BEAM PROPAGATION

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Abstract

The main part of this thesis consists of 15 published papers, in which the numerical Beam Propagating Method (BPM) is investigated, verified and used in a number of applications. The preceding introductory part is divided into six different physical application areas, under which the papers can be sorted. In the introduction a derivation of the nonlinear Schrödinger equation is presented to connect the beginning of the soliton papers with Maxwell's equations including a nonlinear polarization.

This thesis focuses on the wide use of the BPM for numerical simulations of propagating light and particle beams through different types of structures such as waveguides, fibers, tapers, Y-junctions, laser arrays and crystalline solids. We verify the BPM in the above listed problems against other numerical methods for example the Finite-element Method, perturbation methods and Runge-Kutta integration. Further, the BPM is shown to be a simple and effective way to numerically set up the Green's function in matrix form for periodic structures. The Green's function matrix can then be diagonalized with matrix methods yielding the eigensolutions of the structure. The BPM inherent transverse periodicity can be untied, if desired, by for example including an absorptive refractive index at the computational window edges.

We show for example: The amount of laser light coupled into a single-mode fiber can be increased if a rounded taper is placed in between the laser and the fiber. The interaction of two first-order soliton pulses is strongly dependent on the phase relationship between the individual solitons. When optical phase shift keying is used in coherent one-carrier wavelength communication, the fiber attenuation will suppress or delay the nonlinear instability. Further, two star-coupled laser arrays can be made to lase in the in-phase state and by varying the optical path length of the arms of the star, the direction of the output far-field can be scanned. Also, an incoherent light beam can be propagated with the BPM if many field distributions with random grid point phases are propagated and the resulting intensity distributions are later ensemble averaged. Furthermore, simulating coherent random backscattering in the presence of a mirror we find, besides the strongly enhanced backscattered peak, two peaks along the surface of the mirror. All these peaks have an interference structure resulting from interference between the real scatterers and their mirror image.

Descriptors: Beam propagation method
Split-step fast Fourier transform method
Waveguide tapers
Optical solitons
Optical second harmonic generation
Four photon mixing
Band-structure calculations
Green's function
Stripe-geometry-laser arrays
Optical coherence
Optical backscattering

1. Introduction

The biggest application field for optical fibers is communication and almost every communication link placed on the surface of the earth is today made of optical fibers. This is due to two things first their large, approximately 1 THz, bandwidth and secondly their extremely low power losses. We use only around 1 GHz of this bandwidth today leaving room for a potential expansion of the transmission capacity. Although the 1 THz bandwidth might be present there are many unsolved problems to tackle before such a large bandwidth can be used. One of the problems is the stability of the laser light source and another is the tradeoff between transmission length and bandwidth due to linear fiber dispersion. Also, at higher transmission speeds, nonlinear effects are a limiting factor, but in some cases nonlinear effects can actually be used to increase capacity. Higher transmission speed generally means that the intensity of the transmitted pulse has to be increased.

One way of expanding the capacity of already installed fibers is to use coherent optical transmission instead of intensity modulated systems. A second way, for a intensity modulated system, is to employ the nonlinear Kerr-effect, which can either lead to a compression or an expansion in time of the light pulse depending on the sign of the second-order dispersion relation of the fiber. If the group dispersion is negative, which is usually the case, there can be a balance between dispersion and the nonlinear Kerr-effect for soliton propagation. Thirdly, for both coherent and intensity modulated communication, one can send many modulated carrier frequencies in the same fiber.

The fiber limitations are dispersion, damping and, for high powers, nonlinear effects. Further, for close carrier frequencies, nonlinear interaction can be important even at moderate light intensities due to a very long phase-matched interaction length. Dispersion will make a light pulse, if intensity modulated, spread out as it travels along the fiber. If a phase modulated signal (coherent optical communication) instead is sent, the signal will be both intensity and phase modulated due to dispersion after propagation. Damping occurs due to Rayleigh light scattering against irregularities in the fiber. Scattering against temperature dependent fluctuations of the atoms, that build up the fiber, will also clearly reduce the signal power during propagation.

In soliton communication we use the fiber nonlinearity to form a high intensity soliton pulse, which travels more or less undistorted through the fiber. When the second-order dispersion is negative there is a possibility to balance the dispersion with nonlinear effects such as pulse compression. Consequently, the pulse spreading is stopped and we can pack these soliton pulses closer in time to increase the transmission rate. However, there are still problems in that damping will make the pulse spread out in time. Since the intensity always decreases, the pulse will after a while enter the linear domain. Further, due to nonlinear interaction between soliton pulses, a rather large time interval is required between pulses, see **paper III** "Numerical investigation of the soliton interaction". Stimulated Raman amplification of the soliton pulse may be employed to overcome the fiber damping and maintain soliton propagation. Energy is then transferred to the soliton pulse from a strong unmodulated pump beam of a shorter wavelength than the soliton carrier frequency also traveling in the fiber. Unfortunately, a realistic stable and pure high intensity laser which could be used to generate soliton pulses does not yet exist.

A necessity for high speed optical communication is a very stable and pure spectral wavelength laser source. Further, other devices such as couplers, isolators, detectors, switches and Y-junctions have to be of extremely high quality. Therefore all components that build up an optical communication system have to be optimized. Today, on a computer, a numerical calculation or a model simulation of a device performance is a very powerful complement to actual experimental measurements. Furthermore, numerical calculations are an excellent tool for testing out new theories and models, that later can be compared with experimental measurements. Once a theory or a model has been verified experimentally, computer simulations can be used instead of lengthy measurements to understand different device performances or to optimize interesting features. For example, in designing a low-loss high-speed Lithium Niobate ($LiNbO_3$) switch, various parameters such as the form of the waveguide, the component length and geometry must be specified. Clearly it is then easier to run a computer program, maybe for a day, to derive the device behavior than to fabricate and test a new device, which may require a week.

2. Introduction to the Beam Propagation Method

The following study is a collection of 15 published papers in which the Beam Propagation method (BPM) is used to numerically propagate an incoming field distribution through a structure such as a waveguide. The method makes use of the fast Fourier transform algorithm to relate the field at the two planes $z = z_0$ and $z = z_0 + \Delta z$. By repeating this procedure many times we can simulate field evolution in an efficient way within the Fresnel approximation through a waveguide structure.

In order to apply the Beam Propagation Method to a physical problem such as electromagnetic radiation one has to start from Maxwell's equations. For nonmagnetic isotropic materials without free charges the Fourier transform of the electric field $\vec{\mathbf{E}}$ with respect to time obeys

$$\nabla^2\vec{\mathbf{E}} + \frac{\omega^2}{c^2}\left(\epsilon - i\frac{\sigma}{\omega\epsilon_0}\right)\vec{\mathbf{E}} = -\nabla\left(\vec{\mathbf{E}}\cdot\frac{\nabla\epsilon}{\epsilon}\right). \tag{1}$$

Here ω, c, ϵ, σ and ϵ_0 denote the frequency, velocity of light in vacuum, dielectric constant, conductivity and the dielectric permittivity in vacuum respectively. If we restrict ourselves to optical fields and the case when $\vec{\mathbf{E}}$ is effectively orthogonal to the gradient of the dielectric constant, the term on the right hand side is small and therefore can be neglected. In many other applications this term may be small if, for example, the field is negligible when the gradient of ϵ is peaked. When neglecting this term we automatically neglect the coupling between different polarizations and the vector representation of the electric field can be exchanged with a scalar. Thus eq. (1) is replaced with the scalar Helmholtz equation.

$$\nabla^2 E + \frac{\omega^2}{c^2}\left(\epsilon - i\frac{\sigma}{\omega\epsilon_0}\right)E = 0 \tag{2}$$

Further, if in waveguide problems we are interested in the guided light, that travels with small angular distributions around the optical axis, we can expand the electric field E around the wavevector kn_0 in the direction of the positive optical axis thus $E = \mathcal{E}\exp(-ikn_0 z)$, where $k = \frac{\omega}{c}$ and n_0 is equal to the square root of a representative dielectric constant value, for example in a fiber, the cladding value. If we neglect the second derivative of $\mathcal{E}$, with respect to z along the optical axis, compared with the first we get the Fresnel approximation.

$$2ikn_0\frac{\partial\mathcal{E}}{\partial z} = \nabla_\perp^2\mathcal{E} + k^2\left(n^2 - n_0^2 - i\frac{\sigma}{\omega\epsilon_0}\right)\mathcal{E} \tag{3}$$

Here the square of the refractive index n^2 is equal to the dielectric constant ϵ and $\nabla_\perp^2$ is the transverse Laplacian operator. The Fresnel equation above can efficiently be solved for a transverse starting field by the use of the BPM. The neglect of the second derivative of $\mathcal{E}$ is only justified, if the electric field, when decomposed into plane waves, only consists of waves making a small angle with the positive optical axis or when the angular spread of waves around some angle θ is small and the propagation direction of the total electric field makes a small angle with the optical axis. In the latter case, if we are interested in the absolute phase of the electric field, we must correct the phase due to the angle θ. Reflection effects in the backward longitudinal direction will be totally eliminated when incorporating the Fresnel approximation. Notice that eq. (3) is equivalent to the Schrödinger equation, if one lets z be proportional to the time variable and the expression inside the parenthesis in the second term on the right be proportional to the potential. Of course the complex term containing the conductivity has to be set to zero. The BPM is expressed mathematically with two operators i: a propagator and ii: a phase corrector which is used in a split step fashion to increase accuracy. Physically, in the case of light propagation, the material in between two planes is divided for calculational purposes into a number of parallel equidistant planes and the integrated variation of the dielectric constant is collapsed to these planes so that the material is homogeneous in between planes. Now the square of the propagator relates the field in the fictive homogeneous medium from plane to plane, separated by a distance Δz, and the phase corrector takes care of the integrated dielectric variation in a volume $\frac{\Delta z}{2}$ on either side of these planes. Mathematically the propagator is defined as

$$\hat{P} = \exp\left(-i\frac{\Delta z}{4n_0 k}\nabla_\perp^2\right) \tag{4}$$

and the phase corrector

$$\hat{C} = \exp\left(-i\frac{n_0 k}{2}\int_z^{z+\Delta z}\left(\frac{n^2(\vec{r}')}{n_0^2} - 1 - i\frac{\sigma(\vec{r}')}{\omega\epsilon_0 n_0^2}\right)dz'\right) \tag{5}$$

and the electric field at $z + \Delta z$ is

$$\mathcal{E}(x, y, z + \Delta z) = \hat{P}\ \hat{C}\ \hat{P}\ \mathcal{E}(x, y, z) + O((\Delta z)^3). \tag{6}$$

Since the propagator in the transverse Fourier plane is simply an exponential function of the spatial frequencies the propagator is realized by first Fourier transforming the electric field and followed by multiplication with the exponential function before a final inverse Fourier transformation of the resulting electric field. When repeatedly using eq. (6) for field propagation and making the distance Δz between the planes smaller, while keeping the total propagation length constant through increasing the number of planes, the error in the final field distribution will be proportional to $(\Delta z)^2$. When numerically incorporating this theory using the fast Fourier transform we have to, since the fast Fourier transform is discrete, digitalize the transverse directions into equidistant grid points with representative values for the dielectric constant and electric field, see **paper XV** "New formulations of the matrix beam propagating method: application to rib waveguides". Also because of the periodic nature of the fast Fourier transform the resulting electric field is transversely periodic with a period equal to the rectangular window used. Note, that when the conductivity is zero, eq. (6) is composed of hermitian operators and therefore the electric field intensity inside the window is kept constant, which makes the method extremely stable differentiating it from other numerical methods.

3. Spectral analysis

Assume that the dielectric constant and the conductivity are independent of longitudinal distance and that the vector representation of the electric field can be replaced by a scalar, we may then define the normalised electric field distributions $R_j(x, y, \omega)$ and the corresponding j-th Helmholtz propagation constant $\beta_j(\omega)$, i.e. the product $R_j \exp(-i\beta_j z)$ is a solution to eq. (2). All possible time dependent electric field distributions $\hat{E}(x, y, z, t)$ traveling in the direction of the positive optical axis can then be written as

$$\hat{E}(x, y, z, t) = \sum_{\omega} \sum_{j} A_j(\omega) R_j(x, y, \omega) \exp(i\omega t - i\beta_j(\omega) z), \tag{7}$$

where the sum represent an integral where the spectrum is continuous and the functions $A_j(\omega)$ are fixed by the initial excitation. The above conditions apply to an unperturbed fiber or waveguide, if the conductivity term is associated with the fiber damping.

Of great interest in spectral analysis is to determine the eigenfunctions R_j and the related eigenvalues β_j. Fortunately, the Helmholtz modal field distributions are identical to the Fresnel modes and there is the following relationship between the Helmholtz β and the Fresnel propagation constant β'.

$$\beta = kn_0 \left(1 + 2\frac{\beta'}{kn_0}\right)^{1/2} \tag{8}$$

Using the above fact we can determine the eigenfunctions and their related propagation constants with the spectral method based on the BPM for field evolution. The spectral method, see **paper VI** "The accuracy of band-structure calculations based on the split-step fast Fourier transform method", generates the longitudinal spectral content of an input electric field envelope $\mathcal{E}(x, y, z_0)$ in terms of the longitudinal waveguide spectrum. The method makes again use of the fast Fourier transform taken in the longitudinal z direction of a correlation function $F(z)$. The function $F(z)$ equals to the electric field projection onto the complex conjugate of the input field times the Hanning window function $W(z) = \frac{1}{2}[1 - \cos(2\pi(z - z_0)/(M\Delta z))]$, which is zero at the starting point z_0 and the end point $z_0 + M\Delta z$. Mathematically,

$$F(z) = \int\int \mathcal{E}^*(x, y, z_0)\mathcal{E}(x, y, z)W(z)\, dxdy. \tag{9}$$

The electric field is propagated M steps in the z invariant waveguide direction and the value of F is calculated after each step. The Fourier transform of F will be peaked at the values of the waveguide propagation constants with amplitudes corresponding to the spectral intensity content of the incident field. Once the propagation constants β'_j have been located we can extract the related eigenfunctions out of the electric field by multiplying the electric field by $W(z)\exp(i\beta'_j z)$ at each step and adding all the resultant terms. This will project out the j-th eigenfunction which varies as $\exp(-i\beta'_j z)$ as a function of z. The presence of the Hanning window gives the peak, associated with each propagation constant, a characteristic line shape and smooths out the non-periodic behavior of F at the end-points of the propagation length. The peak of this line can then be easily located. The accuracy in such numerical calculations depends on the step-length and the propagation length. The propagation length necessary to resolve the propagation constants is inversely proportional to the spacing of the closest propagation constants and a small step-length must be employed, if the dielectric constant varies largely. The spectral method is most difficult when the parenthesis on the right hand side of eq. (3) is imaginary, see **paper X** "New approach to lossy optical waveguides".

An alternative procedure for finding the propagation constants is provided by the so-called matrix BPM. This formalism may be derived by expressing the electric field at a longitudinal position as an integral over the transverse coordinates of the electric field at another longitudinal point times the Green's function relating the two planes. Replacing the above mentioned integral by a discrete sum over the transverse grid points, the Green's function can easily be constructed. In the matrix BPM we accordingly employ a small step length Δz and propagate N delta functions, where N is equal to the number of points in the window, a single step-length using eq. (6). The (i,j)-th delta function with its peak at the point (i,j) is 1 at this point and zero at all other points in the window. The resulting propagated output functions are collected in a fourth-order tensor $G_{(k,l)(i,j)}$ which represents the Green's function and can be rewritten as a matrix. Evaluation of the eigenvalues and eigenvectors of this tensor gives the propagation constants β'_j, which are related to the eigenvalues $\exp(-i\beta'_j\Delta z)$ of the Green's function matrix. The matrix method is more efficient than the spectral method if the number of points is not too great. Further, it does not share the disadvantage of the spectral method associated with the undetermined number of propagation steps M. Finally, the presence of a non-zero conductivity does not create a problem within the context of the matrix BPM. On the other hand, if the absolute value of the real part of $\beta'_j\Delta z$ is bigger than π, β'_j cannot be determined. This problem can however be solved by propagating in iz rather than z assuming that $|\mathrm{Im}\{\beta'_j\Delta z\}| < \pi$.

4. Soliton analysis

A soliton in optical fibers is a single light pulse envelope that propagates through the fiber with constant speed, the group velocity, and as it travels along the fiber its form is allowed to change periodically as a function of distance along the fiber length. For the zeroth-order soliton the light pulse intensity is constant as it travels down the fiber. The linear group dispersion, in the case of negative group dispersion, is balanced by the nonlinear Kerr-effect which tends to compress the pulse. Solitons are described by the nonlinear Schrödinger equation which will be derived starting from the scalar wave equation

$$\nabla^2\hat{E} - \frac{1}{c^2}\frac{\partial^2(\widehat{\epsilon E})}{\partial t^2} = \frac{1}{c^2}\frac{\partial^2(\widehat{n_{nl}^2 E} - \widehat{\epsilon E})}{\partial t^2}. \tag{10}$$

The nonlinear refractive index for a lossy optical fiber given by

$$n_{nl}(x,y,\omega,E) = n(x,y,\omega) + n_2|E|^2 + n_4|E|^4 + i\chi. \tag{11}$$

Here the hat represents the inverse time Fourier transform since E, $\epsilon = n^2$ and n_{nl} are defined in the frequency domain, χ is the absorption coefficient of the fiber, n_2 and n_4 are related to the third- and fourth-order nonlinear susceptibility tensor. Expanding the dielectric constant around the carrier frequency ω and keeping terms to third-order gives

$$\frac{\partial^2(\widehat{\epsilon E})}{\partial^2 t} \approx e^{-i\omega t}\left[-\epsilon\omega^2 Ee^{i\omega t} - i\frac{\partial(\omega^2\epsilon)}{\partial\omega}\frac{\partial(Ee^{i\omega t})}{\partial t} + \frac{1}{2}\frac{\partial^2(\omega^2\epsilon)}{\partial\omega^2}\frac{\partial^2(Ee^{i\omega t})}{\partial t^2} + \frac{i}{6}\frac{\partial^3(\omega^2\epsilon)}{\partial\omega^3}\frac{\partial^3(Ee^{i\omega t})}{\partial t^3}\right]. \tag{12}$$

and further expanding the right hand side of eq. (10) and keeping only the zeroth-order terms gives

$$\frac{\partial^2(\widehat{n_{nl}^2 E} - \widehat{\epsilon E})}{\partial t^2} \approx -2n\omega^2\left[n_2|E|^2 + \left(n_4 + \frac{n_2^2}{2n}\right)|E|^4 + i\chi\right]E. \tag{13}$$

We write the electric field as

$$\hat{E}(x,y,z,t) = \mathrm{Re}\left\{\Phi(z,t)R_0(x,y)\exp\big(i(\beta_0 z - \omega t)\big)\right\}. \tag{14}$$

Here $R_0(x,y)$ equals to the zeroth-order transverse electric field with propagation constant β_0, i.e.

$$\nabla_\perp^2 R_0 + \left(\frac{\omega^2}{c^2}n^2 - \beta_0^2\right)R_0 = 0 \tag{15}$$

and $\Phi(z,t)$ is the electric field envelope. Inserting then eq. (12) to (15) into eq. (10), multiplying the resulting equation by $\int\int \frac{R^*}{R^* R\, dxdy}$ and integrating over the transverse coordinates x and y gives the following equation

$$\frac{\partial^2 \Phi}{\partial z^2} + 2i\beta_0 \frac{\partial \Phi}{\partial z} + \left(D_2|\Phi|^2 + D_4|\Phi|^4 + i\gamma\right)\Phi + iP_1 \frac{\partial \Phi}{\partial t} - P_2 \frac{\partial^2 \Phi}{\partial t^2} - iP_3 \frac{\partial^3 \Phi}{\partial t^3} = 0 \tag{16}$$

where

$$D_2 = 2\frac{\omega^2}{c^2} \frac{\int\int n n_2 |R_0|^4\, dxdy}{\int\int |R_0|^2\, dxdy} \tag{17}$$

$$D_4 = \frac{\omega^2}{c^2} \frac{\int\int (2nn_4 + n_2^2)|R_0|^6\, dxdy}{\int\int |R_0|^2\, dxdy} \tag{18}$$

$$\gamma = 2\frac{\omega^2}{c^2} \frac{\int\int n\chi |R_0|^2\, dxdy}{\int\int |R_0|^2\, dxdy} \tag{19}$$

$$P_1 = \frac{1}{c^2} \frac{\int\int \frac{\partial(\omega^2 \epsilon)}{\partial \omega} |R_0|^2\, dxdy}{\int\int |R_0|^2\, dxdy} \approx \frac{d\beta_0^2}{d\omega} \tag{20}$$

$$P_2 = \frac{1}{2c^2} \frac{\int\int \frac{\partial^2(\omega^2 \epsilon)}{\partial \omega^2} |R_0|^2\, dxdy}{\int\int |R_0|^2\, dxdy} \approx \frac{1}{2}\frac{d^2\beta_0^2}{d\omega^2} \tag{21}$$

$$P_3 = \frac{1}{6c^2} \frac{\int\int \frac{\partial^3(\omega^2 \epsilon)}{\partial \omega^3} |R_0|^2\, dxdy}{\int\int |R_0|^2\, dxdy} \approx \frac{1}{6}\frac{d^3\beta_0^2}{d\omega^3}. \tag{22}$$

We now neglect the first term in eq. (16), the second derivative of the electric field envelope with respect to time, compared with the second term and by changing variables so that the new time variable travels with the same velocity as the pulse itself, we then obtain the nonlinear Schrödinger equation with a number of correction terms on the right hand side.

$$i\frac{\partial q}{\partial \xi} + \frac{1}{2}\frac{\partial^2 q}{\partial \tau^2} + |q|^2 q = -i\Gamma q + iB_3 \frac{\partial^3 q}{\partial \tau^3} - C_4 |q|^4 q \tag{23}$$

The following change of variables has been made

$$\xi = \frac{10^{-9}}{\lambda} z \tag{24}$$

$$\tau = \frac{10^{-4.5}}{(-\frac{\lambda}{\beta_0} P_2)^{1/2}} \left(t - z\frac{P_1}{2\beta_0}\right) \tag{25}$$

$$q = 10^{4.5}\left(\frac{\lambda}{2\beta_0} D_2\right)^{1/2} \Phi \tag{26}$$

$$\Gamma = 10^9 \frac{\lambda\gamma}{2\beta_0} \tag{27}$$

$$B_3 = 0.5 \times 10^{-4.5}\left(\frac{\beta_0}{\lambda}\right)^{1/2} \frac{P_3}{(-P_2)^{3/2}} \tag{28}$$

$$C_4 = 2 \times 10^{-9}\frac{\beta_0}{\lambda}\frac{D_4}{D_2^2} \tag{29}$$

where λ is the carrier wavelength in vacuum. Approximate expressions of the coefficients in eq. (24) to (29) can be found in **paper II** "Soliton analysis with the propagating beam method".

5. Coupling of laser light into fibers

When a semiconductor laser is used as a light source for optical communications, a maximum amount of light should be coupled into the fiber. The near-field distribution of a semiconductor laser has however a very small spot size compared with a single-mode fiber so that the amount of coupled power is quite low. Various mechanisms have accordingly been developed to increase the coupling efficiency including for example lenses and tapers. Here the laser near-field has to be enlarged and if possible adjusted to resemble the fiber near-field. Since the use of discrete lenses is complicated by the very different small laser and fiber dimensions, Lars Bodén and Lars Thylén at Ericsson in 1982 proposed heating and then stretching the fiber end and subsequently aligning the resulting lens and taper in front of the laser. At that time, the maximum experimental coupling efficiency was 40%. In **paper I** "Propagating-beam-method analysis of two-dimensional microlenses and three-dimensional taper structures" we analyzed a two-dimensional model of the stretched fiber end, which was assumed to have a form of a linear taper with a truncated parabolic dielectric profile just as the unstretched fiber. The lens at tip of the taper was assumed to be elliptical. Our calculations showed that the form of the lens and the distance between the lens and the semiconductor laser were the most critical parameters. The model we used was of course only a rough estimate since we did not know the actual structure of the lens. Furthermore the use of the Fresnel approximation is here not strictly justified nevertheless our calculations agreed reasonably with subsequent measurements at Ericsson. From fig. 5 the resemblance with geometrical optics is clear, leading us to conclude that the optimum distance between the tip of the lens and the laser is attained when the lens focuses the laser spot onto the lowest-order local normal mode of the input part of the taper. The taper then essentially guides all power in the lowest-order local normal mode into the fiber. Recall that the local normal modes at a given longitudinal distance, z, are the eigenmodes of a fiber with a transverse refractive index given by $n(z)$.

A related problem is the coupling of light from a short fiber spliced to a longer fiber. Splicing two single-mode fiber ends together requires precision alignment of the transverse and angular displacement of the two fiber cores to ensure low losses. Transverse alignment of the two cores to within a few microns, is quite difficult to achieve in practice. Accordingly, Leif Stensland at Sieverts Kabelverk suggested to decrease the sensitivity to transverse misalignment by expanding the dimensions of electric field in the fiber through the use of an appropriate taper and then splicing the wide taper ends. While certain radiation losses are associated with the taper, for misalignments bigger than a certain minimum displacement, the presence of the two tapers yields a lower total loss. Paper I also includes a BPM simulation of the losses involved in a tapered splice. The simulations show for example that for short tapers, the precise taper geometry determines to a large extent how fast the field will expand. For example, we find that unless the taper length is very short, the tapered splice made of two Gaussian tapers possesses lower losses than the corresponding splice made of linear tapers.

6. Nonlinear effects in optics

Optical nonlinear effects are due to nonlinear interactions between the optical field and the matter through which the light travels. One type of interaction is due to the polarizability of the medium which is not linear for strong electric fields. Another nonlinear interaction arises when the radiation field interacts with electrons and phonons and mechanical vibrations in the medium either creating or absorbing a phonon and therefore changing the optical wavelength of the scattered photon. Considering the first of the two above interactions, which is non-absorptive and can be represented by a real nonlinear susceptibility tensor. Then, the energy in the optical field is constant although it may be redistributed over many wavelengths after some interaction length. In non-birefringent optical fibers the even orders of the nonlinear susceptibility are zero due to inversion symmetry and the first non-zero order is the third which can give rise to many different effects such as four photon mixing and solitons. When four photon mixing occurs three different photons interact, which in the case of solitons all possess the same wavelength.

For odd nonlinear orders the interaction can be regarded as an intensity dependent dielectric constant. For example, in third-order the dielectric constant increases with a more intense field and therefore gives rise to focusing in space and to pulse compression in time. This interaction can be modelled by the nonlinear Schrödinger equation. Correction terms may however arise from higher-order terms in the expansion of the second time derivative of the electric polarization eq. (12) and (13). In paper II "Soliton analysis with the propagating beam method" we show that BPM can be applied to the nonlinear Schrödinger equation modified to account for damping, third-order linear dispersion and fifth-order nonlinear susceptibility. From our calculations we conclude that only first-order solitons should be excited in a soliton based optical communication system since, for example, only a small third-order linear dispersion will destroy the high-order soliton after a relative small propagation length. Next in paper III "Numerical investigation of soliton interaction" we study the interaction between two first-order soliton pulses separated with a time distance that is less then ten times the half-width of a single soliton pulse. If the two solitons are in phase with respect to each other, there is more intensity in between the pulses than on either side of the two soliton pulses. Consequently, the group velocity of the left pulse will increase while that of the right hand pulse will decrease thus a resulting attraction of the pulses. Since the two pulses now possess different velocities, the pulses oscillate back and forth interchanging their positions. On the other hand if the pulses are 180° out of phase there is less light intensity in between them and higher intensity on the outside boundaries of both pulses, causing a decrease in the velocity of the left-hand and an increase in the velocity of the right-hand pulse. Hence, the two pulses will repel and the pulses therefore separate at a accelerated rate. With this reasoning one would conclude in analogy with mechanics that for some relative phase factor the attractive will balance the repellent force although this configuration is most likely not stable. Consequently, while the soliton-soliton interaction is a limiting factor in high bit rate communications links, if the phase difference between soliton pulses can be precisely controlled, the solitons can be packed closer together.

Except for soliton propagation, nonlinear effects are generally detrimental to optical communication systems. Fortunately, below a certain electric field intensity threshold linear effects are dominant. For having only one-carrier wavelength and a Phase Shift Keyed (PSK) signal, fiber damping increases the nonlinear threshold, see **paper V** "Modulational instability effects in PSK modulated coherent fiber systems and their reduction by optical loss". The modulational instability arises from a combination of second-order dispersion and the nonlinear Kerr-effect. First, second-order dispersion causes the different frequencies in the initial constant amplitude electric field to travel with different relative speeds. Thus, an amplitude variation results after a certain distance. Then once the intensity threshold is reached, the nonlinear Kerr-effect compresses the dispersion induced pulse, but due to damping during the initial linear amplitude pulse formation the intensity is now lower than without damping. Therefore the presence of optical losses delays or suppresses entirely the appearance of the instability.

In non-isotropic crystals the second-order nonlinear susceptibility is not zero and two different wavelengths may interact. For the special case that the two interacting photons have the same energy a photon of twice the input energy may be produced if the two waves of different wavelength travel in phase during a long distance. In **paper IV** "A propagating beam method analysis of nonlinear effects in optical waveguides" we simulate second harmonic generation in a $LiNbO_3$ birefringent waveguide and find that we get a reasonable agreement with experiment. The phase-matching condition is simulated by a temperature dependent refractive index according to ref.(3) of this paper.

In four photon mixing in optical fibers and waveguides, phase-matched coupling must again be present. Paper IV simulates four photon mixing with the BPM, where two of the four photons involved are equal in wavelength. Note that the three waves must be in different waveguide modes for phase-matching to occur. If however the waveguide have irregularities, phase-matching is destroyed due to coupling between modes of the same frequency. We have simulated this effect by including a sinusoidal waveguide perturbation in our BPM calculation.

7. Band-structure calculations

The electronic states in a crystalline solid may be displayed by a band-structure diagram. For example, the band-structure of a semiconductor simplifies calculations of the electron-photon interaction in a quantum-well semiconductor laser. To solve the electron band-structure problem from first principles is an extremely difficult problem and can only be done for the simplest solids. However, since the deeper level electrons screen the positively charged atom at the lattice sites the Coulomb interaction may be replaced by the local empirical pseudopotential felt by a single valance electron. We use in paper VI "The accuracy of band-structure calculations based on the split-step fast Fourier transform method" the periodic nature of the fast Fourier transform to calculate the band-structure of Tin, Sn, from its local empirical pseudopotential. The split-step fast Fourier method is identical to the BPM. The problem is transformed from the unit primitive cell into a three-dimensional rectangle that is taken as the numerical window. We excite the unit cell of Sn with an electron wave packet with a average momentum $\vec{k}$ and propagate this wave packet in time while recording the correlation function in time between the initial electron wave packet and the wave packet after each time step Δt. The Fourier transform of the correlation function will then yield the electron energy eigenvalues for the electron momentum $\hbar\vec{k}$. Doing so for the various values of $\vec{k}$-space, generates the band-structure diagram.

It is highly probable that the numerical method developed in paper XV entitled "New Formulation of the Matrix BPM: Applications to Rib Waveguides" will yield a band-structure calculational method an order of magnitude more efficient than that presented in paper VI. In this case, the method could be successfully applied to superlattices and quantum well structures. However, neither methods can be applied to the singularities in the potential at the atomic positions.

To complement the BPM we also adapted the Finite-element Method (FEM) to band-structure calculations for the local empirical pseudopotential of Aluminum Antimonide, AlSb. We concluded that the FEM is not as efficient for our pseudopotential which was a sum of the lower-order Fourier components of the lattice potential as a variational matrix diagonalization method based on a plane wave expansion or the BPM, cf. **paper VII** "Finite-element approach to band-structure analysis".

8. Numerical realizations of the waveguide Green's function

As discussed earlier in section 3 entitled "spectral analysis", the Green's function expresses the field at a longitudinal distance $z = z_1$ resulting from a delta-function excitation at another $z = z_0$. When the matrix form of the Green's function is obtained after one period of a longitudinally periodic waveguide, we may evaluate the Green's function for a longer distance of 2^M periods in two different ways. The first is to rewrite the Green's function matrix as $\mathbf{G} = \mathbf{U}^{-1}\mathbf{D}\mathbf{U}$, where $\mathbf{D}$ is a diagonal matrix formed from the eigenvalues of the Green's function matrix, and then replacing $\mathbf{D}$ by the matrix obtained by raising the components of $\mathbf{D}$ to the M-th power. The other alternative is to square M times the Green's function matrix for one period as illustrated in **paper IX** "New approach to perturbed optical waveguides". Here we also suggest several methods of constructing the Green's function matrix besides the matrix BPM.

For lossy components like micro-bends or periodically perturbed waveguides which radiate energy, we must generally calculate the energy radiated after a certain distance for a given input field. This can be simply obtained by adding a fictitious absorber a certain transverse distance away from the device and then propagating the input field with the BPM. If the absorber is adjusted so that little power is reflected or transmitted to the opposite window edge, a steady-state field is attained after a long propagation distance. Alternatively, we may construct the complex Green's function matrix for a longitudinal waveguide period L_p, solve for its eigenvalues α_j and compute $\mathrm{Im}\left\{\frac{i}{L_p}\ln\alpha_j\right\}$, which is the power loss per unit length. The eigenvalue with the lowest power loss is that of the stationary field mentioned above. This is done in more detail in paper X "New approach to lossy optical waveguides". A bent fiber can be conformally mapped into a rectangular configuration with a linear ramp added to the square of the refractive index. When an optical lossy component has a relative short length so that no steady-state field arises within the component and the input and output waveguides are single-mode, an interesting parameter is the total radiation losses. The input field can then be propagated through the component in the same manner as mentioned above, cf. paper XV "New formulations of the matrix beam propagation method: Application to rib waveguides" and the Y-junction calculation therein. The guided output field is calculated either, through propagating the component output field in the output waveguides a long enough distance so that the unguided power has been absorbed in the fictitious absorber or by projecting the component output field onto the lowest-order guided modes of the output waveguides. The total radiation losses are then the difference between the input power and the power in the guided output modes.

In paper XV we also discuss the equivalence of BPM to variational methods such as the Rayleigh-Ritz procedure. Our discussion leads to several new features. For example, we employ a transverse average of the dielectric constant over rectangles centered at every grid point rather than simply employ the dielectric constant at every grid point. With this modification we resolve the edges of step index structures such as a rib waveguide with much higher accuracy. Further we show that the BPM basis functions for the electric field in position space may be described by "periodic delta functions", which are a sum of the N lowest Fourier components, where N is the number of grid points. Since these basis functions are orthogonal, an overlap integral between two electric fields collapses to a single sum of products of the electric fields at the given grid point.

Note now that closely spaced propagation constants can only be resolved by using long propagation distances with the method introduced in paper XV since the eigenvalues of the Green's function matrix are $\exp(\beta_j' \Delta z)$ for iz propagation. On the other hand in the FEM, we can obtain the same rate of convergence by shifting the eigenvalues of the matrix to be diagonalized. Unfortunately, when the lowest-order eigenvalues, which are usually of greatest interest, are located far from the maximum value of the refractive index profile, it is difficult to predict the optimum magnitude of the shift. Therefore, if the propagation constants are located close to the profile maximum, the FEM is very efficient while the reformulation of the matrix BPM in paper XV is more efficient for widely spaced propagation constants especially if these are located far from the top of the profile.

9. Semiconductor laser arrays

Semiconductor laser arrays have recently been developed with the aim of producing high intensity beams with small far-field angular divergences. These two conditions may be realized if the individual lasers lase in phase with about equal amplitude. However an equally spaced laser array lases in the highest-order supermode with the neighboring lasers 180° out of phase since the electric field then is most intense in the high-gain active regions and has zeros in the absorptive regions between the laser stripes. Such a laser array will therefore possess a characteristic two-lobed far-field.

When studying equally spaced laser arrays with many individual lasers it is advantageous to consider an infinitely periodic structure using a band-structure analysis as in our paper, **paper VIII** "New fast Fourier transform and finite-element approaches to the calculation of multiple-stripe-geometry laser modes". Here we assume a gain profile with a squared tangent hyperbolic form with an anti-guiding proportional to the gain through the anti-guiding constant. In this paper we calculate the modal field distributions and the associated complex propagation constants for a periodic laser array with the BPM and FEM. These methods were also checked using the standard shooting method, which integrates the differential equation for the periodic part of the Block function and then determines the eigenvalues by matching the numerically integrated solution to the periodic boundary conditions. Here the phase relationship between the lasers is a parameter ranging from 0° to 180°. Therefore for a laser array, the gain and propagation constants of the lasers may be described by continuous bands as a function of the phase relationship between neighboring lasers. The width of the lowest-order bands is related to the degree of mutual interaction of adjacent lasers.

While the zeroth-order supermode may be made to lase by adjusting the spacing between the individual lasers, the stability of the lasing mode is reduced, cf. **paper XI** "Numerical analysis of the modal eigenfunctions of chirped and unchirped multiple-strip-geometry laser arrays". Unfortunately, the near-field intensity distribution is higher over some lasers in the array, reducing the total output power. Although we have not performed a self-consistent calculation, which would take into account the carrier insertion, diffusion, recombination and interaction with the electric field in the active region, we believe that the profiles studied in this paper resemble realistic multiple-stripe-geometry laser arrays.

A better solution to the two-lobed far-field problem is to eliminate the out of phase supermode using destructive interference while promoting the in-phase mode with constructive interference. This can be realized by linking two index-guided arrays with a series of parallel Y-junction waveguides. The length of the arms of every Y-junction is equal and the laser arrays are displaced laterally with respect to each other by half the distance between individual lasers. In **paper XIV** "An analysis of Y-junction and coupled laser arrays" we study an infinite Y-junction laser array with the BPM using a band-structure analysis. We find that for gain-guided laser arrays the above analysis does not apply since the lowest-order modes are then focused into the low-gain region in between the lasers. We also proposed a laser constructed of two weakly index-guided laser arrays coupled with a lens or star-coupler into a short laser and out through another lens or star-coupler into the opposite laser array. When the optical path length of the star is symmetric with respect to the central laser each laser array will lase in phase. Further, by changing the optical path length of the arms of the star-couplers by an electro-optical contact, the far-field can be scanned.

10. Statistical simulations in optics

In the analysis of random phenomena we must perform an ensemble average over a large number of realizations. In optics both the incident optical field and the medium through which the light travels can fluctuate randomly. Since incoherent spontaneous emission processes dominate most light sources, the emitted light is almost fully incoherent, so that no phase relationship exists between different points in the light field at the source. However, because of angular divergence the light field will be partially coherent away from the source. Many media consist of inhomogeneities which fluctuate because of thermal motion or turbulence. If these fluctuations are sufficiently slow, that the inhomogeneities move only a small distance during which the light on the average scatters from several inhomogeneities, it is reasonably accurate to consider the medium frozen for each realization and then record the scattered field resulting from an input field in a steady-state situation. When simulating scattering in the case of a coherent input field, the input field is fixed while the medium changes randomly between realizations. On the other hand if the input field is incoherent then many realizations of the input field with different relative phases at each spatial point must be considered.

Paper XII "Optical coherence calculations with the split-step fast Fourier transform method" introduces the modelling of incoherent and partially coherent light beams with the BPM. Here we have studied a case where the phase of the input field fluctuates randomly while the configuration of the medium is fixed. The BPM is then used to propagate the random field through the configuration. This is possible because of the numerical stability of the algorithm. Every realization of an incoherent light source possesses an identical transverse amplitude distribution but a random complex phase at each grid point. Propagating a large number of realizations through the structure and ensemble averaging the spatial transverse correlation functions allows us to model the propagation of incoherent and partially coherent light. For long z-dependent structures where many propagation steps are required it is however more efficient to construct the Green's function matrix and use matrix multiplication to get the output field for each realization.

When the medium instead is subject to fluctuations and the optical field is coherent, enhanced scattering and enhanced phase fluctuations result in the backward direction. The first effect is due to an interference between a ray passing from a scattering center A to center B with that scattered from B to A. The second effect appears when light is scattered by large scale fluctuations with small refractive index changes to a mirror and then reflected back through the same inhomogeneities. In **paper XIII** "Numerical simulation of coherent backscattering in small two-dimensional systems" we simulate both of these effects. Note that the approximations of slowly moving fluctuating centers or inhomogeneities are assumed valid. Enhanced phase variations are simulated by propagating a coherent electric field consisting of a sum of plane waves in different directions with the aid of the BPM. From our simulations we conclude that as the random inhomogeneities decrease in size, the input plane waves are subjected to more diffraction resulting in a wider angular spread but smaller magnitude enhanced phase variation.

In the second part of paper XIII we find a first-order self-consistent solution for the stationary scattering from a set of very small equal scatterers randomly distributed in a plane containing a non-absorptive background medium. The incident coherent field is a plane wave and the scattered field from every scatterer is assumed to be a circular symmetric field. Our scattering centers are chosen to be infinitely absorptive so that the electric field is zero at the circular boundary of $0.01\mu m$ radius. Ensemble averaging the intensity over a large number of realizations far from the small random distribution allows us to simulate the enhanced backscattering. The width of the enhanced backscattering peak is then related to the size of the distribution. We then repeat the same calculation in the presence of a mirror. In this case the enhanced backscattering is a first-order and therefore more intense process while a second intensity peak is obtained along the surface of the mirror. The structure of the peaks in the backward direction and along the mirror result from interference between a particular center and its mirror image.

Corrections to the papers

Paper I: "Propagating-beam-method analysis of two-dimensional microlenses and three-dimensional taper structures"

i: In fig. 9 of this paper we employed the Simpson integration formula to project the electric field onto the lowest-order normal mode of the taper. The high frequency numerical noise in the figure is a consequence of Simpson's formula and would be suppressed if the projection was implemented by a simple vector product. See paper XV.

ii: While the backward reflection from the tip of the micro-lens is ignored in this paper, it will be less the 4% since the surface has a small radius of curvature.

Pper II: "Soliton analysis with the propagating beam method"

i: Eq. (6) and (7) of this paper should read

$$q = 10^{4.5}(2\pi n_2 A)^{1/2}\Phi$$

$$C_4 = \frac{0.5\times 10^{-9}}{\pi}\left(\frac{n_4}{n_2^2} + \frac{1}{2n_0}\right)\frac{B}{A^2}$$

respectively.

Paper IV: "A propagating beam method analysis of nonlinear effects in optical waveguides"

i: The values for the nonlinear second-order susceptibility tensor are taken from T. Yamada (K-H Hellwege, ed): "Landolt-Börnstein numerical data and functional relationships in science and technology" new series, group III, Vol. 16a (Springer, Berlin, Heidelberg 1981) page 151. Also the lossless electronic polarizability is assumed, resulting in the equality of the tensor elements $\chi^{(2)}_{ijk}$, $\chi^{(2)}_{ikj}$ and $\chi^{(2)}_{jki}$.

ii: The value of the third-order nonlinear susceptibility component is that of Hasegawa and Kodama "Signal Transmission by Optical Solitons in Monomode Fiber" Proc. IEEE 69 (1981) 1145.

Paper V: "Modulational instability effects in PSK modulated coherent fiber systems and their reduction by optical loss"

i: Page 99 it should contain: normalized refractive index $\Delta = 0.002$.

Paper VI: "The accuracy of band-structure calculations based on the split-step fast Fourier transform method"

i: Equation (3) and (4) of this paper should read

$$\begin{aligned}\Psi(\mathbf{x}, t_0 + \Delta t) = \Bigg[1 + i\frac{\Delta t}{2}\nabla^2 - i\int_{t_0}^{t_0+\Delta t} V(\tau)\,d\tau - \frac{(\Delta t)^2}{8}\nabla^4 \\ + \frac{1}{2}\nabla^2\int_{t_0}^{t_0+\Delta t} d\tau\int_{t_0}^{\tau} V(\tau')\,d\tau' + \frac{1}{2}\left(\int_{t_0}^{t_0+\Delta t}(\tau - t_0)V(\tau)\,d\tau\right)\nabla^2 \\ - \int_{t_0}^{t_0+\Delta t} d\tau\, V(\tau)\int_{t_0}^{\tau} V(\tau')\,d\tau' + \ldots\Bigg]\Psi(\mathbf{x}, t_0)\end{aligned}$$

and

$$\begin{aligned}\Psi(\mathbf{x}, t_0 + \Delta t) = \Bigg[\exp\left(i\frac{\Delta t}{4}\nabla^2\right)\exp\left(-i\int_{t_0}^{t_0+\Delta t} V(\tau)\,d\tau\right)\exp\left(i\frac{\Delta t}{4}\nabla^2\right) \\ - \frac{1}{2}\nabla^2\int_{t_0}^{t_0+\Delta t} d\tau\left(\frac{\Delta t}{2}V(\tau) - \int_{t_0}^{\tau} V(\tau')\,d\tau'\right) \\ - \frac{1}{2}\int_{t_0}^{t_0+\Delta t} d\tau\, V(\tau)\left(\frac{\Delta t}{2} - (\tau - t_0)\right)\nabla^2 + O\big((\Delta t)^3\big)\Bigg]\Psi(\mathbf{x}, t_0)\end{aligned}$$

respectively.

Paper VIII: "New fast Fourier transform and finite-element approaches to the calculation of multiple-stripe-geometry laser modes"

i: It should read just above eq. (1): $|n(x) - n_1|^2 \equiv |\Delta n(x)|^2$, where n_1 is a second representative refractive index of the laser,

Paper XI: "Numerical analysis of the modal eigenfunctions of chirped and unchirped multiple-strip-geometry laser arrays"

i: The restriction $-a/2 < x < a/2$, should not apply to eq. (14), but rather to the total refractive index change $\sum_{j=1}^{m}(\Delta n(x - x_j)) - (b - i)(m - 1)(-\eta_0 + \eta_3)$.

ii: Further the value $a = 100\mu m$ was employed in all calculations except that of the infinitely periodic laser array, where $a = 7\mu m$ and $m = 7$ and $x_j = a(j - 4)$.

Paper XII: "Optical coherence calculations with the split-step fast Fourier transform method"

i: The complex degree of spatial coherence should have been defined as

$$\mu(x, x', Z) \equiv \langle E^*(x, Z)E(x', Z)\rangle \,/\, \left(\langle|E(x, Z)|^2\rangle\langle|E(x', Z)|^2\rangle\right)^{1/2}.$$

Paper XIII: "Numerical simulation of coherent backscattering in small two-dimensional systems"

i; Δn in the two integrants of eq. (1) should have been multiplied with the factor $\frac{2\pi}{\lambda}$.

ii; The refractive index of the homogeneous medium n_0 and the wavelength λ is set to one throughout the paper.

iii; The two-dimensional analog, consistent with eq. (7) and (8), of the three-dimensional Green's function $\exp(-ik|\vec{r}_j - \vec{r}_i|)/|\vec{r}_j - \vec{r}_i|$ for the Helmholtz equation is $-i\pi H_0^{(2)}(k|\vec{r}_j - \vec{r}_i|)$.

iv; Further the value of g_j in eq. (4), is the coefficient for the first-order circular symmetric scattered field in term of the above two-dimensional Green's function and should be $\approx 0.96\exp(-1.07i)/(2\pi i) \approx 0.15\exp(-2.64i)$.

v; The left hand side of eq. (5) should be $[1 + g_j G_0(\vec{r}_j, \vec{r}_j{}')]\tilde{E}_j$.