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# HIGGS BOSON HUNTING*

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## ABSTRACT

This is the summary report of the Higgs Boson Working Group. We discuss a variety of search techniques for a Higgs boson which is lighter than the $Z$. The processes $K \to \pi H$, $\eta' \to \eta H$, $\Upsilon \to H\gamma$ and $e^+e^- \to ZH$ are examined with particular attention paid to theoretical uncertainties in the calculations. We also briefly examine new features of Higgs phenomenology in a model which contains Higgs triplets as well as the usual doublet of scalar fields.

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## 1. INTRODUCTION

The search for the Higgs boson is a fundamental quest of current and future accelerators. The Standard Model of weak and electromagnetic interactions is incomplete without the Higgs boson or some similar object whose function is to endow the $W$ and $Z$ gauge bosons with their observed masses. The Standard Model, however, gives no hint as to the mass of the Higgs boson and so it is important to search for it experimentally over as many mass regions as possible. Many experiments are sensitive to Higgs boson production, but the only firm limit which seems completely free from all theoretical and experimental uncertainties is from nuclear physics, $m_H \gtrsim 15$ MeV.[1]

The Higgs Boson Working Group considered various techniques which are useful in the search for a Higgs boson with a mass less than $M_Z$. We begin by discussing light Higgs bosons which can be seen in the decays $K \to \pi H$ and $\eta' \to \eta H$. The derivations of the Higgs boson couplings to pseudoscalar mesons are reviewed and the theoretical uncertainties emphasized. In Section 3, we consider limits on the Higgs boson from the process $\Upsilon \to H\gamma$. Here we pay particular attention to theoretical ambiguities due to large QCD radiative corrections, bound state effects, and relativistic corrections. In Section 4, we turn our focus to a somewhat heavier Higgs boson which will be the object of attention at LEP and SLC. We have examined the contribution of a heavy top quark to the process, $e^+e^- \to Z \to ZH$. Here, we can take one $Z$ to be on-shell and one $Z$ to be off-shell. This permits us to discuss both the decay $Z \to H\mu^+\mu^-$ (relevant for LEP-I and SLC) and the associated production $e^+e^- \to ZH$ (relevant for LEP-II).

In Sections 2, 3, and 4, we consider only a Standard Model Higgs boson. It is important, however, to consider variations of the Standard Model of electroweak interactions, and to determine how they can be distinguished from the minimal version with only one physical Higgs boson. The most thoroughly studied alternatives are models which simply replicate the number of Higgs doublets.[2] In Section 5, we describe a nonminimal model which contains Higgs triplets as well as the usual Higgs doublet. By a clever choice of the particle content and symmetry constraints on the Higgs potential, one can guarantee that the tree-level relation $\rho \equiv M_W^2/M_Z^2\cos^2\theta_W = 1$ is satisfied. We focus on the phenomenological differences between this model and models which contain only Higgs doublet fields. Finally, we briefly summarize some other issues taken up by the Higgs Boson Working Group in Section 6.

## 2. HIGGS PRODUCTION IN $K \to \pi H$ and $\eta' \to \eta H$

We begin by discussing the search for a light Higgs boson in pseudoscalar meson decays. Rare kaon decays at Brookhaven National Laboratory are approaching a sensitivity at which evidence of a light Higgs boson could appear. Precision searches for the process $\eta' \to \eta H$ also exist. As we will see in this section, the problem is not a lack of experimental data, but rather the difficulty in obtaining reliable theoretical calculations.

### 2.1. Low energy theorems for Higgs bosons

The couplings of mesons to a light Higgs boson can be found using a low energy theorem which has been derived by Voloshin and others.[3,4] The derivation of this theorem is discussed in detail elsewhere in these proceedings[5] and we merely summarize the results here.

The effect of a zero-momentum (*i.e.* constant) Higgs field, $H$, is to redefine all mass parameters of the theory,

$$m_i \to m_i\left(1+\frac{H}{v}\right) \tag{1}$$

and to rescale $\alpha_s$,

$$\delta\alpha_s = \left(\frac{\alpha_s^2 N_H}{3\pi}\right)\left(\frac{H}{v}\right) \tag{2}$$

where $\langle H\rangle = v/\sqrt{2} = (\sqrt{2}G_F)^{-1/2}$ and $N_H$ is the number of heavy quarks in the theory with $m_q \gg m_H$. This rescaling of $\alpha_s$ is the result of the gluon-gluon-Higgs triangle diagram with heavy quarks running around the loop. Using eqs. (1) and (2), the coupling of a light Higgs boson to the low energy Lagrangian can be found,

$$\mathcal{L}(H\neq 0) = \left[\left.\frac{\delta\mathcal{L}}{\delta\alpha_s}\right|_{H=0}\delta\alpha_s + \left.\frac{\delta\mathcal{L}}{\delta m_i}\right|_{H=0}\delta m_i\right] H\,. \tag{3}$$

Hence to find the couplings of a light Higgs boson, one needs only to know the scaling of the parameters of the Lagrangian with $\alpha_s$. It is sometimes more convenient to express $\alpha_s$ in terms of $\Lambda_{QCD}$. In the one-loop approximation,

$$\Lambda_{QCD} \simeq \mu \exp\left(\frac{-2\pi}{b\alpha_s(\mu)}\right), \tag{4}$$

where $b = 11 - 2n_L/3$ (for $n_L$ quarks which are light compared to the Higgs mass) is the coefficient of the one-loop QCD $\beta$-function, and $\mu$ is an arbitrary renormaliztion scale. (The renormalization group equations guarantee that physical quantities are $\mu$-independent.) Thus, if the $\Lambda_{QCD}$ and mass dependence of $\mathcal{L}$ is known, then the low-energy Higgs interactions can be determined. Technically speaking, the Higgs interactions obtained in this manner are valid only for a zero-momentum (constant)



Higgs field. We presume that these results can be used to describe the interaction of a light Higgs boson at low energy, since the extrapolation from zero momentum is not great. However, it is clear that the Lagrangian derived in this manner cannot be used to calculate loop effects where the momentum carried by the Higgs boson can be large.

The meson decay rates can most reliably be calculated using chiral perturbation theory.[6] Keeping only the terms with the lowest number of derivatives, the chiral Lagrangian is

$$\begin{aligned}\mathcal{L} = {}& \tfrac{1}{4}f_\pi^2\,\mathrm{Tr}\,\partial^\mu\Sigma^\dagger\partial_\mu\Sigma + \tfrac{1}{2}f_\pi^2\,\mathrm{Tr}\left[\mu M\Sigma^\dagger + \mathrm{h.c.}\right]\\ &+\tfrac{1}{4}f_\pi^2\left[\lambda\,\mathrm{Tr}\,h\partial^\mu\Sigma^\dagger\partial_\mu\Sigma + \mathrm{h.c.}\right]\\ &+\tfrac{1}{2}f_\pi^2\left[A\,\mathrm{Tr}\,h\mu(M\Sigma^\dagger + \Sigma M) + \mathrm{h.c.}\right],\end{aligned} \tag{5}$$

where $f_\pi = 93$ MeV, $\Sigma \equiv \exp(2i\Pi^a T^a/f_\pi)$, $\Pi^a$ is the pseudoscalar meson octet, $T^a$ are $SU(3)$ generators, $M = \mathrm{diag}(m_u, m_d, m_s)$ is the diagonal quark mass matrix, and $h$ picks out the $\Delta S = 1, \Delta I = 1/2$ direction. (We have neglected the $\Delta I = 3/2$ contribution since it is small for the processes we study.) The parameter $\lambda$ is measured in non-leptonic $K$-decays and is approximately[7] $|\lambda| \simeq 3.2\times 10^{-7}$. Using dimensional analysis to find the scaling of the parameters of the Lagrangian with $\Lambda_{QCD}$ and the masses and then using eqs. (3)-(5), we can easily find the coupling of a light Higgs to the chiral Lagrangian.[5] We will list the relevant terms in the next subsections.

### 2.2. $K \to \pi H$

The decay $K \to \pi H$ has been used in the past to restrict the existence of a light Higgs boson. There are three classes of contributions to this decay and we will discuss them each in turn in this subsection.

The first contribution arises from the direct decay $s \to dH$ which occurs at one-loop and results in an effective Lagrangian:

$$\mathcal{L}_{sdH} = \frac{G_F^{3/2}}{2^{1/4}}\frac{3}{16\pi^2}\sum_i m_i^2 V_{id}^* V_{is}[m_s\bar{d}(1+\gamma_5)s + m_d\bar{d}(1-\gamma_5)s]H + \mathrm{h.c.}, \tag{6}$$

where $V_{ij}$ are the elements of the Kobayashi-Maskawa (KM) matrix, $m_i$ are quark masses, and the sum is taken over up-type quarks $(u, c, t)$. This can be easily converted to an effective $K\pi H$ Lagrangian[8] as described elsewhere in these

Proceedings.[5] The corresponding amplitude is given by:

$$\mathcal{M}_1(K^- \to \pi^- + H) = \frac{G_F^{3/2}}{2^{1/4}} \frac{3}{16\pi^2} m_K^2 \sum_i m_i^2 V_{id}^* V_{is} . \tag{7}$$

There are also contributions to the matrix element which correspond to four-quark operators which are generated by non-spectator diagrams such as $s + u \to d + u + H$, in which the Higgs boson is emitted directly from the exchanged $W$ boson. Using the low-energy theorems, the chiral Lagrangian gives a $K\pi H$ coupling,

$$\mathcal{L}_{K\pi H} = -\lambda \frac{H}{v}\left(1 - \frac{2N_H}{3b}\right)\partial^\mu \pi^+ \partial_\mu K^- + \text{h.c.} \tag{8}$$

This yields the result:

$$\mathcal{M}_2(K^- \to \pi^- + H) = \frac{-\lambda}{2v}\left(1 - \frac{2N_H}{3b}\right)(m_K^2 + m_\pi^2 - m_H^2), \tag{9}$$

where $b$ is the coefficient of the one-loop QCD $\beta$ function for $n_L$ light quarks, $b = 11 - 2n_L/3$. For $n_L = N_H = 3$, we find $1 - 2N_H/(3b) = 7/9$. Since we use the experimentally determined value of $\lambda$, we have automatically included the effects of the $\Delta I = \frac{1}{2}$ enhancement seen in kaon decays. The value of $\lambda$ deduced from the data is $|\lambda| = 3.2 \times 10^{-7}$; however the sign of $\lambda$ is not determined from the data. If one matches the chiral Lagrangian to the $\Delta S = 1$ weak Lagrangian in the full theory at 1 GeV, one finds that $\lambda$ is *negative* in the vacuum insertion approximation. To demonstrate this in the simplest possible way, simply consider the effective four-quark operator arising from $s + u \to d + u + H$, where the $H$ is emitted from a $t$-channel $W$ propagator. Sandwiching this operator between a kaon and a pion, and comparing with eq. (9) (with $N_H = 0$), we make the identification:

$$\lambda \to -2\sqrt{2} G_F f_K f_\pi V_{ud}^* V_{us} . \tag{10}$$

Of course, such an estimate is far too naive. It neglects the gluonic corrections which introduce other four-quark operators which contribute to non-leptonic processes. More importantly, we have performed this matching at 1 GeV; it is highly nontrivial to connect this with physics at a momentum scale of a few times $m_\pi$ which is relevant for $K$ decay. Nevertheless, more sophisticated arguments seem to suggest that the above identification correctly determines the sign of $\lambda$, namely, $\lambda$ is negative in the usual convention where $V_{ud}^* V_{us} \equiv \cos\theta_c \sin\theta_c$ is positive.



Finally, there is a contribution which cannot be found in the quark model. If we expand out eq. (5) in terms of the charged pion and kaon fields, we note that there are mixed kinetic energy and mass terms. We can obtain a canonical Lagrangian by diagonalizing the kinetic energy terms, rescaling the fields (by wave function renormalization) and finally diagonalizing the mass matrix. These operations result in the appearance of an extra term in the Lagrangian:

$$\mathcal{L}'_{K\pi H} = A m_K^2 \frac{H}{v}\left(1 - \frac{4N_H}{3b}\right) K^- \pi^+ + \text{h.c.} \tag{11}$$

This generates a contribution to the decay $K \to \pi H$ of the form

$$\mathcal{M}_3(K^- \to \pi^- H) = \frac{A m_K^2}{v}\left(1 - \frac{4N_H}{3b}\right). \tag{12}$$

Since the value of the parameter $A$ can only be determined by measuring a process with a light Higgs boson, it follows that we cannot make a definitive theoretical prediction about the decay rate. When $\mathcal{M}_1$, $\mathcal{M}_2$, and $\mathcal{M}_3$ are added, the possibility of large cancellations between the various contributions exists. However, it was recently pointed out in ref. 9 that there is a prediction for a *lower* limit for $BR(K^\pm \to \pi^\pm H)$ due to the fact that the amplitude for $K^\pm \to \pi^\pm H$ is complex. This suggestion would make sense only if the dominant source of the imaginary part is due to the heavy quark mixing angles which appear *in* $\mathcal{M}_1$. In fact, we believe that this is correct, due to the following argument. The two-quark operator is special in that the amplitude contains an explicit factor of the quark mass squared [see eq. (7)]. The $t$-quark contribution in the sum is greatly enhanced, which permits Im $\mathcal{M}_1$/Re $\mathcal{M}_1$ to be of $\mathcal{O}(1)$. On the other hand, simple dimensional analysis suggests that the contributions to the amplitude from the four-quark operators can at best approach a constant in the large $m_t$ limit. It follows that the ratio of the imaginary to the real part of the four-quark amplitudes can be no larger than (roughly) Im $V_{td}^* V_{ts}/(V_{ud} V_{us})$ which is of order $10^{-3}$. Thus, it follows that:

$$\begin{aligned} \text{Im } \mathcal{M}_2 &\sim \mathcal{O}(10^{-3}) \text{ Re } \mathcal{M}_2 , \\ \text{Im } \mathcal{M}_3 &\sim \mathcal{O}(10^{-3}) \text{ Re } \mathcal{M}_3 . \end{aligned} \tag{13}$$

Since we expect the real parts of $\mathcal{M}_1$, $\mathcal{M}_2$ and $\mathcal{M}_3$ to be roughly of the same order of magnitude, we conclude that Im $\mathcal{M}_1$ should be the dominant source of the imaginary part of the total amplitude.*

* The above argument implies that Im $\lambda \ll$ Re $\lambda$ and Im $A \ll$ Re $A$. We can estimate that Im $\lambda \sim 10^{-4}$ Re $\lambda$ based on the experimentally measured value of $\epsilon'/\epsilon$.

We therefore can obtain a lower limit on $\mathcal{M}(K^\pm \to \pi^\pm H)$ by setting the real part of the total amplitude to zero (which would happen for some choice of the parameter $A$). It is convenient to use the Wolfenstein parameterization[10] of the Kobayashi-Maskawa matrix, where:

$$
\begin{aligned}
V_{ud}^* V_{us} &\simeq -V_{cd}^* V_{cs} \simeq \sin\theta_c \cos\theta_c \\
V_{td}^* V_{ts} &\simeq -A_w^2 \sin^5\theta_c (1-\rho+i\eta)\,.
\end{aligned}
\tag{14}
$$

Experimentally, $\sin\theta_c \simeq 0.22$, the parameter $A_w$ is close to unity ($A_w = 1.05\pm0.17$ according to ref. 11), and $\rho \lesssim 0$ (based on the observed $B$–$\overline{B}$ mixing). In addition, ref. 9 has determined that $\eta = 0.57 \pm 0.19$, based on the recent measurement of $\epsilon'/\epsilon$. Thus,

$$
|\mathrm{Im}\,\mathcal{M}(K^\pm \to \pi^\pm H)| \gtrsim \frac{G_F^{3/2}}{2^{1/4}} \frac{3m_K^2 m_t^2 \eta A_w^2 \sin^5\theta_c}{16\pi^2} \gtrsim 1.5 \times 10^{-10}\ \mathrm{GeV}\,. \tag{15}
$$

We shall take $m_t \approx 80$ GeV, in order that the Linde-Weinberg bound permit a very light Standard Model Higgs boson.[2] Using $\Gamma(K \to \pi H) = B_H\,|\mathcal{M}|^2/(16\pi m_K)$, where $B_H = 2p_H/m_K$ (measured in the kaon rest frame), and normalizing to the total decay rate, $\Gamma(K^\pm) = 5.32 \times 10^{-17}$ GeV, we find:

$$
BR(K^\pm \to \pi^\pm H) \gtrsim 1.7 \times 10^{-5} B_H\,. \tag{16}
$$

We emphasize that this limit is only valid if $\mathrm{Im}\,A \ll \mathrm{Re}\,A$ as argued above.

Let us briefly turn to the decay $K_L^0 \to \pi^0 H$. The above analysis can be repeated, with only minor changes. The most important difference is that $\mathcal{M}_1$ is modified to:

$$
\mathcal{M}_1(K_L^0 \to \pi^0 + H) = \frac{G_F^{3/2}}{2^{1/4}} \frac{3}{16\pi^2} m_K^2 \sum_i m_i^2\ \mathrm{Re}\,(V_{id}^* V_{is})\,, \tag{17}
$$

whereas $\mathcal{M}_2$ and $\mathcal{M}_3$ are unchanged (neglecting an overall minus sign which is common to the three amplitudes). Thus, we cannot obtain such a definitive theoretical bound for $K_L^0 \to \pi^0 H$, since the matrix element for this process is purely real.† Nevertheless, we can examine the branching ratio as a function of the unknown parameter $A$ in eq. (9). We do this by varying $A$ (which is presumably of order $\lambda$) as a function of $m_t^2|\mathrm{Re}\,V_{td}^* V_{ts}|$. The resulting width is normalized with

† The amplitude for $K_L^0 \to \pi^0 H$ is purely real in the approximation where $\epsilon \ll \arg V_{td}^* V_{ts}$. Since $\epsilon \simeq 2 \times 10^{-3}$, the numbers quoted above suggest that this is a good approximation.



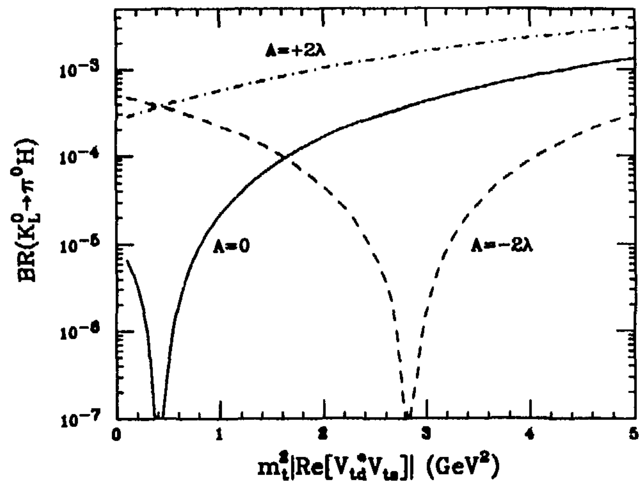


Fig. 1. The $K_L^0 \to \pi^0 H$ branching ratio as a function of $m_t^2|\mathrm{Re}\,V_{td}^* V_{ts}|$, assuming $m_H \ll m_K$. The three curves shown are: $A = 0$ (solid), $A = 2\lambda$ (dot-dashed), and $A = -2\lambda$ (dashes), where $\lambda = -3.2 \times 10^{-7}$.

respect to the total width $\Gamma(K_L^0) = 1.27 \times 10^{-17}$ GeV. According to the discussion above, for $m_t \approx 80$ GeV, we expect $m_t^2|\mathrm{Re}\,V_{td}^* V_{ts}| \gtrsim 2.5\ \mathrm{GeV}^2$, with large uncertainty. Thus, we plot in fig. 1, $BR(K_L^0 \to \pi^0 H)$ *vs.* $m_t^2|\mathrm{Re}\,V_{td}^* V_{ts}|$ for three different values of $A$. We see that only for a very narrow range of $m_t^2|\mathrm{Re}\,V_{td}^* V_{ts}|$, and only for positive values of $A$ does the branching ratio for $K_L^0 \to \pi^0 H$ ever fall below $10^{-6}$. Therefore, barring a very unlikely conspiracy of parameters, we obtain a predicted branching ratio which satisfies:

$$
BR(K_L^0 \to \pi^0 H) \gtrsim 10^{-6} B_H\,. \tag{18}
$$

By a similar computation, we can calculate the amplitude for the $CP$-violating decay $K_S^0 \to \pi^0 H$. If we can neglect $\mathrm{Im}\,A$ and $\mathrm{Im}\,\lambda$ as suggested above, (and neglecting small $CP$-violating effects in the kaon state mixing), then $\mathcal{M}_2$ and $\mathcal{M}_3$ can be ignored. $\mathcal{M}_1$ is modified by replacing $\mathrm{Re}\,V_{id}^* V_{is}$ with $\mathrm{Im}\,V_{id}^* V_{is}$ in eq. (17). Using the numbers quoted above, we find:

$$
BR(K_S^0 \to \pi^0 H) \simeq 10^{-7} B_H\,. \tag{19}
$$

Armed with the above results, we can examine the existing rare kaon decay data and deduce limits on the existence of a light Higgs boson. The experimental situation is reviewed in Refs. 1 and 12. We conclude that high statistics $K$ decay data is close to ruling out the existence of a Standard Model Higgs boson with mass less than $2m_\pi$. The theoretical uncertainties underlying the phrase "is close to" are precisely the ones which we have emphasized above. Clearly, a theoretical calculation which could estimate the value of $A$ would be extremely useful in closing the light Higgs window.

### 2.3. $\eta' \to \eta H$

The $\eta\eta' H$ coupling can be found in a similar manner as the $K\pi H$ coupling. We begin by writing the physical $\eta$ and $\eta'$ states in terms of the $SU(3)_{flavor}$ singlet and octet, $\eta_1$ and $\eta_8$,

$$\begin{aligned} \eta' &= \eta_1 \cos\theta + \eta_8 \sin\theta \\ \eta &= \eta_8 \cos\theta - \eta_1 \sin\theta \, . \end{aligned} \tag{20}$$

The mixing angle $\theta$ can be determined from data, which favor [13] a value of $\theta \simeq -20°$. In terms of the $SU(3)_{flavor}$ states, the mass terms in the Lagrangian are,

$$\mathcal{L}_M = -\tfrac{1}{2} m_1^2 \eta_1^2 - \tfrac{1}{2} m_8^2 \eta_8^2 - \mu^2 \eta_1 \eta_8 \, . \tag{21}$$

Diagonalizing the mass matrix we find,

$$\begin{aligned} \mu^2 &= \sin\theta\cos\theta(m_{\eta'}^2 - m_\eta^2) \\ m_1^2 &= \cos^2\theta \; m_{\eta'}^2 + \sin^2\theta \; m_\eta^2 \\ m_8^2 &= \sin^2\theta \; m_{\eta'}^2 + \cos^2\theta \; m_\eta^2 \, . \end{aligned} \tag{22}$$

The $\eta\eta' H$ coupling can now be derived using the low energy theorem described in eq. (3). Noting that $m_8$ and $\mu$ vanish in the chiral limit, dimensional analysis gives for the scale dependence of the mass parameters:

$$\begin{aligned} m_1^2 &\sim \Lambda_{QCD}^2 \\ m_8^2 &\sim m_s \Lambda_{QCD} \\ \mu^2 &\sim m_s \Lambda_{QCD} \end{aligned} \tag{23}$$

where $m_s$ is the strange quark mass, and $\Lambda_{QCD}$ is defined in eq. (4). Using Eqs. (3) and (23) yields for the $\eta\eta' H$ interaction Lagrangian:

$$\begin{aligned} \mathcal{L} &= \eta\eta' \frac{H}{v} \cos\theta\sin\theta(\cos^2\theta \; m_{\eta'}^2 + \sin^2\theta \; m_\eta^2)\left(-1 + \frac{2N_H}{3b}\right) \\ &= \eta\eta' H (1.1 \times 10^{-3} \text{ GeV})\left(1 - \frac{2N_H}{3b}\right) . \end{aligned} \tag{24}$$



Hence the branching ratio for $\eta' \to \eta H$ is

$$BR(\eta' \to \eta H) = 1.2 \times 10^{-4}\left(1 - \frac{2N_H}{3b}\right)^2 B_H \tag{25}$$

where we have used the particle data group value of $\Gamma(\eta' \to \text{all}) = 2.1 \times 10^{-4}$ GeV [14] and $B_H = 2p_H/m_{\eta'}$ in the $\eta'$ rest frame.

The experiment of ref. 15 has searched for the decay $\eta' \to \eta H \to \eta\mu^+\mu^-$. This experiment is sensitive to 250 MeV $\lesssim m_H \lesssim$ 400 MeV. The branching ratio for $H \to \mu^+\mu^-$ can be found from the chiral Lagrangian. Since the branching ratio for $H \to \pi^+\pi^-$ is enhanced by the Higgs coupling to the gluon content of the pion, the branching ratio to muons is suppressed,[4]

$$\frac{\Gamma(H \to \pi^+\pi^- + \pi^0\pi^0)}{\Gamma(H \to \mu^+\mu^-)} = \frac{m_H^2}{27m_\mu^2}\left(1 + \frac{11m_\pi^2}{2m_H^2}\right)^2 \frac{(1 - 4m_\pi^2/m_H^2)^{1/2}}{(1 - 4m_\mu^2/m_H^2)^{3/2}} . \tag{26}$$

For $m_H = 1$ GeV, this ratio is 4.3. As shown by Raby and West,[16] this result is an underestimate since it neglects enhancements due to final state interactions. Over the mass range of interest, using the results of ref. 16, we find $BR(H \to \mu^+\mu^-) \gtrsim 0.07$. Using $N_H = 3$ and $b = 9$ in eq. (25), we obtain the following theoretical prediction:

$$BR(\eta' \to \eta H \to \eta\mu^+\mu^-) \simeq 7.2 \times 10^{-5} BR(H \to \mu^+\mu^-) B_H , \tag{27}$$

where $B_H \equiv 2p_H/m_{\eta'} < 0.53$ in the kinematical range of interest. This should be compared to the quoted (90% confidence level) experimental upper limit of $1.5 \times 10^{-6}$. Clearly, there is a range of $H$ masses, $250 \; MeV \lesssim m_H \lesssim 2m_\pi$, which is ruled out by the data. The actual Higgs mass upper limit which can be ruled out depends sensitively on $BR(H \to \mu^+\mu^-)$. An improvement by about a factor of 10 of the experimental upper limit is needed to definitively rule out Higgs bosons over most of the above mass range.

## 3. HIGGS PRODUCTION IN Υ RADIATIVE DECAY

We turn now to another source of limits on light Higgs bosons, radiative decays of the Υ. Processes involving the $b$ quark are free of many of the uncertainties which occur in $K$ and $\pi$ decays and can presumably be calculated reliably using the spectator quark model. Unfortunately, other theoretical difficulties arise in Υ decay. The most significant limit on the Higgs boson mass is potentially that of the CUSB collaboration. [17] CUSB has searched for the radiative decays $\Upsilon \to H\gamma$, where

$$R_0 \equiv \frac{\Gamma_0(\Upsilon \to H\gamma)}{\Gamma_0(\Upsilon \to \mu^+\mu^-)} = \frac{G_F m_b^2}{\sqrt{2}\pi\alpha}\left(1 - \frac{m_H^2}{m_\Upsilon^2}\right) \tag{28}$$

is the theoretical prediction computed to lowest order in perturbation theory.[18] CUSB has analyzed $\sim 8 \cdot 10^5$ $\Upsilon(1S)$ and $\sim 6 \cdot 10^5$ $\Upsilon(3S)$ radiative decays and has claimed to rule out a Standard Model Higgs boson in the mass range: [17]

$$600 \text{ MeV} \lesssim m_H \lesssim 5 \text{ GeV}. \tag{29}$$

It is important to understand the theoretical assumptions involved in obtaining this result and we survey them here.

The QCD radiative corrections to eq. (28) have been calculated [19] and take the form,

$$\Gamma_{QCD}(\Upsilon \to H\gamma) = \Gamma_0(\Upsilon \to H\gamma)\left(1 - \frac{4\alpha_s}{3\pi}a_H(z) + \mathcal{O}(\alpha_s^2)\right), \tag{30}$$

where

$$z \equiv 1 - \frac{m_H^2}{4m_b^2}. \tag{31}$$

For $m_H \ll 2m_b$, $a_H$ has the limit, $a_H(1) = 7 + 6\log\ 2 - \pi^2/8 \sim 10$. These corrections are large and reduce the rate by approximately 84% for $m_H \ll m_\Upsilon$ and $\alpha_s = 0.2$. (If the first order corrections to $\Gamma(\Upsilon \to \mu^+\mu^-)$ are also included, then one finds that the ratio $R_0$ [eq. (28)] is reduced by about 50%). Recently, there has been some confusion about the scheme dependence of these corrections. In fact, as emphasized by Nason,[20] eq. (30) is manifestly independent of the subtraction scheme; an alternative choice of scheme can only introduce corrections of $\mathcal{O}(\alpha_s^2)$ in eq. (30). However, a complete $\mathcal{O}(\alpha_s^2)$ calculation has yet to be performed, so one can not *a priori* determine which of two schemes which differ by $\mathcal{O}(\alpha_s^2)$ terms is more appropriate.

In addition to the QCD radiative corrections, there may also be large bound state and relativistic corrections. First, we consider bound state corrections, which were first examined in ref. 21. For $m_H \ll m_\Upsilon$, the photon emitted in the decay $\Upsilon \to H\gamma$ is hard and the perturbative calculation is valid. If, however, $m_H \sim m_\Upsilon$, then the photon is soft and bound state effects must be considered. In this case, after emitting the soft $\gamma$, the propagating quark is still strongly affected by the potential. A key ingredient to the calculation of these bound state effects is that



of gauge invariance: the same $b\bar{b}$ interaction must be used to calculate both the $\Upsilon b\bar{b}$ and $Hb\bar{b}$ vertices. (This may be the source of the problem with the calculation of ref. 22, whose results disagree with all other published calculations.) The importance of gauge invariance has been recently stressed in a detailed calculation of Faldt, Osland and T.T. Wu.[23] Their calculation is valid in the regime $m_H \sim m_\Upsilon$, where the bound state effects are the most important. When bound state corrections are included, it is no longer true that the decay amplitude can be simply expressed as being proportional to the wave function at the origin. Further, the decay rate is controlled by two amplitudes which interfere destructively. The result is that the ratio $R \equiv \Gamma(\Upsilon \to H\gamma)/\Gamma(\Upsilon \to \mu^+\mu^-)$ is smaller than $R_0$ by a substantial amount. The overall suppression factor is always larger than a factor of two, and the suppression can be very large in the region where the destructive interference is maximal. These results are applicable for Higgs masses in the range 7.5 GeV $\lesssim m_H \lesssim m_\Upsilon$, where the approximations employed in ref. 23 are expected to be valid. The observed suppression of the decay rate for $\Upsilon \to H\gamma$ confirms the previous calculations of ref. 21, and suggests that CUSB will never be sensitive to Higgs boson masses much above 7 GeV.

Second, we consider relativistic corrections to the decay rate for $\Upsilon \to H\gamma$ which have been calculated by Aznauryan *et al.*[24] For the $\Upsilon$, $v/c \sim 0.3$; thus these corrections may well be significant. For $m_H \ll m_\Upsilon(1 - \alpha_s^2) \sim 8$ GeV, they calculate the relativistically corrected expression for the ratio given in eq. (28) using the infinite momentum frame and light-cone dynamics.

To leading order in $p^2/m_b^2$, where $p$ is the momentum of the quark in the rest system of the $b\bar{b}$, the relativistic result is:

$$R_{rel} = R_0\left(\frac{M_\Upsilon^2 - m_H^2}{4m_b^2 - m_H^2}\right)^2\left[1 - \frac{\Delta}{3}\left(\frac{10}{z} - 1\right)\right], \tag{32}$$

where

$$m_b^2\Delta \equiv \frac{\int \phi(p^2)p^4 dp}{\int \phi(p^2)p^2 dp}, \tag{33}$$

$z$ is defined in eq. (31), and $\phi(p^2)$ is the radial part of the wave function of the $b$-quarks in the $\Upsilon$. Note that $R_{rel} < R_0$ for all values of $m_H$. For $m_H < 4$ GeV, $R_{rel} \sim \frac{1}{2}R$, while for $m_H \sim 6-8$ GeV, $R_{rel}$ is a factor of $3-4$ times smaller than $R$. It is worth noting that in the region of $m_H$ where the calculations of ref. 23 and 24 overlap, there is rough agreement. For very heavy quarkonium ($Q\overline{Q}$) systems, the mass of the $Q\overline{Q}$ state approaches $2m_Q$ and $\Delta \to 0$, so that the nonrelativistic formula is regained in the appropriate limit.

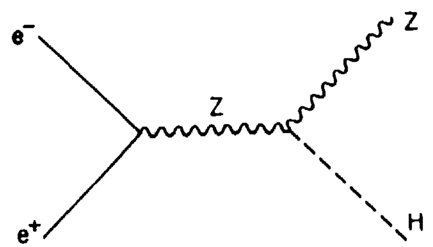


Fig. 2. Feynman diagram for $e^+e^- \to ZH$.

It is not obvious how to combine the QCD and the relativistic corrections since they are both of $\mathcal{O}(v^2/c^2)$. However, the QCD radiative corrections reflect hard gluon effects, whereas the relativistic corrections are due to soft gluon effects. This suggests that it is correct to include the two corrections independently. In particular, the authors of ref. 24 specifically state that their calculation omits the diagrams associated with the hard QCD corrections computed in ref. 19. Thus, it seems that one must include the suppression from relativistic effects on top of the QCD corrected prediction. It would then follow that no limit on $m_H$ can be found from present CUSB data. Clearly, additional theoretical work is needed before a firm limit on the Higgs boson mass can be extracted from the decay $\Upsilon \to H\gamma$. In particular, the relativistic corrections of Aznauryan *et al.*[24] need to be confirmed by an independent calculation.

## 4. RADIATIVE CORRECTIONS TO $e^+e^- \to ZH$

For a Higgs boson in the mass range 5 GeV $\lesssim m_H \lesssim M_Z$, we must turn to SLC and LEP. At the $Z$ factories, with a data sample of $10^6$–$10^7$ $Z$'s, it will be possible to detect Higgs bosons up to $m_H \approx$ 30–50 GeV, by observing the decay $Z \to HZ^* \to H\ell^+\ell^-$. At LEP-II with $\sqrt{s} = 200$ GeV, the Higgs mass discovery range can be extended to close to $M_Z$, by observing the process $e^+e^- \to HZ$ (shown in Fig. 2). In both cases, Higgs production results from the $ZZH$ vertex. As a result, a Higgs boson produced at tree level by these process must be a $CP$-even scalar. In multi-Higgs doublet models, there also exist $CP$-odd scalars (often called "pseudoscalars"), which we generically call $A$. However, $A$ is *not* produced in the above processes at tree-level, due to the absence of tree-level $ZZA$ vertices.

We have investigated the one-loop corrections to $e^+e^- \to ZH$ due to a heavy top quark.[25] These corrections have previously been considered by Fleischer and Jegerlehner, who found large corrections (roughly 10 – 20%) for a top mass of 200 GeV at LEP energies.[26,27] At one loop, both the scalar ($H$) and the pseudoscalar ($A$) can be produced through triangle diagrams in which the internal loop consists of a heavy quark (either the $t$-quark or possibly a fourth generation quark and/or lepton). Because the $t\bar{t}H$ coupling is proportional to $m_t$, heavy fermions do not decouple. In fact, an explicit calculation reveals that the contribution of the top-quark loop to the $HZZ$ vertex results in an amplitude which grows with $m_t$ like $m_t^2/M_W^2$, and so is potentially significant.

This behavior can be deduced on dimensional grounds. The effective operator describing the $HZZ$ vertex, $Z_\mu Z^\mu H$, has dimension $d = 3$. In general, in the heavy quark mass limit, the $t$-quark loop leads to an $m_t^{4-d}$ behavior multiplied by one power of $m_t$ from the $t\bar{t}H$ vertex. The end result is an amplitude which grows as $m_t^{5-d}$. For $d = 3$, we find a result which grows with $m_t^2$ as claimed above. In contrast, let us examine the case of the pseudoscalar $A$. The effective operator which is generated for the $AZZ$ vertex by the heavy top-quark loop is of the form $\epsilon_{\mu\nu\alpha\beta}F_Z^{\mu\nu}F_Z^{\alpha\beta}$. This is a dimension-5 operator, so by the reasoning above, the amplitude for the $AZZ$ vertex only approaches a constant (independent of $m_t$) in the large $m_t$ limit. In fact, the production rate for $e^+e^- \to AZ$ is exceedingly tiny, since it first occurs at one-loop and so is suppressed by a factor of $\mathcal{O}(\alpha^2)$ relative to $e^+e^- \to HZ$. We estimate its cross-section to be roughly a factor of $10^{-6}$ smaller than the production cross-section for $e^+e^- \to HZ$, so we will henceforth restrict our discussion to the production of the scalar Higgs boson $H$.

There are two types of heavy fermion contributions to $e^+e^- \to HZ$. One comes from corrections to the $Z^*ZH$ vertex and the other from corrections to the $Z$ boson propagator. The leading $m_t$ behavior for the decay $Z^* \to HZ$ can be found using the equivalence theorem.[28] The equivalence theorem states that the computation of any process involving external massive gauge bosons can be performed by replacing the external gauge bosons by their corresponding Goldstone scalars and computing the amplitude using $R_\xi$-gauge Feynman rules. Calculations using the equivalence theorem give results for the amplitude which are valid up to terms of order $M_W/E$, where $E$ is the $Z^*$ energy (in the $e^+e^-$ center-of-mass frame).

We have calculated $Z^* \to HZ$, where $Z^*$ is off-shell, using the equivalence theorem. Our amplitudes will therefore be valid up to corrections of $\mathcal{O}(\frac{M_Z}{m_t}, \frac{M_Z}{E})$, and thus will correctly reproduce the leading $m_t$ behavior of the $Z^*ZH$ vertex. Hence we calculate the rate for

$$Z^{*\mu}(k_2) \to H(k_1) + G_0(p) \qquad (34)$$

where $G_0$ is the Goldstone boson which is "eaten" and becomes the longitudinal component of the $Z$.

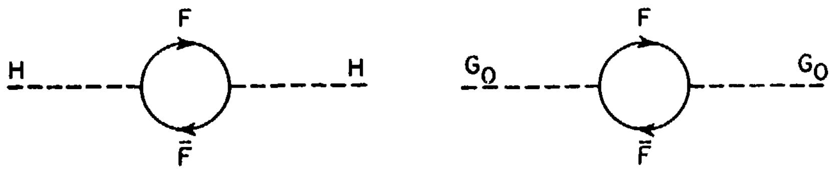


**Fig. 3. Wave function renormalization of the Higgs boson, $H$, and the neutral Goldstone boson, $G_0$.**

We work in Landau gauge where the Goldstone bosons are massless and there is no $G_0 - Z$ mixing. Furthermore, in Landau gauge, the $Z$-propagator is purely transverse; hence we can drop terms which are proportional to $k_2^\mu$ since these are automatically annihilated by the off-shell $Z$ propagator.

The tree level amplitude is,

$$\mathcal{A}_0(Z^{*\mu} \to HG_0) = \frac{g}{\cos\theta_W} p^\mu . \tag{35}$$

There are two sources of $\mathcal{O}(G_F m_t^2)$ corrections to the $G_0 Z^{\mu *} H$ vertex.* The first is the wave function renormalization of $H$ and $G_0$ shown in Fig. 3. We perform the calculation using dimensional regularization in $4 - 2\epsilon$ dimensions. Summing over all possible heavy fermions, $m_j$ we find,

$$\begin{aligned} Z_H &= 1 - \sum_j \left(\frac{G_F}{\sqrt{2}}\right) \frac{m_j^2 N_c}{4\pi^2} \left(\frac{4\pi}{m_j^2}\right)^\epsilon \Gamma(1+\epsilon) \left\{ \frac{1}{\epsilon} - 6 I_3(a_{1j}) - \frac{2}{3} \right\} \\ Z_{G_0} &= 1 - \sum_j \left(\frac{G_F}{\sqrt{2}}\right) \frac{m_j^2 N_c}{4\pi^2} \left(\frac{4\pi}{m_j^2}\right)^\epsilon \Gamma(1+\epsilon) \left\{ \frac{1}{\epsilon} \right\} \end{aligned} \tag{36}$$

where

$$\begin{aligned} a_{ij} &= \frac{k_i^2}{m_j^2} \\ I_3(a) &\equiv \int_0^1 dx\, x(1-x) \log\left[1 - a x(1-x)\right] . \end{aligned} \tag{37}$$

$N_c$ is a color factor and is 3 for quarks and 1 for leptons. The one-loop contribution

* The tadpole contributions to $e^+e^- \to ZH$ cancel exactly among themselves.



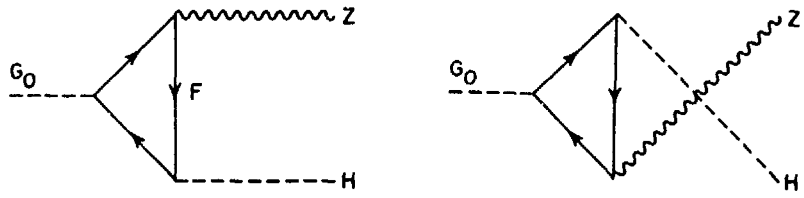


Fig. 4. Corrections to the 3-point function for $Z^* \to HG^0$ due to heavy fermions.

of the wave function renormalization to the three point function is then,

$$\begin{aligned} \mathcal{A}_{11}(Z^{*\mu} \to HG_0) &= \left[ (\sqrt{Z_H} - 1) + (\sqrt{Z_{G_0}} - 1) \right] \mathcal{A}_0 \\ &= -\frac{g}{\cos\theta_W} p^\mu \sum_j \left(\frac{G_F}{\sqrt{2}}\right) \frac{m_j^2 N_c}{8\pi^2} \left(\frac{4\pi}{m_j^2}\right)^\epsilon \Gamma(1+\epsilon) \\ &\quad \times \left\{ \frac{2}{\epsilon} - 6 I_3(a_{1j}) - \frac{2}{3} \right\} . \end{aligned} \tag{38}$$

The corrections to the three point function from triangle diagrams are shown in Fig. 4. The sum of these two diagrams gives a contribution to the amplitude,

$$\begin{aligned} \mathcal{A}_{12}(Z^{*\mu} \to HG_0) = &-\frac{g}{\cos\theta_W} p^\mu \sum_j \left(\frac{G_F}{\sqrt{2}}\right) \frac{m_j^2 N_c}{8\pi^2} \left(\frac{4\pi}{m_j^2}\right)^\epsilon \Gamma(1+\epsilon) \\ &\times \left\{ -\frac{2}{\epsilon} + \frac{2}{k_1^2 - k_2^2} \left( k_1^2 I_1(a_{1j}) - k_2^2 I_1(a_{2j}) \right) \right. \\ &\left. + \frac{4 m_j^2}{k_1^2 - k_2^2} \left( I_2(a_{2j}) - I_2(a_{1j}) \right) \right\} . \end{aligned} \tag{39}$$

Combining these results, we find the radiatively corrected three point function,

$$\Gamma^{(3)}(Z^{*\mu} \to HG_0) = \frac{g}{\cos\theta_W} p^\mu \Big\{ 1 + \sum_j \left(\frac{G_F}{\sqrt{2}}\right) \frac{m_j^2 N_c}{8\pi^2} \Big[ 6 I_3(a_{1j}) + \frac{2}{3} - \frac{2}{k_1^2 - k_2^2}\Big(k_1^2 I_1(a_{1j}) - k_2^2 I_1(a_{2j})\Big) - \frac{4m_j^2}{k_1^2 - k_2^2}\Big(I_2(a_{2j}) - I_2(a_{1j})\Big)\Big]\Big\}$$
$$\equiv \frac{g}{\cos\theta_W} p^\mu \Big(1 + \mathcal{E}(k_1^2, k_2^2)\Big), \tag{40}$$

where

$$I_1(a) \equiv \int_0^1 dx \, \log\Big(1 - ax(1-x)\Big),$$
$$I_2(a) \equiv \int_0^1 \frac{dx}{x} \log\Big(1 - ax(1-x)\Big). \tag{41}$$

Analytic forms for $I_1$, $I_2$, and $I_3$ can be found in ref. 29.

The $Z$ boson propagator receives a contribution from heavy fermion loops and also from the $Z$ mass counterterm. The renormalized $Z$ mass is

$$M_Z^2 \equiv M_{Z0}^2 + \delta M_Z^2, \tag{42}$$

where

$$\frac{\delta M_Z^2}{M_Z^2} = \left(\frac{G_F}{\sqrt{2}}\right) \frac{N_c}{4\pi^2} \sum_i \left(\frac{4\pi}{m_i^2}\right)^\epsilon \Gamma(1+\epsilon) m_i^2 \left(\frac{1}{\epsilon}\right). \tag{43}$$

We define the corrected propagator to be $iA^{\mu\nu}(p)$ and compute the contributions of $\mathcal{O}(G_F m_i^2)$ from the diagrams of Fig. 5. We find,

$$iA^{\mu\nu}(p^2) = \frac{-ig^{\mu\nu}}{p^2 - M_Z^2}\left[1 - \left(\frac{G_F}{\sqrt{2}}\right)\frac{M_Z^2 N_c}{4\pi^2} \sum_i \frac{m_i^2}{p^2 - M_Z^2} I_1\left(\frac{p^2}{m_i^2}\right)\right]$$
$$\equiv \frac{-ig^{\mu\nu}}{p^2 - M_Z^2}\Big(1 + \mathcal{B}(p^2)\Big). \tag{44}$$

Note that we have dropped the $p^\mu p^\nu$ terms since they give contributions of $\mathcal{O}(m_e^2)$ to $e^+e^- \to HG_0$ and of $\mathcal{O}(m_\ell^2)$ to $H \to G_0 \ell^+ \ell^-$.



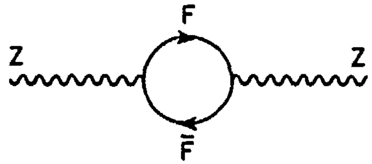


Fig. 5. Contributions of $\mathcal{O}(G_F m_i^2)$ to the $Z$ boson propagator

The contribution of heavy fermion loops to the process, $e^+e^- \to ZH$, is then

$$\mathcal{A}_f(e^+e^- \to HG_0) = \mathcal{A}_0\Big[\mathcal{B}(s) + \mathcal{E}(m_H^2, s)\Big], \tag{45}$$

where $s$ is the center-of-mass energy of the $e^+e^-$ pair and $|\mathcal{A}_0|^2$ is the tree level matrix element squared for $e^+e^- \to HG_0$ with $s \gg M_Z^2$,

$$|\mathcal{A}_0(e^+e^- \to HG_0)|^2 = \frac{g^4}{8\cos^4\theta_W}\left(\frac{ut}{s^2}\right)\Big[(1 - 4\sin^2\theta_W)^2 + 1\Big]. \tag{46}$$

Let us define the total cross-section to be $\sigma_0 + \Delta\sigma_f$, where $\sigma_0$ is the tree-level cross-section and $\Delta\sigma_f$ is the contribution due to heavy fermion loops. Then, we obtain the following ratio

$$R_f = \frac{\Delta\sigma_f}{\sigma_0} = 2\Big[\mathcal{B}(s) + \mathcal{E}(m_H^2, s)\Big]. \tag{47}$$

Fig. 6 shows $R_f$ for a range of energies and Higgs masses. Note that typically the fermionic radiative corrections decrease the cross section by $5 - 10\%$. In the limit, $m_i^2 \gg s \gg M_Z^2, m_H^2$, we find:

$$R_f = -\frac{G_F}{\sqrt{2}} \sum_j \frac{m_j^2 N_c}{3\pi^2}. \tag{48}$$

This is in agreement with ref. 27. For a 400 GeV $t$-quark this is a 12% effect.

Exactly the same calculation can be used to find the effects of the one-loop fermionic corrections to the decay $H \to Z\ell^+\ell^-$. Let $\Gamma_0$ be the tree level result for $H \to Z\ell^+\ell^-$ and let $\Delta\Gamma_f$ be the contribution of the radiative corrections which

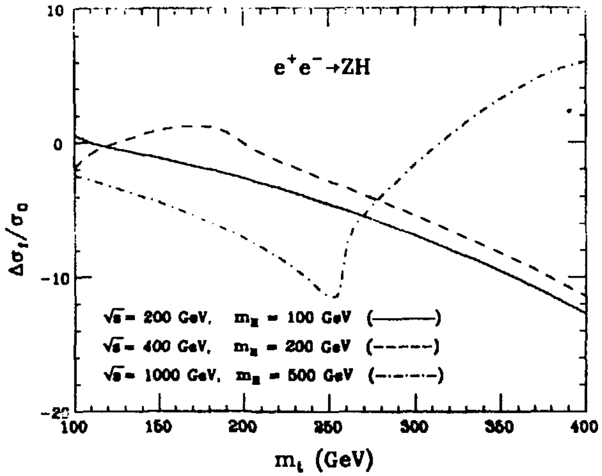


Fig. 6. Ratio of the (one-loop) contribution of a heavy top-quark to $\sigma(e^+e^- \to ZH)$ to the tree level cross section as a function of top-quark mass [see eq. (47)]. The three curves shown are: $\sqrt{s} = 200$ GeV, $m_H = 100$ GeV (solid), $\sqrt{s} = 400$ GeV, $m_H = 200$ GeV (dashed), and $\sqrt{s} = 1000$ GeV, $m_H = 500$ GeV (dot-dashed).

are enhanced for large $m_t^2$. Then,

$$\frac{\Delta\Gamma_f}{\Gamma_0} = 2\Big[\mathcal{B}(k_2^2) + \mathcal{E}(m_H^2, k_2^2)\Big], \tag{49}$$

where as before $k_2$ is the virtual $Z$ momentum. This is the same factor as in eq. (47).

## 5. HIGGS TRIPLET MODEL WITH $\rho = 1$

In this section we consider an extension of the Standard Model which contains Higgs triplet multiplets in addition to the usual complex $SU(2)$ Higgs doublet. Models with only $SU(2)$ doublets and singlets automatically satisfy $\rho = 1$, where $\rho = M_W^2/(M_Z^2 \cos^2\theta_W)$. In models with Higgs triplets, large corrections to the $\rho$ parameter can be avoided by two means: (a) the neutral triplet fields can be given vacuum expectation values which are much smaller than those for the neutral doublet fields; or (b) the quantum numbers of the triplet fields and the vacuum expectation values of their neutral members can be arranged so that a custodial $SU(2)$ symmetry is maintained. The custodial $SU(2)$ is a global symmetry of the Higgs potential which insures that $\rho = 1$ at tree-level.



The model we consider here contains, in addition to the usual complex $SU(2)$ Higgs doublet, one real and one complex triplet of Higgs scalars, denoted by $\xi$ and $\chi$, respectively.[30] The Higgs potential is constructed in such a way that it preserves a custodial $SU(2)$ symmetry; this symmetry is maintained to all orders in the Higgs self-interactions, and is violated only by physics outside the Higgs sector (*e.g.*, a quark doublet with nondegenerate up and down-type masses). Thus, as in the (minimal version of the) Standard Model, $\rho = 1$ at tree-level with only small radiative corrections. Hence, this model provides an attractive example of an extension of the Standard Model Higgs sector which contains Higgs triplets, but no other new physics beyond the Higgs sector. We shall examine this model with regard to the signatures and production mechanisms for the various Higgs bosons.

The Higgs fields are,

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \xi = \begin{pmatrix} \chi^0 & \xi^+ & \chi^{++} \\ \chi^- & \xi^0 & \chi^+ \\ \chi^{--} & \xi^- & \chi^{0*} \end{pmatrix}. \tag{50}$$

If the potential is assumed to have a global $SU(2)_L \times SU(2)_R$ invariance, the vacuum expectation values,

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ a \end{pmatrix}, \qquad \langle\xi\rangle = \begin{pmatrix} b & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & b \end{pmatrix} \tag{51}$$

break the global symmetry to a diagonal $SU(2)$. The $W$ boson mass is fixed,

$$M_W = \frac{g}{2}\sqrt{a^2 + 8b^2} \ . \tag{52}$$

Since Higgs triplets do not couple to fermions, the Yukawa couplings are not forced to be small and there is the possibility that $b \gg a$. In this case, the doublet vacuum expectation value $a/\sqrt{2}$ is much smaller than in the Standard Model and the Yukawa couplings of the doublet to the fermions must be much larger than in the Standard Model in order to obtain the experimentally determined quark masses. In this case, the Higgs bosons that do couple to fermions have much larger fermion-antifermion pair couplings and decay widths than in the Standard Model. This means for example that $\Upsilon$ decays would be an excellent place to search for the new Higgs bosons predicted by this model since they would probably have enhanced $b\bar{b}$ couplings.

The physical Higgs bosons can be classified according to the custodial $SU(2)$. The $W$ and $Z$ bosons obtain their masses from the Goldstone bosons which transform as a [3] of $SU(2)$,

$$
\begin{aligned}
G_3^0 &= \frac{a(\phi^0 - \phi^{0*}) + 2\sqrt{2}b(\chi^0 - \chi^{0*})}{\sqrt{2(a^2 + 8b^2)}} \\
G_3^\pm &= \frac{a\phi^\pm + 2b(\chi^\pm + \xi^\pm)}{\sqrt{2(a^2 + 8b^2)}} .
\end{aligned}
\tag{53}
$$

The remaining 10 physical Higgs bosons are classified as

$$
[5] + [3] + [1] + [1] \tag{54}
$$

under the custodial $SU(2)$. The compositions of the $H$ states are,

$$
\begin{aligned}
[5]: \quad H_5^{++} &= \chi^{++} \\
H_5^{+} &= \frac{\chi^+ - \xi^+}{\sqrt{2}} \\
H_5^0 &= \frac{2\xi^0 - \chi^0 - \chi^{0*}}{\sqrt{6}} \\
[3]: \quad H_3^+ &= \frac{a(\chi^+ + \xi^+) - 4b\phi^+}{\sqrt{2(a^2 + 8b^2)}} \\
H_3^0 &= \frac{a(\chi^0 - \chi^{0*}) - 2\sqrt{2}b(\phi^0 - \phi^{0*})}{\sqrt{2(a^2 + 8b^2)}} \\
[1]: \quad H_1^0 &= \frac{\phi^0 + \phi^{0*}}{\sqrt{2}} \\
[1]: \quad H_1'^0 &= \frac{\chi^0 + \chi^{0*} + \xi^0}{\sqrt{3}} .
\end{aligned}
\tag{55}
$$

Since we have chosen a Higgs potential which preserves the custodial $SU(2)$ symmetry, it follows that there is no mixing of scalar states with different transformation properties under custodial $SU(2)$. Therefore, all the scalar states listed in eq. (55) are mass eigenstates, except for $H_1^0$ and $H_1'^0$ which can mix (with mixing angle depending on the parameters of the Higgs potential).

We now briefly survey the Higgs couplings in this model. The $H_5$ and $H_1'^0$ states cannot couple to fermions. Only $H_3$ and $H_1^0$ couple to fermions; note that $H_3^0$ is a pseudoscalar. The most interesting Higgs couplings of the model are those to vector bosons and are given in Table 1 in terms of the variable,

$$
\sin\theta_3 \equiv \left(\frac{8b^2}{a^2 + 8b^2}\right)^{1/2} . \tag{56}
$$

| | |
|---|---|
| $W^+W^+H_5^{--} + h.c.$ | $\sqrt{2}gM_W \sin\theta_3$ |
| $W^+ZH_5^- + h.c.$ | $-\frac{gM_W \sin\theta_3}{\cos\theta_W}$ |
| $W^+\gamma H_5^-$ | $0$ |
| $W^+W^-H_5^0$ | $\sqrt{\frac{1}{3}}gM_W \sin\theta_3$ |
| $ZZH_5^0$ | $-\sqrt{\frac{4}{3}}\frac{gM_W \sin\theta_3}{\cos^2\theta_W}$ |
| $W^+W^-H_1'^0$ | $\sqrt{\frac{8}{3}}gM_W \sin\theta_3$ |
| $ZZH_1'^0$ | $\sqrt{\frac{8}{3}}\frac{gM_W \sin\theta_3}{\cos^2\theta_W}$ |
| $W^+W^-H_1$ | $gM_W \cos\theta_3$ |
| $ZZH_1$ | $\frac{gM_W \cos\theta_3}{\cos^2\theta_W}$ |

**Table 1:** Feynman rules for the Vector-Vector-Higgs couplings in the Higgs triplet model. (An overall factor of $ig^{\mu\nu}$ is not shown.)

Note the presence of a $H_5^+W^-Z$ coupling, in contrast to models that contain only doublet (and singlet) Higgs multiplets in which the tree-level $H^\pm W^\mp Z$ coupling is absent. If we ignore the $HH$ and $HV$ type channels, the $H_5$ couple and decay only to vector boson pairs, while the $H_3$ couple and decay only to fermion-antifermion pairs. There also exist vector-Higgs-Higgs type couplings: $H_5H_3V$, $H_3H_1^0V$ type couplings are allowed while $H_5H_1^0V$ couplings are forbidden.

We turn now to the phenomenology of this model and begin by considering the decays of the $H_5$ Higgs bosons. At hadron colliders, the $H_5^{++}$ can be made at near Standard Model strength via $W^+W^+$ fusion. The $H_5^{++}$ decays would be spectacular, leading to final states with $W^+W^+$, $H_3^+W^+$ and $H_3^+H_3^-$. For Higgs masses below 1 $TeV$, ref. 30 finds that the $H_5^{++}$ should be detectable above the continuum $W^+W^+$ background at the SSC. $H_5^{++}$ could also be produced in an $e^+e^-$ machine via the process $e^+e^- \to H_5^{++}H_5^{--}$ which contributes one unit to $R$, the cross-section normalized to $\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)$. The singly charged

$H_5^\pm$ also leads to new phenomenology due to the existence of the $H_5^\pm W^\mp Z$ vertex. $H_5^\pm$ can be made with a substantial rate by $e^+e^- \to W^\mp H_3^\pm$ or by $ZW^\pm$ fusion at a hadron collider. We see that the most exciting new physics resulting from this model arises from $ZW^\pm$ and $W^+W^+$ fusion, neither of which occur in the Standard Model. These processes, however, only become important at SSC energies and for relatively heavy Higgs bosons ($m_H \gtrsim 300$ GeV).

## 6. FINAL REMARKS

We have considered a variety of issues affecting the search for a Higgs boson which is lighter than the $Z$. We focused much of our attention on Higgs production in the decays $K \to \pi H$, $\eta' \to \eta H$, and $\Upsilon \to H\gamma$. In each case, precise experimental data exist, whose interpretation is clouded by theoretical uncertainty. The need for further theoretical studies of these processes is obvious!

The working group also addressed a number of issues not reported in this summary. One of these was the possibility of the discovery of a Higgs boson from toponium decay, (toponium) $\to H\gamma$, at LEP. This question has been studied extensively in the CERN Yellow Report on physics at LEP.[31] For 60 $GeV < m_t <$ 76 GeV, this decay can be studied at LEP-II for a Higgs mass up to about 80 GeV if we require 10 events per 100 $pb^{-1}$. Since the current limit on the top quark mass is $m_t \gtrsim 60$ GeV, there is only a very small window for which this process is interesting. Our group also studied QCD radiative corrections to Higgs decay to fermion anti-fermion pairs.[32] The corrections decrease the decay width by as much as a factor of two. This is important when computing the branching ratios for rare decays of the Higgs boson. Finally, some members of the Higgs working group considered the question of whether or not the Weinberg-Salam phase transition can be triggered by cosmic rays, thereby providing a nonperturbative production mechanism for Higgs bosons.[33]

Despite extensive study over the last ten years of the phenomenology of Higgs bosons there still remains interesting and important physics to do!

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