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QED BASED ON SELF–ENERGY: THE RELATIVISTIC $2S_{1/2} \rightarrow 1S_{1/2} + 1\gamma$ DECAY RATES OF HYDROGENLIKE ATOMS *

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ABSTRACT

Within the framework of the recently advanced formulation of QED based on self–energy, we calculate the relativistic rates of the $2S_{1/2} \rightarrow 1S_{1/2} + 1\gamma$ transition in the hydrogen isoelectronic sequence for values of Z ranging between 1 and 92. We compare our results with those of Johnson [*Phys. Rev. Lett.* 29, 1123 (1972)] and Parpia and Johnson [*Phys. Rev. A* 26, 1142 (1982)] and find them to be in good agreement with both.

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I. INTRODUCTION

There has been a renewed interest[1] lately in the study, both theoretically and experimentally, of the decay of the metastable states in hydrogenlike atoms and ions, stimulated mainly by the development of state-of-the-art experimental techniques[2]. The decay of the $2S \rightarrow 1S$ metastable states has been theoretically investigated by many authors[3] working within the framework of the standard theory of Quantum Electrodynamics (QED). The transition has been shown to proceed via two competing processes. Those are the single-photon magnetic dipole ($M1$) transition (dominant at high values of the atomic number Z) and the double–photon electric ($2E1$) transition (dominant at low values of Z). In this work we are interested in the single-photon process, or its equivalent, in the formulation of QED based on self-energy.

According to the standard theory, contribution of the single-photon transition to the decay rates of the metastable $2S$ states of low Z hydrogenic ions is small in comparison with the two-photon transition. Moreover, in 1974 Lin and Feinberg[4] and later in 1978 Barbieri and Sucher[5] have shown that, to lowest order in $Z\alpha$, where α is the fine structure constant, the radiative corrections have no effect on the $M1$ transition matrix element.

A calculation of the single-photon decay rates of the $2S$ states in the hydrogen isoelectronic sequence has been performed in 1972 by Johnson[6] using an analytical closed-form expression. This calculation has been improved in 1982 by Parpia and Johnson[7] by incorporating the finite nuclear size corrections.

On the experimental side, though, no precise measurements of the single photon $M1$ decay rates have yet been reported, but its effect has clearly been identified[8].

The standard treatment, however, is not the only approach. We have recently developed a fully relativistic theory for the decay rates of spontaneous emission based upon the idea that the process is triggered by the electron's self-energy[9]. Our approach has been applied to single-photon atomic transitions in hydrogen and has been found to successfully

reproduce the experimentally well-established results of the standard theory.

Moreover, the formulation of QED based on self-energy has recently been used by Barut and his co-workers[10] to calculate, among other things, such effects as the electron's $g-2$, the Casimir effect, the Unruh effect and the Lamb shift.

In this paper, we employ the closed-form expression we arrived at fully analytically from our entirely relativistic approach in performing a calculation similar to Parpia and Johnson's[7]. Work is also currently in progress[11] to calculate the $2E1$ rates of decay, all within the context of the self-energy approach.

In Section II we give our expression for the $2S \rightarrow 1S$ decay rates. We present the results of our calculation in Section III and give a brief discussion in Section IV.

II. THE DECAY RATES

Apart from a slight change in notation, whose aim is primarily to facilitate the comparison with Johnson's expression, the $2S \rightarrow 1S$ transition rate in a hydrogenlike atom is given in reference [9] by

$$\Gamma_{2S\rightarrow 1S} = -2\alpha\omega\{(R_1+R_2)^2 - 3(R_3+R_4)^2 + 4R_3R_4\} \tag{1}$$

where

$$R_1 = \sqrt{\frac{(2\gamma+1)(1+\epsilon_1)(1+\epsilon_2)}{2(N+1)^3}}\left(\frac{2}{N+1}\right)^{2\gamma}\frac{N^\gamma}{(1+\eta^2)^{2\gamma}}\left(NF_1 - \frac{2}{1+\eta^2}F_2\right) \tag{2a}$$

$$R_2 = \sqrt{\frac{(2\gamma+1)(1-\epsilon_1)(1-\epsilon_2)}{2(N+1)^3}}\left(\frac{2}{N+1}\right)^{2\gamma}\frac{N^\gamma}{(1+\eta^2)^{2\gamma}}\left((N+2)F_1 - \frac{2}{1+\eta^2}F_2\right) \tag{2b}$$

$$R_3 = \sqrt{\frac{(2\gamma+1)^3(1+\epsilon_1)(1-\epsilon_2)}{32(N+1)}}\left(\frac{\omega}{3\lambda}\right)\left(\frac{2}{N+1}\right)^{2\gamma+2}\frac{N^{\gamma+1}}{(1+\eta^2)^{2\gamma}} \times \left(NF_3 - \frac{4(\gamma+1)}{(2\gamma+1)(1+\eta^2)}F_4\right) \tag{2c}$$

$$R_4 = \sqrt{\frac{(2\gamma+1)^3(1-\epsilon_1)(1+\epsilon_2)}{32(N+1)}}\left(\frac{\omega}{3\lambda}\right)\left(\frac{2}{N+1}\right)^{2\gamma+2}\frac{N^{\gamma+1}}{(1+\eta^2)^{2\gamma}} \times \left((N+2)F_3 - \frac{4(\gamma+1)}{(2\gamma+1)(1+\eta^2)}F_4\right) \tag{2d}$$

and where

$$F_1 = {}_2F_1\left(-\gamma+\frac{1}{2}, -\gamma+1, \frac{3}{2}; -\eta^2\right) \tag{3a}$$

$$F_2 = {}_2F_1\left(-\gamma, -\gamma+\frac{1}{2}, \frac{3}{2}; -\eta^2\right) \tag{3b}$$

$$F_3 = {}_2F_1\left(-\gamma+1, -\gamma+\frac{3}{2}, \frac{5}{2}; -\eta^2\right) \tag{3c}$$

$$F_4 = {}_2F_1\left(-\gamma+\frac{1}{2}, -\gamma+1, \frac{5}{2}; -\eta^2\right) \tag{3d}$$

and finally

$$\gamma = \sqrt{1-(Z\alpha)^2}, \quad N = \sqrt{2\gamma+2}, \quad \lambda = Z\alpha m, \quad \eta = \frac{\omega N}{\lambda(N+1)}, \quad \epsilon_1 = \left|\frac{\gamma+1}{2}\right|^{\frac{1}{2}}, \quad \epsilon_2 = \gamma,$$

$$\omega = m(\epsilon_1 - \epsilon_2), \quad m = \frac{m_e}{1+\xi}, \quad \xi = \frac{m_e}{M} \quad \text{and} \quad M = M_{atom} - Zm_e.$$

Parpia and Johnson[6,7], on the other hand, have used the following expression

$$\Gamma(M1) = \frac{4}{9}\alpha\omega^3|\mathcal{M}(\omega)|^2 \tag{4}$$

with

$$\mathcal{M}(\omega) = -\frac{(Z\alpha)^2}{m}\sqrt{\frac{2(N-1)}{N+2}}\left(\frac{2}{N+1}\right)^{2\gamma+1}N^{\gamma-1} \times \left\{{}_2F_1\left(\gamma+\frac{1}{2}, \gamma+1, \frac{5}{2}; -\eta^2\right) + \frac{2}{5}N\eta^2\,{}_2F_1\left(\gamma+\frac{3}{2}, \gamma+2, \frac{7}{2}; -\eta^2\right)\right\} \tag{5}$$

The numerical results reported by Parpia and Johnson[7] are based on a modified version of equations (4) and (5). The modification has been brought about by replacing the Coulomb potential due to a point nuclear charge by the potential produced by a Fermi-Dirac charge distribution.

III. RESULTS

Our calculation differs from that of Parpia and Johnson[7] in two respects. They do not include the reduced-mass corrections in their results but merely show, albiet partly correctly, towards the end of their paper how to incorporate them. In our calculation, however, we use the electron's reduced-mass m everywhere instead of m_e. On the other hand, Parpia and Johnson use the potential of a Fermi-Dirac charge distribution in place of the Coulomb potential of the point nucleus and then use the numerically generated counterparts of the Dirac-Coulomb wavefunctions to calculate the matrix elements. In our calculation, the nucleus is taken as pointlike and exact Dirac-Coulomb wavefunctions are employed in arriving at the analytic expressions (1) and (2).

Our main results are displayed in Table(I). In the second column, we show Johnson's earlier results[6] for $10^6 Z^{-10}$ times the $M1$ decay rates of the metastable $2S \rightarrow 1S$ states of the hydrogenic atoms, for $Z = 1$ to $Z = 92$. In his calculation, Johnson does not take the finite nucleus effects into consideration. The third column contains the results of Parpia and Johnson's 1982 calculation incorporating the corrections due to a nucleus of finite extension. Our results for the same quantity are shown in the fourth column while the last column contains the values in s^{-1} of the decay rates as $a(b)$. In the fifth column $a(b)$ means $a \times 10^b$.

Finally, we remark that, in our calculation, we have used the most recently reported[12] values for the electron mass m_e, the fine structure constant α, Planck's constant $\hbar$, the atomic masses M and the atomic mass unit u.

IV. DISCUSSION

We note that the agreement between the results of our calculation and those of Johnson's for the point nucleus case (compare the second and fourth columns in Table (I)) is almost complete. Like in our calculation, the reduced-mass corrections have been included in Johnson's paper[6].

A comparison of the third and fourth columns, on the other hand, shows quite clearly that the use, by Parpia and Johnson, of numerically generated Dirac-Coulomb wavefunctions, suitable for a finite nucleus potential, amounts only to a decrease in the $M1$ rates that grows with Z to reach the level of 1.1% for $Z = 92$. The agreement, otherwise, between our calculation and that of Parpia and Johnson is quite impressive.

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TABLE (I)

Rates of the $2S_{\frac{1}{2}} \rightarrow 1S_{\frac{1}{2}} + 1\gamma$ transition in hydrogenlike atoms. The rates $\Gamma(M1)$ are given in s^{-1}. Z is the atomic number and $a(b)$ in the last column is to be read as $a \times 10^b$.

	$10^6 Z^{-10}\Gamma(M1)$			
Z	Point nucleus*	Finite nucleus **	This calculation	$\Gamma(M1)$
1	2.4959	2.4958	2.4945	2.4945(-6)
2	2.4964	2.4963	2.4960	2.5559(-3)
3	2.4971	2.4970	2.4968	1.4743(-1)
4	2.4991	2.4980	2.4979	2.6192(00)
5	2.4993	2.4993	2.4992	2.4406(01)
6	2.5009	2.5009	2.5008	1.5121(02)
7	2.5028	2.5027	2.5026	7.0693(02)
8	2.5049	2.5049	2.5048	2.6895(03)
9	2.5073	2.5073	2.5072	8.7422(03)
10	2.5100	2.5100	2.5100	2.5100(04)
12	2.5164	2.5163	2.5163	1.5580(05)
14	2.5239	2.5238	2.5238	7.3002(05)
16	2.5326	2.5325	2.5325	2.7845(06)
18	2.5425	2.5424	2.5424	9.0776(06)
20	2.5537	2.5536	2.5536	2.6149(07)
22	2.5661	2.5660	2.5660	6.8154(07)
24	2.5798	2.5797	2.5798	1.6357(08)
26	2.5949	2.5948	2.5949	3.6631(08)
28	2.6114	2.6112	2.6113	7.7346(08)
30	2.6292	2.6290	2.6292	1.5525(09)
34	2.6694	2.6691	2.6693	5.5105(09)
38	2.7158	2.7154	2.7158	1.7050(10)
40	2.7415	2.7415	2.7415	2.8747(10)
42	2.7690	2.7684	2.7690	4.7294(10)
46	2.8296	2.8287	2.8295	1.2003(11)
50	2.8983	2.8969	2.8982	2.8303(11)
54	2.9759	2.9740	2.9759	6.2741(11)
58	3.0635	3.0609	3.0635	1.3198(12)
60	3.1114	3.1114	3.1114	1.8813(12)
62	3.1623	3.1586	3.1623	2.6541(12)
66	3.2738	3.2687	3.2738	5.1344(12)
70	3.3999	3.3926	3.3998	9.6036(12)
74	3.5425	3.5324	3.5425	1.7443(13)
78	3.7046	3.6903	3.7046	3.0880(13)
80	3.7939	3.7939	3.7939	4.0737(13)
82	3.8894	3.8690	3.8894	5.3459(13)
86	4.1012	4.0719	4.1012	9.0759(13)
90	4.3454	4.3033	4.3453	1.5151(14)
92	4.4817	4.4310	4.4817	1.9468(14)

* Reference 6.
** Reference 7.