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WIGNER-KIRKWOOD EXPANSION OF THE QUASI-ELASTIC NUCLEAR RESPONSES AND APPLICATION TO SPIN-ISOSPIN RESPONSES

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We derive a semi-classical Wigner-Kirkwood expansion ($\hbar$ expansion) of the linear response functions. We find that the semi-classical results compare very well to the quantum mechanical calculations. We apply our formalism to the spin-isospin responses and show that surface-peaked $\hbar^2$ corrections considerably decrease the ratio longitudinal/transverse as obtained through the Los Alamos ($\vec{p}$, $\vec{p}$') experiment.

I. INTRODUCTION

The aim of this paper is twofold. First we present a semi-classical method, going beyond the Thomas-Fermi (TF) approximation, for the calculation of the quasi-elastic nuclear response functions at intermediate momentum transfer ($q \sim 2\text{-}3\ m_\pi$) and second we apply the formalism to the problem of spin-isospin responses which has been one of the central topics of this conference.

In recent years, semi-classical methods have proven to be very convenient and accurate for describing gross nuclear properties where the influence of shell effects has been averaged out. In particular, it has been found that the Thomas-Fermi single-particle response compares extremely well to its quantum mechanical analog[1]. Moreover, realistic TF-RPA calculations give a good description of structure functions measured in electron scattering experiments[2-4].

We will calculate the correction to the T-F response by performing an $\hbar$ (or Wigner-Kirkwood) expansion. At the level of the mean-field (i.e. single particle) response, we will find that the surface-peaked $\hbar^2$ corrections to the TF (i.e. $\hbar^0$) response are, as expected, very small. In addition, we will show that the semi-classical RPA response compares also very well to an exact model calculation which is half quantal in the sense that, for the mean-field response, the semi-classical TF response is used as the input of the integral (Bethe-Salpeter) equation[5]. Again, the TF result is very good and the $\hbar^2$ corrections, although small, improve the agreement.

The main effect of the surface-peaked $\hbar^2$ corrections to the RPA response is, in general, to reduce the collective reshaping predicted by TF theory. This effect remains small as far as we are concerned with volume response, as probed in electron scattering, but may become important for surface responses probed by hadrons. We have applied our formalism[6], to the

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calculation of the ratio R_L/R_T between the longitudinal and transverse surface spin-isospin response functions measured through a $(\vec{p}, \vec{p}\,')$ experiment performed in Los Alamos[7]. Due to the presence of the one-pion exchange potential, TF theory[3] predicts a ratio R_L/R_T much larger than the measured one which is close to one. We will show that the $\hbar^2$ corrections considerably improve the agreement with data.

II. OUTLINE OF THE SEMI-CLASSICAL METHOD

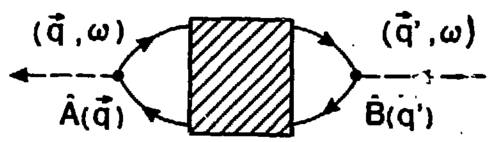

Figure 1 *- The polarization propagator $\pi_{AB}(\vec{q},\vec{q}\,';\omega)$*

The response of the nucleus to any external excitation can be obtained from the polarization propagator $\pi_{AB}(\vec{q},\vec{q}\,';\omega)$ depicted schematically on **fig. 1**. Here the "end-point" operators $\hat{A}(\vec{q})$ and $\hat{B}(\vec{q}\,')$ may involve the identity operator or spin and isospin operators such as σ_α,τ_β and $\sigma_\alpha\tau_\beta$. The polarization propagators are considered as matrix elements in momentum space of one-body operators and are expressible in terms of their Wigner-transforms[8] (WT) $[\pi_{AB}]_W(\vec{R},\vec{K};\omega)$ according to :

$$\pi_{AB}\left(\vec{K}+\frac{\vec{k}}{2}, \vec{K}-\frac{\vec{k}}{2}\,;\,\omega\right) = \int d^3R\; e^{i\vec{k}.\vec{R}/h}\; [\pi_{AB}]\,(\vec{R},\vec{K};\omega) \qquad \text{(II.1)}$$

The semi-classical method consists in an expansion in power of $\hbar$ of the WT $[\pi_{AB}]_W$. In practice we will limit ourselves to order h^2. The response to an external weakly interacting probe, transferring to the nucleus a momentum-energy $(\vec{q};\omega)$ and interacting with the nucleus through the one-body operator $\hat{A}(\vec{q})$ is :

$$R_A(\vec{q};\omega) = \sum_n \left| \;\langle n| \sum_{i=1}^{A} \hat{A}(\vec{q})\, e^{i\vec{q}.\vec{x}}\; |0\rangle \right|^2 \delta\,(E_n-\omega) \qquad \text{(II.2)}$$

where $|0\rangle$ is the ground state and $|n\rangle$ is an excited state with energy E_n. This response is obtained as :

$$R_A(\vec{q};\omega) = -\frac{1}{\pi}\,\mathrm{Im}\left\{ \int d^3R\; [\pi_{AA}]_W\,(\vec{R},\vec{q};\omega) \right\} \qquad \text{(II.3)}$$

In the following we will evaluate semi-classically the response (II.3) at the mean-field level and at the RPA level. For simplicity we will present the formalism at zero temperature but the actual calculations are performed at finite temperature[5] in order to avoid tedious manipulations involving distributions. The zero temperature case is recovered at the end as a limit. In practice the T = 1 MeV result is equal to the T = 0 result with an accuracy better, and most of the time far better, than one per cent.

III. THE PARTICLE-HOLE (OR MEAN-FIELD) RESPONSE FUNCTION

We consider a spherical nucleus with N = Z where the nucleons move in a mean-field schematized by a central local single particle V(R) but all what we will say can be easily extended to a non local mean-field potential. The pure particle-hole response is independent of the spin-isospin structure of the excitation operators and can be written in terms of WT of one-body operators[5]:

$$R_0(\vec{q};\omega) = \int d^3R\, \frac{d^3p}{(2\pi\hbar)^3}\, \frac{dt}{2\pi}\, e^{i\omega t} \Big\{ [\theta(\epsilon_F-\hat{H})e^{i\hat{H}t}]_W\,(\vec{R},\vec{p}) \cdot [\theta(\hat{H}-\epsilon_F)e^{-i\hat{H}t}]_W\,(\vec{R},\vec{p}+\vec{q}) \Big\} \qquad \text{(III.1)}$$

where $\hat{H}$ is the single-particle Hamiltonian and ϵ_F is the Fermi energy. The one-body operators appearing in Eq.(III.1) can be expanded in powers of $\hbar^2$ [8]. We thus write the response as :

$$R_0(\vec{q};\omega) = -\frac{1}{\pi}\,\mathrm{Im}\left\{ \int d^3R \left(\alpha_0(R,q;\omega) + \hbar^2\,\alpha_2(R,q;\omega)\right) \right\} \qquad \text{(III.2)}$$

The familar Thomas-Fermi result[1] is the first term (i.e. $\hbar^0$ term) of the expansion. In the momentum range of interest the surface-peaked $\hbar^2$ correction, which depends on the derivatives of V(R), gives only a few per cent corrections. The exact expression of α_0 et α_2 may be found in ref. 5 and 6. We show on fig. 2 the semi-classical responses together with the exact quantum mechanical result[9] (^{40}Ca, Woods-Saxon potential, q = 2.15 Fm^{-1}). We see that the semi-classical results are very close to the quantum one and that $\hbar^2$ corrections, which are positive (negative) at low (high) energy, are very small. To be complete we have to signal a minor pathology, not shown on the figures, which is inherent to all $\hbar$ corrections of the Wigner-Kirkwood type. At very high energy (where the TF response vanishes) the $\hbar^2$ response may become negative or even divergent in such a way that the S_1 sum rule remains fulfilled[5]. Such a problem could be solved by use of partial resummation methods.

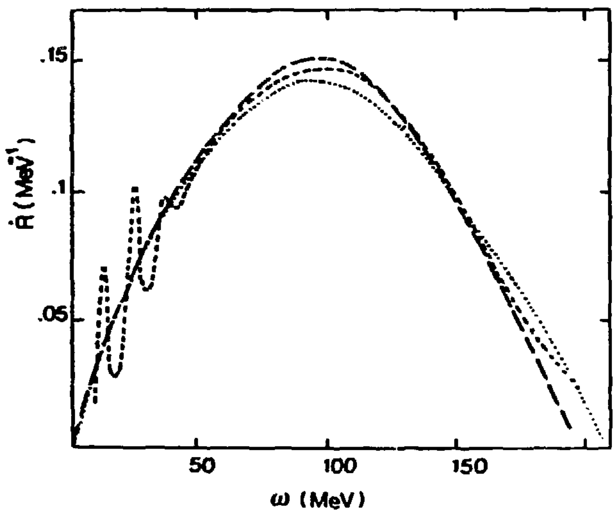

Figure 2 - Mean field response for ^{40}Ca at momentum transfer q = 2.15 Fm^{-1} versus transferred energy ω. The dotted line corresponds to the Thomas-Fermi calculation and the long-dashed line refers to the ($\hbar^0+\hbar^2$) calculation. Also shown (short-dashed line) is the result of the exact quantum mechanical calculation[9].

IV. THE RPA RESPONSE (CENTRAL FORCE)

In order to show how the semi-classical method works, we consider a very simple case. We calculate the response for can external operator $\hat{A} = \mathbb{1}_{\vec{q}}$ in the RPA ring approximation. Thus, we only need the central component $V_c(\vec{k})$ of the residual interaction. The RPA polarization propagators obeys an integral equation :

$$\pi_c(\vec{q},\vec{q}\,';\omega) = \pi_0(\vec{q},\vec{q}\,';\omega) + \int \frac{d^3k}{(2\pi)^3}\,\pi_0(\vec{q},\vec{k};\omega)\,V_c(\vec{k})\,\pi_c(\vec{k},\vec{q}\,';\omega) \qquad \text{(IV.1)}$$

Taking the WT of Eq. (IV.1), we get an equivalent equation, which is solved order by order up to $\hbar^2$. For doing that, we essentially need the WT of mean-field polarization propagator i.e $[\pi_0]_W = \alpha_0 + \hbar^2\,\alpha_2$. Its imaginary part has been obtained in the previous section and its real part is obtainable from a dispersion relation. The RPA response is thus written as[5] :

$$R_c(q;\omega) = -\frac{1}{\pi}\,\mathrm{Im}\left\{ \int d^3R \left(\tilde{\alpha}_0(R,q;\omega) + \hbar^2 \sum_{i=1}^{3} \tilde{\alpha}_2^{(i)}(R,q;\omega) \right) \right\} \qquad \text{(IV.2)}$$

The renormalized TF contribution is formally identical to the nuclear matter RPA response :

$$\tilde{\alpha}_0\ (R,q;\omega) = \alpha_0(R,q;\omega)\ /\ [1 - V_c(q)\ \alpha_0(R,q;\omega)] \tag{IV.3}$$

We have divided the $\hbar^2$ term in three distinct pieces. The first one :

$$\tilde{\alpha}_2^{(1)}\ (R,q;\omega) = \alpha_2\ (R,q;\omega)\ /\ [1 - V_c(q)\ \alpha_0(R,q;\omega)]^2 \tag{IV.4}$$

corresponds to the renormalization of the $\hbar^2$ correction to the mean-field response. The expressions of $\tilde{\alpha}_2^{(2,3)}$, given in ref. 5, depend on derivatives of α_0 with respect to q and R. Here, we simply notice that $\tilde{\alpha}^{(2)}$ is a term wich is proportionnal to V_c or V_c^2, and $\tilde{\alpha}_2^{(3)}$, involving derivatives, with respect to the momentum q, of the residual interaction $V_c(q)$ describes finite range effects and thus disappears in case of a contact force. We have compared the exact RPA response [10], solution of Eq (IV.1) with the semi-classical one, taking in both cases the TF expression of π_0 as an input. On fig. 3 we show the results of the calculations (^{40}Ca, q = 427.3 MeV) for an attractive contact force V_0 = -118 MeV.Fm^3. We see that the semi-classical approximation is excellent and the small $\hbar^2$ correction brings the semi-classical response practically equal to the exact one. Fig. 4 corresponds to the result

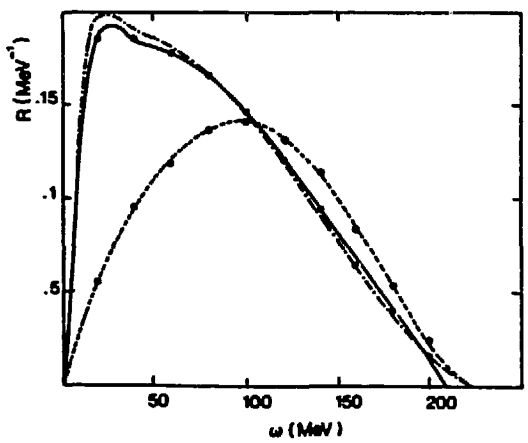

Figure 3 - *Mean field and RPA responses for ^{40}Ca at momentum transfer q = 427.3 MeV versus transferred energy ω. The dashed line is the Thomas-Fermi mean-field result. The open circles represent the same calculation but obtained by J. Delorme after expansion in partial waves. The dot-dashed line is the Thomas-Fermi RPA response and the full line refers to the ($\hbar^0$+$\hbar^2$) RPA response, ignoring $\hbar^2$ corrections coming from the mean-field response. These curves have to be compared with the exact RPA result (full circle) of J. Delorme[10] where the Thomas-Fermi mean-field propagator (expanded in partial waves) is used as an input. The residual interaction is an attractive contact force V_0 = - 118 MeV-Fm^3.*

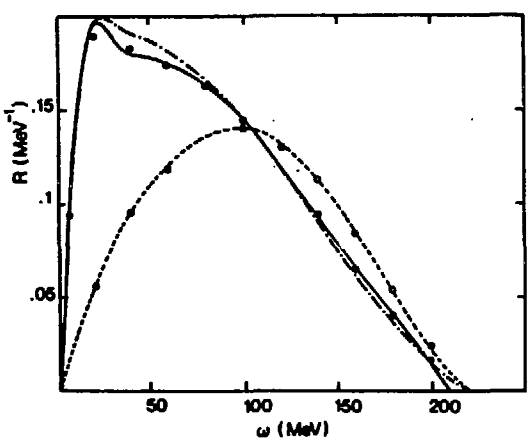

Figure 4 - *The same as fig. 3 but for a finite range force (see text).*

with an interaction of the same strength but with a momentum dependence $\exp(-q^2r_0^2/4)$ with r_0 = 0.5 Fm. If we increase the range of the interaction (r_0 = 1 Fm) the agreement remains excellent apart for the very low energy region where the slight resonance which is present in the exact calculation is overestimated in the semi-classical approach.

V. THE SPIN-ISOSPIN RESPONSES AND INTERPRETATION OF THE LOS ALAMOS $(\vec{p},\vec{p}\,')$ EXPERIMENT

We are now interested in the responses to an external excitation mediated by either the longitudinal spin-isospin operator

$$L(\vec{q}) = \vec{\sigma}.\hat{q}\,\tau^{\alpha} \tag{V.1a}$$

or a component of the transverse spin-isospin operator :

$$T_{\mu}(\vec{q}) = (\vec{\sigma} \times \hat{q})_{\mu}\,\tau^{\alpha} \tag{V.1b}$$

The residual (p-h) interaction $V_L (V_T)$ consists of pion (rho) exchange potential plus the short range g' interaction (see ref. 6 for details). The effect of antisymetrization is assumed to be incorporated in the Landau-Migdal parameter g'. This procedure has been justified by Stroth et al. at the TF level i.e antisymetrization yields a simple redefinition of g'. The longitudinal RPA polarization propagator is obtained as a solution of coupled integral equations which write in a schematic way[6] :

$$\pi_{LL} = \pi^{o}_{LL} + \pi^{o}_{LL}\, V_{LL}\, \pi_{LL} + \Sigma_{\mu} \pi^{o}_{LT_{\mu}}\, V_T\, \pi_{T_{\mu}L} \tag{V.2a}$$

$$\pi_{T_{\mu}L} = \pi^{o}_{T_{\mu}} + \pi^{o}_{T_{\mu}L}\, V_L\, \pi_{LL} + \Sigma_{\nu}\, \pi^{o}_{T_{\mu\nu}}\, V_T\, \pi_{T_{\nu}L} \tag{V.2b}$$

As before, we take the WT of these equations and solve order by order up to $\hbar^2$. Apart from the inclusion of an additionnal $\hbar^2$ correction proportionnal to $\nabla^2_R \alpha_0(R,q;\omega)$ the WT $[\pi^o_{LL}]_W(R,q;\omega)$ is essentially identical to $[\pi_0]_W$ once the Δ-hole contribution, specific to the spin-isospin channel, is taken into account[6]. From Eq. (V.2), we see that there is a coupling between longitudinal and transverse channels.The driving term of this L-T coupling term has the form[6] :

$$\pi^{o}_{T_{\mu}L}(\vec{R},\vec{q};\omega) = -\frac{i\hbar}{q}\,\left(\hat{q} \times \vec{\nabla}_{\vec{R}}\, \alpha_0\,(R,q;\omega)\right) + O(\hbar^2) \tag{V.3}$$

This L-T mixing effect occurs only to order $\hbar$ and thus disappears in the TF approximation. The RPA longitudinal spin-isospin response is found to be

$$R_L(q;\omega) = -\frac{1}{\pi}\,\mathrm{Im}\left\{ \int d^3R \left(\tilde{\alpha}_{oL}\,(R,q;\omega) + \hbar^2 \sum_{i=1}^{4} \tilde{\alpha}^{(i)}_{2L}\,(R,q;\omega)\right)\right\} \tag{V.4}$$

Here $\tilde{\alpha}_{oL}$ is the well-known expression given by Eq. (IV.3) once the central interaction V_c is replaced by the longitudinal spin-isospin residual interaction. Due to the presence of the pion exchange potential, it yields to renormalization and enhancement of the response with a magnitude strongly dependent on the value of g' [3,11]. The surface peaked $\hbar^2$ correction receives four contributions. The first three terms $\tilde{\alpha}^{(1,2,3)}_{2L}$ have already been discussed in the previous section. The last term $\alpha^{(4)}_{2L}$ is specific to the spin-isospin channel and describes the surface coupling of the longitudinal and transverse channels[6] :

$$\tilde{\alpha}^{(4)}_{2L} = \frac{1}{3\,q^2}\,\left[\nabla^2_R\, \alpha_0 + 2\, V_T\, (\vec{\nabla}_R\, \alpha_0)^2 \left(\frac{1}{1-V_T\alpha_0}\right)\right]\left(\frac{1}{1-V_L\alpha_0}\right)^2 \tag{V.5}$$

This is, in our case of interest, the most important $\hbar^2$ term. The net effect of this correction is to attenuate the collective reshaping predicted by T-F theory.

However, as shown on fig. 5 (^{40}Ca, q = 350 MeV/c, g' = 0.7), we see that this reduction is not extremely important. The transverse response R_T, is obtained from R_L, by simply exchanging the role of V_L and V_T. The effect of the $\hbar^2$ terms is now reversed i.e the quenching of R_T, predicted by TF theory is reduced.

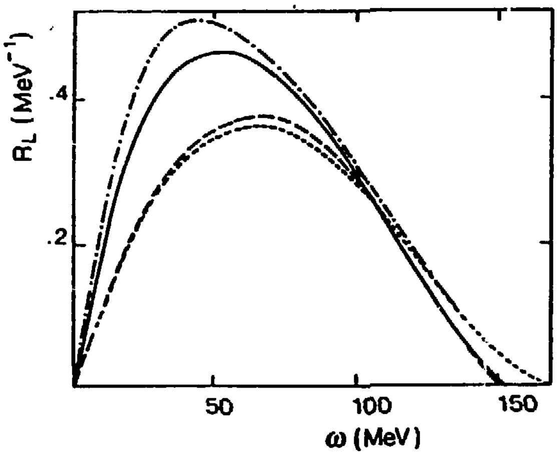

Figure 5 - *Volume longitudinal spin-isospin responses for ^{40}Ca at q = 350 MeV/c. The dotted line (mean-field response) and the dot-dashed line (RPA response) correspond to a Thomas-Fermi calculation. The dashed line(mean-field response) and the full line (RPA response) are obtained after inclusion of the $\hbar^2$ corrections.*

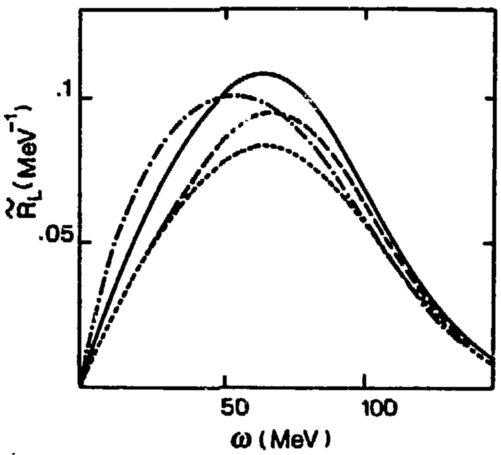

Figure 6 - *The same as fig. 5 but for surface longitudinal responses probed through inelastic scattering of 500 MeV protons.*

Unfortunately, the volume longitudinal response is not known experimentally. The only available information comes from the ($\vec{p}$, $\vec{p}\,'$) Los Alamos experiment[7] which essentially measures the surface response. Thus the surface-peaked $\hbar^2$ term will obviously acquire a much more important relative weight leading to a considerable reduction of the collective effects. In addition the isoscalar spin-responses also enter. In this experiment, Carey and collaborators measured in fact the ratio :

$$\tilde{X} = (1/2.3) \left(3.6\, \tilde{R}_L(\sigma\tau) + \tilde{R}_L(\sigma) \right) / \left(\tilde{R}_T(\sigma\tau) + \tilde{R}_T(\sigma) \right) \qquad (V.6)$$

at momentum q = 350MeV/c. The tilde mean that we are dealing with surface responses and the distorsion effect is taken into account through a position dependent weight factor C(R)[2,6] multiplying the integrand of Eq. (V.4). On fig. 6, we show the surface response $\tilde{R}_L$ (^{40}Ca, q = 350 MeV/c, g' = 0.7). We see that there is still a small enhancement effect but the shift of the peak disappears almost completely due to the $\hbar^2$ terms. In Eq. (V.6) we have approximated the isoscalar response $\tilde{R}_{L,T}(\sigma)$ by the mean-field one. The calculated ratio for calcium (lead) is shown of fig. 7 (fig. 8). It is apparent that the TF theory (dashed line) fails and the $\hbar^2$ corrections considerably improve the agreement with data. Similar conclusion have been reached by Alberico[12] et al. and Ichimura[13] whithin quantum mechanical RPA frameworks. The results for lead are very similar to those for calcium reflecting the fact that the surface properties do not depend very much on the mass number. I also would like to mention that improvements in the theoretical description, such as inclusion of 2p-2h contribution or renormalization of the isoscalar responses, are likely to further reduce the calculated ratio $\tilde{X}$.

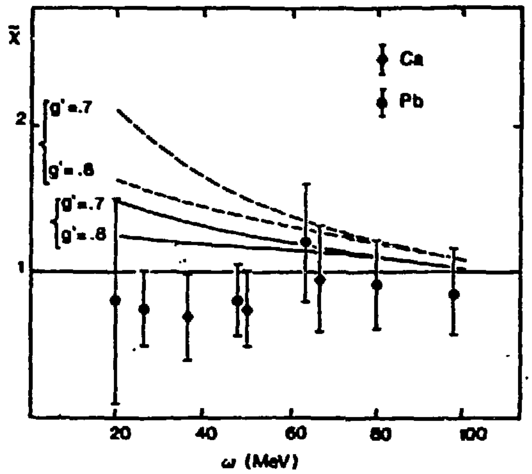

Figure 7 - *The ratio $\tilde{X}$ for ^{40}Ca as a function of the transferred energy ω at q = 350 MeV/c. The dashed curves refer to a TF-RPA calculation and the full curves correspond to the RPA result including $\hbar^2$ corrections. The experimental data are from ref. 6.*

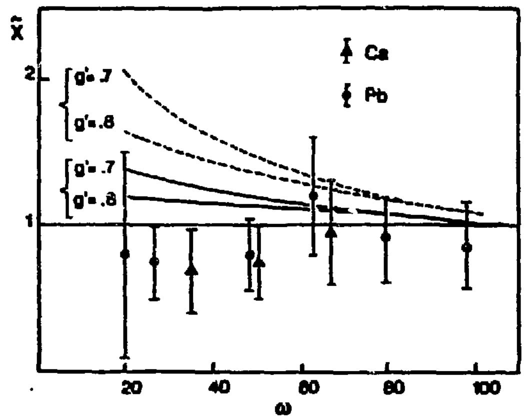

Figure 8 - *The same as fig. 7 but for lead.*

VI. CONCLUSION

As a first conclusive remark, I would like to emphasize the power and the accuracy of the semi-classical method. This approach has the merit of flexibility and allows the calculation of a RPA-type response, at intermediate momentum transfer with very small computer time for any nuclei. This theoretical tool is particularly useful for heavy nuclei for which the exact quantum mechanical calculation requires of huge basis of shell model states and becomes extremely time consuming.

The final remark concerns the experimental knowledge about the longitudinal spin-isospin response. From the arguments developped in this paper, it is thus clear that the only existing data (i.e the Los Alamos data) do not provide decisive information concerning the behaviour of this response. The collectivity, if present, is masked by the isoscalar contamination and by the very important surface effects originating from the hadronic nature of the probe. A first interesting possibility would be the use of (p,n) probes in order to eliminate the isoscalar responses. Further experimental information, less affected by surface effects would be charged electroproduction[14]. It has been shown that (e,e' $\pi^{\pm}$) with pion detected in the direction of the virtual photon yields an exact separation of the longitudinal and transverse spin-isospin responses, through a Rosenbluth plot. In addition, the distorsion of the outgoing pion wave function is less important than the combined distortion of the incident and outgoing proton wave functions. However, such an (e,e' $\pi^{\pm}$) coincidence experiment is feasible only with the next generation of high duty cycle machines.

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