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# Vector Mesons and Chiral Symmetry*

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**Abstract**

The ambiguities in the off-shell behaviour of spin-1 exchange can be resolved to $O(p^4)$ in the chiral low-energy expansion if the asymptotic behaviour of QCD is properly incorporated. As a consequence, the chiral version of vector (and axial-vector) meson dominance is model independent. Additional high-energy constraints motivated by QCD determine the $V$, $A$ resonance couplings uniquely. In particular, QCD in its effective chiral realization successfully predicts $\Gamma(\rho \to 2\pi)$.

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# 1 Introduction

Chiral perturbation theory is a systematic low-energy expansion of the generating functional for Green functions of quark currents. It can be formulated [1,2] in terms of an effective chiral Lagrangian for the matrix-valued mesonic field $U(\varphi)$ which transforms as

$$U(\varphi) \xrightarrow{G} g_R U(\varphi) g_L^\dagger \tag{1}$$

under the chiral group $G = SU(3)_R \times SU(3)_L$. To lowest order in the derivative expansion, the effective chiral Lagrangian is given by the non-linear $\sigma$-model Lagrangian

$$\mathcal{L}^{(2)} = \frac{f^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U\rangle, \tag{2}$$

$$D_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu), \quad \chi = 2B_0(s + ip), \quad \langle A\rangle = \mathrm{tr}\, A$$

in the presence of external $3\times 3$ matrix fields $v_\mu$, $a_\mu$, $s$, $p$. The low-energy constants $f$ and $B_0$ can be related to the pion decay constant and to the quark condensate, respectively [1,2]. The external fields $v_\mu$, $a_\mu$ contain in particular the electroweak gauge vector bosons. The light quark masses are included in the scalar field $s$. With the obvious chiral counting, $\mathcal{L}^{(2)}$ is the effective Lagrangian of $O(p^2)$ and it generates at tree level the standard current algebra relations for Green functions and mesonic amplitudes.

The next step in the low-energy expansion involves two parts [1,2]: the one-loop functional due to $\mathcal{L}^{(2)}$ and tree graphs from an effective chiral Lagrangian $\mathcal{L}^{(4)}$ of $O(p^4)$. In addition to an anomalous piece [3], $\mathcal{L}^{(4)}$ contains the most general chiral invariant Lagrangian $\mathcal{L}_4$ of $O(p^4)$:

$$\mathcal{L}_4 = \sum_{i=1}^{10} L_i P_i \tag{3}$$

$$\begin{aligned}
P_1 &= \langle D_\mu U^\dagger D^\mu U\rangle^2 & P_2 &= \langle D_\mu U^\dagger D_\nu U\rangle\langle D^\mu U^\dagger D^\nu U\rangle \\
P_3 &= \langle D_\mu U^\dagger D^\mu U D_\nu U^\dagger D^\nu U\rangle & P_9 &= -i\langle F_R^{\mu\nu} D_\mu U D_\nu U^\dagger + F_L^{\mu\nu} D_\mu U^\dagger D_\nu U\rangle \\
P_{10} &= \langle U^\dagger F_R^{\mu\nu} U F_{L\mu\nu}\rangle,
\end{aligned}$$

where $F_{R,L}^{\mu\nu}$ are the field strengths for the non-abelian external fields $v_\mu \pm a_\mu$. Only those terms are specified in Eq. (3) which can be produced by vector and axial-vector meson exchange [4].

$L_1, \ldots, L_{10}$ are real coupling constants which absorb the divergences of the one-loop functional due to $\mathcal{L}^{(2)}$. The physically accessible quantities are therefore the renormalized, scale-dependent couplings $L_i^r(\mu)$ where $\mu$ is an arbitrary renormalization scale ($\mu = M_\rho$ for definiteness). In a certain sense, the $L_i^r(\mu)$ are the price one has to pay when passing from the fundamental QCD Lagrangian to the effective chiral Lagrangian of $O(p^4)$. To this order, the only distinction between QCD and any other fundamental theory with a spontaneously broken chiral symmetry lies in the specific values of the $L_i^r(\mu)$ which were determined phenomenologically by Gasser and Leutwyler [2].

## 2 Low-Energy Constants and Resonance Exchange

In principle, the low-energy constants $L_i$ may receive contributions from many different sources, both at short and long distances. In the following, I shall briefly describe the work of Refs. 4,5 where meson resonance contributions of the type $V$, $A$, $S$ and $P$ were considered. I shall restrict myself to $V$ and $A$ resonances concentrating on the problem of different off-shell behaviour for spin-1 exchange [5].

There is a straightforward procedure [6] for including fields with definite $SU(3)$ transformation properties in chiral Lagrangians. For a generic octet field $R$ one has

$$R \xrightarrow{G} h(\varphi)Rh(\varphi)^{\dagger} \tag{4}$$

where the non-linear realization $h(\varphi)$ is defined by

$$u(\varphi) \xrightarrow{G} g_R u(\varphi)h(\varphi)^{\dagger} = h(\varphi)u(\varphi)g_L^{\dagger}, \qquad u(\varphi) \in G/SU(3)_V \tag{5}$$

for the coset element $u(\varphi)$ with $U(\varphi) = u(\varphi)^2$ in the usual representation.

There are various possibilities to describe spin-1 resonance fields. Following Ref. 1, we first consider antisymmetric tensor fields $R_{\mu\nu} = -R_{\nu\mu}$. To lowest order, the chiral resonance Lagrangian takes the form [4]

$$\mathcal{L}_I = \sum_{R=V,A} \mathcal{L}_{kin}(R_{\mu\nu}) + \frac{1}{2\sqrt{2}}\{F_V\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + iG_V\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle + F_A\langle A_{\mu\nu} f_-^{\mu\nu}\rangle\}, \tag{6}$$

$$f_\pm^{\mu\nu} = uF_L^{\mu\nu}u^\dagger \pm u^\dagger F_R^{\mu\nu}u, \qquad u_\mu = iu^\dagger D_\mu U u^\dagger$$

where $F_V$, $G_V$, $F_A$ are real coupling constants and the kinetic part can be found in Ref. 4. The axial field $u_\mu(\varphi)$ is the vierbein field on coset space in the presence of external fields.

The more conventional approach involves vector fields $\hat{R}_\mu$ with a corresponding Lagrangian [5]

$$\mathcal{L}_{II} = \sum_{R=V,A} \mathcal{L}_{kin}(\hat{R}_\mu) - \frac{1}{2\sqrt{2}}\{f_V\langle \hat{V}_{\mu\nu} f_+^{\mu\nu}\rangle + ig_V\langle \hat{V}_{\mu\nu}[u^\mu, u^\nu]\rangle + f_A\langle \hat{A}_{\mu\nu} f_-^{\mu\nu}\rangle\}, \tag{7}$$

$$\hat{R}_{\mu\nu} = \nabla_\mu \hat{R}_\nu - \nabla_\nu \hat{R}_\mu, \qquad \nabla_\mu = \partial_\mu + [\Gamma_\mu, \quad ],$$

$$\Gamma_\mu = \frac{1}{2}\{u^\dagger[\partial_\mu - i(v_\mu + a_\mu)]u + u[\partial_\mu - i(v_\mu - a_\mu)]u^\dagger\}$$

where $\Gamma_\mu$ is the natural connection on coset space with external fields.

Of course, the two Lagrangians are equivalent for on-shell $V$ and $A$ mesons with

$$f_V = F_V/M_V, \qquad g_V = G_V/M_V, \qquad f_A = F_A/M_A \tag{8}$$

where $M_V$, $M_A$ are the $V, A$ octet resonance masses in the chiral limit[1]. However, their off-shell behaviour could hardly be more different. In fact, $V$ and $A$ resonance exchange

[1] $V, A$ singlet resonances do not contribute to $O(p^4)$ [4].

gives rise to a local Lagrangian $\mathcal{L}_4$ in Eq. (3) with

$$L_1^I = \frac{G_V^2}{8M_V^2}, \qquad L_2^I = 2L_1^I, \qquad L_3^I = -6L_1^I, \qquad L_9^I = \frac{F_V G_V}{2M_V^2},$$

$$L_{10}^I = -\frac{F_V^2}{4M_V^2} + \frac{F_A^2}{4M_A^2}, \qquad L_i^I = 0 \qquad (i = 1,5,6,7,8), \tag{9}$$

but

$$L_i^{II} = 0 \qquad (i = 1, \ldots, 10).$$

Although a comparison with the phenomenological values $L_i^r(M_\rho)$ will be postponed to Sect. 3, it is obvious already at this stage that only model I has a chance to describe correctly the $V, A$ resonance effects for the chiral Lagrangian of $O(p^4)$.

Does nature prefer antisymmetric tensor fields for spin-1 resonances? Which are the criteria that determine the correct off-shell behaviour for $V, A$ meson exchange?

# 3 Equivalent Lagrangians for $V, A$ Mesons

The main idea put forward in Ref. 5 is to use the known asymptotic behaviour of QCD at high energies to get additional information for the chiral realization at low energies through dispersion relations. The method can be exemplified most easily for the pion form factor $F(t)$. The perturbatively accessible high-energy behaviour of QCD implies [5] that $F(t)$ obeys a dispersion relation with at most one subtraction

$$F(t) = 1 + \frac{t}{\pi}\int_0^\infty \frac{dt' \,\mathrm{Im}\, F(t')}{t'(t'-t-i\varepsilon)}. \tag{10}$$

With the Lagrangians (6) and (7), $\rho$-exchange in narrow-width approximation leads to

$$\begin{aligned} F^I(t) &= 1 + \frac{F_V G_V}{f^2}\frac{t}{M_V^2 - t} \\ F^{II}(t) &= 1 + \frac{f_V g_V}{f^2}\frac{t^2}{M_V^2 - t} \end{aligned} \tag{11}$$

and thus only model I has the correct low-energy structure (10) dictated by QCD. Model II must be repaired by adding an explicit local term of $O(p^4)$ to $\mathcal{L}_{II}$:

$$\mathcal{L}_{II} \to \mathcal{L}_{\overline{II}} = \mathcal{L}_{II} + \gamma_9^{II} P_9 \tag{12}$$

where $P_9$ is defined in Eq. (3). The resulting pion form factor

$$F^{\overline{II}}(t) = F^{II}(t) + \frac{2\gamma_9^{II}}{f^2}t \tag{13}$$

is consistent with (10) iff

$$\gamma_9^{II} = -\frac{f_V g_V}{2} \tag{14}$$

implying at the same time

$$F^{I}(t) = F^{\overline{II}}(t) \tag{15}$$

in view of Eq. (8).

For a systematic treatment, both models I and II are enlarged to

$$\mathcal{L}_{\overline{N}} = \mathcal{L}_N + \sum_{i=1,2,3,9,10} \gamma_i^N P_i, \qquad N = I, II. \tag{16}$$

The following conditions abstracted from the high-energy structure of QCD are sufficient to determine the new constants $\gamma_i^N$ uniquely [5]:

(i) The $VV - AA$ 2-point function satisfies an unsubtracted dispersion relation;

(ii) The pion form factor needs at most one subtraction;

(iii) The forward amplitudes for elastic meson-meson scattering obey once-subtracted dispersion relations.

The unique solutions are

$$\gamma_i^I = 0 \qquad (i = 1,2,3,9,10)$$

$$\gamma_1^{II} = \frac{g_V^2}{8}, \qquad \gamma_2^{II} = 2\gamma_1^{II}, \qquad \gamma_3^{II} = -6\gamma_1^{II}, \qquad \gamma_9^{II} = \frac{f_V g_V}{2}, \tag{17}$$

$$\gamma_{10}^{II} = -\frac{f_V^2}{4} + \frac{f_A^2}{4}$$

leading to the master relation

$$L_i^{\bar{I}} = L_i^I = \gamma_i^{II} = L_i^{\overline{II}} =: L_i^{V+A} \qquad (i = 1,2,3,9,10). \tag{18}$$

More explicitly, the QCD high-energy constraints ensure that there is a unique $V, A$ resonance contribution to Green functions at least to $O(p^4)$. For consistency with QCD, any other model for $V, A$ mesons [7] must be equivalent to models I and $\overline{II}$.

By making additional plausible assumptions about the high-energy behaviour of form factors, we can get additional information [5]. If both the pion form factor and the axial form factor relevant for $\pi \to e\nu\gamma$ are unsubtracted, resonance saturation gives rise to the approximate relations [5]

$$F_V G_V = f^2, \qquad F_V = 2G_V \tag{19}$$

which appear in different forms in the literature [7] and are sometimes referred to as KSFR relations. Invoking also the Weinberg sum rules [8] in the same approximation, one gets finally ($f \simeq f_\pi = 93.3$ MeV)

$$F_V = \sqrt{2}\, f_\pi, \qquad G_V = f_\pi/\sqrt{2}, \qquad F_A = f_\pi, \qquad M_A = \sqrt{2}\, M_V. \tag{20}$$

Using Eqs. (9) and (18), the $V, A$ resonance contributions are now determined without free parameters ($M_V = M_\rho$). The comparison with the phenomenological coupling constants $L_i^r(M_\rho)$ shown in the Table leads to the chiral version of *vector meson dominance*: whenever vector and axial-vector mesons can contribute at all, they dominate the corresponding low-energy coupling constants. Because of the master relation (18) this result is independent of the choice of $V, A$ resonance fields.

**Table:** Comparison between the renormalized coupling constants [2] $L_i^r(M_\rho)$ in units of $10^{-3}$ and the $V,A$ contributions (18) with resonance parameters (20) and $M_V = M_\rho$.

| | $L_1$ | $L_2$ | $L_3$ | $L_9$ | $L_{10}$ |
|---|---|---|---|---|---|
| $L_i^r(M_\rho)$ | $0.7 \pm 0.3$ | $1.3 \pm 0.7$ | $-4.4 \pm 2.5$ | $6.9 \pm 0.7$ | $-5.2 \pm 0.3$ |
| $L_i^{V+A}$ | 0.9 | 1.8 | −5.5 | 7.3 | −5.5 |

The existence of the vectorial connection $\Gamma_\mu$ on coset space defined in Eq. (7) allows for another popular representation of the spin-1 resonance Lagrangian. Instead of the vector field $\hat{V}_\mu$ used in $\mathcal{L}_{II}$ I introduce a new vector field $\tilde{V}_\mu = \hat{V}_\mu + \frac{i}{g}\Gamma_\mu$ which then transforms like a gauge field under chiral transformations

$$\tilde{V}_\mu \xrightarrow{G} h(\varphi)\tilde{V}_\mu h(\varphi)^\dagger + \frac{i}{g}\, h(\varphi)\partial_\mu h(\varphi)^\dagger. \tag{21}$$

Identifying the "gauge" coupling constant $g$ as

$$g = \frac{M_V}{2f_\pi}, \tag{22}$$

models I and $II$ with resonance parameters (20) can be shown to be equivalent to $O(p^4)$ to the Yang-Mills type Lagrangian [7]

$$\mathcal{L}_{YM} = -\frac{1}{4}\langle \tilde{V}_{\mu\nu}\tilde{V}^{\mu\nu}\rangle + \frac{M_V^2}{2}\langle(\tilde{V}_\mu - \frac{i}{g}\Gamma_\mu)^2\rangle - \frac{1}{4}\langle \hat{A}_{\mu\nu}\hat{A}^{\mu\nu}\rangle + M_V^2\langle(\hat{A}_\mu + \frac{1}{4g}u_\mu)^2\rangle, \tag{23}$$

$$\tilde{V}_{\mu\nu} = \partial_\mu \tilde{V}_\nu - \partial_\nu \tilde{V}_\mu - ig[\tilde{V}_\mu, \tilde{V}_\nu].$$

Since there is no natural axial connection on coset space, the axial-vector field $\hat{A}_\mu$ as well as $u_\mu$ transform homogeneously as in (4). The combination $\hat{A}_\mu + \frac{1}{4g}u_\mu$ corresponds to the field $\hat{A}_\mu$ appearing in $\mathcal{L}_{II}$ in Eq. (7).

Despite the gauge-like appearance of the Lagrangian (23) the $V,A$ mesons are not the gauge bosons of local chiral symmetry. For one, this rôle is already reserved for the external gauge fields, in particular the electroweak gauge bosons. Secondly, there is no Higgs mechanism at work giving masses to the $V,A$ mesons. On the contrary, the explicit mass terms in Eq. (23) exhibit the full chiral symmetry. Although we have nowhere used high-energy constraints for amplitudes with external spin-1 particles, it is remarkable that we arrive automatically at a Lagrangian (23) with the best possible high-energy behaviour for massive $V,A$ mesons [9].

# 4 QCD Prediction for $\Gamma(\rho \to 2\pi)$

The relevant coupling for the decay $\rho \to 2\pi$ is

$$\frac{iG_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle \tag{24}$$

in the anti-symmetric tensor formulation of the Lagrangian (6). The previously obtained relation $G_V = f_\pi/\sqrt{2}$ [Eq. (20)] implies

$$\Gamma(\rho \to 2\pi) = \frac{M_\rho^3}{96\pi f_\pi^2}(1 - \frac{4m_\pi^2}{M_\rho^2})^{3/2} = 141 \text{ MeV}. \quad (25)$$

In the framework of chiral perturbation theory, the success of this prediction is by no means obvious. It may even appear rather puzzling that the $\rho$-width is determined by the lowest-order coupling $G_V$ only, since higher-order terms in the chiral expansion are certainly not negligible a priori for $p^2 = M_\rho^2$. On the other hand, such higher-order couplings would seem to be irrelevant for the effective $O(p^4)$ Lagrangian $\mathcal{L}_4$. If this were the case, the success of chiral vector meson dominance (cf. the predictions for $L_1$, $L_2$, $L_3$ in the Table) would seem rather mysterious.

The solution of this paradox is once again provided by the low-energy vs. high-energy connection discussed before. Neglecting terms of $O(m_\pi^2/M_\rho^2)$ and using the relation

$$[\nabla_\mu, \nabla_\nu] = \frac{1}{4}[u_\mu, u_\nu] - \frac{i}{2}f_{+\mu\nu}, \quad (26)$$

one finds that all possible higher-order couplings have the same effect for both $\Gamma(\rho \to 2\pi)$ (on-shell $V$) and $\pi\pi \to \pi\pi$ (off-shell $V$) as

$$\langle V_{\mu\nu}\Box^n(u^\mu u^\nu)\rangle \sim \langle \Box^n V_{\mu\nu}u^\mu u^\nu\rangle \quad (27)$$

$$\Box = \nabla_\mu\nabla^\mu, \qquad n \geq 1.$$

Due to the Froissart theorem [10], the forward dispersion relation for the $\pi\pi$ scattering amplitude $T(\nu)$ in the crossing symmetric variable $\nu = (s-u)/2$ assumes the once-subtracted form

$$\hat{T}(\nu) := T(\nu) - T(0) = \frac{2\nu^2}{\pi}\int_0^\infty \frac{d\nu'}{\nu'}\frac{\text{Im }T(\nu')}{\nu'^2 - \nu^2}. \quad (28)$$

A naive calculation of $\rho$ exchange leads to

$$\hat{T}_\rho(\nu) = \frac{\nu^2 P_V(\nu)^2}{\nu^2 - M_V^4} \quad (29)$$

where $P_V(\nu)$ is now an arbitrary polynomial in $\nu$ depending on how many terms of the type (27) I want to include in the effective Lagrangian. In comparison, for the single coupling (24) used previously the polynomial $P_V(\nu)$ reduces to a constant proportional to $G_V$. The problem alluded to before can be seen explicitly by comparing Eq. (29) with

$$\Gamma(\rho \to 2\pi) \sim P_V(M_V^2)^2. \quad (30)$$

At low energies (small $\nu$) only the constant term in $P_V(\nu)$ seems to contribute to $\pi\pi$ scattering to $O(p^4)$ while the $\rho$-width is, of course, determined by the complete polynomial at $p^2 = M_V^2$.

However, the naive $\rho$ exchange amplitude (29) is in general inconsistent with the dispersion relation (28). From the previous discussion we expect that $\hat{T}_\rho(\nu)$ has to be modified by a polynomial $P_c(\nu)$:

$$\hat{T}(\nu) = \hat{T}_\rho(\nu) + P_c(\nu). \tag{31}$$

In the narrow-width approximation used in Eq. (29), the absorptive part is given by

$$\mathrm{Im}\, T(\nu)|_{\nu\geq 0} = -\frac{\pi M_V^2}{2} P_V(M_V^2)\delta(\nu - M_V^2). \tag{32}$$

Inserting this absorptive part in the dispersion relation (28), we obtain

$$\hat{T}(\nu) = \frac{\nu^2 P_V(M_V^2)^2}{\nu^2 - M_V^4} = \frac{\nu^2 P_V(\nu)^2}{\nu^2 - M_V^4} + P_c(\nu) \tag{33}$$

which uniquely fixes the "counterterm" polynomial $P_c(\nu)$.

Consequently, both the $\rho$-width and the $\pi\pi$ scattering amplitude to $O(p^4)$ are determined by $P_V(M_V^2)$ reestablishing the previous relation based on the lowest-order coupling $G_V$. It is almost self-evident that I can always perform a field redefinition of the vector meson field $V_{\mu\nu}$ in such a way that the polynomial $P_V(\nu)$ is again replaced by the single coupling constant $G_V$ as in the original treatment.

Finally, the dominant loop effects for $\pi\pi \to \rho \to \pi\pi$ are rescattering corrections which once again do not modify the relation between $\Gamma(\rho \to 2\pi)$ and the low-energy constants $L_1$, $L_2$, $L_3$.

The main conclusion, which extends beyond the specific example discussed here, is that Green functions at low $p^2$ are always sensitive to the resonance decay parameters, independently of higher-order couplings and chiral loop corrections. It is the high-energy behaviour of the underlying field theory QCD which is responsible for this remarkable property. In this sense, QCD does indeed predict the width of the $\rho$ meson.

# 5 Conclusions

i) QCD high-energy constraints enforce a unique off-shell extrapolation for $V, A$ resonances to $O(p^4)$.

ii) With additional plausible assumptions of unsubtractedness, all $V, A$ resonance couplings can be predicted in terms of $f_\pi$ and $M_V = M_\rho$.

iii) The chiral version of vector meson dominance is not an assumption, but is derived in a model independent way.

iv) The most compact representation of the chiral $V, A$ resonance Lagrangian is given by a Yang-Mills type Lagrangian. However, the $V, A$ mesons are *not* the gauge bosons of local chiral symmetry.

v) QCD in its chiral realization successfully predicts $\Gamma(\rho \rightarrow 2\pi)$.

vi) Higher-order resonance couplings do not modify the relation between resonance decay parameters and Green functions at low $p^2$.

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