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A NEW TYPE OF SURFACE ACOUSTIC WAVES IN SOLIDS DUE TO NONLINEAR ELASTICITY *

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ABSTRACT

It is shown that in nonlinear elastic semi-infinite medium possessing a property of self focusing of shear waves , besides bulk non-linear shear waves, new surface acoustic waves exist, localization of which near the boundary is entirely due to nonlinear effects.

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In the past eight years there has been an interest in the properties of surface electromagnetic waves, arising in a medium with dielectric constants depending on the strength of the electric field [1]. The purpose of the present research is to show that the account of the nonlinear elasticity of solids may result in the appearance of a new type of surface acoustic waves which near boundary localization is entirely due to the nonlinear effects as for the electromagnetic waves mentioned above.

The quadratic non-linearity generating the second and higher harmonics is typical for surface and bulk acoustic waves in solids [2,3]. A different situation occurs for the pure shear waves. The shear wave does not generate the second harmonic wave with transverse polarization in unbounded homogeneous isotropic media [2]. This property allows us to consider the cubic non-linearity in the equation of motion for shear waves similar to that for the nonlinear electromagnetic waves.

Consider the propagation of a pure shear wave along a free surface of a homogeneous isotropic half space. Let the polarization of the wave be parallel to the surface, i.e. the wave is of the so-called SH-polarization (shear horizontal). If the medium is linear, the boundary conditions of the elastic stress absence on the surface are satsified by bulk SH wave alone. However, the bulk wave is known [4] to be unstable and may become a surface wave under a small change in the parameters of the problem (for instance, under piezoelectric effect in medium for Gulyaev-Bleustein waves or additional layer on surface in the case of Love waves). What happens to these bulk SH waves if we take into account the non-linearity of real elastic solids?

The equation of motion and the boundary condition of the elastic stress absence on a free boundary of solids with nonlinear elasticity taken into account have the form (see [5])

$$\rho v_{tt} = \mu(v_{zz}+v_{xx}) + \alpha(3(v_x)^2 v_{xx} + 3(v_z)^2 v_{zz} + [(v_x)^2 v_z]_z + [(v_z)^2 v_x]_x) \quad , \tag{1}$$

$$v_z[\mu + \alpha(v_x)^2 + \alpha(v_z)^2] = 0 \quad \text{at} \quad z = 0 \quad . \tag{2}$$

The subscripts denote differentiations by variables x,z,t; $\alpha = (\lambda + 2\mu)/2 + 4(2B + 3C + 4F)$; λ,μ are Lamé constants; B,C,F are nonlinear elasticity constants [5], ρ is the mass of the medium. Eqs.(1) and (2) are written down for the case when the wave propagates along x axis, the displacement vector v is directed along y axis and z axis of Cartesian co-ordinates x,y,z is directed along the normal to the surface $z = 0$. The same equations (1) and (2) also follow from formulas (4.7)-(4.9) obtained by Maradudin [6] for the case of anisotropic media with cubic symmetry after transition to isotropic limit.

We look for a solution of Eq.(1) which in a linear case transforms into the well-known solution for bulk waves in half space with free surface. Hence we can use only one variant of the boundary condition (2): $v_z = 0$. We assume that the solution of the Eq.(1) has the form of a harmonic wave propagating along the surface with frequency ω, i.e. $v = u(z)\sin(\omega t - kx + \varphi)$. Neglecting the interaction of the fundamental frequency wave with higher harmonics as for the electromagnetic waves [7] we can get an equation for u(z)

$$-\rho\omega^2 u = \mu(u_{zz} - k^2 u) + (\alpha/4)[9(u_z)^2 u_{zz} - 3k^4 u^3$$

$$+ k^2 u(u_z)^2 + k^2 u^2 u_{zz}] \quad , \tag{3}$$

As in the linear case this equation has a solution in the form of bulk waves for which $u_z = u_{zz} = 0$ at all z. From that, after substitution into Eq.(3), we find the dispersion relation for nonlinear bulk shear waves, $k^2 = k_t^2 - 3\alpha k^4 u^2/(4\mu)$, where $k_t^2 = \omega^2\rho/\mu$. These bulk waves are the solutions of the boundary problem both for a half space and for a plate with free parallel faces and arbitrary thickness, since u_z is always in these cases zero. If $\alpha < 0$, as it follows from the dispersion equation, the wave velocity $v_p = \omega/k$ is decreasing with the amplitude, so the self focusing effect for the bulk shear waves is possible in such a medium. Further, we will assume that $\alpha < 0$. This case may be quite realistic with the account of the experimental data obtained by Nakagawa [8].

We will now show that besides the bulk shear waves specific surface shear waves exist in the nonlinear self focusing solids. For that we multiply Eq.(3) by u_z and integrate it over z. As a result we find the first integral of this equation

$$(u_z)^2 - (k^2 - k_t^2)u^2 + C = [-\alpha/(8\mu)]\,[3k^4u^4 - 2k^2u^2(u_z)^2 - 9(u_z)^4] \quad , \tag{4}$$

where C is a constant of the integration. For a surface wave solution of Eq.(4) the condition $u = u_z = 0$ must be satisfied at $z = \infty$. From that $C = 0$. The boundary condition $u_z = 0$ at $z = 0$ and Eq.(4) enables us to get the exact dispersion relation for nonlinear surface shear waves (of course, assuming the validity of the approximate equation (3))

$$k^2 = k_t^2 - 3\alpha k^4 u_0^2/(8\mu) \quad , \tag{5}$$

where $u_0 = u(z = 0)$. The additional term in the dispersion relation (5) due to nonlinearity of the medium happens to be exactly twice smaller than that for bulk waves. The smaller value of this nonlinear contribution is understandable if one takes into consideration the decrease of the vibration amplitude of the surface wave with the distance from the boundary

The solution of Eq.(5) for the phase velocity v_p, corresponding to the solution for the bulk shear waves in the linear case, has the form

$$v_p^2/v_t^2 = \left[1 + \sqrt{1 + 3\alpha k_t^2 u_0^2/(2\mu)}\right]/2 \quad , \tag{6}$$

where $v_t = \sqrt{\mu/\rho}$ is the velocity of the small amplitude bulk shear wave. It follows from this formula that at $u_0^2 < -2\mu/(3\alpha k_t^2)$, $\alpha < 0$, v_p is real, i.e. Eq.(4) has actually a solution in the form of harmonic waves travelling along the x axis. The case $u_0^2 > -2\mu/(3\alpha k_t^2)$ will not be considered here because then non-linear contributions become larger than the linear terms and Eq.(3) is not valid.

The simple expressions for phase and group (v_g) velocities of surface nonlinear waves are obtained from Eq.(6) if their amplitudes are small

$$v_p/v_t = 1 + 3\alpha k_t^2 u_0^2/(16\mu), \quad v_g/v_t = 1 + 9\alpha k_t^2 u_0^2/(16\mu) \quad . \tag{7}$$

The dispersion appeared due to nonlinearity of the medium creates more favourable conditions for the validity of the monochromatic approximation accepted here. It is necessary to note that for the higher harmonics the dispersion will be somewhat different from that determined by (7) owing to the strong influence on them by the wave of fundamental frequency.

To determine the dependence of the wave amplitude on a depth, it is convenient to rewrite Eq.(4) in terms of dimensionless variables Z, U, ε, defined by $U = u/u_0$, $Z = z\sqrt{k^2 - k_t^2}$, $\varepsilon^2 = 2(k^2 - k_t^2)/(3k^2) = -\alpha k^2 u_0^2/(4\mu)$. Then (4) takes the form

$$\varepsilon^2[(27/4)\varepsilon^4(dU/dZ)^4 - (1 - \varepsilon^2U^2)(dU/dZ)^2 - U^2(1 - U^2)] = 0 \quad . \tag{8}$$

From Eq.(8) it is easy to find the dependence Z(U). For the half space $Z \leq 0$ it is

$$Z(U) = \int_1^U \frac{dU}{U} \sqrt{\frac{1 - \varepsilon^2U^2 + \sqrt{(1-\varepsilon^2U^2)^2 - 27\varepsilon^4U^2(1-U^2)}}{2(1-U^2)}} \quad . \tag{9}$$

The limits of the integration are chosen to satisfy the condition $U = 1$ at $Z = 0$. A plus sign before the inner square root is chosen to satisfy the condition $U = dU/dZ = 0$ at $Z = \infty$, and the integral sign corresponds to an evanescent wave localized near the surface. The solution (9) exists if the integrand is real. The inner square root is equal to zero at

$$U^2 = [2 + 27\varepsilon^2 \pm 3\sqrt{3(27\varepsilon^4 + 4\varepsilon^2 - 4)}]/(56\varepsilon^2) \quad . \tag{10}$$

So far $\varepsilon < \varepsilon_0$, where $\varepsilon_0 \simeq 0.32$ is the positive solution of the equation $27\varepsilon^4 + 4\varepsilon^2 - 4 = 0$, U^2 determined by (10) is complex and in the region of integration in (9) the integrand is always real. For such $\varepsilon < \varepsilon_0$ the solution exists. For $\varepsilon > \varepsilon_0$ the integrand becomes complex and we have no surface wave solution. The limiting value ε_0 is very close but not equal to the value 1/3, which corresponds to the limiting point of a real solution existence of dispersion relation (6).

For small amplitudes it is possible to neglect the terms involving ε in Eq.(9). Then a simple analytical expression for U may be found

$$U = 1/\cosh Z \quad . \tag{11}$$

This solution is similar to those obtained for electromagnetic nonlinear surface waves [1]. It corresponds to the account of only the first term in the right-hand side of Eq.(4). The other two terms neglected in this approximation have always the sign opposite to that of the first term. Hence under increasing amplitude these two terms will suppress the effect of surface localization of nonlinear shear waves. Taking into account the first two terms in the right-hand side of Eq.(4) one comes to the dependence Z(U)

$$Z(U) = \int_1^U \frac{dU}{U} \sqrt{\frac{1 - \varepsilon^2U^2}{1 - U^2}} \quad , \tag{12}$$

which may also be calculated analytically

$$Z = \ln U - \ln\left(\sqrt{1-\varepsilon^2U^2} + \sqrt{1-U^2}\right)$$

$$+ \varepsilon\ln\left(\sqrt{1-\varepsilon^2U^2} + \varepsilon\sqrt{1-U^2}\right) + c_\varepsilon \quad ,$$

$$c_\varepsilon = (1/2)(1-\varepsilon)\ln(1-\varepsilon^2) \quad . \tag{13}$$

All three terms in question may be taken into account in analytical calculations by expanding the integrand in (9) in power series in ε^2 up to terms of order ε^4

$$Z = \cosh^{-1}/(1/U) - (\varepsilon^2/2)(1+\varepsilon^2/4)(1-U^2)^{1/2} - (13/12)\varepsilon^4(1-U^2)^{3/2} \tag{14}$$

or

$$U = (1/\cosh Z)[1 + (1/2)\varepsilon^2\tanh^2 Z - (1/4)\varepsilon^4\tanh^2 Z +$$

$$+ (19/12)\varepsilon^4\tanh^4 Z]. \tag{15}$$

From these expressions it follows that the second and the third terms on the right-hand side of Eq.(4) oppose a surface localization of the wave as it was expected from qualitative consideration.

At large amplitude, when $\varepsilon = \varepsilon_0$, the nonlinear surface waves transform into the bulk waves, described by the following expression obtained from Eq.(7)

$$U = \cos\left(Z/\sqrt{1-\varepsilon_0^2}\right) . \tag{16}$$

This solution satisfies the boundary condition of the elastic stress absence at a free surface of a half space and it is also a particular solution for nonlinear shear waves in a plate with free parallel faces and a depth equal to $H = \sqrt{1 - \varepsilon_0^2}\,\pi n$, $n = 1,2,3,\ldots$

A few comments on the limitations of the developed theory. As mentioned above, one of them is the requirement that the wave amplitude should be small. On the one hand, it is related to the neglect of the terms above the third order in the amplitude in Eqs.(1) and (2). On the other hand, we neglected the interaction of the fundamental wave with higher harmonics in Eq.(3).

Another approximation, not stated explicitly, is the neglect of one more mechanism for self-action of the nonlinear surface shear waves. Although bulk shear waves do not generate the second harmonic of transverse polarization in an unbounded homogeneous isotropic solid, nevertheless they do generate the longitudinal second harmonic wave. In our case, in contrast with bulk plane waves it should be an inhomogeneous with the depth dilational wave propagating with the same velocity as the fundamental wave. The interaction of this wave with the fundamental one will result in an effective cubic nonlinearity. Some motivation for the omission of this mechanism may be taken from experimental results of Nakagawa [8], who showed that the contribution of the cubic nonlinearity to the generation of the third harmonics can substantially exceed the contribution of the quadratic nonlinearity.

In conclusion let us discuss possibilities of the experimental observation of this type of nonlinear surface shear waves. One of the possible ways is the observation of the amplitude dependence of the SH wave reflection and transmission coefficients in a thick plate with a groove on its back side. For such an experiment the value of penetration depth of the surface wave is essential. Another possible choice is a simultaneous generation of nonlinear surface and bulk waves with the observation of their space interference. The period of their space beating should be proportional to the difference of the wave amplitudes squared as follows from the present results. At last case the interaction between surface and bulk waves must also be taken into account.

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