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Finite Temperature CP^{N-1} Model and Long Range Néel Order

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Abstract

We study in d space-dimensions the finite temperature behavior of long range Néel order (LRNO) in CP^{N-1} model as a low energy effective field theory of the antiferromagnetic Heisenberg model. For $d \leq 1$, or $d \leq 2$ at any nonzero temperature, LRNO disappears, in agreement with Mermin-Wagner-Coleman's theorem. For $d = 3$ in the weak coupling region, LRNO exists below the critical temperature T_N (Néel temperature). T_N decreases as the interlayer coupling becomes relatively weak compared with that within $Cu - O$ layers.

1. Introduction

It is broadly viewd that the spin degree of freedom of electrons on the two-dimensional $Cu-O$ planes may play an essential role in high T_c superconductivity. Many people have therefore concentrated on the properties of antiferromagnetic Heisenberg model in two dimensions.

Some recent studies have shown that this lattice model corresponds, in the continuum, to CP^1 nonlinear σ model as a low energy effective field theory [1] . It is expected that the long-range behavior of the original model could be well described by this more tractable field theoretic model. For example, Chakravarty et al [2] claim that the result of low-temperature renormalization-group analysis of $O(3)$ nonlinear σ model in two space dimensions can be fitted quantitatively with the recent neutron experiments on La_2CuO_4 [3] .

On the other hand, to the Heisenberg model in space-dimensions $d \leq 2$ Mermin- Wagner's theorem [4] applies, and so the spontaneous break down or the appearance of long range order cannot occur at any finite temperature. However, experiments have indeed shown on some dopeless materials that antiferromagnetic long range Néel order (LRNO) does exist below some critical temperature T_N (Néel temperature). This, we think, is the indication that the *third*-space dimension is important for the occurrence of real phenomena in solids such as the presence of LRNO or superconductive state at *finite* temperature.

In this letter, we study in d space-dimensions the finite temperature behavior of LRNO in CP^1 (or CP^{N-1}) model as an effective field theory of the quantum antiferromagnets. Based on an effective potential and a stationary-phase approximation [5,6], we show for $d \leq 2$ how the Mermin-Wagner's theorem manifests itself in the effective field theory. For $d = 3$, extending the model to an anisotropic one, we examine the conditions under which LRNO can appear.

The thermodynamic partition function of $(CP^{N-1})_{d+1}$ model is written as a

Euclidian path integral:

$$Z(\beta) = \int Dn D\overline{n} D\sigma DA_\mu exp\{-\int_0^\beta d\tau \int d^dx[|D_\mu n|^2 + \sigma(|n|^2 - \frac{N}{2f})]\} \quad (1)$$

$$\beta = 1/k_B T, \quad n = (n_1(x), \cdots, n_N(x)), \quad D_\mu n = (\partial_\mu + iA_\mu)n.$$

where the fields are periodic functions of τ. Integrating out the first $N-1$ components of $n(\overline{n})$ fields, we obtain an effective action of n_N, $\overline{n}_N$ and auxiliary fields:

$$S_{eff}(n_N, \overline{n}_N, \sigma, A_\mu) = N\int_0^\beta d\tau \int d^dx[|D_\mu n_N|^2 + \sigma(|n_N|^2 - \frac{1}{2f})] + (N-1)\,\mathrm{Tr}\ln\Delta, \quad (2)$$

where $\Delta \equiv -D_\mu D_\mu^* + \sigma$ and the replacement $n_N \to \sqrt{N}n_N, \overline{n}_N \to \sqrt{N}\overline{n}_N$ has been made. To evaluate the last term on the right hand side (r.h.s.) of (2), it is convenient to write Δ as

$$\Delta_x \delta^{d+1}(x-y) = -\Box_x[\delta^{d+1}(x-y) + (x|L_\beta|y)], \quad (3)$$

where

$$(x|L_\beta|y) = G_\beta(x-y)[i\partial_\mu A_\mu + A_\mu A_\mu + \sigma](y) + 2iA_\mu(y)\partial_\mu^y G_\beta(x-y). \quad (4)$$

Here $G_\beta(x-y)$ is the finite temperature Green's function defined by

$$G_\beta(x_1 - x_2) = \frac{1}{\beta}\sum_{\ell=-\infty}^{\infty}\int \frac{d^d\mathbf{k}}{(2\pi)^d} e^{i\omega_\ell(\tau_1-\tau_2)} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}(\omega_\ell^2 + \mathbf{k}^2)^{-1}, \quad (5)$$

with $\omega_\ell = 2\pi\ell/\beta$ (ℓ : integer). In the following we consider only the constant configurations of $n_N(\overline{n}_N)$ and σ and set $n_N = n_{N_c}, \overline{n}_N = \overline{n}_{N_c}, \sigma = \sigma_c, A_\mu = 0$.

Using (3) ~ (5) we have

$$\begin{aligned}\mathrm{Tr}\ln\Delta+\mathrm{Tr}\ln\Box&=\mathrm{Tr}\ln[\delta^{(d+1)}(x-y)+\langle x|\,L_\beta(\sigma_c)\,|y\rangle]\\&=\sum_{m=1}^{\infty}\frac{(-)^{m-1}}{m}(\sigma_c)^m\int_0^\beta d\tau_1\int d^dx_1\cdots\int_0^\beta d\tau_m\int d^dx_m G_\beta(x_1-x_2)\cdots G_\beta(x_m-x_1)\\&=\left(\int d^dx\right)\sum_{\ell=-\infty}^{\infty}\int\frac{d^d\mathbf{k}}{(2\pi)^d}\ln\left(1+\frac{\sigma_c}{\mathbf{k}^2+\omega_\ell^2}\right).\end{aligned}\tag{6}$$

Thus, up to the constant, we obtain an effective potential for $|n_N|$ and σ:

$$\begin{aligned}&V_{eff}(|n_{N_c}|,\sigma_c)\\&=N\sigma_c(|n_{N_c}|^2-\frac{1}{2f})+(N-1)\frac{1}{\beta}\sum_{\ell=-\infty}^{\infty}\int\frac{d^d\mathbf{k}}{(2\pi)^d}\ln\left(1+\frac{\sigma_c}{\mathbf{k}^2+\omega_\ell^2}\right).\end{aligned}\tag{7}$$

From this, the stationary-phase conditions for σ_c and $\overline{n}_{N_c}$ follow easily

$$\begin{aligned}0&=\frac{1}{N}\frac{\partial V_{eff}}{\partial\sigma_c}\\&=|n_{N_c}|^2-\frac{1}{2f}+\frac{N-1}{N}\frac{1}{\beta}\sum_{\ell=-\infty}^{\infty}\int\frac{d^d\mathbf{k}}{(2\pi)^d}\frac{1}{\omega_\ell^2+\mathbf{k}^2+\sigma_c},\end{aligned}\tag{8}$$

$$0=\frac{1}{N}\frac{\partial V_{eff}}{\partial\overline{n}_{N_c}}=\sigma_c n_{N_c}.\tag{9}$$

From (9) we see that the presence of LRNO (nonzero n_{N_c}) corresponds to that of $\sigma_c=0$ solution. Eq. (8) is the condition that the tree and one-loop σ-tadpole should cancell (stationary phase). We note that in the third term on the right-hand side (r.h.s.) of (8), σ_c plays the role of infrared cut-off. Using the standard trick of finite temperature, this term can be rewritten as the sum of contributions from zero and finite temperature parts:

$$\frac{1}{\beta}\sum_{\ell=-\infty}^{\infty}\int\frac{d^d\mathbf{k}}{(2\pi)^d}\frac{1}{\omega_\ell^2+\mathbf{k}^2+\sigma_c}=\int\frac{d^{d+1}k}{(2\pi)^{d+1}}\frac{1}{k^2+\sigma_c}+\int\frac{d^d\mathbf{k}}{(2\pi)^d}\frac{1}{\omega(e^{\beta\omega}-1)}, \tag{10}$$

with $\omega^2\equiv\mathbf{k}^2+\sigma_c$.

The ultraviolet divergence formally appears for $d\geq 1$ in the first (zero-temperature part) integral on the r.h.s.of (10). But this divergence is regularized by the momentum cut-off Λ related with the lattice spacing($=a$) of the original crystal model as $\Lambda a\sim O(1)$ (the precise value of which depends on the shape of the lattice and the space-dimensions). The cut-off dependence can be absorbed into the coupling constant renormalization:$N^{-1}\partial V_{eff}/\partial\sigma_c(\sigma_c=M^2,|n_{N_c}|=T=0)=-M^{d-2}/2f_R$ (M:a renormalization point, f_R:a dimensionless renormalized coupling) when $d<3$.

On the other hand, if we put $\sigma_c=0$, the case we are interested in, the infrared divergence appears in the first integral when $d\leq 1$ and in the second (finite temperature part) when $d\leq 2$. Hence, for $d\leq 1$, or $d\leq 2$ at any finite temperature, any solution with $\sigma_c=0$ is absent in (8). This is the manifestation of Mermin-Wagner-Coleman's theorem [4], [7] .

Let us more explicitly see the $d=2$ case. After integration and renormalization eq.(8) becomes for $d=2$

$$0=|n_{N_c}|^2-\frac{M}{2}\Big(\frac{1}{f_R}-\frac{N-1}{N}\frac{1}{2\pi}\Big)-\frac{N-1}{N}\frac{\sqrt{\sigma_c}}{4\pi}-\frac{N-1}{N}\frac{1}{2\pi\beta}\ln\Big(1-e^{-\beta\sqrt{\sigma_c}}\Big), \tag{11}$$

where for simplicity we have taken the infinite cut-off limit. In the absence of the last term, that is, $T=0$, we find the following solutions [6] ;(i)for $f_R>\frac{2N\pi}{N-1}$, $|n_{N_c}|=0,\sqrt{\sigma}_c=M(1-\frac{N}{N-1}\frac{2\pi}{f_R})$ (symmetric phase) ;(ii)for $f_R=\frac{2N\pi}{N-1}$, $|n_{N_c}|=\sigma_c=0$;(iii)for $f_R<\frac{2N\pi}{N-1}$, $|n_{N_c}|^2=\frac{M}{2}(\frac{1}{f_R}-\frac{N-1}{N}\frac{1}{2\pi}),\sigma_c=0$ (broken phase). Here $f_R = \frac{2N\pi}{N-1}$, or $\hat{f}(\equiv f\Lambda) = \hat{f}_c = \frac{2N\pi}{N-1}$ is the critical coupling. LRNO exists in the weak coupling region (*iii*).

However once we incorpolate the finite temperature effect, the last term of eq.(11)with $\sigma_c = 0$ diverges logarithmically and gives no solution with $\sigma_c = 0$. LRNO is absent in any coupling region. As seen before this catastrophe is due to the infrared divergence in the finite temperature one-loop integral. It appears because of the presence of $n_i (i = 1, \cdots, N-1)$ zero mode $\omega_{\ell=0} = 0$. We note that n_i is a Goldstone boson.

This is in sharp contrast to the similar analyses of four-fermi models [8] where the infrared divergence does not appear at finite temperature since fermions obey the antiperiodic boundary condition and do not have zero frequency mode. $\sigma_c = 0$ phase observed at zero-temperature persists to some finite critical temperature where the phase transition occurs.

In our bosonic model, however, the thermal phase transition occurs exactly at $T = 0$. The value of σ_c at finite temperature is obtained universally by

$$\sigma_c(\beta) = M\left(\frac{N}{N-1}\frac{2\pi}{f_R} - 1\right) - \frac{2}{\beta}\ln\left(\frac{-1 + [1 + 4e^{-\beta M(1 - \frac{N}{N-1}\frac{2\pi}{f_R})}]^{\frac{1}{2}}}{2}\right).$$

To conclude the $d \leq 2$ case, our effective theory with a mean-field type approximation shows no LRNO (no spontaneous breakdown of $SU(N)$) when $d \leq 1$, or $d \leq 2$ at any nonzero temperature. The result is consistent with the Mernin-Wagner's rigorous theorem about the absence of long range order in the Heisenberg spin model [4]. (The $d = 1$ case corresponds to the Coleman's theorem[7].)

Then we are naturally led to suppose that the *third* space-dimension may be important for the real appearance of LRNO observed experimentally in some dopeless materials. We are now urged to extend our analyses to the $d = 3$

case where the infrared divergence is absent in the one-loop integral (10). But before proceeding to the concrete analysis, we should recall that in real metarials showing the superconductivity such as La_2CuO_4 the interlayer coupling $(= f_z^{-1})$ between $x-y$ planes ($Cu-O$ layers) is known from the experiments to be very weak compared with that$(= f^{-1})$ within planes. (In ref. [2], for example, the ratio has been estimated to be of order 10^{-5}.) Taking this fact into consideration we extend our model to a following general anisotropic one:

$$\mathcal{L} = \sum_{j=x,y,\tau} |D_j n|^2 + \frac{f}{f_z}|D_z n|^2 + \sigma(|n|^2 - \frac{N}{2f}). \qquad (12)$$

In this model the stationary-phase equation (8) is replaced by

$$0 = |n_{N_c}|^2 - \frac{1}{2f} + \frac{N-1}{N\beta}\sum_{\ell=-\infty}^{\infty}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{\omega_\ell^2 + k_1^2 + k_2^2 + \alpha k_3^2 + \sigma_c}, \qquad (13)$$

where we have set $f_z = \alpha^{-1} f$ $(0 \le \alpha \le 1)$. Evaluating the integrals gives

$$\begin{aligned} 0 =& |n_{N_c}|^2 - \frac{1}{2f} + \frac{N-1}{N}\frac{1}{8\pi^2}[\Lambda^2 A(\alpha,\gamma) + B(\sigma_c,\Lambda,\alpha,\gamma)] \\ &+ \frac{N-1}{N}[\frac{\alpha^{-\frac{1}{2}}}{12\beta^2} + C(\sigma_c,\beta,\Lambda,\alpha,\gamma)], \end{aligned} \qquad (14)$$

where

$$\begin{aligned} &A(\alpha,\gamma) = \gamma^{\frac{1}{2}}(\sqrt{\alpha\gamma+1} - \sqrt{\alpha\gamma}) + \alpha^{-\frac{1}{2}}\log(\sqrt{\alpha\gamma+1} + \sqrt{\alpha\gamma}), \\ &B(0,\Lambda,\alpha,\gamma) = C(0,\beta,\infty,\alpha,\gamma) = 0. \end{aligned} \qquad (15)$$

Here, we have introduced the two-dimensionally symmetric cut-off Λ in the k_1, k_2 integral and $\Lambda_3(\equiv \gamma^{\frac{1}{2}}\Lambda,\ \gamma^{\frac{1}{2}} \sim O(1))$ in the k_3 integral.

In the following we will restrict ourselves to the $\sigma_c = 0$ phase where LRNO could appear. In this case the term $B(\sigma_c,\Lambda,\alpha,\gamma)$ vanishes, and we will neglect the term $C(0, \beta, \Lambda, \alpha, \gamma)$ appearing in the finite-temperature part of the one-loop integral, which vanishes if we let Λ go to infinity. This approximation is valid unless $\beta\hbar c\alpha^{\frac{1}{2}}\Lambda_3$ is very small, where c is the spin wave velocity and we have set $\hbar c = 1$ so far. Then renormalizing the remaining quadratic cut-off dependence by the condition:

$$\frac{1}{N}\frac{\partial V_{eff}}{\partial \sigma_c}\bigg|_{\sigma_c = n_{N_c} = T = 0} = -\frac{1}{2f_R}, \tag{16}$$

where f_R is a dimensional renormalized coupling in $x - y$ plane, we can write (14) as

$$|n_{N_c}|^2 = \frac{1}{2f_R} - \frac{N-1}{N}\frac{\alpha^{\frac{1}{2}}}{12\beta^2}. \tag{17}$$

For this equation to be meaningful f_R needs to be positive. This means

$$\hat{f} < \hat{f}_c \equiv \frac{N}{N-1}\frac{8\pi^2}{A(\alpha, \gamma)}, \tag{18}$$

with $\hat{f} \equiv f\Lambda$. Note that $\hat{f}_c$ has a finite limit if α goes to zero. When the above condition is satisfied, we find from (17) that LRNO exists if and only if

$$T < T_N \equiv \alpha^{\frac{1}{4}}\sqrt{\frac{6N}{N-1}}\sqrt{\frac{1}{\hat{f}} - \frac{1}{\hat{f}_c}}\frac{\Lambda}{k_B}, \tag{19}$$

where T_N is the so called Néel temperature and is well defined if the coupling $\hat{f}$ is so weak to satisfy (18). It follows from (19) and (18) that T_N decreases as α becomes small, i.e., the interlayer coupling becomes relatively small compared with that within $Cu - O$ layers. We note that although eq.(19) holds approximately *only* when α is not very small, T_N indeed goes to zero in the limit $\alpha \to 0$, as is seen in eq.(13) from the beginning. In the $\alpha = 0$ limit the stationary-phase condition (13) becomes essentially the one in $(2 + 1)$ dimensions.

LRNO is given by

$$|n_{N,}|^2 = \frac{k_B^2}{12\alpha^{\frac{1}{3}}}(T_N^2 - T^2). \tag{20}$$

To summarize, we have studied in d space-dimensions the finite temperature behavior of LRNO in CP^{N-1} model as an effective field theory of the antiferromagnetic Heisenberg model. Using the stationary-phase method, we have shown that LRNO exists at $d = 2(T = 0)$ or $d = 3(0 \leq T < T_N)$ in the weak coupling region. For $d = 3$ we have obtained the conditions under which LRNO exists. The results are summarized in eqs. (18)$\sim$ (20).

In the rest we add some comments.

In this letter we have so far considered only the dopeless case. We comment that the doping effects on the spin dynamics can be incorpolated in our analysis by replacing the coupling $\hat{f}$ with an effective coupling $\hat{f}(\delta)$, where δ is the concentration of holes and $\hat{f}(\delta)$ can be calculated from the microscopic analysis of Hubbard mod l [9] .

For $d = 3$, to make our discussion simple, we have neglected the existence of C term in eq.(14). Hence the eqs.(19) and (20) are approximately good only when α is not very small. For the range $\alpha = 10^{-5} \sim 10^{-2}$ where the comparison with experiments is possible, the C term in (13) may not be negligible. More quantitative arguments near $\alpha = 0$ including the doping effect, needs therefore more careful treatment of evaluating the finite-temperature part of the one-loop integral in (13), and will appear in the forthcoming paper.

Although a two-dimensional $Cu - O$ plane is said to be important in high-T_c superconductivity, our results of field t ory analysis implies that the *three*-dimensional effect is also indispensable to the spontaneous symmetry breaking (LRNO or superconductive state) at *fi ite* temperature.

We have found that the spontaneous breakdown occurs for $d = 3$ in the low temperature, weak coupling region. The results may support the general expectation that the low-temperature phase in three-space dimensions would qualitatively be the same with the zero-temperature phase in two dimensions. Hence we could say that for simplicity we should first study the $d = 2$, $T = 0$ case in order to approach the high-T_c superconductivity, as is thought by many people.

Note added: After we have written up this work we received a preprint by Rosenstein, Warr and Park [10] in which the thermodynamics of the (2+1) dimensional $O(N)$ invariant sigma-model including the $T_c = 0^+$ transition is discussed. They claim the similar conclusion to ours but do not mention about the (3+1) dimensional case.

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