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FORMALISM**

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THERMAL CONDUCTION

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REACTOR KINETICS

REACTOR KINETICS EQUATIONS

REACTIVITY

MATHEMATICAL MODELS

ONE-DIMENSIONAL CALCULATIONS

MESH GENERATION

FINITE DIFFERENCE METHOD

BOUNDARY CONDITIONS

NARORA-1 REACTOR

NARORA-2 REACTOR

REACTOR ACCIDENTS

POINT-KINETICS MODEL WITH ONE-DIMENSIONAL (RADIAL) HEAT CONDUCTION FORMALISM

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ABSTRACT

A point-kinetics model with one-dimensional (radial) heat conduction formalism has been developed. The heat conduction formalism is based on corner-mesh finite difference method. To get average temperatures in various conducting regions, a novel weighting scheme has been devised. The heat conduction model has been incorporated in the point-kinetics code MRIF-FUEL. The point-kinetics equations are solved using the method of real integrating factors. It has been shown by analysing the simulation of hypothetical loss of regulation accident in NAPP reactor that the model is superior to the conventional one in accuracy and speed of computation.

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1. INTRODUCTION

An accurate prediction of various temperatures in a nuclear reactor in an accidental condition is very important from the point of view of reactor safety. The centre-line fuel temperature, for instance, decides the process of fuel melting. The phenomena of coolant-clad interaction and coolant voiding are decided by coolant and clad temperatures. Doppler feedback reactivity, an important parameter from the point of view of power and temperature rise, depends upon the average temperatures in various media, particularly fuel.

A one-dimensional (radial) heat conduction model based on the corner-mesh finite difference scheme has been developed to get detailed temperature distribution in fuel, clad and coolant of a nuclear reactor. This model has been incorporated in the point-kinetics code MRIF-FUEL⁽¹⁾. The code solves the point-kinetics equations with temperature feedback in a reactivity transient in a reactor using the method of real integrating factors⁽²⁾.

The point-kinetics equations are described in Section 2. The heat conduction model is highlighted in Section 3. The corner mesh finite difference equations are derived in Section 4. The model to get average temperatures from the nodal temperatures is described in Section 5. To test the effectiveness of the model, we have selected two examples of simulating loss of regulation accident⁽³⁾ in NAPP reactors. The test-cases are described in Section 6. The results are discussed in Section 7. Finally, some conclusions are drawn.

2. POINT-KINETICS EQUATIONS

The point-kinetics equations are given by

$$dn(t)/dt = (\rho(t) - \beta)n(t)/\lambda + \sum_{i=1}^G \lambda_i C_i(t) + S_0 \quad (1a)$$

and

$$dC_i(t)/dt = \beta_i n(t)/\lambda + \lambda_i C_i(t) \quad (1b)$$

$i = 1, 2, \dots, G$

where G is the total number of delayed neutron groups and

$n(t)$ = amplitude factor

$\rho(t)$ = net reactivity

β = total delayed neutron fraction = $\sum_{i=1}^G \beta_i$

β_i = delayed neutron fraction for i^{th} delayed neutron group

λ_i = decay constant for i^{th} delayed neutron group

S_0 = external source strength

The net reactivity $\rho(t)$ is given by

$$\rho(t) = \rho_0 + \rho_{ex}(t) + \rho_{trip}(t) + \rho_{fb}(t) \quad (2)$$

where

ρ_0 = initial reactivity

$\rho_{ex}(t)$ = external reactivity

$\rho_{trip}(t)$ = trip reactivity

$\rho_{fb}(t)$ = feedback reactivity

The feedback reactivity is given by

$$\rho_{fb}(t) = \sum_{j=1}^{NT} a_j [\tau_j(t) - \tau_j(0)] \quad (3)$$

where a_j is the feedback coefficient of reactivity due to j^{th} temperature. NT is the total number of temperatures T_j . The temperatures are given by the heat balance equation

$$dT(t)/dt = AT(t) + \gamma n(t) + d \quad (4)$$

where

- T = temperature vector of order NT
- γ = power distribution vector of order NT
- d = source vector of order NT
- A = square matrix of order $NT \times NT$

The point kinetics equations are solved by the method of real integrating factors⁽²⁾.

3. HEAT CONDUCTION MODEL

The fuel assembly in a nuclear reactor consists of cylindrical rods of fissile material like UO_2 or metallic uranium sheathed in a cladding material like zircalloy, aluminium or steel. Between clad and fuel, there is some gap to accommodate fission gases. The sheath is surrounded by coolant to take away heat generated in the fuel rods.

The time-dependent heat conduction equation in one-dimensional cylindrical geometry is given by

$$\rho c_p (\partial/\partial t)T(r,t) = q(r,t) + (1/r)(\partial/\partial r)[rk(\partial/\partial r)T(r,t)] \quad (5)$$

where, ρ , c_p , and k represent respectively the material density, specific heat at constant pressure, and thermal conductivity at the space-point r . $q(r,t)$ and $T(r,t)$ represent respectively the volumetric heat generation rate and the temperature at the point r at time t . In a conducting medium the temperature

is continuous. At the interface of two conducting materials A and B the temperature satisfies the condition

$$k_A (\partial T / \partial r)_A = k_B (\partial T / \partial r)_B \quad (6)$$

The derivatives are taken at the two sides of the interface. At the interface of a conducting medium and convective(gap) material the boundary condition is given by

$$k_A (\partial T / \partial r)_A = h(T_s - T_o) \quad (7)$$

where, the derivative is taken at the interface r_s . 'h' gives the heat transfer coefficient (gap conductance) for the material. T_s and T_o represent the temperatures at the interface and bulk (the other side of the gap). At the outer surface, two types of boundary conditions can be used. They are:

- (i) Outer surface temperature is specified,
- (ii) The coolant is flowing with a specified speed and inlet temperature.

4. CORNER-MESH APPROACH

The one-dimensional (radial) heat conduction model is based on the corner mesh finite difference scheme to discretize the space variable in time-dependent heat conduction equation. The corner mesh scheme has been preferred as it gives better information about nodal temperatures. To get the finite difference form of the heat conduction equation in space, the space-domain is divided into subregions, called meshes. The material properties (ρ , c_p , k etc,) are assumed to be uniform within each mesh. There may be any number of meshes in a conducting medium but the convective(gap) material may have only one mesh per material. There can not be two convective materials side by side.

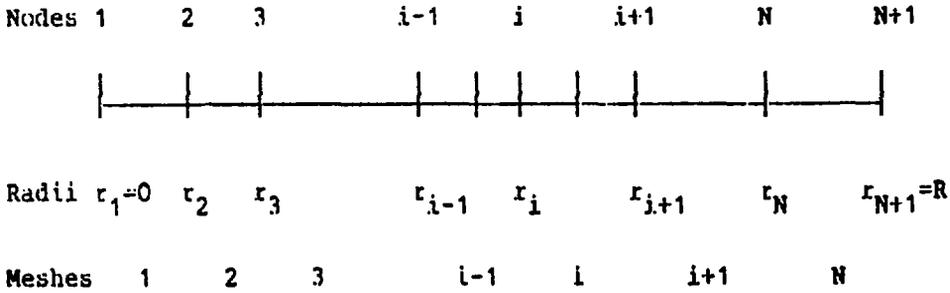


Fig. 1. One-dimensional Cylindrical Geometry

We divide the domain (0 to R) in N meshes as shown in Fig. 1. Thus we will have N+1 endpoints. The temperatures are defined at these endpoints, commonly known as nodes. For example, T_i represents the temperature at the node i having the coordinate r_i and situated at the left-hand endpoint of the i^{th} mesh.

To get the finite difference equation for the temperature T_i , we multiply the equation (5) by $2\pi r$ and integrate it from $r=r_{i-1/2}$ to $r=r_{i+1/2}$, where, $r_{i+1/2}=(r_i+r_{i+1})/2$ denotes the coordinate of the midpoint of the mesh i. We obtain

$$[(\rho c_p v^R)_{i-1} + (\rho c_p v^L)_i] dT_i / dt = [(q v^R)_{i-1} + (q v^L)_i] + k_i S_i^R (T_{i+1} - T_i) / \Delta r_i - k_{i-1} S_{i-1}^L (T_i - T_{i-1}) / \Delta r_{i-1} \quad (8)$$

where

$$v_i^J = \pi (r_{i+1/2}^2 - r_i^2) \quad (9)$$

$$v_i^R = \pi (r_{i+1}^2 - r_{i+1/2}^2) \quad (10)$$

$$\Delta r_i = r_{i+1} - r_i \quad (11)$$

$$S_i^J = 2\pi r_{i-1/2} \quad (12)$$

and

$$S_i^R = 2r_{i+1/2} \quad (13)$$

The subscript i on ρ , c_p , k , q , V_i^L , and V^R denotes that the properties refer to the mesh i . T_i denotes the temperature at the node i . S_i^L and S_i^R denote the surfaces at the midpoints of the mesh $i-1$ and i respectively. Depending on the properties of the meshes i and $i-1$, the following cases are to be treated separately.

4.1 CASE 1

First, we consider the equation for node 1. Here, there is no contribution from left hand side of node 1. The equation (5) is integrated from $r=r_i=0$ to $r=r_{i+1/2}$ only. The last term on the RHS of equation (8) is also not there as $r=0$. In this case we get

$$(\rho c_p V^L)_i (dT_i/dt) = (qV^L)_i + k_i S_i^R (T_{i+1} - T_i) / \Delta r_i \quad (14)$$

This can be made equivalent to the equation (8) by putting $V_{-1}^R=0$ and $S_1^L=0$.

4.2 CASE 2

In this case we consider the situation when both i^{th} and $(i-1)^{\text{th}}$ meshes have conducting materials. The differential equation for the temperature T_i is given by the equation (8).

4.3 CASE 3

In this case, we consider the situation when $(i-1)^{\text{th}}$ mesh has a conducting material whereas the mesh ' i ' has a nonconducting material. In this case, the integration of equation (5) is carried out from $r=r_{i-1/2}$ to $r=r_i$. We also make use of the interface boundary condition between a conducting and a nonconducting material represented by equation (7). Finally, we get

$$(\rho c_p V^R)_{i-1} dT_i/dt = (qV^R)_{i-1} + h_i 2\pi r_i (T_{i+1} - T_i) - k_{i-1} S_i^J (T_i - T_{i-1}) / \Delta r_{i-1} \quad (15)$$

This can be made equivalent to the equation (8) by defining

$$V_i^L = 0. \quad (16)$$

$$S_i^R = 2\pi r_i \quad (17)$$

and $k_i = h_i \Delta r_i \quad (18)$

4.4 CASE 4

In this case, we consider the situation when i^{th} mesh has a conducting material and $(i-1)^{\text{th}}$ mesh has a nonconducting material. The integration, in this case, is carried out from $r=r_i$ to $r=r_{i+1}/2$. After making use of equation (7), we get,

$$(\rho c_p V^J)_i (dT_i/dt) = (qV^J)_i + k_i S_i^R (T_{i+1} - T_i) / \Delta r_i - h_{i-1} 2\pi r_i (T_i - T_{i-1}) \quad (19)$$

This can be made equivalent to the equation (8) by defining

$$V_{i-1}^R = 0. \quad (20)$$

$$S_i^J = 2\pi r_i \quad (21)$$

$$k_{i-1} = h_{i-1} \Delta r_{i-1} \quad (22)$$

4.5 CASE 5

In this case, we consider the situation when boundary condition 1 (T_{N+1} is specified) is applicable. The equation for the temperature T_N remains identical to the equation (8), (15) or (19) depending on the properties of the $(N-1)^{\text{th}}$ and N^{th} meshes. The only difference is that now we will have only N

unknown temperatures. The temperature T_{N+1} is specified and so the term containing T_{N+1} is constant.

4.6 CASE 6

Finally we consider the situation when the boundary condition 2 is applicable. In this case the outermost mesh has a coolant material flowing with a specified speed and the inlet temperature of the coolant is also specified. In this case, the equations for T_i for $i=1,2,\dots,N$ are identical to the equation (8), (15) or (19) depending on the properties of the meshes. For $(N+1)^{th}$ node the coolant equation in lumped parameter form is used. The coolant temperature equation is given by

$$\rho_N C_{PN} V_N dT_{N+1}/dt = S_N^R h_N (T_N - T_{N+1}) - V_N u_N C_{PN} \partial T_{N+1}/\partial z \quad (23)$$

where V_N and u_N represent respectively volume per unit length, and speed of coolant. Writing $\partial T_i/\partial z$ as $(T_o - T_I)/l = 2(T_i - T_I)/l$ where the subscripts o and I stand for outlet and inlet conditions. l stands for length of the channel. Thus for coolant temperature the equation is given by

$$\rho_N C_{PN} V_N dT_{N+1}/dt = S_N^R h_N (T_N - T_{N+1}) - 2V_N u_N C_{PN} (T_{N+1} - T_I)/l \quad (24)$$

We have seen that in all the cases the time-dependent differential equation for the nodal temperature can be written in the form of equation (4). For cases 1 to 4, the elements of λ , γ and d are given by

$$\lambda_{i,i-1} = k_{i-1} S_i^L / \Delta r_{i-1} / \rho_i \quad (25)$$

$$\lambda_{i,i+1} = k_i S_i^R / \Delta r_i / \rho_i \quad (26)$$

$$\lambda_{i,i} = -(\lambda_{i,i-1} + \lambda_{i,i+1}) \quad (27)$$

$$\gamma_i = [(qV^R)_{i-1} + (qV^I)_i] / \rho_i \quad (28)$$

$$d_i = 0. \quad (29)$$

$$D_i = [(\rho c_p v^R)_{i-1} + (\rho c_p v^L)_i] \quad (30)$$

For case 5, the definitions for the elements of matrix A and column vectors γ and d remain unchanged except for the definition of d_N . There are N equations. In this case d_N is defined as

$$d_N = A_{N,N+1} T_{N+1} / R_N \quad (31)$$

For case 6, the total number of temperature equations are N+1. The elements of A, γ and d for nodes upto N remain unchanged. The coefficient for (N+1)th temperature equation are defined as

$$A_{N+1,N} = S_{N,N}^R h / (G_N V_N C_{PN}) \quad (32)$$

$$A_{N+1,N+1} = -A_{N+1,N} - 2u_N / l \quad (33)$$

$$\gamma_{N+1} = 0. \quad (34)$$

and
$$d_{N+1} = 2u_N / l \quad (35)$$

In the steady state condition the temperature distribution is given by the solution of

$$AT + \gamma n + d = 0 \quad (36)$$

This matrix equation is solved by the Gauss elimination or by the method of sweeps i.e. forward elimination and backward substitution.

5. AVERAGE TEMPERATURE MODEL

Doppler feedback reactivity, an important parameter from the point of view of power and temperature rise, depends upon the average temperatures in various media, particularly fuel. The fuel assembly in a nuclear reactor usually consists of cylindrical rods of fissile material sheathed in a cladding material. Between clad and fuel, there is some gap to accommodate fission gases. The sheath is surrounded by coolant to take away heat generated in the fuel rods.

The steady state temperature distribution can be obtained analytically for one-dimensional cylindrical geometry by solving equation (5) by putting $\partial T(r,t)/\partial t = 0$. The general solution is given by

$$T(r) = -(q(r)r^2/4k) + C_1 \ln(r) + C_2 \quad (37)$$

where C_1 and C_2 are some constants to be determined by boundary conditions. In the central region with uniform heat generation (which is the usual case in a nuclear reactor) the temperature distribution is given by

$$T(r) = T(0) - qr^2/(4k) \quad (38)$$

where $T(0)$ stands for the centre-line temperature. The average temperature between any two points r_i and r_{i+1} can be obtained by multiplying the equation (38) by $2\pi r$, integrating it between the two points and dividing by $\pi(r_{j+1}^2 - r_i^2)$. We get

$$\langle T \rangle = T(0) - (r_i^2 + r_{i+1}^2)q/(8k) \quad (39)$$

which can be rewritten as

$$\langle T \rangle = (T_i + T_{i+1})/2 \quad (40)$$

where T_i and T_{i+1} are temperatures at the endpoints.

Thus we notice that the average temperature between two points in the central region with uniform heat generation is just the arithmetic average of the endpoint temperatures. We have used this idea for defining the average temperatures for various materials like fuel and clad for the purpose of Doppler feedback. It may be noted that this is exact only in the steady state conditions and in the central region with uniform heat generation. But as the main contribution to Doppler feedback comes from fuel, this approximation seems to be good. To see the effectiveness of this formulation, we have also considered the conventional way of defining average temperature which is given by

$$\langle T \rangle = [T_i (r_{i+1/2}^2 - r_j^2) + T_{i+1} (r_{j+1}^2 - r_{i+1/2}^2)] / [r_{i+1}^2 - r_j^2] \quad (41)$$

i.e. the domain of T_i is from r_j to midpoint of i and $i+1$ and that of T_{i+1} is from the midpoint to r_{j+1} .

To get a feeling of the differences in the two averaging models we take a simple case of a solid cylinder with uniform heat generation. We divide the cylinder in one mesh only i.e. we have only two temperatures. The average temperature according to the proposed model is contributed equally by its centre-line and surface temperature whereas in the conventional model the contribution of central line temperature is only 25% and that of surface temperature is 75%. Thus the average temperature is underpredicted by the conventional scheme.

The feedback coefficient of reactivity due to T_i can thus be written as:

Present model
$$a_i = \alpha_I V_{i-1} / 2V_I + \alpha_J V_i / 2V_J \quad (42)$$

Conventional model
$$a_i = \alpha_I V_{i-1}^R / V_I + \alpha_J V_i^I / V_I \quad (43)$$

where, V_I and V_J represent respectively the volumes of the materials wherein the meshes $i-1$ and i are situated. The parameters α_I and α_J are the corresponding Doppler feedback coefficient of reactivity for the two materials.

6. TEST-CASES

To test the model, we have taken two cases of loss of regulation accident (LORA) in NAPP reactors⁽³⁾. In the loss of regulation accident, it is assumed that the regulation system is inadvertently withdrawn from the reactor core introducing 9 mk of positive reactivity. The point-kinetics parameters and other data are listed in Table 1. The reactor is assumed to be operating at 100% full power (770 MWth). The rod radius is 0.7185 cm, the clad inner and outer radii are taken to be 0.7225 and 0.76 cm respectively. The coolant flow rate is 3520 kg/s with an inlet temperature of 249°C. The effective delayed neutron fraction is 0.0069 and prompt neutron life time is 0.64 μ s. The fuel temperature and coolant temperature feedback coefficients are -2.48×10^{-5} and 5.60×10^{-6} $\Delta k/k/^\circ C$ respectively. Six delayed neutron groups were used in the analysis.

In the first case, we have assumed a reactivity insertion rate of 0.05 mk/s which very closely simulates the actual withdrawal of the regulation system. In the second case, a reactivity insertion rate of 0.005 mk/s corresponding to the removal of boron from the moderator has been considered.

To arrest the unintended rise of power and temperatures, there are two shutdown systems in NAPP reactors. They are actuated whenever power or rate of power rise exceed some preset value. The shutdown systems introduce a large amount of negative reactivity at a very fast rate. The reactor becomes subcritical. But we have assumed that the power and temperatures go on rising. The two test-cases were analysed upto 500 and 2000 s respectively.

7. RESULTS

In both the schemes two nodes in clad and one node in coolant were taken. For both the test-cases, the analysis was carried out with 2, 3, 5, and 9 nodes respectively in fuel. The salient results (power, centre-line and average fuel temperatures) are given in the Tables 2 and 3 for test-case 1 and 2 respectively. C2, C3, and C9 stand for cases with 2, 3, and 9 nodes respectively in fuel with conventional method. The results with 5 nodes in fuel region have not been included in the Table as they are very close (within 1%) to those obtained with 9 nodes which are taken as standard. For the present model, the results with only two nodes in fuel (one node each at the centre and surface of the fuel region) are included as no significant change was noticed when nodes in fuel were increased. The results are marked with the symbol P2.

We notice from the results that in both the schemes the steady state centre-line fuel temperature remains identical irrespective of the number of nodes taken in fuel. This is mainly due to the corner mesh finite difference scheme used for spatial discretization. The average fuel temperature, on the other hand, is underpredicted by about 20%, 5% and 1% when we take 2, 3, and 5 nodes respectively in fuel in the conventional scheme. But with the present model, the average fuel temperature also does not change with the number of nodes in the fuel.

The fuel temperature feedback reactivity and consequently the power and temperature rise depend on the average fuel temperature. We notice that in the conventional scheme, the results are different when we vary the number of nodes. For instance, in test-case I, at 180 s the power is overpredicted by about 25% when 2 nodes are taken in fuel. With the present model, it was noticed that the results do not change with number of mesh points and they are in good agreement with those obtained with fine subdivision of fuel in the conventional scheme. The total CPU time for the two test-cases was found to be 1287, 1323, 1835 and 2873 s with 2, 3, 5 and 9 nodes respectively in fuel for the conventional model compared to 1253 s with 2 nodes in the fuel region in the present model.

8. CONCLUSIONS

The following conclusions can be drawn from the study. At steady state conditions the number of nodes do not matter as far as the centre-line fuel temperature is concerned. For average fuel temperature, the fuel has to be divided into a large number of nodes in the conventional scheme whereas only two nodes (one in the centre and the second on the surface) are sufficient in the present model. It is also shown that the the transient results with two nodes with the present model are as good as those with 9 nodes in the conventional model. The computer time depends on the number of temperature equations. Thus, with the present model, there is a net saving in computer time without any loss of accuracy.

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TABLE 1

PARAMETERS USED IN THE ANALYSISA. Point-Kinetics Parameters

Prompt neutron gen. time(ms)	0.640
Delayed neutron fraction	0.0069
Fuel temp. coeff. (mk/C)	-2.48E-2
Coolant temp coeff. (mk/C)	5.60E-3

B. Heat Conduction

	<u>Fuel</u>	<u>Gap</u>	<u>Clad</u>	<u>Coolant</u>
Outer radius (m)	7.185E-3	7.225E-3	7.6E-3	9.475E-3
Density (kg/m ³)	10.6E-3		6.5E-3	8.426E-4
Sp. Heat (J/kg/K)	300.0		320.	5120.2
Th. Cond. (watt/m/K)	4.1		15.85	
Heat tr. coeff. (watt/m ² /K)		7.356E+3		5.6784E+4
Heat generation rate in fuel at FP		1.6324E+8	watt/m ³	
Total coolant mass in core		2.48E+3	kg	
Coolant flow rate		3.52E+3	kg/s	
Inlet coolant temperature		249	C	

* Fuel and clad parameters are taken at 800 K and 550 K respectively.

TABLE 2

SALIENT RESULTS FOR TEST-CASE 1

Time (s)	Neutron Power(%FP)				Centre-line Fuel Temp. (C)				Average Fuel Temp. (C)			
	C2	C3	C9	P2	C2	C3	C9	P2	C2	C3	C9	P2
0	100	100	100	100	896	896	896	896	511	607	637	639
20	117	114	113	113	956	948	945	944	538	638	668	670
40	133	125	123	124	1053	1020	1012	1011	578	677	708	710
60	150	137	134	134	1156	1094	1079	1078	618	717	748	750
80	166	149	145	145	1260	1169	1148	1147	660	758	789	791
100	183	161	156	156	1366	1244	1217	1215	701	799	830	832
120	199	172	166	167	1472	1320	1287	1284	743	841	871	873
140	216	184	177	178	1578	1395	1356	1354	785	882	912	915
160	233	196	188	189	1684	1471	1426	1423	827	924	954	956
180	249	208	199	199	1789	1547	1495	1492	869	965	995	997
500	247	206	199	197	1831	1569	1515	1511	881	975	1005	1007
CPU Time in seconds					250	291	555	254				

C2, C3, and C9 stand for cases with 2, 3, and 9 nodes in fuel in the conventional model. P2 stands for 2 nodes in fuel with present model.

TABLE 3

SALIENT RESULTS FOR TEST-CASE 2

Time (s)	Neutron Power(%FP)				Centre-line Fuel Temp.(C)				Average Fuel Temp.(C)			
	C2	C3	C9	P2	C2	C3	C9	P2	C2	C3	C9	P2
0	100	100	100	100	896	896	896	896	511	607	637	639
200	116	112	111	111	992	967	961	961	550	646	676	678
400	132	124	122	122	1097	1042	1030	1030	591	687	717	719
600	149	135	132	132	1201	1117	1099	1098	632	728	758	760
800	165	147	143	143	1306	1192	1168	1167	673	769	799	801
1000	182	159	154	154	1410	1267	1237	1235	715	810	840	842
1200	198	171	165	165	1514	1342	1306	1304	756	851	881	883
1400	214	182	176	175	1619	1417	1375	1373	797	892	922	924
1600	231	194	187	186	1723	1492	1444	1441	838	933	963	965
1800	247	206	197	197	1827	1567	1513	1510	880	974	1004	1006
2000	247	206	197	197	1831	1569	1515	1511	881	975	1005	1007
CPU Time in seconds					1037	1132	2318	999				

C2, C3, and C9 stand for cases with 2, 3, and 9 nodes in fuel in the conventional model. P2 stands for 2 nodes in fuel with present model.

