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COMPUTATION OF 3D FORM FACTORS IN COMPLEX ENVIRONMENTS

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1- DIFFUSE RADIATIVE HEAT TRANSFER IN COMPLEX ENVIRONMENTS

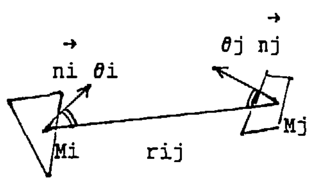
The calculation of radiant interchange among opaque surfaces in a complex environment poses the general problem of determining the visible and hidden parts of the environment. In many thermal engineering applications, surfaces are separated by radiatively non-participating media and may be idealized as diffuse emitters and reflectors. Consequently the net radiant energy fluxes are intimately related to purely geometrical quantities called form facfors, that take into account hidden parts: the problem is reduced to the form factor evaluation.

This paper presents the method developed for the computation of 3D form factors in the finite-element module of the system **TRIO** , which is a general computer code for thermal and fluid flow analysis. The method is derived from an algorithm devised for synthetic image generation. A comparison is performed with the standard contour integration method also implemented and suited to convex geometries. Several illustrative examples of finite-element thermal calculations in radiating enclosures are given.

2- HIDDEN-SURFACE ALGORITHM FOR FORM FACTORS

Definition of form factors

The environment is a mesh composed of a set of finite planar polygonal surfaces or elements, generated by the preprocessor linked to the system. The form factor between two surfaces i and j is defined as the fraction of the radiant energy leaving surface i which strikes on surface j. According to the radiative properties of surfaces, assumed to be isothermal, the form factor reduces to:



$$F_{ij} = \frac{1}{Si} \int_{Si} \int_{Sj} \frac{\cos\theta i \cos\theta j}{\pi (rij)^2} \, vij \, dsi \, dsj \qquad (1)$$

where $\vec{ni}$, $\vec{nj}$, Si, Sj denote the surface local normals and areas.

The vij term takes into account the possible occlusion of surface j due to intervening surfaces in the direction MiMj defined by two current points Mi on surface i and Mj on surface j: vij = 0 if there is an occlusion in local direction MiMj, vij = 1 if not.

Complexity due to hidden parts

The difficulty stems from the fact that the evaluation of form factors requires for any couple of surfaces to sort all other surfaces in order to predict if they see each other entirely, partially or not at all. The number of operations to perform would thus grow as the cube of the number of elements, which is prejudicial to the performance of the method when this number is high. This is the main reason to work out a more efficient sorting algorithm.

Algorithm

The visibility problem can be tackled as follows: consider a surface i and a current point Mi on this surface. This surface has, from point Mi, a view of its environment accross the hemisphere of directions surrounding its normal. The problem is to determine for each direction which surface is visible from point Mi and calculate the corresponding elementary form factors.

The numerical implementation of this method requires:

-the subdivision of surfaces, which define within each surface a set of view points and corresponding sub-elements

- the approximation of the hemisphere of directions.

The algorithm is greatly simplified by considering a cubic surface, the center of which is the current view point. Each face of the cube is divided into square regular cells, each cell defining a viewing direction and an elementary solid angle. The number of cells per face is called resolution.

The algorithm belongs to the class of 'z-buffer' algorithms: for every surface and for each view point on that surface, it projects every other surface on the projection cube and calculates the depth 'z' of each corresponding cell, that is the distance between the projected cell and the view point along the cell direction. The projection of the entire environment solves simultaneously the problems of visibility in each cell thanks to the depth and numerical calculation of form factors:

The form facteur of surfaces i and j is the weight average of individual form facfors corresponding to the view points k taken on surface i:

$$(2) \qquad F_{ij} = \sum_k \frac{(dsi)^k}{Si} F^k_{di-j} \qquad \text{with} \quad Si = \sum_k (dsi)^k$$

$(dsi)^k$ denotes the area of the sub-element associated with view point k

For the form factor relative to a view point, expresion (1) can be approximated by:

$$(3) \qquad F_{di-j} = \int_{Sj} \frac{\cos\theta i \cos\theta j}{\pi\,(rij)^2}\; vij \; dsj$$

in so far as the area of the sub-element is small.

For a given view point, the individual form factor is computed by the sum of the elementary solid angles of the cells through which surface j is visible, this sum being weight by the dot product of the surface normal and the orientation of the cells.

$$(4) \qquad F_{di-j} = \sum_{c} \cos\theta i \; \omega c$$

$$\omega c = \frac{\cos\theta c \; sc}{\pi (rc)^2} \qquad \text{solid angle of cell c}$$

with sc : cell area
rc : distance cell/center of the cube
θc : angle direction /local cell normal

Remarks

The energy conservation principle depends only on the resolution. It is well checked, generally at less than 0.01 per cent, for a minimum resolution of 20x20.

The reciprocity rules are linked to the global accuracy of the calculation, given by both resolution and subdivision of elements.

A high resolution is suited to elements of small dimension whereas the subdivision of elements is necessary only for 'great' elements.

The algorithm automatically divides the elements of great area, for a given value of theorical subdivision assigned to the mean area of all elements.

Comparison with the contour integration method: convex case

The general hidden-surface algorithm is compared with the contour integration method suited to convex geometries, that provides with very accurate values since it is semi-analytical:

- area integrals are replaced by contour integrals by using Stokes' theorem, thus reducing the order of integration.

- one integral is integrated analytically (polygonal contour).

The comparison performed on a set of triangular elements shows the global convergence of the algorithm with the resolution and the factor of subdivision.

3- APPLICATIONS: FINITE-ELEMENT CALCULATION IN RADIATIVE ENCLOSURES

The standard radiosity method gives the matrix relationship between the mean net radiant fluxes and the mean temperatures over the elements, the matrix depending upon the form factors and the wall total emissivities. This relation constitutes a non-linear boundary condition to the steady state heat conduction equation. After linearization, the finite-element formulation leads to a non-linear system that is solved implicitly by a block triangulation.

Several illustrative examples are presented in which a cubic enclosure contains inner structures.

4- CONCLUSION

Although the method requires relatively large computational resources due to the volume of information that must be processed for complex environments, it is very attractive for several reasons:

- the method is general and easily transposable to plane geometry

- sorting is efficient

- the method warrants the energy conservation principle

- the accuracy is controlled by the resolution and subdivision of the elements.

- elementary form facfors defined by the projection surface are evaluated once.

- a great number of operations performed on the projection surface are done on integers, which is poor time-consuming.