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FERMILAB-Conf-89/137

Finite Difference Time Domain Modelling of Particle Accelerators *

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March 1989

* Presented at the 1989 IEEE Particle Accelerator Conference, Chicago, Illinois, March 20-23, 1989.

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Abstract: Finite Difference Time Domain (FDTD) modelling has been successfully applied to a wide variety of electromagnetic scattering and interaction problems for many years. Here the method is extended to incorporate the modelling of wake fields in particle accelerators. Algorithmic comparisons are made to existing wake field codes, such as MAFIA T3.

Brief Review of FDTD

The FDTD algorithm is a finite difference solution of the time-dependent Maxwell's equations defined on a lattice of points which discretizes a volume of space containing a scatterer. This is depicted in Figure one. In order to compute the electric field value at a particular grid point we apply Ampere's law.

$$\oint_C \vec{H} \cdot d\vec{l} = \iint_S \sigma\vec{E} \cdot d\vec{S} + \frac{\partial}{\partial t} \iint_S \vec{D} \cdot d\vec{S} \qquad (1)$$

Similarly for an magnetic field value we apply Faraday's law.

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \iint_S \vec{B} \cdot d\vec{S} \qquad (2)$$

Wave propagation, scattering, and penetration phenomena are modeled in a self-consistent manner by marching in time, that is, repeatedly implementing the finite-difference analog of Maxwell's equations. This results in a simulation of the continous actual waves by sampled-data numerical analogs propagating in a data space stored in a computer. Space and time sampling increments are selected to avoid aliasing of the continuous field distribution, and to guarantee stability of the time-marching algorithm. Time marching is completed when the desired steady-state field behavior is observed. Truncation of the computational domain is done by implementing absorbing boundary conditions.

The FDTD code has been applied to a wide range of wave propagation problems both scattering and coupling [1]. A good review of the code and its applications is provided in [2]. Some of these areas are outlined below. Given the demonstrated capabilities of the FDTD code, this paper extends its scope of applications to accelerator physics.

Many codes have been written for the purpose of modelling accelerator physics. The MAFIA code T3, developed by Weiland [3] to handle time varying fields in accelerators, is also based on a finite difference algorithm. It employs a stair step approximation of surfaces with curvature in more than one orthogonal direction. The modelling of thin wires with subcell radii is not implemented in the code.

Application to Accelerator Problems

Incorporating of the physics of the moving particle bunch into the FDTD formalism is accomplished by altering Ampere's law. A term corresponding to the movement of charge is added to the right side of the equation.

$$\oint_C \vec{H} \cdot d\vec{l} = \iint_S \sigma\vec{E} \cdot d\vec{S} + \frac{\partial}{\partial t} \iint_S \vec{D} \cdot d\vec{S} + \iint_S \rho\vec{v} \cdot d\vec{S} \qquad (3)$$

The movement of charge is modeled by exciting the FDTD grid at appropriate spatial and temporal locations, given the desired path and shape of the particle bunch. In our simulations, curved surfaces are conformably modeled. Figure two illustrates the differences between the stepped edge and contour representations of two dimensional accelerating cavity. Accurate geometry representation is especially important in high field regions such as found at accelerating cavity nose cones. In Figures three to seven the geometrical axis of symmetry is located at a vertical ordinate of 45.

Line Charge Between Parallel Plates: The first and simplest problem we have considered is a gaussian distribution of positive charges travelling at

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relativistic speed ($v = c$) along the plane of symmetry of a 2-D smooth pipe. Since our model employs cartesian coordinates, our 2-D problem is represented by two parallel plates. The analytical analysis of the parallel plates geometry is similar to that of the circular pipe, and predicts electric field lines to be transverse to the beam and normal to the plate surface. The magnetic field lines are also transverse to the beam and normal to the electric field. At $v = c$ no longitudinal fields are present. The magnitudes of the electric and magnetic fields are equal. Our numerical solution is shown in Figures three and four. The longitudinal E field component is not depicted since it was found to be negligible. For these results, the bunch is propagating in the positive x-direction. This initial simulation shows the ability of the code to reconstruct the proper physics of the problem.

Excitation of a Two Dimensional Cavity: The next example in our simulation is the excitation of an accelerating cavity by a gaussian distribution of positive charges moving at the speed of light along the plane of symmetry. Again a 2-D model in cartesian coordinates is used. Figures five and six are contour plots of the E_x and H_z field components, respectively, which were excited by the passage of the particle bunch along the beam pipe. The field plots show that the accelerating mode of the cavity has been excited: the E field (capacitance) of the cavity is concentrated near the nose cones, while the H field (inductance) is maximum in the upper portion of the cavity. In this simulation, the E_y field components in the cavity were found to be small.

The final simulation, illustrated in Figure seven, is the excitation of an acclerating cavity in the same manner as reported above, with the exception that the charge distribution's path is off the plane of symmetry. It is located so that the distance from the bottom plate to the path is three times the distance from the top plate to the path. The depicted results show the expected enhancement of the E field in the upper nose cone gap with respect to the lower gap.

Capabilities of the FDTD Algorithm

The advent of the contour integral approach has increased the variety of structures that can be analyzed with the FDTD algorithm. Within the past few years EM scattering from and coupling to objects with thin slots, wires, curved surfaces, biological bodies, relativistically moving surfaces and surfaces with nonlinear time varying parameters have been investigated. Details of these investigations are summarized below. The contour integral approach is employed in the accurate modelling of the object and is not related to the modelling of the EM scattering source. This implies that the successful modelling of the above structures in particle accelerators is a straightforward application of the contour FDTD method. The computational overhead incurred in using the contour approach is small: it is proportional to the surface area of the object for three dimensional structures and to the boundary length for two dimensional structures.

Curved Surfaces: Until recently, all finite difference methods approximated a curved surface with some degree of stepped edging. The contour FDTD approach allows comformable modelling of curved surfaces. As reported by Jurgens [4], this better representation of the structure increases the accuracy of resulting field data, especially near a surface.Objects with nonzero surface impedance have also been modeled. The numerical analysis of other subcell physics phenomena, such as surface roughness now appears tractable.

Wires: Beam pickups and monitors are structures which consist of variously configured plates and wires located in the beam pipe and connected to signal processing electronics usually by coaxial cables. The plates, wires and coaxial cables (including the dielectric insulator) are all amenable to the contour FDTD method. Indeed, the FDTD analysis of coupling to multiconductor bundles has been validated [5].

Thin Slots and Joints: The contour FDTD method has also been applied to the EM modelling of narrow slots and joints [6]. The FDTD algorithm has accurately predicted field behavior in and behind slots located in conducting screen. Complicated joints such as the lapped joints were modeled. Slot and joint path length resonances were observed. Particle accelerators are assembled from smaller pieces and therefore possess joints.

Biological Bodies: Biological materials have been analyzed using the FDTD algorithm [7]. This application illustrates the extreme flexiblity of the algorithm in dealing with inhomogeneous objects. An arbitrary value for σ, ϵ and μ can be assigned to each lattice cell, where σ, ϵ and μ need not be scalars. It is worthwhile to note here that the contour FDTD algorithm is not restricted to metal objects [4].

Moving Surfaces: In [8] it is shown that a surface moving at relativistic speeds can be modelled with the FDTD code after some modifications. The model was tested for uniformly moving mirrors and vibrating mirrors. Even for complicated cases like scattering from a vibrating mirror at oblique incidence the code was able to reconstruct the proper physics of the problem. No system transformation is used and the results are obtained directly in the observer frame. This model provides a strong tool to study many interesting problems where analytical solutions are impossible to obtain. A moving, dense plasma front is generally treated as a moving conducting wall and can therefore be modeled with this code.

Time Varying Parameters: The FDTD code also allows for the modelling of wave interactions with media of time varying parameters. Time varying parameters can occur for different reasons and it is important to know the effects they might have on wave scattering and propagation. In [9] a study is made of waves scattering from a planar media having a time varying conductivity. To validate the numerical results an analytical approximate solution was derived. Both results, numerical and analytical, were shown to be in good agreement. Again this model makes it possible to study many problems, such as active surfaces, that are not possible to solve analytically.

Summary and Future Investigations

This initial investigation examined the propagation of a line charge through two dimensional structures. The two examples selected exhibited the correct physics resulting from a beam bunch excitation. The extension to three dimensions is straightforward. Considering the wide capabilities of this code it is now possible to model accelerating cavities, beam monitoring devices and kicker magnets. We hope to address these topics in the near future as our needs for such a model becomes more urgent every day.

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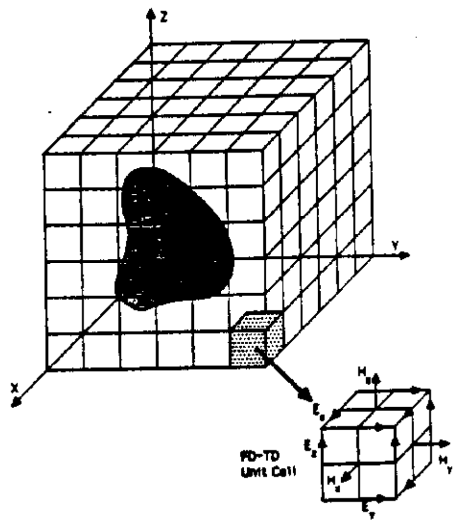

Figure 1: The FDTD Unit Cell

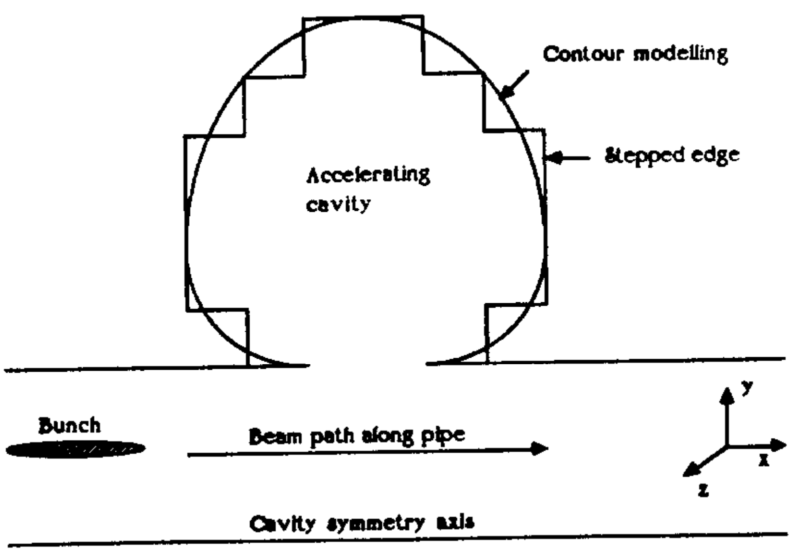

Figure 2: Stepped Edge v.s. Contour Geometries

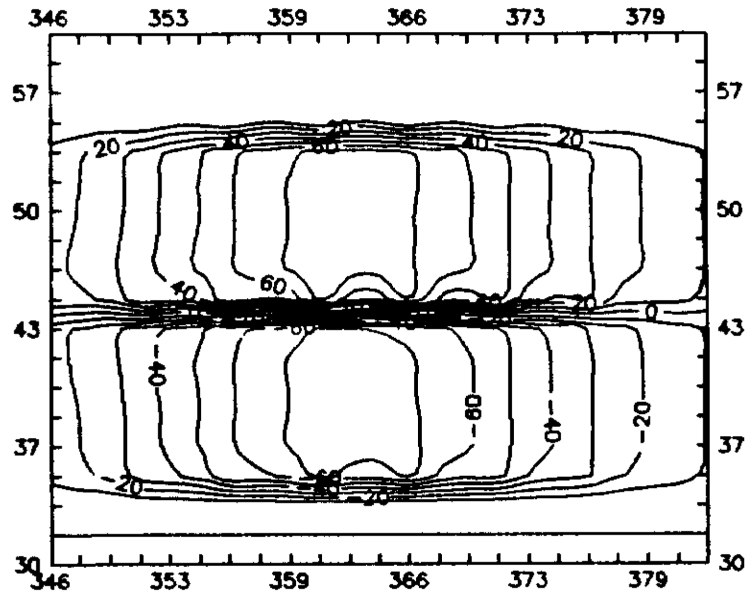

Figure 3: Contour Plot of E_y in a Smooth Pipe

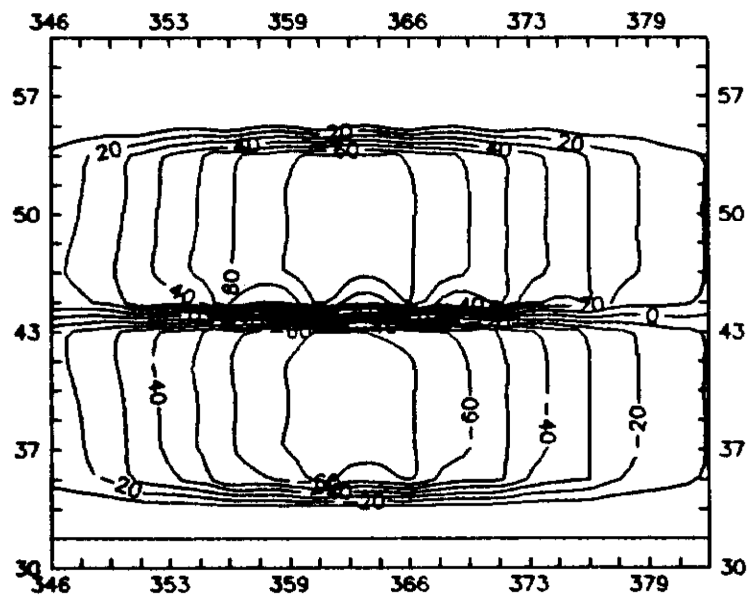

Figure 4: Contour Plot of H_z in a Smooth Pipe

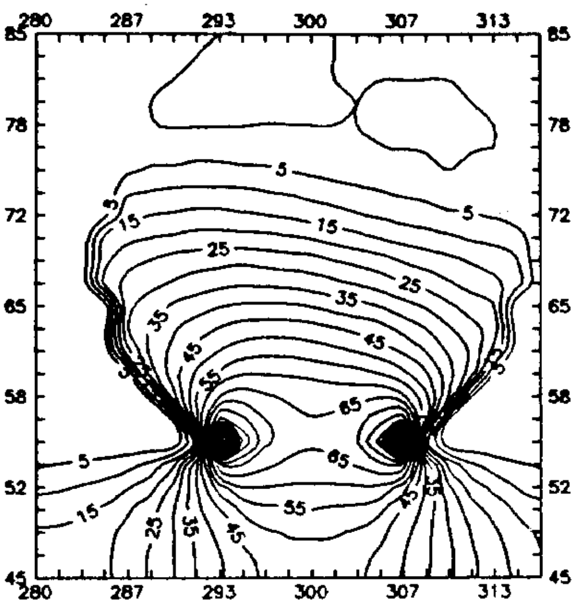

Figure 5: Contour Plot of E_z in the Cavity

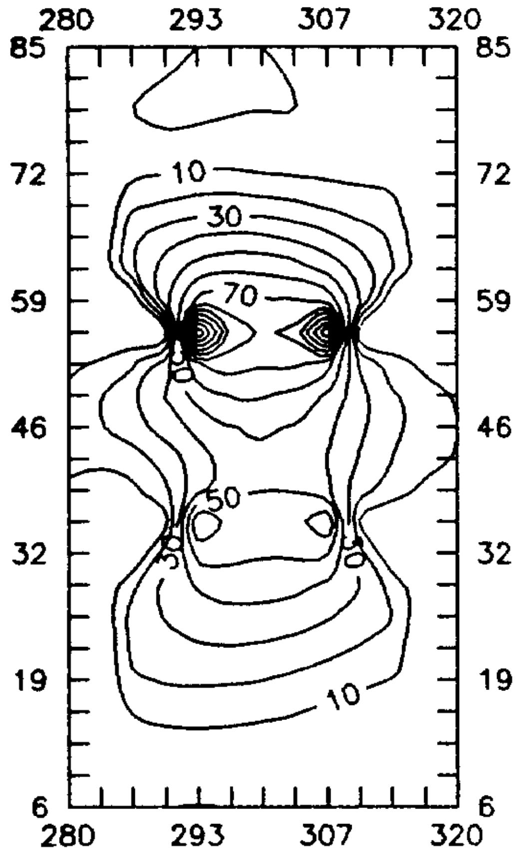

Figure 7: Off Axis Excitation of E_z

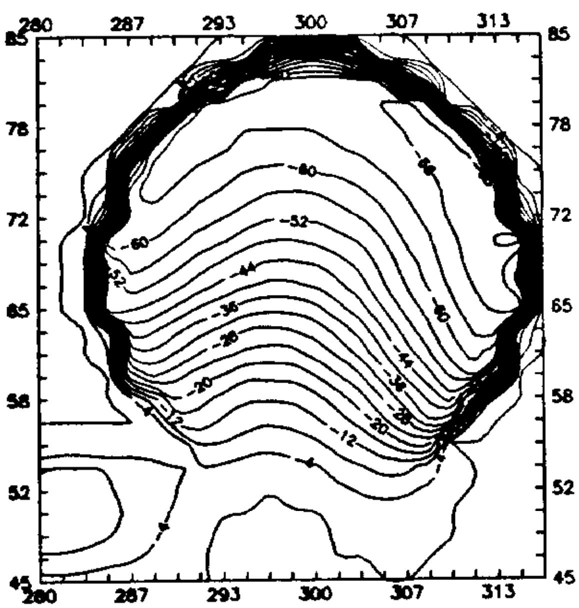

Figure 6: Contour Plot of H_z in the Cavity