ON FIELD LINE RESONANCES OF HYDROMAGNETIC ALFVÉN WAVES IN DIPOLE MAGNETIC FIELD

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Abstract

Using the dipole magnetic field model, we have developed the theory of field line resonances of hydromagnetic Alfvén waves in general magnetic field geometries. In this model, the Alfvén speed thus varies both perpendicular and parallel to the magnetic field. Specifically, it is found that field line resonances do persist in the dipole model. The corresponding singular solutions near the resonant field lines as well as the natural definition of standing shear Alfvén eigenfunctions have also been systematically derived.

I. INTRODUCTION

The theory of resonant field lines as developed independently by Chen and Hasegawa [1974] as well as Southwood [1974] has successfully accounted for many important observed features of long-period geomagnetic pulsations; see the review by Lanzerotti and Southwood [1979]. In the original theoretical models, only one-dimensional inhomogeneity perpendicular to **B** (the magnetic field) is assumed. Thus, the corresponding eigenfrequencies for the transverse (shear) Alfvén waves standing along the field lines also vary spatially and constitute the so-called *shear* Alfvén continuum [Grad, 1969]. For an excitation frequency matching an eigenfrequency inside the shear Alfvén continuum, we then have wave resonance and, correspondingly, the perturbations become singular near the particular resonant field line.

Since in realistic plasmas, such as the Earth's magnetosphere, the Alfvén velocity is nonuniform at least in the parallel-to-B direction in addition to the radial direction, one wonders about the nature of field line resonances (FLR) in magnetic field geometries with two inhomogeneous coordinates. This issue is particularly interesting since it also has implications to the heating of the solar corona (see, for example, Ionson [1978]). Southwood and Kivelson [1986] employed the rectangular box model and initiated investigations in the effects of parallel inhomogeneity. In the present work, we will develop the FLR theory in the more realistic dipole magnetic field geometry [Chen and Cowley, 1988]. In particular, we show, via properly defined standing shear Alfvén eigenfunctions and eigenfrequencies, resonant field lines do persist in the dipole model and the theoretically predicted features are consistent with recent numerical results [Lee and Lysak, 1988].

II. THEORETICAL MODEL

We consider a cold nonuniform plasma in the following magnetic field

$$\mathbf{B} = \nabla\phi \times \nabla\psi = \nabla\chi, \tag{1}$$

where ψ and ϕ are the two perpendicular to $\mathbf{B}$ coordinates and χ is the coordinate along $\mathbf{B}$. For a dipole field, we may adopt the (ν, ϕ, μ) coordinates used by Radoski [1967]. Here, ϕ is the West-East azimuthal symmetry direction, $\psi = M\nu$, $\chi = M\mu$, and $M = 8 \times 10^{25}\,\mathrm{Gauss} \times \mathrm{cm}^3$. In terms of the (r, ϕ, θ) spherical coordinates, we then have

$$\nu = \frac{\sin^2\theta}{r},$$

$$\mu = \frac{\cos^2\theta}{r^2},$$

$$B = M|\nabla\mu| = \frac{M(1 + 3\cos^2\theta)^{1/2}}{r^3},$$

and

$$|\nabla\phi| = \frac{1}{r\sin\theta}.$$

In the present work, it is sufficient to describe the plasma using the ideal magnetohydrodynamic (MHD) equations. In terms of the electric field perturbations,

$\delta\boldsymbol{\eta} = \delta\boldsymbol{\xi} \times \mathbf{B} = \delta\eta_\phi \nabla\phi + \delta\eta_\psi \nabla\psi$ where $\delta\boldsymbol{\xi}$ is the usual one-fluid displacement vector and $\delta\boldsymbol{\eta} = \delta\boldsymbol{\eta}(\psi, \chi)\exp(-i\omega t + im\phi)$, as well as the perturbed compressional magnetic component $\delta B_{||}$, the coupled MHD equations are

$$D_{A\psi}\delta\eta_\psi \equiv \left[\frac{\partial}{\partial\chi}(|\nabla\psi|^2)\frac{\partial}{\partial\chi} + \frac{\omega^2}{V_A^2 B^2}|\nabla\psi|^2\right]\delta\eta_\psi = im\frac{\delta B_{||}}{B}, \tag{2}$$

$$D_{A\phi}\delta\eta_\phi \equiv \left[\frac{\partial}{\partial\chi}(|\nabla\phi|^2)\frac{\partial}{\partial\chi} + \frac{\omega^2}{V_A^2 B^2}|\nabla\phi|^2\right]\delta\eta_\phi = \frac{\partial}{\partial\psi}(\frac{\delta B_{||}}{B}), \tag{3}$$

and

$$\frac{\delta B_{||}}{B} = im\delta\eta_\psi - \frac{\partial}{\partial\psi}\delta\eta_\phi. \tag{4}$$

Here, $V_A = V_A(\chi, \psi)$ is the Alfvén velocity.

III. THEORETICAL APPROACH

Noting that perturbations become singular near the resonant field lines, we, therefore, adopt the approach of assuming the existence of singular solutions and then deriving the necessary compatibility conditions. This approach is thus similar to the one first employed by Pao [1975] in his investigations of the continuous spectrum in axisymmetric tokamaks.

Since we have azimuthal symmetry, let ψ_0 label the resonant layer. To systematically construct solutions near ψ_0, we shall denote $\epsilon \equiv |\psi - \psi_0|/|\psi_0| \ll 1$ as the smallness parameter and note the following orderings:

$$\left|r\nabla\psi\frac{\partial}{\partial\psi}\right| \sim O\left(\frac{1}{\epsilon}\right) \gg 1, \tag{5}$$

$$\left|r\nabla\chi\frac{\partial}{\partial\chi}\right| \sim |m| \sim O\left(\frac{\omega^2 r^2}{m^2 V_A^2}\right) \sim O(1), \tag{6}$$

and

$$|D_{A\psi}\delta\eta_\psi|/|\delta\eta_\psi| \sim O(\epsilon). \tag{7}$$

Equation (7) is, of course, motivated by the fact that FLR corresponds to the shear Alfvén continuum. Expanding the solutions asymptotically as

$$\begin{aligned}
\delta\eta_\psi &= \delta\eta_\psi^{(-1)} + \delta\eta_\psi^{(0)} + \delta\eta_\psi^{(1)} + ... \\
\delta\eta_\phi &= \delta\eta_\phi^{(0)} + \delta\eta_\phi^{(1)} + ... \\
\delta B_\parallel &= \delta B_\parallel^{(0)} + \delta B_\parallel^{(1)} + ...,
\end{aligned} \tag{8}$$

where the superscripts denote orderings in ϵ, we then have from Eq. (4)

$$im\delta\eta_\psi^{(-1)} = \frac{\partial}{\partial\psi}\delta\eta_\phi^{(0)}, \tag{9}$$

and from Eq. (2)

$$D_{A\psi}\delta\eta_\psi^{(-1)} = im\frac{\delta B_\parallel^{(0)}}{B}. \tag{10}$$

From Eq. (3) we deduce that $\delta B_\parallel^{(0)}$ is a constant on the short "ϵ" scale.

IV. FLR EIGENFUNCTIONS

To proceed further, let us define $\{a_n(\chi,\psi)\}$ as the FLR eigenfunctions with $\{\omega_n^2(\psi)\}$ the FLR eigenfrequencies; that is, for $n =$ integers,

$$\begin{aligned}
&D_{A\psi}(\psi,\chi,\omega_n^2)a_n(\chi,\psi) \\
&\equiv \left\{\frac{\partial}{\partial\chi}|\nabla\psi|^2\frac{\partial}{\partial\chi} + \frac{\omega_n^2(\psi)}{V_A^2B^2}|\nabla\psi|^2\right\}a_n(\chi,\psi) = 0,
\end{aligned} \tag{11}$$

with the standing-wave boundary conditions

$$a_n(\chi,\psi) = 0 \text{ at } \chi = \pm\chi_0(\psi), \tag{12}$$

where $\pm\chi_0(\psi)$ denote the two end points of the field line. $\{a_n\}$ and $\{\omega_n^2\}$ possess several important properties. First, one can readily show that the eigenfrequencies $\{\omega_n^2(\psi)\}$ are real. Furthermore, one can prove that the eigenfunctions $\{a_n\}$ are complete and orthogonal with the orthogonality condition given by

$$\int_{-\chi_0}^{\chi_0} a_n a_{n'} \frac{|\nabla\psi|^2}{V_A^2 B^2} d\chi = \delta_{nn'}. \tag{13}$$

Here, $\delta_{nn'}$ is the Kronecker delta.

Let us emphasize that, since $D_{A\psi}$ is a smooth function of ψ, both the eigenfunctions $\{a_n(\chi,\psi)\}$ and the eigenfrequencies $\{\omega_n^2(\psi)\}$ are *smooth* functions of ψ; i.e., they do not contain (radial) singularities in ψ. For each n = integer, $\omega_n^2(\psi)$ thus constitutes the shear Alfvén continuous spectrum. Note also that, for a dipole **B**, Eq. (11) corresponds to the toroidal equation and has been solved numerically by Cummings *et al.* [1969].

V. SINGULAR SOLUTIONS NEAR THE RESONANT FIELD LINES

Let the resonant field lines be located at $\psi = \psi_{on}$; that is, $\omega^2 = \omega_n^2(\psi_{on})$ and ω may be regarded as the excitation frequency. We can then express $\delta\eta_\psi^{(-1)}$ in terms of the FLR eigenfunctions as

$$\delta\eta_\psi^{(-1)}(\psi,\chi) = \sum_n \lambda_{n\psi}(y,\psi) a_n(\chi,\psi), \tag{14}$$

where $y = (\psi - \psi_{on})/\epsilon$ and ψ dependencies denote, respectively, fast and slow spatial variations in ψ. Note $\delta B_{\parallel}^{(0)}$ depends on ψ but not on y. Substituting Eq. (14) into Eq. (10) and applying the orthogonality condition, Eq. (13), we then have

$$[\omega^2 - \omega_n^2(\psi)]\lambda_{n\psi}\left\langle \frac{|\nabla\psi|^2 a_n^2}{V_A^2 B^2} \right\rangle = im\left\langle \frac{\delta B_{\parallel}^{(0)} a_n}{B} \right\rangle \equiv im\delta\hat{B}_n^{(0)}(\psi), \qquad (15)$$

where

$$\langle A \rangle \equiv \int_{-\chi_o}^{\chi_o} A\, d\chi. \qquad (16)$$

Noting that $\omega^2 = \omega_n^2(\psi_{on})$, Eq. (15) readily shows the singular nature of $\lambda_{n\psi}$; i.e.,

$$\lambda_{n\psi}(y, \psi) \propto \frac{\delta\hat{B}_n^{(0)}(\psi)}{(\omega_n^2)'(\psi - \psi_{on})} \propto \frac{\delta\hat{B}_n^{(0)}(\psi)}{y}. \qquad (17)$$

Meanwhile, taking

$$\delta\eta_\phi^{(0)} = \sum_n \lambda_{n\phi}(y, \psi) a_n(\chi, \psi), \qquad (18)$$

we then have, from Eq. (9),

$$\frac{\partial}{\partial y}\lambda_{n\phi} = im\lambda_{n\psi}, \qquad (19)$$

or, noting Eq. (15) or (17),

$$\lambda_{n\phi} = c_1(\psi) + c_2\delta\hat{B}_n^{(0)}(\psi) ln(\psi - \psi_{on}). \qquad (20)$$

Equations (17) and (20), thus, indicate that, with the properly defined FLR eigenfunctions $\{a_n(\chi, \psi)\}$, the nature of singular solutions near the resonant field lines is similar to that with perpendicular one-dimensional inhomogeneity [Chen and Hasegawa, 1974; Southwood, 1974]. Consequently, theoretical predictions on wave properties can also be similarly carried over. Finally, it is worth noting that, since the resonant layers are determined by $\omega^2 = \omega_n^2(\psi_{on})$ for $n =$ integers, there exist in general multiple resonant layers for a given excitation frequency ω. On the other hand, the strength of coupling to field line resonances depends on $\delta\hat{B}_n^{(0)}$ which, according to Eq. (15), corresponds to the projection of the compressional perturbation $\delta B_{\|}$ onto the FLR basis eigenfunctions $\{a_n(\chi,\psi)\}$ at the local resonant layer ψ_{on}. Thus, quantitatively, the coupling depends on the field-aligned structures of $\delta B_{\|}$; which, in turn, depend on how the compressional oscillations are excited; e.g., whether $\delta B_{\|}$ corresponds to compressional cavity eigenmodes or steady-state excitations due to some instability sources. These features, we note, are consistent with those observed in recent numerical calculations [Lee and Lysak, 1988].

VI. SUMMARY AND DISCUSSIONS

In summary, we have demonstrated analytically that field line resonances of standing hydromagnetic Alfvén waves do persist in a realistic dipole magnetic field. Furthermore, the theoretically predicted wave properties near the resonant field lines are similar to those of the previous one-dimensional model. The only qualitative difference is that, in the present analysis, the standing shear Alfvén waves are defined in terms of the FLR eigenfunctions $\{a_n(\chi,\psi)\}$ with corresponding eigenfrequencies $\{\omega_n^2(\psi)\}$; i.e., Eqs. (11) and (12).

We note that, while we have employed the dipole magnetic field model here, the present analyses and conclusions can be readily seen to remain valid in general magnetic field geometries. Implications due to effects such as finite plasma pressure, magnetic trapping, and azimuthal asymmetry are currently under investigation and the results will be reported in a future publication.

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