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Limitations on Tests of Quantum Flavour Dynamics from Quark Confinement*

dedicated to Deszö Kiss on the occasion of his 60th birthday

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1 Introduction

Dedicating an article to the birthday of a good old friend provides particular pleasure also to the author; it allows him to expand on those essential, but often suppressed, subtleties in the research process which are sometimes incorrectly called "philosophical". If the friend is an experimentalist (and the author a theorist), the spicy side of the mutual fertilization of theory and experiment is best suited for this exercise.

In what follows, I shall choose a very concrete example of our days, in which Dezsō Kiss as well as the author have been interested throughout their life: weak interactions which – by unification with electromagnetism – may now be called Quantum Flavour Dynamics (QFD). It provides one of the best theories of physics in general, if this is measured by the success of precisely predicting experimental results [1]. However, the basic Lagrangian is formulated for leptons and quarks; since the latter are not accessible to direct experimentation, predictions are model dependent and pose a limit for the predictive power of the theory. It is precisely this fact which I shall put in the center of interest in this article.

2 Quantum Flavour Dynamics and Leptonic Processes

The theory to be tested – QFD – is defined by the Lagrangian [1]

$$\mathcal{L}_{QFD} = \mathcal{L}_0 + \mathcal{L}_{GB} + \mathcal{L}_{em} + \mathcal{L}_{CC} + \mathcal{L}_{NC} + \mathcal{L}_H, \tag{1}$$

where $\mathcal{L}_0$ is the free Lagrangian including mass terms, $\mathcal{L}_{GB}$ is the gauge boson self-interaction, $\mathcal{L}_{em}$ is the electromagnetic interaction of QED, $\mathcal{L}_H$ is the Higgs Lagrangian responsible for spontaneous symmetry breaking; and – finally – the charged current and neutral current weak interactions of interest here are

$$\mathcal{L}_{CC} = \frac{g}{2\sqrt{2}}(J_\lambda^{CC} W^\lambda + h.c.) \tag{2}$$

$$\mathcal{L}_{NC} = \frac{g}{4\cos\Theta_W} J_\lambda^{NC} Z^\lambda, \tag{3}$$

where g is the triplet coupling constant related to the electric charge e and the weak mixing angle Θ_W by

$$e = g\sin\Theta_W. \tag{4}$$

The charged current of eq. (2) is given by

$$J_\lambda^{CC} = \sum_\ell \bar{\nu}_\ell \gamma_\lambda (1+\gamma_5)\ell + \sum_{q_1 q_2} \bar{q}_1 \gamma_\lambda (1+\gamma_5) U_{q_1 q_2} q_2, \tag{5}$$

where q_1 and q_2 refer to the mass eigenstates of the quarks, (u, c, t) and (d, s, b), respectively. $U_{q_1 q_2}$ is the weak quark mixing matrix of Kobayashi and Maskawa [2].

The neutral current of eq. (3) is given by

$$J_\lambda^{NC} = \sum_\ell [\bar\nu_\ell \gamma_\lambda (1+\gamma_5)\nu_\ell - \bar\ell \gamma_\lambda (C_V^\ell + \gamma_5)\ell] + \sum_{q_1} \bar q_1 \gamma_\lambda (C_V^1 + \gamma_5) q_1 - \sum_{q_2} \bar q_2 \gamma_\lambda (C_V^2 + \gamma_5) q_2 \quad (6)$$

with

$$\begin{aligned} C_V^\ell &= 1 - 4\sin^2\Theta_W \\ C_V^1 &= 1 - \frac{8}{3}\sin^2\Theta_W \quad (7) \\ C_V^2 &= 1 - \frac{4}{3}\sin^2\Theta_W . \end{aligned}$$

Having so defined our theory, we can now begin to confront it with experiment. (We shall not expand on the technical problem of renormalization schemes and higher order corrections unless it is imperative for the argument.) In the leptonic sector, where confinement plays no role, the theory is indeed remarkably powerful in predicting data. From the wealth of experimental information I shall only pick some very typical examples.

The weak mixing angle $\sin^2\Theta_W$ can be determined from purely leptonic processes, though accuracy is, of course, not as good as from overall fits. It may be worth noting that it is persistently about $(3 \pm 2)\%$ smaller than that derived from W, Z masses.

The relative minus sign (destructive W-Z interference) of the neutral current terms in eq. (6) is tested rather well [3] to be $-0.85 \pm 0.3 \pm 0.1$.

γ-Z interference is found at the predicted level by asymmetry measurements in $e^+e^- \to \ell^+\ell^-$ processes [4].

Curiously enough, it is one of the typical precision experiments in pure QED, the $g-2$ measurement of leptons, where limitations due to quark confinement become relevant. Virtual hadron contributions are negligibly small for the electron, but they are of the order of the α^3 correction for the muon [5]. For the τ-lepton, their contribution is, of course, even more prominent, but experimental information is not yet very good. Indeed, even the present value [6]

$$(g-2)_\tau = 0.0052 \pm 0.0048 \quad (8)$$

is impressive, though it does not yet test the Schwinger correction $\alpha/2\pi$.

The τ-lepton is in itself a rich source of information since it is heavy enough to decay also into hadrons. (The mode $\tau \to \nu_\tau \pi$ formed historically the first obstacle for it was not found in the original attempts.) it is not yet completely clear, whether persistent differences of 2 standard deviations are due to experimental problems or to as yet unaccounted physical effects. An example is the leptonic branching ratio $\tau \to \ell\bar\nu_\ell\nu_\tau$ which tends to be too low when compared to lepton universality [1,7]:

$$G_\tau / G_\mu = 0.963 \pm 0.023. \quad (9)$$

The τ-lepton provides more puzzles of this kind, but that leads into the next section.

3 Semi-Leptonic Processes

Let us begin with the τ-lepton. As early as 1985 it was noted by Gilman and Rhie [8] that the exclusive one-prong decay modes show a deficit of a few per cent with respect to the inclusive one-prong modes. In spite of quite some effort, this deficit still persits [9]. It is not clear where it can come from and this is, in turn, at least partially due to the fact that firm computations are only possible on the quark level whereas hadrons are, of course, in the observed exclusive decay modes. On the other hand, it stimulated theoretical ideas even if they turned out not to be the right answer [10].

Another problem, which exhibits the limitations due to quark confinement more explicitly, is the semi-leptonic branching ratio of beauty particles. One should expect a rather good prediction from a comparison of quark diagrams [1] with and without leptons in the final state [11]. But this gives about 17% which is reduced to about 15% by QCD-corrections. Further reduction to about 13% is possible by a mixed quark-hadron model [12]. But experiments give at most about 10% [13]. It may be that baryonic decay modes add a little, but certainly not more than $1-2\%$. This is clearly a case where the limitations due to quark confinement become painful. It will be interesting to observe the development of this gap [14] once experiments with top particles become possible.

4 Non-Leptonic Decay Processes

It is probable that the problem of the semi-leptonic branching ratio in B-decay stems from the denominator, i.e. the pure quark diagrams supposed to describe non-leptonic decays. This assumption is also supported from charmed particle decays, where the D^+ and D^0 lifetime ratio [1]

$$\tau(D^+)/\tau(D^0) = 2.28 \pm 0.12 \tag{10}$$

agrees well with their semi-leptonic branching ratio [1]

$$\frac{\Gamma(D^+ \to e^+\nu_e X^0)}{\Gamma(D^0 \to e^+\nu_e X^-)} = 2.27 \pm 0.44 \tag{11}$$

implying that the difference in lifetime has its origin in the hadronic sector.

Since we do not yet agree on how the $\Delta I = 1/2$ rule can be explained, there is little hope to gain unanimous insight into purely hadronic processes.

5 Radiative Decay Processes

Weak radiative decays form in a way a bridge between semi-leptonic and hadronic processes. The matrix element to be computed is a Lorentz vector and can be decomposed into form factor like parts. Thus it was expected that predictions for simple cases should not be too far off experimental results. That, however, led to a disappointment.

Let us consider the decay

$$\Sigma^+ \to p + \gamma. \tag{12}$$

Its matrix element can be written as (for notation see Ref. 1)

$$T_{fi} = \frac{Ge}{\sqrt{2}} \bar{u}_p (a + b\gamma_5) i\sigma^{\mu\nu} k_\nu u_\Sigma \cdot \varepsilon_\mu(k) \tag{13}$$

leading to the decay rate

$$\Gamma(\Sigma^+ \to p\gamma) = \frac{G^2\alpha}{4}(|a|^2 + |b|^2)\left(\frac{m_\Sigma^2 - m_p^2}{m_\Sigma}\right)^3 \tag{14}$$

and to the angular spectrum with respect to the axis of polarization P_Σ

$$\frac{d\Gamma}{d\cos\vartheta} = \frac{\Gamma}{2}(1 + \alpha_\gamma P_\Sigma \cos\vartheta) \tag{15}$$

with the asymmetry parameter

$$\alpha_\gamma = \frac{2\,\mathrm{Re}(ab^*)}{|a|^2 + |b|^2}. \tag{16}$$

As early as 1964, it was predicted from flavour $SU(3)$ that α_γ should vanish [15]. $SU(3)$-breaking effects on the quark level give

$$\alpha_\gamma = \frac{m_s^2 - m_d^2}{m_s^2 + m_d^2} \tag{17}$$

which may be as large as 1/2 but note that it is positive!

The surprise came from more recent precise experiments which gave [16]

$$\alpha_\gamma = -0.86 \pm 0.13 \pm 0.04. \tag{18}$$

This led B.L. Roberts to say [17] that "weak radiative decays are of interest because they represent the last remaining low q^2 frontier of weak interaction physics. They are not well understood either theoretically or experimentally, and it is not yet clear whether they fit nicely into the standard model or not".

Needless to say that theoretical creativity was stimulated by this discrepancy. The whealth of ideas and papers is nicely summarized in Ref. 17. It is surprising, comforting and exciting at the same time that such interesting questions came up in a field seemingly covered and almost closed by progress in our research field.

With this happy message I would like to end my article dedicated to Dezső Kiss at his 60th birthday.

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