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Quark jets, gluon jets and the three-gluon vertex

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ABSTRACT

Using hadronic jets in electron-positron annihilation, we suggest a simple and model-independent method to see the differences between quark and gluon jets. We define and analyse special energy dependent moments of jets and choose those which are the most characheristic to the jet type. The method handles the energy of a jet in an adequate way. We discuss new methods using jet flavor tagging, ordinary flavor tagging of a definite quark jet or discrimination between quark and gluon jets, to test the triple- gluon vertex in electron-positron annihilation. An enriched sample of gluon jets, jets with the smallest energy in four-jet events, as well as a continuous tagging variable are also studied.

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1. Introduction

Our high energy experiments produce hadrons. The detailed properties of hadronisation processes are unknown. Hadrons forming a jet are believed to mark the path of a quark or a gluon. There has been remarkable succes in correlating the data for two or three-jet variables with theoretical predictions based on partons. The next step to have a deeper insight into QCD is to see less inclusive processes. Thus we should determine the parton ancestor of a jet. At this point the most important question is to see the differences between the jets produced by a spin-half quark and a spin-one gluon.

It is even more important to see the differences between quark and gluon jets if we take into consideration that theory has definite predictions for these differences. Gluons carry a stronger color charge than quarks, and one expects this to produce a difference in their fragmentation, namely higher multiplicity, softer hadron spectrum and broader p_t spectrum. Thus lattice gauge calculations show a gluon string tension 8/3 times the corresponding quark string tension [1]. Due to the larger color charge QCD predicts $r = 9/4$ times larger multiplicity for gluon jets than for quark jets at the same infinite limit energy range [2]. This ratio is valid for parton multiplicities. Second order calculations, finite energy corrections and heavy quark effects are intensively studied [3]. These corrections at present energies reduce the value of r to approximately 1.3. On the other hand there are several theoretical papers to give methods by which the experimental data may be analysed and the above mentioned differences between quark and gluon jets can be seen [4].

Large amount of data exist on fragmentation of quark and antiquark jets.

Far less is known experimentally on fragmentation of high energy gluon jets. The experimental problem in these studies is that in e^+e^- annihilation quark jets occur predominantly in the two jet ($q\bar{q}$) topology, but gluon jets occur only in three or more jet topologies. There is thus a large kinematic difference superimposed on any dynamical difference.

Because of the simplicity of the parton level processes, a large part of our experimental knowledge on gluon jet fragmentation comes from $e^+e^- \to q\bar{q}g$. However, most studies of gluon jet fragmentation led to inconclusive results.

One of the possibilities is to study symmetric three-jet events (in this case the quark and gluon jets have the same enrgy) and to compare the results with the results of the two-jet events gained at $\frac{2}{3}$ CM energies [5-7]. The statistics is a very serious problem in these studies. There are several approaches based on studies of jets with different energies [8-10]. Clearly a kinematical problem is superimposed on the dynamical ones.

The study of symmetric three-jet and two-jet events is a very promising possibility to see the differences between quark and gluon jets, because in symmetric three-jet events where the jets are of nearly equal energy, a mistake in assigning a soft particle to a particular jet is irrelevant. Nevertheless in order to have a reasonable statistics one has to study nearly-symmetric three-jet events in a large angular range eg.: $100^o < \phi_{ij} < 140^o$ for all jet pairs. (This is the choice of the TASSO Collaboration [7].) It is easy to show that in this sample there are kinematic configurations where the energy of the most energetic jet is more than 40% higher than that of the less energetic jet. Another serious problem is the statistics. The nearly-symmetric three jet selection criteria has reduced the multihadronic TASSO events from 45852 to 396.

Despite a considerable amount of experimental effort, unambiguous differences between quark-induced and gluon-induced jets have not yet been established.

Thus, there is a basic necessity to reach a better statistics and to take the energy of the jets into consideration. The main result of our work is that it shows how to do it. It is our purpose in this paper to introduce a method capable to see the difference of an energy dependent jet-variable $A_{q/g}(E_j)$ (e.g. multiplicity, average p_t, rapidity etc.) between quark (q) and gluon (g) jets at a given jet energy (E_j) studying two-jet and non-symmetric three-jet events in e^+e^- annihilation.

The determination of the number, the energies and the directions of the jets can be done by using cluster algorithms (e.g. the d_{join} algorithm in the standard Lund 6.3 Monte-Carlo [11]). For simplicity first we suppose that our cluster algorithm identifies n-parton events as n-jets. Of course the jet number, as determined by the cluster algorithm, does not always coincide with the parton number. Events where it is not are called background. (Effects due to background will be studied later.)

2.Differences between quark and gluon jets

First, one has to determine the energy dependence of the chosen variable for quark jets ($A_q(E_j)$). This can be done by analysing two jet events at different jet-energies (e.g 22 GeV, 29 GeV, 35 GeV ,44 GeV and 57 GeV etc.). The jet energies in this case are half of the CM energies. $A_q(E_j)$ is simply the average of the jet variable.

$$A_q(E_j) = \frac{1}{N_2(E_j)} \sum_{i=1}^{N_2(E_j)} A^{(i)}(E_j)$$
$$D_q(E_j) = \frac{\sigma_q}{\sqrt{N_2(E_j)}} \quad , \tag{1}$$

where $N_2(E_j) \gg 1$ is the number of jets in our two jet sample at E_j jet energy, the summation runs over the jets in our sample, $\sigma_q^2(E_j)$ is the variance of the $A_q(E_j)$ variable and $D_q(E_j)$ is the statistical error of our determination. $A^{(i)}(E_j)$ is the actual value of the variable for the i^{th} jet having an energy E_j. To give the function of $A_q(E_j)$ for the energies between these fixed energies an interpolation must be used [5].

If we know $A_q(E_j)$ it is easy to give $A_g(E_j)$. Let us denote $p_g(x_a \mid x_b, x_c)$ the probability that a jet (in a three-jet event) with a given $x_a = 2E_a/E_{CM}$ energy fraction is a gluon jet (the two other jets have x_b and x_c energy fractions). For instance in first order of α_s this probability is clearly:

$$p_g(x_a \mid x_b, x_c) = \frac{x_b^2 + x_c^2}{(1-x_b)(1-x_c)} \cdot$$
$$\left(\frac{x_a^2 + x_b^2}{(1-x_a)(1-x_b)} + \frac{x_b^2 + x_c^2}{(1-x_b)(1-x_c)} + \frac{x_c^2 + x_a^2}{(1-x_c)(1-x_a)} \right)^{-1} \tag{2}$$

(The second order formula is available as a FORTRAN code in the e^+e^- Lund Monte-Carlo.) The probability that this jet is a quark or an antiquark jet is:

$$p_q(x_a \mid x_b, x_c) = 1 - p_g(x_a \mid x_b, x_c) \quad . \tag{3}$$

We distribute the jets into different samples according to their jet energy. For these mixed quark-gluon samples we calculate the averages of the jet variable:

$$A_{q+g}(E_j) = \frac{1}{N(E_j)} \sum_{i=1}^{N(E_j)} A^{(i)}(E_j) \quad , \tag{4}$$

where $N(E_j)$ is the numer of the jets in the sample containing jets with E_j jet energy. In these samples the expectation values for the numbers of gluons and quarks are:

$$N_g(E_j) = \sum_{i=1}^{N(E_j)} p_g^{(i)} \quad , \qquad N_q = N(E_j) - N_g(E_j) \quad , \tag{5}$$

where $p_g^{(i)}$ is the probability for the i^{th} jet to be a gluon jet, given by the second order form of eq. (2). Using the value of $A_q(E_j)$ it is possible to give $A_g(E_j)$ and its variance $\sigma_g^2(E_j)$:

$$\begin{aligned} A_g(E_j) &= \frac{1}{N_g(E_j)}\big(N(E_j)A_{q+g}(E_j) - N_q(E_j)A_q(E_j)\big) \\ \sigma_g^2(E_j) &= \frac{1}{N_g(E_j)}\Big(\sum_{i=1}^{N(E_j)} A^{(i)^2}(E_j) - N_q(E_j)\big(A_q^2(E_j) + \sigma_q^2(E_j)\big)\Big) - A_g^2(E_j). \end{aligned} \tag{6}$$

Determining both $A_q(E_j)$ and $A_g(E_j)$ the differences between quark and gluon jets can be seen as a function of the jet energy. The square of the statistical error of the determination of $A_g(E_j)$ is:

$$D_g^2 = \frac{1}{Np_g^2}\left(p_g\sigma_g^2 + (1-p_g)\sigma_q^2 + (1-p_g)^2 N\frac{\sigma_j^2}{N_2}\right) \quad . \qquad (7)$$

(For simplicity we have not indicated the energy dependence.) Eq. (7) is valid in the approximation where the $p^{(i)}$ probabilities are constant. Eg. for symmetric three-jet events $p \approx 1/3$.

To handle the large angle range and the kinematical problems mentioned above, this method should be used even in the nearly-symmetric three-jet event studies.

In the following we will apply the method outlined above to determine special moments of quark and gluon jets. In our studies we have used the JETSET 6.3 Lund Monte-Carlo program [13] in the form of second order matrix element, for finite jet resolution parameters y_{min}. Therefore in our treatement all partons are well separated so the fixed order perturbative results are applicable in this region of the phase space. We have changed the QCD part of the JETSET 6.3 to get rid of the GKS [12] approximation, by the method of the MARK J Collaboration [13]. The string fragmentation model has been used. We have analysed the moments

$$M_{nm}(E_j) = \sum \left(\frac{p_t}{E_j}\right)^n \eta^m \qquad (8)$$

at the previously mentioned 22 GeV, 29 GeV, 35 GeV, 44 GeV, and 57 GeV CM energies. 100 000 hadronic events have been analysed at each beam energy. The sum in (8) goes over the outgoing particles in a jet, (η is the pseudorapidity,

p_t is the transverse momentum of a definite particle with respect to the jet axis). Similar moments were studied in [14-16]. It is easy to see that M_{00} is the multiplicity of the jet. Clearly these moments are sensitive to the softer fragmentation, higher multiplicity and broader p_t of the gluon jets as compared to the quark jets. There is a small dependence on the jet finding algorithm parameter. However varying it in wide ranges, the changes in M_{nm} are an order of magnitude smaller than the differences between the moments for quark and gluon jets. We have used the d_{join} algorithm of the standard Lund Monte-Carlo with the same d_{join} value for both the two and three-jet events. We have chosen d_{join} the parameter of the jet finding algorithm in a way that we have approximately the same amount of background for two-jet events (from three and four-parton events) and for three-jet events (from two and four-parton events). Increasing the CM energy one has to increase d_{join}. Thus eg. for E_{CM}=29 GeV d_{join}=1.7 GeV, for E_{CM}=44 GeV d_{join}=1.9 GeV and for E_{CM}=57 GeV d_{join} =2.15 GeV. Using these two and three-jet events we have determined the M_{nm} moments for quark and gluon jets according to eq. (1) and (6) respectively.

We have studied the moments M_{nm} (where n,m=0,...,10) and concluded that M_{10} M_{13}, M_{14}, M_{15}, M_{26} and M_{27} are the most sensitive to the jet type. The jet-energy dependence of the average values of some of these moments (M_{10}, and M_{14}) as well as the jet-energy dependence of the mean jet multiplicity are shown in Fig. (1-3) for gluon and quark jets, respectively.

σ_q and σ_g (the square roots of the variances of the moments) have the same order of magnitude as the differences of the moments between quark and gluon jets.

$$|A_q(E_{jet}) - A_g(E_{jet})| = O(1)\frac{\sigma_g + \sigma_q}{2} \quad . \tag{9}$$

The best variable in our model calculation is M_{14}. For this moment the difference is 1.4 times larger than $\frac{\sigma_g+\sigma_q}{2}$ for 20 GeV jets. For the multiplicity this ratio is only 0.9.

In our detailed analysis we have examined a sample having the same amount of gluon and quark jets. A jet with an energy E_{jet} is tagged as a gluon jet with respect to a moment M_{ij} if the appropriate M_{ij} moment of the jet was closer to the average gluon value than to the average quark value at the appropriate E_{jet}. Both the efficiencies of the discrimination and the purity of the obtained samples are 60%, 69% and 69% using M_{00},M_{10} and M_{14}, respectivcely. (E.g. the discrimination based on the value of M_{14} leads to a sample containing 69% of the original gluon jets, and 69% of this new sample consists of gluon jets).

We could increase the efficiencies using multidimensional discrimination analysis. The most succesful possibility is to take $M_{14}, M_{13}, M_{10}, M_{15}$, M_{27}, and M_{26} into consideration. The linear discrimination function is defined as

$$F_{gluon/quark} = C_0 + \sum_i C_i X_i \tag{10}$$

where X_i-s are the above mentioned moments, C_i-s are jet energy dependent coefficients and chosen in a way to obtain optimal discrimination. To determine the appropriate C_i-s one can use well-known computer codes. (For the algorithm see [21].) If for a set of M_{ij} moments, corresponding to an observed jet event, the $F_{gluon/quark}$ is greater then 0 the jet is tagged as a gluon jet otherwise it is tagged as a quark jet. The efficiency and the purity of this method are slightly better (approximately 73%) than those of the single moment analysis. Examining the misidentified jets we exclude a part of the moments' space –where most of the misidentified jets are situated– in order to increase the purity, while the efficiency is decreasing. (It can be seen in Fig.(5) that excluding larger and larger part of the moments' space we study less and less jets , but the jets are more and more characteristically quark or gluon jets.)

3. Experimental tests of the three-gluon vertex

The self-interaction of gluons is one of the most important properties of quantum chromodynamics, a consequence of the nonvanishing color charge of gluons. There are lots of indirect indications, that Abelian strong interaction theories are inconsistent with the data (eg. the jet-cross section in proton-antiproton collision is large due to the triple-gluon vertex) It is customary to compare QCD with an Abelian theory defined in [17]. In this Abelian ("QED" like) theory the cross sections and distributions in electron-positron annihilation are made identical in leading order of the strong coupling constant to the cross sections and distributions of QCD by defining

$$\alpha_{"QED"} = C_F \alpha_S = \frac{4}{3}\alpha_s. \tag{11}$$

The triple-gluon vertex enters only in second and higher order of the strong coupling constant, thus testing the triple-gluon vertex requires a detailed study of four or more jet events. Several methods have been proposed to find observables sensitive to the triple gluon vertex in $e^+e^- \to 4$ partons [18-20]. The most recent succesful attempt [11] relies on counting tagged events. It is not suffering from fragmentation effects and it is very sensitive to the presence of the triple-gluon vertex.

The essential difference between QCD and an Abelian theory is the presence or the nonexistence of the triple-gluon vertex. As a result of the $g \to g + g$ splitting the rate of four-quark final states is low in QCD, while in "QED" the lack of the triple-gluon vertex leads to more four-quark final states. We get that 4% of the four-parton final states are $q_a\bar{q}_a q_b\bar{q}_b$ in QCD, while in the QED-like model 53% of them are $q_a\bar{q}_a q_b\bar{q}_b$ states.

The increase of the relative rate of the four quark states in "QED" leads to an increase of the number of quarks having a definite flavor for a fixed number of four parton final states, and a decrease of the number of gluons. Flavor tagging was introduced in [11] and it has been shown that using the above mentioned properties and flavor tagging one gets an extremely good method to verify the presence or the absence of the triple-gluon vertex at SLC/LEP energies. One of the possibilities (single tag) is to count a definite flavor (which can be tagged) in four jet events. The other method has been based on using double tags (i.e. detecting two flavors in a single four-jet event). We obtain e.g. for b quarks we have 31.4% single tag in "QED" and 21.1% in QCD, while the double bb-tag values are 2.24% in "QED" and 0.17% in QCD.

It is interesting to study the jets with the smallest energy. The differences between the two models, appearing at the level of the secondary particles, are more visible. The single quark flavor tag of the jet with the smallest energy gives that 1.1% of them are bottom, 2.6% of them are strange and 6.7% of them are charm quarks in QCD. The appropriate percentages are in "QED", 3.2%, 14% and 17%, respectivcely.

We can use gluon tagging defined in Section 2 to see the difference between the two models. Supposing 73% tagging efficiency and purity, the probability that we will not see any gluon jet in a four-jet event (no gluon tag), is 5.3% in QCD while it is 17% in "QED". Increasing the number of the excluded events in order to have better purity these probabilities are changing (Fig.[6]). Large differences between QCD and "QED" can be seen. Another observable is the probability of finding two gluon jets simultaneously in a four-jet event. The answer for QCD is 35% while for "QED" it is 24%. It is also worth to study how these numbers depend on the excluded jet-rate.

The most succesful method is to calculate the average of a continuous tagging variable.This variable is defined for a jet having energy E_{jet} as the distance of a given moment of the jet from the mean value of the quark and gluon average moments at E_{jet}, normalized by half of the difference of the quark and the gluon average moments at E_{jet}:

$$S_{nm} = \frac{1}{N} \sum_{i=1}^{N} \frac{M_{nm}^{(i)}(E_{jet}) - \frac{\bar{M}_{nm}^{q}(E_{jet}) + \bar{M}_{nm}^{g}(E_{jet})}{2}}{\frac{\bar{M}_{nm}^{q}(E_{jet}) - \bar{M}_{nm}^{g}(E_{jet})}{2}} \qquad (12)$$

where $M_{nm}^{(i)}(E_{jet})$ is the i^{th} jet's moment, $\bar{M}_{nm}^{q(g)}(E_{jet})$ means the average value of a moment at the energy E_{jet} for quarks (gluons), the sum goes over the jets and N is the number of the jets in the sample. It is clear that for a sample of only quark jets $S_{nm} = 1$, while for only gluon jets $S_{nm} = -1$. Analysing 10^4 quark (gluon) jets we have $S_{00} = +1(-1) \pm 8.2 \cdot 10^{-2}$, $S_{10} = +1(-1) \pm 3.0 \cdot 10^{-2}$ and $S_{14} = +1(-1) \pm 2.9 \cdot 10^{-2}$. Converting it to an arbitrary number of four-jet events, we have for QCD

$S_{00} = 0.03 \pm 4.1 \frac{1}{\sqrt{N_{four-jet}}}$, $S_{10} = 0.03 \pm 1.5 \frac{1}{\sqrt{N_{four-jet}}}$ and
$S_{14} = 0.03 \pm 1.4 \frac{1}{\sqrt{N_{four-jet}}}$, while for "QED"
$S_{00} = 0.47 \pm 4.1 \frac{1}{\sqrt{N_{four-jet}}}$, $S_{10} = 0.47 \pm 1.67 \frac{1}{\sqrt{N_{four-jet}}}$
and $S_{14} = 0.47 \pm 1.49 \frac{1}{\sqrt{N_{four-jet}}}$, ($N_{four-jet}$ is the number of four-jet events.) Large differences can be seen between QCD and "QED" predictions, while the statistical errors help us to determine the necessary number of four-jet events needed for a clear distinction. Thus, e.g. for a hypotetical experimental result $S_{14} = 0$, obtained from only 100 pure four-jet event, one can confirm QCD with a confidence level of 99.7%.

4.Conclusions

In this paper we have suggested a simple method to see the differences between quark and gluon jets. The full method is model independent in the sense that all the M_{nm} moments , or any other jet variable, should be determined from the experimental data. The differences between quark and gluon jet variables arise from the perturbtive QCD prediction of eq.(2) (more precisely its second order version). We have tested our method to calculate special moments of quark and gluon jets in the Lund Model. Remarkable differences can be seen between them. In this model calculation the square roots of the variances of the moments have the same order of magnitude as the difference between the moments for quark and gluon jets. Comparing our method to other methods (eg. symmetric three-jet event studies), we can say that our method compares gluon jets with quark jets having exactly the same energy, thus there is no kinematical difference superimposed on the dynamical ones. The other advantage of our method is that it reaches high statistics.

In the triple-gluon vertex search we want to find observables very sensitive to the non-Abelian features of QCD. The clearest manifestations of the triple-gluon vertex, as it was emphasised many times in the literature, are the details of the four-parton final states in electron-positron annihilation.It is customary to compare QCD with an Abelian model, "QED", defined in [3].

In this paper we have worked out several methods to test the triple-gluon vertex , elaborating and broadening the flavor tagging method proposed in [21] . Two basically different ways have been worked out to solve this problem. We have shown that (due to the larger number of gluon-jets than quark-jets with definite flavor) tagging gluons we can discriminate between QCD and the Abelian model. Simple gluon-jet tagging has been improved by introducing continuous tagging variables. The differences between the average values of these continuous tagging variables for the two models are very large. We have analysed the jets with the smallest energy because this sample is richer in secondary particles (secondary quarks and gluons).

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Figure Caption

Figure 1. Dependence of mean multiplicities (M_{00}) of quark and gluon jets on the jet energy.

Figure 2. Dependence of the average M_{10} value of quark and gluon jets on the jet energy.

Figure 3. Dependence of the average M_{14} value of quark and gluon jets on the jet energy.

Figure 4. Mean multiplicities of primary and secondary quark jets as a function of the jet energy.

Figure 5. Gluon tagging efficiency as a function of the purity of the gained sample.

Figure 6. Dependence of the probability of not tagging any gluon jet in four-jet events (P_{no-g}) for QCD and "QED" on the ratio of the excluded jets.

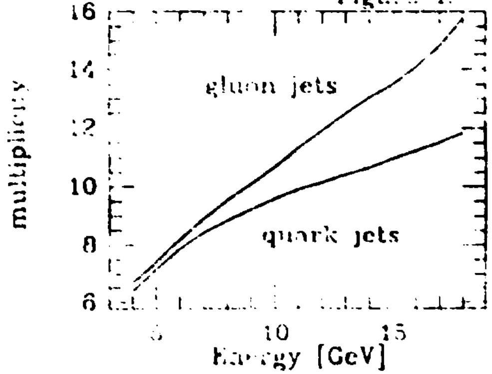


Figure 1.

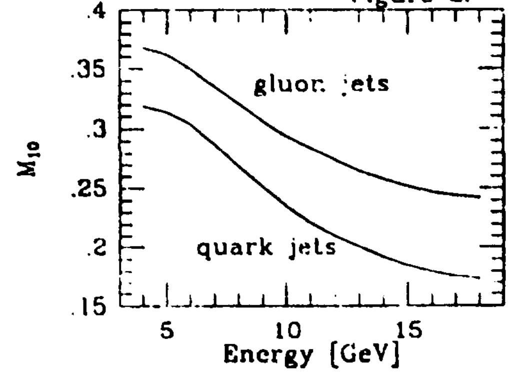
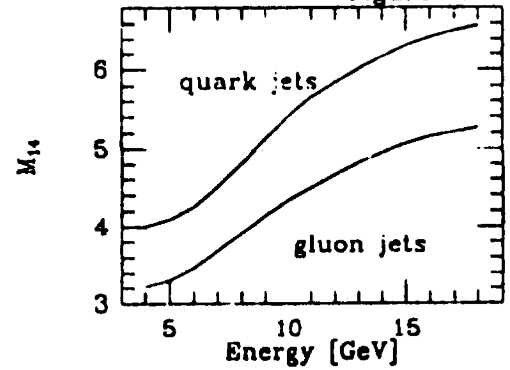


Figure 2.



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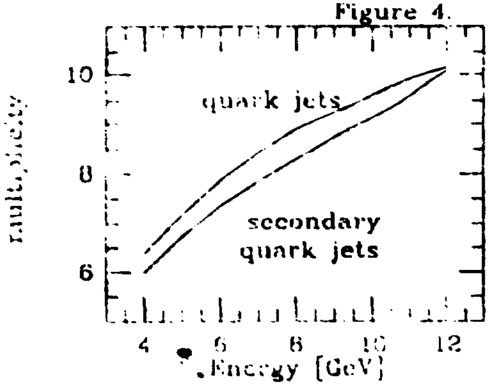

Figure 4.

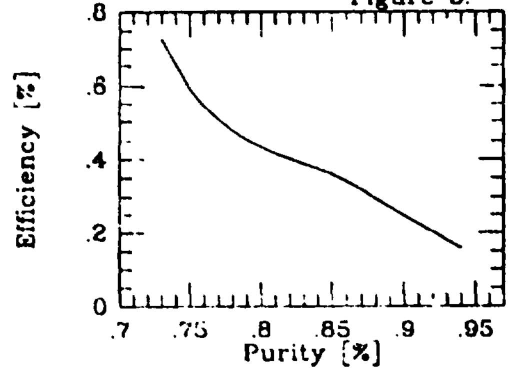

Figure 5.

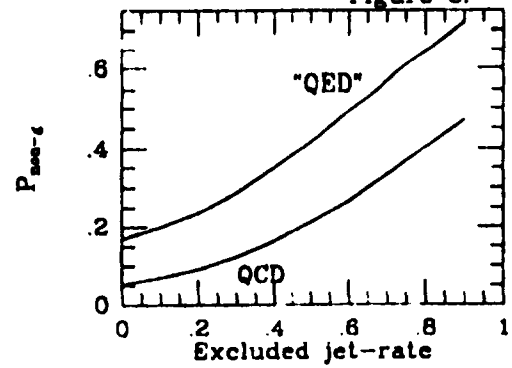

Figure 6.

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