Received by OSTI

NOV 22 1989

ANL/PPRNT--89-192

DE90 003003

Pion distribution in the nucleon

T.-S. H. Lee

Physics Division, Argonne National Laboratory, Argonne, IL 60439-4843

ABSTRACT

A model is presented for calculating the pion wave function inside the nucleon. By assuming that all pions around a core of the nucleon are in the lowest eigenstate of the system, it is shown that both the bound state and πN scattering amplitude can be consistently described by an exactly soluable model defined in the subspace spanned by the core state and the physical πN state. The parameters of the model are determined by fitting the data of the nucleon mass, πNN coupling constant and low energy πN scattering phase shifts. The model predicts that the probability of finding the pion component inside the nucleon is about 20%. The calculated πNN form factor differs significantly from the conventional monopole form. The dynamical consequences of the differences are demonstrated in a calculation of electromagnetic production of pions from the nucleon and the deuteron.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

MASTER

I. INTRODUCTION

It is generally believed[1] that the nucleon has a core with three valance quarks and a pion cloud on its surface. The physics of the valance quarks was unambiguously revealed from the deep inelastic lepton scattering, while the existence of the pion cloud provides a simple explanation of the observed Chiral dynamics of low energy nuclear interactions. In this paper we present a model, based on this physical picture, to describe the following experimental observables: (i) the mass of the nucleon, (ii) the πNN coupling constant $f^2/4\pi=0.081$, and (iii) πN scattering phase shifts in the P_{11} channel below the pion production threshold. The main outcome of the model is a prediction of the pion distribution inside the nucleon. This quantity is needed to answer many pressing questions raised in recent nuclear studies, such as the mesonic explanation of the EMC effect,[2] absorption of low energy pions by nuclei[3] and the electromagnetic production of pions from nuclei.[4]

In Sect. II we define our model and show that the model provides an internally consistent description of both the pion wave function inside the nucleon and the πN scattering amplitude. In Sect. III we introduce parameterizations of the model to fit the data of the three basic observables mentioned above, and present the predicted pion wave function inside the nucleon. We will demonstrate in Sect. IV the implications of our model in a impulse approximation calculation of the electromagnetic production of pions from the nucleon and the deuteron. In Sect. V we summarize our study.

II. THE MODEL

We start with the assumption that the pions around the core of the nucleon are all in the lowest eigenstate of the system. In this simplest picture, one can immediately see that if Z_0 is the probability of finding the core state, denoted as $|N_0\rangle$ and also called the bare particle state, then the probability of finding n mesons moving around the core is

$$P_n = Z_0(1 - Z_0)^n \ . \tag{2.1}$$

We now notice that such a probability distribution can be obtained from a wave function of the following form

$$|N\rangle = Z_0^{1/2} \left\{ |N_0\rangle + \sum_{n=1}^{\infty} \frac{1}{\sqrt{n!}} \left[\prod_{i=1}^{n} \int d\vec{k}_i Z_0^{1/2} A(\vec{k}_i) a^+_{\vec{k}_i} \right] |N_0\rangle \right\} , \tag{2.2}$$

with

$$Z_0 = \frac{1}{1 + D} , \tag{2.3a}$$

and

$$D = \int |A(\vec{k})|^2 d\vec{k} \ . \tag{2.3b}$$

Here $a^+_{\vec{k}}$ is the creation operator of a pion with momentum $\vec{k}$, and $A(\vec{k})$ is the pion wave function to be determined from the assumed dynamics. To simplify the presentation, all spin-isospin quantum numbers are suppressed throughout the paper. To verify Eq. (2.1), we use the wave function Eq. (2.2) to calculate the probability of finding a n-meson configuration

$$P_n = \int d\vec{k}_1 \ldots d\vec{k}_n |\langle N| \frac{1}{\sqrt{n!}} a^+_{\vec{k}_1} |N_0\rangle|^2 \tag{2.4}$$

By using Eqs. (2.2) and (2.3), Eq. (2.1) is verified as follows

$$P_n = Z_0 \left[Z_0 \int |A(\vec{k})|^2 d\vec{k} \right]^n$$

$$= Z_0 \left[Z_0 D \right]^n$$

$$= Z_0\ (1-Z_0)^n \tag{2.5}$$

The considered model wave function Eq. (2.2) is clearly well normalized since

$$\langle N|N\rangle = \sum_{n=0}^{\infty} P_n$$

$$= Z_0 \sum_{n=0}^{\infty} (1-Z_0)^n$$

$$= Z_0 \frac{1}{1-(1-Z_0)} \equiv 1$$

Our task now is to find a way to calculate the pion wave function $A(\vec{k})$ by solving the bound state problem

$$H_B|N\rangle = m|N\rangle \tag{2.6}$$

where m is the measured mass of the nucleon, and H_B is the "bare" Hamiltonian describing the basic $\pi N_0 \leftrightarrow N_0$ pion emission and absorption mechanism and possible $\pi N_0 \leftrightarrow \pi N_0$ two-body interactions due to other mechanisms such as the exchange of heavier mesons. In Appendix A we describe the procedure to solve Eq. (2.6). It involves a highly nonlinear self-consistent problem which can not be easily solved in practice. The problem becomes even more intractable if we demand a consistent description of both the bound state and πN scattering amplitude, as required in a realistic prediction of mesonic components of the nucleon.

The starting point of the formulation of our model is the observation that Eq. (2.2) can be written as an integral equation form

$$|N\rangle = Z_0^{1/2} \left\{ |N_0\rangle + \int d\vec{k}\ A(\vec{k})\ S_b a^{+}_{\vec{k}} |N\rangle \right\} \tag{2.7}$$

Here we have introduced the boson symmetrization operator S_b, which symmetrizes and normalizes each multi-meson state resulted from iterating Eq. (2.7). The above equation can be simply considered as the definition of the core state $|N_0>$ in terms of the physical states $|N>$ and $|\pi N>$. In this representation the physical nucleon, defined originally by Eq. (2.2), can be obtained by solving the bound state problem with a suitable phenomenological Hamiltonian defined within the subspace $N_0 \oplus \pi N$. The form of the phenomenological Hamiltonian is suggested by the matrix elements $<N_0|H_B|\pi N>$ and $<\pi N|H_B|\pi N>$, which are given explicitly in the Appendix A. The phenomenological Hamiltonian is then taken as

$$H = H_0 + H' \tag{2.8}$$

with

$$H_0 = \int d\vec{p} \left[\sqrt{m_0^2+\vec{p}^2}\; C^+_{\vec{p}} C_{\vec{p}} + \sqrt{m^2+\vec{p}^2}\; b^+_{\vec{p}} b_{\vec{p}} \right] + \int d\vec{k}\, \sqrt{\mu^2+\vec{k}^2}\; a^+_{\vec{k}} a_{\vec{k}} \tag{2.9a}$$

$$H' = \int d\vec{k} d\vec{p} \left[h(\vec{\kappa})\; C_{\vec{p}+\vec{k}}\, a^+_{\vec{k}} b^+_{\vec{p}} + h^+(\vec{\kappa})\; C^+_{\vec{p}+\vec{k}}\, a_{\vec{k}} b_{\vec{p}} \right] + \int d\vec{p} d\vec{p}' d\vec{k} d\vec{k}' \delta(\vec{k}+\vec{p}-\vec{k}'-\vec{p}') v(\vec{\kappa}',\vec{\kappa}) b^+_{\vec{p}'} b_{\vec{p}} a^+_{\vec{k}'} a_{\vec{k}} \tag{2.9b}$$

Here m_0, m and μ are respectively the mass of the core N_0, the physical nucleon and the pion. Their creation operators are denoted as $C^+_{\vec{p}}$, $b^+_{\vec{p}}$ and $a^+_{\vec{k}}$ respectively. $\vec{\kappa}$ is the πN relative momentum. The parameterizations of the vertex function $h(\vec{\kappa})$ and the two-body matrix element $v(\vec{\kappa}',\vec{\kappa})$ will be given explicitly in Sect. III.

In this simplified model the bound state and the πN scattering problems can be solved exactly. In the following three subsections we will present explicit solutions for calculating the nucleon mass pion wave function, πNN coupling constant and πN scattering amplitude. All solutions will be given in the center of mass (c.m.) frame in which the one-particle state $|N_0\rangle$ is at rest and the πN relative momentum $\vec{\kappa}$ in Eq. (2.9) becomes $\vec{\kappa}=\vec{k}=-\vec{p}$.

1. The nucleon state

To obtain the pion wave function $A(\vec{k})$ of the nucleon, we need to solve the following bound state equation in c.m. frame

$$(H-m)|N\rangle = (H-m)\ Z_0^{1/2} \left\{|N_0\rangle + \int d\vec{k}\ A(\vec{k})\ a^+_{\vec{k}} b^+_{-\vec{k}}|0\rangle\right\} = 0 \qquad (2.10)$$

In writing down Eq. (2.10), we have used the property that in the subspace $N_0 \oplus \pi N$ multi-meson states are suppressed and hence

$$S_b a^+_k |N\rangle \equiv a^+_k |N\rangle$$

$$+\ a^+_{\vec{k}} b^+_{-\vec{k}} |0\rangle\ .$$

The one-particle state N_0 is of course at rest; i.e. $|N_0\rangle \equiv C^+_0|0\rangle$. By using the definition of H (Eqs. (2.8) and (2.9)) and projecting Eq. (2.10) onto the basis states $|N_0\rangle$ and $a^+_k b^+_{-k}|0\rangle$, we have

$$m_0 + \int d\vec{k} h^+(\vec{k}) A(\vec{k}) = m \qquad (2.11)$$

$$W(k)\ A(\vec{k}) + h(\vec{k}) + \int d\vec{k}' v(\vec{k},\vec{k}') A(\vec{k}') = mA(\vec{k}) \qquad (2.12a)$$

with

$$W(k) = \sqrt{m^2+k^2} + \sqrt{\mu^2+k^2} \qquad (2.12b)$$

It is straightforward to show that the solution of the above coupled integral equations can be written as

$$A(\vec{k}) = \frac{1}{m-W(k)}\left[h(\vec{k}) + \int \frac{t(\vec{k},\vec{k}',m)\, h(\vec{k}')}{m-W(k')}\, d\vec{k}'\right] \tag{2.13}$$

where t-matrix is determined only by the two-body interaction v(k,k')

$$t(\vec{k}',\vec{k},E) = v(\vec{k}',\vec{k}) + \int v(\vec{k}',\vec{k}'') \frac{d\vec{k}''}{E-W(\vec{k}'')}\, t(\vec{k}'',\vec{k},E) \tag{2.14}$$

The normalization constant Z_0 is then calculated from wave function $A(\vec{k})$ by using Eq. (2.3).

2. πNN form factor

With the bound state solution Eq. (2.7), the πNN vertex function can be calculated from

$$F_{\pi NN}(\vec{k}) = \langle \pi N | H' | N\rangle = \langle 0 | a_{\vec{k}} b_{-\vec{k}} H' | N\rangle$$

$$= Z_0^{1/2}\left[h(\vec{k}) + \int d\vec{k}'\, v(\vec{k},\vec{k}') A(\vec{k}')\right] \tag{2.15}$$

By using the properties Eqs. (2.13)-(2.14), it is easy to express Eq. (2.15) in terms of the πN scattering t matrix

$$F_{\pi NN}(\vec{k}) = Z_0^{1/2}\left[h(\vec{k}) + \int d\vec{k}'\, \frac{t(\vec{k},\vec{k}',m)\, h(\vec{k}')}{m-W(\vec{k}')}\right] \tag{2.16}$$

The physical meaning of Eq. (2.16) is clear. The πN two-body interaction v plays an important role in determining the transition from the physical nucleon to a physical πN state. In the zero momentum limit, the vertex function is related to the measured πNN coupling constant $f^2/4\pi=0.081$ by

$$F_{\pi NN}(\vec{k}) \xrightarrow[k\to 0]{} \frac{i}{(2\pi)^{3/2}} \frac{1}{\sqrt{2E_\pi(k)}} \frac{f}{\mu}\, \vec{\sigma}\cdot\vec{k}\, (I\tau_{-I}) \tag{2.17}$$

where I is the z-component of pion isospin, $\vec{\sigma}$ and $\vec{\tau}$ are the usual Pauli operators. We also have defined

$$\tau_{\pm} = \frac{\mp 1}{\sqrt{2}} (\tau_1 \pm i\tau_2)$$

3. πN scattering

The πN scattering matrix is defined by

$$T(\vec{k}',\vec{k},E) = \langle \vec{k}' | T(E) | \vec{k} \rangle \tag{2.18}$$

with

$$|\vec{k}\rangle = a^{+}_{\vec{k}} b^{+}_{-\vec{k}} |0\rangle$$

where the scattering operator is defined by the model Hamiltonian Eqs. (2.8)-(2.9)

$$T(E) = H' + H' \frac{1}{E-H+i\epsilon} H' \tag{2.19}$$

By using standard projection techniques, it is straightforward to see that the solution of Eq. (2.19) can be expressed in terms of the t matrix defined by Eq. (2.14) and the vertex function h(k)

$$T(\vec{k}',\vec{k},E) = t(\vec{k}',\vec{k},E+i\epsilon) + \frac{H(\vec{k}',E)\; H^{+}(\vec{k},E)}{E-m_0-\Sigma(E)} \tag{2.20}$$

with

$$\Sigma(E) = \Sigma_0(E) + \Sigma_I(E) \;.$$

The self-energies of the bare particle N_0 are

$$\Sigma_0(E) = \int \frac{h(\vec{k})\; h^{+}(\vec{k})}{E-W(k)+i\epsilon}\, d\vec{k}$$

$$\Sigma_I(E) = \int h(\vec{k}) \frac{d\vec{k}}{E-W(k)+i\epsilon}\, t(\vec{k},\vec{k}',E) \frac{d\vec{k}'}{E-W(\vec{k}')+i\epsilon}\, h^{+}(\vec{k}')d\vec{k}' \tag{2.21}$$

The renormalized vertex interaction is

$$H^+(\vec{k},E) = h^+(\vec{k}) + \int d\vec{k}' \frac{h^+(\vec{k}')\ t(\vec{k}',\vec{k},E)}{E-W(k)+i\epsilon}$$

$$H(\vec{k},E) = h(\vec{k}') + \int d\vec{k} \frac{t(\vec{k}',\vec{k},E)\ h(\vec{k}')}{E-W(\vec{k})+i\epsilon} \tag{2.22}$$

Clearly at E=m, the renormalized vertex function $H(\vec{k},m)$ is related to the πNN form factor defined by Eq. (2.16)

$$H(\vec{k},m) \equiv \frac{1}{Z_0^{1/2}} F_{\pi NN}(\vec{k}) \tag{2.23}$$

To end this section, we want to show explicitly that the scattering solution given above is consistent with the bound state condition; i.e. the scattering amplitude has a pole at E=m and its residue is uniquely determined by the physical πNN coupling constant f. This is done by expanding the self-energy of N_0 to write the second term of the πN amplitude Eq. (2.20) as

$$\frac{H(\vec{k},E)\ H^+(\vec{k},E)}{E-m_0-\Sigma(E)} = \frac{H(\vec{k},E)\ H^+(\vec{k},E)}{E-m_0-\Sigma(m)-(E-m)\left.\frac{\partial\Sigma(E)}{\partial E}\right|_{E=m} - \Sigma_R'(E)} \tag{2.24}$$

where

$$\Sigma_R'(E) = \sum_{n=2}^{\infty} (E-m)^n \left.\frac{\partial^n\Sigma(E)}{\partial E^n}\right|_{E=m}$$

By using the Eqs. (2.11) and (2.13) one can show that

$$m = m_0 + \Sigma(m) \tag{2.25}$$

In Appendix B, we show explicitly that

$$1 - \left.\frac{\partial\Sigma(E)}{\partial E}\right|_{E=m} \equiv Z_0^{-1} \tag{2.26}$$

Since

$$\Sigma'(m)_R = 0 \ ,$$

we can use Eqs. (2.23)-(2.26) to show that

$$\frac{H(\vec{k},E)\ H^+(\vec{k},E)}{E-M_0-\Sigma(E)}$$

$$\xrightarrow[E\to m]{} \frac{F_{\pi NN}(\vec{k}')\ F^+_{\pi NN}(\vec{k})}{E-m}$$

$$\xrightarrow[\substack{\vec{k}\to 0\\ \vec{k}'\to 0}]{} \frac{f^2}{(2\pi)^3} \frac{1}{\sqrt{2E_\pi(k)}} \frac{1}{\sqrt{2E_\pi(k')}} \frac{k}{\mu} \frac{k'}{\mu} \frac{(\sigma\cdot\vec{k}')(\sigma\cdot\vec{k})}{E-m} (\tau_{-I}\tau^+_{-I}) \qquad (2.26)$$

The constructed πN t-matrix therefore has a pole at E=m. The residue of the pole is uniquely defined by the πNN coupling constant f. Our model therefore has the required properties of a physical πN amplitude, and can be used to confront the scattering data.

III. NUMERICAL RESULTS

To proceed, we need to define the vertex function h(k) and the two-body matrix element v(k,k'). We parameterize them according to the forms deduced from using the wave function defined by Eq. (2.2) to evaluate the matrix elements $\langle N_0|H_B|\pi N\rangle$ and $\langle \pi N|H_B|\pi N\rangle$, where H_B is a more fundamental Hamiltonian defined by a bare vertex $\pi N_0 \leftrightarrow N_0$ and a two-body $\pi N_0 \to \pi N_0$ interactions. The resulting forms of these two matrix elements are given in Appendix A, where we also discuss the self-consistent problem involved in obtaining the bound state solution of the form Eq. (2.2). For our present construction of the model in the subspace $N_0 \oplus \pi N$, it is only necessary to note that such a more fundamental consideration suggests that the $\pi N \to \pi N$ interaction can be taken as a separable form. Taking into account spin-isospin quantum numbers explicitly, we write

$$v(\vec{k}',\vec{k}) = \sum_{\alpha} \mathcal{Y}^{J_\alpha M_\alpha T_\alpha M_{T_\alpha}^{+}}_{\ell_\alpha 1/2}(\hat{k}')\ v_\alpha(k',k)\ \mathcal{Y}^{J_\alpha M_\alpha T_\alpha M_{T_\alpha}}_{\ell_\alpha 1/2}(\hat{k}) \qquad (3.1)$$

with

$$v_\alpha(k',k) = \sum_{i,j=1}^{2} g_i^\alpha(k')\ C_{ij}^\alpha g_j^\alpha(k)$$

where is the usual normalized spin-isospin-angular function for the πN channel α. For the P_{11} channel considered here, we have $J_\alpha = T_\alpha = 1/2$ and $\ell_\alpha = 1$. We parameterize g(k) as

$$g_i^\alpha(k) = \frac{1}{\sqrt{2}} \frac{1}{(m+\mu)} \left(\frac{k}{\mu}\right)^{\ell_\alpha} e^{-(k/a_\alpha^i)^2} \qquad (3.2)$$

The vertex interaction is parameterized as

$$h(\vec{k}) = \frac{1}{(2\pi)^{3/2}} \frac{i}{\sqrt{2E_\pi(k)}} \frac{f_0}{\mu} \vec{\sigma} \cdot \vec{k} \left[\frac{\Lambda_{\pi NN}^2}{\Lambda_{\pi NN}^2 + k^2}\right] (I\tau_{-I}) \qquad (3.3)$$

where I indicates the z-component of the pion isospin.

The free parameters of the model are the mass of the bare particle m_0, coupling constant f_0 and range $\Lambda_{\pi NN}$ of the vertex interaction h(k), coupling constants C_{ij}^α and ranges a_i^α of the two-body interaction v(k,k'). We adjust these parameters to fit the data of the nucleon mass, πNN coupling constant and πN scattering phase shifts below the pion production threshold. In the χ^2 fit to the data, we look for solutions with a bare mass $m_0 \sim 1000$ Mev. This probably will lead to a simple interpretation that the core is the object with three valance quarks, as described by the standard nonrelativistic quark model or the Bag model. We found that no fit can be obtained if the two-body πN interaction v is set to zero. This is consistent with the findings of Ref. 5. In Table I we list the determined parameters. The calculated phase shifts are compared with the data in Fig. 1.

Because of the simplicity of the separable parameterization of v(k,k'), the radial part of the pion wave function, defined by Eq. (2.13), can be explicitly written as

$$A_\alpha(k) = \frac{1}{m-W(k)} \left[\tilde{h}(k) + \sum_{i=1}^{2} \beta_i g_i^\alpha(k)\right] \tag{3.4}$$

where

$$\tilde{h}(k) = \frac{1}{(2\pi)^{3/2}} \frac{f_0}{\sqrt{2E_\pi(k)\mu}} \frac{k}{\mu} \left(\frac{\Lambda^2_{\pi NN}}{\Lambda^2_{\pi NN}+k^2}\right)$$

The calculated coefficients β_i are also given in Table I. In Fig. 2 we display the calculated radial wave function $|kA_\alpha(k)|$.

By using Eq. (2.3), we can calculate Z_0 and also the probability functions from Eq. (2.1). The results are:

$P_0 = Z_0 = 0.80$

$P_1 = 0.16$

$P_2 = 0.032$

$P_3 = 0.0064$

This means that the mesonic component of the nucleon is 20% and is dominated by the one-pion component.

IV. PREDICTIONS OF ELECTROMAGNETIC PRODUCTION OF PIONS

The need of a two-body πN interaction to fit the πN scattering data leads to the presence of the off-shell πN scattering amplitude in determining the πNN form factor, as seen in Eq. (2.16). The importance of this πN correlation effect is illustrated in Fig. 3a, where we compare our form factor (solid curve) and the conventional monopole parameterization with same $\Lambda_{\pi NN}$=980 MeV/c (dashed) and 500 MeV/c (dash-dotted). It is clear that the theoretical results cannot be represented adequately by the conventional monopole form. We found that a reasonable representation of our result is (Fig. 3b)

$$F_D(k) = \frac{1 + \exp(-k^2/t_1^2)}{1 + \exp(+k^2/t_2^2)} \tag{4.1}$$

with t_1=5000 MeV/c, t_2=480 MeV/c. The significance of our prediction is already notable in the low momentum part of the form factor, which can be explored in the study of low and intermediate energy nuclear reactions. To illustrate this point, we have studied electromagnetic production of pions from nuclear targets.

In the impulse approximation, the electromagnetic production of pions from a nuclear target can be calculated from the amplitude of the elementary $\gamma N \to \pi N$ process. If we follow the conventional approach and calculate this amplitude from the Born term deduced from the field theoretical Largrangian of pion nucleon interaction, the calculated cross section is directly proportional to the πNN form factors. Such a first-step calculation of $\gamma p \to \pi^+ n$ and $(e,e'\pi^+)$ from the deuteron has been carried out for the present study by using the approach developed in Ref. 6. In addition to the Born term taken from the work by Larget,[4] we also add the Δ excitation term following the approach by Koch and Moniz.[7] In the $d(e,e'\pi^+)$ calculation, we concentrate on the kinematics that the outgoing pion is in the direction parallel to the exchanged virtual photon, so that the dominant mechanism is the interaction between the photon and the pion emitted from the nucleon in the deuteron. The calculated cross section of $d(e,e'\pi^+)$ is therefore a direct measure of the πNN form factor.

Two typical results are shown in Fig. 4a for $\gamma p \to \pi^+ n$ and Fig. 4b for $d(e,e'\pi^+)$. The dashed curves are calculated from the conventional monopole πNN form factor with $\Lambda_{\pi NN}$=1000 MeV/c. The solid curves are calculated from our form factor. Our results suggest that the conventional monopole parameterization is not adequate for a precise study of pionic effects. We want to emphasize here that the differences shown in Fig. 4 are the errors one can make in interpreting the data, if the observed low energy πN scattering dynamics is not taken into account in describing πNN form factor.

V. SUMMARY

We have presented a model for a consistent description of pion distribution inside the nucleon and the low energy πN scattering. The underlying dynamics is assumed to be a vertex interaction for describing the absorption and emission of pions by a core (N_0) of the nucleon, and a two-body direct interaction between the core and the pion. We further assume that the pions are all in the same lowest eigenstate of the system, as described by the model wave function Eq. (2.2). It is then possible to solve the problem in a subspace spanned by the core state N_0 and the physical πN state. By fitting the data of the nucleon mass, πNN coupling constant and the low energy πN scattering data, we have determined the vertex interaction describing the transition from the core state to the πN state, and a two-body πN interaction. The relationships between these two phenomenological quantities and the underlying more fundamental "bare" Hamiltonian for the pion and the core are briefly discussed in the appendix.

The main outcome is a pion wave function given explicitly in Eq. (3.4), which can be used in other studies. As an example we have shown, in an impulse approximation calculation of the electromagnetic production of pions from the nucleon and the deuteron, that the conventional monopole parameterization of the πNN form factor is not adequate. Our studies of other pionic nuclear reactions will be published elsewhere.

APPENDIX A

In this appendix, we briefly discuss the self-consistent problem involved in obtaining the model wave function Eq. (2.2) from the following "bare" Hamiltonian

$$H_B = H_{OB} + H_B' \tag{A.1}$$

$$H_{OB} = \int d\vec{k}\, \sqrt{\mu^2+k^2}\; a^+_{\vec{k}} a_{\vec{k}} + \int d\vec{p}\, \sqrt{m_0^2+\vec{p}^{\,2}}\; C^+_{\vec{p}} C_{\vec{p}} \tag{A.2}$$

and

$$H_B' = \int d\vec{p}\,d\vec{k} \left[h_0(\vec{\kappa})\; C_{\vec{p}+\vec{k}}\, C^+_{\vec{p}}\, a^+_{\vec{k}} + h_0^+(\vec{\kappa})\; C^+_{\vec{p}+\vec{k}}\, C_{\vec{p}}\, a_{\vec{k}}\right]$$

$$+ \int d\vec{p}\,d\vec{k}\,d\vec{p}\,'d\vec{k}\,'\; \delta(\vec{p}\,'+\vec{k}\,'-\vec{p}-\vec{k})\; C^+_{\vec{p}\,'}\, a^+_{\vec{k}\,'}\, C_{\vec{p}}\, a_{\vec{k}}\; v_0(\vec{\kappa}\,',\vec{\kappa}) \tag{A.3}$$

All of the notations in the above equations are explained in Sect. II. Projecting the bound state equation in c.m. frame ($\vec{k}=-\vec{p}=\vec{\kappa}$)

$$(H_B-m)\; Z_o^{1/2}\left[|N_0\rangle + \sum_{i=1}^{\infty} \frac{1}{\sqrt{n!}} \left[\prod_{i=1}^{n} \int (Z_0^{1/2}A(\vec{k}_i))\,d\vec{k}_i\, a^+_{\vec{k}_i}\right] |N_0\rangle\right] = 0 \tag{A.4}$$

onto the basis states $|N_0\rangle$ and $a^+_{\vec{k}}|N_0\rangle$, we get the following nonlinear coupled integral equations

$$m_0 + \int (Z_0^{1/2} h_0^+(\vec{k}))A(\vec{k})\; d\vec{k} = m \tag{A.5a}$$

$$(Z_0^{1/2} h_0(\vec{k})) + \sqrt{2}\; Z_0 A(\vec{k}) \int (Z_0^{1/2} h_0(\vec{k}))\; A(\vec{k}\,')\; d\vec{k}\,'$$

$$+ Z_0 \left(\sqrt{m_0^2+\vec{k}^2} + \sqrt{\mu^2+\vec{k}^2}\right) A(\vec{k})$$

$$+ Z_0 \int d\vec{k}\ v_0(\vec{k}',\vec{k})\ A(\vec{k}) = Z_0 m A(\vec{k}) \tag{A.5b}$$

The normalization condition yields another condition

$$Z_0 = \frac{1}{1+\int |A(\vec{k})|^2 d\vec{k}} \lesssim 1 \tag{A.5c}$$

In practice, we have very little information about the bare interactions. The task is to find h_0 and v_0 so that the above equations can be satisfied. It is interesting to note that if the two-body interaction v_0 is neglected, the above equations can be cast into the following form

$$Z_0 = Z_0^2 + \int \frac{(Z_0^{1/2} h_0(\vec{k}))(Z_0^{1/2} h_0^+(\vec{k}))d\vec{k}}{m - \sqrt{2}\ (m-m_0) - \sqrt{m_0^2+k^2} - \sqrt{\mu^2+k^2}}$$

$$Z_0 m_0 + \int \frac{(Z_0^{1/2} h_0(\vec{k}))(Z_0^{1/2} h_0^+(\vec{k}))d\vec{k}}{\left[m - \sqrt{2}\ (m-m_0) - \sqrt{m_0^2+\vec{k}^2} - \sqrt{\mu^2+k^2}\right]^2} = mZ_0$$

If we assume that the vertex interaction $h_0(\vec{k})$ is known, then the existence of a bound state solution can be determined by examining whether the above set of nonlinear equations has solutions with positive definite values of Z_0 and m_0.

The above equations illustrate the nontrivial self-consistent problem involved in getting the wave function Eq. (2.2) from the "bare" Hamiltonian of the form of Eq. (A.1). The problem becomes even more complicated if we also want to describe the πN scattering data. This is the motivation of constructing the simplified model described in the text. The relationship between our model and this probably more fundamental model can be established by equating our

$\pi N \leftrightarrow N_0$ vertex function and two-body πN matrix element to the matrix elements calculated from H_B'. The results are

$$h(\vec{k}) = \langle N_0 | H_B' a^+_{\vec{k}} | N \rangle \equiv Z_0^{1/2} h_0(\vec{k})$$

$$v(\vec{k}',\vec{k}) = \langle N | a_{\vec{k}'} H_B' a^+_{\vec{k}} | N \rangle$$

$$= \Bigg\{ (Z_0^{1/2})^3 \left[\sum_{n=1}^{\infty} \sqrt{n+1}\ (Z_0 D)^n \right] \left[h_0^+(\vec{k}') A(\vec{k}) + A^+(\vec{k}') A(\vec{k}) H(Z_0 D) \right]$$

$$+ v_0(\vec{k}',\vec{k}) + (Z_0^{1/2})^4\ U^+(\vec{k}') A(\vec{k})$$

$$\times \left[1 + (D\ Z_0) + \sum_{n=3}^{\infty} \frac{n!(n-2)!}{2!} (D\ Z_0)^{n-1} \right] \Bigg\} + \text{h.c.}$$

Where D and Z_0 have been defined in Eq. (2.3), and

$$U^+(\vec{k}') = \int A^+(\vec{k}) v_0(\vec{k}',\vec{k}) d\vec{k}$$

We see that the bare vertex interaction is directly proportional to the vertex interaction in the subspace $N_0 \oplus \pi N$. If the bare two-body interaction v_0 can be neglected, $v(k,k')$ is of the form of a separable interaction. This motivates the parameterizations of $h(k)$ and $v(k,k')$ in Sect. III.

APPENDIX B

To show Eq. (2.25), we first note that the t-matrix, Eq. (2.14), can be written as

$$t(\vec{k}',\vec{k},E) = \langle\vec{k}'|t(E)|\vec{k}\rangle \ , \tag{B.1}$$

where

$$t(E) = v + v\,\frac{1}{E-H_0}\,t(E) \ . \tag{B.2}$$

Equation (B.2) leads to the following relations

$$t(E) = v + v\,\frac{1}{E-H_0-v}\,v \ , \tag{B.3}$$

and hence

$$v\,\frac{1}{E-H_0-v} = t(E)\,\frac{1}{E-H_0} \ , \tag{B.4a}$$

$$\frac{1}{E-H_0-v}\,v = \frac{1}{E-H_0}\,t(E) \ . \tag{B.4b}$$

We then have

$$\frac{\partial t(E)}{\partial E} = -\,v\,\frac{1}{(E-H_0-v)^2}\,v$$

$$\equiv -\,t(E)\,\frac{1}{(E-H_0)^2}\,t(E) \ . \tag{B.5}$$

Now we also write the self energy (Eq. (2.21))

$$\Sigma(E) = \langle N_0|\hat{\Sigma}(E)|N_0\rangle \tag{B.6}$$

where

$$\hat{\Sigma}(E) = \frac{\hat{h}^+\,\hat{h}}{E-H_0} + \hat{h}^+\,\frac{1}{E-H_0}\,t(E)\,\frac{1}{E-H_0}\,\hat{h} \tag{B.7}$$

here, we have introduced a vertex operator $\hat{h}$ by $\langle\vec{k}|\hat{h}|N_0\rangle = h(\vec{k})$.

By using Eqs. (B.5) and (B.7), we have

$$\frac{\partial\hat{\Sigma}(E)}{\partial E} = -\left[\frac{\hat{h}^+ \hat{h}}{(E-H_0)^2} + \frac{h^+}{(E-H_0)^2}\, t(E)\, \frac{h}{E-H_0}\right.$$

$$+ \frac{h^+}{E-H_0}\, t(E)\, \frac{h}{(E-H_0)^2}$$

$$\left. + \frac{h^+}{E-H_0}\, t(E)\, \frac{1}{(E-H_0)^2}\, t(E)\, \frac{1}{E-H_0}\, h\right]$$

$$= \hat{A}^+(E)\ \hat{A}(E) \qquad \text{(B.8)}$$

where

$$\hat{A}(E) = \frac{\hat{h}}{E-H_0} + \frac{1}{E-H_0}\, t(E)\ \hat{h}$$

$$\hat{A}^+(E) = \frac{\hat{h}^+}{E-H_0} + \hat{h}^+\ t(E)\ \frac{1}{E-H_0} \qquad \text{(B.9)}$$

From Eq. (B.9) it is easy to see that the wave function Eq. (2.15) can be written as

$$A(\vec{k}) = \langle\vec{k}|\hat{A}(E)|N_0\rangle \qquad \text{(B.10)}$$

We finally arrive at

$$1 - \left.\frac{\partial\Sigma(E)}{\partial E}\right|_{E=m} = 1 - \langle N_0\left|\left.\frac{\partial\hat{\Sigma}(E)}{\partial E}\right|_{E=m}\right|N_0\rangle$$

$$= 1 + \langle N_0|\hat{A}^+(E)\ \hat{A}(E)|N_0\rangle$$

$$= 1 + \int |A(\vec{k})|^2 d\vec{k}$$

$$= 1 + D$$

$$\equiv 1 + \frac{1-Z_0}{Z_0}$$

$$= \frac{1}{Z_0} \qquad \text{Q.E.D.}$$

Here we have used the definitions of D and Z_0 given in Eq. (2.3).

This works supported by the U. S. Department of Energy, Nuclear Physics Division, under contract W-31-109-ENG-38.

REFERENCES

[1] See review by A. W. Thomas, in Advances in Nuclear Physics, ed. J. W. Negele and E. Vogt (Plenum, NY 1983) Vol. 13, pp. 1-137.

[2] See review by E. Berger and F. Coester, in Annal Review of Nuclear and Particle Science, Vol. 37, 463 (1987).

[3] See review by D. Ashary and J. P. Schiffer, in Annal Review of Nuclear and Particle Science, Vol. 36, 207 (1986).

[4] See review by J. M. Larget, in New Vistas in Electro-Nuclear Physics, ed. E. L. Tomusiak, H. S. Caplan and E. T. Dressler (Plenum, NY 1986), p. 361.

[5] J. Johnstone and T.-S. H. Lee, Phys. Rev. C 34, 243 (1986).

[6] C. R. Chen and T.-S. H. Lee, to be published.

[7] J. H. Koch and E. Moniz, Phys. Rev. C 27, 751 (1983).

TABLE I. Parameters of the model with the parameterizations given in Eqs. (3.2) and (3.3), and the coefficients β_i for calculating pion radial wave function Eq. (3.4).

	m_0 MeV)		$\Lambda_{\pi NN}$ (MeV/c)	f_0		
h(k)	1254.20		980.83	0.6715		
	a_1 (MeV/c)	a_2 (MeV/c)	C_{11}	C_{12}	C_{21}	C_{22}
v(k',k)	412.634	1760.453	-13.861	4.602	4.602	-2.082
	$Z_0^{1/2}$ β_i (MeV$^{1/2}$)			$Z_0^{1/2}$ β_2 (MeV$^{1/2}$)		
Z_0=0.80	19.582			-5.070		

FIGURE CAPTIONS

Fig. 1 Fit to the πN P_{11} phase shifts.

Fig. 2 Calculated radial Pion wave function $|kA(k)|$ of the nucleon.

Fig. 3 (a) Calculated πNN form factor (solid curve) is compared with the conventional monopole form factor with $\Lambda_{\pi NN}$=980 MeV/c (dashed) and 500 MeV/c (dash-dotted); (b) parameterization Eq. (4.1) (dashed) is compared with the calculated πNN form factor (solid).

Fig. 4 (a) the differential cross sections of $\gamma p \to \pi^+ n$ calculated from two πNN form factors are compared, (b) same as (a) but for the coincidence cross sections of $d(e,e'\pi^+)$, with the outgoing pion in the direction of the exchanged virtual photon. Solid curves: the present model; dashed curves: monopole form with $\Lambda_{\pi NN}$=1000 MeV/c.

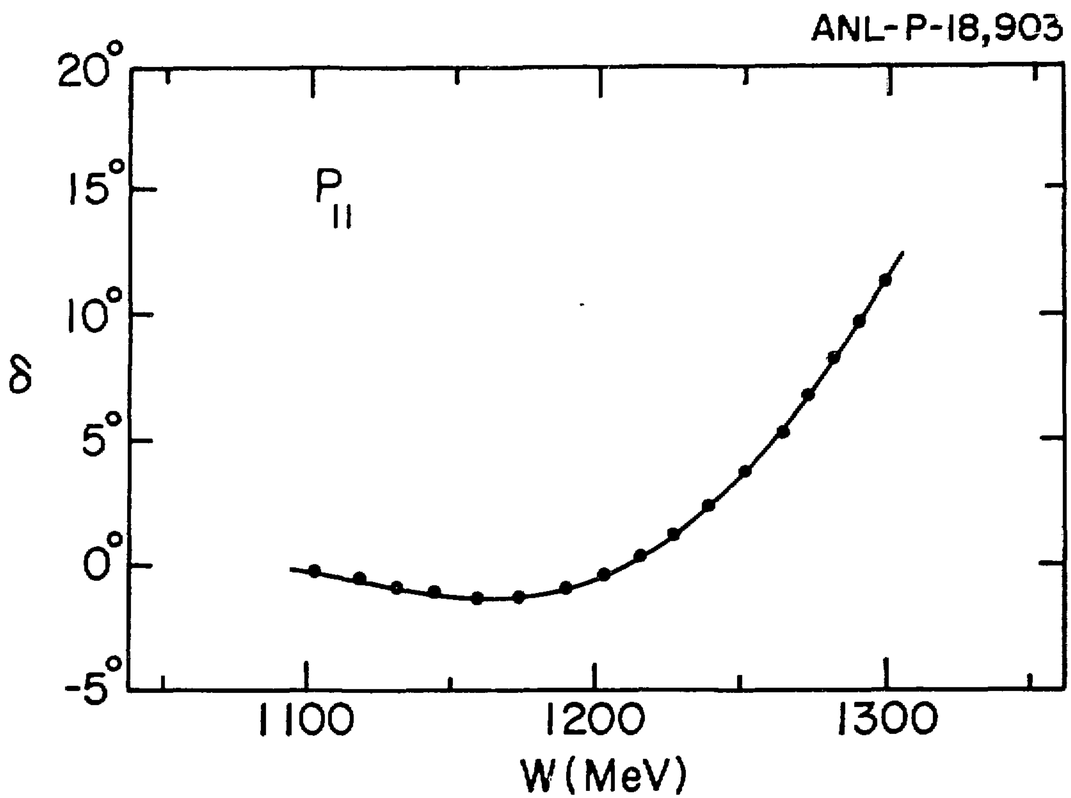

Fig. 1

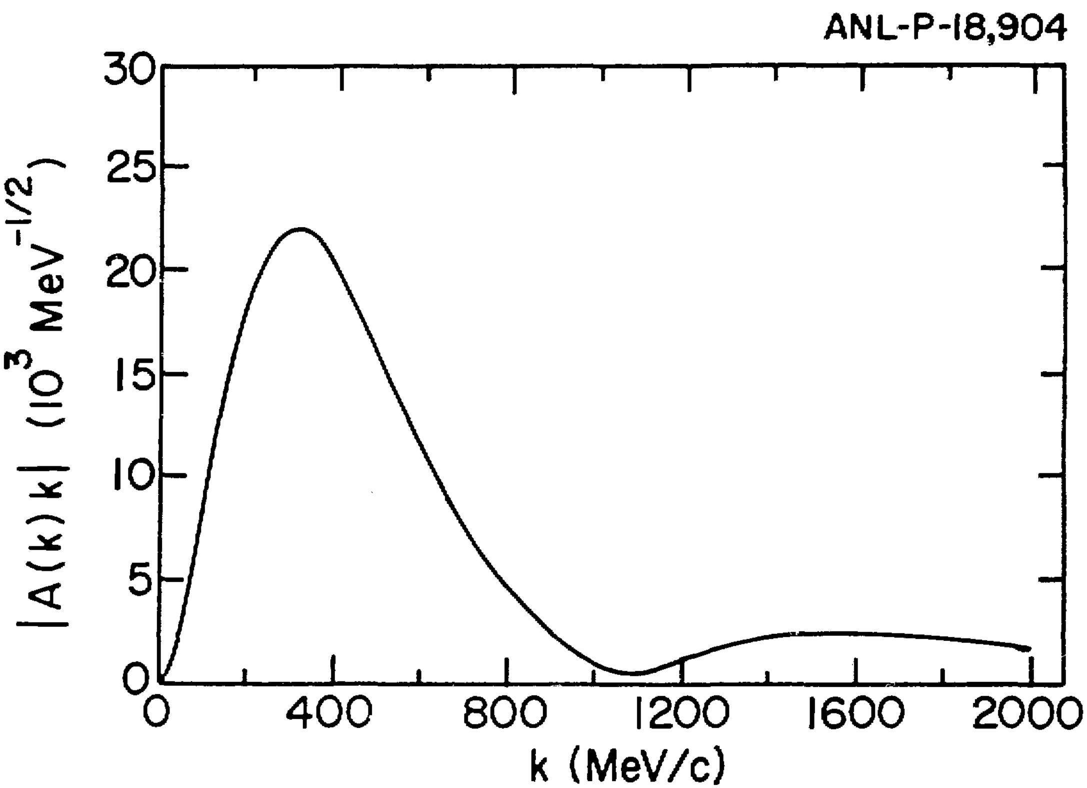
ANL-P-18,904
30
25
20
15
10
5
0
0
400
800
1200
1600
2000
k (MeV/c)
|A(k)k| (10^3 MeV$^{-1/2}$)

Fig. 2

ANL-P-18,905

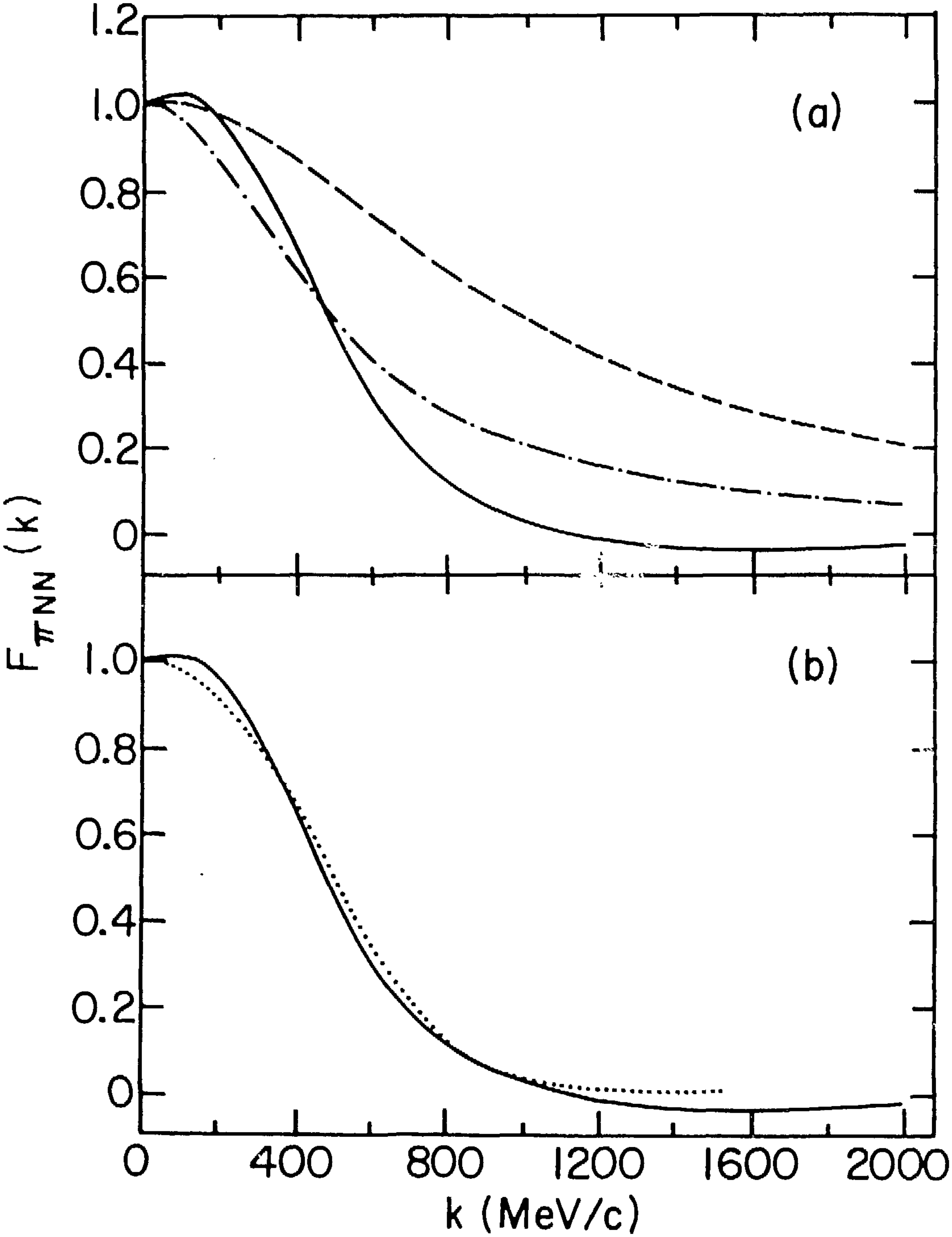

Fig. 3

ANL-P-18,906

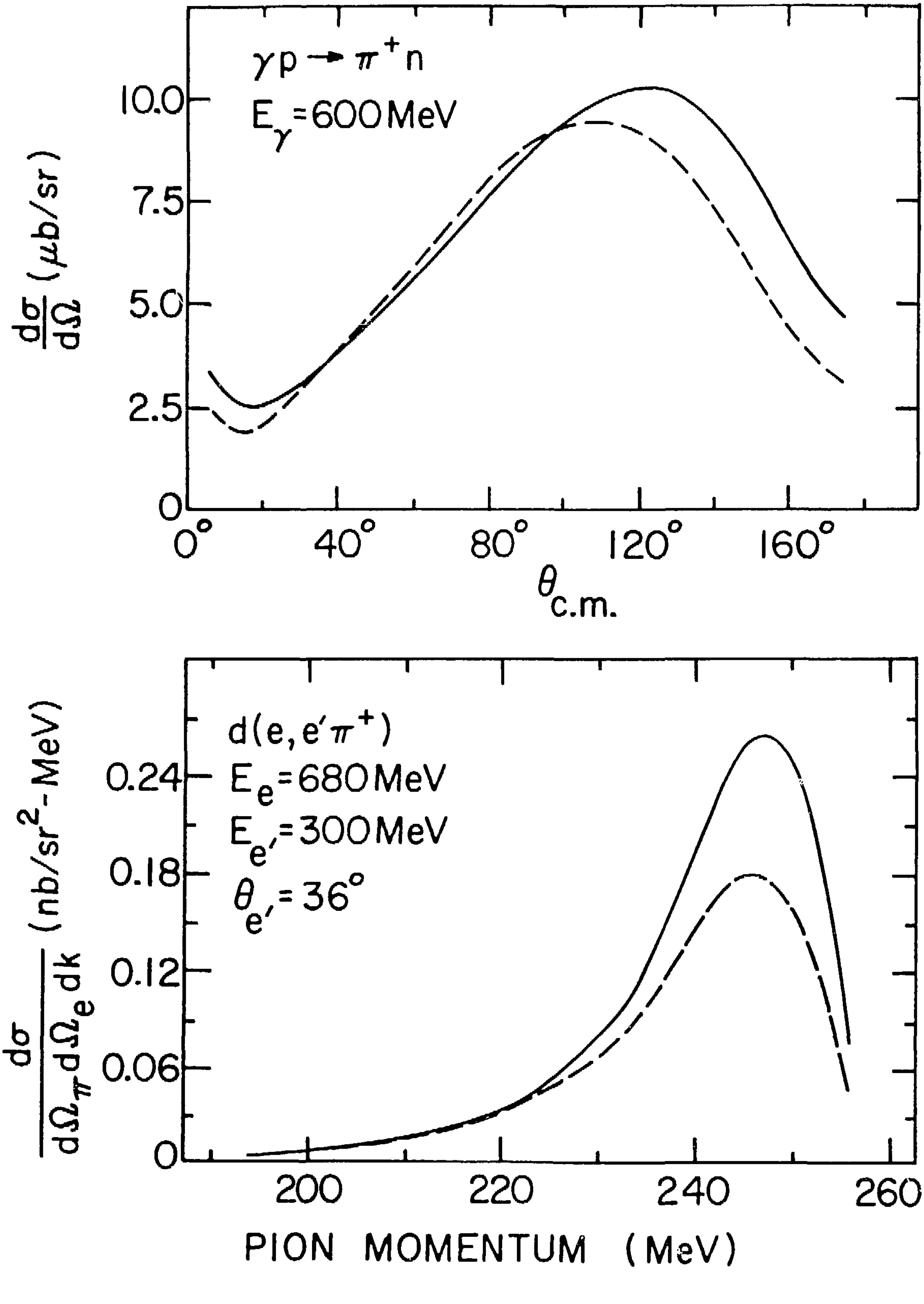

Fig. 4