Fermi National Accelerator Laboratory

Fermilab-Conf-89/211-T
December 29, 1989

APPLICATION OF HEAVY-LIGHT METHODS TO B MESON PHYSICS*

Estia EICHTEN, G. HOCKNEY and H. B. THACKER†

Theoretical Physics Group, Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, IL 60510, USA

The heavy-light method is applied to the study of the B meson spectrum, the pseudoscalar decay constant f_B, the mixing (B) parameter, and exclusive semileptonic B meson decays. Preliminary results are discussed for f_B and the B parameter at $\beta = 5.7$ and $\kappa = 0.165$ on a $12^3 \times 24$ lattice and at $\beta = 5.9$ and $\kappa = 0.158$ on a $16^3 \times 32$ lattice.

1. INTRODUCTION

Heavy-light methods[1,2] were proposed to allow the calculation of the properties of meson systems with one light and one heavy quark - such as B mesons. The basic idea is simply to use an effective interaction which arises in the static[3,5] or nonrelativistic limit[4] of the heavy quark motion. The only difference from the usual lattice action is in the treatment of the heavy quark. The heavy quark action in the leading order in $1/m_b$ is[5]:

$$ia^3 \sum_N [b^\dagger(n)(b(n) - U_0(n-\hat{0})^\dagger b(n-\hat{0})] \qquad (1.1)$$

where $\hat{0}$ is a unit step in the time direction. Renormalization for matrix elements of interest can be performed in perturbation theory using a two step process. First, the matrix elements calculated in the covariant continuum theory are compared to their value for the static (or nonrelativistic) effective continuum action. Second the values are compared in the continuum and lattice version of the static effective action.

In the remaining sections we will discuss the applications to spectrum, decay constant and B parameter and semileptonic B decays. Some preliminary numerical results are presented and discussed.

2. HEAVY-LIGHT MASSES

The most basic measurement to study in the heavy-light approach is the spectrum of B mesons. In lowest order in $1/m_b$ the excitation spectrum can be calculated on the lattice by standard techniques. The lowest mass state for each distinct set of conserved quantum numbers can be obtained using the two point correlation function of any operator with these quantum numbers. In the heavy quark limit, the good quantum numbers are the total angular momentum (J), parity, and total angular momentum of the light quark (j_l). Different values of j_l are *not* degenerate even in the static limit. In the usual nonrelativistic terminology, the S-wave B mesons (the pseudoscalar and vector) are degenerate but the P- wave mesons are split. There are two degenerate states with $(j_l, J) = (3/2, 2)$ and $(3/2, 1)$ and two with $(j_l, J) = (1/2, 1)$ and $(1/2, 0)$. The difference between the mass measured by this method for the various B meson states and that of the pseudoscalar B meson (the true ground state) is the excitation energy and is physical. In order $1/m_b$, all remaining degeneracies are broken. It is to be expected that corrections of order $1/m_b$ to the spectrum with a given (j_l, J) should be of the order of ten percent of the splitting between ground and first excited state.

In the heavy-light approach, the actual mass parameter extracted from the two point correlation is a binding energy. For the pseudoscalar B meson this binding energy $\mathcal{E}$ is defined by:

$$\mathcal{E} = M_B - m_b \qquad (2.1)$$

where M_B is the physical B meson mass and m_b is the bare b quark mass in the lattice theory. Care must be exercised in applying this formal argument. The bare mass on the lattice never directly appears in the heavy-light formulation. It would be more physical to replace it by the renormalized mass defined at some convenient

*Talk presented by Estia Eichten at the 1989 Symposium on Lattice Field Theory, Capri, Italy, September 18-21, 1989.

†Present address Department of Physics, University of Virginia, Charlottesville, VA 22901, USA

normalization scale in the usual covariant continuum lagrangian. The relation between these quantities can actually be calculated in perturbation theory by using the static continuum action as an intermediate step[5]. Since the mass renormalization in the covariant continuum theory is standard and there is no mass renormalization in the static continuum effective theory, it only remains to relate the bare and renormalized mass on the lattice. In the lattice theory, the mass renormalization for the static heavy quark is[6]

$$\delta m = -\frac{g^2}{12\pi^2}\frac{1}{a}C \qquad (2.2)$$

where

$$C = \frac{1}{\pi}\int d^3l \frac{1}{4\sum_{i=1}^{i=3}\sin^2(l_i/2)} \approx 19.95 \qquad (2.3)$$

Note that the measured quantity

$$M_B - m_b(\text{bare}) = M_B - m_b(\text{renormalized}) - \delta m \quad (2.4)$$

therefore has dependence on the lattice spacing a. We will come back to this later.

3. DECAY CONSTANT

The simplest matrix element to study is f_B, the decay constant of the B meson, defined by

$$-i < 0|j_A^\mu|B(p_B) >= p_B^\mu f_B \qquad (3.1)$$

Here j_A is the axial current

$$j_A^\mu = \overline{\psi}_l \gamma^\mu \gamma^5 \psi_b \qquad (3.2)$$

where ψ_l is the light quark field and ψ_b is the b quark field. Using the effective lattice action (Eq. 1.1) for the heavy quark the two point correlation function for the axial current is related to f_B and the binding mass $\mathcal{E} = M_B - m_b(\text{bare})$ of the ground state B meson as follows:

$$< Tr[(\frac{1-\gamma^0}{2})S_l(t,0)\prod_{t=0}^{ta=T} U_0(ta)] >$$
$$\rightarrow \frac{1}{2}M_B f_B^2 a^3 \exp(-at\mathcal{E}) \qquad (3.3)$$

where $S_l(t,0)$ is the light quark propagator.

Another standard parameter of the B meson system that can be calculated by heavy-light methods is the $B^0 - \overline{B^0}$ mixing parameter. The extraction of this B parameter uses the same method[1] as used for the K and D mesons systems.

4. PRELIMINARY RESULTS

In this section we will present results for two sets of runs :

- 75 configurations separated by 500 pseudo-heatbath sweeps on a $12^3 \times 24$ lattice at $\beta = 5.7$ and with $\kappa = 0.165$ for the light quark propagator.
- 50 configurations separated by 2000 sweeps on a $16^3 \times 32$ lattice at $\beta = 5.9$ and with $\kappa = 0.158$.

We have checked using the multiple-elimination jackknife method that this number of sweeps between configurations is sufficient to produce uncorrelated results. As was found previously[7], there is a large contribution from excited states for the two point correlation using the point operators for the interpolating field ($< XX >$). Here we report on one simple trick that can be employed for heavy-light systems to allow smearing over various paths for the heavy quark propagator and is very effective in reducing the contributions from excited states. For the correlation between the light quark source point $(0,0)$ and the sink point $(0, T = ta)$ the point axial current operator can be replaced by a smeared operator at either or both ends ($< SX >, < XS >$ and $< SS >$) without having to calculate the light quark propagator for a smeared source. The smearing operator used here is:

$$\psi_l^\dagger(0)\gamma^5 \sum_P [\prod_P U] W(P)\psi_b(0 + n_P), \qquad (4.1)$$

which is gauge invariant. A weight $W(P)$ can be assigned to each product of gauge links along a spatial path P (starting at 0 and ending at n_P). Not only can the mass of the ground state be extracted using these smeared operators but the value of the decay constant can also be obtained by using the relation

$$< XX > \simeq < SX >< XS > / < SS > \qquad (4.2)$$

for the ground state contribution to $< XX >$. In the results presented here we have summed over all spatial paths of two or less links and assigned unit weight to each path.

Smearing results in a dramatic improvement in the ability to extract the value of binding mass $\mathcal{E}$ and f_B. To show this we define a binding mass at timeslice t, $\mathcal{E}(t)$, from the correlation function F(t) ($=< XX >$ or $< SX >$) by

$$\mathcal{E}(t) = \frac{1}{a} ln(F(t)/F(t+1)). \qquad (4.3)$$

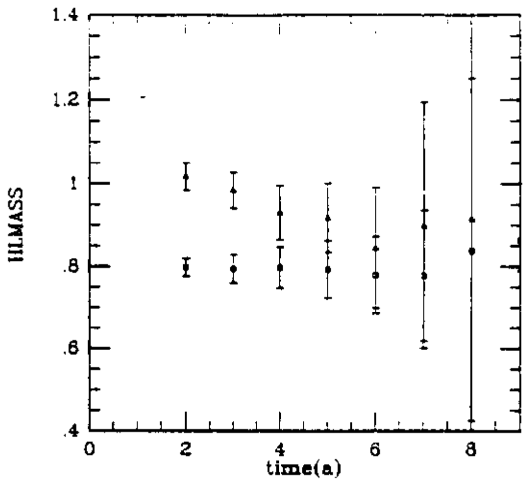

Figure 1: The binding mass $a\mathcal{E}(t)$ measured for the unsmeared (triangles) and smeared (squares) correlator as a function of time slice t. The data is for 50 configurations on a $16^3 \times 32$ lattice with $\beta = 5.9$ and $\kappa = 0.158$.

Source	Sink	$t \geq 2$	$t \geq 3$
S	X	$.795 \pm .035$	$.797 \pm .050$
X	S	$.817 \pm .095$	$.794 \pm .160$
S	S	$.795 \pm .057$	$.796 \pm .100$

Table 1: Results for $a\mathcal{E}$ with various choices for the beginning timeslices and smearing methods. S denotes smearing and X denotes no smearing on source or sink timeslice of light quark propagator.

The results for $16^3 \times 32$ are shown in Figure 1. The stability of results to choice of the beginning time slice and to the smearing procedure is showing in Table 1 for the $16^3 \times 32$ lattices. Smearing only the source gives the least noisy results.

A summary of our preliminary results is given in Table 2. The behavior of the $< XX >$ correlation function is completely consistent with the data of Boucaud et. al.[7]. However our results for f_B are in clear disagreement with their upper bound.

size	β	κ	$a\mathcal{E}$	$f_B a^{1.5}$
$12^3 \times 24$	5.7	.165	$.802 \pm .050$	$.405 \pm .165$
$16^3 \times 32$	5.9	.158	$.795 \pm .035$	$.350 \pm .180$

Table 2: The results for the binding mass and decay constant in lattice units a (in GeV^{-1}).

In physical units $\beta = 5.7$ corresponds approximately to $1/a = 1.0(GeV)$ and $\beta = 5.9$ is approximately $1/a = 1.7(GeV)$. Therefore the mass, $\mathcal{E}$, is not constant in physical units. This can be understood if it is remembered that the bare mass on the lattice has a term which behaves as $1/a$ as a goes to zero. The behaviour of the decay constant, f_B, is more surprising as it is large and $f_B a^{1.5}$ does not show much tendency to scale properly with varying lattice spacing, a. The effects of renormalization of f_B are giving by the renormalization constant Z_A defined by

$$f_B(physical) = Z_A^{-1} f_B. \tag{4.4}$$

This renormalization constant has been calculated by two groups[8,6]. Their results do not completely agree. Using the lattice value of $\alpha_s(1/a)$ the one loop values of Z_A are .98 for $\beta = 5.7$ and 1.10 for $\beta = 5.9$ are obtained by Eichten and Hill[6].

We can check our results for f_B by using an different method to extract the decay constant. If the point-to-point correlation is fit to two exponentials with the lower mass obtained from the smeared result then the excited state and the value of f_B for the ground state can be extracted. The form is

$$A \exp(-at\mathcal{E})(1 + B \exp(-at\delta\mathcal{E})) \tag{4.5}$$

The data and fit for our $16^3 \times 32$ data is shown in Figure 2. We obtain $A = 1.06 \pm .44$. This corresponds to $f_B = .360 \pm .150(a^{-1.5})$ GeV which is consistent with the results in Table 2. The value for the excitation energy $\delta\mathcal{E} = .350 \pm .170$ (1/a) GeV and for the relative strength of the excited state $B = 3.9 \pm 1.4$. As expected we see significant mixing with excited states in $< XX >$.

Finally, the values for the B parameter extracted for these lattices are consistent with one. For example, for the $12^3 \times 24$ we obtain:

$$\begin{aligned} B &= 3/4(B_{direct} + B_{exchange}) \\ B_{direct} &= 1.02 \pm .10 \\ B_{exchange} &= 0.34 \pm .12 \end{aligned} \tag{4.6}$$

5. SEMILEPTONIC B DECAYS

The final set of applications consider transition matrix elements between heavy-light meson states and ordinary light meson states. One important example of this class is the semileptonic decay of the B meson.

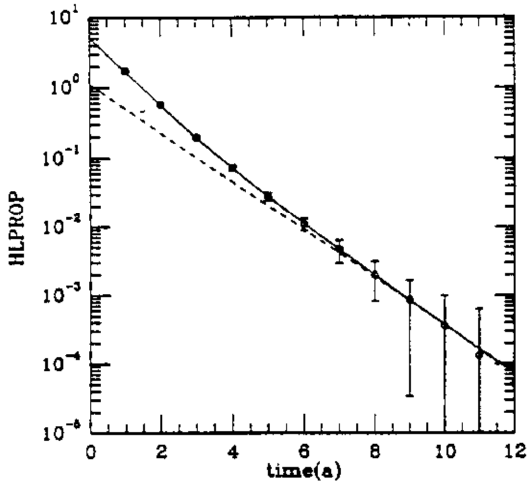

Figure 2: The axial current correlation without any smearing for our $16^3 \times 32$ lattices. The dashed curved shows the ground state exponential with $a\mathcal{E} = .795$ fixed from the analysis of the smeared propagator. The solid curve is the two exponential fit with the ground state mass $a\mathcal{E} = .795$ fixed from the analysis of the smeared propagator. The mass of the excited state and the amplitude for both exponentials are fit from this data. The error bars shown are only the diagonal elements of the error matrix.

The $B \to e^+ + \nu + X$ decay rate is

$$\begin{aligned} d\Gamma(B \;\; \to \;\; & e^+ + \nu + X) \equiv \\ & |V_{cb}|^2 d\hat{\Gamma}(B \to e^+ + \nu + X_c) \\ + \;\; & |V_{ub}|^2 d\hat{\Gamma}(B \to e^+ + \nu + X_u) \end{aligned} \quad (5.1)$$

where the $d\hat{\Gamma}$'s are, in principle, calculable within the standard model. The calculation of $d\hat{\Gamma}$ allows the extraction of $|V_{cb}|$ and $|V_{ub}|$ from the measurements of B meson semileptonic decays. In the free quark model

$$d\hat{\Gamma}(B \to e^+ + \nu + X_q) = \frac{G_F^2 M_B^5}{192\pi^3} f(r) \quad (5.2)$$

where $r = m_q/m_b$ and $f(r) \equiv 1 - 8r^2 + 8r^6 - r^8 + 24r^4 \ln r$. In reality these partial decay amplitudes are difficult to calculate because of nonperturbative QCD effects.

However, near the end-point of the positron energy spectrum, the relevant amplitudes can be calculated using heavy-light methods. This allows lattice calculations of exclusive final hadronic states $X = \pi, \rho, \omega, \ldots$. For positron energy E_e greater than the endpoint for D meson final state

$$E_e > (M_B^2 - M_D^2)/2M_B \quad (5.3)$$

there is no contribution from final states involving charmed quarks. Thus the contribution to $b \to u$ quark transitions can be isolated, providing a measurement of V_{bu}.

The kinematics of these endpoint decays has been calculated by Isgur et. al.[9]. For $x \equiv 2E_e/M_B$ and $y \equiv (p_B - p_X)^2/M_B^2$ near one, the b propagator can be replaced with the heavy quark limit because the weak current extracts a momentum close to m_b. Hence in the effective lattice action, no momentum large compared to the QCD scale remains.

6. SUMMARY

There are a rich and varied set of applications of heavy-light methods to the B meson system. In this brief report we have only mentioned some of the most obvious examples. The one-loop renormalization calculations required to the match the lattice and physical quantities are proceeding. The preliminary numerical results at $\beta = 5.7$ and $\beta = 5.9$ are somewhat mysterious. The values for f_B are unexpectedly large and the proper scaling behavior in lattice spacing is not seen. These issues are likely to be understood and resolved in the coming year.

REFERENCES

1. E. Eichten, in Field Theory on the Lattice, Nucl. Phys. B (Proc. Suppl.) 4 (1988) 170.
2. G.P. Lepage and B.A. Thacker, in Field Theory on the Lattice, Nucl. Phys. B (Proc. Suppl.) 4 (1988) 199.
3. E. Eichten and F. Feinberg, Phys. Rev. D **23** (1981) 2724.
4. W. E. Caswell and G. P. Lepage, Phys. Lett. **167B** (1986) 437.
5. E. Eichten and B. Hill, FERMILAB-PUB-89/184T, to appear in Phys. Lett. B.
6. E. Eichten and B. Hill, FERMILAB-PUB-89/209T, submitted to Phys. Lett. B.
7. Ph. Boucaud, O. Pene, V. J. Hill, C. T. Sachrajda and G. Martinelli, Phys. Lett. **220B** (1989) 219.
8. Ph. Boucaud, C. L. Lin and O. Pene, Phys. Rev. D **40** (1989) 1529.
9. N. Isgur, D. Scora, B. Grinstein and M. Wise, Phys. Rev. D **39** (1989) 799.