ISSN 0469-4732

INSTITUTE OF PLASMA PHYSICS

NAGOYA UNIVERSITY

Resistivity of Flame Plasma in an Electric Field

K. IKUTA

(Received — Dec. 22, 1988)

IPPJ- 897 Jan. 1989

RESEARCH REPORT

NAGOYA, JAPAN

Resistivity of Flame Plasma in an Electric Field

Kazunari IKUTA

Abstract

A generalized Ohm's law is obtained for a flame plasma in an electric field for the study of arc resistivity in an electromagnetic launcher (EML). The effective resistivity of flame plasma is reduced by the source, which suggests the injection of premixed combustible fuel into the arc plasma in EML in order to reduce the electron energy of the arc. The reduction of electron energy in the arc is desirable to minimize the damage of electrodes in EML.

Further communication about this report is to be sent to the Research Information Center, Institute of Plasma Physics, Nagoya University, Nagoya 464-01 Japan.

§1. Introduction

The acceleration of projectiles to hyper-velocities by electro-magnetic means is an exciting application of plasma physics. The favorable results of the railgun work done in Canberra have demonstrated the possibility of electromagnetically accelerating projectiles with masses in the range of interest of fusion experiments.[1] The advantages of the railgun are its simplicity and its demonstrated acceleration capability. It consists of two parallel rails connected to the terminals of a charged capacitor bank or uses inductive energy storage systems or explosive flux compressors as power sources. The current flowing in the rails generates a magnetic flux density between the rails. This magnetic field interacts with the current flowing in the armature which is at the base of the projectile. This armature is a dense plasma and the resulting Lorentz $\mathbf{J} \times \mathbf{B}$ force accelerates the armature together with the projectile along the rails.

However, as far as the velocities required for impact fusion[2] are concerned, it has three fundamental limitations. The first is that, as a railgun is lengthened, the resistance and inductance of the rails eventually absorb a dominant fraction of the energy. This results in a second limitation, that of velocity saturation, observed to begin at a length of about five meters in the Canberra tests. A third fundamental length limitation is the back-emf, which increases with velocity. An intermediate energy-storage inductor will ensure continued current flow even when the back-emf exceeds the output voltge of the homopolar generator or compulsator, but there is a practical limit to the voltage which can be withstood by the gap between the rails, thus creating a practical velocity limit depending on the scale size. A scheme for circumventing the limitations of the classical railgun is therefore needed. The use of a segmented railgun[3] eliminates the two length limitations imposed by rail resistance and inductance, but not the speed limitaiton of the back-emf. The use of an axisymmetric device, however, would eliminate all three limitations, since the strong magnetic field surrounding the plasma column in the electrodes would act as a magnetic insulator.

A novel kind of electro-magnetic launcher (EML) with axisymmetry, called the ablation mass driver (AMD), was designed by the use of a z-pinch between the cylindrical electrodes.[4] The axisymmetric configuration can overcome the possible disadvantages of the railgun given above although the force distribution on the base of a projectile will be uniform in the railgun so long as the current in the armature is not filamented. An important new feature of AMD is that the dense armature plasma can propagate with the thermal velocity of plasma along the null line of the azimuthal magnetic field in the direction opposite to the projectile acceleration. The reaction of this plasma ejection on the base of the projectile would be significantly different from the case of not having this ablation acceleration. Some of the experimental results on AMD are reported in a literature[5].

Even in the improved configuration of EML the damage of electrodes in the low velocity sections is quite severe. This damage comes from the use of the arc discharge between the electrodes because the discharge is sustained by an electric potential of about 50eV in the arc plasma. From the view point of the material science the particle energy of about 50eV is sufficiently high to give strong damage to the electrode materials. Some means of reducing the arc voltage must be discovered in order to use EML as a practical method of obtaining hyper-velocities for many applications.

A possible way of reducing the arc voltage will be the introduction of electron source in the armature plasma of EML. This will be realized by injecting premixed combastible fuel in the armature plasma. The injected fuel should be ignited by the armature plasma between the electrode of EML and the electrons together with the high pressure can be generated in the armature. This electron source can reduce the arc voltage in the armature. We can expect, therefore, that AMD with the injectors of combustible fuel can accelerates a projectile to hyper-velecities without any serious damages on the electrode surface.

The purpose of the present work is to consider the behavior of flame plasma in an electric field, E, together with the magnetic field, B. The flame plasma is created by the injection of premixed combustible fuel in the armature plasma. Since the components of the plasma must be quite complicated for the analysis of chemically reacting flame, we should introduce a model for the plasma in order to facilitate the consideration.

The most of fuels includes hydrogens as its components. This means that the fuel is the source of protons and electrons, although the flame must be sustained by oxygens. In this work the fuels and the oxygens are assumed to be a single static fluid which interacts with electrons and protons through collision process. The effects from the generated radicals is neglected in this work. Based on this assumption a generalized Ohm's law will be established in §2. We will see the resistivity of this flame plasma in electric field in §3.

§2. Generalized Ohm's Law

In order to develop a theory of flame plasma, the components of plasma other than electron and proton are assumed to be a background collisional fluid which creates electrons and protons. Let m_e be the mass of an electron, $-e$ its charge ; let n_e be the number of electrons per unit volume, $\mathbf{V}_e$ the local velocity of the electron gas, p_e its pressure and n_B the number of background neutrals per unit volume. The source density of electron is given by S. And ξ_e corresponds to the collision frequency between electrons and neutrals. Then the hydrodynamic equations for this electron fluid are

$$\frac{\partial n_e}{\partial t} + \nabla \cdot n_e \mathbf{V}_e = S \,, \tag{1}$$

$$n_e m_e \left\{ \frac{\partial \mathbf{V}_e}{\partial t} + (\mathbf{V}_e \cdot \nabla) \mathbf{V}_e \right\} = -\nabla p_e - n_e e\mathbf{E} - n_e e\mathbf{V}_e \times \mathbf{B} + \mathbf{P} - n_e n_B \xi_e \mathbf{V}_e . \tag{2}$$

We have omitted the gravitational force, but on the other hand we have added a term **P**, which represents the momentum transfer per unit time from the ion gas to the electron gas through collisions. Similarly the ion gas obeys the equations

$$\frac{\partial n_i}{\partial t} + \nabla \cdot n_i \mathbf{V}_i = S , \tag{3}$$

$$n_i m_i \left\{ \frac{\partial \mathbf{V}_i}{\partial t} + (\mathbf{V}_i \cdot \nabla) \mathbf{V}_i \right\} = -\nabla p_i + n_i e\mathbf{E} + n_i e\mathbf{V}_i \times \mathbf{B} - \mathbf{P} - n_i n_B \xi_i \mathbf{V}_i . \tag{4}$$

The charge of each proton is taken to be e. and ξ_i corresponds to the collision frequency between protons and neutrals.

These equations have to be supplemented with Maxwell's equations and with two equations of state connecting the pressures with the densities,

$$p_e = f_e(n_e) , \quad p_i = f_i(n_i) . \tag{5}$$

Finally one has to find **P** ; we shall assume

$$\mathbf{P} = \zeta n_e n_i (\mathbf{V}_i - \mathbf{V}_e) . \tag{6}$$

The constant ζ corresponds to the collision frequency between electrons and protons.

Before proceeding, we should firstly define the mass density, ρ, the current density, J, and the average velocity, V, of plasma ; i.e.

$$\rho = n_i m_i + n_e m_e$$

$$\mathbf{J} = \left(n_i \mathbf{V}_i - n_e \mathbf{V}_e\right) e.$$

and

$$\mathbf{V} = \frac{n_i m_i \mathbf{V}_i + n_e m_e \mathbf{V}_e}{n_i m_i + n_e m_e}. \tag{7}$$

A generalized ohm's law is obtained by calculating $\partial J / \partial t$ as in a text book[6]. This can be done by combining (1), (2), (3) and (4) with the approximations that quadratic terms in the velocities are neglected and that the ratio m_e / m_i is neglected with respect to unity together with the requirement of quasi-neutrality being $n_i = n_e$. And also the term P reduces to

$$\mathbf{P} = \frac{\zeta}{e} n_i \mathbf{J}. \tag{8}$$

The generalized ohm's law, finally, becomes

$$\frac{\partial J}{\partial t} = \frac{n_e e^2}{m_e}(\mathbf{E} + \mathbf{V} \times \mathbf{B}) - \left(\frac{n_e \zeta + n_B \xi_e}{m_e} - \frac{S}{n_e}\right)\mathbf{J}$$

$$+ \frac{e}{m_e}\left(\nabla p_e - \mathbf{J} \times \mathbf{B}\right)$$

$$+ \frac{e n_e n_B \xi_e}{m_e}\mathbf{V}. \tag{9}$$

In the same way we also have an equation for force balance, i.e.

$$\frac{\partial}{\partial t}(\rho \mathbf{V}) = -\nabla(p_i + p_e) + \mathbf{J} \times \mathbf{B}$$

$$- \left[n_e n_B (\xi_i + \xi_e) - m_i S \right] \mathbf{V}$$

$$+ \frac{n_B \xi_e}{e} \mathbf{J} , \tag{10}$$

and the continuity equation, i.e.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 2S . \tag{11}$$

In the generalized ohm's law (9), we see that the source ,S, has a tendency to reduce the effective resistivity and that the average velocity ,V, can contribute to the current ,J,. The similar role of the source ,S, is seen in the force balance equation, (10) The source ,S, reduces the frictional force from the back ground fluid.

§3. Resistivity of Flame Plasma

In a steady dense flame the generalized ohm's law is further reduced to

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} - \eta \mathbf{J} + \zeta \mathbf{V} = 0 . \tag{12}$$

where $$\eta \equiv \frac{\zeta + \xi_e (n_B / n_e)}{e^2} - \frac{m_e S}{n_e^2 e^2} .$$

and

$$\zeta \equiv \frac{n_B \xi_e}{e} .$$

The continuity equation becomes

$$\nabla \cdot (n_e \mathbf{V}) = \frac{2S}{m_i} . \tag{13}$$

It should be noted that the quantity ,η, reduces to the resistivity of the old ohm's law when $\xi_e = S = 0$.[6]

A simple but important solution is found from (12) and (13), if **V** = (V, 0, 0), **J** = (0, 0, J), **E** = (0, 0, E) and **B** = (0, B, 0) . Here a standard cylindrical coordinate system, (r, θ, z), is used.

Then, from (12), the ohm's law, now, is

$$E + VB - \eta J = 0 . \tag{14}$$

and, from (13), we have

$$V = \frac{S}{m_i n_e} r . \tag{15}$$

for a uniform source, S. We can conclude from (14) and (15) that the current ,J, can be enhanced by the source, S. Even if the electric field ,E, is absent at r $\neq$ 0, the current is induced by the source. Moreover, there is a possibility that the resistivity ,η, can vanish for a large S which is realizable but is dependent on the choice of the fuel.

§4. Discussions and Conclusions

In the real flame many radicals are produced by chemical reactions. Even in the simplest case of hydrogen-oxygen reaction the flame is composed of protons, ionized oxygen, OH radicals, electrons and others.[7] We can develop the many-component theory for any kind of flame, although it is rather complicated one.[6] Qualitatinely, a similar solution to the present demonstration should be obtained in any cases.

A decisively important result from this work is that a weak electric field can induce a large current in a flame plasma by making the resistivity ,η,small. This can be done by taking large S which will be realized by the choice of the fuel.

We therefore conclude that there is a possibility of reducing the voltage in an arc discharge with a finite length, provided that a properly premixed combustible fuel is injected into the current-carrying arc plasma. In other words, the energy of electrons sustaing the arc discharge can be made low enough in order not to give a fatal damage to the electrodes. A long life electrodes in EML should be realized by following this technique suggested by this work.

References

1) S.C. Rashleigh and R.A. Marshall : J. Appl. Phys. 49 (1978) 2540.

2) E.R. Harrison : Phys. Rev. Lett. 11 (1963) 535.

3) R.A. Muller, R.L. Garwin and B. Richter : Proc. the Impact Fusion Workshop, Los Alamos, LA8000-C (1979) p. 156.

4) K. Ikuta : Jpn.J. Appl. Phys. 24 (1985) 862.

5) K. Ikuta : Bulletin of the Japan Institute of Metals (in Japanese) 27 (1988) 360.

6) For example, N.G. Van Kampen and B.U. Felderhof : Theoretical methods in Plasma physics (North-Holland Publishing Company, Amsterdam, 1967) p.102.

7) I. Glassman : Combustion (Academic Press, INC., Orland, 1987) p. 51.