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EVALUATION OF SIGNAL PROCESSING FOR BOILING NOISE DETECTION

Further work on the Karlsruhe data and a preliminary analysis of the BOR-60 tapes

J.L. BLACK, T.J. LEDWIDGE
Darling Downs Institute of Advanced Education,
Toowoomba, Queensland, Australia

Abstract

As part of the co-ordinated research programme on the detection of sodium boiling some further analysis has been performed on the data from the test loop in Karlsruhe and some preliminary analysis of the data from the BOR 60 experiment.

The work on the Karlsruhe data is concerned with the search for a reliable method by which the quality of signal processing strategies may be compared. The results show that the three novel methods previously reported are all markedly superior to the mean square method which is used as a benchmark. The three novel methods are nth order differentiation in the frequency domain, the mean square prediction based on nth order conditional expectation and the nth order probability density function.

A preliminary analysis on the data from the BOR 60 reactor shows that 4th order differentiation is adequate for the detection of signals derived from a pressure transducer and that the map of spurious trip probability (S) and the probability of missing an event (M) is consistant with the theoretical model proposed herein, and the suggested procedures for evaluating the quality of detection strategies.

1.0 GENERAL SIGNAL PROCESSING STRATEGY – Discrimination and Quality

1.1 Basic Assumptions

It is assumed that the flow noise is spatially distributed over the whole plant giving rise to a signal at the sensor which is sampled appropriately to generate a series x(j). On the other hand the noise from the boiling is assumed to be localised and thus not correlated with the spatially distributed flow noise. The boiling noise gives rise to a signal at the sensor which is similarly sampled to generate a series y(j), regarded as the target signal.

The sampled signal at the sensor may thus be written

$$z(j) = x(j) + Ry(j) \qquad (3)$$

where R = 0 corresponds to no boiling

and R = 1 corresponds to boiling in progress

x(j) and y(j) are assumed to have zero mean value and, as previously stated, to be uncorrelated.

With these assumptions it is easy to show that the mean square of the signal at the sensor is given by

$$\text{mean square } \overline{z^2(j)} = \overline{x^2(j)} + \overline{R^2y^2(j)} \qquad (4)$$

The mean square is one obvious measure of the signal that may be used to assess whether or not boiling is in progress.

1.2 Discrimination

A measure of the discrimination Δ inherent in using the mean square as a detection strategy is given by

$$\Delta = \frac{|\overline{z^2(j)}|_{R=1}}{|\overline{z^2(j)}|_{R=0}} = \frac{\overline{x^2(j)} + \overline{y^2(j)}}{\overline{x^2(j)}} = 1 + \text{signal to noise ratio} \qquad (5)$$

This then can be used as a benchmark against which the discrimination of any proposed strategy may be evaluated.

In general if the signal z(j) is described by some feature of it, say W, then we may define the discrimination of the signal processing strategy in general terms as

$$\Delta = \frac{|W|_{R=1}}{|W|_{R=0}} = \frac{W_1}{W_0} \qquad (6)$$

1.3 Quality of the Signal Processing Strategy

In practice the estimate of W (whatever that may be) is enacted over an averaging time corresponding to N samples. Usually it is required to reach some decision concerning the likelihood of incipient boiling in as short a time as possible, but with a reasonable assurance that the decision is neither spurious nor fails to meet a real demand. This assurance can only be formulated in probabilistic terms and depends on the probability density functions $f_1(W)$ and $f_0(s)$. For example if x(j) and y(j), and hence z(j) were all derived from a Gaussian process, then the estimate of the mean square would be described by a Chi-Square distribution with a mean of N and a variance of 2N. The normalised value of the variance can be made as small as we choose, but only at the expense of increasing the number of samples, N and hence the time required to make a decision.

Whilst the display of the output of a signal processing unit can take many forms, the ultimate use of the output is one of the determinants in a decision making process. In some cases the output of the signal processing unit may be used in an automatic control sense, in others as an indication to an operator who may decide to take appropriate action, often after considering other valid inputs.

In the final analysis the operator of the plant will judge the quality of the signal processing unit in terms of the likelihood or otherwise of presenting valid or invalid statements. In extreme cases the unit may indicate that action should be taken when in fact no malfunction of the plant is either incipient or has taken place; such an output would be spurious (S). Conversely it may eventuate that the signal processing unit fails to indicate that a malfunction is incipient or has taken place; i.e. the unit would miss an event (M).

In the case of automatic trip systems for plant protection (which must provide the goal for all protection systems) stringent targets are imposed to ensure good equipment design. For example in the UK an integrated spurious trip rate of less than 0.1 per annum is demanded together with a probability of failing to meet a demand, M of less than 10^{-3}. For the required maximum spurious trip rate of 0.1 per annum a probability S of less than 1.9×10^{-6} of tripping is needed for each 60 second decision time.

It is believed that S and M can often be formulated in probabalistic terms, which would be inter-connected and functions of both the time over which the signal is processed before an output is indicated (T) and the signal to noise ratio. Some complicated, multi-parameter, classification methods may be difficult to compare in probabalistic terms because of the nature of the outputs.

It is proposed that a map of S or M or some suitable combination together with T could form a consistent frame of reference against which differing signal processing strategies might be judged. The mathematical background to this proposal follows.

1.4 Mathematical Framework

In order to establish the nomenclature we shall first consider that the signal processing unit deals with only one feature of one signal, providing estimates of this feature at equal intervals of time T apart, where T is the averaging time of the unit. Examples of possible features would include the mean square, power within a defined frequency band and so on. Clearly on the assumption that the plant is operating, without malfunction, at steady state the statistics of the estimates of the feature may be described by a probability density function $f_1(\hat{W})$ say.

Once the plant moves to a malfunction situation then it is expected that the statistics associated with the monitored feature would change to $f_2(\hat{W})$ say. Action would normally be initiated if the estimated value of the feature exceeded some pre-set threshold level or decision boundary. This situation is shown graphically in Figure 1.

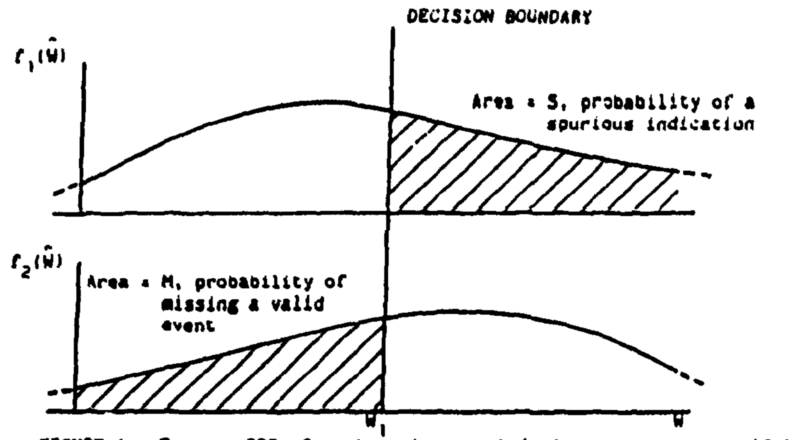

FIGURE 1. Feature PDFs for plant in normal (f_1) and malfunction (f_2) states.

Clearly from the diagram S and M are conjointly dependent on the position of the decision boundary, with the possibility that either may be made as small as desired, but at the expense of the other.

S the probability of a spurious indication and M the probability of missing a valid event are given by the following.

$$S = \int_{W_1}^{\infty} f_1(\hat{W})\, dW \qquad (7)$$

$$M = \int_{-\infty}^{W_1} f_2(\hat{W})\, dW \qquad (8)$$

As indicated earlier it is believed that both S and M will depend on the time T over which the signal is averaged. It is suggested that a simple map of S and M either singly or in some combination as a function of T could form a useful way by which signal processing qualities may be compared.

Using the above nomenclature we may now formalise the description of S and M using N-fold probability density functions when the signal processing technique relies on the utilisation of N features derived from one or more signals. Such features might be, as before, the mean square, skewness, kurtosis, power in a specific frequency band and so on.

Unlike the one dimensional case a number of different situations may occur depending on whether or not a valid output of the signal processing unit depends on all or some of the estimates of the N features exceeding set threshold levels.

The two extremes are given below:

(i) All levels to be exceeded at the same time.

then
$$S = \int_{W_1}^{\infty}\int_{W_2}^{\infty} \text{-----} \int_{W_N}^{\infty} f_1(\hat{W}_1, \hat{W}_2 \text{ -----} \hat{W}_N)\, dW_1\, dW_2 \text{---} dW_N \qquad (9)$$

and
$$M = 1 - \int_{W_1}^{\infty}\int_{W_2}^{\infty} \text{-----} \int_{W_N}^{\infty} f_2(\hat{W}_1, \hat{W}_2, \hat{W}_3 \text{ -----} \hat{W}_N)\, dW_1, dW_2 \text{---} dW_N \qquad (10)$$

where f(.) is the joint probability density function of the N estimates of the desired features and the suffices $_1$ and $_2$ have the same meaning as before.

The other extreme case is

(ii) Any one level exceeded at any time, in which case

$$S = 1 - \int_{W_1}^{\infty}\int_{W_2}^{\infty} \text{----} \int_{W_N}^{\infty} f_1(\hat{W}_1, \hat{W}_2 \text{ -----} \hat{W}_N)\, dW_1, dW_2 \text{---} dW_N \qquad (11)$$

$$M = \int_{-\infty}^{W_1}\int_{-\infty}^{W_2} \text{-----} \int_{-\infty}^{W_N} f_2(\hat{W}_1, \hat{W}_2 \text{ -----} \hat{W}_N)\, dW_1, dW_2 \text{---} dW_N \qquad (12)$$

In both case (i) and case (ii) the N dimensional decision boundry is defined by specifying the threshold values of W_1, W_2 ------ W_N.

2.0 Specific Features used in the Signal Processing

In this section we assume that the data is available as a time series at equal intervals of time and denoted as Z(j). The four features used in this work are defined below.

2.1 Mean Square Estimate

$$\overline{z^2}(j) = \frac{1}{N}\sum_{k=0}^{k=N-1} z^2(j-k)$$

2.2 rth Order Differentiation

$$\overline{z_r^2}(j) = \frac{1}{N}\sum_{k=0}^{k=N-1} [z_{r-1}(j-k) - z_{r-1}(j-k-1)]^2$$

2.3 Mean Square of the Prediction Error

$$\overline{\varepsilon^2}(j) = \frac{1}{N}\sum_{k=0}^{k=N-1} \{Z(j-k) - E[Z(j-k)|Z(j-k-1), \ldots , Z(j-k-n)]\}^2$$

where E (.) is the expectation estimated using data in the non-boiling "learning" phase of the process.

2.4 Mean Square of the 4th Order PDF Cost Function

$$\overline{\eta^2}(j) = \frac{1}{N}\sum_{k=0}^{k=N-1} \{A - f_x(z(j), \ldots , z(j-n+1))\}^2$$

where $A = \sup\{a|a = f_x(.)\}$ and f_x is the nth order PDF of the signal x(j).

3.0 Results froom File 2 of the Karlsruhe Tape

Figure 2(a) shows a plot of $\overline{Z^2}(j)$ with an averaging time of 1.25 milliseconds. Figures 2(b), 2(c) and 2(d) show plots of the same signal from the accelerometer processed in terms of $z^2_6(j)$; $\varepsilon^2(j)$ and $\eta^2(j)$ respectively. In each case there appears to be little doubt as to the time in the record at which boiling commences.

The pesky nature of the mean square quantities in these figures stems from the use of a short averaging time for the processes observed here. In terms of the context of this work it is appropriate to define a modified discrimination as

$$\Delta' = \frac{\text{Height of 1st significant peak after boiling on-set}}{\text{Maximum peak value for flow noise only}} \qquad (34)$$

This then allows the comparison of the various methods shown in Table 1 below.

TABLE 1. COMPARISON OF DISCRIMINATION

Method	Symbol	Discrimination ($10 \log_{10} \Delta'$)
Mean Square	$\overline{z^2}(j)$	0.37 dB
6th Order Difference	$\overline{z_r^2}(j)$	10.22 dB
Prediction Error	$\overline{\varepsilon^2}(j)$	9.48 dB
P.D.F. Cost Function	$\eta^2(j)$	8.42 dB

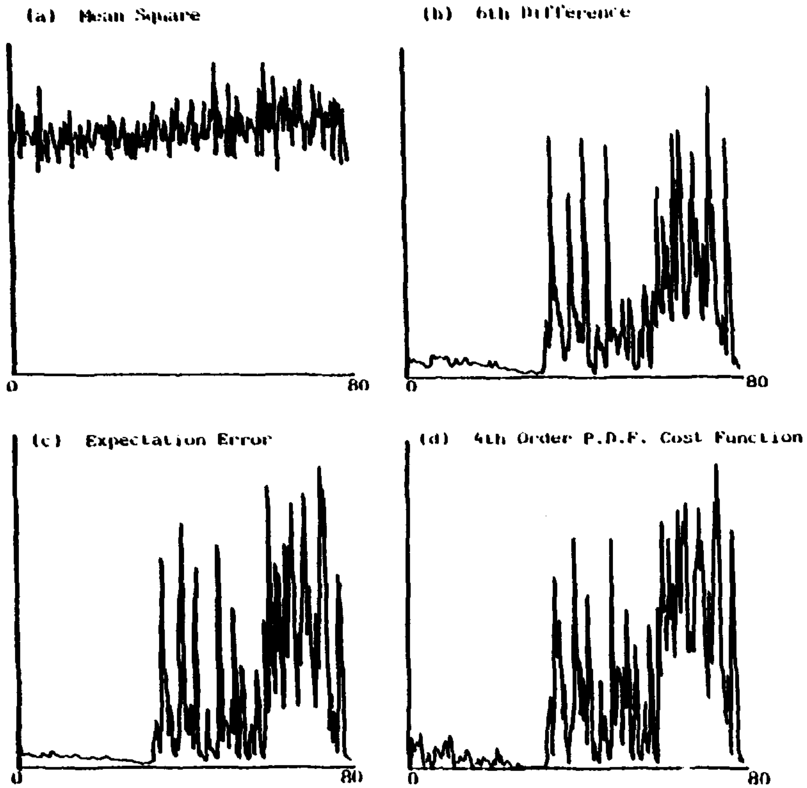

FIGURE 2.

Boiling onset during 80 second period - Karlsruhe benchmark test file Z.

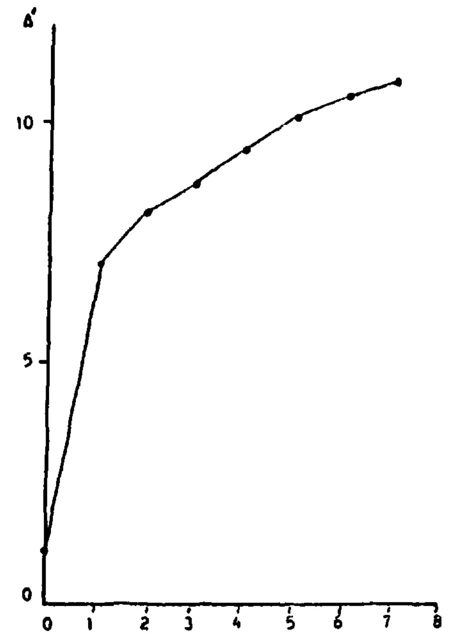

FIGURE 3.

Discrimination vs order of difference

3.1 Relationship Between $z_r^2(j)$ and the Order of Difference r

Figure 3 shows the variation of the modified discrimination Δ' with the order, r, of the difference process. This indicates that there is little to be gained in going beyond the 6th difference. The trend of this graph is similar to that predicted theoretically for continuous differentiation reported previously.

3.2 Probability Density Functions and Their Relationship Between Spurious Trips and the Failure to Detect Malfunction.

Typical probability density functions for each of the four signal processing strategies are shown in Figures 4 to 7 for a range of averaging times corresponding to 20, 200 and 2000 data samples.

The probability density function for the mean square of the signal in the non-boiling phase seems to be reasonably represented by a Chi-Square distribution, but deviating from that distribution when boiling commences. The probability density function for the other three strategies are markedly different from the Chi-Square.

In each of the methods recorded the effect of increasing the averaging time, over which the estimates were formed, led to an increase in separation in the probability density functions between the non-boiling and boiling phases. As indicated in Section 4, this separation directly determines the value S of a spurious trip and the value of M the probability of failing to detect a malfunction.

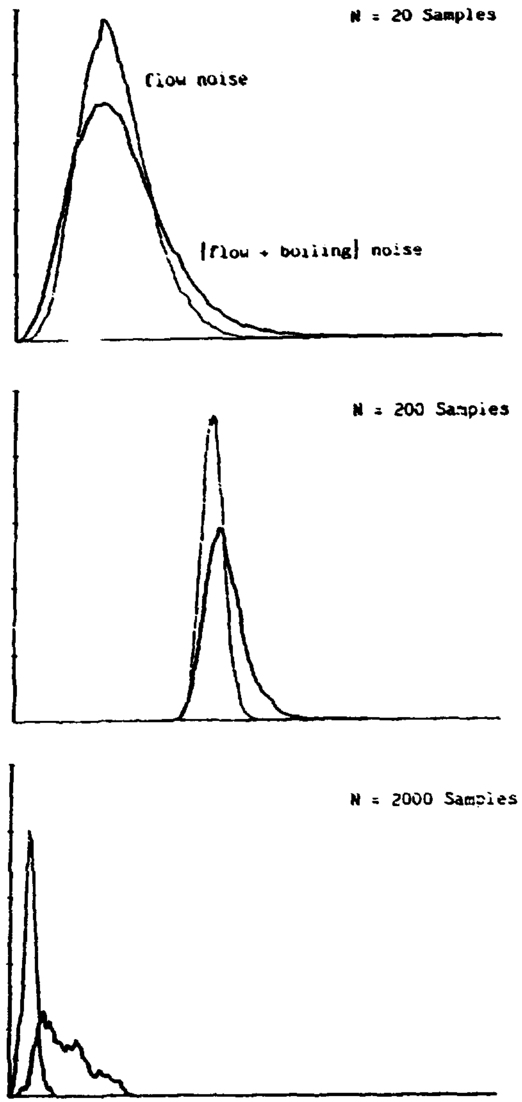

FIGURE 4.

Separation of mean square feature PDFs: various N
Karlsruhe benchmark test file 2.

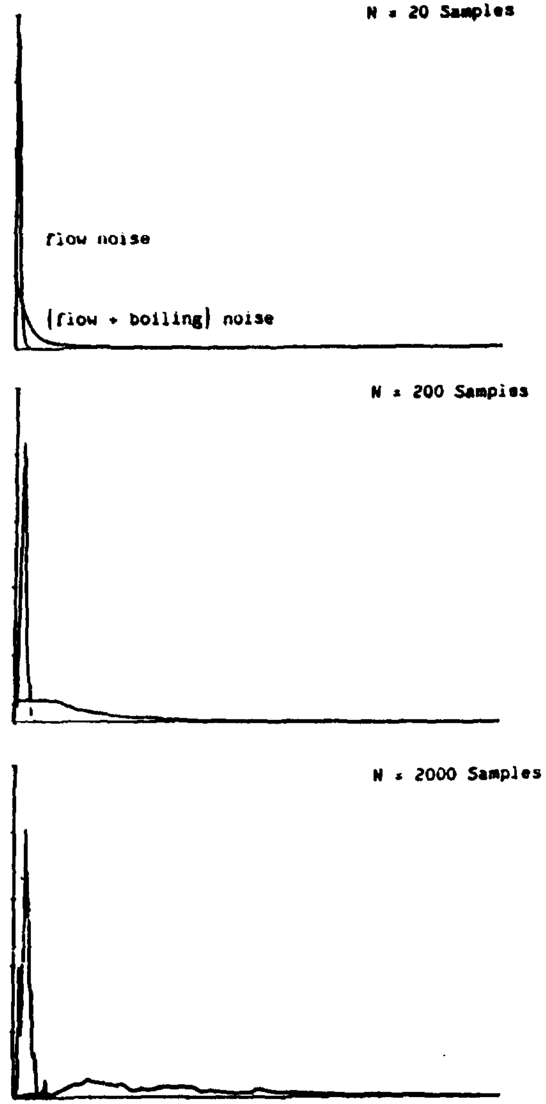

FIGURE 5.

Separation of 6th difference feature PDFs: various N
Karlsruhe benchmark test file 2

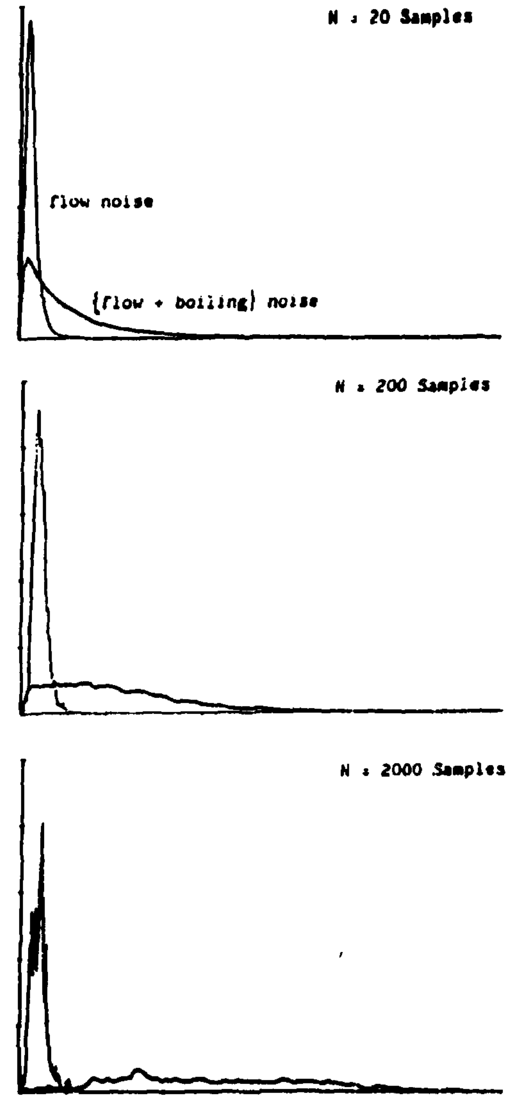

FIGURE 6.

Separation of expectation error PDFs: various N
Karlsruhe benchmark test file 2

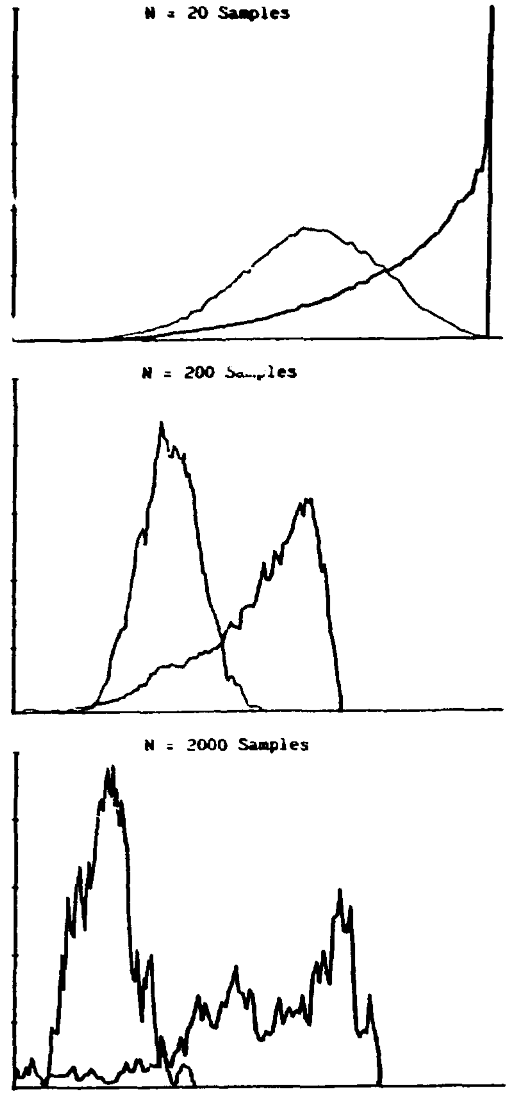

FIGURE 7. Separation of feature PDFs for 4th PDF cost function: various N.
Karlsruhe benchmark test file 2

In order to compare the results on a common base the decision boundry was adjusted to minimise the sum of S + M. The result of this comparison is given in figure 8.

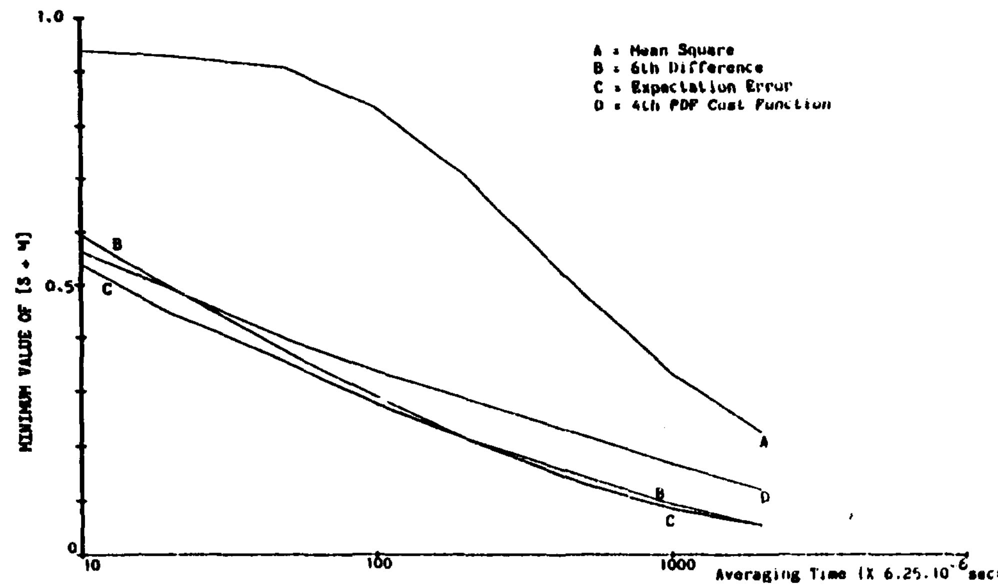

FIGURE 8. Comparison of 4 different signal processing strategies.
Karlsruhe benchmark test file 2

4.0 Preliminary Analysis of the BOR 60 Tapes

The basic idea in examining the BOR 60 tapes was to test whether or not the signal processing strategies outlined in section 2.0 of this report were valid and to examine if the concept of evaluating the quality of the strategy was consistant with results obtained from the Karlsruhe data and shown in figure 8.

For the purposes of the preliminary testing of the two hypotheses indicated above the four signals on tape II were chosen for analysis. Tape II was a complete run in which a transition from non-boiling to boiling was defined to have taken place.

The RMS, averaged over 0.06 seconds, for each of the signals from the four sensors is shown in figure 9 covering the whole of the data on the tape. It will be noted that both pressure transducers behave in similar

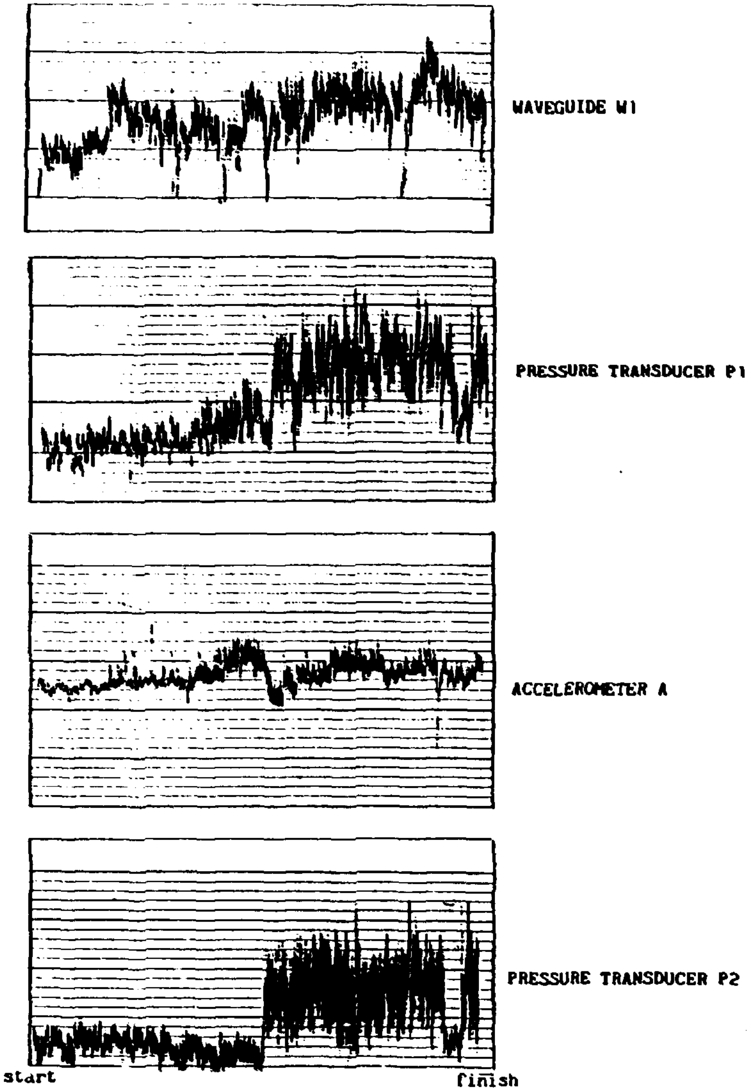

FIGURE 9 · RMS signals from BOR 60 Tape II (file 4) - Averaging time 0.06 secs.

ways suggesting that boiling commenced about half way through the record and ceased for a short while towards the end. The data from the waveguide and accelerometer appeared to be inconclusive.

Figures 10 and 11 show the evolution of the Power Spectral Density as the reactor power is raised, again there are significant changes to the spectra for the two pressure transducers but little strong evidence in the spectra associated with the waveguide or the accelerometer.

On the basis of this preliminary examination of the data, it was decided to analyse the output of pressure transducer 2 in some detail.

4.1 Analogue Results with 4th Order Differentiation

On the basis of work reported in section 3 above, in which the strategies defined as nth order differentiation, conditional expectation and nth order probability density function were shown to have similar discrimination and quality measures, it was decided to commence with an examination of the 4th order differentiation method as being the easiest to implement.

Figure 12 shows the RMS of the 4th order differential for four representative averaging times in the range 0.06 to 1.20 seconds. As predicted the discrimination is adequate to estimate the epoch of the commencement of boiling and the reduction of the variance of the estimate is evident as the averaging time is increased.

Figures 13 and 14 show the data in the probability density and the cumulative probability domain based on samples covering the whole of the non-boiling and boiling sections respectively. These data were obtained with a number of averaging times over the range 0.042 ms to 103.4 ms. The overlaps of the pdfs shown in figure 13 indicate a consistency with the ideas on quality discussed in section 3 and illustrated diagrammatically in figure 1.

Because of the self defined scaling of the cumulative probability function (its asymptote is 1.0), figure 14 was used to estimate the position of the decision boundry which minimises the sum of S (the probability of a spurious trip) and M (the probability of missing a valid event). The value of the minimisation of (S+M) was also obtained from this figure.

It is emphasised that the minimisation of (S+M) was arbitrarily chosen by the authors as giving equal weight to S and M. Clearly, operational considerations may dictate otherwise in which case it would be a trivial extension of the present concept to seek to minimise $\alpha S + \beta M$ where α and β are defined weights. It is believed, however, that for the purpose of comparing the quality of different signal processing strategies an equal weight model is valid and has the advantage of being simple to apply.

The result of minimising (S+M) over the range of averaging times is shown in figure 15 which is of similar form to figure 8 illustrating the data from the Karlsruhe tests.

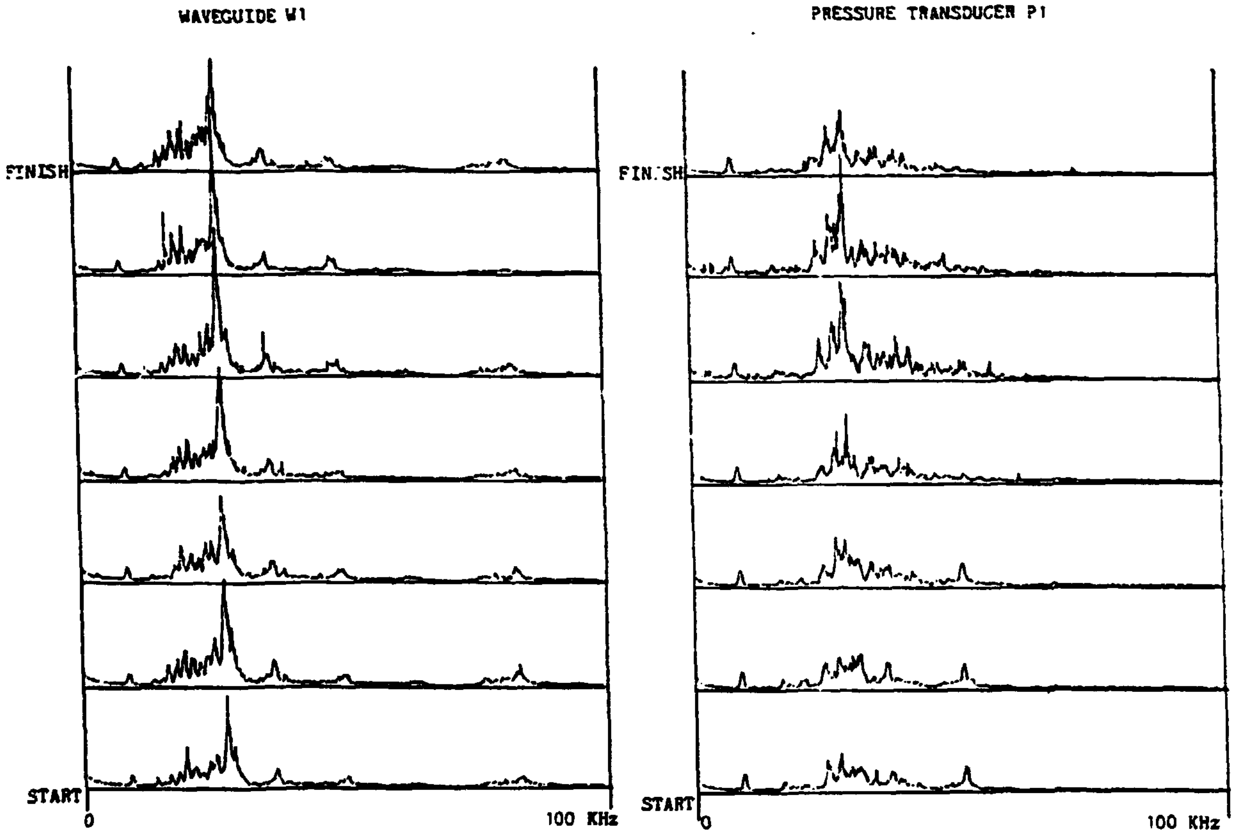

FIGURE 10: Signal Spectrum Development from Start of BOR 60 Tape II to finish of record.

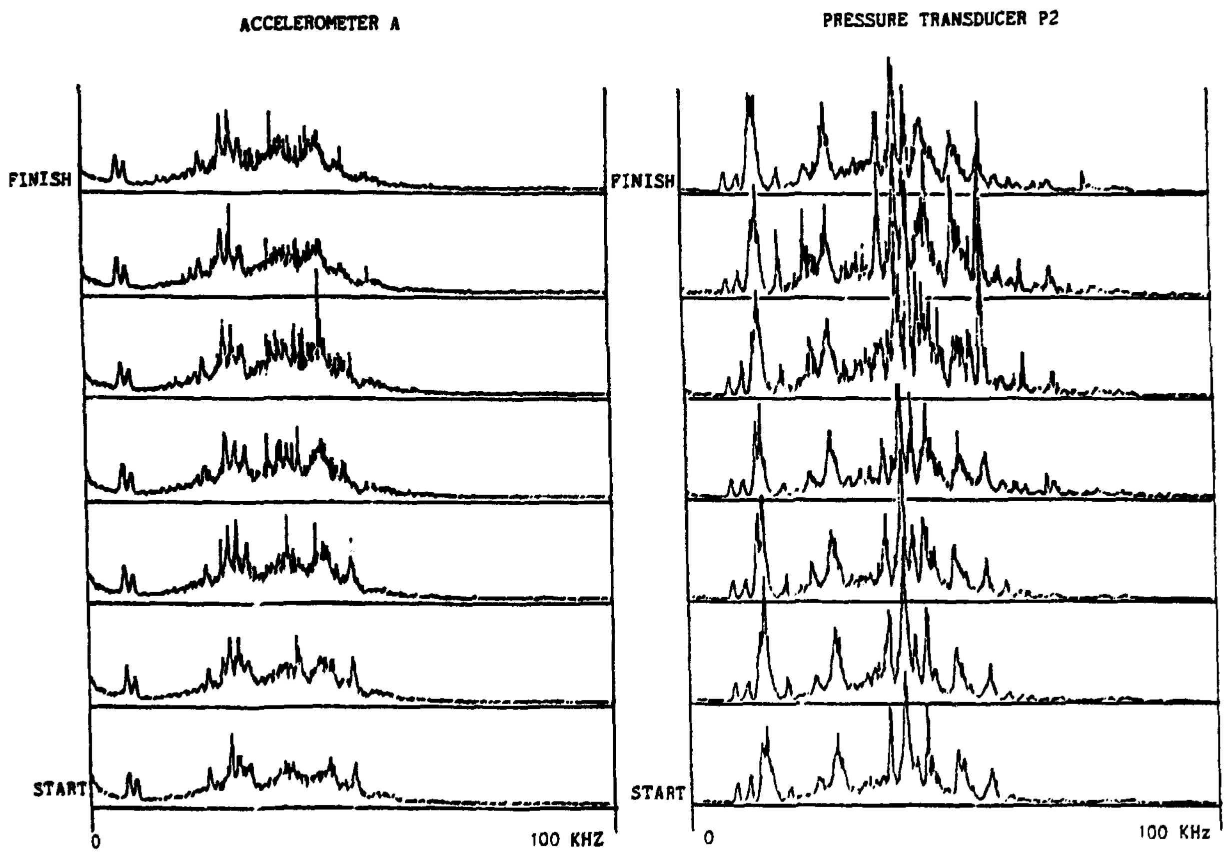

FIGURE 11: Signal Spectrum Development from Start of BOR 60 Tape II to finish of record

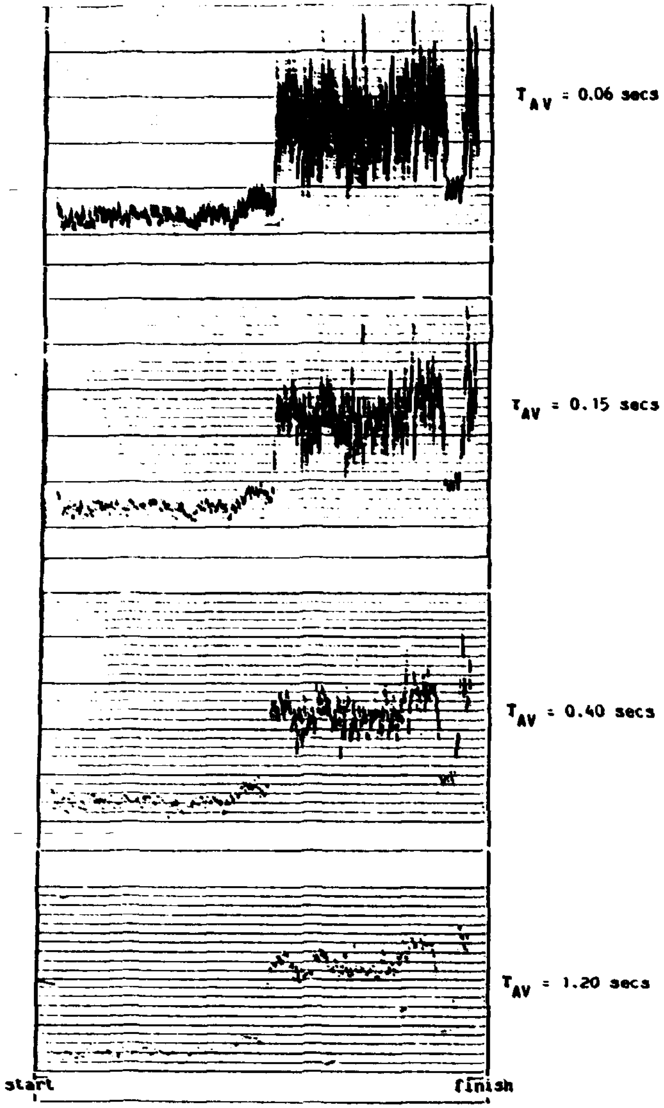

FIGURE 12:

Pressure transducer P2 (BOR 60 Tape II): RMS signal with 4th order differentiation - various averaging times.

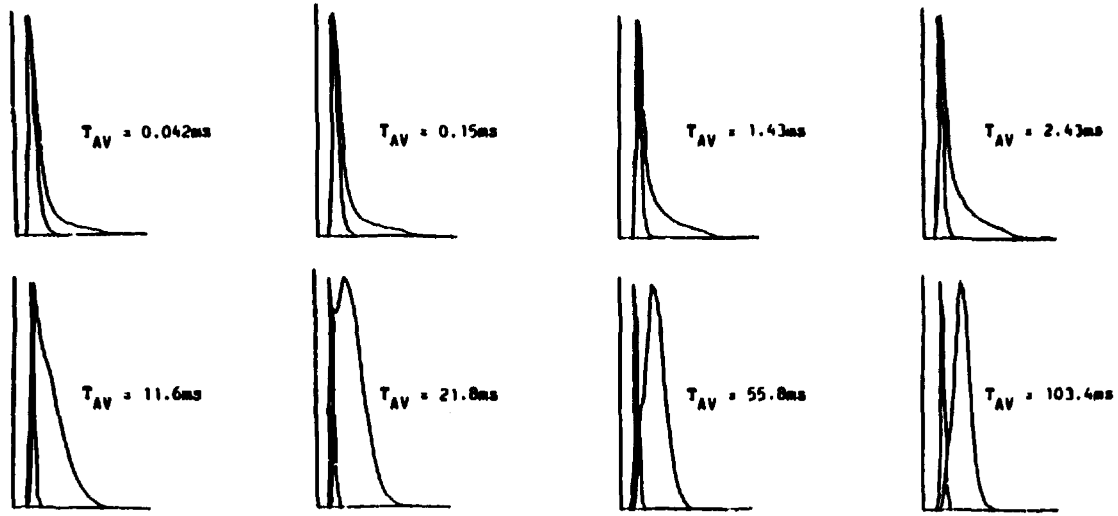

FIGURE 13: Pressure transducer P2 (BOR 60 Tape II) with 4th order differentiation: Separation of flow (a) and boiling (b) feature PDFs with increased averaging time.

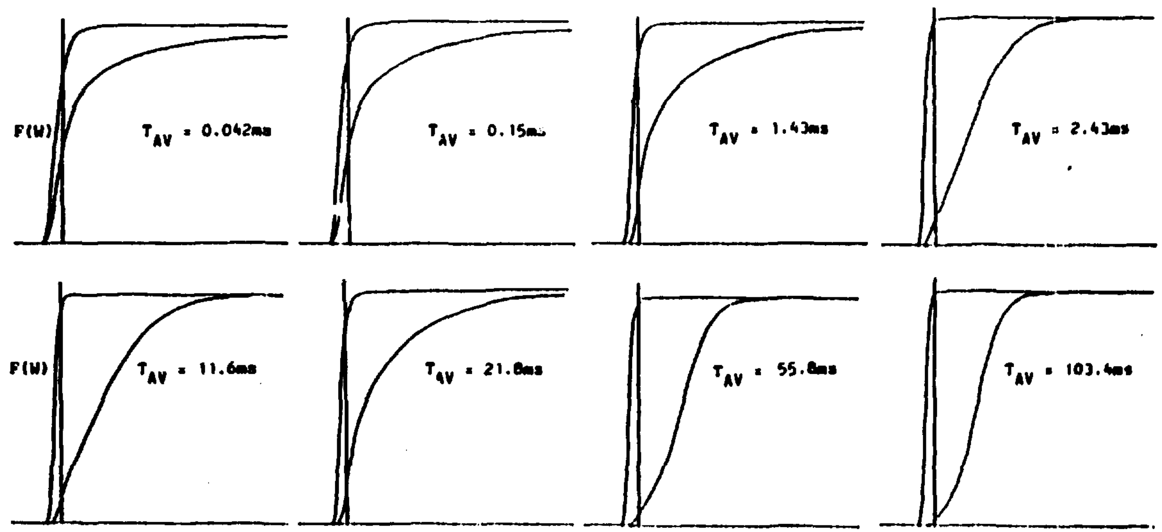

FIGURE 14: Transducer Ps (BOR 60 Tape II) with 4th order differentiation: Cumulative feature PDFs showing decision boundry W1 for minimum (S+M).

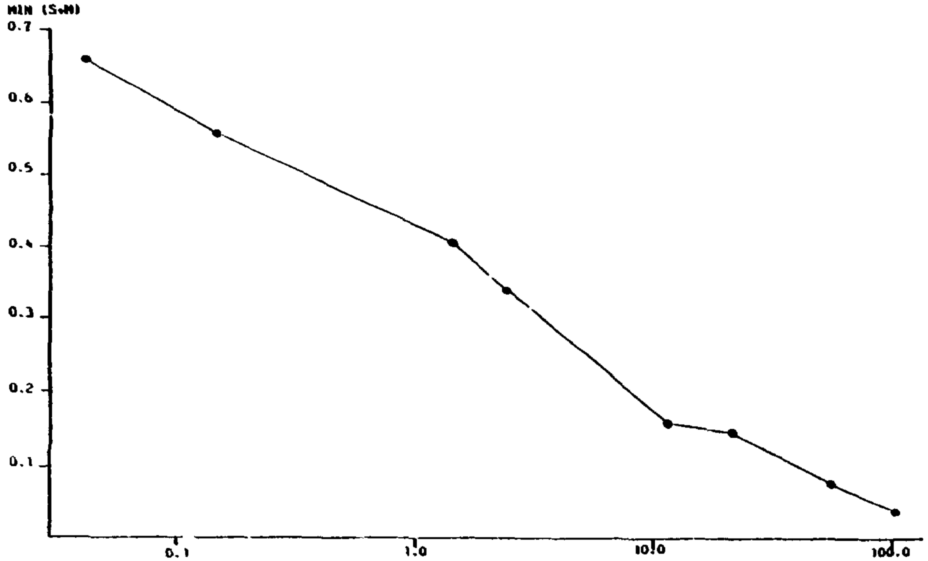

FIGURE 15:

Transducer P2 (BOR 60 Tape II) with 4th order differentiation: Minimum (S+M) vs Averaging Time

5. Conclusions

The tentative conclusions that may be reached on the basis of the work reported so far are:

- A map of the minimum value of (S+M) is a reasonable means by which the quality of different signal processing strategies may be compared.
- The 4th (or higher order) differential method or its related methods reported herein are adequate for the detection of boiling using the pressure transducer number 2.
- The minimum value of (S+M) can be made as small as we choose by increasing the averaging time over which the estimates of the output of the decision making algorithm are determined.

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