρ^2) is the proportion of the total variation in L/L_0 that is explained by the relationship between L/L_0 and the particular parameter. Hence, ρ^2 is termed the coefficient of determination, since it is the proportion of the variation in L/L_0 that is determined by the variation in the particular parameter. The calculated variability compositions of L/L_0 in the absence and presence of trapped water are given in Figures 6 and 7, respectively. Figures 6 and 7 show that no single parameter dominates the system, although L_0/W is somewhat less important than $\Delta\alpha$ or W.

As was noted in Section 4, L/L_0 is slightly less sensitive to the distributed input parameters in the presence of trapped water, as compared to the case of the absence of trapped water. Figures 6 and 7 show that the variability in L/L_0 due to factors other than the input distributions has increased from 19.02% without trapped water, to 19.09% in the presence of trapped water. This is because of additional microcracking caused by the expansion of trapped water. This additional microcracking is not reflected in the correlations to the distributed input parameters.

We have seen, therefore, that the rank correlation coefficient is used to determine the variability composition, and hence, answer one of the questions required of sensitivity analysis. There remains the question of how individual parameters can influence the system consequence. This question is answered by the technique demonstrated in the next section.

6 INDIVIDUAL PARAMETER SENSITIVITY ANALYSIS

Of the many parameters in a multi-parameter probabilistic system, some of the parameters have the potential to be modified by human intervention. This intervention may be in the form of some engineering or chemical treatment, or some form of institutional control. In the case of the MCROC model, different geological formations can allow some control over input parameter values. In order that the optimum intervention can be undertaken, it is desirable to understand how modifications to particular parameters can alter the system consequence.

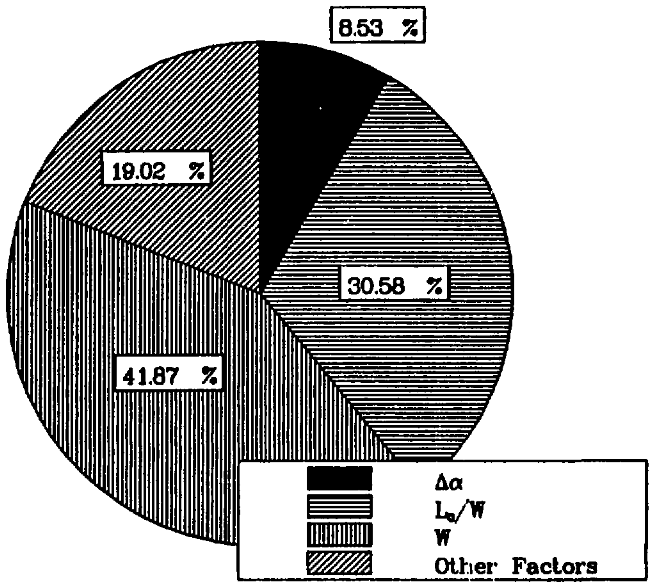

FIGURE 6: Variability Composition of L/L_0 (No Trapped Water)

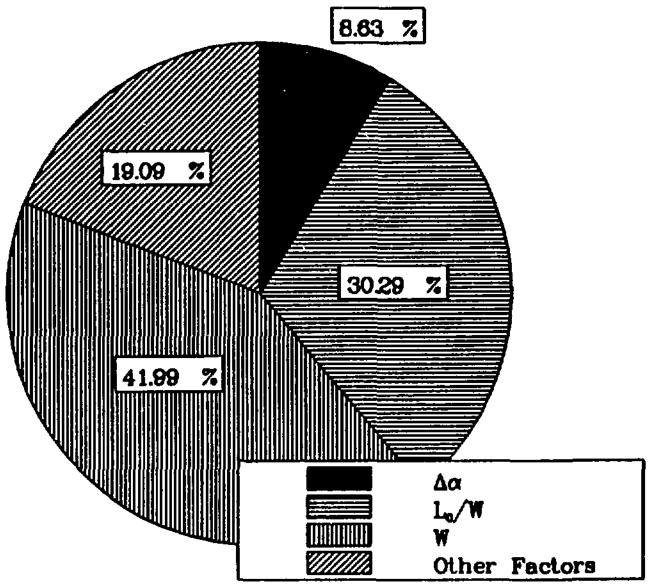

FIGURE 7: Variability Composition of L/L_0 (Trapped Water)

An effective way of determining the effect of an individual parameter (X_i) is to fix the parameter at its minimum value and calculate the consequence ($Y_{i_{low}}$), while allowing all other parameters to be sampled from their PDFs. Then fix the parameter under investigation at its maximum value and again calculate the consequence ($Y_{i_{high}}$), allowing all other parameters to be sampled from their PDFs. The difference ($Y_{i_{high}} - Y_{i_{low}}$), represents the increase in consequence that can be achieved by increasing the value of parameter X_i from its minimum value to its maximum value.

Figure 8 shows E/E_0 as a function of time with W set at its extreme values (Table 1), while allowing $\Delta\alpha$ and L_0/W to be sampled from their PDFs. A summary of results of this procedure, at 10^5 years after vault closure, in the absence and presence of trapped water, is given in Tables 7 and 8, respectively. These results were calculated using 10 000 realizations of the MCROC model in each case. Therefore, the values given in Tables 7 and 8 are accurate to within approximately four parts in 10^4, at a 95% confidence level. The parameter importance ranking, as determined by ($Y_{i_{high}} - Y_{i_{low}}$), is $\Delta\alpha > W > L_0/W$, with or without the presence of trapped water.

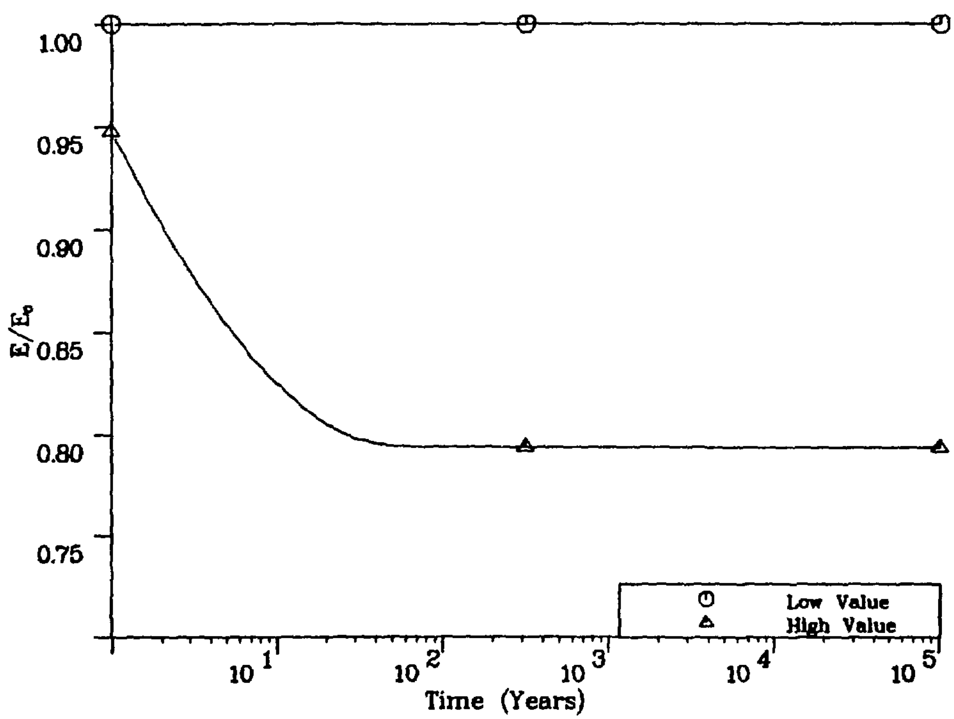

FIGURE 8: E/E_0 with W Fixed at Extreme Values (No Trapped Water)

TABLE 7: INDIVIDUAL PARAMETER SENSITIVITY (NO TRAPPED WATER)

	E/E_0			B/B_0		
Parameter	Low	High	Difference	Low	High	Difference
L_0/W	1.0000	0.8176	−0.1822	1.0000	2.4152	1.4152
W	0.9993	0.7933	−0.2060	1.0194	8.6970	7.6776
$\Delta\alpha$	1.0000	0.7441	−0.2559	1.0000	11.114	10.114

TABLE 8: INDIVIDUAL PARAMETER SENSITIVITY (TRAPPED WATER)

	E/E_0			B/B_0		
Parameter	Low	High	Difference	Low	High	Difference
L_0/W	1.0000	0.8172	−0.1828	1.0000	2.4209	1.4209
W	0.9989	0.7927	−0.2060	1.0422	8.7305	7.6883
$\Delta\alpha$	0.9996	0.7440	−0.2556	1.0221	11.119	10.010

7 SUMMARY OF MCROC SENSITIVITY ANALYSIS

- The extent of microcracking is small, and decreases rapidly as distance from the vault increases.
- The consequence L/L_0 is monotonically non-decreasing with respect to $\Delta\alpha$ and W. L_0/W effects L/L_0 in a non-monotonic manner. Since E/E_0 and B/B_0 are related to L/L_0, the effects of microcracking can be minimized by choosing geological formations that have small $\Delta\alpha$ and W.
- No single parameter dominates the system, although L_0/W is somewhat less important than $\Delta\alpha$ or W.
- Individual parameter sensitivity analysis shows that the largest change in microcracking extent is obtained by changing $\Delta\alpha$ from its minimum to its maximum value. Thus, the optimum route to microcracking reduction is to choose geological formations that have low differential thermal expansion between the crystal types that compose the rock.
- The sensitivity analysis techniques consistently identify the parameter importance ranking as being $\Delta\alpha > W > L_0/W$, with or without the presence of trapped water.
- The production of additional microcracking by the expansion of trapped water is quite small, and the sensitivity to the distributed parameters is only slightly reduced by trapped water.

8 CONCLUSIONS

The sensitivity analysis of the MCROC rock microcracking model has been performed using a package of sensitivity analysis techniques. The techniques consistently identify the same parameter importance ranking.

The sensitivity analysis provides:

- The combination of parameters that maximizes (and minimizes) the consequence,
- The variability composition of consequence with respect to parameter variability,
- The effect of changes in individual parameter values.

The sensitivity analysis techniques have known confidence limits, and provide a comprehensive methodology for the sensitivity analysis of multi-parameter probabilistic systems. The techniques are not specific to the MCROC system, and can be applied to a wide range of engineering problems.

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