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V.V.Ezhela, O.P.Yushchenko

# HEISENBERG RISE OF TOTAL CROSS SECTIONS

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Abstract Ezhela V.V., Yushchenko O.P. Heisenberg Rise of the Total Cross Sections: IHEP Preprint 88-198. - Serpukhov, 1988. - p.16, figs.7, refs. 14.

It is shown that on the basis of the original idea of Heisenberg on the quasiclassical picture of extended particle interactions one can construct a satisfactory description of the total cross sections, elastic cross sections, elastic diffractive slopes and mean charged multiplicities in the cm energy range from 5 to 900 GeV, and produce reasonable extrapolations up to several tens of TeV.

Аннотация Ежела В.В., Ющенко О.П. Гайзенбергов рост полных сечений. Препринт ИФВЭ 88-198. - Серпухов, 1988, 16 с., 7 рис., библиогр.: 14.

Показано, что на основе идеи Гайзенберга о квазиклассическом характере взаимодействия протяженных частиц при высоких энергиях можно построить удовлетворительную процедуру описания полных сечений столкновений, упругих сечений, наклонов дифракционного конуса и средних заряженных множественностей в области энергий от 5 до 900 ГэВ в с.ц.м. и получить разумные экстраполяции в область десятков ТэВ.

1. Eikonal models of hadron interactions at small momenta transfer are rather attractive due to their simplicity and relative naturality of the basic assumptions. Successful description of experimental data in a wide energy range ($5. < \sqrt{s} < 1000.$ GeV)/1/, simplicity and conceptual clarity of eikonal models are the driving motives for the further development with the view to find connection with QCD and to reduce the number of input artefacts.

The eikonal parametrization is one of such artefacts. As a rule, the eikonal is chosen in the factorised form:

$$\Omega(s,b) = f(s)*\Phi(b) \tag{1}$$

The function $f(s)$ is determined by technical requirements of smoothness, monotonous behaviour and true s-asymptotics rather than by physical assumptions. At the same time the factorised eikonal assumption is not supported by the current experimental data /2/.

2. In this note we develop the method of constructing the physically motivated eikonal parameterisation starting with Heisenberg's idea on interaction of extended particles /3/.

It was first pointed out by Heisenberg, that it is possible to have the total inelastic cross section growing as $\ln^2 s$ by considering hadrons as extended objects. He introduced the assumption that protons interact only when

$$m_p^2\gamma_p^2 \int\rho_1(\vec{r})\rho_2(\vec{r}-\vec{b})d^2r \geq m_\pi^2, \tag{2}$$

where $\gamma$ is the Lorentz factor of the proton in the c.m.s., $m_p$ - proton's mass, $m_\pi$ - pion's mass, $\rho_i(\vec{r})$ - mass density space distribution.

The integral in (2) is estimated at the moment when protons are passing through the impact parameter plane. The cross section value is calculated as $\sigma = \pi b^2_{max}$, where $b_{max}$ is estimated from the equality in (2). It can be shown by very simple calculations that $\sigma_{tot} \approx \ln^2 s$ when $\rho_i(r) = \exp(r/r_0)$.

It was pointed out in /4/ that since the interaction time behaves as $1/E_{cm}$ and the energy of interaction from (2) is fixed the condition of applicability of quasiclassical approach

$$\Delta E * \Delta T > 1. \tag{3}$$

is violated.

3. If we consider the local condition of the interaction emergency then it is possible to remove the violation of (3), and, hence, criticism of /4/. One can then define the eikonal function in the realistic manner and to connect the asymptotics of total and elastic cross sections with the "static hadron" parameters.

The model proposed is based on two main assumptions. The first one states that any free hadron can be considered as an extended object with the energy-momentum tensor

$$T^{\mu\nu}(x,u,b) = \rho\{((x-b)u)^2-(x-b)^2\}u^\mu u^\nu, \tag{4}$$

where $u^\mu$ is 4-velocity of a hadron, and $b^\mu$ is impact 4-vector. In the hadron rest system $T^{\mu\nu}$ has $T^{00}$ component only and coincides with $\rho(r)$ - matter space distribution.

As energy-momentum for noninteracting subsystems is additive, let us write the energy-momentum tensor for two particles at large distance as

$$T^{\mu\nu}(x,u_+,u_-) = T^{\mu\nu}_+(x,u_+,b_+) + T^{\mu\nu}_-(x,u_-,b_-) \ , \tag{5}$$

where $T^{\mu\nu}_\pm$ are in form (4), and subscripts $\pm$ mark boundary conditions at $t \to \pm\infty$ (see (15) and (16)).

This tensor obeys the equation

$$\partial_\mu T^{\mu\nu} = 0, \tag{6}$$

and can be expressed in a canonical hydrodynamical form

$$T^{\mu\nu} = (\varepsilon+p)U^\mu U^\nu - pg^{\mu\nu}, \qquad U^\mu U_\mu = 1, \tag{7}$$

where $\varepsilon$ is the density of the rest energy, $p$ is the pressure density. From (4), (5) and (7) we have two equations

$$T^{\mu\nu}U_\mu = \varepsilon U^\nu; \quad U^\mu U_\mu = 1, \tag{8}$$

and

$$\rho_+ + \rho_- = \varepsilon + 3p. \tag{9}$$

Finally we have

$$\varepsilon = \{\sqrt{(\rho_+ + \rho_-)^2 + \rho_+\rho_-\Lambda(s,m_+^2,m_-^2)/(m_+^2 m_-^2)} + (\rho_+ + \rho_-)\}/2 \tag{10}$$

$$p = \{\sqrt{(\rho_+ + \rho_-)^2 + \rho_+\rho_-\Lambda(s,m_+^2,m_-^2)/(m_+^2 m_-^2)} - (\rho_+ + \rho_-)\}/6 \tag{11}$$

where

$$\Lambda(s,m_+^2,m_-^2) = s^2 + m_+^4 + m_-^4 - 2sm_+^2 - 2sm_-^2 - 2m_+^2 m_-^2$$

Now let us suppose that interaction can arise only in the region where the density of rest energy exceeds some critical value $\rho_c$. This is our second assumption. It seems quite realistic: it is impossible to accumulate an infinite amount of energy in the limited volume so that nothing would happen.

According to our second assumption, we can assert that in the regions where $\varepsilon > \rho_c$ expression (5) is not valid. We must add the part describing the interaction. At present we have no good ideas about such part but it is not the aim of this work to discuss this question.

The value of $\varepsilon$ is a function of space coordinates, time, impact parameter and energy:

$$\varepsilon = \varepsilon(s,\vec{r},t,\vec{b})$$

This value gives the maximum amount of colliding hadrons energy that can be converted into net rest mass of produced hadrons at the given impact parameter.

Now we proceed to the construction of the scattering amplitude. Let us define the classical average of available "collision rest energy" :

$$\bar{M}(s,b) = \frac{1}{T_{max}} \int_0^{T_{max}} dt \cdot \int \varepsilon\Theta(\varepsilon - r_c) d^3r \tag{12}$$

where $T_{max}$ is defined by

$$\varepsilon(s,\vec{\theta},T_{max},\vec{b}) = \rho_c.$$

We connect this value (12) with absorptive part of opaqueness used in eikonal models by relation

$$Disc_s[\Omega(s,b)] = \alpha(\bar{M}(s,b)/M_0)^{\beta}(T_{max}/T_0)^{\gamma}. \qquad (13)$$

Values $M_0$ = 1 GeV and $T_0$ = 1 1/GeV will be further omitted for brevity.

We have some motivation for expression (13): absorptive part must be larger for larger rest mass and interaction time. Parameters $\beta$ and $\gamma$ must be determined from the state equation of hadronic matter in interaction region.

To construct the real part we use dispersion relations at fixed $b$. Two subtraction constants have been put equal to zero.

$$\Omega = s^2\int_{s_0}^{\infty}\frac{\alpha\bar{M}(s',b)^{\beta}T(s',b)^{\gamma}}{s'^2(s'^2-s^2)}ds'$$

where $s_0$ is defined from $\varepsilon(s_0,\vec{\theta},0,\vec{b}) = \rho_c$. Then we put

$$A_c(s,b) = 1 - e^{i\Omega}$$

$$A_c(s,t) = i\cdot\int A_c(s,b)\cdot J_0(b\cdot\sqrt{-t})bdb \qquad (14)$$

4. For the comparison of the model proposed with experiment let us suppose

$$\rho_{\pm}(R_{\pm}^2) = A_{\pm}\cdot e^{-(R_{\pm}^2/r_0^2)} \qquad (15)$$

and

$$R_{\pm}^2 = (tP_{\pm}^z/m_{\pm}\pm zP_{\pm}^0/m_{\pm})^2 + x^2 + (y\pm b/2)^2 \qquad (16)$$

This parametrization gives a possibility to perform a large amount of calculations in analytical form and is very convinient to show the applicability of the model. Normalization constants $A_{\pm}$ are defined as follows:

$$m_i = A_i \int \rho_i(r_i(\vec{P}=0))d^3r \qquad (17)$$

where $m_i$ are the colliding hadron masses.

To evaluate the applicability of the model constructed let us turn to equation (3) which in our case gives:

$$\bar{M}(s,b)\cdot T_{max}(s,b) > 1.$$

It can be shown that starting from the energy of Serpukhov accelerator this equation is valid in a sufficiently wide b-range ($0 < b < 2r_0$, see Fig. 1).

To fit the data on total and elastic cross sections and slopes of diffractive cones we combine the classical and reggeon opaquenesses

$$\Omega(s,b) = \Omega_c(s,b) + \Omega_R^j(s,b)/s,$$

where the second term is reggeon amplitudes, index "j" marks the projectile type. $\Omega_R$ is conversion from

$$A_R^j(s,t)=a_j(s\cdot e^{i\pi/2})^{1+\alpha' t}\cdot e^{B_{j1}t}+c_j^{\pm}(s\cdot e^{i\pi/2})^{0.5+\alpha'_2 t}\cdot e^{B_{j2}t} \qquad (18)$$

Experimental values are calculated as follows:

$$\begin{aligned}
\sigma_{tot}(s) &= Im(A(s,t)) && (mb) \\
d\sigma/dt &= |A(s,t)|^2/16\pi && (mb/GeV^2) \\
B(s) &= d/dt[\ln(d\sigma/dt)]\Big|_{t=0} && (GeV^{-2}) \\
\sigma_{el}(s) &= (d\sigma/dt\Big|_{t=0})/B(s) + d_i^{\pm}/\sqrt{s} && (mb)
\end{aligned} \qquad (19)$$

This model can describe the interaction of different hadrons, but we treat proton-proton and antiproton-proton collisions only because of large amount of computer calculations. We shall return to the case of pions and kaons later.

To obtain parameters of the model we first fit the data on total cross sections and slopes. We have obtained reasonable data description $X^2/NDF=152/100 = 1.5$ (Fit.1). We found the value of parameter $\gamma$ (see 13) compatible with zero, so we put it equal to zero in further analysis. Then we add the data on elastic cross sections. It turns out that to get reasonable fit at low energies we need additional term (see 19). We fix the parameters of the model from Fit.1 and parameters $d^2$ have been obtained from the elastic cross sections data only (Fit.2). We have an overall description with $X^2/NDF = 330/170 = 2.0$.

Note that in Fit.1 we exclude the data from /13/ because these data do not affect the final parameters but give $X^2/NDF = 304/104$.

5. To describe data on multiplicities we use usual hydrodynamical approach /4/ in the interaction region at fixed b, where

$$n = \overline{V(s,b)^{1/4}M(s,b)^{3/4}}.$$

Mean charged multiplicity is obtained then by averaging with inelastic overlap function

$$\bar{n} = \frac{N\cdot\int_0^\infty (\tilde{n})\,\mathrm{Im}(1-e^{2i\Omega})\,b\,db}{\int_0^\infty \mathrm{Im}(1-e^{2i\Omega})\,b\,db} + 2 \qquad (20)$$

So, we have only one additional parameter N to describe total hadron multiplicities. Additional 2 in (20) stresses the fact that our model describes emmision of newly produced hadrons only, so we have $\bar{n} \to 0$ at $s \to (m_1+m_2)^2$

Data description is presented on figs.2-7. The curves have been drawn with the parameters from Tab.1. It should be noted, that we did not fit the data on real-to-imaginary part ratio. But still we have a reasonable description of the data. An attempt to fit these data did not give a better description. We believe, that we must use more realistic matter distribution with Ukawa's tale to have better fit of elastics.

6. The proposed model gives a good description of experimental data. As was expected, the eikonal (13) is a factorised one at asymptotically high energies only.

Note, that our model does not contradict the quark-gluon picture of hadron. There are some structure changes during hadron collisions and domain of high rest energy density arises. It is the domain where virtual many-parton configurations are produced. These configurations give then final hadrons. The domain of high rest energy density expands as the energy of collision rises, so the amount of partons and many-parton configurations rise also. Assumption that the critical density of the rest energy is the density when parton configuration can be materialized in produced hadrons, gives us the way to introduce the expanding region of inelastic interaction with growing energy.

Mean squared radius of hadron calculated with $\rho(r)$ used here are in agreement with the radii obtained from inelastic profile functions in /5/.

Moreover, our nucleon radius is in a good agreement with different calculations in potential models /6-8/ and with radii from the bag models/9/ :

| $r_p$ (fm) | Ref. |
|---|---|
| 0.37 | / 5/ |
| 0.4-0.77 | / 6/ |
| 0.46 | / 7/ |
| 0.48-0.62 | / 8/ |
| 0.4 | / 9/ |
| 0.40 | this ref. |

Here we considered spinless hadrons. Matter distribution has been chosen "ad hoc". We plan to find more physically motivated parametrizations for $\rho(r)$. If it leads to the vanishing of the first term in (18), this will mean that the plateau in the total cross section at Serpukhov energies and the subsequent growth is the manifestation of quasiclassical regime only and the asymptotical behaviour of the cross section is controlled by the static hadron's properties as follows from the Hiesenberg's idea.

The parameter $\rho_c$ expressed in geV/fm$^3$ is compatible with those typical for speculations on the phase transition into quark-gluon plasma:

| $\rho_c$ (Gev/fm$^3$) | Ref. |
|---|---|
| 5-10 | /10/ |
| 5 | /11/ |
| 4 | /12/ |
| 7 | this ref. |

Numerical data used have been extracted from COMPAS data bases.

We express our gratitude to our collegues from COMPAS group for fruitful discussions and help in data collection and evaluation. We are also much obliged to anonymous referee of "Yadernaya Fizika" for constructive criticism of the first version of this work.

Tab. 1 Parameters

| Classical | | Pomeron | |
|---|---|---|---|
| $r_0 = 0.320(1)$ fm | | $a_n = 7.10(6)$ | |
| $\lambda_c = 9.5(1)$ GeV/fm$^3$ | | $B_n = 3.02(5)$ GeV$^{-2}$ | |
| $\alpha = 0.082(8)$ | | $\alpha'_p = 0.382(8)$ GeV$^{-2}$ | |
| $\beta = 0.440(7)$ | | | |
| $N = 7.95(3)$ | | | |
| **Regge** | | | |
| $a_-$ | $a_+$ | $B_R$ [GeV$^{-2}$] | |
| 42.7(9) | 20.3(4) | 3.74(7) | |
| $\alpha'_R = 0.50(0.26)$ GeV$^{-2}$ | | | |
| **Correction (mb)** | | | |
| $\sigma_-$ | $\sigma_+$ | | |
| -8.4(4) | 2.2(1) | | |

Tab. 2 Extrapolations

| | FNAL | UNK | LHC | SSC |
|---|---|---|---|---|
| Ecm | 1800. | 6000. | 18000. | 40000. |
| $\sigma_{tot}$ | 79.0 | 98.3 | 121.0 | 138.9 |
| $\sigma_{el}$ | 19.7 | 29.0 | 41.1 | 51.1 |
| B | 16.6 | 17.5 | 18.6 | 19.7 |
| $n_{ch}$ | 46.3 | 71.0 | 107.7 | 145.6 |

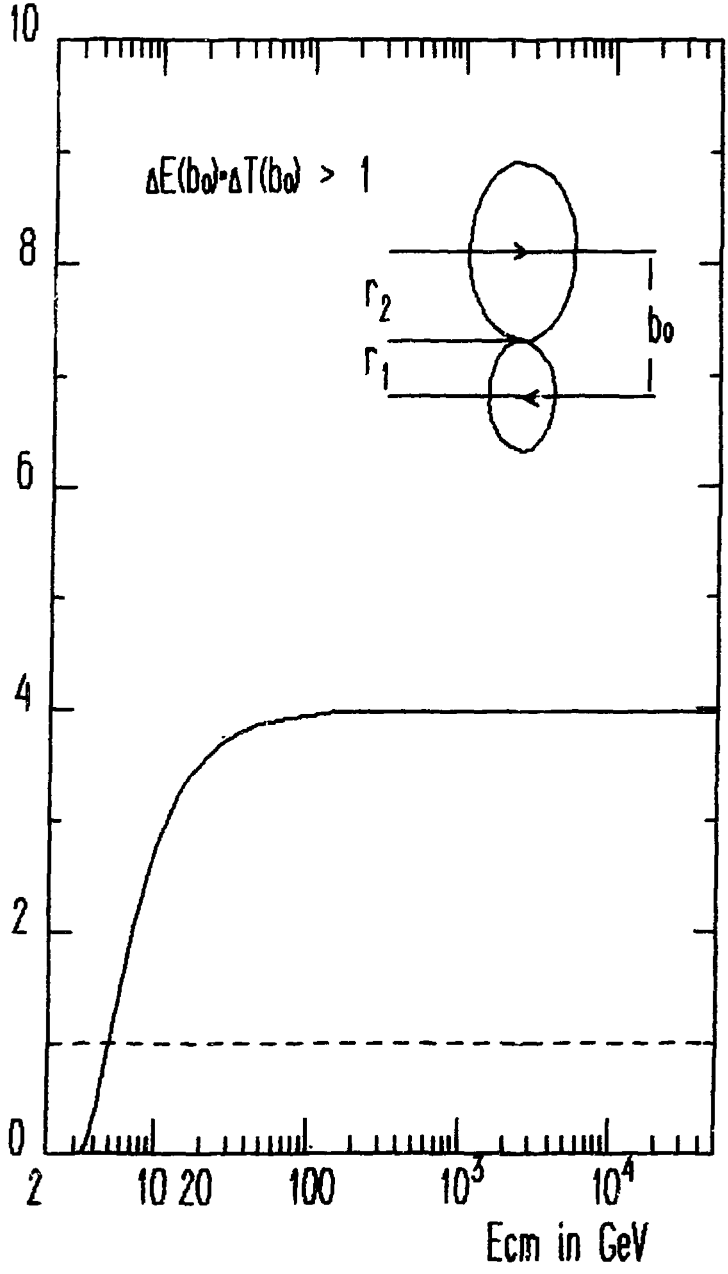



Fig.1

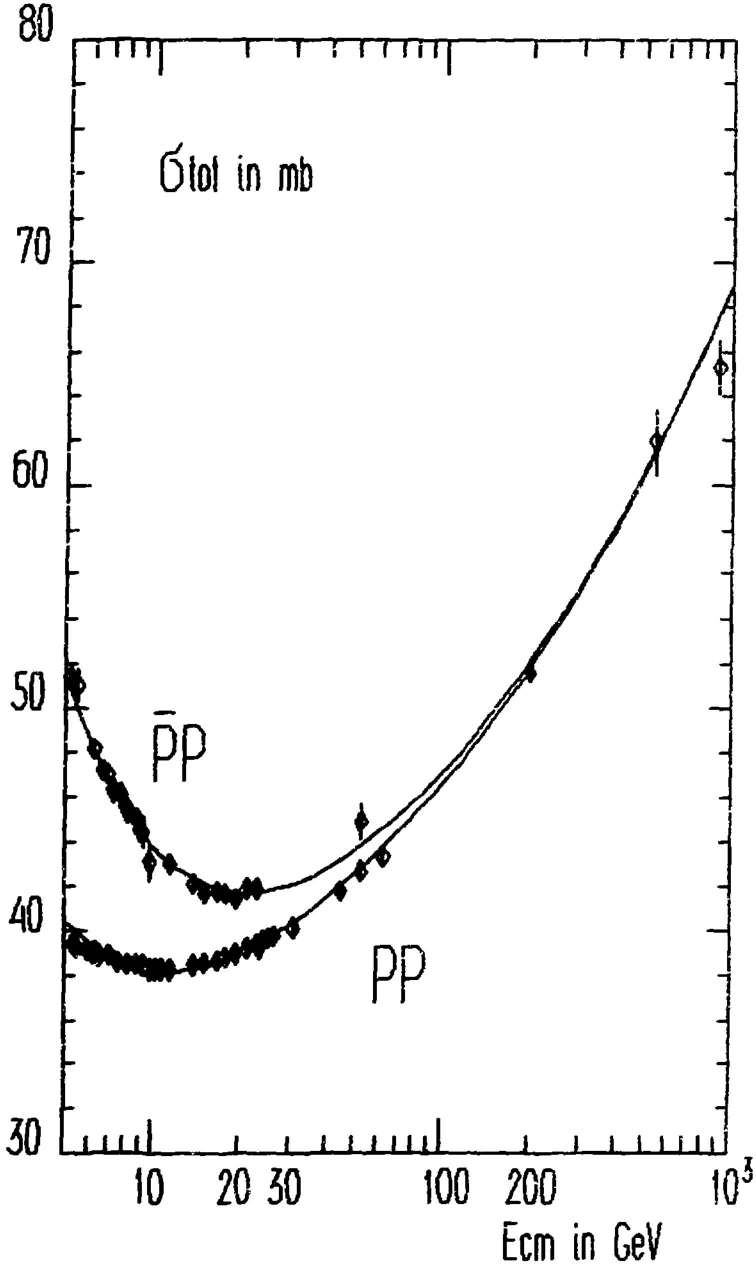



Fig.2

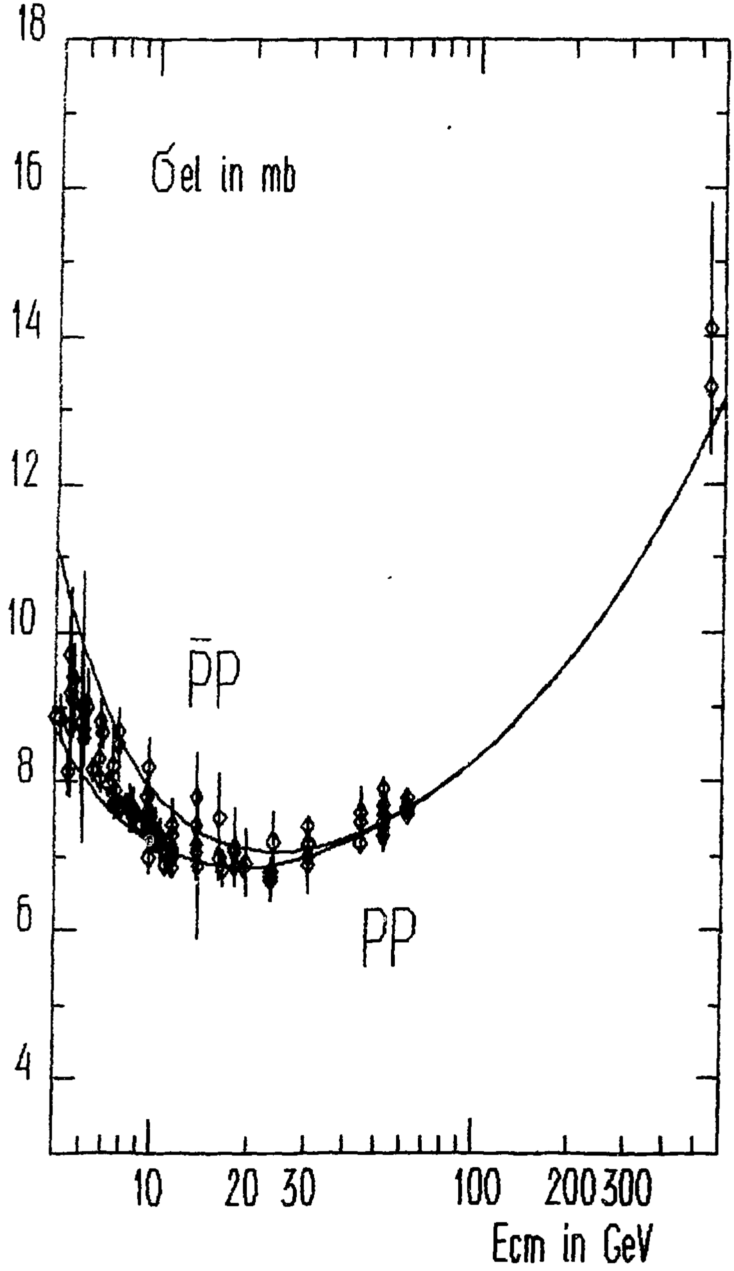
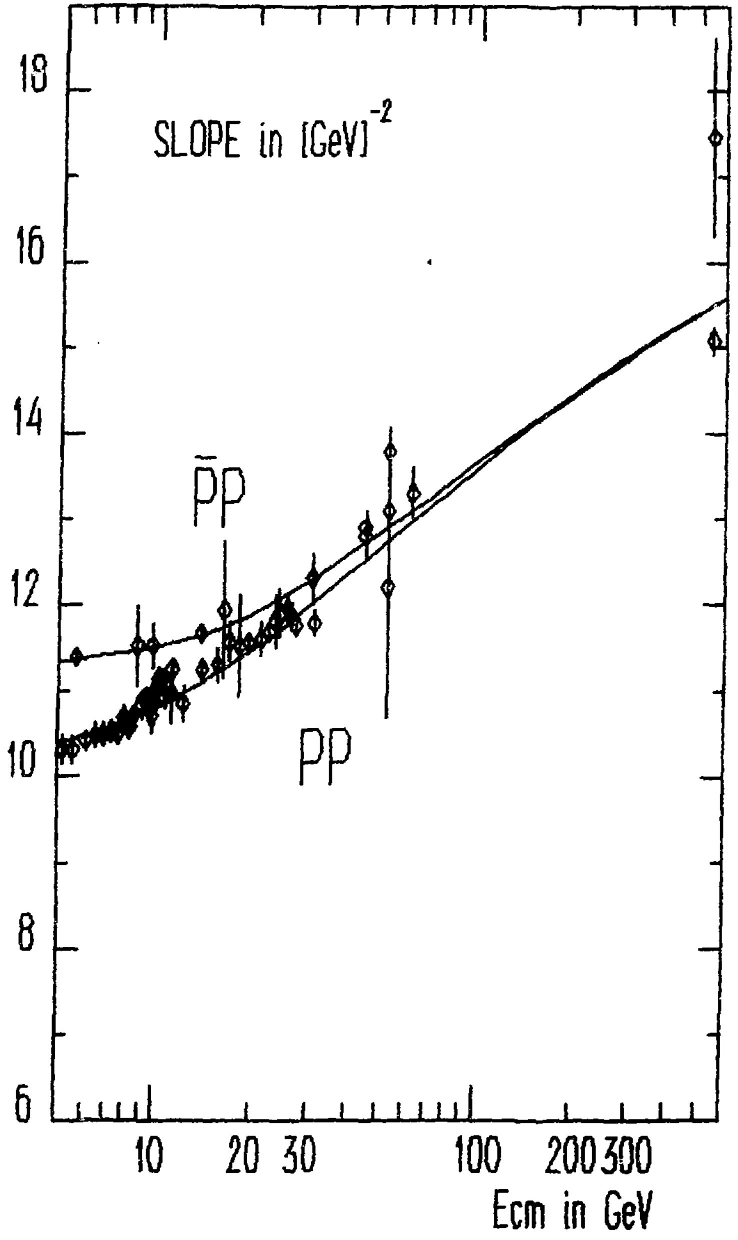



Fig.3



SLOPE in [GeV]$^{-2}$
18
16
14
12
10
8
6
$\bar{P}P$
PP
10
20
30
100
200
300
Ecm in GeV


Fig.4

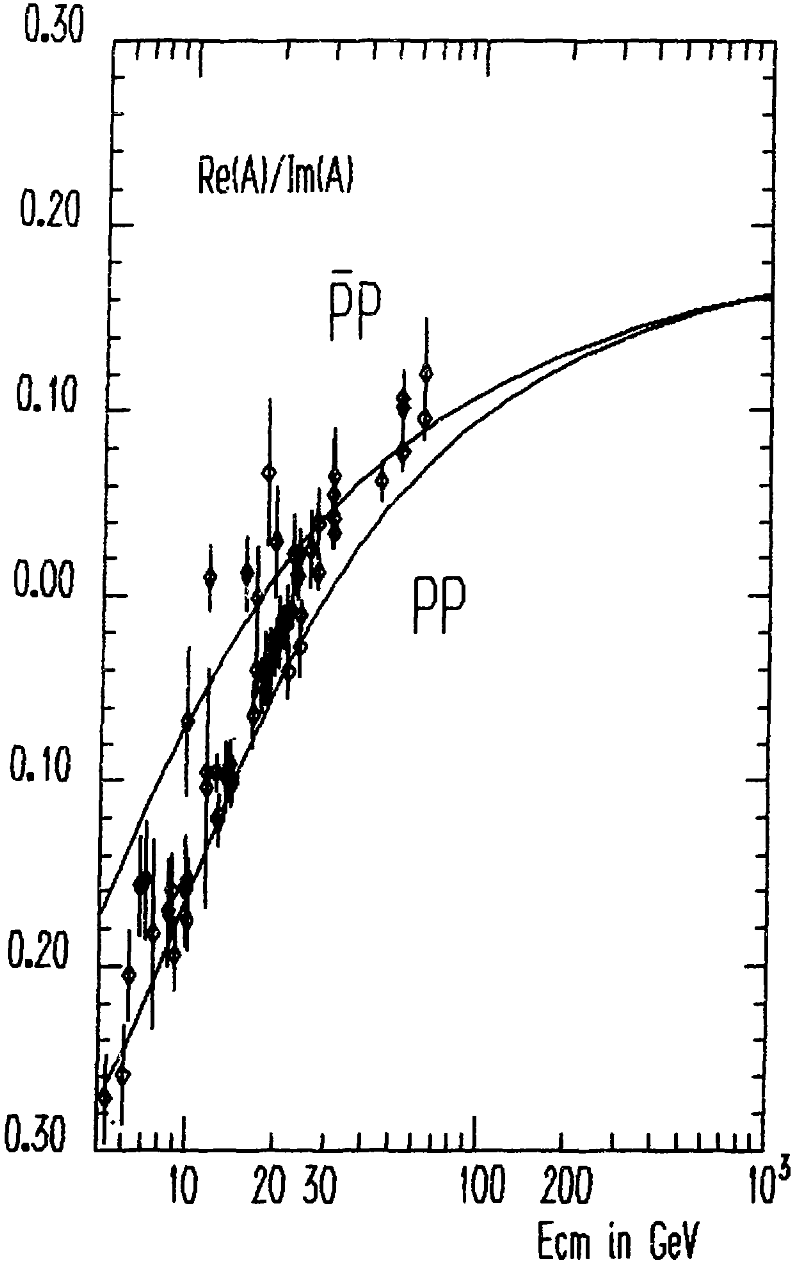

0.30
0.20
0.10
0.00
0.10
0.20
0.30
Re(A)/Im(A)
P̄P
PP
10
20 30
100
200
10^3
Ecm in GeV

Fig.5

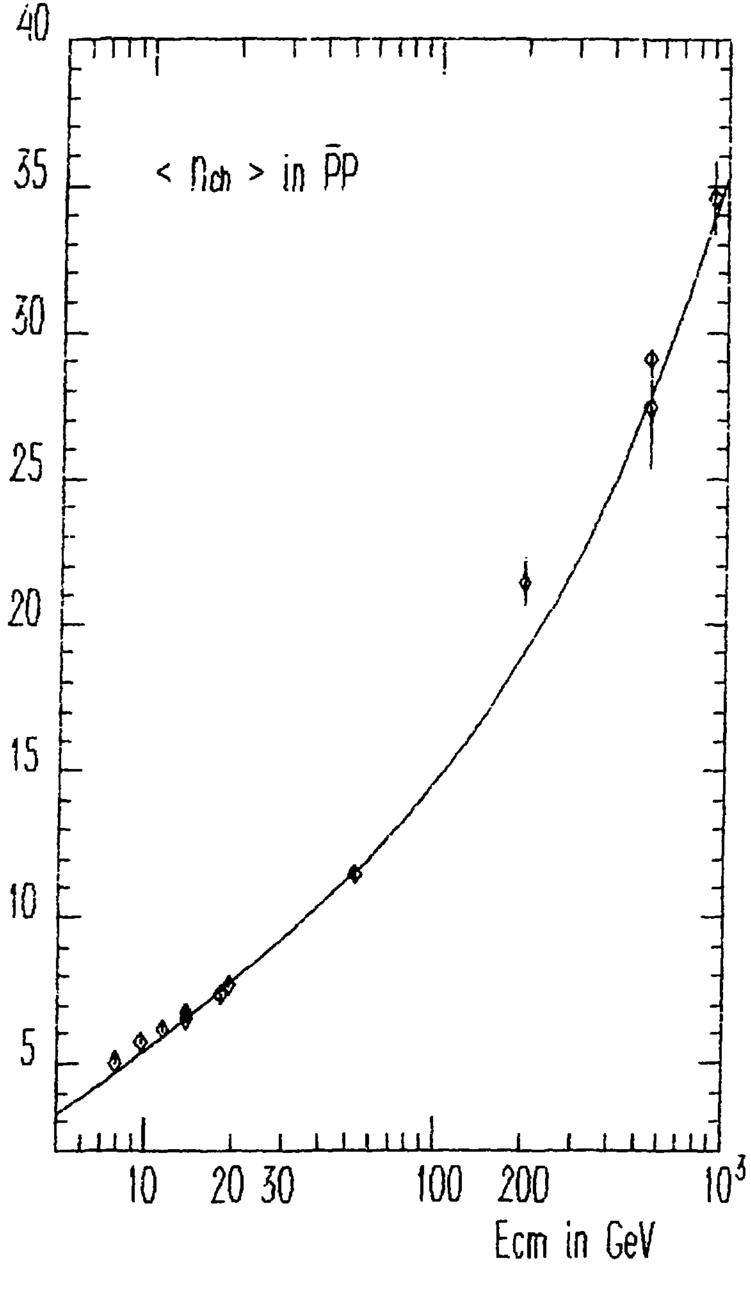



Fig.6

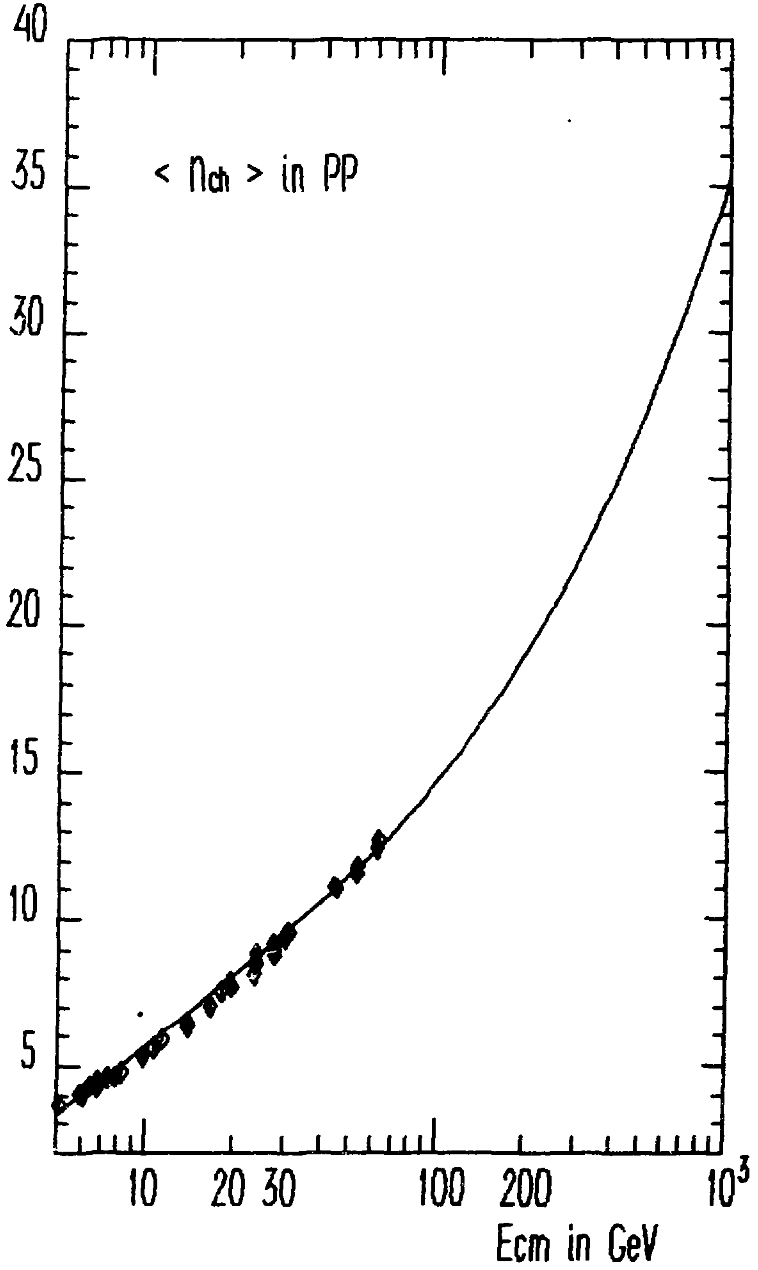



Fig.7

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В.В.Ежела, О.П.Ющенко
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