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THE NEUTRON ELECTRIC DIPOLE MOMENT

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Abstract

We have made a systematic study of the electric dipole moment (EDM) of neutron D_n in various models of CP violation.

We find that

(i) in the standard KM model with 3 families the neutron EDM is in the range $1.4 \times 10^{-33} \leq |D_n| \leq 1.6 \times 10^{-31}$ e.cm,

(ii) the two Higgs doublet model has approximately the same value of D_n as the standard model,

(iii) D_n in the Weinberg model is predicted to satisfy $|D_n| > 10^{-25}$ ecm,

(iv) in a class of left-right symmetric models D_n is of the order of $10^{-26 \pm 1}$ e.cm,

v) in supersymmetric models, D_n is of order $10^{-22} \phi$ ecm with ϕ being the possible phase difference of the phases of gluino mass and the gluino-quark-squark mixing matrix,

vi) the strong CP parameter θ is found to be $\theta < 10^{-9}$, using the present experimental limit that $|D_n| < 2.6 \times 10^{-25}$ ecm with 90% confidence.

1) INTRODUCTION

The electric dipole moment (EDM) of neutron (D_n) has been of interest to physicists for a long time. In 1950, Purcell and Ramsey^[1] first considered the problem of the existence of the neutron EDM. However at the time parity-P was assumed to be an exact conservation law, and in order to have a nonzero D_n , Purcell and Ramsey had to construct a rather unconventional P even EDM for neutron. In 1957 Landau^[2] observed that a non vanishing D_n was a signal of P and T (time reversal) non-conservation^[3]. It was at about this time that P invariance was found not to be an exact conservation law of nature. The EDM of neutron violates both P and T. Even after P invariance was found to be violated by the weak interactions, the prejudice was that T would be conserved, and if the CPT theorem is valid it must preserve the product of P and C(charge conjugation). When CP violation was observed in Kaon system^[4] in 1964, the subject of the neutron EDM became of considerable theoretical and experimental interest. A recent review has been provided by Shabalin^[5]. The current experimental situation is that two groups have reported values of $-(1.4 \pm 0.6) \times 10^{-25}$ ecm^[6] and $-(1.1 \pm 0.7) \times 10^{-25}$ ecm^[7] which correspond to an upper bound of $|D_n| \leq 2.6 \times 10^{-25}$ e.cm at the 90% Confidence Level(C.L). There are many theoretical models which attempt to account for the observed CP violation in the K meson system, and in general they give different predictions for the neutron EDM^[5]. This has provided an impetus for the experimenters - particularly as some of the theories give values which are tantalizingly close to the present experimental upper bound.

This fact means that it is important to be satisfied that the calculation of D_n is reliable, and that the effect in different theories is calculated consistently, so that comparisons may be made in a consistent way. Surprisingly, a systematic, consistent study of the value of the neutron EDM in the different theories of CP violation has not been performed. It is our purpose in this paper to provide such a study, by integrating the various methods that have been used to calculate D_n , applying them to all of the models, and establishing the regions in which each method is valid.

The most commonly used technique is the valence quark model in which the EDM of each valence quark is calculated and summed to obtain the neutron EDM. However in some models, most notably the standard model, the valence quark contribution is suppressed (in the standard model the quarks acquire a non-vanishing EDM only at the three loop order^[8,9]). In this case both quark level exchange diagrams and hadronic level loop diagrams must also be considered, and the hadronic level loops turn out to give the dominant contribution^[10], which is perhaps not surprising since it is known that the hadronic loops involving pions are logarithmically singular in the soft pion limit^[11]. In some models, due to different origin of CP violation, quark EDM occurs at the one loop level and the calculations for the neutron EDM are different from the standard model. In this paper we consider all possible contributions to D_n in a systematic way, for the standard Kobayashi-Maskawa model, the standard model with two Higgs doublets, the Weinberg Higgs model, the left-right symmetric model and the supersymmetric standard model. We also for completeness briefly discuss the neutron EDM from the strong CP violation characterised by the θ parameter.

In section 2 we discuss possible contributions to the neutron EDM in a general, model independent way and set out the formalism for later use. These techniques are then applied in sections 3 to 8 to the Standard K-M model, the two Higgs doublet model, the Weinberg model, the left-right symmetric model and the supersymmetric standard model respectively. For completeness we also briefly discuss the neutron EDM in the strong CP violation model in section 8. Section 9 is devoted to a discussion of our results and in it we summarize the conclusions.

2) METHODS FOR CALCULATING ELECTRIC DIPOLE MOMENTS

There are different model dependent contributions to the neutron EDM which we describe in this section. We also collect some general results in a form which can be readily particularized to specific models in the ensuing sections.

2.1) The Valence Quark Contribution

This is conceptually the simplest contribution to the neutron EDM, in that one simply computes the quark EDM and sums to obtain D_n . If the EDM of the u and d quarks are D_u and D_d , the neutron EDM is then given by

$$D_n^{(V)} = \frac{1}{3} (4D_d - D_u) \quad (2.1.1)$$

The calculation of the individual quark moments is of course model dependent and is deferred to the subsequent sections.

2.2) Quark CDM contributions to the EDM^[12]

A P and T violating coupling of the gluon to a quark gives rise to a colour dipole moment (CDM) of the quark. We write the coupling in the form

$$H_g = g_s f_q \frac{1}{2} \bar{q} \sigma_{\mu\nu} \gamma_5 \frac{\lambda^a}{2} q G^{\mu\nu}_a, \quad (2.2.1)$$

where g_s is the strong QCD coupling constant, λ^a are the SU(3) Gell-Mann matrices, $G^{\mu\nu}_a$ is the gluon field strength and f_q is the quark CDM. This P and T violating interaction induces a change in the neutron wave function which results in a neutron EDM. To estimate this contribution to the neutron EDM we use the non-relativistic quark model and treat H_g perturbatively. Non-relativistically, H_g becomes

$$H_g = -g_s \sum_k f_q \sigma_k (-\nabla G_0^a) \frac{\lambda_k^a}{2}, \quad (2.2.2)$$

where G_0^a is the time component of the gluon field potential, and the summation is over the spin. From equation (2.2.2) or equation (2.2.3) the P and T violating nature of H_g is immediately apparent. Now we introduce the unperturbed QCD Hamiltonian, H_0 , in this approximation :

$$H_0 = \sum_k \left(\frac{p_k^2}{2m} + g_s \frac{\lambda_k^a}{2} G_0^a \right). \quad (2.2.3)$$

and we may write

$$H_g = i \sum_k f_q \sigma_k \left[p_k, H_0 \right]. \quad (2.2.4)$$

To the lowest order in perturbation theory, the modified neutron wave function is

$$\begin{aligned} |\tilde{n}\rangle &= |n\rangle + \sum_m \frac{|m\rangle \langle m| H_g |n\rangle}{E_n - E_m} \\ &= \left(1 + i \sum_k f_q \sigma_k \cdot p_k \right) |n\rangle \end{aligned} \quad (2.2.5)$$

Then the neutron EDM generated by the CDM, $D_n^{(c)}$ is given by

$$\begin{aligned} D_n^{(c)} &= \langle \tilde{n} | \sum_i Q_i r_i | \tilde{n} \rangle \\ &= i \langle n | \left[\sum_i Q_i r_i, \sum_k f_q \sigma_k \cdot p_k \right] | n \rangle \\ &= -\langle n | \sum_k Q_k f_q \sigma_k | n \rangle. \end{aligned} \quad (2.2.6)$$

where Q_i is quark charge operator. We then immediately see that

$$D_n^{(c)} = \frac{1}{3} e \left(\frac{4}{3} f_d + \frac{2}{3} f_u \right) \quad (2.2.7)$$

2.3) Hadronic loop contributions to the neutron EDM.

It was shown by Barton and White^[13] that mesonic loops at the hadron level can lead to contributions to the EDM proportional to $\ln(m_M)$ which are thus singular in the soft meson limit. Two classes of diagrams, illustrated in Figures 2.1 and 2.2, may contribute. In figure 2.1 CP violation occurs at first order in the weak interactions, and in the diagrams of Figure 2.2 CP violation occurs only at second order in the weak interaction.

Writing the strong interaction effective lagrangian as

$$L_s = -\sqrt{2}g\bar{B}\gamma_5 B'M + \text{H.C.}, \quad (2.3.1)$$

and weak $BB'M$ vertices as

$$L_w = \sqrt{2}f e^{-i\phi} \bar{B}B'M + \text{H.C.}, \quad (2.3.2)$$

we evaluate these loop diagrams, obtaining

$$D_n^{(l)} = D_{n,1}^{(l)} + D_{n,2}^{(l)} \quad (2.3.3)$$

where

$$D_{n,1}^{(I)} = \frac{egf}{4\pi^2} \frac{m_B}{m_n} \sin(-\phi) G_B(m_M^2) \quad (2.3.4)$$

and

$$D_{n,2}^{(I)} = \frac{egf}{4\pi^2} \frac{k_B}{2m_B} \sin(-\phi) F_B(m_M^2) \quad (2.3.5)$$

where m_B , m_n and m_M are the masses of the baryon B, neutron n and meson M respectively. We have split $D_n^{(I)}$ into two parts, the first of which arises from the sum of the graphs with the photon coupled to the meson and to the baryon through Dirac γ_μ coupling, and the second part arises from the graphs with the photon coupled to the baryon through Pauli $\sigma_{\mu\nu}$ coupling. The anomalous magnetic moment coupling is written as k_B , and we have assumed for simplicity that the Dirac and Pauli form factors are constant off the Baryon mass-shell^[10]. The following functions appear on performing the integrals:

$$G_B(m_M^2) = \frac{1}{2} \ln\left(\frac{1+v}{s}\right) + \frac{s-v}{2} f(s,v), \quad (2.3.6)$$

$$s = m_M^2/m_n^2, \quad v = (m_B^2 - m_n^2)/m_n^2, \quad (2.3.7)$$

$$f(s,v) = \frac{1}{\sqrt{s - \frac{(s-v)^2}{4}}} \tan^{-1} \left| \frac{\sqrt{s - \frac{(s-v)^2}{4}}}{\frac{s+v}{2}} \right|, \quad (2.3.8)$$

and

$$F_B(m_M^2) = \frac{3}{2} - (s-v) + \frac{3s-v-(s-v)^2}{2} \ln\left(\frac{1+v}{s}\right) + \frac{(s-v)[5s-v-(s-v)^2]-4s}{2} f(s,v). \quad (2.3.9)$$

When two weak vertices are necessary to have a non vanishing EDM, one must evaluate the diagrams of figure 2.2, which give the neutron EDM $D_n^{(II)}$. Assume that the MM' weak interaction lagrangian is

$$L' = g e^{i\theta} D^\mu M'^+ D_\mu M + \text{H.C} \quad (2.3.10)$$

where $D_\mu = \partial_\mu + iQA_\mu$ with Q being the electric charge of the meson. We obtain

$$D_n^{(II)} = D_{n,1}^{(II)} + D_{n,2}^{(II)}. \quad (2.3.11)$$

Now $D_{n,1}^{(II)}$ is given by

$$D_{n,1}^{(II)} = \frac{eghf}{4\pi^2 m_n} m_B \sin(\theta-\phi) R_B \quad (2.3.12)$$

where

$$R_B = \frac{m_{M'}^2 G_B(m_{M'}^2) - m_M^2 G_B(m_M^2)}{m_{M'}^2 - m_M^2} \quad (2.3.13)$$

The anomalous magnetic moment contribution is given by

$$D_{n,2}^{(II)} = \frac{eghf}{4\pi^2} \frac{k_B}{2m_B} \sin(\theta-\phi) H_B \quad (2.3.14)$$

where H_B is defined similarly to R_B , but with the function $F_B(\mu^2)$ replacing the function $G_B(\mu^2)$.

In these relations the CP violating transition amplitude of MM' is $h \sin \theta$, so the EDM contribution $D_n^{(II)}$ is second order weak, and thus very small. It makes a significant contribution only when the first order weak contributions to the neutron EDM vanish. We will concentrate on the three effects discussed in detail above, viz:

- (i) The Valence Quark Contribution
- (ii) Quark colour dipole moment contributions to the EDM
- (iii) Hadronic loop contributions to the neutron EDM.

We will study the neutron EDM determined by these contributions in the standard KM model, the two Higgs doublet model, the Weinberg spontaneous CP violation model, the left right

symmetric model, and supersymmetric model. While it does not fit precisely into our classification, we also for completeness briefly discuss the neutron EDM due to strong CP violation.

3) THE NEUTRON EDM IN THE STANDARD MODEL

In the standard $SU(2)_L \times U(1)_Y$ model with one Higgs doublet, CP violation is due to the phase in the quark mixing matrix V_{KM} of the charged current, the Kobayashi-Maskawa model[14]. There must be at least three generations of quarks in order to have non zero CP violating phase. The charged current interaction lagrangian is

$$L_W = \frac{g}{2} \bar{U}_L \gamma^\mu V_{KM} D_L W_\mu + \text{H.C.} \quad (3.1)$$

where W is the w -gauge boson, and $U = (u, c, t, \dots)$ and $D = (d, s, b, \dots)$ are the charge $2/3$ and charge $-1/3$ quark fields. In the three generation case, V_{KM} can be parametrized as

$$V_{KM} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \quad (3.2)$$

where $c_i = \cos\theta_i$ and $s_i = \sin\theta_i$. The mixing angles are determined by the analysis of many experiments[15]. The range of the CP violating phase δ is determined from the observed CP violation in $K^0 - \bar{K}^0$ system and one finds[16]

$$2 \times 10^{-4} < s_2 s_3 s_\delta < 2 \times 10^{-3} \quad (3.3)$$

when one varies m_t from 40 to 180 GeV with the maximum being reached for small values of the top quark mass $m_t \cong 40$ GeV.

Already there have been many discussions in the literature of the neutron EDM in the standard 3 generation model^[5], and the values quoted range from 10^{-30} ecm to 10^{-34} ecm^[8-10,17-23]. Most of these calculations are characterized by the consideration of only one of the mechanisms considered above. Clearly all of these effects should be considered, as long as one is convinced that there is no double counting in doing so. In the case of the standard model one is on safe ground in adding effects, as they give contributions of different orders of magnitude. The quark level effects (i) and (ii) above give contributions of order 10^{-33} to 10^{-34} ecm^[5]. A number of early calculations of hadronic loop effects^[5,21-23] (type (iii)) gave contributions of order 10^{-30} ecm. However as we discussed in Ref^[10], some of these calculations are not invariant under changes in phase of the strange quark wavefunction. Our rephasing invariant calculation of the type (iii) effects shows that they dominate the quark level effects, but give results in the range 10^{-31} to 10^{-33} ecm. From section 2.3 we see that the essential ingredient of the calculation is to obtain the parameters h, f, δ and ϕ for diagrams in Fig.(2.2) with the appropriate internal hadrons. We include all SU(3) octet pseudoscalars M and baryons B since in the SU(3) limit all these contributions are logarithmically divergent and we also use SU(3) relations for various couplings. The possible internal hadrons can be $(M, M', B) = (\pi^-, K^-, p), (K^+, \pi^+, \Sigma^-)$ for Fig. (2.2. a, b, c and d); $(M, M', B, B') = (\pi^-, K^-, p, p), (K^+, \pi^+, \Sigma^-, \Sigma^-), (\pi^0, K^0, n, n), (K^0, \pi^0, \Lambda^0, \Lambda^0), (K^0, \pi^0, \Sigma^0, \Sigma^0), (K^0, \eta, \Sigma^0, \Sigma^0), (K^0, \eta, \Lambda^0, \Lambda^0), (\eta, K^0, n, n), (K^0, \pi^0, \Sigma^0, \Lambda^0), (K^0, \pi^0, \Lambda^0, \Sigma^0), (K^0, \eta, \Sigma^0, \Lambda^0)$ and $(K^0, \eta, \Lambda^0, \Sigma^0)$ for Fig. (2.2. d). For the strong vertices, we have^[24]

$$L_s = -\sqrt{2}g_{\pi NN}[\text{Tr}(\bar{B}i\gamma_5 MB) + (2\alpha-1)\text{Tr}(\bar{B}i\gamma_5 BM)] \quad (3.4)$$

where $\alpha = 0.64$ and $g_{\pi NN}^2/4\pi = 14$ [24].

Neglecting small terms, the relevant weak parity-violating $\bar{B}B$ interaction lagrangian can be written as[25]

$$L_w = \sqrt{2}\{f_3[e^{i\phi_3}\text{Tr}(BM\lambda_+\bar{B}) + e^{-i\phi_3}\text{Tr}(BM\lambda_-\bar{B})] \\ + f_4[e^{i\phi_4}\text{Tr}(BM\lambda_+B) + e^{-i\phi_4}\text{Tr}(BM\lambda_-B)]\}, \quad (3.5)$$

For the weak $M'M$ interaction, we use

$$L' = h e^{i\theta}\text{Tr}(\lambda_+ D^\mu M D_\mu M) + \text{H.C.}, \quad (3.6)$$

where

$$\lambda_+ = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_- = \lambda_+^T \quad (3.7)$$

$$D_\mu = \partial_\mu + iQA_\mu$$

We relate h and θ to $\langle \pi^+\pi^- | H_W | K \rangle$ using PCAC:

$$hm_K^2 = i\sqrt{2} f_\pi \langle \pi^+\pi^- | H_W | K_S \rangle \quad (3.8)$$

$$i\theta = i \frac{\text{Im}A_0}{\text{Re}A_0} + \epsilon' = i \frac{\text{Im}A_0}{\text{Re}A_0}$$

Using the experimental values for $\langle \pi^+\pi^- | H_W | K_S \rangle$ and the calculation of $\frac{\text{Im}A_0}{\text{Re}A_0}$ from Ref.[26] we obtain

$$h = 1.49 \times 10^{-7}. \quad (3.9)$$

$$\theta = 0.32 s_2 s_3 s_8.$$

The parameters f_3 and f_4 are related to the hyperon decay amplitudes $A(\Sigma^+ \rightarrow p\pi^0)$ and $A(\Lambda^0 \rightarrow n\pi^-)$: [15]

$$\sqrt{2}f_3 = -\sqrt{2} A(\Sigma^+ \rightarrow p\pi^0), \quad (3.10)$$

$$\sqrt{2}f_4 = \frac{1}{\sqrt{2}} A(\Sigma^+ \rightarrow p\pi^0) - \sqrt{\frac{3}{2}} A(\Lambda^0 \rightarrow n\pi^-).$$

These relations give

$$f_3 = -3.2 \times 10^{-7}, \quad \text{and} \quad f_4 = 1.18 \times 10^{-7}. \quad (3.11)$$

The phases ϕ_3 and ϕ_4 are similarly obtained from the calculated CP violating amplitudes for hyperon decays [27]:

$$\phi_3 = 0.029 s_2 s_3 s_8, \quad \text{and} \quad \phi_4 = 0.061 s_2 s_3 s_8. \quad (3.12)$$

Using these values of the parameters, we find the contributions to D_n are

$$D_{n,1}^{(II)} = 4.95 \times 10^{-29} s_2 s_3 s_8 \text{ ecm} \quad (\text{Dirac coupling to baryon}),$$

$$D_{n,2}^{(II)} = 1.95 \times 10^{-29} s_2 s_3 s_8 \text{ ecm} \quad (\text{Pauli coupling to charged baryon}),$$

$$D_{n,2}^{(III)} = -2.7 \times 10^{-29} s_2 s_3 s_8 \text{ ecm} \quad (\text{Pauli coupling to neutral baryon}).$$

In obtaining the results for Pauli coupling contribution to D_n , we have used the experimental data on the anomalous magnetic moments of $k_n = -1.91$, $k_p = 1.79$, $k_{\Sigma^-} = -0.14$, $k_{\Lambda^0} = -0.613$, the transition magnetic moment of $\Sigma^0 \Lambda^0$, $k_{\Sigma\Lambda} = -1.82$ determined from the experimental data from $\Sigma^0 \rightarrow \Lambda^0 \gamma$, and for k_{Σ^0} we use the mass corrected SU(3) relation $k_{\Sigma^0} = -(m_\Lambda/m_\Sigma)k_\Lambda = 0.57$.

There is another class of diagrams as shown in Fig.(3.1). which also contribute to the neutron EDM at the same level. With very little modification, we can use the formulae given in section 2. This time one of the weak vertices correspond to the radiative transition $BB'\gamma$ which has Parity conserving and violating amplitudes of approximately the same order of magnitude. For the other weak vertex, the Parity conserving amplitude dominates over the parity violating amplitude^[12], and so the P-wave contribution to the neutron EDM will be dominant in this case. The P-wave amplitudes with CP violating phases relevant to our calculation have been evaluated in Ref.[20]. The only new calculation needed is that of the Parity violating $\Delta S=1$ radiative decay amplitudes of Λ^0 , Σ^0 and Σ^- ,

$$L = D_{\Lambda n} \bar{\Lambda} i \sigma_{\mu\nu} Q^\nu \gamma_5 n e^\mu + D_{\Sigma n} \bar{\Sigma} i \sigma_{\mu\nu} Q^\nu \gamma_5 n e^\mu$$

(3.13)

$$+ D_{\Sigma^- p} \bar{\Sigma} i \sigma_{\mu\nu} Q^\nu \gamma_5 p e^\mu + \text{H.C.}$$

Using the experimental decay amplitude and asymmetry for $\Sigma^+ \rightarrow p\gamma$, we have $\text{Re}D_{\Sigma p} = 4.7 \times 10^{-8} \frac{e}{2m_n}$. For $\Lambda^0 n\gamma$ we use the experimental decay amplitude and theoretical estimates for asymmetry parameter^[21] $\alpha \cong 0.8$, then we find $\text{Re}D_{\Lambda n} = 1.01 \times 10^{-8} \frac{e}{2m_n}$ and for $\text{Re}D_{\Sigma n}$ we use SU(3) relation $\text{Re}D_{\Sigma n} = \sqrt{3}\text{Re}D_{\Lambda n}$. To estimate the CP violating part of D amplitude, $\text{Im}D$, we use the electromagnetic penguin diagram calculation of $\Delta S=1$ radiative decay amplitudes^[22]

$$L = \text{Im}K \bar{d}i\sigma_{\mu\nu}q^\nu \gamma_5 s \epsilon^\mu + \text{H.C.}, \quad (3.14)$$

where

$$\text{Im}K \cong \frac{eG_F}{\sqrt{2}(4\pi)^2} m_s \left(\frac{m_t}{m_W}\right)^2 \frac{27}{6} s_1 s_2 s_3 \quad (3.15)$$

and using the quark model estimate, we obtain

$$\text{Im}D_{\Sigma n} : \text{Im}D_{\Lambda n} : \text{Im}D_{\Sigma p} = 1 : 3\sqrt{3} : \sqrt{2}. \quad (3.16)$$

using these values and summing over all diagrams in Fig.(3.1), we have

$$D_n \cong 2.4 \times 10^{-29} s_2 s_3 s_3 \text{ ecm}, \quad (3.17)$$

We mentioned above that the calculation of $D_n^{(II)}$ depends on the assumption that the electromagnetic formfactors are constant off the baryon mass shell. We have argued in Ref[10] that the variation

of the magnetic formfactor is such that one should expect our value of $D_n^{(II)}$ to be an upper limit. We therefore give a range of values of D_n in the standard model to accommodate this variation of $D_n^{(II)}$.

Our final result is

$$8 \times 10^{-29} s_2 s_3 s_\delta \text{ ecm} \geq |D_n| \geq 6.6 \times 10^{-29} s_2 s_3 s_\delta \text{ ecm} \quad (3.18)$$

The final result depends on the value of $s_2 s_3 s_\delta$ which is function of top quark mass m_t . Using the range of $s_2 s_3 s_\delta$ in eq.(3.3), we have

$$1.6 \times 10^{-31} \text{ ecm} \geq |D_n| \geq 1.4 \times 10^{-33} \text{ ecm} \quad (3.19)$$

for m_t between 40 to 180 GeV with the maximum value reached by small m_t .

We now comment on the calculations of Gavela et. al. in Refs.[22,23] where contributions from Fig.(3.2) are considered. In each of the papers, only one of the weak phases is discussed and the results is therefore not phase invariant. However, if one includes both weak phases by using the results of both calculations, the result is phase invariant. Using harmonic oscillator potential model calculation for the weak $\Delta s=1$ baryon transition amplitude $a_{BB'}$ and penguin diagram calculation for CP violating phases, in Ref.[23] it is found that the phases from $a_{BB'}$ dominate the phases from the $\Delta S=1$ radiative decay amplitude by several orders of magnitude and the result for D_n is of order 10^{-30} ecm. We believe that the phases of $a_{BB'}$ in Ref.[23] are over estimated^[9]. We have carried a calculation for the phases in $a_{BB'}$.

for the SU(3) octet baryons using the calculated Hyperon decay amplitude and pole model calculation, we find that the phases in $a_{BB'}$ are considerably smaller than the calculation in Ref.[23]. Our calculation gives

$$D_n \equiv 10^{-29} s_2 s_3 s_8 \text{ ecm.} \quad (3.20)$$

This is in agreement with Ref.[9,21] and is the same order of magnitude as the result we obtained earlier. Since one can argue that the contributions of fig.(3.2) with N^* as the intermediate state are dual to those of figs.(2.2) and (3.1) (see ref.[11]), we also believe that the estimate of the N^* contribution in Ref.[23] is over estimated. We conclude that in the standard model the neutron EDM is approximately given by eq.(3.19).

The neutron EDM in the standard model with three generations is in the range of 1.6×10^{-31} to 1.4×10^{-33} ecm which is several orders of magnitude smaller than the present experimental upper bounds. With four generations, the calculated neutron EDM may be larger in magnitude may be improved. In this case there are six angles and three CP violating phases in the KM matrix. The CP violation in the $K^0 - \bar{K}^0$ system does not determine all the CP violating phases. Using allowed values from experiments, Barroso et al.[29] calculated the neutron EDM with four generations for diagrams of Fig.(3.2) type and found that a factor of 20 enhancement with respect to the three generation model is possible. The same enhancement factor is also expected for the other diagrams. The experimental measurement of the neutron EDM at the level of 10^{-29} to 10^{-30} could be an indication of the presence of the fourth generation.

4) The Neutron EDM in the Two Higgs Doublet Model

CP violation in the two Higgs doublet model^[30] can arise from different origins, CP violation from the phases in the KM matrix, or from spontaneous symmetry breaking. However, it can be shown that in the two Higgs doublet model, it is not possible to have spontaneous CP violation if one requires neutral flavor conservation at the tree level^[31]. We will therefore consider the case where CP violation arises only from the the phases in V_{KM} . Consequently the calculation of the neutron EDM is similar to that in the standard model, with the new feature in the two Higgs doublet model that the charged physical Higgs gives additional contributions.

In this model there are two Higgs doublets, $\phi_1^T = (\phi_1^+ \phi_1^0)$ and $\phi_2^T = (\phi_2^+ \phi_2^0)$. After the spontaneous breakdown of the $SU(2) \times U(1)$ symmetry at the m_W scale, there remains one charged physical Higgs H^+ which couples to the quarks through the interaction^[30]

$$L_H = \frac{g_2}{2\sqrt{2}m_W} H^+ [\alpha U M_U V_{KM} (1 - \gamma_5) D + \beta U V_{KM} M_D (1 + \gamma_5) D] + H.C. \quad (4.1)$$

The parameters α and β take one of two possible sets of values:

- (i) if ϕ_1^0 couples to the up quark and ϕ_2^0 to the down quark,

$$\alpha = v_1/v_2 \text{ and } \beta = 1/\alpha = v_2/v_1,$$

(ii) if only ϕ_1^0 couples to the up and down quarks,

$$\alpha = -\beta = v_1/v_2.$$

(v_1 and v_2 are the vacuum expectation values of the neutral Higgs fields). This Lagrangian will induce effective $q_i q_j \gamma$ and $q_i q_j g$ couplings at the one loop level by the diagrams of Figure 4.1. It is clear that at one loop level, no quark EDM or CMD are generated. However the new interaction of the physical charged Higgs with quarks will modify the standard model calculations. We now proceed to calculate the additional contributions to the neutron EDM. Evaluating diagrams in Fig. (4.1.b), we obtain the quark-gluon effective lagrangian

$$L_{q_i q_j G} = -\frac{G_F}{\sqrt{2}} \frac{\theta_s}{16\pi^2} \{A + B\} \quad (4.2)$$

where

$$A = \bar{q}_i \sigma_{\mu\nu} \epsilon^{\mu\nu\alpha\beta} \frac{\lambda^a}{2} [(m_i(1-\gamma_5) + m_j(1+\gamma_5))] q_j \sum_k V_{ik}^* V_{jk} g(x_k),$$

$$B = \bar{q}_i (q^2 \gamma^\mu - q^\mu \gamma) (1 - \gamma_5) \frac{\lambda^a}{2} q_j \epsilon_{\mu\nu} \sum_k V_{ik}^* V_{jk} f(x_k).$$

Here

$$x_k = (m_k / m_H)^2,$$

(4.3)

$$g(x) = \alpha^2 H_1(x) + \alpha\beta H_2(x),$$

with

$$H_1(x) = -\frac{x}{6(1+x)^3} \left(2+5x-x^2 + \frac{6x \ln x}{1-x} \right), \quad (4.4)$$

$$H_2(x) = \frac{x}{(1-x)^3} (3-4x+x^2+2 \ln x).$$

$$f(x) = -\frac{\alpha^2 x}{16(1-x)^3} \left(16-29x+7x^2 - \frac{6(3x-2) \ln x}{1-x} \right).$$

This effective interaction will contribute to the CP violating $\Delta S = 1$ amplitude, and thus the CP violating phases ϕ_3 , ϕ_4 and θ will be different to those of the standard KM model. We denote the standard phases as $\phi_{3,4}^{St}$, and the additional phases in the 2 Higgs doublet model as $\phi_{3,4}^{H1}, \phi_{3,4}^{H2}$, etc., where the superscripts 1 and 2 refer to the contributions of the first and second terms in $L_{q_i q_j G}$, respectively. The contribution of the first term can be obtained by using the matrix elements $\langle \pi N | m_S \bar{q}^a \sigma_{\mu\nu} (1+\gamma_5) d G_a^{\mu\nu} + m_d \bar{s} \lambda^a \sigma_{\mu\nu} (1+\gamma_5) d G_a^{\mu\nu} | P \rangle$ calculated in Ref.[32] In this way we obtain

$$\begin{aligned} \phi_3^{H1} &= \frac{\text{Im}A(\Sigma_0^+) |_1}{\text{Re}A(\Sigma_0^+)} \\ &= 1.1 \times 10^{-3} g_s (g(x_1) - g(x_c)) s_2 s_3 s_6 \\ \phi_4^{H1} &= \frac{\text{Im}A(\Sigma_0^+) + \sqrt{3} \text{Im}A(\Lambda_0^0) |_1}{\text{Re}A(\Sigma_0^+) + \text{Re}A(\Lambda_0^0)} \end{aligned} \quad (4.5)$$

$$= 1.3 \times 10^{-3} g_s (g(x_i) - g(x_c)) s_2 s_3 s_8$$

$$\theta^{H1} = \frac{\text{Im} \langle \pi^0 \pi^0 | K^0 \rangle_1}{\text{Re} \langle \pi^0 \pi^0 | K^0 \rangle}$$

$$= 7.7 \times 10^{-3} g_s (g(x_i) - g(x_c)) s_2 s_3 s_8.$$

We compute the contribution of the second term in $L_{q_i q_j G}$ by using it to generate a four-quark interaction by gluon exchange with the usual QCD interaction at the other vertex. This interaction is

$$H_G^{\text{eff}} = -\frac{G_F}{\sqrt{2}} \frac{g_s^2}{16\pi^2} \bar{q}_i \gamma^\mu (1 - \gamma_5) \frac{\lambda^a}{2} q_j \bar{q} \gamma^\mu \frac{\lambda^a}{2} q \sum_k V_{ik}^* V_{jk} f(x_k). \quad (4.6)$$

The necessary matrix elements of the four quark operator in this equation are also evaluated in Ref.[32]. We thus obtain

$$\phi_3^{H2} = 2.21 \times 10^{-4} g_s^2 (f(x_i) - f(x_c)) s_2 s_3 s_8$$

(4.7)

$$\phi_4^{H2} = 1.86 \times 10^{-3} g_s^2 (f(x_i) - f(x_c)) s_2 s_3 s_8$$

$$\theta^{H2} = -0.015 g_s^2 (f(x_i) - f(x_c)) s_2 s_3 s_8$$

The parameter α and β entering into $g(x)$ and $f(x)$ may be bounded using experimental information. The best bound is from experimental bound on^[33] $B \rightarrow K^* \gamma$. Using the bound on α from this, we have

$$g(x_i)_{\max} = -0.4 \sim -1.0 \quad \text{for model (i)}$$

$$g(x_i)_{\max} = 2.0 \sim 2.8 \quad \text{for model (ii),} \quad (4.8)$$

$$f(x_i)_{\max} = 1.0 \sim 1.2 \quad \text{for both models,}$$

for values of $(m_\mu/m_H)^2$ in the range $0.01 \leq (m_\mu/m_H)^2 \leq 1$.

For these values of the parameters we find that the standard KM contributions completely dominates the new contributions from the charged Higgs H^+ , the new contribution being only at the few percent level relative to the standard contribution. For practical purposes we may take the neutron EDM in the two Higgs doublet model to be the same as that of the standard KM model discussed in §3, viz

$$1.6 \times 10^{-31} \text{ ecm} \geq |D_n| \geq 1.4 \times 10^{-33} \text{ ecm} \quad (4.9)$$

5) THE NEUTRON EDM IN THE WEINBERG HIGGS MODEL OF SPONTANEOUS CP VIOLATION

In this section we will discuss the neutron EDM in models with spontaneous CP violation. The minimal model of this type is the Weinberg model of three Higgs doublets^[34]. In this model the KM matrix is real and CP violation is due to complex vacuum expectation values of the Higgs fields. The relevant Lagrangian is

$$\begin{aligned}
L_Y = & 2^{3/4} G_F^{1/2} \sum_i U_{iK} [V_{KM} M_u \frac{1+\gamma_5}{2} X_i H_i^\dagger \\
& + M_d V_{KM} \frac{1+\gamma_5}{2} Y_i H_i^\dagger] D + H.D
\end{aligned} \tag{5.1}$$

Here X_i and Y_i are related to the vacuum expectation values of the Higgs fields, $\text{Im}(X_1 Y_1) = -\text{Im}(X_2 Y_2)$ and $H_{1,2}^\dagger$ are the physical charged Higgs fields.

It is evident that this Lagrangian will induce a quark EDM at the one loop level through the mechanism of diagram similar to figure 4.1. The effective CP violating Lagrangian for the qqg and $qq\gamma$ vertices are [31,35]

$$\begin{aligned}
L_{qqg} = & - \frac{G_F g_s}{16\sqrt{2}\pi^2} \sum_{k,l} [\text{Im}(X_l Y_l) V_{ik} V_{jk} \frac{df(x_{k,l})}{dQ} \\
& \bar{q}_i m_j \sigma_{\mu\nu} \epsilon_a^\mu q^{\nu} (1+\gamma_5) \frac{\lambda^a}{2} q_j] \\
& + \text{H.C.}
\end{aligned} \tag{5.2}$$

$$\begin{aligned}
L_{qq\gamma} = & - \frac{G_F g_s}{16\sqrt{2}\pi^2} \sum_{k,l} [\text{Im}(X_l Y_l) V_{ik} V_{jk} f(x_{k,l}) \bar{q}_i m_j \sigma_{\mu\nu} \epsilon_a^\mu q^{\nu} (1+\gamma_5) \frac{\lambda^a}{2} q_j] \\
& + \text{H.C.}
\end{aligned}$$

where $l = 1, 2$ (the physical charged Higgs) and k sums over the internal quarks, $x_{k,l} = (m_k/m_l)^2$ and

$$f(x) = \frac{x}{(1-x)^3} \{ Q(3-4x+x^2+2 \ln x) - (1-x^2+2x \ln x) \} \tag{5.3}$$

At the one loop level it is clear that there will be quark EDMs, quark CDMs, and also CP violating $K\Sigma n$ and πn transitions. It is therefore important to examine all the possible contributions to the neutron electric dipole moment carefully.

(i) The quark EDM contribution

From the effective Lagrangian $L_{qq\gamma}$ we immediately obtain

$$D_q = \frac{eG_F}{8\sqrt{2}\pi^2} \sum_{l,k} m_q \text{Im}(X_l Y_l^*) V_{qk}^2 f(x_{k,l}). \quad (5.4)$$

Employing the valence quark model as outlined in §2, we obtain the contribution $D_n^{(V)}$ to the neutron EDM from this source.

$$\begin{aligned} D_n^{(V)} &= \frac{1}{3} (4D_d - D_u) \\ &= \frac{G_F e}{24\sqrt{2}\pi^2} (4m_d \text{Im}(X_1 Y_1^*) \sum_i V_{di}^2 [f(x_{i,1}) - f(x_{i,2})] \\ &\quad - m_u \text{Im}(X_1 Y_1^*) \sum_i V_{ui}^2 [f(x_{i,1}) - f(x_{i,2})]) \end{aligned} \quad (5.5)$$

(ii) The quark CDM contribution

The quark color dipole moment contribution can be read off from the effective CP non-conserving qqg coupling in exactly the same way, and then use it in eq. (2.2.10). In this way we obtain $D_n^{(C)}$ The quark CDM contribution to the neutron EDM

$$D_n^{(C)} = \frac{1}{3} e(4f_d + 2f_u) \quad (5.6)$$

where

$$f_q = \frac{G_F}{8\sqrt{2}\pi^2} \sum_i m_q \text{Im}(X_1 Y_i) V_{qi}^2 \left(\frac{df(x_{i,1})}{dQ} - \frac{df(x_{i,2})}{dQ} \right) \quad (5.7)$$

(iii) Since CP is violated at the one loop level for the flavor conserving interaction, it is possible to generate an effective CP violating meson-nucleon vertex at the one loop level. We now estimate the CP violating $K\Sigma n$ coupling $f_{K\Sigma n}$. The effective lagrangian responsible for $f_{K\Sigma n}$ can be obtained from eq.(5.2), we have

$$\sqrt{2}f_{K\Sigma n} = \langle K\Sigma | L_{sdg} | n \rangle \quad (5.8)$$

with

$$L_{sdg} = -\bar{s} \frac{g_s}{2} \sigma_{\mu\nu} \gamma_5 \frac{\lambda^a}{2} s G_{\mu\nu}^a \quad (5.9)$$

It is also possible to generate a CP violating πnp coupling from an effective Hamiltonian similar to eq.(5.2). However, because L_{sdg} is proportional to the quark mass involved, the πnp coupling is suppressed by a factor of $m_{u,d}/m_s$ and is small. We will therefore consider only the contribution from $K\Sigma n$ coupling. The amplitude $f_{K\Sigma n}$ has been evaluated in ref.[12] by relating the matrix element to the baryon mass differences and also by other methods[36]. Here we estimate $f_{K\Sigma n}$ by using current algebra and PCAC to connect the matrix element involved in the $K\Sigma n$ vertex to the one with $\Sigma\pi n$, we have

$$\langle K\Sigma | \bar{s} \sigma_{\mu\nu} \gamma_5 \lambda^a s | n \rangle = \frac{f_K}{f_\pi} \langle \pi\Sigma | \bar{s} \sigma_{\mu\nu} \gamma_5 \lambda^a d | n \rangle \quad (5.10)$$

where f_π and f_K are the pion and Kaon decay constants respectively and $f_K/f_\pi = 1.27$. The matrix element on the right hand side has been evaluated in the bag model yielding

$$\langle \pi | \bar{s} m_s \frac{g_s}{2} \sigma_{\mu\nu} (1 + \gamma_5) \frac{\lambda^a}{2} d G_{\mu\nu}^a | n \rangle \cong -0.24 \text{ (GeV)}. \quad (5.11)$$

To get numerical results, we have to know $\text{Im}(X_1 Y_1^*)$. For this we follow Ref[37] in assuming that ϵ is mainly due to the long-distance dynamics. Then

$$2m_K \sqrt{2} |\epsilon| \delta m_{K-S} \cong 2 \times 10^{-7} \langle \pi | L_{sdg} | K \rangle \quad (5.12)$$

and translating this in terms of $\text{Im}(X_1 Y_1^*)$ and using $\langle \pi^0 | \bar{s} \sigma_{\mu\nu} \lambda^a d G_{\mu\nu}^a | K^0 \rangle = 0.4 \text{ GeV}^3$ as calculated in the bag model, we have

$$\text{Im}(X_1 Y_1^*) = - \frac{2.9 \times 10^{-10} (1/\text{GeV}) 32 \sqrt{2} \pi^2}{G_F g_s m_s \sum_i V_{si} V_{di} (df(x_{i,1})/dQ - df(x_{i,2})/dQ)} \quad (5.13)$$

Inserting all this information in the expressions for $D_n^{(V)}$, $D_n^{(C)}$ and $D_n^{(I)}$, we obtain

$$D_n^{(V)} \cong -1.9 \times 10^{-25} \text{ ecm} \quad (5.14)$$

$$D_n^{(C)} \cong -0.32 \times 10^{-25} \text{ ecm}$$

$$D_{n,1}^{(I)} \cong -1.1 \times 10^{-24} \text{ ecm}$$

$$D_{n,2}^{(1)} \equiv -0.44 \times 10^{-25} \text{ ecm}$$

It is obvious that the largest contribution is from the hadron loop and the predicted value for neutron EDM is larger than the experimental upper bound. However, the hadron loop contribution can be easily wrong by a factor of two or three due to uncertainties in the calculation of the matrix element. Furthermore, it has been pointed out[38,39] that the exchange of neutral Higgs in this model violates CP due to the mixing between the real and imaginary parts of the neutral Higgs fields. The quark EDM from Fig.(5.1) has been evaluated in Ref.[38] and found to be

$$D_q = \frac{Q_q e G_F}{\sqrt{2} \pi^2} m_q \sum_i X_i Y_i \frac{m_q^2}{m_{H_i}^2} \ln \frac{m_q^2}{m_{H_i}^2} \quad (5.15)$$

where Q_q is the electric charge of the q-quark, m_{H_i} the neutral Higgs masses, and X_i and Y_i are the mixings of the neutral Higgs particles. If one assumes that all the neutral Higgs have approximately the same masses and the average mixing $\langle X'Y' \rangle$ is of the same order as $\text{Im}(X_1 Y_1)$, then this contribution to D_n is about 10^{-24} ecm. In Ref.[39], a different treatment of the neutral Higgs contribution to neutron EDM is carried out by evaluating the effective coupling for the scalar and pseudoscalar Higgs with nucleon and then using eq.(2.3.5). It is found that

$$D_n = C k_n g_\sigma g_H \langle X'Y' \rangle m_N^2 \quad (5.16)$$

where $C \approx 3.34 \times 10^{-3}$ from the loop integral, $g_\sigma \approx (8/29)m_n(2G_F)^{1/2}$ is the scalar Higgs-nucleon coupling, and $g_H \approx 2.5m_n(2G_F)^{1/2}$ is the

pseudoscalar Higgs-nucleon coupling. If one assumes again that the mixing $\langle X'Y' \rangle$ is approximately the same as $\text{Im}(X_1 Y_1^*)$, then D_n is about two to three orders of magnitude larger than the experimental bound. Since the exchange of the neutral Higgs particles conserves flavor, they do not contribute to the only observed CP violation in $K^0-\bar{K}^0$ system, and thus no constraint can be put on the parameters. While it is possible that the assumption $\langle X'Y' \rangle \sim \text{Im}(X_1 Y_1^*)$ over estimates the neutral Higgs contribution and some cancelation may occur between the contributions from the neutral and the charged Higgses, it is unlikely that this will reduce the neutron EDM by an order of magnitude. If such happy cancellations do not occur, the Weinberg model for CP violation may be in trouble with the present experimental bound. If on the other hand the neutron EDM turns out to be of order $\sim 10^{-25}$ ecm, the Weinberg model, with cancellations and an overestimate in eq.(5.14) could still be viable.

6) THE NEUTRON EDM IN THE LEFT-RIGHT SYMMETRIC MODEL

In this section we discuss the neutron EDM in the left-right symmetric model[40]. Left-right symmetric models are based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ with quarks and leptons being assigned to irreducible representations of the gauge group as below

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L : (2, 1, 1/3), \quad Q_R = \begin{pmatrix} u \\ d \end{pmatrix}_R : (1, 2, 1/3)$$

$$L_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L : (2, 1, -1), \quad L_R = \begin{pmatrix} \nu \\ e \end{pmatrix}_R : (1, 2, -1)$$
(6.1)

where the first two numbers in the parentheses are the transformation properties under $SU(2)_L$ and $SU(2)_R$ respectively and the last number is the $U(1)_{B-L}$ charge.

The gauge boson-quark interaction lagrangian is

$$L_{W-F} = \frac{i}{2g_L} \bar{Q}_L \gamma_\mu \vec{\tau} Q_L \vec{W}_L^\mu + \frac{i}{2g_R} \bar{Q}_R \gamma_\mu \vec{\tau} Q_R \vec{W}_R^\mu + \frac{i}{2g} \bar{Q}_L \gamma_\mu \vec{\tau} Q_R B^\mu \quad (6.2)$$

where $\vec{W}_{L,R}^\mu$ and B^μ are the gauge bosons corresponding to the groups $SU(2)_{L,R}$ and $U(1)_{B-L}$ respectively; $g_{L,R}$, and g' are the corresponding gauge coupling constants. To spontaneously break the gauge symmetry down to $SU(2)_L \times U(1)_L$ and then $U(1)_{em}$ and to generate fermion masses, Higgs multiplets have to be introduced. The most economical Higgs representations are^[40]: the bi-doublet $\phi = (2,2,0)$ which is necessary for breaking of $SU(2)_L$ at the m_{W_L} scale and to give masses to quarks and leptons with its VEV $\langle \phi \rangle = \begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix} e^{i\alpha}$; $\Delta_L = (3,1,2)$ and $\Delta_R = (1,3,2)$ which are necessary to break the $SU(2)_R$ at a higher scale $\langle \Delta_{R,L} \rangle = \begin{pmatrix} 0 & 0 \\ v_{R,L} & 0 \end{pmatrix}$. This choice of Higgs representation also allows interesting neutrino masses. Other Higgs representations are possible^[40]. In general, \vec{W}_L^μ and \vec{W}_R^μ will mix. For example, in the model mentioned above, the mixing angle ζ is approximately given by $\zeta \equiv \arctan(kk'/v_R^2)$. The mass eigenstates $W_{1,2}$ are

$$W_1 = W_L \cos \zeta + W_R \sin \zeta \quad (6.3)$$

$$W_2 = -W_L \sin \zeta + W_R \cos \zeta$$

In the left-right symmetric models, there are right handed charged currents carrying new phases and hence there are new sources for CP violation[41]. In the mass eigenstate basis of gauge bosons and quarks, we have

$$\begin{aligned}
 L_{Wq} = & \frac{1}{\sqrt{2}} g_L \bar{U}_L \gamma_\mu V_L D_L \cos\zeta + \frac{1}{\sqrt{2}} g_R \bar{U}_R \gamma_\mu V_R D_R \sin\zeta \} W_1^\mu \\
 & + \{ \frac{1}{\sqrt{2}} g_L \bar{U}_L \gamma_\mu V_L D_L \sin\zeta + \frac{1}{\sqrt{2}} g_R \bar{U}_R \gamma_\mu V_R D_R \cos\zeta \} W_2^\mu + H.C
 \end{aligned}
 \tag{6.4}$$

where V_L is the KM matrix and V_R is an analogous KM matrix involving the right handed current. If we parametrize V_L in the usual KM way, then for n -generation of quarks, there are $(n-1)(n-2)/2$ CP violating phases. However, V_R can have different phases depending on the models. In manifest left-right symmetric models, with $V_L = V_R$, the phases in V_L and V_R are equal. In pseudo-manifest left-right symmetric models, there are $2n-1$ additional phases in V_R . If there is no relations between V_L and V_R , there are, in general, $n(n+1)/2$ phases in V_R . It is no longer necessary to have three generations of quarks in order to have CP violation. In the two generation case, V_L and V_R can be parametrized as

$$\begin{aligned}
 V_L = & \begin{pmatrix} \cos\theta_L & \sin\theta_L \\ -\sin\theta_L & \cos\theta_L \end{pmatrix} \\
 V_R = & e^{i\eta} \begin{pmatrix} e^{-i\delta_2} \cos\theta_R & e^{-i\delta_1} \sin\theta_R \\ -e^{i\delta_1} \sin\theta_R & e^{i\delta_2} \cos\theta_R \end{pmatrix}
 \end{aligned}
 \tag{6.5}$$

Due to the mixing of W_L and W_R , the quark EDM as well as the quark CDM will be generated at the one loop level[42,43]. The one loop diagrams for the quark EDM are shown in Fig.(6.1). After evaluating the diagrams, one obtains[42,43]

$$L_Y^{a+b} = \sum_{kl} \frac{em_l}{8\pi^2 m_{Wk}^2} \text{Im}(a_{il}^k b_{jl}^{k*}) f_a(x_{lk}) \bar{q}_j \sigma_{\mu\nu} (1-\gamma_5) q_l q^\nu \epsilon^\mu \quad (6.6)$$

$$L_Y^{c+d} = - \sum_{kl} \frac{em_l}{8\pi^2 m_{Wk}^2} \text{Im}(c_{il}^k d_{jl}^{k*}) f_c(x_{lk}) \bar{q}_j \sigma_{\mu\nu} (1-\gamma_5) q_l q^\nu \epsilon^\mu$$

where $x_{lk} = \left(\frac{m_l}{m_{Wk}}\right)^2$

$$f_a(x) = (Q_l - Q_i) \frac{1}{(1-x)^2} \left(2 - \frac{11x}{2} + \frac{x^2}{2} - \frac{3x \ln x}{1-x} \right) + \frac{Q_l}{(1-x)^2} \left(2 + \frac{x}{2} + \frac{x^2}{2} + \frac{3x \ln x}{1-x} \right) \quad (6.7)$$

$$f_c = (Q_l - Q_i) \frac{1}{2(1-x)^2} \left(1 + x + \frac{2x \ln x}{1-x} \right) - \frac{Q_l}{2(1-x)} \left(3 - x + \frac{2 \ln x}{1-x} \right)$$

with

$$a_{ij}^1 = \frac{1}{2\sqrt{2}} (g_L V_{Lij} \cos \zeta + g_R V_{Rij} \sin \zeta)$$

$$b_{ij}^1 = \frac{1}{2\sqrt{2}} (-g_L V_{Lij} \cos \zeta + g_R V_{Rij} \sin \zeta)$$

$$a_{ij}^2 = \frac{1}{2\sqrt{2}} (-g_L V_{Lij} \sin \zeta + g_R V_{Rij} \cos \zeta)$$

$$b_{ij}^2 = \frac{1}{2\sqrt{2}} (g_L V_{Lij} \sin \zeta + g_R V_{Rij} \cos \zeta)$$

(6.8)

Similar calculations give the quark CDM[35]

$$L_g^b = \sum_k \frac{em_l}{8\pi^2 m_{Wk}^2} \text{Im}(a_{l,j}^k b_{j,l}^{k*}) g_b(x_{lk}) \bar{q}_j \sigma_{\mu\nu} (1-\gamma_5) \frac{\lambda^a}{2} q_i q^{\nu\mu} \epsilon_a^\mu \quad (6.9)$$

$$L_g^d = -\sum_k \frac{em_l}{8\pi^2 m_{Wk}^2} \text{Im}(c_{l,j}^k d_{j,l}^{k*}) g_d(x_{lk}) \bar{q}_j \sigma_{\mu\nu} (1-\gamma_5) \frac{\lambda^a}{2} q_i q^{\nu\mu} \epsilon_a^\mu$$

where $m_{W1,2}$ are the eigenmasses of the charged gauge bosons and

$$g_b = \frac{1}{(1-x)^2} \left(2 + \frac{x}{2} + \frac{x^2}{2} + \frac{3x \ln x}{1-x} \right) \quad (6.10)$$

$$g_d = \frac{1}{2(1-x)^2} \left(3 - x + \frac{2 \ln x}{1-x} \right)$$

It is clear that the contributions from diagrams c and d are suppressed by a factor of $(m_l/m_{Wk})^2$. If quark masses are small compared with the W-mass, we can safely neglect these contributions.

Applying the formulae developed in section 2, we have

i) Valence quark contribution

$$D_n^{(V)} = -\frac{eg_L g_R}{192\pi} \sin 2\zeta \left\{ 4 \sum_i m_i \text{Im}(V_{Ldi} V_{Rdi}^*) \left[\frac{f_a(x_{i1})}{m_{W1}^2} - \frac{f_a(x_{i2})}{m_{W2}^2} \right] \right. \\ \left. - \sum_i m_i \text{Im}(V_{Lui} V_{Rui}^*) \left[\frac{f_a(x_{i1})}{m_{W1}^2} - \frac{f_a(x_{i2})}{m_{W2}^2} \right] \right\} \quad (6.11)$$

ii) The neutron EDM due to CDM is given by

$$\begin{aligned}
D_n^{(C)} = & \frac{1}{3} \frac{e g_L g_R}{192\pi} \sin 2\zeta \left\{ 4 \sum_i m_i \operatorname{Im}(V_{Ldi} V_{Rdi}^*) \left[\frac{g_a(x_{i1})}{m_{W1}^2} - \frac{g_a(x_{i2})}{m_{W2}^2} \right] \right. \\
& \left. + 2 \sum_i m_i \operatorname{Im}(V_{Liu} V_{Ru}^*) \left[\frac{g_a(x_{i1})}{m_{W1}^2} - \frac{g_a(x_{i2})}{m_{W2}^2} \right] \right\}
\end{aligned} \tag{6.12}$$

and iii) for the CP violating $K\Sigma n$, we have

$$\begin{aligned}
f_{K\Sigma n} = & \frac{0.24 \text{ GeV}}{\sqrt{2}} \frac{f_\pi}{f_k} g_s \sin 2\zeta \frac{g_L g_R}{64\pi^2} \sum_i m_i \operatorname{Im}(V_{Lsi} V_{Rsi}^*) \left[\frac{g_a(x_{i1})}{m_{W1}^2} \right. \\
& \left. \frac{g_a(x_{i2})}{m_{W2}^2} \right]
\end{aligned} \tag{6.13}$$

The CP violating $pn\pi$ vertex is $\sin^2\theta_c$ times smaller than the $K\Sigma n$ vertex, so we neglect it in our calculation.

To obtain an order of magnitude estimate and to simplify the problem, in the following we will consider the two generation case. Inserting in the expressions for $V_{L,R}$, we have

$$\begin{aligned}
D_n^{(V)} = & \frac{e}{9\sqrt{2}\pi^2} G_F \sin 2\zeta \{ 5[m_c \sin\theta_L \sin\theta_R \sin(\gamma+\delta_1) \\
& + m_u \cos\theta_L \cos\theta_R \sin(\gamma-\delta_2)] - [m_s \sin\theta_L \sin\theta_R \sin(\gamma-\delta_1) \\
& + m_d \cos\theta_L \cos\theta_R \sin(\gamma-\delta_2)] \} \\
D_n^{(C)} = & \frac{e}{16\sqrt{2}\pi^2} G_F \sin 2\zeta \{ 2[m_c \sin\theta_L \sin\theta_R \sin(\gamma+\delta_1) \tag{6.14}
\end{aligned}$$

$$+ m_u \cos\theta_L \cos\theta_R \sin(\gamma - \delta_2)] + [m_s \sin\theta_L \sin\theta_R \sin(\gamma - \delta_1) \\ + m_d \cos\theta_L \cos\theta_R \sin(\gamma - \delta_2)]$$

$$D_{n,1}^{(I)} = \frac{e g_N \pi}{4\sqrt{2}\pi^2 m_f^2} m_\Sigma (2\alpha - 1) G(m_K^2) \frac{0.24 \text{ GeV}}{\sqrt{2}} \frac{f_\pi}{f_K} f_s (g_s G_F, 4\sqrt{2}\pi^2)$$

$$\times \sin 2\zeta \{m_c \sin\theta_L \sin\theta_R \sin(\gamma - \delta_1) + m_u \cos\theta_L \cos\theta_R \sin(\gamma + \delta_2)\}$$

Here we have neglected the $D_{n,2}^{(I)}$ contribution due to small anomalous magnetic moment of Σ^- .

Let us now work in the pseudo-manifest left-right models in which $g_L = g_R$ and $\sin\theta_L = \sin\theta_R$. Using $\sin\theta_c = 0.22$, $m_u = 5.1$ MeV, $m_d = 10$ MeV, $m_s = 170$ MeV, $m_c = 1.4$ GeV and $\alpha_s = 0.4$, we obtain

$$D_n^{\text{total}} = \sin 2\zeta \{4.5 \sin(\gamma - \delta_2) + 74 \sin(\gamma + \delta_1) - 1.1 \sin(\gamma - \delta_1) \\ + 11.2 \sin(\gamma + \delta_2)\} \times 10^{-23} \text{ (ecm)} \quad (6.15)$$

It is clear that

$$|D_n^{\text{total}}| \leq 7.6 \times 10^{-24} \text{ (ecm)} \quad (6.16)$$

It is difficult to give a definite prediction for D_n since the phases γ , δ_i and mixing angle ζ are not known. To get some idea on the possible bound on D_n , we can relate D_n to ϵ'/ϵ . We now briefly discuss ϵ'/ϵ . Evaluating possible $\Delta S = 1$ diagrams up to one loop level, we have^[44]

$$\begin{aligned} \epsilon' \equiv e^{i(\delta_1 - \delta_2 + \pi/2)} \frac{\text{Re}A_2}{\sqrt{2\text{Re}A_0}} \{-19.5[\sin(\gamma - \delta_2) + \sin(\gamma - \delta_1)] \\ + 1.8[\sin(\gamma + \delta_2) + \sin(\gamma + \delta_1)]\} \tan\zeta \end{aligned} \quad (6.17)$$

Using the experimental values $\text{Re}A_2/\text{Re}A_0 = 1/22$ and $|\epsilon| = 2.27 \times 10^{-3}$, we have

$$\begin{aligned} |\epsilon'/\epsilon| &= 276 \tan\zeta \{\sin(\gamma - \delta_2) + \sin(\gamma - \delta_1) - 0.1(\sin(\gamma + \delta_1) + \sin(\gamma + \delta_2))\} \\ &\leq 550 \tan\zeta \end{aligned} \quad (6.18)$$

Then the observed value $|\epsilon'/\epsilon| = (3.2 \pm 1.1) \times 10^{-3}$ ^[45] implies

$$\tan\zeta \geq 2.3 \times 10^{-6} \quad (6.19)$$

Unfortunately the angles γ, δ_i occur differently in ϵ' and D_n , so we can not estimate D_n without additional assumptions. In a particularly simple model with spontaneous CP violation due to Chang^[46], the phases γ, δ_i are given by

$$\begin{aligned} \delta_1 &\equiv -\frac{3}{2} \frac{k'}{k} \frac{m_c}{m_s} \sin 2\alpha \\ \delta_2 &\equiv -\frac{1}{2} \frac{k'}{k} \frac{m_c}{m_s} \sin 2\alpha \\ \gamma &\equiv 2\alpha + \frac{1}{2} \frac{k'}{k} \frac{m_c}{m_s} \sin 2\alpha \end{aligned} \quad (6.20)$$

In the case $k'/k \ll 1$, δ_i are negligible compared with γ , and we have

$$|D_n^{\text{total}}| = 3.6 \times 10^{-24} \left| \frac{\epsilon'}{\epsilon} \right| \text{ (ecm)} \quad (6.21)$$

If on the other hand, $k'/k \gg 1$, then

$$\gamma = -\delta_2 = -\frac{1}{3}\delta_1 = \frac{1}{2} \frac{k'}{k} \frac{m_c}{m_s} \sin 2\alpha \quad (6.22)$$

leading to

$$|D_n^{\text{total}}| = 1.7 \times 10^{-24} \left| \frac{\epsilon'}{\epsilon} \right| \quad (6.23)$$

this is about a factor of 3 less than the estimate in eq.(6.21). Using the experimental value for ϵ'/ϵ with two standard deviations, we estimate

$$1.9 \times 10^{-26} \text{ (ecm)} \geq |D_n^{\text{total}}| \geq 1.9 \times 10^{-27} \text{ (ecm)} \quad (6.24)$$

There is the exceptional case when $k'/k \cong 1$ and 2α lies near $3\pi/2$; then $\gamma \cong \delta_2$ and D_n can be as large as 10^{-25} ecm. This has been discussed in Ref. [47]. Otherwise, this bound is respected.

Measurement of ϵ'/ϵ and D_n with slightly improved sensitivity will constrain pseudo-manifest left-right symmetric theories of CP violation tightly and can verify or rule out some models.

There are other class of left-right models which contain heavy quarks can given large neutron EDM. One class of models in which heavy singlet vector-like quarks $P = (1,1,4/3)$, $N = (1,1,-2/3)$ and singlet charged lepton $E = (1,1,-2)$ are introduced can explain the smallness of the fermion masses in comparison with the W_L

mass^[48] or can explain the observed fermion mass hierarchy^[49] and the smallness of the neutrino mass^[50]. The Higgs needed for symmetry breakings are $H_L = (2,1,1)$ and $H_R = (1,2,1)$. Also it is essential to introduce a parity odd Higgs scalar $\sigma = (1,1,0)$ in order to obtain the desired symmetry breaking.^[50] Because of the absence of the bidoublet $\phi = (2,2,0)$ the W_L and W_R mixing is vanishing at the tree level. The CP-violating effects induced by the right-handed W_R -exchange satisfy the "isoconjugate" relations leading to $\epsilon' = 0$. At one loop level, W_L mixes with W_R and the mixing is given by

$$\xi = \frac{\alpha}{4\pi} \sin^2\theta_W \frac{m_b m_t}{m_{W_R}^2} \quad (6.25)$$

which is of order 10^{-6} for $m_{W_R} \approx 1$ Tev. This gives too small ϵ'/ϵ and also small D_n . However, the mixing of ordinary quarks with the vector-like singlet quarks through H_L and H_R will contribute to ϵ'/ϵ and D_n significantly. With proper choice of the parameters, the experimentally observed value of ϵ'/ϵ can be easily reproduced^[51]. The one loop diagram which contributes to neutron EDM is shown in Figure (6.2). Evaluating such diagrams, one typically obtains $D_n = 10^{-25}$ to 10^{-26} ecm^[51].

Another class of models involving mirror quarks has also been introduced. Mixing of the ordinary quarks with the mirror quarks with a CP violating phase can give a neutron EDM at the one loop level (Figure (6.3)). With the experimentally allowed mixings^[52], it is possible to have neutron EDM of order 10^{-25} ecm^[52].

7) THE NEUTRON EDM IN SUPERSYMMETRIC MODELS

In this section we discuss the NEDM in supersymmetric models. Supersymmetry has many appealing features^[53]. It solves the hierarchy problem and also provides a working model for unifying strong, electroweak and gravity if supersymmetry is localized. Supersymmetry transforms fermions into bosons and vice versa. Each particle has its superpartner with the same mass. Since experimentally no superparticles have been found, supersymmetry must be broken. Because there are new particles in the theory, their interactions will give new signatures. We will concentrate on possible new contributions to the neutron EDM in supersymmetric models. To illustrate how, in supersymmetric models, a new CP violating source arises and contribute to the neutron EDM, we will study a low energy supersymmetric model resulting from an N=1 supergravity model^[54] after local supersymmetry breaking by the so called super Higgs mechanism. In this model, besides the gauge vector superfields and the superfields which contain the ordinary fermions, two superfields each of which contains an SU(2)_L doublet Higgs and an appropriate hidden sector are introduced. After the break down of supergravity due to a super Higgs mechanism, a unique soft supersymmetric breaking term is generated to break the global supersymmetry. Evaluating the relative parameters down to m_W by the use of the renormalization group equations, the squarks \tilde{d}_L, \tilde{d}_R associated with the down quarks (d_L, d_R) acquire a mass matrix of the form^[55]

$$\begin{pmatrix} \mu_L^2 + c_d M_d M_d^\dagger + c_u M_u M_u^\dagger & A m_{3/2} M_d \\ A m_{3/2} M_d^\dagger & \mu_R^2 + c_d' M_d' M_d' \end{pmatrix} \quad (7.1)$$

where M_d and M_u are the up and down quark mass matrices respectively, c_i are constants, $m_{3/2}$ is the gravitino mass, $\mu_{L,R}$ are mass parameters which are the same order of magnitude as $m_{3/2}$, and A is a typical complex soft supersymmetric breaking parameter with phase $-2\phi_A$. It is clear that in the quark mass eigenstate basis, the squark mass matrix eq.(7.1) is not diagonal and is a complex matrix. It will generate new CP violating interactions between quarks, squarks and gluinos. In the quark mass eigenstate basis, the quark-squark-gluino interaction lagrangian for the down sector is

$$L_{\tilde{g}d\bar{d}} = i\sqrt{2}g_s\bar{d}^+\tilde{G}_2\lambda^a\left(\Gamma_L\frac{1-\gamma_5}{2}+\Gamma_R\frac{1+\gamma_5}{2}\right)d \quad (7.2)$$

where g_s is the strong gauge coupling constant, λ^a are the $S(3)$ generators, $\tilde{d} = (\tilde{d}_L, \tilde{d}_R)$, \tilde{G} is the gluino field, and the coupling matrices Γ_L, Γ_R are 6×3 matrices which are related to the squark mass matrix eq.(7.1). In the basis where the quark mass matrices are diagonal ($\hat{M}_{d,u}$), eq.(7.1) becomes

$$\hat{M}_d^2 = \begin{pmatrix} \mu_L^2 + c_d \hat{M}_d^2 + c_u V^\dagger \hat{M}_u^2 V & A m_{3/2} \hat{M}_d \\ A m_{3/2} \hat{M}_d & \mu_R^2 + c_d \hat{M}_d^2 \end{pmatrix} \quad (7.3)$$

where V is the KM-matrix.

Let U being the matrix which diagonalizes \hat{M}_d^2 , that is $U^\dagger \hat{M}_d^2 U = \text{Diag}$, we have

$$(\Gamma_L, \Gamma_R) = U^+ \begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix} \quad (7.4)$$

where $\phi = \phi_g - \phi_A$ with ϕ_A being a possible phase in the gluino mass ($m_{\tilde{g}} = |m_{\tilde{g}}| e^{-i2\phi_g}$). It is interesting to notice that CP is violated in this interaction even if the KM-matrix is real due a non-zero phase ϕ . Since our goal is to show how new CP violating sources can affect the physics in supersymmetric model, we will later assume the KM-matrix is real. In this model also it is not necessary to have three generations of quarks in order to have CP violation. For simplicity, we will assume that the mixings of the third generation to the first two are negligible. In this case, Γ_L and Γ_R can be written as 4x2 matrices

$$\Gamma_L \equiv e^{i\phi} \begin{pmatrix} c\theta_{CP1} & -s\theta_{CP2} \\ s\theta_{CP1} & c\theta_{CP2} \\ s\rho_1 & 0 \\ 0 & s\rho_2 \end{pmatrix} \quad (7.5)$$

$$\Gamma_R \equiv e^{i\phi} \begin{pmatrix} -c\theta_{CP1} & s\theta_{CP2} \\ -s\theta_{CP1} & -c\theta_{CP2} \\ c\rho_1 & 0 \\ 0 & c\rho_2 \end{pmatrix}$$

where

$$\tan(2\rho_1) = \frac{-2|A|m_{\tilde{g}}m_{3/2}}{\mu_L^2 - \mu_R^2} \quad (7.6)$$

and s, c are sin and cos respectively.

We are ready to calculate the NEDM due to the new phase in eq.(7.2). The one loop diagrams which generate quark electric and color dipole moments are shown in Fig.7.1. Evaluating these diagrams, we have for down quark EDM[56]

$$d_d = -\frac{2e\alpha_s}{9\pi m_{\tilde{g}}} \text{Im}(\Gamma_L^{id} \Gamma_R^{id*}) D(z) \quad (7.7)$$

$$D(z) = \frac{1}{2(z-1)^2} \left(1+z + \frac{2z}{z-1} \ln z\right)$$

with $z = \tilde{m}_i^2 / m_{\tilde{g}}^2$, the repeated index being summed over.

For the down quark CDM, one has[56].

$$L = \frac{g_s^3}{16\pi^2 m_{\tilde{g}}} \text{Im}(\Gamma_L^{iq} \Gamma_R^{iq*}) (C_2(G)E(z) - 2C_2(R)D(z)) \frac{1}{2} \bar{q} \sigma_{\mu\nu} \gamma_5 \frac{\lambda^a}{2} q G_a^{\mu\nu} \quad (7.8)$$

$$E(z) = \frac{1}{(1-z)^2} \left(\frac{1}{3} + \frac{1}{2}z - z^2 + \frac{1}{6}z^3 + z \ln z \right)$$

In the above expression, $C_2(G) = N$ and $C_2(R) = (N^2-1)/2N$ are the Casimir operators of the adjoint and the fundamental representations of $SU(N)$.

Using the same method as in section (5), we can calculate the effective $K\Sigma n$ CP violating vertex. Putting all the above information in the formula for the neutron EDM in section (2), we obtain

$$D_n^{(V)} = -\frac{4}{3} \frac{2e\alpha_s}{9\pi m_{\tilde{g}}} c p_1 s p_1 \sin 2\phi \frac{\mu_L^2 - \mu_R^2}{m_{\tilde{g}}^2} D'(z)$$

$$D_n^{(C)} = -\frac{4}{9} \frac{e\alpha_s}{4\pi m_{\tilde{g}}} g_s c p_1 s p_1 \sin 2\phi \frac{\mu_L^2 - \mu_R^2}{m_{\tilde{g}}^2} (C_2(G)E'(z) - 2C_2(R)D'(z))$$

$$D_{n,1}^{(I)} = \frac{e g_{\pi NN}}{4\pi^2 m_n^2} m_{\Sigma} (-(2\alpha_-)) \frac{f_{K\Sigma n}}{\sqrt{2}} G_{\Sigma}(m_K^2)$$
(7.9)

where $f_{K\Sigma n}$ is the effective CP violating $K\Sigma n$ vertex which is calculated to be

$$f_{K\Sigma n} = \frac{0.24 \text{ GeV}}{\sqrt{2}} \frac{f_{\pi}}{f_K} \frac{m_s}{m_d} \frac{e\alpha_s}{4\pi m_{\tilde{g}}}$$

$$\times g_s c p_1 s p_1 \sin 2\phi \frac{\mu_L^2 - \mu_R^2}{m_{\tilde{g}}^2} (C_2(G)E'(z) - 2C_2(R)D'(z))$$
(7.10)

Here $z = \tilde{m}^2 / m_{\tilde{g}}^2$ and \tilde{m}^2 is the averaged squark mass and $D'(z)$, $E'(z)$ indicate the derivatives of the functions $D(z)$, $E(z)$ respectively. We have used eq.(5) to express $\Gamma_{L,R}$ in terms of $c p_1$ and $s p_2$. Since the squark masses and the angles p_i are not known, no precise prediction can be made. For an order of magnitude estimate, we take $\mu_L^2 - \mu_R^2 \cong \tilde{m}^2 \cong m_{\tilde{g}}^2 \cong m_W^2$. We find that $D_n^{(V)}$, $D_n^{(C)}$, and $D_{n,1}^{(I)}$ are comparable, no one term dominates the others. The total NEDM in this model is

$$D_n \cong 10^{-22} \phi \text{ ecm} \quad (7.11)$$

As found in Ref.[56,57].

Comparing this result with experimental bound, we have $\phi < 10^{-3}$. CP violation due to ϕ also contributes to ϵ in the Kaon system, however with $\phi < 10^{-3}$, this contribution is negligibly small ($< 10^{-8}$)[55]. Hence if the sole source of CP violation were the phase ϕ it would not be enough to explain the observed CP violating effects in $K^0 - \bar{K}^0$ system. Inclusion of the third generation does not change this situation. Other CP violating sources have to be included to restore agreement with experiment.

8) THE NEUTRON EDM IN QCD

In this section we study the neutron EDM in quantum chromodynamics(QCD). It has long been realized that due to instanton effects, in non-abelian gauge theory, the total divergence term

$$\frac{1}{2} \epsilon_{\mu\nu\alpha\beta} G^{\alpha\beta} G^{\mu\nu} = \tilde{G}_{\alpha\beta} G^{\alpha\beta} \quad (8.1)$$

constructed from the field strength $G^{\mu\nu}$ has nonvanishing physical effects[58]. In the case of QCD, $G^{\mu\nu}$ is the gluon field strength. The full QCD Lagrangian is then

$$L_{\text{QCD}} = \frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \bar{q} (D_{\mu} \gamma^{\mu} - m) q - \theta \frac{g^2}{32\pi^2} \tilde{G}_{\mu\nu} G^{\mu\nu} \quad (8.2)$$

where q is the quark field, m is the quark mass and D_{μ} is the covariant derivative and θ is a constant.

The last term in L_{QCD} violates P and CP. One may attempt to impose P and CP conservation by setting $\theta = 0$. However, this

choice of parameter is unstable with respect to renormalization effects[59], in general θ need not be zero. The physical effects of non-zero θ have been studied by several authors[1,58,60]. Here we shall concentrate our attention on the effect of the θ -term on the neutron EDM. The calculation is most readily carried out by introducing an equivalent θ -term in the quark matrix to remove the $\tilde{G}_{\mu\nu}G^{\mu\nu}$ term in the Lagrangian. In this convention, the effective quark mass term in the Lagrangian becomes

$$L_{\text{mass}} = -(m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s) + \delta L_{\text{CP}} \quad (8.3)$$

$$\delta L_{\text{CP}} = i\theta \frac{m_u m_d m_s}{m_u m_d + m_u m_s + m_d m_s} (\bar{u} \gamma_5 u + \bar{d} \gamma_5 d + \bar{s} \gamma_5 s)$$

Several methods have been employed to evaluate the effect of δL_{CP} on the neutron EDM. The first investigation on this subject is carried out in Ref.[60] in which it is noted that in the presence of δL_{CP} the nucleon becomes a mixture of states of opposite parity

$$|n\rangle = |N\rangle + \sum \frac{\langle N' | \delta L_{\text{QCD}} | N \rangle}{M' - M} |N'\rangle \quad (8.4)$$

where M and M' are the masses of the states $N(1/2^+)$ and $N'(1/2^-)$ respectively, and the summation is over all possible physical states with the quantum number $(1/2^-)$. The NEDM is then given by

$$\vec{D} = 2 \sum \text{Re} \langle N | \vec{d} | N' \rangle \frac{\langle N' | \delta L_{\text{QCD}} | N \rangle}{M' - M} \quad (8.5)$$

where \vec{d} is the dipole moment operator which can induce transitions $(1/2^+) \rightarrow (1/2^-)$ without CP violation. Taking into

account of the nearest low-lying resonances $N^*(1/2^-) = S_{11}^{\prime}, S_{11}^{\prime\prime}$ and performing a bag model calculation for the matrix elements, one obtains

$$D_n \approx 2.7 \times 10^{-16} \theta \text{ ecm} \quad (8.6)$$

Another way of evaluating of the effect of the θ -term on D_n is to calculate the CP violating nucleon-meson interaction due to δL_{QCD} using current algebra and PCAC[61]

$$\langle P_a B_f | \delta L_{\text{CP}} | n \rangle = \frac{1}{F_\pi} \theta \frac{m_u m_d m_s}{m_u m_d + m_u m_s + m_d m_s} \langle B_f | \bar{q} \lambda^a q | n \rangle \quad (8.7)$$

where P_a and B_f are the pseudo-scalar-meson and baryon octets respectively, $\langle B_f | \bar{q} \lambda^a q | n \rangle$ is related by ordinary SU(3) symmetry to the mass difference of the baryon octet particles. Considering only the $\pi \bar{B} n$ interaction, one obtains, as in Ref.[11]

$$L_{\pi NN} = -\sqrt{2} \pi N \tau (i \gamma_5 g_{\pi NN} + f_{\pi NN}) N, \quad (8.8)$$

$$f_{\pi NN} = \frac{1}{\sqrt{2}} 2(m_\Xi - m_\Sigma) \frac{m_u m_d m_s}{F_\pi (m_u + m_d)(2m_s - m_u - m_d)}$$

Inserting these numerical values in eq.(2.3.3), one has

$$D_n = -3.8 \times 10^{-16} \theta \quad (8.9)$$

In this calculation, contributions from the other pseudoscalars (η, η', K) are not included. However, the θ -term is closely related to the generation of these pseudoscalar masses[62], and it is

therefore necessary to include contributions of these particles in the calculation. To this end, we use the method developed in Ref.[63] to construct an effective CP violating Nucleon-Meson coupling from a nonlinear Chiral model containing a nonet of baryons and a nonet of pseudoscalar mesons, we have

$$\begin{aligned}
 L_{CP} = & -\sqrt{2}(f_{\pi NN} \bar{n} n \left(-\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} \right) + f_{nn\eta} \bar{n} n \left(-\frac{2\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} \right) \\
 & + (f_{\pi NN} \bar{p} n \pi^+ + f_{\Lambda n K} \left(-\frac{2}{\sqrt{6}} \bar{\Lambda} n \bar{K}^0 \right) + f_{\Sigma n K} \bar{\Sigma} n K^- \\
 & + f_{\Sigma n K} \left(-\frac{\bar{\Sigma}^0}{\sqrt{2}} + \frac{\bar{\Lambda}}{\sqrt{6}} \right) n \bar{K}^0 + \text{H.C})
 \end{aligned} \tag{8.10}$$

where

$$f_{\pi NN} \equiv \frac{1}{F_\pi} \frac{1}{2m_s} \left(\frac{3}{2}(m_\Sigma - m_\Lambda) - (m_\Xi - m_n) \right) \frac{m_u m_d}{m_u + m_d} \theta = -0.027$$

$$\begin{aligned}
 f_{\Lambda n K} & \equiv \frac{1}{F_\pi} \frac{1}{2m_s} \left(\frac{3}{2}(m_\Sigma - m_\Lambda) \frac{m_n}{m_n + m_\Xi - m_\Sigma} - (m_\Xi - m_n) \right) \frac{m_u m_d}{m_u + m_d} \theta \\
 & = -0.028
 \end{aligned}$$

(12)

$$\begin{aligned}
 f_{nn\eta} & \equiv \frac{1}{F_\pi} \frac{1}{2m_s} \left(\frac{3}{2}(m_\Sigma - m_\Lambda) \left(\frac{2m_\Sigma}{m_n} - 1 \right) + (m_\Xi - m_n) \right) \frac{m_u m_d}{m_u + m_d} \theta \\
 & = 0.057
 \end{aligned}$$

$$f_{\Sigma n K} \equiv \frac{1}{F_\pi} \frac{1}{2m_s} \left(\frac{3}{2}(m_\Sigma - m_\Lambda) \frac{m_\Sigma}{m_n} + (m_\Xi - m_n) \right) \frac{m_u m_d}{m_u + m_d} \theta = 0.054$$

The CP conserving nucleon-meson couplings can also be easily obtained using this method. In particular, we obtain the ratio of πNN coupling $g_{\pi NN}$ to $\eta_1 NN$ coupling $g_{\eta_1 NN}$

$$\frac{g_{\pi NN}}{g_{\eta_1 NN}} \equiv \frac{\sqrt{3}m_\eta}{m_\eta + m_\Sigma} = 0.76 \quad (8.12)$$

Including contributions from all the pseudoscalars in the nonet to the neutron EDM which arise from diagrams with photon coupled to the meson and to the baryon with Dirac-type γ_μ coupling, only π^+ and K^- contribute, and we have

$$D_{n,1}^{(1)} \equiv D_{\pi^+} + D_{K^-} = (-3.8 + 0.8) \times 10^{-16} \theta \text{ ecm} \quad (8.13)$$

While for the contributions arise from the Pauli-type $\sigma_{\mu\nu}q^\nu$ coupling of the photon to the baryon, all ($\pi^0, \pi^+, K^-, K^0, \eta_8, \eta_1$) contribute, we have

$$\begin{aligned} D_n^{(2)}(1) &\equiv D_{\pi^-} + D_{K^-} + D_{\pi^0} + D_{K^0} + D_\eta + D_{\eta_1} \\ &= (-2.0 - 0.05 + 1.06 - 0.36 + 0.26 - 0.46) \times 10^{-16} \theta \text{ ecm} \end{aligned} \quad (8.14)$$

Combining the total contributions, we finally have

$$2.45 \times 10^{-16} \theta \text{ ecm} \leq |D_n| \leq 4.55 \times 10^{-16} \theta \text{ ecm} \quad (8.15)$$

In order to satisfy the experimental upper bound, θ must be $\leq 10^{-10}$. Such a small value of the parameter θ leads one to construct theories in which θ is automatically zero. There are

several ways to achieve this. For example, if one of the quarks has zero mass^[64] or the theory contains light pseudoscalar particles, the Axions^[64,65].

9) CONCLUSION

We have systematically studied the neutron EDM in several CP violating models. It is important to notice that the predictions for the neutron EDM are very sensitive to the choice of models. We summarize our results below:

(i) in the standard KM model with 3 families the neutron EDM is in the range $1.4 \times 10^{-33} \leq |D_n| \leq 1.6 \times 10^{-31}$ e.cm. This is several orders of magnitude below the present experimental bound. If the standard KM model is the correct description of CP violation, the experimental measurement of the neutron EDM will be very difficult. A fourth family can raise the neutron EDM by as much as a factor of 20.

(ii) the two Higgs doublet model has approximately the same value of D_n as the standard model.

(iii) D_n in the Weinberg Higgs model is predicted to be $|D_n| > 10^{-25}$ ecm. The calculated result is already in conflict with the experimental bound and therefore the Weinberg model may be in trouble. However there are uncertainties in the contribution to the neutron EDM from the neutral Higgs particles which are hard to pin down, and there are some uncertainties in the calculation. A happy choice of parameters could make the Weinberg model consistent with the present experiments.

(iv) In the left-right symmetric model D_n is of the order of $10^{-26 \pm 1}$ e.cm. This value is just below the present experimental

bound. A slight improvement in the experimental measurements will provide us with crucial information in support of left-right symmetric models.

v) In supersymmetric models, D_n is of order $10^{-22}\phi$ ecm with ϕ being the possible phase difference of the phases of gluino mass and the gluino-quark-squark mixing matrix. Since there is no bound for the phase ϕ from other experiments, no prediction for the neutron EDM can be made. However, an important message from this calculation is that the phase ϕ is bounded from the experimental bound on the neutron EDM to be $< 10^{-3}$. Such a small value of ϕ can not reproduce the observed CP violation in the K^0 - \bar{K}^0 system. This implies that the new phase ϕ can not be the only source for CP violation.

vi) the strong CP violation parameter θ is found from the bound on the neutron EDM to be limited by $\theta < 10^{-4}$.

Already useful information on the parameters of the models of CP violation is available from the experimental bound on the neutron EDM. Improved precision in both experiments and calculations will provide much more information and is strongly encouraged.

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Figure Captions

- Fig. 2.1. The hadronic loop diagrams for the neutron EDM (first order in G_F). The vertex with the blob (\bullet) is the P or CP violating vertex.
- Fig. 2.2. The hadronic loop diagrams for the neutron EDM (second order in G_F).
- Fig. 3.1. The hadronic loop diagrams contribution to the neutron EDM with an $\Delta s = 1$ radiative CP violating vertex.
- Fig. 3.2. The pole model diagram for the neutron EDM.
- Fig. 4.1. The one loop diagrams for $qq\gamma$ and qqg due to the physical charged Higgs.
- Fig. 5.1. The one loop diagram for quark EDM due to neutral Higgs exchange.
- Fig. 6.1. The one loop diagrams for quark EDM and CDM in the left-right symmetric model in the unitary gauge. Figs.(a) and (b) are gauge boson loop diagrams and (c) and (d) are scalar loop diagrams.
- Fig. 6.2. The one loop contribution to neutron EDM from exchange of heavy singlet quarks Q .
- Fig. 6.3. The one loop contribution to neutron EDM from exchange of mirror quarks Q .
- Fig. 7.1. The one loop diagrams for quark EDM and CDM due to Squark and gluino exchange.

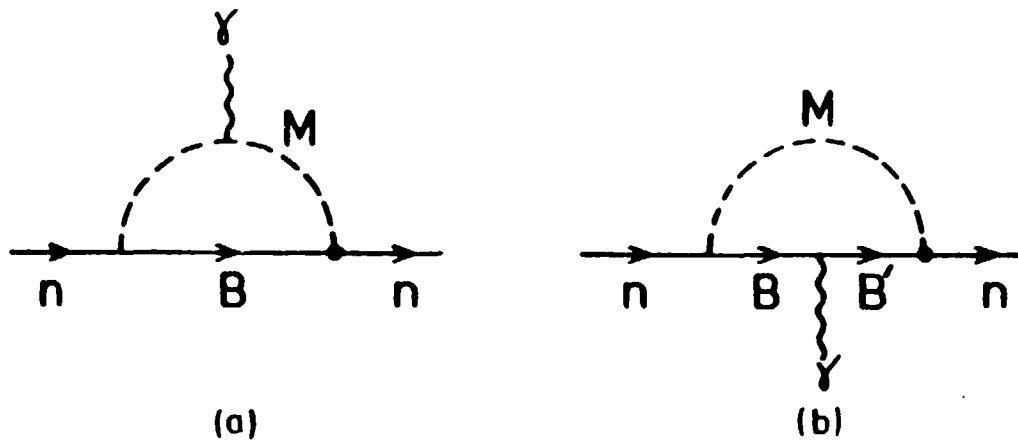


Fig. 2.1

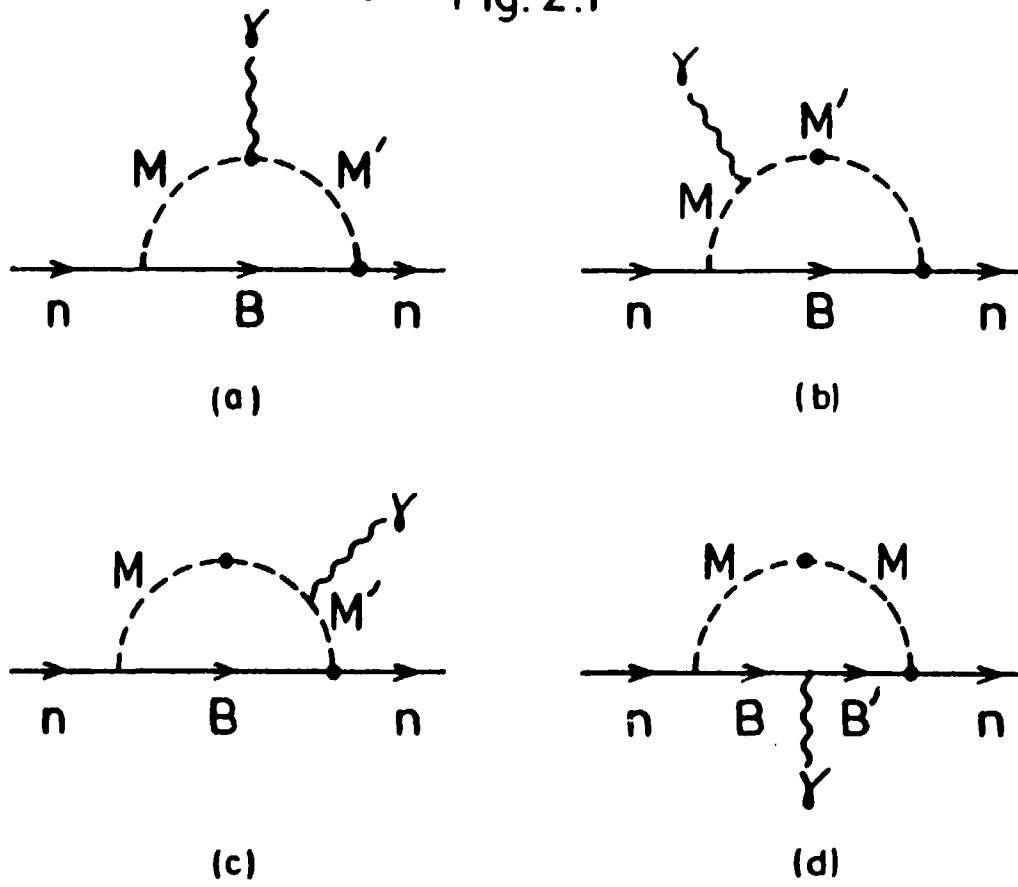


Fig. 2.2

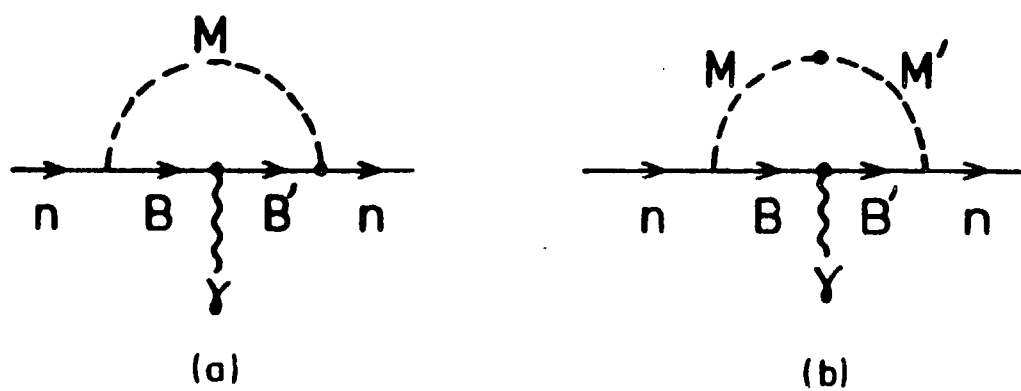


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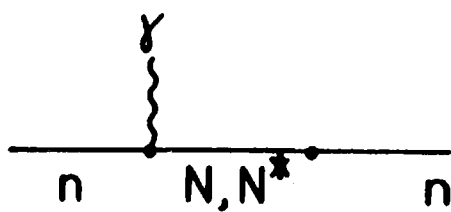


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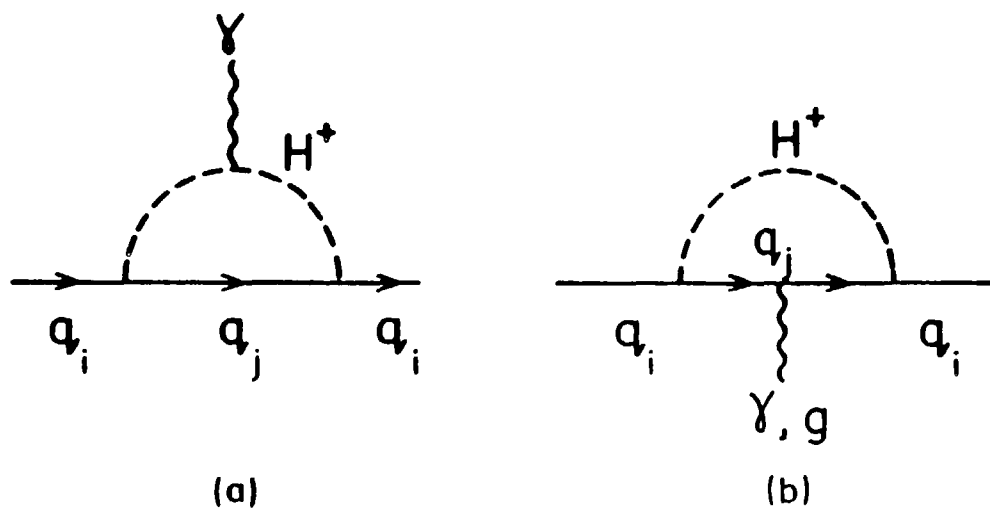


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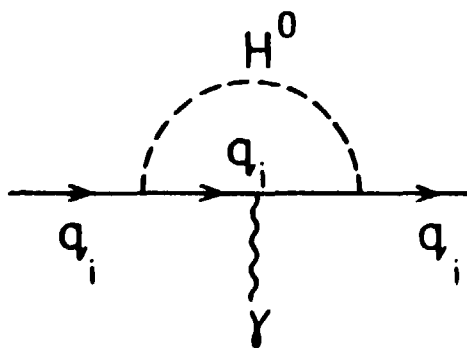
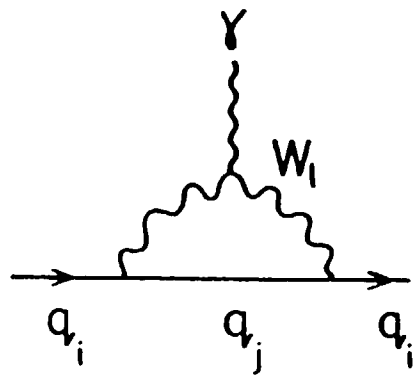
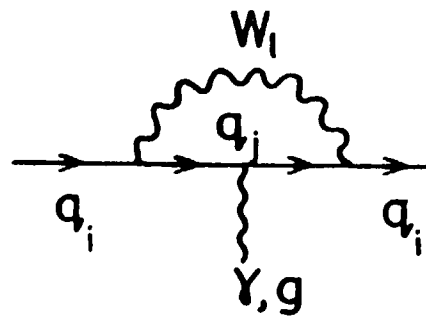


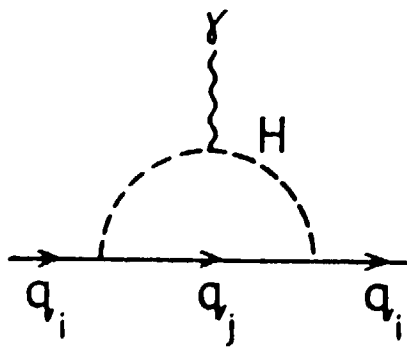
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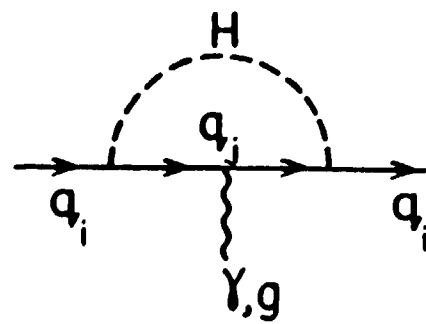
(a)



(b)



(c)



(d)

Fig.6.1

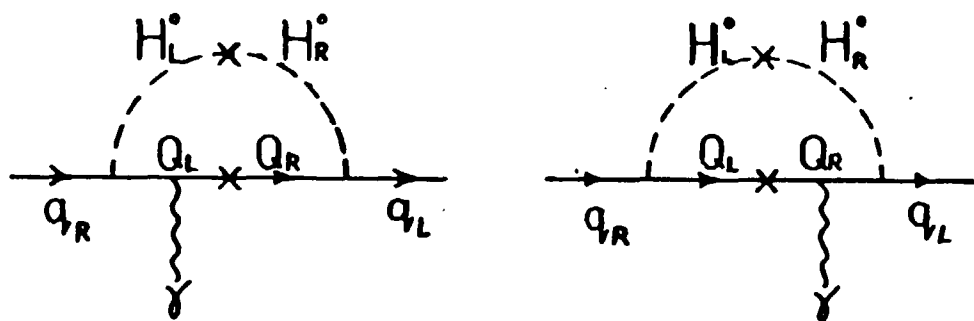


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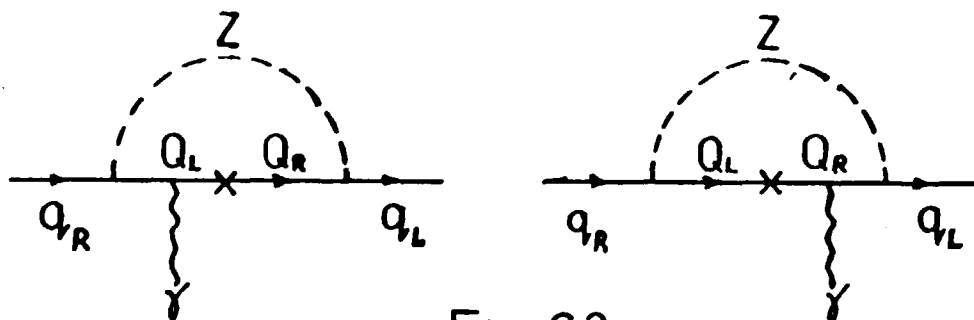


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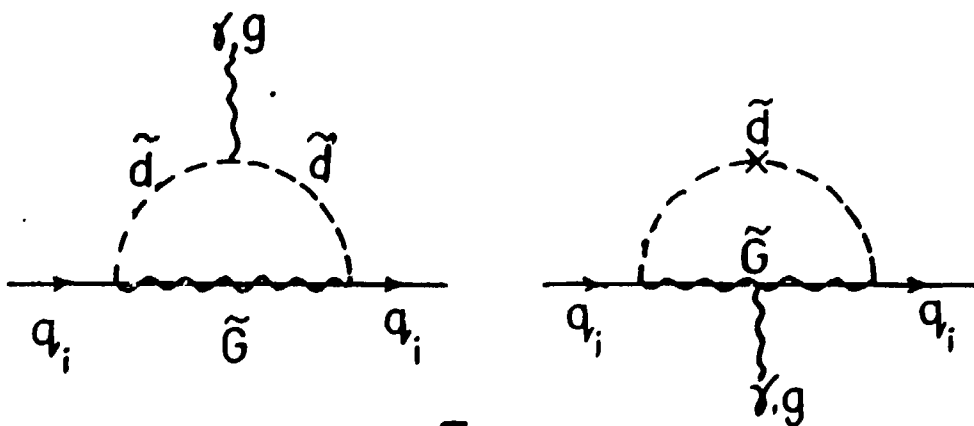


Fig. 7.1