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ABSTRACT

The emission of ion-sound waves by an accelerated Langmuir soliton is studied. The acceleration of the soliton is due to an inhomogeneous density barrier. On the assumption that the kinetic energy of the Langmuir solition is smaller than the potential energy created by the barrier. The basic equations describing the dynamic behaviour of the soliton and the emission of the ion-sound waves are formulated. The qualitative spatial distributions of the perturbed concentration in the ion-sound waves are analyzed at different characteristic points of the soliton. The energy lost by the soliton, as a result of the emission, is estimated.

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I Introduction

The study of Langmuir soliton and its propagation in a plasma is very useful with regard to the understanding of plasma diagnostic , plasma heating , as well as ionospheric communications . Several authors , for example Zakharov et al. (1972) and Scott et al. (1973), have shown that a soliton preserves its shape and velocity in moving in homogeneous plasma , i.e .the soliton behaves like a particle . However , in an inhomogeneous plasma , the Langmuir soliton is found to get accelerated , Chen and Lui (1978). The analogy between solitons and the charged particles appeared to be a very important feature in the sense that the accelerated Langmuir soliton can emitt ion-sound waves , in the same manner by which an accelerated charged particle radiates electromagnetic waves . As it was mentioned in the work of Kaw et al. (1982), in emitting ion-sound waves by an accelerated Langmuir soliton, the energy of the ions was taken into account, as well as the recoil interaction of the ion-sound waves on the soliton was neglected.

Recently El-Ashry and Tskhakaya (1989) have investigated the problem of ion-sound emission by a Langmuir soliton moving with acceleration due to a barrier of a density inhomogeneity in plasma , when the kinetic energy of the soliton was higher than the potential energy created by the barrier.In the present work, we investigate the case in which the kinetic energy of the soliton is smaller than the potential energy created by the inhomogeneity so that the soliton is reflected back from the barrier.

In the first part of this paper, the basic equations describing the dynamic behavior of Langmuir soliton and the emission of the ion-sound waves are formulated, and the problem of the reflection of the soliton at the barrier is studied. The second part is devoted to the time- space distribution of the ion-sound waves radiated by the soliton. The estimation of the energy gained by the radiated waves is carried out in the third part. Meanwhile, it should be mentioned here that the dimensionless notations introduced in the papaers of both Kaw et al. (1982) and El-Ashry (1989) will be used in the present work.

I. Let us first of all limit ourselves to the case in which the recoil effects of the radiated field on the dynamic behavior of Langmuir soliton are neglected. Thus, the total perturbed density of ions could be presented in the form:

$$\delta n = \frac{|E|^2}{(\dot{\bar{x}}^2 - 1)} + N \qquad (1)$$

where the first term in the right hand side of equation (1) corresponds to the density perturbation localized in the region where the soliton exists , the second term describes the ion-sound field , E is the electric intensity of Langmuir oscillations, and $\bar{x}(t)$ is the coordinate of the center of the soliton ($\dot{\bar{x}}(t) = \partial_t \bar{x}(t)$). According to what was mentiond before, we assume that

$$N << |E|^2 \qquad (2)$$

$$|E|^2 = \xi^2(x - \bar{x}(t)) \tag{3}$$

where ξ is the amplitude of the Langmuir soliton.The soliton velocity $\dot{\bar{x}}$ is supposed to be much smaller than the ion-sound velocity,i.e.,

$$\dot{\bar{x}}^2 << 1 \tag{4}$$

Hereafter, we shall consider also that the change of the soliton during a period of time τ is much less than the sound velocity,i.e.

$$\ddot{\bar{x}} \ \Delta l << 1 \tag{5}$$

where τ is the time required by the ion-sound waves to travel a distance equal to the width of the soliton Δl .

Accordingly , the localization of Langmuir waves and the radiation of the ion-sound waves are described by the following equations:

$$\partial_x^2 \ \xi - \frac{1}{2} \ \xi^2{}_m \ \xi + \xi^3 = 0 \tag{6}$$

$$(\partial_t^2 - \partial_x^2) \ N \ = - \ \ddot{\bar{x}} \ \partial_x \ \xi^2 \tag{7}$$

where ξ_m is the maximum value of the Langmuir soliton.

If the width of the inhomogeneous plasma ΔL was much greater than the soliton width Δl (equations (6) and (7) are valid only in this case), then the motion of the center of the soliton could be described by:

$$\ddot{\bar{x}} = -\frac{1}{2} \quad \partial_x \ n \quad |_{x\ =\bar{x}} \tag{8}$$

where $n(x)$ is a small deviation of the ion concentration (from its equilibrium value n_0) caused by the inhomogeneity,($n(x) \ll n_0$).

The solitary solution of equation (6) has the form:

$$\xi = \frac{\xi_m}{cosh(\frac{x-\bar{x}(t)}{\Delta l})} \tag{9}$$

where the maximum value ξ_m is related to the soliton width Δl by the relation

$$\Delta l = \frac{\sqrt{2}}{\xi_m} \tag{10}$$

Equation (7) describes the radiation of ion-sound waves.It clearly shows that only the soliton moving with acceleration can radiate ion-sound waves. However, when $t \to -\infty$, the soliton was moving with constant velocity , and the radiation was absent.

On considering the change of acceleration in the period τ to be small , Kaw et al. (1982), then the solution of (7) yields:

$$N = \frac{1}{2}\xi_m^2 \ \Delta l \ \{\ddot{\bar{x}}(t + x - \bar{x}(t)) \ [tanh\frac{x - \bar{x}(t)}{\Delta l} - 1] \ +$$

$$\ddot{\bar{x}}(t - x + \bar{x}(t))[tanh\frac{x - \bar{x}(t)}{\Delta l} + 1]\} \tag{11}$$

Let us now consider the case in which the plasma inhomogeneity has the form of a barrier of width ΔL :

$$n(x) = \frac{n_m}{4 \ cosh^2 \frac{x}{\Delta L}} \tag{12}$$

With the help of equation (8), the following expression is obtained for the soliton velocity:

$$\dot{\bar{x}}^2 = V^2 - n(x) \tag{13}$$

where V is the initial constant velocity of the soliton when it was far from the barrier, $|x| \gg \Delta L$. To be more definite , we shall consider that initially the soliton was far to the left and moving toward the barrier .i.e $\bar{x}(t) < 0$, $\dot{\bar{x}}(t) > 0$ and $|\bar{x}(t)| \gg \Delta L$, when $t \to -\infty$.

Hence forward, we shall consider the kinetic energy of the soliton to be much less than the potential energy of the plasma inhomogeneity, i.e. $V^2 < n_m$. This condition means that the soliton is reflected at a certian point , say x_0 , of the barrier. The point x_0 could be determined from the condition $\dot{\bar{x}} = 0$. It follows from (13) that

$$V^2 = n(x_0), \tag{14}$$

The same calculation could be done if the point of reflection was at the right wing of the barrier,i.e.when $x_0 < 0$, $|x_0| > \Delta L$. This is the reason why , in equation (8) , we can use the asymptotical distribution of the left wing of the barrier (12) ,i.e.

$$n(x) = n_m \ e^{\frac{2x}{\Delta L}} \tag{15}$$

$$x \ \leq \ x_0 \ < \ 0 \tag{16}$$

Substituting (15) in (8), the equation which describes the coordinate of the center of soliton is written as:

$$\bar{x}(t) = -|x_0| - \Delta L \ \ ln \ cosh \ \frac{Vt}{\Delta L} \tag{17}$$

$$\dot{x}(t) = - \; V \; tanh(\frac{V\,t}{\Delta L}) \quad (18)$$

and

$$\ddot{x}(t) = -(\frac{v^2}{\Delta L}) \; \frac{1}{cosh^2 \frac{Vt}{\Delta L}} \quad (19)$$

We shall choose the moment $t = 0$ to be the moment at which the soliton exists at the point of reflection x_0 . According to (14), and (15), the point of reflection is determined by :

$$|x_0| = \frac{\Delta L}{2} \; ln(\frac{n_m}{V^2}) \quad (20)$$

It is clear from (18) that when $t < 0$, the soliton moves from the left to the right toward the barrier, while, when $t > 0$, the soliton moves from the right to the left going away from the barrier. At the moment $t = 0$, the soliton velocity $\dot{x}(0)$ equals zero, while its acceleration has its maximum value.

2. On substituting (19) in (11), the following expression is derived for the perturbed concentration in the ion-sound waves:

$$N = -\frac{1}{2} \; \xi_m^2 \; V^2 \; \frac{\Delta L}{\Delta l} \; \{ \frac{tanh\frac{x - x(t)}{\Delta l} - 1}{cosh^2[\frac{V}{\Delta L}(t + x - x(t))]} + \frac{tanh\frac{x - x(t)}{\Delta l} + 1}{cosh^2[\frac{V}{\Delta L}(t - x + x(t))]} \} \quad (21)$$

Let us determine the spatial distribution of the ion-sound field in the following characteristic points of the soliton:

(a) Initially, when the soliton exists far from the barrier,i.e.

$$|x(t)| \gg \Delta L \quad , \quad |t| \gg \frac{\Delta L}{V} \quad (22)$$

It is easy to show from (18) and (21) that the soliton moving with constant velocity does not radiate, i.e.,

$$N = 0 \quad (23)$$

(b) When the soliton exists at the left of the barrier, moving from the left to the right, but still does not reach the point of reflection, i.e. when

$$|t| < \frac{\Delta L}{2V} \quad , \quad t < 0 \quad (24)$$

The qualitative spatial distribution of the perturbed concentration corresponds to this moment is shown in figure (1). In front of the soliton, the perturbed concentration is negative,($N < 0$), while it is positive, ($N > 0$) , behind the soliton . The concentration changes its sign at the center of the soliton. The fronts of the waves are at a distance Δs_1 $(= \frac{\Delta L}{2V} - |t|)$ from the center of the soliton.

(c) When the soliton exists at the point of reflection,i.e.

$$x(t) = x_0 \quad (25)$$

In this case the spatial distribution of the perturbed concentration preserves its general feature obtained in case (b). The only difference, as was expected, is that : the fronts of the waves are at a relative larger distance from the center of the soliton. The corrospnding qualitative spatial distribution of the perturbed concentration is shown in figure (2).

(d) When the soliton is already reflected from the barrier , and being moved from the right to the left, but still exists at the left wing of the barrier, i.e.

$$t < \frac{\Delta L}{2V} \quad , \quad t > 0 \quad (26)$$

The corresponding qualitative spatial distribution is given in figure (3).The fronts of the waves become at a more larger distance from the soliton. This distance approximately equals

$$\Delta s_3 \; = \; \frac{\Delta L}{2\,V} \; + \; t$$

But, in this case, in front of the soliton, the perturbed concentration is positive, ($N > 0$) , and it is negative, ($N < 0$) , behind the soliton. This characteristic distribution is in agreement with the general feature of radiation of [4], where it was found that in front of the soliton moving with positive acceleration, the concentration is positive, and it is negative behind it. If the soliton was decelerated, then the perturbed concentration changes its sign. In our case, after reflection, the soliton moves with positive acceleration, and therefore the concentration is positive in front of the soliton. Before reflection, the perturbed concentration was also positive in the same region. Thus, the general feature of the qualitative spatial distribution of the perturbed concentration has the same form as obtained in case (b).

(e) When the soliton, after reflection, becomes at a large distance far from the barrier and it continues its motion with constant velocity, i.e.

$$vt \; \gg \; \Delta L \quad (27)$$

In this case the soliton stops to radiate, since its acceleration becomes zero. According to (18), the soliton velocity in this case equals:

$$\dot{x} = -V \tag{28}$$

After reflection and far from the barrier, two pulses are separated. They are propagating to the right and to the left of the soliton with the ion- sound velocity.The centers of these waves are located at a distance Δs_4 (= t) from the center of the soliton. The width of each pulse is approximately equal to $\frac{\Delta L}{V}$. The qualitative spatial distribution of the perturbed concentration in this case is shown in figure (4).

3. According to (21), the maximum value of the perturbed concentration is :

$$N \simeq \xi_m^2 (\frac{\Delta l}{\Delta L}) V^2 \tag{29}$$

The ratio $(\frac{\Delta l}{\Delta L})$ is small, and also $V^2 \ll 1$, then it is clear from (29) that the condition (2) is satisfied. This condition was necessary to neglect the recoil effects of the radiation on the soliton. This condition allows us to consider that the energy gained by the radiation is small in comparison with the energy of the soliton.

With the help of the law of energy consevation , we can thus estimate the energy flux gained by the radiation. The law of energy conservation of the system" soliton + radiation field " has the following form (Kaw et al. and Degtyarev et al.)

$$\partial_t Q = -\partial_x \{ \frac{i}{2} [\partial_x E \partial_x \partial_x E^* - \partial_x E^* \partial_x \partial_x E]$$

$$+ \frac{i}{2} [n(x) + \delta n] [E \partial_x E^* - E^* \partial_x E] + \delta n V + V |E|^2] \} \tag{30}$$

where

$$Q = \partial_x E \partial_x E^* + n(x) \ |E|^2 + \delta n \ |E|^2 + \frac{V^2}{2} + \frac{\delta n^2}{2} (31)$$

$$\partial_t \delta n = -\partial_x V, \tag{32}$$

We shall determine the total energy in a volume bounded by two perpendicular (to the x-axis) surfaces. These two surfeces are supposed to be located at the points $-\Delta X$ and $+\Delta X$, which are chosen in such a way that the soliton, after reflection, and the radiation exist inside the volume (Kaw et al.) :

$$\Delta X \gg |x(t)| \quad , \quad t \gg \frac{\Delta L}{V} \tag{33}$$

With the help of (1) , (9) , (21) , and (30) the following equation is derived to express the change in the total energy :

$$\partial_t W = -Nu \ |_{-\Delta X}^{+\Delta X} = -2P \tag{34}$$

where

$$W = \int_{-\Delta X}^{+\Delta X} Q dx \tag{35}$$

the velocity (u) of the particle in the ion-sound waves is determined from the continuity equation :

$$\partial_x u = -\partial_t N \tag{36}$$

and P is the energy flux through each surface.

Finally, from (21) and (36), the energy flux P is given by :

$$P = 2 \xi_m^2 \ \frac{V^4}{(L)^2} \ \frac{1}{cosh^4 \frac{V}{(\Delta L)^3} (t - \Delta L)} \tag{37}$$

From (37) it is clear that P is proportional with V^4 .

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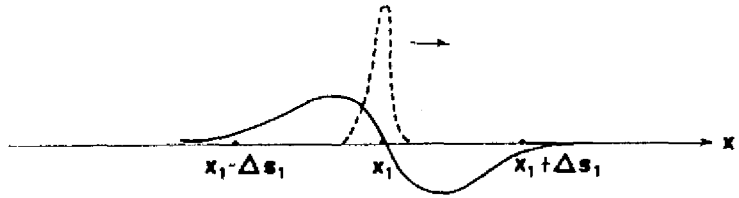
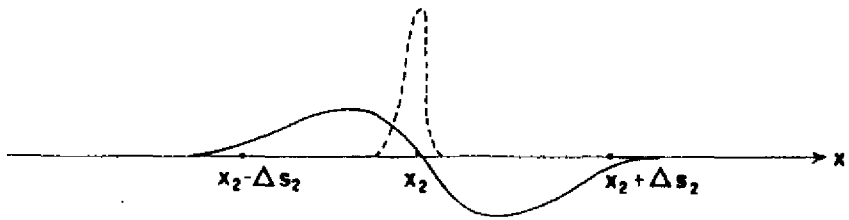
FIGURE CAPTIONS

Qualitative spatial distribution of the radiation field at different positions of the soliton.



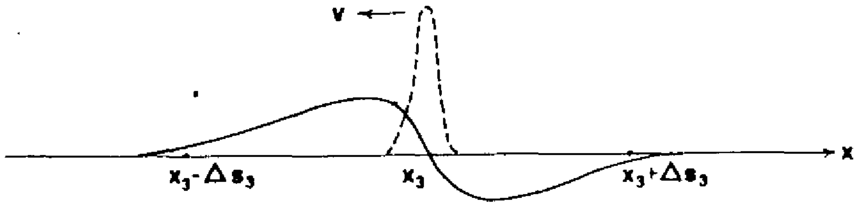
$x_1 = -|\bar{x}(t)|$, $\Delta s_1 = \frac{\Delta L}{2v} - |t|$

Fig.1



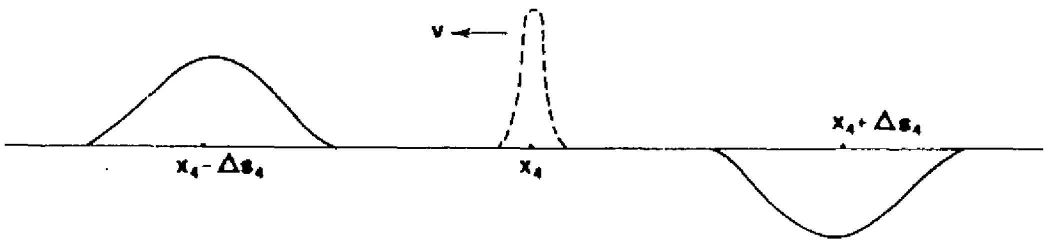
$x_2 = -|x_0|$, $\Delta s_2 = \frac{\Delta L}{2v}$

Fig.2



$x_3 = - |\bar{x}(t)|$, $\Delta s_3 = \frac{\Delta L}{2v} + t$

Fig.3



$x_4 = \bar{x}(t)$, $\Delta s_4 = t$

Fig.4

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