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Singularity Theory and $N = 2$ Superconformal Field Theories*

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Abstract: The $N = 2$ superconformal field theories that appear at the fixed points of the renormalization group flows of Landau-Ginsburg models are discussed. Some of the techniques of singularity theory are employed to deduce properties of these superconformal theories. These ideas are then used to deduce the relationship between Calabi-Yau compactifications and tensored discrete series models. The chiral rings of general $N = 2$ superconformal theories are also described.

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My intention in this paper is to give an overview of the application of singularity theory to the classification and understanding of $N = 2$ superconformal field theories (SCFTs). This paper is based upon the work described in [1-3], and details may be found in these references. Closely related work has also appeared in [4-6].

The original motivation for this work was to find some universal characterization of $N = 2$ superconformal field theories. There are many known methods of constructing such theories: Calabi-Yau compactifications [7], orbifolds [8], lattices [9], free fermions [10], and tensoring of discrete series characters [11]. There are some complex inter-relationships between these constructions—perhaps the most startling of which is the relationship between Calabi-Yau and tensored discrete series models [12]. One of my aims here is to show how, by using Landau-Ginsburg models, one can characterize many of the $N = 2$ superconformal models that have been constructed to date. Moreover, this formalism captures the essentials of these SCFT's in such a simple way that the interrelationships between them frequently becomes manifest. In particular, it is easy to understand the mysterious relationship of Calabi-Yau manifolds and tensored discrete series models.

My second purpose here is to introduce an essentially topological characterization of $N = 2$ SCFT's, that is, the notion of a chiral* ring. This structure does not, in general, completely characterize the theory but it does give one an easily computable method of determining when two models, however constucted, can be isomorphic and when they cannot be. One should view the chiral ring as a natural parallel in superconformal field theories of the cohomology algebra of a manifold. On the other hand, for the $N = 2$ SCFT's coming from Landau-Ginsburg theories, the chiral ring takes on a special form that does completely characterize the SCFT.

The basic idea [1,5] is to consider the mean field theory, or Landau-Ginsburg theory, corresponding to a $N = 2$ supersymmetric statistical mechanical system [4]. That is, consider an action of the form

$$S = \int d^2z d^4\theta K(\Phi^i, \overline{\Phi}^i) + \left(\int d^2z d^2\theta^- W(\Phi^i) + \int d^2z d^2\theta^+ W(\overline{\Phi}^i)\right) \qquad (1)$$

where K is some Kähler form, and W is an arbitrary, analytic superpotential. The superfields Φ^i are complex $N = 2$ chiral superfields satisfying

$$D^+\Phi^i = \overline{D}^+\Phi^i = 0 \qquad (2)$$

* In this paper the notion of chiral will be used only in the sense of $N = 2$ supersymmetry, and not in the sense of holomorphic or anti-holomorphic.

In components, the potential for the bosons, $V(\phi^i)$, is equal to $|\nabla W|^2$ and thus critical points of W correspond to zero energy minima of V or vacuum states of the theory. As we approach the critical point of the statistical mechanical system the minima of V all come together or coalesce and the long range correlations of the statistical mechanical system correspond to some $N = 2$ SCFT. Putting it another way, we can consider (1) as defining a (non-conformal) $N = 2$ field theory. We can then study the behavior of (1) under renormalization group flow towards some infra-red fixed point. (I shall always assume that there is such a fixed point.) At the fixed point, the action will define an $N = 2$ SCFT and the potential will be degenerate in the mathematical sense, *i.e.*

$$\det\left(\frac{\partial^2 W}{\partial\Phi^i\partial\Phi^j}\right)\Bigg|_{\Phi^i=0} = 0 \tag{3}$$

where I take the critical point to be at $\Phi^i = 0$. The problem is that, at the fixed point, the action might, in principle, look nothing like the one with which I started.

For $N = 2$ Landau-Ginsburg theories there is a major simplification. First, the D-terms are irrelevant or marginal in the renormalization group flow. This is because, at the fixed point, the lowest components of a superfield must have non-negative conformal dimension (by unitarity), and so the highest component must have conformal dimension greater than or equal to 2, and are thus irrelevant or marginal [13]. The D-terms are precisely the highest components of $K(\Phi^i,\overline{\Phi}^i)$. Thus the only relevant operators appear in the F-term, or superpotential. The second important point is that if we assume the non-renormalization theorems hold, then, apart from wave function renormalization, the superpotential, W, does not renormalize. These two observations were made in [4]. Their importance is that W is an invariant characteristic of the flow *and* W completely dictates the flow. One can start with any choice of $K(\Phi^i,\overline{\Phi}^i)$ and some choice of W. The resulting W at the fixed point will be unchanged (up to wave function renormalizations), and the form of K at the fixed point will be some fixed function that is entirely determined by the choice of W. It is this idea that forms the basis of the classification scheme proposed in [1]. To every analytic function, W, there corresponds an $N = 2$ superconformal field theory.

We must put some physical restrictions on W. First, W must have at least one critical point—which we take to be at $\Phi^i = 0$. Moreover, we require that W be multi-critical (3), for otherwise all fields will be massive and so freeze out in the infra-red limit. If all its fields had masses the result would be a trivial theory (with the central charge zero) whose correlation functions are delta functions. (The Hilbert space would also be trivial.) We only need to classify W up to field re-definitions since such field re-definitions would only modify the action (1) by irrelevant D-terms. Moreover, because massive fields freeze out at the infra-red fixed point, the $N = 2$ SCFT theory arising from $W(\Phi^i)$ is unchanged if we add some new fields to the theory with a potential for these new fields that give masses to all the new fields. Having done this we can also make changes of variables that intermix the old and new fields. This will still not change the fixed point theory. The foregoing is the textbook definition of the statement that the $N = 2$ superconformal fixed point only depends on the stable, singularity type of W.

There are two further conditions that we need to impose on W. The first is to require that the critical point be isolated, that is, in a small enough neighborhood of $\Phi^i = 0$ the only solution to $\frac{\partial W}{\partial\Phi^i} = 0$ is to take $\Phi^i = 0$. This means that there are no flat directions in the potential. It also enables us to define the multiplicity, μ, of the critical point: this is the winding number of the map $\Phi^i/|\Phi^i| \to (\partial W/\partial\Phi^i)/|\partial W/\partial\Phi^i|$ considered as a map from $S^{2n-1} \leq C^n$ to $S^{2n-1} \leq C^n$. Because of the foregoing condition, μ is always finite. There is another important consequence of this condition. As we will see, it guarantees that all fields have strictly positive conformal dimension, and thus the D-terms have dimension strictly greater than 2. The D-terms are therefore truly irrelevant and contain no marginal operators. This finite multiplicity condition is made for convenience and it would be interesting to see what would happen if one were to relax it.

The second condition is also made for convenience, but is physically well motivated. We require that W be quasi-homogeneous. That is, there are weights, ω_i, such that

$$W(\lambda^{\omega_i}\Phi^i) = \lambda W(\Phi^i)\ . \tag{4}$$

There are several ways of explaining why we do this. The simplest is to observe that this requirement is necessary to give all the fields, Φ^i, a well defined conformal dimension. Indeed, if h_i and $\overline{h}_i$ denote the conformal weights of the scalar field Φ^i, then*

$$h_i = \overline{h}_i = \frac{1}{2}\omega_i\ . \tag{5}$$

* Equation (5) follows because under the scaling $z \to \lambda^{-1}z$, $\theta \to \lambda^{-1/2}\theta$ the measure scales according to $d^2zd^2\theta \to \lambda^{-1}d^2zd^2\theta$ and hence (4) and (5) ensure that the F-term is unrenormalized.

If we had chosen an arbitrary superpotential then wave function renormalization would mean that $W(\Phi^i)$ would flow to its lowest dimension. quasihomogeneous part. For example, given a potential $\Phi^3 + \Phi^5$, the scaling at the fixed point means that only the Φ^3 term survives. Mathematically, many of the stable singularity types can be represented by quasi-homogemeous functions and thus this physical restriction is not as strong as it might, at first, seem. The exceptions to this have the property that the lowest dimensional quasi-homogeneous parts, taken in isolation, have non-isolated critical points. These exceptions may well correspond to an interesting class of models, but I shall not consider them here. We can now apply the vast machinery of singularity theory to these Landau-Ginsburg models.

Given an analytic function, $W(\Phi^i)$, with a critical point at $\Phi^i = 0$, one defines the local ring, $\mathcal{R}$ of this function by

$$\mathcal{R} = P/J \tag{6}$$

where P is the ring of power series about $\Phi^i = 0$, and J is the ideal in P generated by all the partial derivatives $\frac{\partial W}{\partial \Phi^i}$ of W. The ring, $\mathcal{R}$, has dimension equal to the multiplicity, μ. Moreover, $\mathcal{R}$ plays an important rôle in singularity theory. Physically, it corresponds to all the chiral, primary fields of the theory. This is because one only gets chiral fields by taking power series in the Φ^i, however, these can only become descendant fields if the power series in question is related to super-derivatives via an equation of motion. This can only happen if the power series in Φ^i has some factor of $\frac{\partial W}{\partial \Phi^j}$. Therefore $\mathcal{R}$ consists of precisely the chiral fields that are not descendants, i.e., primary.

One of the basic theorems of singularity theory tells us that the local ring of W contains a unique element, ρ, of maximal scaling dimension. This element is, in fact,

$$\rho = \det\left(\frac{\partial^2 W}{\partial \Phi^i \partial \Phi^j}\right) \tag{7}$$

and has conformal dimension

$$h_{max} = \overline{h}_{max} = \sum_i \left(\frac{1}{2} - \omega_i\right) \tag{8}$$

There are many other concepts in singularity theory that carry over to physics, but the foregoing will suffice for my present purposes. I shall now start at the other end. We know that we have to get an $N = 2$ superconformal theory at the critical point, so we must get a representation of the $N = 2$ superconformal algebra. The commutation relations of this algebra are

$$\begin{aligned}
\{G^+_{n+a}, G^-_{n-a}\} &= 2L_{n+m} + (n - m + 2a)J_{n+m} + \frac{1}{3}c\left[(n+a)^2 - \frac{1}{4}\right]\delta_{m+n,0} \\
[L_n, L_m] &= (n-m)L_{n+m} + \frac{1}{12}n(n^2-1)\delta_{m+n,0} \\
[L_n, J_m] &= -mJ_{m+n} \\
[L_n, G^\pm_{m\pm a}] &= (\frac{1}{2}n - m \mp a)G^\pm{}_{n+m\pm a} \\
[J_n, G^\pm_{m\pm a}] &= \pm G^\pm_{m+n\pm a} \\
[J_n, J_m] &= \frac{c}{3}\delta_{m+n,0}
\end{aligned} \tag{9}$$

where m and n are integers, and a is a real parameter. In the Neveu-Schwarz sector one takes $a \in Z + \frac{1}{2}$ and in the Ramond sector $a \in Z$. Shifting a by an integer generates isomorphic algebras.

The Landau-Ginsburg theories have an $N = 2$ superconformal algebra for the left-moving and right-moving sectors. The generators of the left-moving algebra will be denoted by G, L and J, and will correspond to holomorphic world-sheet coordinates, while the right-moving algebra will be denoted by $\overline{G}$, $\overline{L}$, and $\overline{J}$ and will correspond to anti-holomorphic coordinates. In the sequel, I will largely suppress discussion of the right-movers as their treatment exactly parallels the discussion of the left movers. I shall also work only in the Neveu-Schwarz sector, but one should bear in mind that there is also a parallel discussion for the Ramond sector. (The corresponding theory in the Ramond sector can be obtained by spectral flow from the Neveu-Schwarz sector.) For the present I will *not* restrict myself to Landau-Ginsburg theories, but will consider an arbitrary (2,2) theory.

A primary state $|\Psi>$ is defined by

$$G^\pm_r|\Psi> = 0, \quad r \geq \frac{1}{2} \tag{10}$$

$$L_n|\Psi> = J_n|\Psi> = 0, \quad n \geq 1 \tag{11}$$

(with similar conditions for the right-movers). However, condition (11) is redundant given (9) and (10). A primary, chiral field, $|\Phi>$, is defined to satisfy (10) and

$$G^+_{-1/2}|\Phi> = \overline{G}^+_{-1/2}|\Phi> = 0 \tag{12}$$

More precisely, $|\Phi>$, is the lowest component of a chiral superfield, the other components will be obtained by acting with $G^-_{-1/2}$ and $\overline{G}^-_{-1/2}$ on $|\Phi>$. Equation (12) means that $|\Phi>$ is chiral because it has no θ^+ or $\overline{\theta}^+$ super-partners.

An immediate consequence of (9), and unitarity, is that for any state $|\Psi>$ of conformal weight h and $U(1)$ charge q, one has

$$h \geq \frac{1}{2}|q| \tag{13}$$

This is a trivial consequence of the inequality

$$\begin{aligned} 0 &\leq |G^{\pm}_{-1/2}|\Psi>|^2 + |G^{\mp}_{+1/2}|\Psi>|^2 \\ &= <\Psi|\{G^{\pm}_{-1/2}, G^{\mp}_{1/2}\}|\Psi> \end{aligned} \tag{14}$$

Moreover, for chiral, primary fields one has $h = \frac{1}{2}q$. Conversely, it is also elementary to see that $h = \frac{1}{2}q$ if *and only if* $|\Psi>$ is chiral and primary.

One can establish a generalization of the Hodge-decomposition theorem: given any state $|\Psi>$, it may be written in the form

$$|\Psi> = |\Phi> + G^+_{-1/2}|\Psi_1> + G^-_{+1/2}|\Psi_2> \tag{15}$$

where $|\Phi>$ is chiral and primary. This decomposition is unique in that $|\Phi>$, $G^+_{-1/2}|\Psi_1>$ and $G^-_{+1/2}|\Psi_2>$ are unique given $|\Psi>$. (For details see [3].)

Consider, now, a chiral primary state $|\Phi>$. Using (13), and the fact that $<\Phi|G^-_{+3/2}G^+_{-3/2}|\Phi> \geq 0$, it follows that

$$h \leq \frac{c}{6} \tag{16}$$

and that $h = c/6$ if and only if

$$G^+_{-3/2}|\Phi> = 0 \tag{17}$$

in addition to (10) and (12).

Now consider the spectral flow of the vacuum. That is, consider the $SL(2,\mathcal{R})$ invariant vacuum of a Hilbert space representing (9) for some value of $a \in Z + 1/2$. Consider what happens to this state as we smoothly change $a \to a-1$. The Hilbert space flows to some other Hilbert space that represents the original $N = 2$ superconformal algebras. Moreover, the vacuum state flows to some state $|\rho>$ in the new Hilbert space. Since the vacuum satisfies

$$G^{\pm}_r|0> = 0 \qquad r \geq -\frac{1}{2} \tag{18}$$

it follows that

$$\begin{aligned} G^+_r|\rho> &= 0 \qquad r \geq -\frac{3}{2} \\ G^-_r|\rho> &= 0 \qquad r \geq +\frac{1}{2} \end{aligned} \tag{19}$$

In other words, $|\rho>$ is a chiral primary state saturating the bound (16). If a given $N = 2$ super-conformal field theory is invariant under this spectral flow then $|\rho>$ is the unique state satisfying (19) since it is the spectral flow of the unique $SL(2,R)$ invariant vacuum.

It is important to remember at this point, that the foregoing spectral flow is to be performed simultaneously in both the left-moving and right-moving sectors. Thus $|\rho>$ will be paired with the corresponding right moving state. It is clearly of interest to know when an $N = 2$ SCFT is invariant under this spectral flow. One simple, sufficient criterion is that the theory be modular invariant, and that every state satisfy

$$q_L - q_R \in \boldsymbol{Z} \tag{20}$$

where q_L and q_R are the left-moving and right-moving $U(1)$ changes respectively.*

For theories that are invariant under the spectral flow we have thus shown that the Hilbert space of the theory contains a unique, chiral primary field of maximal conformal dimension

$$h = \overline{h} = \frac{c}{6} \tag{21}$$

The $N = 2$ SCFT's coming from Landau-Ginsburg theories obviously satisfy (20) and are manifestly modular invariant, and hence the element, ρ, of the chiral ring, defined by (7) must coincide with the state $|\rho>$ defined above. Therefore, combining (8) and (21), we have shown that the central charge of the $N = 2$ SCFT arising from a Landau-Ginsburg theory is given by

$$c = 6\sum_i \left(\frac{1}{2} - w_i\right) \tag{22}$$

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* This suffices because modular invariance can be used to relate invariance under spectral flow to invariance under the corresponding $U(1)$ twist, and (20) guarantees that all states are invariant under the twist.

Returning to a general $(2,2)$ superconformal field theory, one can show quite generally that the chiral, primary fields also define a finite dimensional, polynomial ring. Moreover, the operator product induces the naive polynomial multiplication on this ring. Let $\Phi_1(w)$ and $\Phi_2(w)$ be two chiral, primary fields with conformal weights h_1 and h_2. Let $\Psi(w)$ be the leading term in the operator product of these two fields, *i.e.*,

$$\Phi_1(z)\Phi_2(w) = (z-w)^{h-h_1-h_2}\Psi(w) + \cdots \tag{23}$$

where h is the conformal weight of $\Psi(w)$. Let q be the $U(1)$ charge of $\Psi(w)$, and recall that the $U(1)$ charges of $\Phi_1(z)$ and $\Phi_2(w)$ are $2h_1$ and $2h_2$. Thus $\frac{1}{2}q = (h_1 + h_2)$. However, $\frac{1}{2}q \leq h$, with equality if and only if $\Psi(w)$ is chiral and primary. Therefore, define

$$(\Phi_1 \cdot \Phi_2)(z) = \lim_{z \to w} (\Phi_1(z)\Phi_2(w)) \tag{24}$$

This limit is always finite, and is non-zero if and only if the result is both chiral and primary. In a general $N = 2$ superconformal field theory I shall define the chiral ring to be the set of chiral, primary fields with multiplication defined by (24).

More precisely, in any $(2,2)$ SCFT one may define four chiral rings since one can consider all pairings of chiral and anti-chiral fields in both the left and right moving sectors. These rings are usually denoted by (c,c), (a,c), (c,a) and (a,a). One should also note that (c,c) and (a,a) are conjugate to each other, as are (a,c) and (c,a). Thus, there are only two independent rings. For Landau-Ginsburg theories, the (a,c) ring is trivial, consisting only of the vacuum state.

Like chiral rings in Landau-Ginsburg theories, these general chiral rings can be thought of as quotients P/J, where J is the ideal of vanishing relations, which consists of all polynomials of chiral, primary fields that vanish as a consequence of the operator product algebra. Note that any polynomial, Ψ, in J (that is, any vanishing polynomial of chiral primary fields), is necessarily chiral but *not* primary.* One can decompose the state corresponding to Ψ according to (15) and because it is chiral and non-primary one has

$$|\Psi\rangle = G^+_{-1/2}|\Psi_1\rangle \tag{25}$$

for some $|\Psi_1\rangle$. Thus, in general, the vanishing relations are necessarily D^+ of something. Moreover, one can always isolate a set of Ψ_j that generate the ideal J, and each of these Ψ_j must also be D^- of something. In a Landau-Ginsburg theory we know that these Ψ_j's can be integrated to give a function superpotential. That is, there is a function $W(\Phi^i)$ and a choice of the Φ^i's and Ψ_j's such that $\Psi_j = \frac{\partial W}{\partial \Phi^j}$. In general it appears that one can integrate the Ψ_j's to get a superpotential, W if (i) the (a,c) and (c,a) rings are trivial, (ii) equation (20) is satisfied for all states in the theory, (iii) all states in the theory are generated from the operator products of elements of the (c,c) and (a,a) rings, and (iv) all (chiral, chiral), but not necessarily primary, states of the theory can be obtained from operator products of a finite number of left-right symmetric chiral, primary fields Φ^i. A heuristic proof of this statement may be found in [3].

The point I wish to stress, however, is that in general these (c,c) and (a,c) rings are simple (and from experience, easily computable) substructures of an $N = 2$ superconformal theory that will help to characterize the theory. For Landau-Ginsburg theories they completely characterize the theory. Moreover, chiral rings appear to be natural "topological" objects. There is an obvious formal similarity between chiral rings and the Dolbeault complex, with $G^+_{-1/2}, G^-_{+1/2}$ playing the role of ∂ and δ (while $\overline{\partial}$ and $\overline{\delta}$ correspond to the right moving $\overline{G}^+_{-1/2}, \overline{G}^-_{+1/2}$). Moreover, under spectral flow to the Ramond sector, one sees that chiral, primary fields correspond to zero-modes the Dirac-Ramond operators:

$$G_0^{\pm}|\Phi\rangle = 0 \Leftrightarrow (G_0^+ + G_0^-)|\Phi\rangle = 0 \tag{26}$$

For level one, $N = 2$ coset models on hermitian symmetric spaces [14], one can show [3] that the correspondence with the Dolbeault complex is exact. If one grades the ring of chiral, primary fields, $\mathcal{R}$, according to their charge (or conformal weight) then there is a one to one correspondence between the chiral, primary fields of charge q and the elements of $H^{q,q}(G/H)$ (with q suitably normalized).* Moreover, the ring structure of $H^*(G/H)$ appears to coincide with that of $\mathcal{R}$, though this has not been checked in general. In [3] it was also shown that the level one, $N = 2$ coset models on hermitian symmetric spaces are, in fact, Landau-Ginsburg models. For general $N = 2$ coset models one can show that they are, in general, *not* Landau-Ginsburg models.** On the other hand, these models do have large chiral, primary rings that can be completely characterized. The details may be found in [3]

* In general, a chiral field is only required to satisfy (12), and does not necessarily satisfy (10).

* For hermitian symmetric spaces $H^{p,q}(M) = 0$ when $p \neq q$.

** However, it is possible that some twisted form of these theories might be Landau-Ginsburg.

As a final application of the Landau-Ginsburg formalism, I shall briefly review the ideas of [2] that relate compactifications on Calabi-Yau spaces to exactly solvable $\mathcal{N} = 2$ superconformal theories. The basic idea is to consider the Landau-Ginsburg path integral within fields, Φ^i:

$$\int d\Phi^i d\overline{\Phi}^j \, e^{iS[\Phi^i, \overline{\Phi}^i]} \tag{27}$$

where S is given by (1). One starts with a formal calculation in which one neglects the kinetic term. One then changes variables according to

$$\chi = (\Phi^1)^{1/\omega_1}, \qquad \xi^i \equiv \Phi^i/(\Phi^1)^{\omega_i/\omega_1} \tag{28}$$

Note that $\xi^1 \equiv 1$ and the ξ^i, $i = 2, 3, \cdots, n$, define a coordinate patch on an $n-1$ dimensional weighted projective space. This change of variables enables us to factor χ out of the superpotential, *i.e.*, $W(\Phi^i) = \chi W(\xi^i)$. The change of variables also introduces a purely algebraic Jacobian into the path integral. One can show that the Jacobian is independent of χ if and only if

$$c = 6\sum_{i=1}^{n}\left(\frac{1}{2} - w_i\right) = 3(n-1) \tag{29}$$

Assuming that (29) is satisfied, one can integrate out the field χ to obtain a path integral over the ξ^i and $\overline{\xi}^i$ with a term $\delta(W)$ in the integrand. This delta function fixes the bosonic part of ξ^i to lie on the hypersurface $W(\xi^i) = 0$, and requires the fermionic part of ξ^i to be tangent to this hypersurface. We thus have converted the path integral (27) into one over an algebraic hypersurface in an $(n-1)$-dimensional weighted projective space. One can show that (29) is also precisely equivalent to requiring the first Chern class of this algebraic surface to vanish. Note also that the right-hand-side of (20) is $3(n-1)$, which is the correct central charge for the $\mathcal{N} = 2$ superconformal theory defined by an $(n-1)$-dimensional Calabi-Yau compactification. Finally, also note that the change of variables, (28), is only single valued if we divide the original Landau-Ginsburg model by the symmetry,

$$\Phi^j \rightarrow e^{2\pi i \omega_j} \Phi^j \tag{30}$$

Thus the Calabi-Yau compactification is really equivalent to the twisted Landau-Ginsburg model.*

* Dividing by the symmetry (30) is the analogue of Gepner's $U(1)$ projection [11].

It is straightforward to generalize the foregoing calculation to Calabi-Yau manifolds that are described by the vanishing of several polynomials in products of weighted projective spaces.

This calculation is extremely suggestive, but so far, somewhat formal. Obviously one cannot really drop the kinetic term. However, if one were to keep the kinetic term and go through the foregoing calculation one would no longer arrive at a δ-function, but would get an extremely complicated Gaussian. The width of the Gaussian would be proportional to the momentum scale at which one was working. Thus, in the infra-red limit, one would once again approach a δ-function of W. Therefore the proper way to interpret the foregoing calculation is that it shows how to relate the universality classes of Calabi-Yau manifolds to the universality classes of Landau-Ginsburg theories. The two theories are not identical, but their $N = 2$ superconformal infra-red fixed points are.

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