FR 8902499

# WEAKLY INTERACTING MASSIVE PARTICLES AND STELLAR STRUCTURE


Alain Bouquet*
LAPP, BP909, 74019 ANNECY LE VIEUX Cedex, France

Jean Kaplan and François Martin
LPTHE, Université Paris VII, 75251 PARIS Cedex 05, France



ABSTRACT : The existence of weakly interacting massive particles (WIMPs) may solve both the dark matter problem and the solar neutrino problem. Such particles affect the energy transport in the stellar cores and change the stellar structure. We present the results of an analytic approximation to compute these effects in a self-consistent way. These results can be applied to many different stars, but we focus on the decrease of the $^8B$ neutrino flux in the case of the Sun.



LPTHE Paris 88-22 LAPP TH-220/88


* On leave of absence from LPTHE, Université Paris VII, 75251 PARIS Cedex 05

**1-Introduction**

The dark matter problem is one of the most controversial issues of modern astrophysics (see for instance Kormandy and Knapp, 1987). The nature of this dark matter remains unknown, but the existence of a weakly interacting massive particle (hence the acronym WIMP) is one of the most interesting possibilities suggested to solve this problem (Silk, 1984). Candidates for such a particle include a heavy neutrino, the photino $\tilde{\gamma}$ or the sneutrino $\tilde{\nu}$ of supersymmetric theories (Nilles, 1984) or more exotic particles (Raby and West, 1987 and 1988; Gelmini, Hall and Lin, 1987; Ross and Segré, 1987).

It was realised (Steigman et al., 1978; Spergel and Press, 1985; Press and Spergel, 1985; Krauss, 1985; Faulkner and Gilliland, 1985) that these particles may also solve the long standing solar neutrino problem (Davis, 1978; Davis, 1986; Bahcall, Davis and Wolfenstein, 1988). The basic idea is that particles constituing the dark matter present in the halo of our Galaxy can be trapped when they cross the Sun, and accumulate in the course of time. Their number remains quite small, less than one per $10^9$ nuclei, nevertheless they can carry enough energy out of the core to cool it. The star readjusts its structure, due to this energy drain. If the central temperature decreases by a few percent, the flux of neutrinos coming from the decay of $^8B$ may go down by a factor of about 3 and agree with the experimental results (Davis, 1978; Davis, 1986; Koshiba, 1988). This change of structure is very difficult to compute in a self-consistent way. In principle, the best way would be to run a stellar numerical code (Gilliland et al., 1986) because it takes into account the complexities of the structure and the chemical evolution of the star. However, it takes a very long time on big computers, and the results are sometimes difficult to understand [1]. On the contrary, an analytic approximation allows a fast study of many different stars, and a quick scan of the WIMP parameter domain (mass and cross-sections). There is a price to pay : the stellar evolution cannot be followed. Moreover, it is only an approximation.

We studied the effects of WIMPs on a stellar structure, pushing the analytic resolution of the structure equations as far as possible. It turns out that 2 parameters, that we called W and $\eta$, govern WIMP effects : W measures the efficiency of WIMPs in transfering energy compared to photons, and in all interesting cases W>>1 , while $\eta$ is a measure of the relative variation of stellar parameters (temperature, density, etc) over the WIMP zone. In all interesting cases $\eta < 0.1$ : therefore, we can make an expansion in $\eta$ and neglect $\eta^2$, $\eta^3$, ... terms, which considerably simplifies the equations. Up to first order in $\eta$, the relative changes due to WIMPs ($\delta T/T$, $\delta\rho/\rho$...) in the stellar parameters are universal (independant of the star) when expressed as functions of W and $\eta$. We start

---

1 Moreover, numerical instabilities appear when WIMP energy transport ceases to be negligible (Bouquet, Salati, Cassé and Prantzos,1987; Faulkner and Swenson, 1988).

from a stellar structure without WIMPs, with given density and temperature profiles, solutions of the equations of hydrostatic and radiative equilibrium (Chandrasekhar, 1938; Clayton, 1968). We add WIMP energy transport to the equation of radiative equilibrium and we compute the new equilibrium state of the star. The solution is completely analytic for small W, but requires one (universal) numerical quadrature in the case of large W. We find that the temperature gradient is indeed lowered due to WIMP energy transport, and that the central temperature decreases while the central density increases : this leads to a decrease of the total luminosity and of the solar neutrino flux, as expected from qualitative arguments. Interestingly enough, we find a kind of saturation : the changes in the stellar structure are linear in W for small W, but become logarithmic in W for large W.

Section 2 recalls the dynamics of the capture of WIMPs from the galactic halo, the equilibrium reached by the WIMP gas in the star, and the energy transfer due to them. Section 3 presents the analytic approach to the change in the stellar structure, when WIMPs are a small perturbation, and then extends these results to the more interesting case of large effects. Section 4 applies these results to the case of the Sun, and we compute the reduction of the flux of neutrinos from $^8$B as a function of the mass and cross-sections of the WIMP.

**2-WIMP effects on stars**

Let us consider the capture of WIMPs from the galactic halo by a star. Particles constituting the halo have large velocities and eventually cross the surface of a star. Inside the star, they may collide with nuclei and lose enough kinetic energy to be unable to escape to infinity : they are trapped.

The halo is usually modelled by an isothermal sphere (Bahcall and Soneira, 1980; Bahcall and Casertano, 1985; Burstein and Rubin, 1985; Blumenthal et al., 1986) with an isotropic maxwellian distribution of velocities, of mean value $V_{halo} = 300$km/s. For definiteness, we take the density of the halo to be $\rho_{halo} = 0.01 M_{\odot}/pc^3$ in the solar neighbourhood. To compute the number of WIMPs captured, we take the number of incoming particles, multiply by the probability of a collision in which enough energy is lost to lead to capture, and sum over all possible trajectories inside the star (Press and Spergel, 1985). The number of captures per unit time strongly depends on the scattering cross-section $\sigma_i$ of a WIMP on a nucleus i (i = H, He, C, N, O...) : for low cross-section, the probability of capture is weak and proportional to $\sigma_i \langle X_i \rangle / m_i$, where $\langle X_i \rangle$ is the mean mass fraction of nucleus i of mass $m_i$ . On the contrary, for large cross-sections, any WIMP which enters the star is captured, and the captured flux then saturates at the level of the incoming flux. For a star of mass M, the turnover takes place at a "critical" value $\sigma_c(M)$ of the cross-section which roughly corresponds to a mean free path of the same order as the radius R of the star, that is :

$$\sigma_c(M) \simeq m_p \frac{R^2}{M} \simeq 4\ 10^{-36}\,\mathrm{cm}^2 \left(\frac{R}{R_\odot}\right)^2 \left(\frac{M}{M_\odot}\right)^{-1} \qquad (1)$$

where $R_\odot$ and $M_\odot$ are the solar radius and mass. The capture rate is the sum of two components : one, $F_{geo}$, is proportional to the geometric section $\pi R^2$ of the star, and the other one, $F_{grav}$, is proportional to the mass M of the star, and is due to the gravitational bending of the trajectory of the particle. For a large range of stellar masses M (and corresponding radii R) and of WIMP masses $m_w$ and cross-sections $\sigma_i$ , the capture rate of WIMPs is well approximated by :

$$F(M,m_w,\sigma_i) = F_{geo} + F_{grav} \qquad (2a)$$

$$F_{geo} \simeq 1.64\ 10^{29}\,\mathrm{s}^{-1}\ \frac{\rho_{halo}}{0.01\ M_\odot/\mathrm{pc}^3} \frac{V_{halo}}{300\ \mathrm{km/s}} \frac{m_p}{m_w} \frac{R^2}{R_\odot^2}\ \mathrm{Min}\left\{1, \sum_i \frac{\sigma_i \langle X_i \rangle}{\sigma_c(M)} \frac{m_p}{m_i}\right\} \qquad (2b)$$

$$F_{grav} \simeq 1.04\ 10^{30}\,\mathrm{s}^{-1}\ \frac{\rho_{halo}}{0.01\ M_\odot/\mathrm{pc}^3} \frac{300\ \mathrm{km/s}}{V_{halo}} \frac{m_p}{m_w} \frac{R}{R_\odot} \frac{M}{M_\odot}\ \mathrm{Min}\left\{1, \sum_i \frac{\sigma_i \langle X_i \rangle}{\sigma_c(M)} \frac{m_p}{m_i}\right\} \qquad (2c)$$

where $m_p$ is the proton mass. The geometric part $F_{geo}$ is only important for light stars ($M \ll M_\odot$). An enhanced capture rate might be expected in binary systems (Spergel, 1988) where dynamics may allow a particle to orbit many times in their neighbourhood before being captured by one of the stars. Note also that if WIMPs make up the dark matter of the galactic disk, which has a smaller mean velocity ($V_{disk} \simeq 30$ km/s) and a larger density ($\rho_{disk} \simeq 0.1\ M_\odot/\mathrm{pc}^3$), the capture rate is much larger. However, the very existence of dark matter in the disk is debated (Kormandy and Knapp, 1987; Robin, 1988) and in any case, it is unlikely that non-dissipative matter like WIMPs contribute much to it.

It is important to note that the capture rate depends only weakly on the internal structure of the star (Bouquet and Salati, 1987). We performed an exact computation of the capture rate, using polytropic approximations as well as "real" structures , i.e. given by numerical evolution codes (Cahen et al., 1986; Cassé et al., 1986), and found nearly identical results. For a red giant with a very dense core surrounded by a diffuse atmosphere, only the core contributes to the capture (unless the cross-sections $\sigma_i$ are unreasonably large), and R and M in the above formulas should be interpreted as the mass and radius of the core. We also took into account the motion of the star relative to the dark matter halo, but this only decreases the capture rate by 10% to 20% (Bouquet and Salati, 1987), which is smaller than the uncertainties in the halo density.

Once captured, a WIMP experiences collisions with nuclei. From time to time, a collision may give sufficient energy to a WIMP to escape from the star : the particle evaporates. For cross-sections of the order of the "critical" cross-section $\sigma_c$, the evaporation rate is exponentially suppressed for WIMPs heavier than a few GeV/$c^2$, and can be neglected (Gould, 1987). The WIMP gas tends towards thermalization with baryonic matter, with a time scale (a few collisions) much shorter than the time scale of stellar evolution. If the cross-sections $\sigma_i < \sigma_c$, the mean free path is large and two successive collisions are widely separated. We then assume that WIMPs behave as an isothermal gas with temperature $T_w$, which is an average of the temperatures of the regions in the star through which the particles move (Spergel and Press, 1985). The spatial WIMP distribution $n_w(r)$ is then simply the barometric equilibrium density :

$$n_w(r) = n_w(0)\, e^{-\frac{m_w U(r)}{kT_w}}$$

where $U(r)$ is the gravitational potential at distance r from the center of the star (normalized to zero at the center). This exponential decrease of the WIMP density $n_w(r)$ means that WIMPs are concentrated in the core of the star : if the WIMP mass is larger than a few GeV/$c^2$, most of them stay in the inner core of the star. We showed (Bouquet and Salati, 1987) that the WIMP distribution is Gaussian to a very good approximation, simply because their equilibrium temperature $T_w$ is nearly equal to the central temperature $T_c$ of the star (in numerical computations, we can replace $T_w$ by $T_c$ with a negligible error), the density $\rho(r)$ of stellar material is nearly equal to the central density $\rho_c$, and therefore the gravitational potential is nearly parabolic :

$$n_w(r) \approx n_w(0)\, e^{-\frac{r^2}{r_w^2}} \tag{3a}$$

where :

$$r_w^2 = \frac{3kT_w}{2\pi G \rho_c m_w} \tag{3b}$$

measures the spatial extension of WIMPs. For the Sun, $r_w^2 \approx 0.01\, R_\odot^2 m_p/m_w$.

The central density of WIMPs $n_w(0)$ depends whether WIMPs can annihilate by pairs (like photinos) or not. If WIMPs annihilate by pairs, an equilibrium will be reached between capture and annihilation in a few million years, and there will be as many WIMPs annihilating than accreting. Let $\sigma_{ann}$ be the annihilation cross-section and V the mean relative velocity of WIMPs. The derivation of the central density of WIMPs is straightforward and gives :

$$n_w(0) \approx 1.13\ 10^{13}\ \mathrm{cm}^{-3} \left(\frac{m_p}{m_w}\right)^{-0.25} \left[\frac{R_\odot^2}{R^2} \frac{M}{M_\odot} \frac{10^{-26}\mathrm{cm}^3/\mathrm{s}}{\sigma_{ann} V}\ \mathrm{Min}\left\{1, \sum_i \frac{\sigma_i \langle X_i \rangle}{\sigma_c(M)} \frac{m_p}{m_i}\right\}\right]^{0.5}$$

Most of the candidate particles annihilate by pairs, and the resulting density is too low to be of any effect on the stellar structure (Krauss, 1985; Krauss et al., 1985). To avoid this, one must assume the existence of a conserved quantum number (Gelmini, Hall and Lin, 1987; Raby and West, 1987) and of a cosmic asymmetry with a negligible number of anti-particles (Griest and Seckel, 1987). The total number then increases steadily with time, and WIMPs accumulate inside the star. The stellar mass and the stellar radius remain roughly constant during the evolution, and the capture rate depends only weakly on the internal structure of the star. Therefore the total number of WIMPs is approximately given by $N(t) \approx F\,t$, where the capture rate F is given by Equ.2. The WIMP distribution $n_w(r)$ is nearly Gaussian (Equ.3), and we may write :

$$N(t) \approx n_w(0)\,\pi\sqrt{\pi}\,r_w^3 \approx F\,t$$

For homologous stars, the WIMP spatial extension $r_w$ scales as the radius R of the star, and the central density $n_w(0)$ of WIMPs is approximately given at time t by :

$$n_w(0) \approx 3.7\ 10^{16}\,\mathrm{cm}^{-3}\,\frac{t}{10^{17}\mathrm{s}}\left(\frac{m_w}{m_p}\right)^{0.5}\frac{R_\odot^2}{R^2}\,\frac{M}{M_\odot}\,\mathrm{Min}\left\{1,\sum_i \frac{\sigma_i\langle X_i\rangle}{\sigma_c(M)}\,\frac{m_p}{m_i}\right\} \qquad (4)$$

for main sequence stars (Bouquet and Salati, 1987). For comparison, the central density of hydrogen in the Sun is $n_H(0) \approx 3.2\ 10^{25}\ \mathrm{cm}^{-3}$, $10^9$ times larger.

Since WIMPs interact weakly with nuclei, two successive interactions occur in separate regions of the star. A collision with a nucleus at temperature $T(r) > T_w$ will (on the average) transfer some energy from the nucleus to the WIMP, while a collision with a nucleus at temperature $T(r) < T_w$ will transfer energy from the WIMP to the nucleus. WIMPs therefore transfer heat from the hot inner core to the colder outer core of the star, supplementing the normal energy transfer by radiation or convection. The energy transferred per unit time and unit mass from nuclei to WIMPs can be written as (Spergel and Press, 1985) :

$$\varepsilon_w(r) = 8\sqrt{\frac{2}{\pi}}\,n_w(r)\sum_i \sigma_i\,X_i(r)\,\frac{m_w}{(m_w+m_i)^2}\left(\frac{m_w T(r)+m_i T_w}{m_w\,m_i}\right)^{1/2} k^{3/2}\,[T(r)-T_w] \qquad (5)$$

where $X_i(r)$ is the r-dependant mass fraction of nuclei i and the sum runs overs all species of nuclei. Numerical simulations (Nauenberg, 1987) show that this equation slightly overestimates the WIMP energy transfer, but the general behaviour is correctly reproduced.

If WIMPs do not evaporate, the total luminosity $L_w$ carried out of the star by WIMPs is zero :

$$L_w(R) = \int_0^R 4\pi r^2 \, \varepsilon_w(r) \, \rho(r) \, dr = 0$$

and this fixes the equilibrium temperature $T_w$ appearing in $\varepsilon_w(r)$. The maximal efficiency for cooling requires the largest possible number of WIMPs, and therefore a scattering cross-section $\sigma_i \gtrsim \sigma_c(M)$ from Equ.2, but also a mean free path large enough to transfer energy between inner and outer regions of the star, and therefore requires $\sigma_i \lesssim \sigma_c(M)$ from Equ.1. For cross-sections larger than about $25\sigma_c(M)$, the energy transfer proceeds by thermal conduction, scales as $1/\sigma_i$ (Gilliland et al., 1986) and is much less efficient. When the WIMP mass increases, the energy transfer decreases, as Equ.5 shows, and the range $r_w$ over which energy is transferred also decreases. Maximal efficiency therefore requires cross-sections of the order of $\sigma_c(M)$, and a WIMP mass not much larger than the minimum required for non-evaporation. This means $\sigma_i \approx 4\ 10^{-36}\ cm^2$ and $m_w \gtrsim 4\ m_p$ in the case of the Sun (Gould, 1987). Therefore, we shall study the changes in the stellar structure under these assumptions, which are the most favourable to the WIMP hypothesis. To get some feeling of the numbers involved, Equ.5 approximately gives in the case of the Sun :

$$\varepsilon_w(r) \approx 400\ \text{erg/g/s} \left(\frac{4\ m_p}{m_w}\right)^{1.5} \left(\frac{\sigma_p}{4\ 10^{-36}\ cm^2}\right)^2 \left(\frac{3}{2} - \frac{r^2}{r_w^2}\right) e^{-r^2/r_w^2}$$

for non-annihilating WIMPs with $\sigma_p < 4\ 10^{-36}\ cm^2$ (neglecting for a while possible changes in the solar structure). This can easily be larger than the nuclear energy produced in the core $\varepsilon_N \approx 17$ erg/g/s (Turck-Chièze et al., 1988), and therefore nuclei are more efficiently cooled by the WIMP energy drag than heated by nuclear reactions. The whole problem that we address in this paper is : how does the star adjust itself to this quite unusual situation ?

What may be expected ? WIMPs supply another way of transfering energy from the energy producing core to the outside, and thus supplement the usual transfer by photon diffusion. Under favourable circumstances, we just saw that WIMP energy transfer could be very efficient : it may avoid convection, energy transfer by bulk movements of the material of the star which takes place when photons cannot carry away all the nuclear energy generated in the center. Since WIMPs are concentrated in the core, they may suppress core convection in main sequence stars (core hydrogen burning) and horizontal branch stars (core helium burning). No effect can be expected in stars which already have a nearly isothermal core (stars of the asymptotic giants branch, white dwarves). Convection has an important side effect on the lifetime of the star : by mixing fresh nuclear fuel with burnt one, it increases the amount of usable fuel, and therefore the lifetime of the star (for a given energy generation rate). Preventing convection will probably shorten the lifetime of horizontal branch stars (Renzini, 1987; Spergel and Faulkner, 1988; Bouquet, Raffelt, Salati and Silk, 1988) in contradiction with observations. For main sequence stars, a simple computation (Bouquet, Kaplan and Salati, 1987) shows that WIMPs can suppress convection for low mass stars ($M < M_{\odot}$), but since these stars already have a very long lifetime, it is not clear that the suppression of convection can be observable. On the other hand, convection remains for heavy stars because they have a short lifetime and cannot accrete a large number of WIMPs, and moreover the energy generation rate increases faster with temperature than the WIMP transport rate.

A subtler effect is a small change in the structure of a star in *radiative* equilibrium, but sensitive quantities such as the $^8B$ neutrino flux from the Sun may change dramatically. This requires a computation of the structure change, and will be done in the next section.

## 3 - Analytic approach to a perturbed stellar structure

Because WIMPs supply another way of transfering nuclear energy from the core of a star to the outside, their presence changes the radiative equilibrium and decreases the temperature, while the hydrostatic equilibrium is recovered by an increase in density. These effects are purely mechanical, and the star reacts instantaneously to the perturbation brought about by WIMPs. Our analytic approach aims at a computation of such changes of the stellar structure. But the stellar evolution, also, is different in the presence of WIMPs : a reduction of the central temperature leads to a lower consumption rate of hydrogen (or helium), and therefore a different chemical composition $X_i(r)$ after a time t. Our approach cannot take into account these evolutionary effects, which are very difficult to estimate but can be simulated by changing the chemical composition.

### 3.1 Stellar structure equations

We compare a star of mass M without WIMPs to a star of same mass but having accreted WIMPs during its life. In both cases we assume that the star is in hydrostatic equilibrium, and that energy transport proceeds through photons (and WIMPs when they are present). In the following, unless otherwise stated, quantities with index 0 refer to the star without WIMPs : the star without WIMPs has a radius $R_0$, a density profile $\rho_0(r)$ and a temperature profile $T_0(r)$ , completely determined as functions of the mass M and the chemical composition by the equations of hydrostatic and radiative equilibrium (Chandrasekhar, 1938; Clayton, 1968), supplemented by the boundary conditions $\rho_0(R_0) = T_0(R_0) = 0$ [(1)]. As a result one gets a relation between M and $R_0$ (which defines for

---

1 In fact the surface temperature is not zero, but it is so low compared to the central temperature that for our purpose it is the same.

instance the main sequence). The star with WIMPs has a radius R, a density profile $\rho(r)$ and a temperature profile $T(r)$, again determined by equations of hydrostatic and luminosity equilibrium supplemented by the boundary conditions $\rho(R) = T(R) = 0$.

Because the number density of WIMPs is so low compared to nuclei (Equ.4), they do not contribute directly to the equation of hydrostatic equilibrium :

$$\frac{d}{dr} P(r) = -\frac{G}{r^2} \rho(r) \int_0^r 4\pi r'^2 \rho(r')\, dr' \qquad (6)$$

They contribute indirectly, of course, through the changes induced in the density, temperature and pressure profiles. The pressure $P(r)$ is related to the density and temperature by the equation of perfect gases, which is a very good approximation to the true equation of state for stars of mass around the solar mass (Clayton, 1968) :

$$P(r) = \frac{\rho(r)\, k\, T(r)}{\mu(r)\, m_p}$$

where $\mu(r)$ is the mean molecular weight in units of the proton mass $m_p$ :

$$\mu(r)^{-1} = \sum_i \frac{Z_i+1}{A_i} X_i(r)$$

$Z_i$ and $A_i$ being the charge and mass numbers of nucleus i. In the treatment that we propose, we cannot take into account the change in chemical composition induced by the accretion of WIMPs. Therefore, we take $X_i(r)$ and $\mu(r)$ to be the same in the star with and without WIMPs [(1)] :

$$X_i(r) = X_{i0}(r)$$

The equation of radiative equilibrium can be written as :

$$L_N(r) = L_\gamma(r) + L_w(r) \qquad (7)$$

where $L_N$ is the luminosity generated by nuclear reactions, $L_\gamma$ the luminosity carried by photon radiation, and $L_w$ the luminosity transported by WIMPs. The nuclear luminosity $L_N$ is defined by :

---

[1] Of course, since the stellar evolution is different in the presence of WIMPs, the run $X_i(r)$ of the chemical composition must be different. However, the difference will not show up in the changes of the stellar structure (only the central value $X_i(0)$ appears in the definition of W).

$$L_N(r) = \int_0^r 4\pi r'^2 \, \varepsilon_N(r') \, \rho(r') \, dr' \qquad (8)$$

We parametrize the total nuclear energy generation rate $\varepsilon_N$ by the following approximate analytic form, valid in a limited temperature range (Clayton, 1968) :

$$\varepsilon_N(r) \approx \varepsilon_{norm} \, \hat{\varepsilon}(X_i(r)) \left(\frac{\rho(r)}{\rho_{norm}}\right)^m \left(\frac{T(r)}{T_{norm}}\right)^n$$

$\varepsilon_{norm}$ , $\hat{\varepsilon}$, m and n are known quantities which depend on the dominant nuclear cycle. $\rho_{norm}$ and $T_{norm}$ are normalization points that we choose to be $\rho_{norm}$ = 148 g/cm$^3$ and $T_{norm}$ = 15.5 MK, and $\hat{\varepsilon}$ is normalized so that $\hat{\varepsilon}(X_i(0)) = 1$ . For instance, for the pp chain in hydrogen burning :

$$\hat{\varepsilon} \approx \frac{X_H^2(r)}{X_H^2(0)} \qquad \text{and} \qquad \varepsilon_{norm} \approx 14.3 \text{ erg/g/s} \quad \text{for} \quad X_H(0) \approx 0.36$$

The luminosity $L_\gamma$ carried by photons is given by :

$$L_\gamma = -\frac{64\pi\sigma_{Stefan} T^3 r^2}{3\,\kappa\rho} \frac{dT}{dr}$$

where $\sigma_{Stefan}$ is the Stefan-Boltzmann constant, and $\kappa(r)$ is an opacity function which we approximate by :

$$\kappa(r) \approx \kappa_{norm} \, \hat{\kappa}(X_i(r)) \left(\frac{\rho(r)}{\rho_{norm}}\right)^a \left(\frac{T(r)}{T_{norm}}\right)^b$$

$\kappa_{norm}$ , $\hat{\kappa}$, a and b are known quantities which depend on the density and temperature ranges, and also on the chemical composition. $\hat{\kappa}$ is normalized so that $\hat{\kappa}(X_i(0)) = 1$ . Opacity is one of the main source of uncertainties in our results (as it always is in stellar studies).

The WIMP luminosity $L_w$ is obtained by integrating Equ.5 :

$$L_w(r) = \int_0^r 4\pi r'^2 \, \varepsilon_w(r') \, \rho(r') \, dr'$$

$$L_w = 32\sqrt{2\pi} \int_0^r r'^2 n_w(r') \sum_i \sigma_i X_i(r') \frac{m_w}{(m_w+m_i)^2} \left(\frac{m_w T(r') + m_i T_w}{m_w\, m_i}\right)^{1/2} k^{3/2} [T(r') - T_w] \rho(r') dr'$$

In the remaining of this section, for simplicity, we consider only one species i of nucleus, namely hydrogen, and replace $m_i$ by $m_p$, $\sigma_i$ by $\sigma_p$, and $X_i$ by $X_H \equiv X$ (hydrogen abundance). Some WIMP candidates, such as the magnino (Raby and West, 1987) couple more strongly on hydrogen than on helium. Some do not even couple to spin zero nuclei, such as $^4He$, $^{12}C$, $^{16}O$. We shall see later how we can generalize to many species of nuclei.

### 3.2 Search for a perturbative solution

These equations have no analytical solution, hence the necessity to resort to computers. However, starting from a known solution in the absence of WIMPs, we can look for *corrections* brought about by WIMPs. We obtain more tractable differential equations, which can be solved under favourable circumstances. WIMPs change both the shape and normalisation of the density and temperature profiles. It is convenient to work with dimensionless quantities, and we introduce the dimensionless normalized profiles f, g and h :

$$\rho(r) = \rho_c\, f(r) \quad (9a)$$

$$T(r) = T_c\, g(r) \quad (9b)$$

$$\mu(r) = \mu_c\, h(r) \quad (9c)$$

and their counterpart with index 0 for the star without WIMPs. Our assumption that WIMPs do not change the chemical distribution implies that $\mu(r) = \mu_0(r)$. By definition $f(0) = g(0) = h(0) = 1$ . We first compute the change in the profile shapes ($f_0 \to f$ and $g_0 \to g$), and use them to compute the change in the normalisations ($\rho_{c0} \to \rho_c$ and $T_{c0} \to T_c$).We use the dimensionless variable $z = r/r_w$ instead of r ($r_w$ measures the spatial extension of WIMPs, Equ.3). The density of WIMPs is then large for $\lesssim 1$ and negligible for $z >> 1$ . The equation of hydrostatic equilibrium (Equ.6) writes :

$$\frac{d}{dz}\left[\frac{f(z)g(z)}{h(z)}\right] = -\frac{6A}{z^2}\frac{r_w^2}{R^2} f(z) \int_0^z dz'\, z'^2 f(z') \quad (10)$$

and the equation of radiative equilibrium (Equ.7) becomes :

$$\frac{d}{dz}g(z) = -\frac{6B}{z^2}\frac{r_w^2}{R^2}\frac{f^{1+a(z)}}{g^{3-b(z)}}\hat{\kappa}(z)\int_0^z dz'\, z'^2 f^{1+m}(z')\, g^n(z')\, \hat{\varepsilon}(z') \; + \tag{11}$$

$$+\, 4\,W\frac{f^{1+a(z)}}{g^{3-b(z)}\,z^2}\hat{\kappa}(z)\int_0^z dz'\, z'^2\left(\frac{g(z')+\frac{m_p}{m_w}\frac{T_w}{T_c}}{1+\frac{m_p}{m_w}\frac{T_w}{T_c}}\right)^{1/2}\frac{X(z')}{X(0)}\,f(z')\left(g(z')-\frac{T_w}{T_c}\right)e^{-z'^2}$$

where the dimensionful parameters of the problem are grouped in the dimensionless quantities A, B, and W :

$$A = \frac{2\pi\, G\, R^2\, \rho_c\, m_p\, \mu_c}{3\, k\, T_c} \tag{12}$$

$$B = \frac{\varepsilon_N(0)\,\kappa(0)\,\rho_c^2\,R^2}{32\,\sigma_{Stefan}\,T_c^4} \tag{13}$$

$$W = \frac{3\sqrt{2/\pi}\; n_w(0)\,(kT_c)^{3/2}\kappa(0)\,\rho_c^2\, r_w^2}{8\,\sigma_{Stefan}\, m_w \sqrt{m_p}\, T_c^4}\,\sigma_p\, X(0)\left(\frac{m_w}{m_p+m_w}\right)^2\left(1+\frac{m_p}{m_w}\frac{T_w}{T_c}\right)^{1/2} \tag{14}$$

We have used the Gaussian WIMP distribution (Equ.3) $n_w(z) = n_w(0)\exp(-z^2)$. The second term in the r.h.s. of Equ.11 is the correction due to WIMPs, and is weighted by the quantity W which plays a central role in our analysis. Physically, W is the ratio, at the center of the star, of the luminosities $L_w$ carried by WIMPs and $L_\gamma$ carried by photons (in the limit $W\to 0$). The WIMP equilibrium temperature $T_w$ is determined by the condition that WIMPs do not evaporate :

$$L_w(R) = 0 \Rightarrow \int_0^\infty dz\, z^2\left(g(z)+\frac{m_p}{m_w}\frac{T_w}{T_c}\right)^{1/2} X(z)\, f(z)\left(g(z)-\frac{T_w}{T_c}\right)e^{-z^2} = 0 \tag{15}$$

The problem now is to solve the equations of hydrostatic equilibrium, Equ.10, and radiative equilibrium, Equ.11, subject to the non-evaporation condition Equ.15, for the unknown functions f(z) and g(z), for a given chemical composition X(z) (or $h(z) = h_0(z)$). As we said, there is no analytic solution to this set of equations, not even in the absence of WIMPs. Our procedure starts from a known solution $f_0(z)$ and $g_0(z)$ of these equations without WIMPs, and treats theWIMP contribution as a perturbation. For the star without WIMPs the structure equations Equ.10-11 become :

$$\frac{d}{dz}\left[\frac{f_0(z)g_0(z)}{h_0(z)}\right] = -\frac{6A_0}{z^2}\frac{r_w^2}{R^2}f_0(z)\int_0^z dz'\, z'^2 f_0(z') \tag{16a}$$

$$\frac{d}{dz}g_0(z) = -\frac{6B_0}{z^2}\frac{r_w^2}{R^2}\frac{f_0^{1+a}(z)}{g_0^{3-b}(z)}\hat{\kappa}(z)\int_0^z dz'\, z'^2 f_0^{1+m}(z')\, g_0^n(z')\,\hat{\varepsilon}(z') \tag{16b}$$

where $A_0$ and $B_0$ [1] are given by Equ.13-14 with $\rho_c$, $T_c$ and R replaced by $\rho_{c0}$, $T_{c0}$ and $R_0$. It is worth noticing that boundary conditions $f_0(0)=g_0(0)=1$ and $f_0(R_0)=g_0(R_0)=0$ overdetermine the functions $f_0(r)$ and $g_0(r)$, and Equ.16 therefore determines $A_0$ and $B_0$ for a given chemical composition (and therefore known functions $\hat{\varepsilon}(z)$, $\hat{\kappa}(z)$ and $h_0(z)$). Thus $\rho_{c0}$, $T_{c0}$, and $R_0$ are fixed as functions of the star mass M by the very definitions of the mass M, $A_0$ and $B_0$.

Let us introduce the parameter $\eta$ :

$$\eta = B_0\frac{r_w^2}{R^2} \tag{17}$$

which, as we shall see, characterizes the WIMP influence (together with W). We search for a solution of the complete equations in the form :

$$f(z) = f_0(z)\big(1+\eta W f_w(z)\big) \tag{18a}$$

$$g(z) = g_0(z)\big(1+\eta W g_w(z)\big) \tag{18b}$$

Since $f(0) = g(0) = 1$ and $f_0(0) = g_0(0) = 1$, one has $f_w(0) = g_w(0) = 0$ .We have explicitly factorized out $\eta W$ because f(z) and g(z) should reduce to $f_0(z)$ and $g_0(z)$ when the effect of WIMPs vanishes.This means that when either W or $\eta$ goes to zero, $f_w$ and $g_w$ should not be singular at fixed $r = r_w z$ .This may happen either when W goes to zero (vanishing WIMP number, vanishing WIMP extension $r_w$ , infinite WIMP mass $m_w$ , or vanishing cross-section) or when $\eta$ goes to zero (vanishing WIMP extension $r_w$, or vanishing temperature "gradient" [2] $B_0$ of the unperturbed star). We expect $f_w(z)$ and $g_w(z)$ to decrease fast for $z >> 1$ , where WIMPs are very diffuse, and that is what we find. In all cases that we encounter, $\eta$ turns out to be smaller than 0.1 (see Section 4), which allows us to keep *only terms up to first order in* $\eta$ . Let us consider first the case of small W.

---

1 The $r_w^2$ dependance in Equ.16 for a star without WIMPs is spurious, and due to the definition of z.

2 Without WIMPs, $B_0 = -\lim_{r\to 0} R^2\frac{1}{T}\frac{dT}{dr^2}$

### 3.3 Solution for small W

We plug Equ.18 into the structure equations (Equ.10-11), keep only the terms of first order in $\eta$ and W, and use the fact that $f_0$ and $g_0$ are solutions of Equ.16, and that for small z :

$$f_0(z) \approx g_0(z) \approx h_0(z) \approx \hat{\varepsilon}(z) \approx \hat{\kappa}(z) \approx 1$$

The resulting differential equations for $f_w$ and $g_w$ can easily be integrated (details are given in the Appendix), and give :

$$f_w(z) \; = \; - g_w(z) \; = \; \exp(- z^2) \; - \; 1 \quad (+ \text{ terms of order } \eta \,)$$

Note that these correction factors do not depend on the choice of the initial profiles $f_0(z)$ and $g_0(z)$. The WIMP equilibrium temperature $T_w$, to first order in $\eta$ and W is given by :

$$T_w \; = \; T_c \left\{ 1 - \frac{3}{2}\eta \right\}$$

We get the normalized density and temperature profiles f(z) and g(z) for small z, to first order in $\eta$ and W :

$$f(z) = f_0(z) \left( 1 - \eta W (1 - e^{-z^2}) + \eta^2 W f_{w1}(z) \right) \tag{19a}$$

$$g(z) = g_0(z) \left( 1 + \eta W (1 - e^{-z^2}) + \eta^2 W g_{w1}(z) \right) \tag{19b}$$

These formulae have been derived in the $z < 1$ region, and we now turn to the region $z >> 1$ where WIMPs are very few. There the WIMP term in Equ.11 is very small (as $\exp(-z^2)$) because of the non-evaporation condition Equ.15, and can be forgotten. Therefore, in the large z region, f(z) and g(z) are solutions of the equations of hydrostatic and radiative equilibrium without WIMPs, but with parameters A and B instead of $A_0$ and $B_0$ . Let $\tilde{f}(z)$ and $\tilde{g}(z)$ be these solutions of Equ.10-11 without the WIMP term. These functions still vanish at the surface of the star, but are necessarily different from 1 at the center (otherwise, we would have $A = A_0$ and $B = B_0$, as we remarked before). It is easy to see that $\tilde{f}(z)$ and $\tilde{g}(z)$ only differ from $f_0(z)$ and $g_0(z)$ by a normalization factor :

$$\tilde{f}(z) = \tilde{f}(0)\, f_0(z)$$

$$\tilde{g}(z) = \tilde{g}(0)\, g_0(z)$$

Functions f(z) and g(z) tend very quickly to $\tilde{f}(z)$ and $\tilde{g}(z)$ at large z (up to exponentially small terms), and this precisely tells us that the functions $f_{w1}(z)$ and $g_{w1}(z)$ in Equ.19 are practically constant in z for large z . Thus the last terms in Equ.19 are truly of order $\eta^2$, and can be neglected. Therefore Equ.19 are valid for all z, to first order in $\eta$ and W, and, to this order, one can deduce :

$$\tilde{f}(0) = (1 - \eta W) \qquad (20a)$$

$$\tilde{g}(0) = (1 + \eta W) \qquad (20b)$$

Not only do we need the shape of the density and temperature profile, but we also need their normalisation. In our dimensionless approach, the central density $\rho_c$ and central temperature $T_c$ only appear in the definitions of A and B. So we must compute first the changes in A and B. We use the fact that $\tilde{f}(z)$ and $\tilde{g}(z)$ are solutions of Equ.16 with A and B replacing $A_0$ and $B_0$, and are normalized according to Equ.20, to get [1] :

$$\frac{\delta A}{A_0} = 2\,\eta W \qquad (21a)$$

$$\frac{\delta B}{B_0} = (6\text{-}b\text{+}a\text{+}m\text{-}n)\,\eta W \qquad (21b)$$

From the definition of A and B, we also obtain :

$$\frac{\delta A}{A_0} = \frac{\delta \rho_c}{\rho_{c0}} - \frac{\delta T_c}{T_{c0}} + 2\,\frac{\delta R}{R0} \qquad (22a)$$

$$\frac{\delta B}{B_0} = (2\text{+}m\text{+}a)\,\frac{\delta \rho_c}{\rho_{c0}} + (b\text{+}n\text{-}4)\,\frac{\delta T_c}{T_{c0}} + 2\,\frac{\delta R}{R0} \qquad (22b)$$

We need to know the change in the radius, which is given by the fact that the stars with and without WIMPs have the same mass M :

$$M = \frac{\rho_c R^3}{k_1} = \frac{\rho_{c0} R_0^3}{k_{1_0}}$$

where

$$k_1 = R^3 \left( 4\pi \int_0^R dr\, r^2 f(r/r_w) \right)^{-1}$$

which means that

$$\frac{\delta \rho_c}{\rho_{c0}} + 3\,\frac{\delta R}{R_0} = \frac{\delta k_1}{k_{1_0}} \qquad (23)$$

---

1 $\delta A$ is defined to be $A\text{-}A_0$, $\delta B$ to be $B\text{-}B_0$ and so on.

To get the change in $k_1$, we use Equ.19a and obtain :

$$\frac{\delta k_1}{k_{1_0}} \approx \eta W \left( 1 - 4\pi\, k_{1_0} \frac{r_w^3}{R^3} \int_0^{r_w/R} dz\, z^2 e^{-z^2} \right)$$

$$\approx \eta W \left( 1 - \pi\sqrt{\pi}\, k_{1_0} \frac{r_w^3}{R^3} \right) \tag{24}$$

In most cases the last term in Equ.24 will be negligible with respect to 1. For instance, for the Sun, taking $\rho_c = 148$ g/cm$^3$, $T_c = 15.5$ MK, and a WIMP mass $m_w = 4\, m_p$, one finds :

$$\pi\sqrt{\pi}\, k_{1_0} \frac{r_w^3}{R^3} \approx 0.013$$

Therefore, to the accuracy we are aiming at :

$$\frac{\delta k_1}{k_{1_0}} = \eta W = \frac{\delta\rho_c}{\rho_{c0}} + 3\frac{\delta R}{R_0} \tag{25}$$

The solution of Equs.21-25 is :

$$R = R_0 \tag{26}$$

and

$$\frac{\delta\rho_c}{\rho_{c0}} = -\frac{\delta T_c}{T_{c0}} = \eta W \tag{27}$$

Using Equ.27 with the definition of f(z), Equ.9a, and its relation to $f_0(z)$, Equ.19a, we can write the density profile of a star with WIMPs, to first order in $\eta$ and W as :

$$\rho(r) = \rho_{c0}\,(1 + \eta W)\, f_0(z) \left( 1 - \eta W + \eta W\, e^{-z^2} \right)$$

and a similar equation for the temperature profile, which can be rewritten as :

$$\rho(r) = \rho_0(r) \left( 1 + \eta W\, e^{-\frac{r^2}{r_w^2}} + O(\eta^2) \right) \tag{28a}$$

$$T(r) = T_0(r) \left( 1 - \eta W\, e^{-\frac{r^2}{r_w^2}} + O(\eta^2) \right) \tag{28b}$$

As shown by these equations, inside the WIMP region $r < r_w$, the effect of WIMPs can be sizeable and increases the density and lowers the temperature. Outside the WIMP region ($r >> r_w$), the density and temperature profiles are not sensitive to the presence of WIMPs in the star. Notice that the contraction of the star is *not* homologous, since the *shapes* of the density and temperature profiles are changed by the WIMPs.This is shown in Figures 1 . Moreover Equ.26 states that the radius of a star of given mass with and without WIMPs is the same. These results stem from the fact that the exponential distortion induced by WIMPs on the density distribution contributes by $\eta^2$ terms to the total mass of the star. Thus the relation between mass and radius of stars is practically insensitive to the presence of WIMPs.

Other observable quantities which have a stronger dependence on the structure of the stellar core, such as the total luminosity or (even more) the neutrino counting rate, are more affected. However, interesting values of the WIMP mass and cross-sections lead to large values of W, as we shall see in section 4 .

### 3.4 Solution for all W

The reasoning above has been held to first order in W, however it generalizes for any W up to minor changes as we explain now. To compute the change of the temperature and density profile shapes, we again look for solutions of the equilibrium equations of the form :

$$f(z) = f_0(z)\big(1 + \eta W\, f_w(z)\big)$$
$$g(z) = g_0(z)\big(1 + \eta W\, g_w(z)\big)$$

When W is large, the linearized hydrostatic equilibrium equation (i.e. to the first order in $\eta$) is identical to the small W case, and we still get :

$$f_w(z) = - g_w(z)$$

The main difference is that the linearized radiative equilibrium equation (Equ.A.4 of the Appendix) can no longer be solved fully analytically, and the computation of $g_w(z)$ requires a numerical quadrature (details are given in the Appendix). Figure 2 shows the function $g_w(z)$ for various values of W, and one can see that $g_w(z)$ behaves in a way very similar to the result to first order in W, which was $(1 - \exp(-z^2))$, but its asymptotic value for large z becomes much *smaller* than one, thus decreasing strongly the effect of WIMPs as compared to the expectation from first order in W. Note that the correction factor $g_w(z)$ still does not depend on the choice of the initial profiles $f_0(z)$ and $g_0(z)$.

To compare more precisely with the first order result, we rewrite $g_w(z)$ as :

$$g_w(z) = g_w(\infty)\,(1 - \hat{g}_w(z))$$

$\hat{g}_w(z)$ behaves in a way very similar to $\exp(-z^2)$ as Figure 3 shows, it is always smaller than 1 and falls off exponentially to zero for $z >> 1$, although somewhat more slowly. The no-evaporation condition fixes the WIMP equilibrium temperature $T_w$, to first order in $\eta$ and all orders in W, to be :

$$T_w = T_c \left\{ 1 - \left( \frac{3}{2} + W\,G(0) \right) \eta \right\}$$

The function G(z) is defined in the Appendix, and we found a reasonable fit of G(0) for $W > 10$ :

$$\frac{3}{2} + W\,G(0) \approx \frac{1}{0.71W - 1.36}$$

which shows that $T_w \rightarrow T_c$ when W gets large. A temperature inversion then develops at the center of the star (see Figure 1).

It is now easy to generalize to all W the first order discussion following Equ.19 to get the changes in the normalizations of the density and temperature profiles. Equations 21-27 remain true for any $W$, with the replacement $W \rightarrow W g_w(\infty) \equiv W_{eff}$. To all orders in W and first order in $\eta$, the radius of the star is not changed by WIMPs. The changes in the central quantities are simply :

$$\frac{\delta\rho_c}{\rho_{c0}} = -\frac{\delta T_c}{T_{c0}} = \eta W_{eff}$$

The density and temperature profiles then are :

$$\rho(r) = \rho_0(r)\ \left(1 + \eta W_{eff}\ \hat{g}_w(r/r_w)\right) \tag{29a}$$
$$T(r) = T_0(r)\ \left(1 - \eta W_{eff}\,\hat{g}_w(r/r_w)\right) \tag{29b}$$

where $W_{eff}$ and $\hat{g}_w(r/r_w)$ depend on W. Since $0 < \hat{g}_w(r/r_w) < 1$ , this means that :

$$0 < \frac{\delta\rho(r)}{\rho_0(r)} = -\frac{\delta T(r)}{T_0(r)} = \eta W_{eff}\,\hat{g}_w(r) < \eta W_{eff}$$

up to terms of order $\eta^2$. The quantity $W_{eff} \equiv W g_w(\infty)$ plays the same role as W in the linear case, and we found a reasonable fit for $W > 4$ :

$$W_{eff} \approx 1.87 \log(W) + 0.75$$

The very slow logarithmic increase of $W_{eff}$ means that the effects of WIMP on the stellar structure saturate in some way. Unfortunately, we were unable to get a practical fit for $\hat{g}_w(z)$. The best one is :

$$\hat{g}_w(z) \approx \exp\{ -\alpha(W)\, z^2 + \beta(W)\, z^4 \}$$

with $\alpha(W) \approx 1 - 0.34 \log(W)$

and $\beta(W) \approx -0.006 - 0.019 \log(W)$

but the accuracy of this fit can be as low as 15%, and we hope that Figure 3 will be precise enough to allow an interpolation for any value of W and z one may need.

To compute the change in luminosity $\delta L(R)/L_0(R)$, we integrate numerically Equ.8 using the density and temperature profiles given by Equ.29. If we remember that :

$$L_0(R) = \int_0^R dL_0(r) = \int_0^R 4\pi r^2 \varepsilon_{norm} \hat{\varepsilon}(X_i(r)) \left(\frac{\rho_0(r)}{\rho_{norm}}\right)^m \left(\frac{T_0(r)}{T_{norm}}\right)^n \rho_0(r)\, dr$$

the change in luminosity is just :

$$\frac{\delta L(R)}{L_0(R)} = \frac{1}{L_0(R)} \int_0^R dL_0 \left\{ (1 + \eta W_{eff} \hat{g}_w)^{m+1} (1 - \eta W_{eff} \hat{g}_w)^n - 1 \right\} \qquad (30)$$

which can be easily evaluated. Here, and only here, does the choice of $\rho_0(r)$ and $T_0(r)$ matter.

### 4-The Sun and the solar neutrino problem

We now apply these results to the present Sun. We want to compute the changes in the density and temperature profiles, in the total luminosity and in the neutrino flux from $^8B$ decay. Our "standard" solar model of reference is borrowed from the Saclay group (Cassé et al., 1986; Cohen et al., 1986; Turck -Chièze et al., 1988).

As shown in Section 3, the changes in density and temperature only depend on the parameters W and $\eta$ (to first order in $\eta$) but, from Equ.14 and Equ.17, these "internal" parameters depend both on the physical WIMP parameters (mass $m_w$ and cross-sections $\sigma_i$) and on the physical conditions at the center of the Sun (density, temperature, opacity, nuclear energy generation rate). Strictly speaking, W, $\eta$ and $r_w$ depend on the physical conditions at the center of the *perturbed* Sun, i.e. on $\rho_c, T_c, \varepsilon_N(0)$ and $\kappa(0)$ and not on $\rho_{c0}$, $T_{c0}$, $\varepsilon_{N0}(0)$ and $\kappa_0(0)$. However, the perturbed values differ from the unperturbed ones by terms of order $\eta W_{eff}$, therefore the error on W, $\eta$ and $r_w$ is also of order $\eta W_{eff}$, and the resulting error on $\delta\rho/\rho_0$ (for instance) is of order $\eta^2$ and can be neglected as we always did.

From Section 2, we know that the range of interest for the WIMP parameters is :

$$m_w > 4\, m_p \qquad \text{and} \qquad 10^{-35}\, cm^2 > \sigma_p > 10^{-37}\, cm^2$$

For the central density and temperature of the Sun, we take $\rho_{c0} = 148$ g/cm$^3$ and $T_{c0} = 15.5$ MK, but it is easy to rescale all following results if one prefers other values. The WIMP extension parameter $r_w$, defined by Equ.3 then is :

$$r_w \approx 0.057\, R_\odot \left(\frac{4\, m_p}{m_w}\right)^{0.5}$$

WIMP effects are important up to $r \approx 2\, r_w$, as Figure 1 shows, and the region of the solar core affected by WIMPs includes less than 10% of the solar mass (less than 2% for $m_w = 10\, m_p$). The parameter $\eta$ involves the extension $r_w$ and the dimensionless parameter $B_0$. This parameter $B_0$ is defined by Equ.13 with indices 0, but this equation involves the central opacity $\kappa(0)$ which is poorly known. Opacity is known to be one of the main sources of uncertainty in stellar models, and particularly in the computation of the flux of neutrinos from the decay of $^8$B. It is better to work backwards, and to extract $B_0$ from the temperature profile of the standard Sun (Turck-Chièze et al., 1988) by the relation:

$$B_0 = -\lim_{r\to 0} R_\odot^2 \frac{1}{T_0}\frac{dT_0}{dr^2}$$

which gives :

$$B_0 \approx 22$$

The parameter $\eta = B_0 \frac{r_w^2}{R_\odot^2}$ then is :

$$\eta \approx 0.070 \left(\frac{4\, m_p}{m_w}\right)$$

which *justifies our neglect of terms of order $\eta^2$ in our calculations*. In the standard solar model (Turck -Chièze et al.,1988), the energy generation rate at the center of the Sun is $\varepsilon_N(0) \approx 17.5$ erg/g/s. Then, from Equ.13, we get for the central opacity :

$$\kappa(0) \approx 1.24\ \mathrm{cm^2/g}$$

This value is slightly higher than the analytic approximation used by the Global Oscillation Network Group (GONG) :

$$\kappa_{GONG} \approx 0.893\ \mathrm{cm^2/g} \left(\frac{\rho(r)}{148\ \mathrm{g/cm^3}}\right)^{0.138} \left(\frac{T(r)}{15.5\ \mathrm{MK}}\right)^{-1.98}$$

To assess the consequences of a change of $\kappa(0)$, we shall keep the $\kappa$-dependence explicitly in the following . For instance :

$$\eta \approx 0.070 \left(\frac{4\, m_p}{m_w}\right)\left(\frac{\kappa(0)}{1.24\ \mathrm{cm^2/g}}\right)\left(\frac{\varepsilon(0)}{17.5\ \mathrm{erg/g/s}}\right) \tag{31}$$

Let us now turn to the other parameter, W From Equ.4, assuming the Sun to be 4.6 $10^9$ years old, the central density of non-annihilating WIMPs is :

$$n_w(0) \approx 1.07\ 10^{17}\ \mathrm{cm^{-3}} \left(\frac{m_w}{4\, m_p}\right)^{0.5} \mathrm{Min}\left\{ 1\ ,\frac{\sigma_p\langle X\rangle}{4\ 10^{-36}\ \mathrm{cm}^2}\right\}$$

Then, from Equ.14, W is equal to :

$$W \approx 188 \left(\frac{4m_p}{m_w}\right)^{1.5} \frac{\kappa(0)}{1.24\ \mathrm{cm^2/g}} \frac{\sigma_p X(0)}{4\ 10^{-36}\mathrm{cm}^2} \left(\frac{m_w}{m_p+m_w}\right)^{1.5}\ \mathrm{Min}\left\{ 1\ ,\frac{\sigma_p\langle X\rangle}{4\ 10^{-36}\ \mathrm{cm}^2}\right\} \tag{32}$$

assuming $T_w \approx T_c$, which is a very good approximation at this level. Equ.32 shows that W is easily much larger than 1, and therefore that the solution to all orders in W is needed. We saw that this solution to all orders was very similar to the solution to first order in W, the main change being the replacement $W \rightarrow W_{eff} \equiv W g_w(\infty)$. But for large W, $W_{eff} \ll W$ : for instance, when $W= 188$, $W_{eff} \approx 5$ .

Note also that it is now easy to generalize our results to the more realistic case where a WIMP can scatter over many different nuclei of mass $m_i$, with a scattering cross-section $\sigma_i$ . In Equ.32, the cross-section $\sigma_p$ appears twice, once from the capture rate (Equ.2) and once from the energy transfer rate $\varepsilon_w$ (Equ.5), but weighed differently by the proton mass $m_p$ [1] and mass fraction X. To generalize Equ.32, we just make the replacements :

$$\sigma_p X(0) \left(\frac{m_w}{m_p+m_w}\right)^{1.5} \quad\rightarrow\quad \sigma_{transfer} \equiv \sum_i \sigma_i X_i(0) \left(\frac{m_w}{m_i+m_w}\right)^{1.5} \left(\frac{m_p}{m_i}\right)^{0.5}$$

$$\sigma_p\langle X\rangle \quad\rightarrow\quad \sigma_{capture} \equiv \sum_i \sigma_i\langle X_i\rangle \frac{m_p}{m_i}$$

We can also define an effective cross-section $\sigma_{eff}$ by :

$$\sigma_{eff}\ \mathrm{Min}\left\{1\ ,\frac{\sigma_{eff}}{\sigma_c(M)}\right\} \equiv \sigma_{transfer}\ \mathrm{Min}\left\{1\ ,\frac{\sigma_{capture}}{\sigma_c(M)}\right\} \tag{33}$$

One can check that this equation defines a unique value of $\sigma_{eff}$ for every couple of values $(\sigma_{transfer},\sigma_{capture})$, and we can then write Equ.32 as :

---

[1] The factor $(4m_p/m_w)^{1.5}$ in Equ.32 just sets the scale for the WIMP mass, and is *not* related to the mass of the scattering nucleus.

$$W \approx 188 \left(\frac{4m_p}{m_W}\right)^{1.5} \frac{\kappa_0}{1.24\ \mathrm{cm}^2/\mathrm{g}} \frac{\sigma_{eff}}{\sigma_c(M)} \mathrm{Min}\left\{1, \frac{\sigma_{eff}}{\sigma_c(M)}\right\}$$

with $\sigma_c(M_\odot) \approx 4\ 10^{-36}\mathrm{cm}^2$. One may use its favorite WIMP candidate to compute the cross-sections on heavy nuclei and obtain $\sigma_{eff}$.

The effects of WIMPs in a star (measured by $W_{eff}$ or $\eta$) do not become large, and in the range of interest for the WIMP parameters ($m_w > 4\ m_p$, and $10^{-35}\ \mathrm{cm}^2 > \sigma_p > 10^{-37}\ \mathrm{cm}^2$), the product $\eta W_{eff}$ remains small :

$$\eta W_{eff} < 0.4$$

In some sense, the star is very stable, and the first order WIMP effects are cancelled to a large extent (but not completely) by higher order effects [1]. We conjecture that the order of magnitude of $\eta^2$ terms will be :

$$\eta^2 W_{eff} \lesssim 0.03$$

which, among other things, imply that WIMPs do not change the radius of the Sun by more than 3%. As we said, $\eta$ and W actually depend on $\rho_c$ and $T_c$ and not on $\rho_{c0}$ and $T_{c0}$, but the error induced on the changes of the luminosities and the stellar radius is of order $\eta^2$ (i.e. less than 3% in the worst situation). For any given values of the WIMP parameters $m_w$ and $\sigma_{eff}$, and any central values $\rho_{c0}$ and $T_{c0}$ for a star, it is easy to compute the corresponding values of $\eta$ (Equ.31) and W (Equ.32), and deduce $W_{eff}$ from its logarithmic approximation.

Now than we can link W and $\eta$ to the WIMP parameters, let us examine the physical effects of the presence of WIMPs. Let us consider, for instance, the two cases shown in Figure 1, i.e. a light WIMP with a mass of 4 $m_p$ and a small cross-section $\sigma_{eff}$ of $4.6\ 10^{-37}\ \mathrm{cm}^2$, and a heavier WIMP with a mass $m_w = 10.6\ m_p$ and a larger cross-section of $10^{-35}\ \mathrm{cm}^2$. In the first case we get :

$$W = 2.5 \rightarrow W_{eff} \approx 1.4 \quad \text{and} \quad \eta = 0.070 \Rightarrow \eta W_{eff} = 0.10$$

and, in the second one :

$$W = 108 \rightarrow W_{eff} \approx 4.5 \quad \text{and} \quad \eta = 0.026 \Rightarrow \eta W_{eff} = 0.12$$

---

[1] Faulker and Swenson (Faulkner and Swenson, 1988) suggest that a rough but easy way to take WIMP effects into account is to replace the opacity by a smaller effective opacity (i.e., when $L_W = 10^3 L_\gamma$, one may define $\kappa_{eff} = \kappa/10^3$ and forget about WIMPs). But then $W \approx 10^3$ and we see that the effective value is much smaller $W_{eff} \approx 6.4$). We therefore think that the procedure suggested by Faulker and Swenson may considerably overestimate the effect of WIMPs.

The relative changes in the central density and temperature are equal to $\eta W_{eff}$, and we get nearly the same results in both cases. The density and temperature profiles in the presence of WIMPs are given in Figure 1, and show the large increase in density in the core, and the correlative temperature decrease relative to the profiles in the absence of WIMPs. For large values of W, a temperature inversion appears in the core : the temperature increases, roughly from $r = 0$ to $r = 2r_W$, and decreases afterwards. The temperature $T_W$ of the WIMP gas is almost equal to the central temperature $T_c$. For $r \lesssim 2.5r_W$, $T_W < T(r)$ and WIMPs act as an energy sink in this region, and an energy source outwards.

Let us now turn to luminosities. The main point of assuming the presence of WIMPs in the Sun is to explain the low neutrino flux detected by the $^{37}Cl$ experiment (Davis, 1986) and by the Kamiokande group (Koshiba, 1988). The pp chain which dominates energy generation in the Sun has 3 branches (ppI, ppII and ppIII) which do not have the same temperature dependence (Clayton, 1968). The main part of the total luminosity of the Sun comes from the ppI and ppII branches, and its temperature dependence scales roughly as $T^5$. The ppIII branch adds a negligible contribution to the energy, but the neutrinos detected by Davis' experiment and by the Kamiokande underground detector only come from the decay of $^8B$, which belongs to the ppIII branch. The temperature dependence of this branch is very large in the range $14.5 < T < 15.5$ MK, and is well approximated by $T^{21}$. The neutrino flux $L_\nu$ is roughly proportional to the luminosity due to the ppIII branch, and, neglecting the r-dependence of the chemical composition, we can write :

$$L_\nu \propto \int_0^R r'^2 \rho(r')^2 T(r')^{21} dr'$$

The precision is much better in the relative change $\frac{\delta L_\nu}{L_{\nu 0}}$ than in the absolute values $L_{\nu 0}$ and $L_\nu$ because errors on $\rho(r)$ and $T(r)$ cancel in the difference. Using Equ.30, we can compute $\frac{\delta L_\nu}{L_{\nu 0}}$ for any value of $\eta$ and W, that is for any value of the WIMP mass $m_W$ and effective cross-section $\sigma_{eff}$. There are uncertainties both in the experimental results and on the theoretical prediction. Because of their different detection probabilities, the result of Davis'experiment is expressed in Solar Neutrino Units, and the result of the Kamiokande experiment is an upper limit on the neutrino flux :

$$\text{Davis} \Rightarrow L_\nu = 2.07 \pm 0.25 \text{ SNU}$$
$$\text{Kamiokande} \Rightarrow L_\nu < (1.4 \pm 1)\, 10^6 \text{ cm}^{-2}\text{s}^{-1}$$

The corresponding predictions in standard solar models are :

$L_{\nu 0} = 5.8 \pm 1.2$ SNU and $L_{\nu 0} = (3.8 \pm 1.1)\ 10^6\ cm^{-2}s^{-1}$ (Turck-Chièze et al., 1988)
$L_{\nu 0} = 7.9 \pm 2.6$ SNU and $L_{\nu 0} = (5.8 \pm 2.2)\ 10^6\ cm^{-2}s^{-1}$ (Bahcall and Ulrich, 1988)

The discrepancy between theory and experiment is larger than the uncertainties, and this is the "solar neutrino problem". Therefore, we look for WIMP parameter values $m_w$ and $\sigma_{eff}$ such that the decrease in the solar core temperature leads to a drop of the resulting neutrino flux down to the low observed values. The upper limit on the neutrino flux from the Kamiokande experiment is not precise enough yet to put useful limits on the WIMP parameters, although it confirms the results of Davis'experiment. Due to slightly different choices, the solar model of Bahcall and Ulrich predicts a higher neutrino flux than the Saclay model, and therefore requires a larger reduction to agree with experiment :

$$-0.74 < \frac{\delta L_\nu}{L_{\nu 0}} < -0.50 \qquad \text{(Turck-Chièze et al., 1988)}$$

$$-0.83 < \frac{\delta L_\nu}{L_{\nu 0}} < -0.56 \qquad \text{(Bahcall and Ulrich, 1988)}$$

We computed the values of $m_w$ and $\sigma_{eff}$ required to obtain a given reduction $\delta L_\nu / L_\nu$ of the neutrino flux. Figure 4 shows the corresponding curves in the WIMP parameter space ($m_w, \sigma_{eff}$). The two extreme points of the curve labelled ".6" (i.e. a 60% reduction of the flux) correspond to the two cases shown in Figure 1. It is amusing to see that $\eta W_{eff}$ remains almost constant along these curves (although $\eta$ and W vary much). The WIMP mass range is quite narrow, and the required cross-sections (remember that they are effective cross-sections, which sum over all nuclei) are rather large for WIMP model building (Raby and West, 1987 and 1988; Gelmini, Hall and Lin, 1987; Ross and Segré, 1987).

Let us now turn to the total luminosity $L_\odot$, which is given by :

$$L_\odot = \int_0^R 4\pi r'^2\, \varepsilon_N(r')\, \rho(r')\, dr'$$

To compute the change in luminosity, we need the energy generation rate $\varepsilon_N(r)$ over the whole star, and moreover the dependence of $\varepsilon_N$ on the temperature and density. The main energy source in the Sun is the pp chain, with about 1.75% due to the CNO cycle (Turck-Chièze et al., 1988). We reproduce the total energy generation rate $\varepsilon_N$, with a good precision, over most of the energy producing region ($r < 0.25\ R_\odot$), by a sum of two terms :

$$\varepsilon_N(r) = \varepsilon_{pp}(r) + \varepsilon_{CNO}(r)$$

with $$\varepsilon_{pp}(r) = 14.3 \text{ erg/g/s} \left(\frac{X(r)}{0.355}\right)^2 \left(\frac{\rho(r)}{148 \text{ g/cm}^3}\right)\left(\frac{T(r)}{15.5 \text{ MK}}\right)^{4.8}$$

and $$\varepsilon_{CNO}(r) = 3.2 \text{ erg/g/s} \left(\frac{X(r)}{0.355}\right)\left(\frac{\rho(r)}{148 \text{ g/cm}^3}\right)\left(\frac{T(r)}{15.5 \text{ MK}}\right)^{25}$$

The precision is much better on the difference $L_\odot - L_{\odot 0}$ than on the total luminosity itself. Equ.30 is easily generalized to account for two sources $\varepsilon_{pp}(r)$ and $\varepsilon_{CNO}(r)$ with different temperature dependences, and we get a change in the total luminosity of the order of $\eta W_{eff}$, namely $\frac{\delta L_\odot}{L_\odot} \approx -0.075$, and $\frac{\delta L_\odot}{L_\odot} \approx -0.080$ in the two cases shown in Figure 1. Such a decrease of the solar luminosity is of course unacceptable, but it is very difficult to correct it (by an increase of the central hydrogen mass fraction X(0) for instance) without suppressing at the same time the reduction of the neutrino flux. The decrease of the total luminosity cannot be blamed upon the neglect of higher order terms : we evaluated the error due to the neglect of $\eta^2, \eta^3, \ldots$ terms to be $\approx \eta^2 W_{eff}$, which is less than 1% along the curves of Figure 4. We see that the change in the total luminosity is larger than the errors of our procedure, and may therefore be an actual problem.

## 5 - Conclusions

We did an analytical study of the perturbations brought about by WIMPs on the structure of a star in radiative equilibrium at the center. We showed that these perturbations could be expressed as universal functions of two parameters, that we called $\eta$ and W. We gave the relation between $\eta$ and W, the WIMP parameters (the mass $m_w$ and the effective cross-section $\sigma_{eff}$) and the central density $\rho_c$ and temperature $T_c$ of the star. We showed that WIMP energy transport reduced the temperature gradient, and could even lead to a temperature inversion in the core. In the case of the Sun, we computed the WIMP parameters required to decrease the neutrino flux (from the decay of $^8B$) down to the observed values. We found a narrow range :

$$4\, m_p < m_w < 12\, m_p \quad \text{and} \quad 10^{-36} \text{cm}^2 < \sigma_{eff} < 10^{-35} \text{cm}^2$$

We also found a decrease of the total solar luminosity, larger than the error due to our approximations. We do not think that this analytical study can replace a full evolutionary solar code, but we think that it can help to understand the physics involved and to control the numerical code. It can also be very useful to explore different WIMP models, and to study stars of different masses, and in different surroundings (for instance, in places where the density of dark matter is larger than in the solar neighbourhood).

**Appendix : Solution to first order in η and all orders inW**

We search for a solution of the structure equations (21) and (22), in the presence of WIMPs, in the form :

$$f(z) = f_0(z)\left(1+\eta W\, f_w(z)\right) \quad \text{(A.1a)}$$
$$g(z) = g_0(z)\left(1+\eta W\, g_w(z)\right) \quad \text{(A.1b)}$$
$$h(z) = h_0(z) \quad \text{(A.1c)}$$

We plug Equ.A.1 in Equ.10-11, keeping only the terms linear in η, and use the fact that $f_0(z)$ and $f_0(z)$ are solutions of Equ.16. Equ.10 for hydrostatic equilibrium becomes :

$$\eta W \left\{\frac{d}{dz}\left(\frac{f_0 g_0}{h_0}\right)\left[\left(1-\frac{A}{A_0}\right)f_w(z)+g_w(z)\right]+\frac{f_0(z)g_0(z)}{h_0(z)}\left[\frac{d}{dz}f_w(z)+\frac{d}{dz}g_w(z)\right]\right\}$$
$$+\left(1-\frac{A}{A_0}\right)\frac{d}{dz}\left(\frac{f_0 g_0}{h_0}\right) = -\eta W\,\frac{6A\, r_w^2}{z^2 R^2} f_0(z) \int_0^z dz'\, z'^2 f_0(z')\, f_w(z'). \quad \text{(A.2)}$$

This equation is a good approximation only in the $z \lesssim 1$ region, where one can write as a consequence of Equ.16 :

$$\frac{f_0(z)\, g_0(z)}{h_0(z)} \approx 1 - A_0 \frac{r_w^2}{R^2} z^2 = 1 - a_0\, \eta\, z^2 \quad \text{(A.3a)}$$
$$g_0(z) \approx 1 - B_0 \frac{r_w^2}{R^2} z^2 = 1 - \eta\, z^2 \quad \text{(A.3b)}$$

where $a_0 = \frac{A_0}{B_0}$ is of order 1. Using Equ.A.3, Equ.A.2 can be rewritten :

$$\eta W \left\{-2\, a_0\eta\, z\left[\left(1-\frac{A}{A_0}\right)f_w(z)+g_w(z)\right]+\frac{d}{dz}f_w(z)+\frac{d}{dz}g_w(z)\right\}$$
$$+\ 2\, a_0\eta\, z\left(\frac{A}{A_0}-1\right) \approx -6\eta^2 W \frac{a_0}{z^2}\frac{A}{A_0}\int_0^z dz'\, z'^2 f_w(z')$$

Keeping only terms of order one in η, this equation reduces to :

$$W\left[\frac{d}{dz}f_w(z)+\frac{d}{dz}g_w(z)\right] \approx 2\, a_0\left(1-\frac{A}{A_0}\right)z$$

which can be easily integrated to give :

$$f_w(z)+g_w(z) \approx \frac{a_0}{W}\left(1-\frac{A}{A_0}\right)z^2 = \frac{A_0}{\eta W}\left(1-\frac{A}{A_0}\right)\frac{r^2}{R^2}$$

using $f_w(0) = g_w(0) = 0$ (see Equ.18). But remember that $\eta W$ has been factorized out in Equ.A.1 because f(z) and g(z) should reduce to $f_0(z)$ and $g_0(z)$ when either W or $\eta$ go to zero. In other words, at fixed radius r, $f_w(z)$ and $g_w(z)$ should not be singular when either W or $\eta$ go to zero. Therefore one must have :

$$1 - \frac{A}{A_0} = 0 \ \text{ up to terms of order } \eta W$$

So that, in the region of WIMPs where $\lesssim 1$, to lowest order in $\eta$ and for *any* value of W, one has :

$$f_w(z) = - g_w(z)$$

Turning now to the equation for luminosity balance Equ.11, we neglect terms of order $\frac{m_p}{m_w}$ compared to 1 [(1)]. Then we can handle it in the same way as Equ.10, i.e. we make use of Equ 16b for the star without WIMPs, use the small z expansions Equ.A.3 and keep only terms in first order in $\eta$ . In this way we again conclude that

$$1 - \frac{B}{B_0} = 0 \ \text{ up to terms of order } \eta W$$

Keeping only terms of first order in $\eta$ Equ.11 becomes :

$$\frac{d}{dz} g_w(z) = \frac{4}{\eta z^2} \int_0^z dz' \left(1 - \frac{T_w}{T_c} - \eta\, z'^2 + \eta W\, g_w(z')\right) z'^2\, e^{-z'^2} \qquad \text{(A.4)}$$

and the condition of non evaporation of WIMPs Equ.15 reads :

$$\int_0^\infty dz \left(1 - \frac{T_w}{T_c} - \eta\, z^2 + \eta W\, g_w(z)\right) z^2\, e^{-z^2} = 0 \qquad \text{(A.5)}$$

For the sake of simplicity, and because it helps understanding the physics, we first solve these equations to first order in W However this is not a realistic approximation if WIMPs are to play a significant role in stars, and we shall solve these equation later for any W, but only numerically. To lowest order in W, Equ.A.5 determines $T_w$ to be :

---

1 To first order in $\eta$ , terms of order $m_p/m_w$ contribute only to the definition of W in Equ.14.

$$1 - \frac{T_w}{T_c} = \frac{3\eta}{2}$$

To the same order in W, plugging this value of $T_w$ into Equ.A.4, we get :

$$\frac{d}{dz} g_w(z) = 2 z e^{-z^2}$$

which is easily integrated to give :

$$g_w(z) = 1 - e^{-z^2} + o(W; \eta)$$

using $g_w(0) = 0$. Starting from a known solution for the star without WIMPs, these equations give us the normalized density and temperature profiles f(z) and g(z) to first order in $\eta$ and W :

$$f(z) = f_0(z) \left[ 1 - \eta W (1 - e^{-z^2}) + \eta^2 W f_{w1}(z) \right]$$
$$g(z) = g_0(z) \left[ 1 + \eta W (1 - e^{-z^2}) + \eta^2 W g_{w1}(z) \right]$$

The previous result generalizes for any W up to minor changes. We still have $f_w(z) = - g_w(z)$. To solve Equ.A.4, subject to condition $g_w(0) = 0$ and Equ.A.5, it is convenient to introduce an auxiliary function G(z) defined by the conditions :

$$g_w(z) = G(z) - G(0)$$
$$\int_0^\infty dz\, z^2 e^{-z^2} G(z) = 0 \qquad \text{(A.6)}$$

Equation A.5 then reduces to :

$$1 - \frac{T_w}{T_c} = \eta \left( \frac{3}{2} + W\, G(0) \right)$$

and then Equ.A.4 to :

$$\frac{d}{dz} G(z) = 2 z e^{-z^2} + \frac{4W}{z^2} \int_0^z dz' z'^2 e^{-z'^2} G(z') \qquad \text{(A.7)}$$

An iterative solution as a series in W can be constructed, but is not very helpful for large W because already at order two in W the integrations must be performed numerically.

Therefore it is more economical to solve numerically Equ.A.7, transforming it into a second order differential equation :

$$\frac{d}{dz}\left[z^2 \frac{d}{dz} G(z)\right] = 4W\, z^2\, e^{-z^2} G(z) + 2\, z^2 (3 - 2\, z^2)\, e^{-z^2} \qquad \text{(A.8)}$$

which is equivalent to Equ.A.6 and A.7 for an G(z) non singular at $z = 0$ if it is supplemented by the boundary conditions :

$$\lim_{z \to 0} \left[\frac{d}{dz} G(z)\right] = 0 \text{ or equivalently in this case, } \lim_{z \to 0} \left[z^2 \frac{d}{dz} G(z)\right] = 0$$

and

$$\lim_{z \to \infty} \left[z^2 \frac{d}{dz} G(z)\right] = 0 . \qquad \text{(A.9)}$$

To avoid spurious singularities near $z = 0$ it is convenient to make the following change of function :

$$\hat{G}(z) \equiv z\, G(z)$$

Equation A.8 then reads :

$$\frac{d^2}{dz^2} \hat{G}(z) - 4We^{-z^2} \hat{G}(z) = 2z\,(3 - 2z^2)\, e^{-z^2} \qquad \text{(A.10)}$$

and boundary conditions A.9 :

$$\lim_{z \to {}^{\infty}_{0}} \left[z \frac{d}{dz} \hat{G}(z) - \hat{G}(z)\right] = 0. \qquad \text{(A.11)}$$

Finally Equ.A.10 with boundary conditions A.11 is solved numerically using the Merson method. Knowing $\hat{G}(z)$ for a given value of W, it is easy to get $G(z) = \frac{\hat{G}(z)}{z}$ , then $g_w(z) = G(z) - G(0)$ , and finally $g_w(\infty)$ and $\hat{g}_w(z) = 1 - \frac{g_w(z)}{g_w(\infty)}$.

**FIGURE CAPTIONS**

Figure 1a : Temperature profile $T_0$ in the solar core for the standard model, and temperature profile T with a light WIMP of mass $m_w = 4\ m_p$ and small cross-section $\sigma = 4.6\ 10^{-37}\ cm^2$. The corresponding WIMP equilibrium temperature $T_w$ is also shown.

Figure 1b : Density profile $\rho_0$ in the solar core for the standard model, and density profile $\rho$ with a light WIMP of mass $m_w = 4\ m_p$ and small cross-section $\sigma = 4.6\ 10^{-37}\ cm^2$.

Figure 1c : Temperature profile $T_0$ in the solar core for the standard model, and temperature profile T with a heavy WIMP of mass $m_w = 10.6\ m_p$ and large cross-section $\sigma = 10^{-35}\ cm^2$. A temperature inversion clearly appears in the center, and the WIMP equilibrium temperature $T_w$ is nearly equal to the central temperature. Note that the temperature is *always smaller* in the perturbed model than in the standard model, which explains the reduction in luminosity.

Figure 1d : Density profile $\rho_0$ in the solar core for the standard model, and density profile $\rho$ with a heavy WIMP of mass $m_w = 10.6\ m_p$ and large cross-section $\sigma = 10^{-35}\ cm^2$.

Figure 2 : The correction factor $g_w(z)$ as a function of $z=r/r_w$ for different values of W. For small values of W, $g_w(z)$ is nearly equal to $1 - e^{-z^2}$,but for large values of w, $g_w(z)$ remains much smaller than 1, which strongly decreases the effect of WIMPs on the stellar structure.

Figure 3 : The function $\hat{g}_w(z)$ for W = 0, 2.5, 5, 10, 20, 40, 80, 160, 320 and 640. The qualitative behaviour of $\hat{g}_w(z)$ is very similar to $e^{-z^2}$ (W=0), but the decrease at large z is much slower. This implies that the effect of WIMPs extends farther away from the center for large W (all things being otherwise equal).

Figure 4 : Curves in the WIMP parameter space $(m_w, \sigma_{eff})$ showing the values which yield a given reduction $|\delta L_\nu/L_\nu|$ of the solar neutrino flux. The curves correspond to $|\delta L_\nu/L_\nu| = 0.5, 0.6, 0.7$ and 0.8. The WIMP mass $m_w$ is given in unit of the proton mass $m_p$, and the effective cross-section $\sigma_{eff}$ (defined in Equ.33) is given in picobarns (1 pb = $10^{-36} cm^2$).

**REFERENCES**

Allen, C.W.: 1973, *"Astrophysical Quantities"* (London, Athlone Press)

Bahcall, J.N., Soneira, R.M.: 1980, *Astrophys. J. Supp.* **44**, 73

Bahcall, J.N., Casertano, S.: 1985, *Astrophys. J.* **293**, L7

Bahcall, J.N., Ulrich, R.K.: 1988, to appear in *Rev. Mod. Phys.*

Bahcall, J.N., Davis, R. Jr., Wolfenstein, L.: 1988, *"Solar neutrinos : a field in transition"*, Princeton preprint IASSNS-AST 88/18, to appear in *Nature*

Blumenthal, G., Faber, S., Flores, R., Primack, J.: 1986, *Astrophys. J.* **301**, 27

Bouquet, A., Salati, P., Cassé, M., Prantzos, N.: 1987, *"Weakly interacting massive particles and solar neutrinos"* , Communication to the XXth Cosmic Ray Conference (Moscow 1987)

Bouquet, A., Salati, P.: 1987, *"Life and death of cosmions in stars"*, Paris and Annecy preprint LPTHE 87-20/LAPP 192-87, submitted to Astronomy and Astrophysics

Bouquet ,A., Kaplan, J., Salati,P.: 1987, *"Cosmions and Stars"* in Proceedings of the Rencontre sur la Masse Cachée et la Matière Noire, Annecy, July 1987 (to appear in *Annales de Physique* )

Bouquet, A., Kaplan, J., Martin, F., Salati, P.: 1988, *"WIMPs and Stellar structure"* in Proceedings of the XXIII$^{rd}$ Moriond Astrophysics Meeting, Les Arcs, March 1988 (Editions Frontières, J. Audouze, ed.)

Bouquet, A., Raffelt, G., Salati, P., Silk, J.: 1988, *"Dark matter, solar neutrinos and horizontal branch stars"*, LBL-TH-25215, submitted to Phys. Rev. Lett.

Burstein, D., Rubin, V.C.: 1985, *Astrophys. J.* **297**, 423

Cahen, S., Cassé, M., Doom, C.: 1986, in *Massive neutrinos in astrophysics and particle physics*, Proceedings of the Sixth Rencontre de Moriond, O. Fackler and J. Tran Thanh Van (eds) (Editions Frontières, Gif sur Yvette, 1986)

Cassé, M., Cahen, S., Doom, C.: 1986, in *Neutrinos and the present day universe*, Colloque de la Société Française de Physique, T. Montmerle and M. Spiro (eds) (CEN-Saclay 1986)

Chandrasekhar, S.: 1938, *"Stellar structure"* (Cambridge University Press)

Clayton, D.D.: 1968, *"Principles of stellar evolution"* (University of Chicago Press)

Davis, R. Jr.: 1978, in Proceedings of the Informal Conference on Status and Future of Solar Neutrino Research (G. Friedlander ed, Brookhaven National Laboratory Report 50879)

Davis, R. Jr.: 1986, Report to the Seventh Workshop on Grand Unification (ICOBAN 1986, Toyoma, Japan)

Faulkner, J., Gilliland, R.L.: 1985, *Astrophys. J.* **299**, 994

Faulkner, J., Swenson, F.J.: 1988, *Astrophys. J.* **329**, L47

Gelmini, G., Hall, L., Lin: 1987, Nucl. Phys. **B281**, 726

Gilliland, R.L., Faulkner, J., Press, W.H., Spergel, D.N.: 1986, *Astrophys. J.* **306**, 703

Gould, A.: 1987, *Astrophys. J.* **321**, 560

Griest, K., Seckel, D.: 1987, *Nucl. Phys.* **B283**, 681

Koshiba, M.T..: 1988, in *"Neutrinos and exotic phenomena"*, Moriond Worskhop (J. Tran Thanh Van, ed.) Editions Frontières

Kormandy, J., Knapp ,G.R. (eds): 1987, Proceedings of the IAU Symposium n°117, *"Dark matter in the Universe"* (Reidel 1987)

Krauss, L.M.: 1985, *"Cold dark matter candidates and the solar neutrino problem"* Harvard preprint HUTP-85/A008a, *unpublished*

Krauss, L.M., Freese, K., Spergel, D.N., Press W.H.: 1985, *Astrophys. J.* **299**, 1001

Nilles, H.P.: 1984, *Phys. Rept* **C110** , 2

Press, W.H., Spergel, D.N.: 1985, *Astrophys. J.* **296**, 679

Raby, S., West, G.: 1987, *Nucl. Phys.* **B292**, 793

Raby, S., West, G.: 1988, *Phys. Lett.* **200B**, 547

Renzini, A.: 1987, *Astron. Astrophys.* **171**, 121

Robin, A.: 1988, Proceedings of the XXIII[rd] Moriond Astrophysics Meeting, Les Arcs, March 1988 (Editions Frontières, J. Audouze, ed.)

Ross, G.G., Segrè, G.C.: 1987, *Phys. Lett.* **197B**, 45

Silk, J.: 1984, Proceedings of the Inner space- Outer space conference, held at Fermilab in May 1984 (The University of Chicago Press, 1986)

Spergel, D.N., Press, W.H.: 1985, *Astrophys. J.* **294** , 663

Spergel ,D.N., Faulkner, J.: 1988, *"WIMPs in horizontal branch stars"*, IAS Princeton preprint

Spergel, D.N.: 1988, private communication

Steigman, G., Sarazin, C.L., Quintana, H., Faulkner ,J.: 1978, *Astrophys. J.* **83** , 1050

Stein, R.F., Cameron, A.G.W.: 1966, *"Stellar evolution"* (Plenum Press, 1966)

Turck-Chièze, S., Cahen, S., Cassé, M., Doom, C.: 1988, *"Revisiting the standard solar model"* , submitted to *Astrophys. J.*

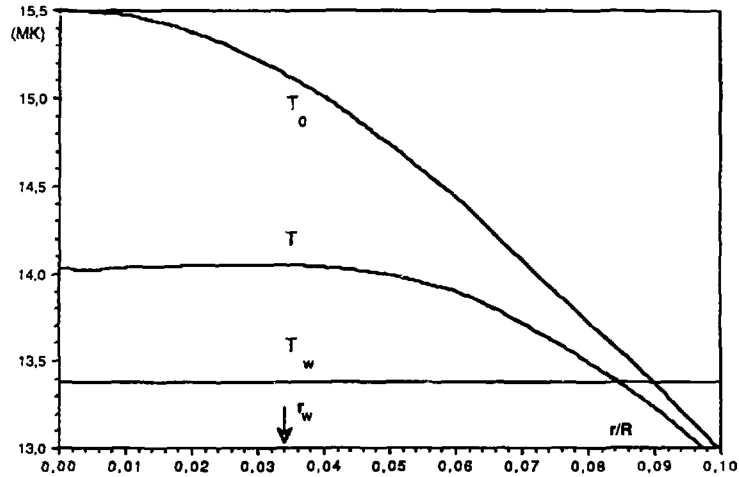


**Figure 1a**

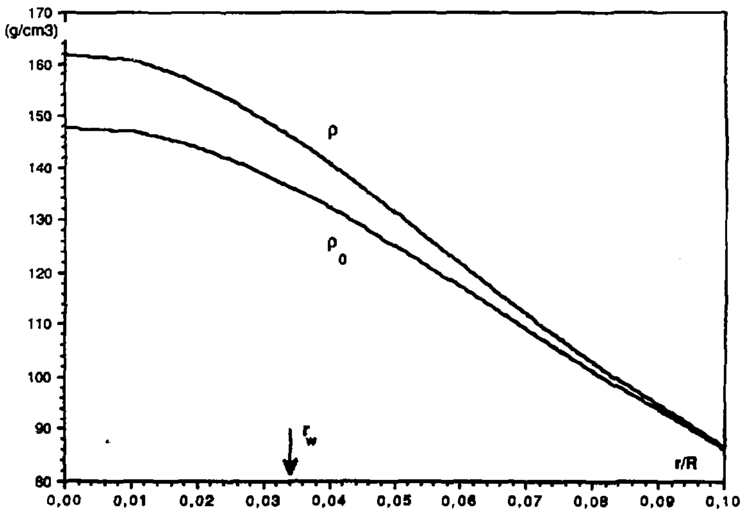


**Figure 1b**

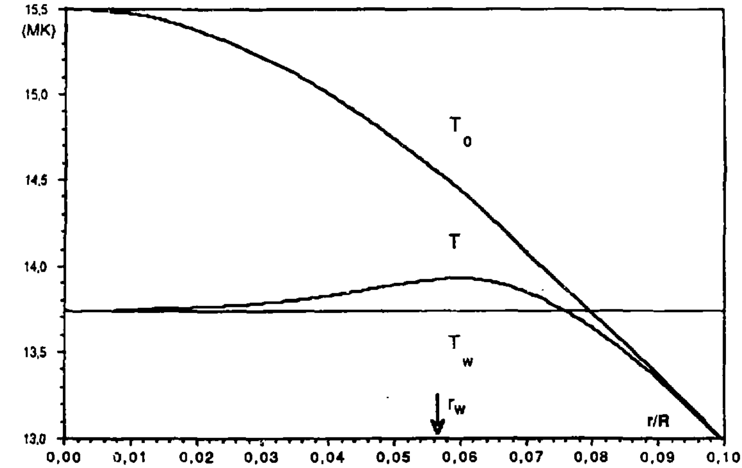


**Figure 1c**

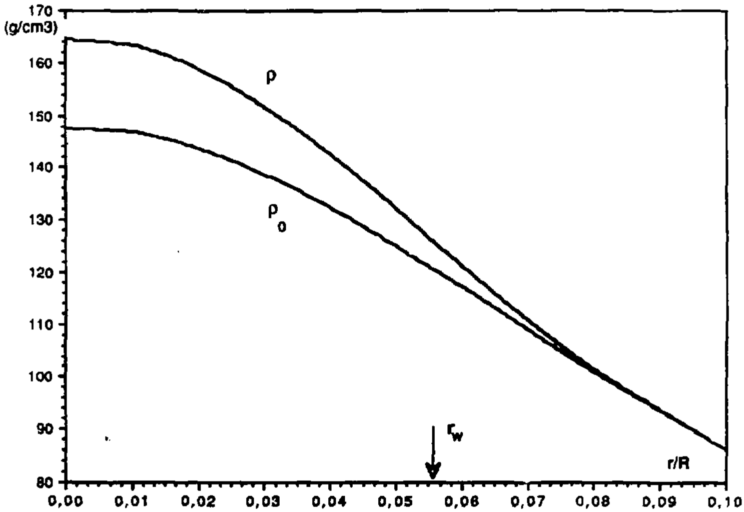


**Figure 1d**

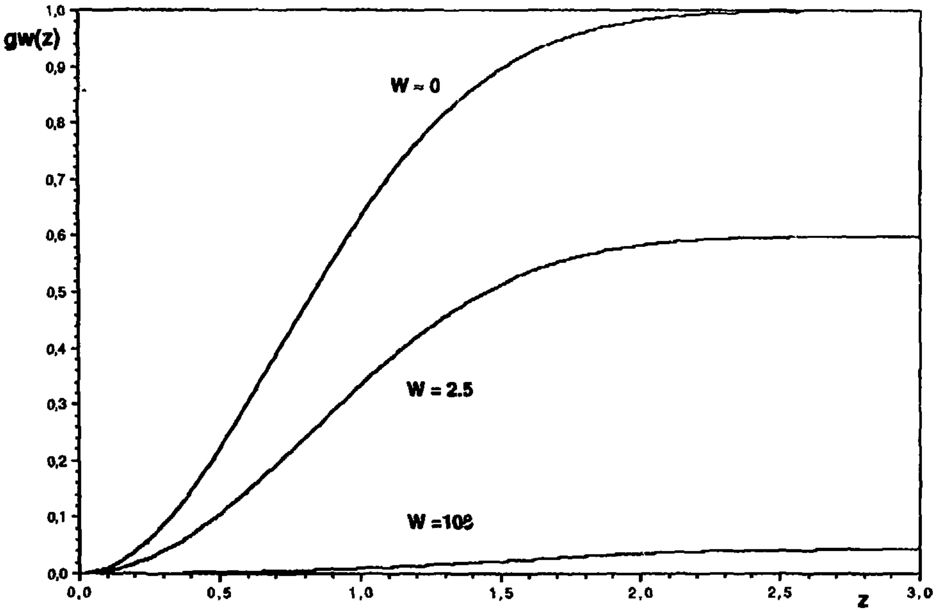


**Figure 2**

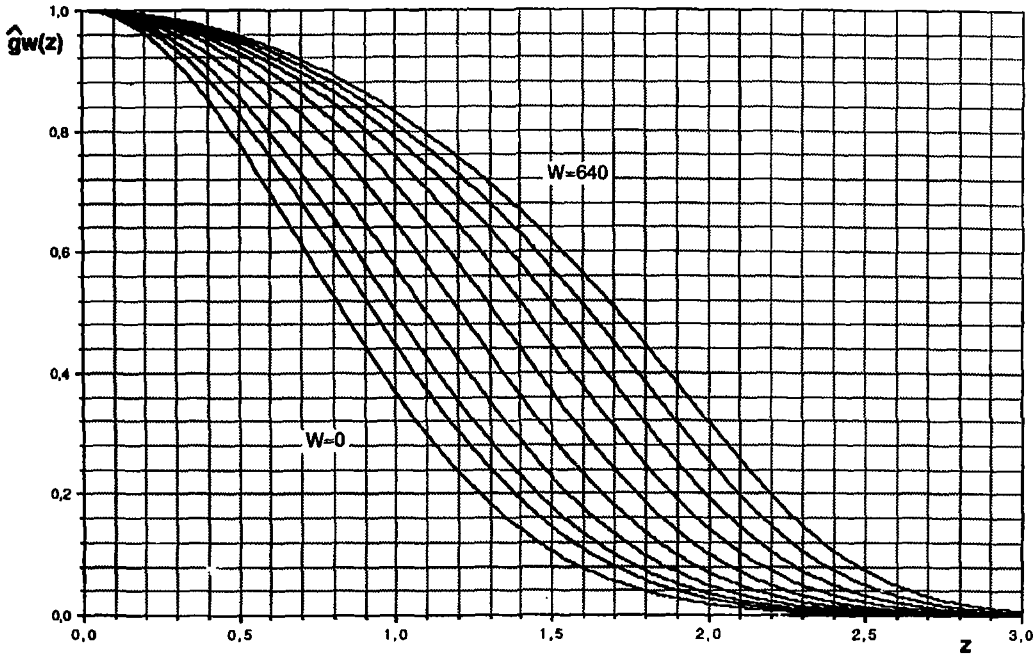


Figure 3

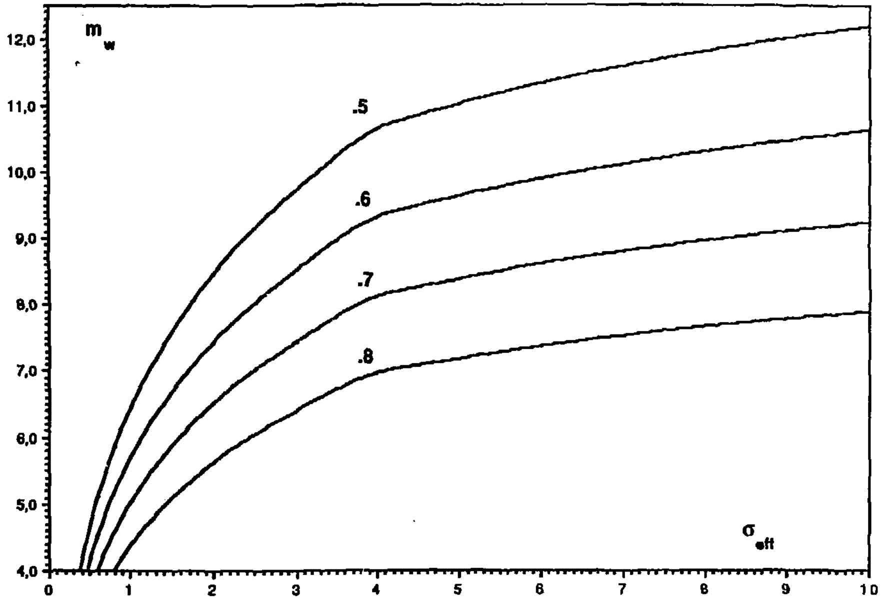


Figure 4