ENERGETIC PARTICLE EFFECTS ON GLOBAL MHD MODES*

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Abstract

The effects of energetic particles on MHD type modes are studied by analytical theories and the nonvariational kinetic-MHD stability code (NOVA-K). In particular we address the problems of (1) the stabilization of ideal MHD internal kink modes and the excitation of resonant "fishbone" internal modes and (2) the alpha particle destabilization of toroidicity-induced Alfven eigenmodes (TAE) via transit resonances. Analytical theories are presented to help explaining the NOVA-K results. For energetic trapped particles generated by neutral-beam injection (NBI) or ion cyclotron resonant heating (ICRH), a stability window for the n=1 internal kink mode in the hot particle beta space exists even in the absence of core ion finite Larmor radius effect (finite ω_{*i}). On the other hand, the trapped alpha particles are found to resonantly excite instability of the n=1 internal mode and can lower the critical beta threshold. The circulating alpha particles can strongly destabilize TAE modes via inverse Landau damping associated with the spatial gradient of the alpha particle pressure.

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1. Introduction

With the introduction and increased use of powerful auxiliary heating techniques such as ion cyclotron resonant heating (ICRH) and neutral-beam injection (NBI), present-day tokamak experiments and future fusion reactors will involve significant amount of energetic particles. The energetic particles can significantly affect the MHD stability because some of the basic assumptions of MHD theory become invalid for energetic particles. In tokamaks, particle dynamics typically satisfy the condition ω_t, $\omega_b >> \omega_d$ (ω_d is the energetic particle magnetic drift frequency, ω_t the circulating particle transit frequency, and ω_b the trapped particle bounce frequency). For the low frequency MHD modes with $\omega << \omega_d$, the energetic particle dynamics are no longer governed by the $(\vec{E} \times \vec{B})/B^2$ drift, but rather by the magnetic (∇B and curvature) drifts. The trapped particles precess very rapidly across $\vec{B}$ field, and their motions become very rigid with respect to the MHD perturbations. Since trapped particles tend to move on constant-B surfaces, the trapped particle pressure is approximately a function of B, $P_{\perp h} = P_{\perp h}(B)$. The perturbed perpendicular energetic trapped particle pressure is given by $\delta p_{\perp h} \approx (\partial P_{\perp h}/\partial B)\, \delta B_{\parallel} = \beta_h(\omega_*/\omega_B)_h\, B\delta B_{\parallel}$, where ω_B is the ∇B drift frequency. From the perpendicular force balance equation we have $\nabla_\perp \delta p_c \approx -\nabla_\perp[(1+\beta_h \omega_{*h}/2\omega_{Bh})\, B\delta B_{\parallel}]$, where δp_c is the perturbed core plasma pressure. This means that if $(\omega_*/\omega_B) > 0$, it requires an extra restoring force from $\nabla_\perp \delta p_c$ to maintain the force balance. Therefore, the effect of the energetic trapped particles is stabilizing, and the corresponding potential energy δW_K is positive. If the MHD wave has a real frequency such that $\omega \approx \omega_d$, then the energetic trapped particle motion is no longer rigid with respect to the MHD perturbation and wave particle resonance can occur. Magnetic drift resonant "MHD" instabilities can be excited by tapping free energy from the energetic particle pressure nonuniformity. For the high frequency shear Alfven waves with $\omega \approx \omega_t$, ω_b, the ideal MHD stable global Alfven modes can be driven unstable by the circulating energetic particles resonanting with the background waves via inverse Landau damping.

In the high-power, nearly perpendicular neutral-beam injection experiments, bursts of large amplitude m=n=1 (m is the poloidal mode number and n is the toroidal mode number) MHD fluctuations, dubbed "fishbones" were observed in the Mirnov coil and x-ray signals[1-3]. These fishbone bursts are believed to be resonant internal kink modes and are found to be correlated with the reductions of neutron emissivity which correspond to significant losses of energetic beam ions, and thus play an important role in limiting the β-values of tokamak devices. On the other hand, ICRH heated plasmas with an energetic minority ion component on JET[4,5] seemed to have demonstrated a sawtooth-free domain of tokamak operation. The JET experiments are interpreted as the energetic particle stabilization of both the resistive internal kink mode and the fishbone mode. In future tokamak reactors, alpha particles may destabilize the toroidicity-induced shear Alfven eigenmodes (TAE)[6] through circulating particle resonance by taping the free energy associated with the α-particle pressure nonuniformity. The TAE modes have theoretically been shown to cause very serious α-particle loss[7].

In the following, we first briefly describe in Sec. 2 the formulation of the kinetic-MHD eigenmode equations for our nonvariational treatment. In Sec. 3 we study the effects of neutral beam injection (NBI) and α-particles on the stability of the internal mode and the excitation of resonant fishbone modes. We first derive and studty an analytical dispersion relation. For the NBI case, comparisons with numerical results involving analytical approximations indicate that the critical values of the energetic trapped particle beta for the stabilization of the ideal MHD internal kink and the destabilization of the resonant fishbone mode can be an order of magnitude different from those computed by the "non-approximated" NOVA-K code. In addition, a necessary condition for the excitation of fishbone mode is found that the total plasma beta must be in the domain of the unstable ideal MHD internal kink modes. This is contrary to the analytical results of no total beta threshold. For trapped α-particles, the resonant internal mode can lower the total beta threshold for instability. In Sec. 4, we present the NOVA-K results of the α-particle destabilization of TAE modes via transit resonances. The summaries and discussion are given in Sec. 5.

2. Kinetic-MHD Model and NOVA-K Code

We will consider an axisymmetric toroidal plasma consisting of the core isotropic and the hot anisotropic components with $n_h << n_c$ and $T_h >> T_c$ so that $\beta_h < \beta_c$. In terms of the flux coordinate system (ψ,θ,ζ), the equilibrium magnetic field with nested flux surfaces can be written as

$$\vec{B} = \nabla\zeta \times \nabla\psi + q(\psi)\,\nabla\psi \times \nabla\theta, \qquad (1)$$

where $2\pi\psi$ is the poloidal flux within a magnetic surface, $q(\psi)$ is the safety factor, θ is the generalized poloidal angle varying between 0 and 2π, and ζ is the generalized toroidal angle varying between 0 and 2π. Summing the collisionless equations of motion for each species, we obtain the linear momentum equation

$$\omega^2 \rho\, \vec{\xi} = \nabla\delta p_c + \nabla \bullet \delta\overleftrightarrow{p}_h + \vec{b} \times (\nabla \times \vec{B}) + \vec{B} \times (\nabla \times \vec{b}) \ , \qquad (2)$$

where $\vec{\xi}$ is the usual fluid displacement vector, $\vec{b}$ is the perturbed magnetic field, δp_c is the perturbed core plasma pressure, $\delta\overleftrightarrow{p}_h$ is the perturbed hot plasma pressure tensor, and ρ is the total plasma mass density. The ideal MHD description is adopted for the core plasma and the following ideal MHD relations hold

$$\delta p_c + \vec{\xi} \bullet \nabla P_c + \Gamma_s P_c\, \nabla \bullet \vec{\xi} = 0 \ , \qquad (3)$$

$$\vec{b} = \nabla \times (\vec{\xi}_\perp \times \vec{B}) \ , \qquad (4)$$

and

$$\delta\vec{E} = i\,\omega\, \vec{\xi} \times \vec{B} \ , \qquad (5)$$

where $\delta\vec{E}$ is the perturbed electric field, $\Gamma_s = 5/3$ is the ratio of specific heat, and P_c is the core plasma pressure.

The drift kinetic description neglecting the Finite-Larmor-radius correction is employed for the hot particle dynamics. The drift kinetic equation can be solved by the method of characteristics with the total perturbed particle distribution given by

$$\delta f = -\vec{\xi}_\perp \bullet \nabla F - \frac{\mu b_\parallel}{B} \frac{\partial F}{\partial \mu} + \hat{g} \ , \qquad (6)$$

where the nonadiabatic particle distribution is given by

$$\hat{g} = \int_{-\infty}^{t} dt' \left(-\frac{ieF}{T}\right)\left(\tilde{\omega} - \omega_*^T\right)\left[\frac{i\,\vec{v}_d \bullet \delta\vec{E}_\perp}{\omega} + \frac{M_h \mu b_{\parallel}}{e}\right], \tag{7}$$

$\tilde{\omega} = -(T\omega/M_h)\partial \ln F/\partial\varepsilon$, $\omega_*^T = -i\,(T/M_h\omega_c)\,(\hat{b} \times \nabla \ln F \bullet \nabla)$ only perates on the perturbed quantities, $\vec{v}_d = (\hat{b}/\omega_c) \times [\,\nabla(\mu B) + \vec{\kappa}\, v_{\parallel}^2\,]$ is the magnetic drift velocity, c is the speed of light, T is the average temperature of the hot particle, ω_c is the hot particle cyclotron frequency, $F(\varepsilon,\mu,\psi_g)$ is the guiding-center particle distribution. The time integration in Eq. (7) is along the unperturbed guiding-center trajectory. $\delta\overleftrightarrow{p}_h$ can be expressed as

$$\delta\overleftrightarrow{p}_h = -\,\vec{\xi}_\perp \bullet \nabla \overleftrightarrow{P} + b_{\parallel}\left(\frac{\partial \overleftrightarrow{P}}{\partial B}\right)_{\psi} + \delta\hat{p}_\perp \overleftrightarrow{I} + (\,\delta\hat{p}_{\parallel} - \delta\hat{p}_\perp)\,\hat{b}\,\hat{b}\,, \tag{8}$$

where $\delta\hat{p}_{\parallel}$ and $\delta\hat{p}_\perp$ are given by

$$\begin{pmatrix} \delta\hat{p}_{\parallel} \\ \delta\hat{p}_\perp \end{pmatrix} = \sum_{\sigma_{\parallel}} \sqrt{2}\,\pi \int_0^{\infty} d\varepsilon\, \varepsilon^{1/2} \int_0^{h(\psi,\theta)} \frac{d\Lambda\, \hat{g}}{h\sqrt{1-\Lambda/h}} \begin{pmatrix} 2(\varepsilon - \mu B) \\ \mu B \end{pmatrix}, \tag{9}$$

$\Lambda = \mu B_o/\varepsilon$ is the pitch angle, B_o is the vacuum magnetic field at X = R, and h = $B_o/B(\psi,\theta)$. On a flux surface, circulating particles correspond to $0 \le \Lambda \le h_{min}(\psi)$, and trapped particles to $h_{min}(\psi) \le \Lambda \le h$ at a given θ, where $h_{min}(\psi)$ = Min $[h(\psi,\theta)]$ on the ψ surface.

In terms of the dependent variables, $\xi_\psi = \vec{\xi} \bullet \nabla\psi$, $\delta p_1 = \delta p_c + \vec{b} \bullet \vec{B}$, $\xi_s = \vec{\xi} \bullet (\vec{B} \times \nabla\psi/|\nabla\psi|^2)$, and $\nabla \bullet \vec{\xi}$, Eqs. (2) - (9) can be cast into a set of non-Hermitian integro-differential eigenmode equations which are solved by a nonvariational kinetic-MHD stability code (NOVA-K)[8]. In general, a flux coordinate (ψ,θ,ζ) system with an arbitrary Jacobian, the NOVA-K code employs Fourier expansion in the poloidal angle θ direction, and cubic B-spline finite elements in the radial ψ direction. An arbitrary nonuniform ψ-mesh can be set up to provide the option of zoning the mesh to allow more finite elements near rational surfaces, the plasma edge, and the magnetic axis. The boundary condition at the magnetic axis is $\xi_\psi = 0$. For fixed boundary modes the boundary condition is $\xi_\psi = 0$ at the plasma-wall interface. In general, the boundary condition at the plasma-vacuum interface is given by $\vec{b}_v \bullet \nabla\psi = \vec{B} \bullet \nabla\xi_\psi$, where $\vec{b}_v$ is the perturbed vacuum magnetic field which must be solved from the divergence-free equation $\nabla \bullet \vec{b}_v = 0$ with proper wall boundary condition.

3.1 Quadratic Form

By taking an inner product of Eq. (2) with $\vec{\xi}^*$ and integrating over the all plasma volume with the assumption of a fixed conducting boundary, we obtain a quadratic form

$$D(\omega) = \delta W_f + \delta W_k - \delta K = 0 \,, \tag{10}$$

where the inertial energy is given by

$$\delta K = \omega^2 \int d^3x \; \rho \left|\vec{\xi}\right|^2 , \tag{11}$$

the total fluid potential energy due to both the core and hot components is

$$\delta W_f = \int d^3x \; \Big\{ \Gamma_s P_c \left|\nabla\bullet\vec{\xi}\right|^2 + \left|\vec{b}_\perp\right|^2 + \left|\nabla\bullet\vec{\xi}_\perp + 2\,\vec{\kappa}\bullet\vec{\xi}_\perp\right|^2 + \left(\frac{\vec{J}\bullet\vec{B}}{B^2}\right)\left(\vec{b}\times\vec{B}\right)\bullet\vec{\xi}_\perp^{\,*} - 2\left(\vec{\kappa}\bullet\vec{\xi}_\perp^{\,*}\right)\left(\vec{J}\times\vec{B}\right)\bullet\vec{\xi}_\perp \Big\} , \tag{12}$$

and in the limit $\omega << \omega_b$, the the energetic particle potential energy is mainly contributed by the trapped particles and is given by

$$\delta W_k \approx -\, 2^{7/2} M \pi^2 \int d\psi \int_0^\infty d\varepsilon \, \varepsilon^{5/2} \int_{h_{min}}^{h_{max}} d\Lambda \, K_b$$

$$\times \frac{\left(\omega \frac{\partial F}{\partial \varepsilon} + \hat{\omega}_* F\right)}{\left(\omega - \langle \omega_d \rangle\right)} \left| \left\langle \left(1 - \frac{3\Lambda}{2h}\right) \vec{\kappa} \bullet \vec{\xi}_\perp - \frac{\Lambda}{2h} \nabla \bullet \vec{\xi}_\perp \right\rangle \right|^2 , \qquad (13)$$

where $K_b = \sqrt{\varepsilon/2}\, \tau_b$, τ_b is the trapped particle bounce period and is a function of ε and Λ, and $\hat{\omega}_* = (-i/\omega_c)\,(\hat{b} \times \nabla \ln F \bullet \nabla)$. In the low-frequency limit $\omega << \omega_d, \omega_*$, the sign of the trapped particle potential energy is proportional to $(\hat{\omega}_*/\omega_d)$ and the energetic trapped particles have a stabilizing effect if $(\hat{\omega}_*/\omega_d) > 0$ and vice versa. On the other hand, if $\omega \approx \omega_d$ the energetic trapped particle can excite a resonant instability. Similarly, the nonadiabatic contributions of energetic circulating particles on the potential energy can be obtained and is too complicated to be presented here. Note that in deriving the quadratic form we have neglected the pressure anisotropy terms, i.e., the $(P_\perp - P_\parallel)_h$ terms. The quadratic form is useful in providing the stability properties of the system in certain limits.

3. Energetic Trapped Particle Effects on the n=1 Internal Mode

The previous analytical theories[9-13] of energetic particle effects on the n=1 internal kink mode were performed for large aspect ratio tokamak plasmas with circular, concentric magnetic surfaces, and the radial plasma displacement ξ_r was taken as the cylindrical solution of the $m = n = 1$ mode with ξ_r = constant for $q \le 1$ (or $r \le r_s$), and $\xi_r = 0$, otherwise. Other approximations made in calculating energetic particle contributions to δW_k involve setting $1-q(r)=0$, assuming $(\omega_*/\langle\omega_d\rangle) > 0$ for the whole minor radius, and neglecting toroidal couplings of neighboring poloidal harmonics. These approximations led to the cancellation of the hot particle contributions from δW_f with part of δW_k so that δW_k is proportional to ω. This cancellation has prevented the previous theories from predicting a stability window in β_h for the stabilization of both the internal kink and the fishbone mode. In addition, those approximations also failed to give quantitative predictions of the critical β_h's for both the stabilization of the internal kink and the excitation of the fishbone mode.

Since the behavior of MHD modes depends strongly on both the energetic particle distributions and the plasma equilibria, it is important to have a numerical code (e.g., NOVA-K code) that computes the stability of low-n MHD modes for realistic energetic particle distributions and plasma equilibria.

In Section 3.1, we will first derive and study an analytic dispersion relation that does not cancel the hot particle contributions from δW_f with δW_K so that δW_K is not proportional to ω. Then in Section 3.2, we will present numerical studies of the effects of energetic particles on the n=1 internal kink mode by using the NOVA-K code for NBI energetic trapped particles with singular pitch angle distribution. The effects of alpha particles with uniform pitch angle distribution on the n=1 internal kink mode is presented in Sec. 3.3.

3.1 Analytic Dispersion Relation for the n=1 Internal Kink Mode

To derive the analytic dispersion relation for the n=1 internal kink mode including energetic trapped particle contributions we consider the large aspect ratio orderings with the small parameter $\varepsilon_o \equiv a/R << 1$. We will limit the consideration to the parameter range of the first stability boundary of the internal kink mode, and we order $\beta_{pc} \sim O(1)$, $\beta_{ph} \sim O(\varepsilon_o)$, where β_p is the poloidal beta. The relative temperatures of the core and hot components is $T_c / T_h \sim O(\varepsilon_o^2)$, which implies $n_h / n_c \sim O(\varepsilon_o^3)$ and overall charge neutrality may be assumed. The usual internal kink ordering of $(\omega/\omega_A) \sim (<\omega_d>/\omega_A) \sim O(\varepsilon_o^2)$ is also assumed. With the present ordering we find from Eqs. (10)-(13) that $\delta K/\delta W_f \sim \delta W_k/\delta W_f \sim O(\varepsilon_o^2)$. We note that the contribution of δK is significant only in a singular layer of a width Δ, which has the ordering $(\Delta/a) \sim (\omega/\omega_A) \sim O(\varepsilon_o^2)$, but the contribution of δW_k is mainly from outside the singular layer. Therefore, outside the singular layer the contribution to the dispersion is $D_o \approx \delta W_f + \delta W_k$ and inside the singular layer the contribution to the dispersion is $D_i \approx \delta W_f - \delta K$. Since δW_f and δK are self-adjoint, they can be variationally minimized to $O(\varepsilon_o^2)$ by constructing an appropriate trial function. For the case of circular cross sections, the minimization of δW_f outside the singular layer is nontrivial and can be done by using the trial function given by Bussac et al. [14], and we have

$$\delta W_f = \frac{2\pi^2 R B_o^2 r_s q_s' |\xi_{ro}|^2}{\omega_A} \gamma_B + O(\varepsilon_o^4) \quad , \tag{14}$$

where γ_B contains the contribution from both the core and the hot particles, and near the first stability boundary it is proportional to $[\beta_p{}^2 - (\beta_{pcrit})^2]$, where β_{pcrit} is the critical β_p for the ideal MHD internal kink. By choosing the radial plasma displacement ξ_r as the cylindrical solution of the m = n = 1 mode with ξ_r = constant for $q \le 1$ (or $r \le r_s$), and $\xi_r = 0$, otherwise, the hot trapped particle kinetic contribution is

$$\delta W_k \approx \frac{2\pi^2 R B_o^2 r_s q_s' |\xi_{ro}|^2}{\omega_A} \delta\widehat{W}_k + O(\varepsilon_o^4) \quad , \tag{15}$$

where only m = n = 1 contribution is kept and

$$\delta\widehat{W}_k = \frac{2^{3/2}\pi^2 M \omega_A}{R^2 B_o^2 r_s q_s'} \int_0^{r_s} dr\, B \int_{1-\varepsilon_o}^{1+\varepsilon_o} d\Lambda \int_0^\infty d\varepsilon\, \varepsilon^{5/2} \frac{|K_2|^2}{K_b} \left(\frac{\omega \frac{\partial F}{\partial \varepsilon} + \omega_*^{(1)}}{\langle \omega_d \rangle - \omega} \right) \quad , \tag{16}$$

$$K_2 = \int_{-\theta_T}^{\theta_T} \frac{d\theta \cos(\theta) \exp[i(1-q)\theta]}{\sqrt{1-\Lambda/h}} \quad . \tag{17}$$

From Eqs. (14) and (15) the total contribution of the dispersion relation from outside the singular layer is

$$D_o = \frac{2\pi^2 R B_o^2 r_s q_s' |\xi_{ro}|^2}{\omega_A} \left(\gamma_B - \delta\widehat{W}_k\right) + O(\varepsilon_o^4) \quad . \tag{18}$$

Inside the singular layer, $D_i = \delta W_f - \delta K$ is variational and can be minimized. We should note that $\nabla \bullet \vec{\xi} \neq 0$ so that compressibility is finite in the singular layer. The minimizing ξ_r, that satisfies the Euler-Lagrange equation obtained by varying D_i with respect to ξ_r and matches to the outside solution, is given by

$$\xi_r = \xi_{ro}\left[\frac{1}{2} - \frac{1}{\pi}\tan^{-1}\left(\frac{r-r_s}{\Delta}\right)\right] , \tag{19}$$

where the singular layer width is

$$\Delta^2 = \frac{\omega^2}{\left(q_s'\omega_A\right)^2}\left[\frac{\omega_T^2}{\left(\omega^2-\omega_s^2\right)} - 1\right] \xrightarrow[\omega<<\omega_s,\ \varepsilon_o\to 0]{} \frac{-\omega^2\left(1+2q^2\right)}{\left(q_s'\omega_A\right)^2} . \tag{20}$$

Then D_i becomes

$$D_i = \frac{2\pi^2 R B_0^2 r_s q_s' \left|\xi_{ro}\right|^2}{\omega_A}(i\,\omega)\left[\frac{\omega_T^2}{\left(\omega_s^2-\omega^2\right)} + 1\right]^{1/2} . \tag{21}$$

where

$$\omega_T^2 = \left\langle\frac{\Gamma_s P_c B^2 \kappa_s^2}{\rho\left|\nabla\psi\right|^2}\right\rangle , \quad \omega_s^2 = \left\langle\frac{\Gamma_s P_c}{\rho B^2 J^2}\right\rangle , \quad \omega_A^2 = \left\langle\frac{1}{\rho J^2}\right\rangle , \tag{22}$$

and $\langle\cdots\rangle$ means surface average at the $q = 1$ surface. Finally, combining Eqs. (18) and (21), we arrive at the dispersion relation

$$-i\,\omega\left[1 + \frac{\omega_T^2}{\left(\omega_s^2-\omega^2\right)}\right]^{1/2} = \left(\gamma_B - \delta\widehat{W}_k\right) . \tag{23}$$

If we consider neutral-beam-injected (NBI) plasmas, the equilibrium hot particle distribution F_h is taken as a slowing-down distribution in energy and a delta function in pitch angle, i.e., $F_h(\varepsilon,\Lambda,\psi) = n_h(\psi)\,\delta\left(\Lambda-\Lambda_0\right) /$

$(\varepsilon_c^{3/2} + \varepsilon^{3/2})$ for $\varepsilon \le \varepsilon_b$, and $F_h(\varepsilon,\Lambda,\psi) = 0$ otherwise, where ε_b is the neutral beam injection energy, and ε_c is the cutoff energy where the beam ions lose an amount of energy to the core ions equal to that lost to electrons. For simplicity, we take $\varepsilon_c = 0$ and assume $|\omega| \ll \omega_s$, then the dispersion relation takes the form

$$-i\left(\frac{\omega}{\omega_A}\right) = \left(\frac{\gamma_B}{\omega_A}\right) - \hat{\beta}_h\left\{\frac{\omega_*}{\omega_d} + \frac{\omega}{\omega_d}\left(\frac{\omega_*}{\omega_d} - 1\right)\left[\ln\left(1 - \frac{\omega_d}{\omega}\right) + i\pi\sigma\right]\right\}, \tag{24}$$

where the hot trapped particle terms are volume averaged, and the detailed forms are not important in determing the qualitative stability properties. For $0 < \mathrm{Re}(\omega) < \omega_d$, we set $\sigma = 0$ for $\mathrm{Im}(\omega) > 0$, $\sigma = 1$ for $\mathrm{Im}(\omega) = 0$, and $\sigma = 2$ for $\mathrm{Im}(\omega) < 0$. ω_* and ω_d are the typical hot particle diamagnetic drift frequency and the bounce-averaged magnetic drift frequency, respectively. γ_B is the MHD growth rate including the contribution from hot particles, and $\hat{\beta}_h$ is proportional to the hot particle beta. We should reiterate here that the first (ω_*/ω_d) term in Eq. (24) does not cancel with the hot particle contribution to γ_B. Equation (24) predicts that if $\gamma_B > 0$, the ideal branch will be stabilized for $\hat{\beta}_h > \hat{\beta}_{h1}$, where $\beta_{h1} = (\gamma_B\,\omega_d / \omega_A\,\omega_*)$. At $\hat{\beta}_h = \hat{\beta}_{h1}$, $\omega = 0$. The resonant fishbone branch will be destabilized for $\hat{\beta}_h > \hat{\beta}_{h2}$, where $\hat{\beta}_{h2} = \omega_d/[\pi\omega_A(\omega_*/\omega_d - 1)]$. At $\hat{\beta}_h = \hat{\beta}_{h2}$, ω *is purely real. Figure 1(a) shows the* growth rate and Fig. 1(b) shows the real frequency versus $\hat{\beta}_h$ for the fixed parameter $\omega_*/\omega_d = 10$. Several values of $\bar{\omega}_C \equiv \omega_d/\gamma_B$ are used. The curves labeled $\bar{\omega}_C \ge 3$ correspond to the ideal branch. We see that for $(\omega_*/\gamma_B) \le \pi(\omega_*/\omega_d - 1)$ (i.e., $\hat{\beta}_{h1} \ge \hat{\beta}_{h2}$), the internal kink is unstable for all $\hat{\beta}_h$. For $\hat{\beta}_{h1} < \hat{\beta}_{h2}$, there is a stability window for $\hat{\beta}_{h1} \le \hat{\beta}_h \le \hat{\beta}_{h2}$. It is important to note that previous studies [9-13] have concluded that $\hat{\beta}_{h1} \ge \hat{\beta}_{h2}$ so that there is no stability window. The erroneous conclusion is mainly due to the mistake of making exact cancellation of the first (ω_*/ω_d) term in Eq. (24) with the hot particle contribution to γ_B.

3.2 Effects of Neutral-Beam-Injected Particles on the n=1 Internal Kink Mode

For $\beta_c >> \beta_h$, the equilibrium is approximately isotropic. We first consider an equilibrium with circular plasma surface with the profiles $P(y) = P_o\left(1-y^{\lambda}\right)^{\mu}$, $q(y) = q(0) + y\{q(1) - q(0) + \left[q'(1) - q(1) + q(0)\right]\left(1-y_s\right)\left(y-1\right)/\left(y-y_s\right)\}$, where $y_s = [q'(1) - q(1) + q(0)]/\{q'(0) + q'(1) - 2[q(1) - q(0)]\}$, $y = (\psi - \psi_0)/\Delta\psi$, $\Delta\psi = \psi_{lim} - \psi_0$, ψ_{lim} is evaluated at the limiter, and ψ_0 is evaluated at the magnetic axis. *The parameters for the "PDX05" equilibrium are* $P_o = 0.018$, $\lambda = 2$, $\mu = 2$, $\Delta\psi = 0.061$, $\beta = 2 <P>/<B^2> = 0.625\%$, $R = 1.43$, $R/a = 3.4$, $q(0) = 0.8$, $q(1) = 2.85$, $q'(0) = 13.82$ and $q'(1) = 106.6$, and $\beta_{pol} = 2 <P>/<B_{pol}^2> = 0.4278$. Note that for a given pitch angle Λ_o, $<\omega_d>$ and hence $(\omega_*/<\omega_d>)$ may change from being negative to positive as r changes. If the trapped particles are destabilizing to a certain MHD mode for $(\omega_*/<\omega_d>) > 0$ in a certain radial region, the trapped particles in the radial region with $(\omega_*/<\omega_d>) < 0$ would be stabilizing. Since both ω_* and $<\omega_d>$ are proportional to $\mathcal{E}_b$, we will introduce an energy scaling factor C_T in ω_* and $<\omega_d>$. If $C_T = 0$, $\omega_{*h} = <\omega_d> = 0$ and the quardatic form, Eq. (10), corresponds to the Kruskal-Oberman energy principle[15]. For $C_T = \infty$, Eq. (10) corresponds to the case studied by Rosenbluth et al.[16]. For simplicity we let $\mathcal{E}_c = 0$, $n_h(\psi)\mathcal{E}_b = \alpha_h P_c(\psi)$, and vary α_h to change the hot particle pressure. The other fixed parameters for the energetic particles are $\mathcal{E}_b/T_c(0) = 10$, $R/\rho_h(0) = 100$, $M_h/M_c = 1$, $\Lambda_o = 1.1$, where M_c and M_h are the core and hot ion mass, respectively. $\rho_h(0)$ is the hot ion gyroradius at the magnetic axis. For this equilibrium, the volume-averaged hot particle beta, $\beta_h = 2 <P_h>/<B^2>$, is related to the total volume-averaged beta by $\beta_h = 1.38\,\alpha_h\beta$. In the absence of energetic particles (α_h=0), the n = 1 fixed boundary internal kink mode is unstable with the growth rate $\gamma/\omega_A = 1.195 \times 10^{-2}$. In varying β_h (or α_h) the total β is fixed so that as β_h is increased, the core plasma β_c is decreased by the same amount.

In Figs. 4(a) and 4(b), the growth rates and the negative real frequencies, respectively, versus α_h are plotted for several values of the hot particle injection energy scaling factor C_T. For $C_T < 0.4$, the hot particle pressure is stabilizing for small values of α_h, but becomes destabilizing as α_h becomes larger ($\alpha_h > 0.036$). The destablizing effect is accompanied by an increase in $|\omega_r|$ and is associated with $\omega - <\omega_d> \approx 0$ resonance. However, when C_T becomes larger with $C_T \approx 1$, the ideal branch is stabilized for $\alpha_h > \alpha_{h1} \approx 0.036$ (for C_T=1). But the resonant fishbone branch is destabilized for $\alpha_h > \alpha_{h2} \approx 0.06$ for $C_T = 1$. For larger C_T, α_{h1} is smaller, but α_{h2} becomes larger.

The results presented in Figs. 4(a) and 4(b) are qualitatively similar to Figs. 1(a) and 1(b) obtained from the analytical dispersion relation given by Eq. (24). Figure 1(a) clearly shows these behaviors through the variations in ω_d with ω_*/ω_d being held fixed. We should note that the real frequency shown in Fig. 1(b) is somewhat different from Fig. 4(b) for the resonant branch in the limit $\alpha_h < \alpha_{h2}$. This is because the analytical dispersion, Eq. (24), does not describe the MHD continuous spectrum that is contained in the NOVA-K code. The above comparison clearly demonstrates that the NOVA-K code gives correct results of the energetic particle effects on the n=1 internal kink mode.

Next, we examine the validity of the approximations made in the analytical dispersion, Eq. (24). Our calculations show that although the analytical dispersion gives qualitatively correct results, it fails to provide the correct values of the critical hot particle betas for both the stabilization of the ideal branch and the destabilization of the resonant fishbone branch. To make comparisons on the values of critical β_h, we impose similar approximations in computing the perturbed hot particle pressures $\delta \overleftrightarrow{P}_h$ by retaining only the m=1 poloidal harmonics and taking 1-q(r) = 0 in the nonadiabatic hot particle contribution. We find that the approximations have produced smaller values of β_{h1} and β_{h2} by roughly a factor of 4 in this particular example. The difference is mainly due to the omission of the m=2,3 harmonics which are not negligible outside the q=1 surface and have opposite contributions to stability from the m=1 harmonic.

Another important conclusion from the calculations of the NOVA-K code, that is different from that of the analytical theory, is that when the ideal MHD internal kink is stable, the resonant fishbone mode is also stable. Our conclusion is derived by computing the stabilities of several equilibria with different β. As β is closer to the ideal MHD critical betas, $\beta_{1,2}$, the growth rates of both the ideal branch and the resonant fishbone branch also decrease to zero. For $\beta \leq \beta_1$ and $\beta \geq \beta_2$, no instabilities are found by the NOVA-K code. On the other hand, it can be easily seen from the analytical dispersion relation, Eq. (24), that the critical β_{h2} for the resonant fishbone branch is independent of γ_B, and there is no constraint on the total plasma β. The breakdown of the analytical analysis for both the ideal and resonant fishbone branches in the ideally stable β domain is that the eigenfunction becomes singular near q=1 surface and the analytical dispersion is invalid. Finally, the effects of the neutral beam injection or ICRH heatings on the n=1 internal kink mode can be summarized schematically by the stability window in the $\beta-\beta_h$ space shown in Fig. 5. This stability window provides an explanation of the experimental observations of the stabilization of sawtooth in JET[4,5]. Here, we emphasize that unlike the previous analysis [9-13] the stability window exists even in the absence of core ion finite Larmor radius effect (finite ω_{*i}).

3.3 Alpha Particle Effects on the n=1 Internal Mode

The stabilization of the internal kink mode by trapped α-particles had been reported by several authors[10,17] who studied the analytic dispersion relations by employing large aspect ratio tokamak equilibria. However, this conclusion can not be applicable to finite aspect ratio tokamaks because the analytical studies failed to take into account the pitch angle variations of the bounce-averaged magnetic drift frequency $\langle\omega_d\rangle$ and the contributions from m=2 and 3 poloidal harmonics. In fact, the numerical studies using the NOVA-K code indicated that the trapped α-particles can excite the resonant internal mode and lower the critical beta threshold of the n=1 internal mode.

In the NOVA-K code, we will employ the slowing-down distribution with $\mathcal{E}_c = 0$ for α-particles, but with uniform pitch angle distribution. We will study ignition type tokamak equilibria which have noncircular plasma surfaces defined by $X = R + a \cos[\theta + \delta \sin(\theta)]$, $Z = \kappa a \sin(\theta)$. The fixed parameters are the ellipticity $\kappa = 2$, the triangularity $\delta = 0.4$, $a = 0.65$, and $R = 2.1$. The total pressure and q-profiles have the parameters: $\lambda = 1.05$, $\mu = 2$,

$q(1) = 3.2$, $q'(0) = 0.25$, and $q'(1) = 7$, and $\beta_{pol} = 2 <P>/<B_{pol}^2> = 1.0376$. P_o and $q(0)$ are varied to find stability boundary in the $(\beta, q(0))$ space. Note that for a radial position, $<\omega_d>$ and hence$(\omega_*/<\omega_d>)$ may change from being negative to positive as Λ decreases. Thus, the net effect of the trapped α-particles must be integrated over the entire trapped α-particle population with proper weightings of different poloidal harmonics. The parameters for the α-particles are $\mathcal{E}_\alpha/T_c(0) = 100$, $R/\rho_\alpha(0) = 100$, $M_\alpha/M_c = 4$, where M_c and M_α are the core and alpha mass, respectively, and $\rho_\alpha(0)$ is the α-particle gyroradius at the magnetic axis. The α-particle pressure is taken to be $P_\alpha(\psi) = \alpha_h P_o [P(r)/P_o]^{7/2}$. For $P_o = 1.0$ and $q(0) = 0.8$, the volume-averaged α-particle beta, $\beta_\alpha = 2 <P_\alpha>/<B^2>$, is related to the total volume-averaged beta by $\beta_\alpha = 0.6214\ \alpha_h\beta$ where $\beta = 4.0159\%$. In varying β_α (or α_h) we will keep the total β fixed so that as β_α is increased, the core plasma β_c is decreased by the same amount. Figure 4 shows the stability boundaries in the $(\beta, q(0))$ space for the n=1 internal mode obtained by varying P_o and $q(0)$. From the ideal MHD stability boundary curve, the critical beta has a sharp transition as $q(0)$ approaches 1. For $q(0) < 0.95$ the solution behaves like an internal kink mode, for $0.95 < q(0) < 1$ the solution becomes more localized near q=1 surface and is a quasi-interchange type, and for $q(0) > 1$ the solution resembles the infernal mode. The lower stability boundary curve is obtained with $\alpha_h = 0.1$ and is determined by the stability of the resonant internal instabilities driven by α-particles.

4. Alpha Particle Destabilization of Toroidicity-Induced Alfven Eigenmodes

High frequency shear Alfven instabilities could be excited in a burning plasma by the expansion free energy associated with the spatial gradient of the alpha particle pressure through the alpha particle diamagnetic drift frequency $\omega_{*\alpha}$. For typical ignition parameters the alpha particle velocity $V_\alpha = (\varepsilon_\alpha/M_\alpha)1/2 = 9 \times 10^8$ cm/sec for an energy ε_α of 3.5 MeV is comparable to the Alfven speed $V_A = B/(N_i M_i)^{1/2}$. Thus, the transiting alpha particles could destabilize shear Alfven waves via inverse Landau damping through the $\omega = \omega_t$ wave-particle resonance. To satisfy the resonance condition, it requires that $V_\alpha > V_A$. To overcome the Landau damping by the inverse Landau damping associated with $\omega_{*\alpha}$, it requires that $\omega_{*\alpha} > \omega_A = V_A/qR$.

Briefly, let us describe the various types of shear Alfven waves in a tokamak plasma. The kinetic Alfven wave associated with the shear Alfven resonance condition has a nonsingular radial structure when electron parallel dynamics and ion finite Larmor radius effects are included. Its mode structure is fairly localized, and hence it experiences strong electron Landau damping[18]. There are two global types of shear Alfven waves that have radially extended mode structure. Both types have low mode numbers n and m. The first type of global shear Alfven wave is a regular, spatially nonresonant wave whose frequency lies just below the minimum of the continuum, i.e. $\omega < k_{\parallel} V_A$ and $k_{\parallel} = (m-nq)/qR \neq 0$. This wave is called the Global Alfven Eigenmode (GAE)[19]. When finite toroidicity is included, GAE modes with different poloidal mode numbers will become coupled. Such toroidal mode coupling tends to stabilize the GAE modes completely[6,20]. Another type of global shear Alfven wave, one that exists only in toroidal geometry. Its frequency lies within "gaps" in the shear Alfven continuum that are created due to toroidal coupling. The existence of this so-called Toroidicity-Induced Alfven Eigenmode (TAE) was previously shown in the ideal MHD limit without alpha particles[21,22]. In this section, we show that this TAE mode can be strongly destabilized by alpha particles in a burning tokamak plasma.

The TAE mode exists inside gaps, due to toroidal coupling, in the shear Alfven continuum spectrum. For example, modes (n,m) and (n,m+1) couple at radial location r_o, where $q(r_o) = (m + 1/2)/n$, to form a gap. For n=m=1, this gap is bounded by

$$\omega_{\pm}^2 = \omega_o^2 + \left\langle \frac{\Gamma_s P B^2 \kappa_s^2}{|\nabla\psi|^2} \right\rangle \pm 2\,\omega_o^2 \left(\frac{r_o}{R} + \Delta'(r_o)\right) , \tag{25}$$

where $\omega_o^2 = (V_A/2qR)^2$ at $r = r_o$, $\Delta(r)$ is the Shafranov shift of the nonconcentric flux surfaces and $\Delta' > 0$. The pressure and q- profiles of the "EC42.3D" equilibrium have the parameters: $P_0 = 0.45$, $\lambda = 2$, $\mu = 2$, $q(0) = 1.05$, $q(1) = 2.3$, $q'(0) = 0.75$, and $q'(1) = 3$, $\beta = 1.893\%$, $a = 0.25$, $R = 1$. The real frequency of the n=1 fixed boundary TAE mode is $\omega_r/\omega_A = -0.739$, which lies within the continuum gap, $1.25 \geq (\omega/\omega_A)^2 \geq 0.53$, formed by the toroidal coupling of the m=1 Alfven mode and m=2 Alfven mode at q=1.5. As β

increases, the frequency of the discrete TAE mode may move downward into the continuum and suffer Landau dampings through wave-particle resonances. Similar results of the finite β effect on the exitence of the TAE modes has also been found in the high-n limit[23].

The alpha particle perturbed pressure tensor $\delta\overleftrightarrow{p}_\alpha$ in the momentum balance equation contains both transit and bounce resonance effects. Core plasma kinetic effects on this mode are neglected here. The alpha particle equilibrium distribution function was taken to be isotropic in pitch angle Λ and slowing down in energy ε. Figure 5 shows the growth rate computed from the NOVA-K code for the n=1 fixed boundary TAE mode[6] as a function of the alpha particle diamagnetic drift frequency $\omega_{*\alpha}$ (evaluated at $r = 0.5$ *and for m=1) normalized to* $\omega_A = V_A(0)/q(a)R$, *for* the "EC42.3D" equilibrium with $\beta_\alpha = 0.4\%$. Here ε_α is the alpha particle birth energy and is taken to be $100T_e(0)$, and $n(\psi)$ is proportional to the alpha particle density, which was taken to be functionally related as $n(\psi) \propto [P(\psi)]^{7/2}$ to the total plasma pressure $P(\psi)$. The wave functions for the m=1 and m=2 poloidal harmonics show that these modes peak near $r = 0.5$; hence $\omega_{*\alpha}$ is a good measure of the alpha particle free energy that can be tapped via inverse Landau damping. Figure 5 shows that $\omega_{*\alpha}$ is large enough to overcome the usual Landau damping when $\omega_{*\alpha}/\omega_A > 1.5$ and that beyond this threshold the growth rate is *approximately linearly proportional to* $\omega_{*\alpha}$. *For typical* ignition parameters, the growth rate of the n=1 TAE mode can be of the order of 10^{-2} of the real frequency.

The numerical results can be interpretated from the quadratic form, Eq.(10). *For the mode frequency, we write* $\omega = \omega_r + i\gamma$ *and assume that the* growth rate is small ($|\gamma| << |\omega_r|$). Then Eq. (10) yields

$$\omega_r^2 \approx \frac{\delta W_f}{\delta K}, \tag{26}$$

and

$$\gamma = -\mathrm{sign}(\omega_r) \int d\psi\, d\Lambda \left(\frac{9\pi^2 \tau_t P(\psi)}{4\,\delta K} \right.$$

$$\times \sum_{m',m,p=-\infty}^{\infty} \left\{ \omega_r - \frac{\omega_{*\alpha}(m)\ \omega_r^2}{\left[(p-nq)\ \omega_t\right]^2} \right\} \frac{\mathrm{Re}\left\{\langle G(m',p)\rangle^* \langle G(m,p)\rangle\right\}}{\left[(p-nq)\ \omega_t\right]^2} , \tag{27}$$

where $G(m,p,\theta)$ depends on the pitch angle, the equilibrium ψ-variation and the mode amplitude, ω_t and τ_t are the alpha particle transit frequency and transit time, respectively, and $<G(m,p,\theta)>$ is the transit average of $G(m,p,\theta)$. The transit harmonic number is p, where p values close to m contribute the most. We define $\omega_{*\alpha}(m) = m\omega_{*\alpha}$, where, as before, $\omega_{*\alpha}$ is the diamagnetic frequency for m=1. Equation (27) shows that the instability condition is $\omega_{*\alpha}(m) > \omega_r$; here the alpha particle free energy drive overcomes usual Landau damping. This marginal stability condition agrees with the numerical result, shown in Fig. 5, from which $\omega_{*\alpha}/\omega_A \approx 1.5$ is marginal. Above this threshold, Eq. (27) indicates that the growth rate γ will tend to scale linearly with $\omega_{*\alpha}$ which again agrees with Fig. 5.

It should be noted that if the edge plasma density value is reduced, the toroidicity-induced mode may possibly resonate with the shear Alfven continuum near the plasma periphery, an effect which could be stabilizing and should be investigated in the future.

5. Summary and Conclusion

Employing the NOVA-K code, we have studied the problems of energetic trapped particle stabilization of the n=1 internal kink mode and the excitation of the resonant fishbone mode by resonating with the energetic trapped particle magnetic drifts. An analytical dispersion relation involving large aspect ratio orderings was derived and studied to help understanding the numerical results of the NOVA-K code. Comparisons with the results of the analytical dispersion for energetic trapped particles with singular pitch angle distribution, such as in the cases of NBI and ICRH heatings, showed that the analytical results are qualitatively similar to the NOVA-K results which indicate the existence of a stability window in the β_h space even in the absence of core ion finite Larmor radius effect (finite ω_{*i}). Imposing analytical approximations on the NOVA-K code would give incorrect values of critical β_h, which can be an order of magnitude different from the correct values. In addition, the results of the NOVA-K code indicate that a necessary condition to excite the resonant fishbone mode is $\beta_2 > \beta > \beta_1$, where $\beta_{1,2}$ are the ideal MHD critical total betas for the n=1 internal kink. This necessary condition is not predicted by the analytical dispersion relation. For the trapped α-particles with a uniform pitch angle distribution, the numerical results of the NOVA-K code are different from the case of energetic particles with singular pitch angle distribution, and we found that the trapped α-particle driven resonant instabilities of the n=1 internal mode can lower the critical beta.

We have also studied the transit α-particle destabilization of the toroidicity-induced shear Alfven eigenmodes (TAE) via inverse Landau damping associated with the spatial gradient of the α-particle pressure. The TAE modes are shown to be strongly unstable with the growth rate being approximately linearly proportional to $\omega_{*\alpha}$ and typically of the order of $10^{-2}\,\omega_A$. Other types of global Alfven waves are stable in ignition tokamaks due to toroidal coupling effects. Therefore, primary attention - especially experimental - should be focused on the toroidicity-induced shear Alfven eigenmodes (TAE), which can be strongly destabilized by alpha particles.

Finally, we believe that the tokamak fusion research has evolved into a new era that the global plasma behaviors will be greatly affected by energetic particles. The kinetic-MHD models, such as the one presented in this paper, will eventually replace the MHD model for understanding the global plasma behaviors. Therefore, the nonvariational kinetic-MHD stability codes would become indispensable tools for studying the global plasma behaviors. In the NOVA-K code we have layed out the groundwork for implementing energetic particle physics into a fluid MHD code. The energetic particle physics included in the present version of the NOVA-K code can still be improved. Some important energetic particle physics that need to be included in the future are finite banana width, pressure anisotropy, finite particle density, and more realistic particle distribution functions generated from Fokker-Planck code. On the other hand, the core plasma non-ideal MHD effects such as finite core ion FLR, electron and ion Landau resonances, and plasma resistivity, should also be considered for certain problems.

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REFERENCES

1. K. McGuire, et al., Phys. Rev. Lett. 50 (1983), 891; D. Johnson et al., in Proceedings of the Ninth International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Baltimore, 1982 (IAEA, Vienna, 1983).
2. Doublet-III Group, Heating in Toroidal Plasmas 1984, edited by H. Knoepfel and E. Sindoni, ENEA (Rome), Vol. I, p. 21 (1984)
3. R. Kaita, R. B. White, A. W. Morris, E. D. Fredrickson, K. M. McGuire, S. S. Medley, and S. D. Scott, Princeton Plasma Physics Laboratory Report PPPL-2493 (January 1988).
4. J. Jacquinot, et al., 11th International Conference on Plasma Physics and Controlled Nuclear Fusion Research (kyoto, Japan),IAEA, Vienna, 1987, Vol. I, p 449.
5. D. J. Campbell, et al., 12th International Conference on Plasma Physics and Controlled Nuclear Fusion Research (Nice, France), IAEA-CN-50/A-7-2 (1988).
6. C. Z. Cheng, G. Y. Fu, and J. W. Van Dam, Princeton Plasma Physics Laboratory Report PPPL-2585 (December 1988).
7. D. J. Sigmar, C. T. Hsu, R. White, and C. Z. Cheng, Bull. Am. Phys. Soc. 34,1971 (1989).
8. C. Z. Cheng, Princeton Plasma Physics Laboratory Report PPPL-2604 (April 1989).
9. L. Chen, R.B. White, and M.N. Rosenbluth, Phys. Rev. Lett 52, 1122 (1984).
10. R. B. White, L. Chen, R. Romanelli, and R. Hay, Phys. Fluids 28, 278 (1985).
11. B. Coppi, R. J. Hastie, S. Migliuolo, F. Pegararo, and F. Porcelli, Phys. Lett. A132, 267 (1988).
12. R. B. White, M. N. Bussac, and F. Romanelli, Princeton Plasma Physics Laboratory Report PPPL-2540 (1988).
13. Y. Z. Zhang, H. L. Berk, and S. M. Mahajan, University of Texas report IFSR #326 (1988).
14. M. N. Bussac, R. Pellat, D. Edery, and J. L. Soule, Phys. Rev. Lett. 35, 1638 (1975); D. Edery, G. Laval, R. Pellat, and J. L. Soule, Phys. Fluids 19, 260 (1976).
15. M.D. Kruskal and C. Oberman, Phys. Fluids 1, 275 (1958).
16. M.N. Rosenbluth, S.T. Tsai, J.W. Van Dam, and M.G. Engquist, Phys. Rev. Lett. 51 (1983), 1967.
17. B. Coppi, S. Migliuolo, F. Pegoraro, and F. Porcelli, Massachusetts Institute of technology Report # PTP-88/4 (1988).
18. Y.M. Li, S.M. Mahajan, and D.W. Ross, Phys. Fluids 30, 1466 (1987).

19. K. Appert, R. Gruber, F. Troyon, and J. Vaclavik, Plasma Physics 24 , 1147 (1982); S. M. mahajan, D. W. Ross, and G. L. Chen, Phys. Fluids 26, 2195 (1983).
20. G.Y. Fu and J. W. Van Dam, Phys. Fluids B 1, 1949 (1989).
21. C.Z. Cheng and M.S. Chance, Phys. Fluids 29, 3695 (1986).
22. C.Z. Cheng, L. Chen and M.S. Chance, Ann. Phys. (NY) 161, 21 (1984).
23. G. Y. Fu and C. Z. Cheng, University of Texas Report IFSR #385 (1989).

Figure Captions

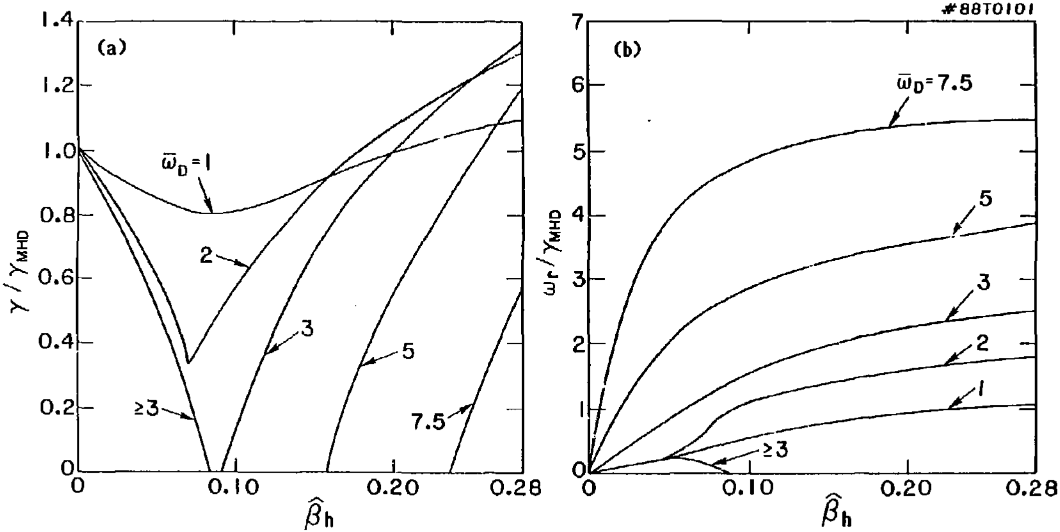
Fig. 1 (a) The growth rates γ/γ_B computed from the analytical dispersion expression, Eq. (24), versus $\hat{\beta}_h$ for several values of $\bar{\omega}_D = <\omega_d>/\gamma_B$. The curve labelled with $\omega_D \geq 3$ represents the ideal branch. (b) The corresponding real frequencies (ω_r/γ_B) versus $\hat{\beta}_h$.

Fig. 2 (a) The growth rates, (γ/ω_A), of the n = 1 mode versus α_h for several values of C_T for the "PDX05" equilibrium with average beta β = 0.625%, q(0) = 0.8, q(1) = 2.85, R/a = 3.4, R = 1.43. The hot particle parameters are $\Lambda_o = 1.1$, $\mathcal{E}_b/T_c = 10$, $R/\rho_h(0) = 100$, $M_h/M_c = 1$, and thus the volume-averaged $\beta_h = 1.38\alpha_h\beta$. (b) The corresponding negative real frequencies, $-(\omega_r/\omega_A)$, versus α_h.

Fig. 3 The schematic stability window in the $\beta-\beta_h$ space,which exists even in the absence of core ion finite Larmor radius effect (finite ω_{*i}), due to the NBI or ICRH heated energetic trapped particles on the n=1 internal kink mode .

Fig. 4 The stability boundaries in $(\beta,q(0))$ space for the n=1 fixed boundary intrenal modes. The lower stability boundary is due to trapped alpha particles.

Fig. 5 Growth rate γ for the n=1 toroidicity-induced Alfven eigenmode (TAE) as a function of the alpha particle diamagnetic drift frequency $\omega_{*\alpha}$ (normalized to the shear Alfven frequency ω_A) for the "EC42.3D" equilibrium.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(b)
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ω_r/γ_{MHD}
$\hat{\beta}_h$
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3
5
7.5
≥3
$\bar{\omega}_D$=7.5
5
3
2
1
≥3
0
0.10
0.20
0.28
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6
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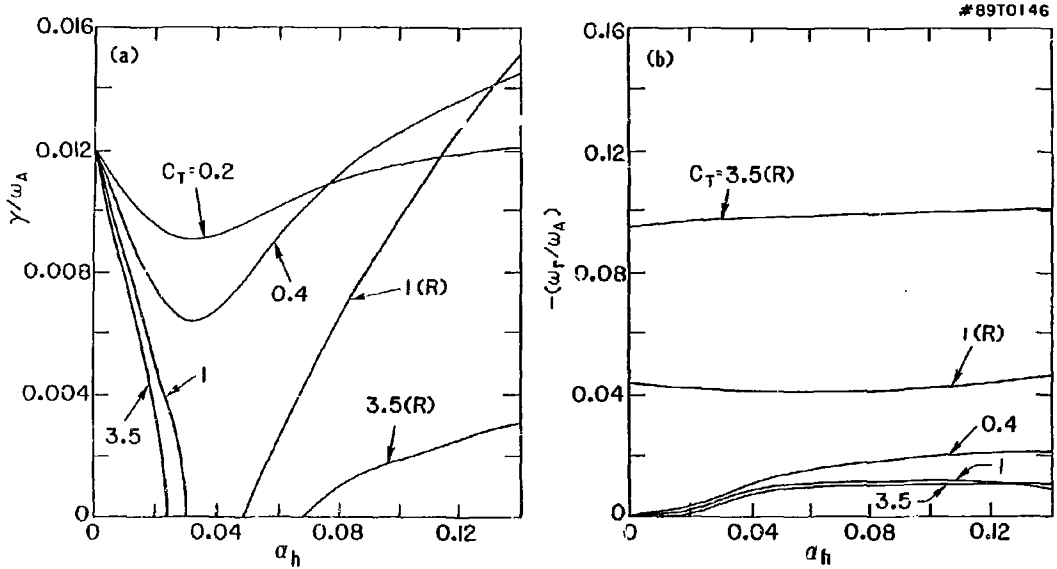
4
3
2
1

Fig. 1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1
3.5
3.5(R)
0
0.04
0.08
0.12
α_h
(b)
0.16
0.12
0.08
0.04
0
$-(\omega_r/\omega_A)$
C_T=3.5(R)
1(R)
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3.5
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0.08
0.12
α_h

Fig. 2

#89X0072

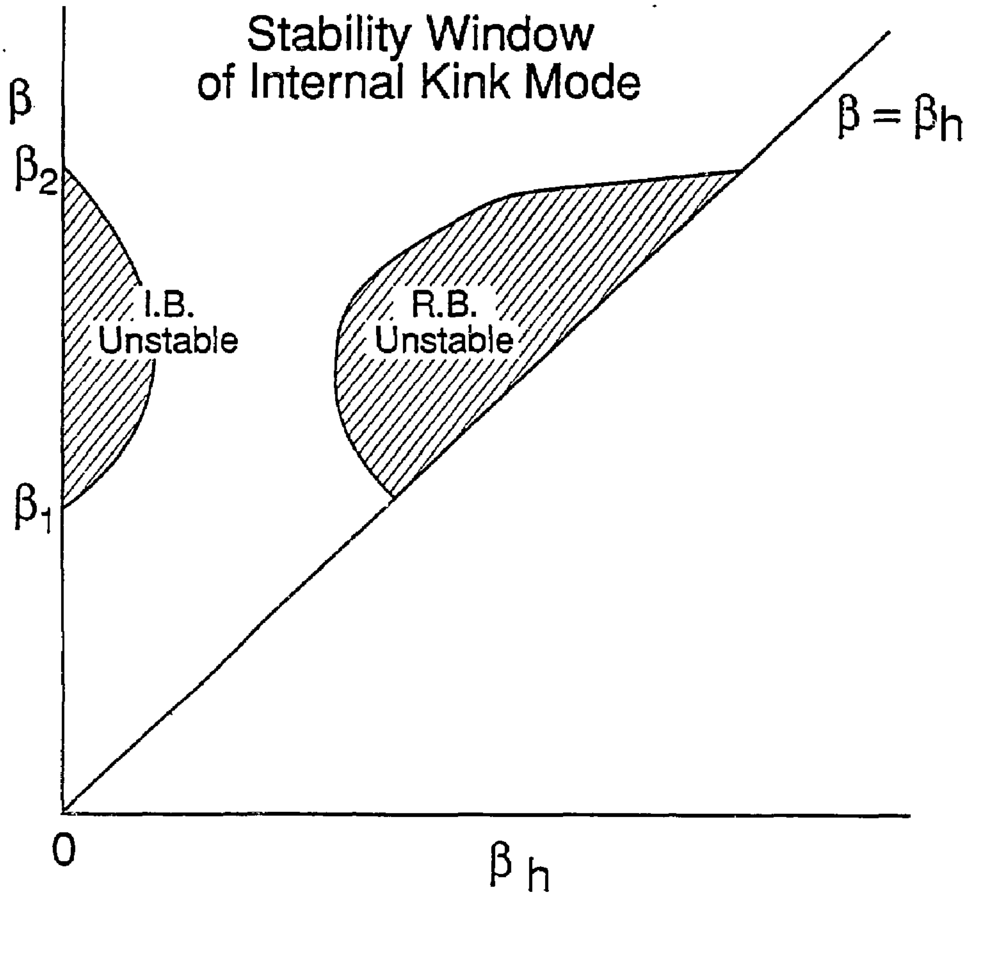
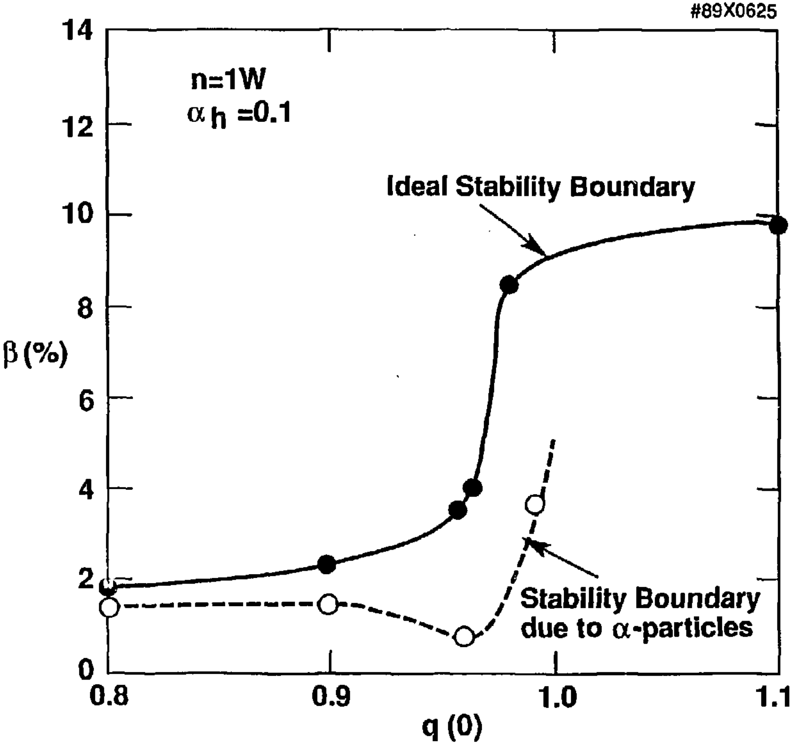


Fig. 3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α h =0.1
Ideal Stability Boundary
Stability Boundary
due to α-particles
β (%)
q (0)
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10
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6
4
2
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0.8
0.9
1.0
1.1

Fig. 4

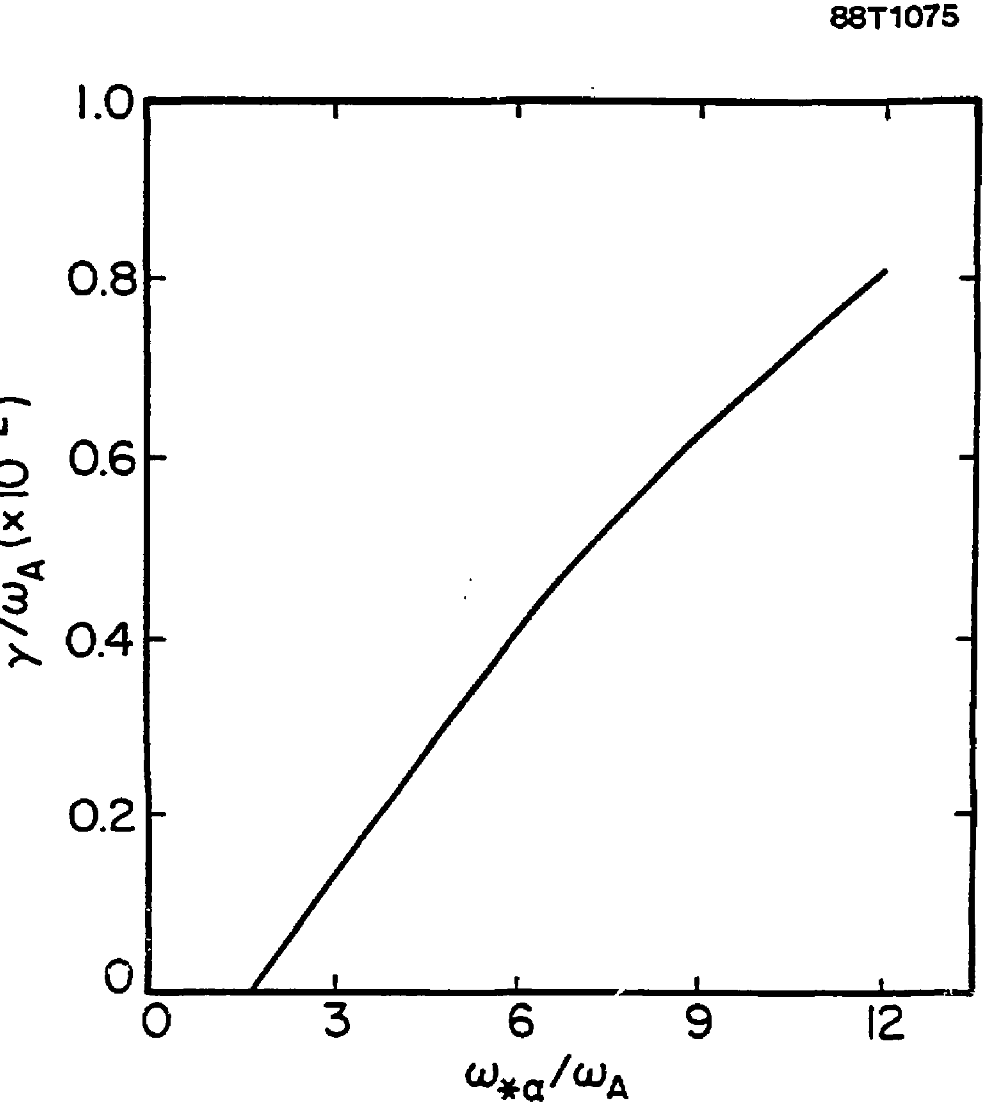

Fig. 5

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