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The Status of the Electric Dipole Moment of the Neutron

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Abstract

The electric dipole moment of particles in quantum mechanics and quantum field theory is discussed. Furthermore, calculations of the neutron electric dipole moment in the standard model and several of its low-energy extensions are reviewed.

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1 Introduction

The discovery of a non-zero electric dipole moment (EDM) of an elementary particle would be an unequivocal observation of CP violation like the discovery of the decay $K_L \to \pi\pi$ more than 25 years ago [1]. Both observables belong to the privileged class of observables which have zero expectation value if CP is conserved and which therefore represent a direct measure of CP violation. Up to now CP violation has only been found in the $K^0\bar{K}^0$ system but according to many theories the EDM of the neutron (EDMN) is rather large and should therefore be found too in the near future. However, the prediction of the Kobayashi-Maskawa (KM) mechanism [2] of CP violation in the standard model (SM) [3] is far from the present limits. All this favours the persistent interest in the EDMN.

This review article is organized as follows. In the rest of the introduction general features of the EDM in quantum mechanics and field theory are discussed and the experimental results are quoted. In Sect. 2 general formulas for the EDM at the one-loop level are presented. A discussion of the EDMN in various models, namely the SM (KM mechanism and strong CP violation), the supersymmetric extension of it, left-right symmetric models and multi-Higgs models, is given in Sect. 3. The conclusions are contained in Sect. 4.

1.1 The EDM in Non-Relativistic Quantum Mechanics

In non-relativistic quantum mechanics the interaction of an EDM with an electric field $\vec{E}$ is described by

$$H_{\mathrm{EDM}} = -\vec{d}\cdot\vec{E} \tag{1}$$

where $\vec{d}$ is the EDM vector operator. It has been known for a very long time that without parity (P) violation the EDM of a particle is zero. However, even before the discovery of P violation it was pointed out by Purcell and Ramsey [4] that the question of a possible existence of an EDM of a nucleus or particle is a purely experimental matter. Later on Landau [5] argued that also time reversal (T) invariance must be violated to have a non-zero EDM. This is equivalent to CP violation in the light of the CPT theorem which we assume to hold in this paper. At first, let us discuss the connection between an EDM and P violation and prove the following statement.

(P) If the EDM of a particle is non-zero then P must be violated.

Proof: We assume P invariance of all interactions. If P is conserved the transformation properties of $\vec{E}$ and $\vec{d}$ are fixed:

$$P : \vec{E} \to -\vec{E}, \quad \vec{d} \to -\vec{d}. \tag{2}$$

The second relation follows from the invariance of H_{EDM}. Furthermore, every physical state ψ has a definite parity. Consequently,

$$\langle\psi|\vec{d}\psi\rangle = \langle P\psi|(P\vec{d}P^{-1})P\psi\rangle = -\langle\psi|\vec{d}\psi\rangle$$

and the EDM of ψ is zero.

Next we review the argument of Landau [5] concerning T violation.

(T) If the EDM of a particle is non-zero then T must be violated too.

Argument: If all interactions are T invariant then we have

$$T : \vec{E} \to \vec{E}, \quad \vec{J} \to -\vec{J}, \quad \vec{d} \to \vec{d} \tag{3}$$

where the last relation again follows from invariance of (1). For an "elementary" particle its orientation is completely specified by the angular momentum $\vec{J}$. Therefore, necessarily the proportionality relation

$$\langle\psi|\vec{d}\psi\rangle \sim \langle\psi|\vec{J}\psi\rangle \tag{4}$$

must be fulfilled. Since $\vec{d}$ and $\vec{J}$ transform differently under T the EDM of ψ is zero.

This argument does not seem to apply to non-pointlike particles where the following objections can be raised [6]:

i) For bound states (e.g. the neutron can be considered as a bound state of quarks) there might be other vectors available than $\vec{J}$.

ii) Since $\vec{d}$ is a vector operator the relation [7]

$$\langle\tau jm|\vec{d}|\tau jm\rangle = \frac{\langle\tau jm|\vec{J}\cdot\vec{d}|\tau jm\rangle}{j(j+1)}\langle\tau jm|\vec{J}|\tau jm\rangle \tag{5}$$

holds with $\psi = |\tau jm\rangle$ where j, m are the quantum numbers of angular momentum and τ contains the other quantum numbers which characterize ψ. Now both sides of Equ. (5) transform equally under T and the argument given after Equ. (3) does not apply.

These two objections can, however, be invalidated and one can show that statement (T) also holds for bound states [7,8]. Here we want to express this fact as a theorem with all the necessary assumptions and sketch the proof of Ref. [6].

Theorem: If time reversal and rotational invariance hold then the EDM of any stationary state non-degenerate in energy apart from degeneration due to rotational invariance must be zero.

Proof: In quantum mechanics the antiunitary operator of time reversal for a system with an arbitrary number of particles with arbitrary spins is given by [9]

$$T = e^{-i\pi S_y}K. \tag{6}$$

K denotes the operation of complex conjugation. We assume that the spin operator $\vec{S}$ is given in its usual basis where S_x, S_z are real and S_y is imaginary. It can easily be checked that T acts on the total angular momentum $\vec{J}$ in the correct way (3). Since T contains the spin operator we get $T^2 = 1$ only if the number of half-integer particles in the system is even. However, for the operator

$$K_y = e^{-i\pi J_y}T \tag{7}$$

one can easily derive the relations

$$\begin{aligned} &K_y^2 = 1 \\ &[K_y, \vec{J}^2] = 0, \qquad [K_y, J_z] = 0 \\ &K_y d_z K_y^{-1} = -d_z. \end{aligned} \tag{8}$$

The last relation follows from Equ. (3) and from the vector operator property of $\vec{d}$. Of course, K_y commutes with the Hamiltonian and can therefore be used to construct a "real" standard basis of energy eigenstates defined by [9]

$$K_y|\tau mj\rangle = |\tau mj\rangle. \tag{9}$$

Consequently, we obtain

$$\langle\tau jm|d_z|\tau jm\rangle = \langle\tau jm|K_y d_z K_y^{-1}|\tau jm\rangle^* = -\langle\tau jm|d_z|\tau jm\rangle.$$

The expectation values of d_x, d_y must be zero because of rotational invariance. Therefore the EDM of the state $|\tau jm\rangle$ is zero. Since any state non-degenerate in energy must correspond to a unique τ the proof is finished.

1.2 The EDM in Quantum Field Theory

Here we confine ourselves to the case of a fermion f with spin 1/2. The general matrix element of the electromagnetic (e.m.) current is given by

$$\langle p_2, s_2|j_\mu^{em}(x)|p_1, s_1\rangle = \frac{1}{(2\pi)^3}\left(\frac{m^2}{E_2E_1}\right)^{1/2} e^{iqx}$$

$$\cdot\bar{u}(p_2, s_2)[F_Q\gamma_\mu + F_M i\sigma_{\mu\nu}q^\nu + F_A(\frac{q^2}{2m}\gamma_\mu - q_\mu)\gamma_5 - F_E\sigma_{\mu\nu}\gamma_5 q^\nu]u(p_1, s_1) \tag{10}$$

where $|p_i, s_i\rangle$ $(i = 1,2)$ denote states with momentum p_i and polarization vector s_i, $E_i = p_i^0$, $q = p_2 - p_1$ and m is the mass of f. The form factors F_i $(i = Q, M, A, E)$ are functions of q^2. In Equ. (10) current conservation has been taken into account. As a consequence of the hermiticity of the e.m. current the form factors are real:

$$j_\mu^{em\dagger} = j_\mu^{em} \Rightarrow F_i \text{ real} \quad (i = Q, M, A, E). \tag{11}$$

Assuming T invariance the e.m. current transforms as

$$Tj_\mu^{em}(x)T^{-1} = \varepsilon(\mu)j_\mu^{em}(-t, \vec{x})$$

with

$$\varepsilon(\mu) = \begin{cases} 1, & \mu = 0 \\ -1, & \mu = 1,2,3 \end{cases} \tag{12}$$

and one can show that

$$\text{T invariance} \Rightarrow F_Q, F_M, F_A \text{ real, } F_E \text{ imaginary.} \tag{13}$$

Putting Equs. (11) and (13) together we obtain

$$\text{T invariance} \Rightarrow F_E = 0. \tag{14}$$

There are three gauge-invariant physical quantities contained in the form factors [10], namely $F_Q(0)$, $F_M(0)$ and $F_E(0)$ which correspond to the e.m. charge, the magnetic moment and the EDM of f, respectively. Note that $F_A(0)$ is not gauge-invariant [10] (the dependence of the e.m. form factors comes from the non-abelian gauge group of the SM). Defining

$$F_E(0) \equiv d_f \tag{15}$$

we call d_f the EDM of f. To establish the connection between (15) and (1) we write down the effective Hamiltonian interaction density induced by d_f and take the non-relativistic limit:

$$\mathcal{H}_{\text{EDM}} = \frac{i}{2} d_f \bar{\psi} \sigma_{\mu\nu} \gamma_5 \psi F^{\mu\nu} \xrightarrow{\text{NR}} H_{\text{EDM}} = -d_f \vec{\sigma} \cdot \vec{E} \tag{16}$$

with

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

being the e.m. field strength tensor and $\vec{\sigma}$ the Pauli matrices.

From Equ. (16) we can draw the following important conclusions which corroborate the non-relativistic discussion:

i) the EDM $\vec{d}$ is proportional to the spin and

ii) the EDM interaction $\mathcal{H}_{\text{EDM}}$ violates P, T and CP.

Finally we quote the experimental limits on the various EDMs.

1.3 Experimental Bounds on d_f

neutron	d_n	$=$	$(-14 \pm 6) \cdot 10^{-26}$ e.cm	Leningrad	86 [11]
			$(-3 \pm 5) \cdot 10^{-26}$ e.cm	Grenoble	89 [12]
			or $\lvert d_n \rvert < 12 \cdot 10^{-26}$ e.cm	(95% c.l.)	

The experimental information on other fermions is taken from Ref. [13]:

electron	$\lvert d_e \rvert < 3 \cdot 10^{-24}$ e.cm (90% c.l.)
muon	$d_\mu = (3.7 \pm 3.4) \cdot 10^{-19}$ e.cm
proton	$d_p = (9 \pm 14) \cdot 10^{-21}$ e.cm
Λ	$\lvert d_\Lambda \rvert < 1.5 \cdot 10^{-16}$ e.cm (90% c.l.)

2 One-Loop EDM

In this section we will discuss the general formulas for an EDM generated at the one-loop level by the exchange of vector bosons W^μ and scalars H. The general interaction Lagrangian for a vector ψ of spin-1/2-fields is given by

$$-\mathcal{L}_{int} = \bar{\psi}\gamma_\mu[G_L^W P_L + G_R^W P_R]\psi W^\mu + \bar{\psi}[G_L^H P_L + G_R^H P_R]\psi H + h.c. \tag{17}$$

with

$$P_L = \frac{1}{2}(1-\gamma_5), \qquad P_R = \frac{1}{2}(1+\gamma_5).$$

The $G_{L,R}^{W,H}$ are matrices in the (flavour, colour, ...) space of all fermions. There is one term in (17) for each mass eigenfield W^μ or H. For hermitian W^μ, H the corresponding coupling matrices are hermitian and $h.c.$ has to be omitted in (17).

Before writing down the general one-loop expressions for the EDM of a fermion f we want to derive a very useful estimate. As can be seen e.g. from Equ. (16) the EDM changes chirality:

$$\bar{\psi}\sigma_{\mu\nu}\gamma_5\psi = \bar{\psi}_L\sigma_{\mu\nu}\psi_R - \bar{\psi}_R\sigma_{\mu\nu}\psi_L.$$

Therefore the contribution to d_f generated by the exchange of a fermion F and a boson B (Figs. 1a,b) must be proportional to

$$G_{R,fF}G^*_{L,fF} - G_{L,fF}G^*_{R,fF} = -2i\ \mathrm{Im}(G_{L,fF}G^*_{R,fF}). \tag{18}$$

Furthermore, looking at Fig. 1a and taking $B = W$ for definiteness this Feynman diagram contains the factor

$$\gamma^\sigma P_R(\not{p}' + m_F)\gamma^\mu(\not{p} + m_F)\gamma^\rho P_L \sim m_F. \tag{19}$$

The same proportionality to the mass m_F of the exchanged fermion is also obtained for Fig. 1b where we have

$$\gamma^\sigma P_R(\not{q} + m_F)\gamma^\rho P_L \sim m_F. \tag{20}$$

p, p', q denote momenta of F. For the exchange of scalars ($B = H$) the same reasoning is valid. Taking into account the e.m. coupling constant e and a factor $(16\pi^2 M_B^2)^{-1}$ from one-loop integration we obtain the estimate

$$d_f(F,B) \sim \frac{e}{16\pi^2}\frac{m_F}{M_B^2}\ \mathrm{Im}(G_{L,fF}G^*_{R,fF}) \tag{21}$$

where M_B is the mass of the exchanged boson B. To get a feeling for possible orders of magnitude of one-loop EDMs we take $m_F = 100$ MeV and $M_B = 81$ GeV, the mass of the W boson. With these values $em_F/(16\pi^2 M_B^2)$ corresponds to $1.9 \cdot 10^{-21}$ e.cm.

Exact formulas for the EDM are given by [14]

$$\begin{aligned} d_f(W) &= \frac{1}{16\pi^2 M_W^2}\sum_F m_F\ \mathrm{Im}(G^W_{L,fF}G^{W}_{R,fF}{}^*)[(Q_F - Q_f)I_1(x_F,y_f) + Q_F I_2(x_F,y_f)] \\ d_f(H) &= \frac{1}{16\pi^2 M_H^2}\sum_F m_F\ \mathrm{Im}(G^H_{L,fF}G^{H}_{R,fF}{}^*)[(Q_F - Q_f)I_3(x_F,y_f) + Q_F I_4(x_F,y_f)] \end{aligned} \tag{22}$$

with $x_F = m_F^2/M_B^2$, $y_f = m_f^2/M_B^2$. Of course, in the first formula $B = W$ whereas in the second one $B = H$. Q_F, Q_f are electric charges. The integral expressions for I_i ($i = 1, \ldots, 4$) can be found in Ref. [14]. In most cases m_f can be neglected. Then the functions I_i are obtained as [14]

$$\begin{aligned} I_1(x,0) &= \frac{2}{(1-x)^2}\{1 - \frac{11}{4}x + \frac{1}{4}x^2 - \frac{3}{2}\frac{x^2 \ln x}{1-x}\} \\ I_2(x,0) &= \frac{2}{(1-x)^2}\{1 + \frac{1}{4}x + \frac{1}{4}x^2 + \frac{3}{2}\frac{x \ln x}{1-x}\} \\ I_3(x,0) &= \frac{-1}{2(1-x)^2}\{1 + x + 2\frac{x \ln x}{1-x}\} \\ I_4(x,0) &= \frac{1}{2(1-x)^2}\{3 - x + 2\frac{\ln x}{1-x}\}. \end{aligned} \tag{23}$$

3 The EDMN in Different Models

3.1 Standard Model

We take the notion SM in the strict sense, e.g. it contains only one Higgs doublet and three generations and CP violation is hard through complex Yukawa couplings. This results in the KM mechanism where CP violation appears solely in the charged current interaction

$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}}\bar{u}\gamma^\mu V P_L d W_\mu^+ + h.c. \tag{24}$$

apart from a possible strong CP violation. g denotes the $SU(2)$ coupling constant, u and d are the vectors of the up- and down-quark mass eigenfields, respectively, and the unitary matrix V is the KM matrix which contains one physical complex phase factor giving rise to CP violation [2]. In the following we will first discuss the EDM of single quarks, then exchange contributions of the neutron and finally the connection between strong CP violation and the EDMN.

EDM of single quarks: Since there is no right-handed coupling in Equ. (24) we immediately conclude from (21) that the EDM of a single quark q at the one-loop level vanishes:

$$d_q(\text{1-loop}) = 0. \tag{25}$$

At the two-loop level one has to look for diagrams with a complex product of KM factors. This rules out graphs with Z^0 and neutral scalar exchange and leaves only those graphs with W or unphysical charged scalar exchange which do not contain a closed fermion loop [15]. One such graph is depicted in Fig. 2. The other possibilities are obtained by attaching the photon line in the interior of the diagram wherever possible. The corresponding product of KM factors

$$V_{\ell d}^* V_{\ell k} V_{jk}^* V_{jd} \tag{26}$$

is complex in general. Despite of this it has been shown [16] that, surprisingly, the two-loop EDM of a quark is still zero:

$$d_q(\text{2-loop}) = 0. \tag{27}$$

A non-zero result is obtained by adding a gluon G exchange (see Fig. 3 for an example), however, the result is far below experimental limits [17,18]:

$$|d_q| \lesssim 10^{-34} \text{ e.cm}. \tag{28}$$

Exchange contributions: The total EDM of the neutron

$$d_n = \frac{4}{3}d_d - \frac{1}{3}d_u + d_n^{ex} \tag{29}$$

also contains exchange contributions. In the simplest case two W bosons participate (see Fig. 4 for an example). It has been shown that such contributions give a very small EDMN [19]

$$d_n^{ex}(2W) \sim 10^{-34} \text{ e.cm}. \tag{30}$$

A contribution two to three orders of magnitude larger than (30) is obtained by considering the additional effect of a gluon. Interpreting the diagram of Fig. 5 and a similar one with the lower end of the gluon line attached to the lower quark line in terms of baryon poles one gets [20] $d_n \sim 10^{-31}$ e.cm. The diagram Fig. 5 (and one with crossed gluon and W lines) calculated as an ordinary Feynman diagram gives [21] $d_n \sim 10^{-32}$ e.cm. This consideration is somehow complementary to the picture of Ref. [20] where the momenta of the intermediate quark lines have to be small and the crossed diagram cannot contribute. Fig. 6 shows the so-called photopenguin diagram. Its contribution to d_n lies in the same range [21,22] as those of Figs. 4 and 5. Thus the exchange contribution to d_n containing two W bosons and one gluon can be estimated to be

$$d_n^{ex}(2W, 1G) \sim 10^{-32} - 10^{-31} \text{ e.cm} \tag{31}$$

which is the largest contribution of the KM mechanism. Further references and details concerning this subject and also discussions of strong CP violation and extensions of the SM can be found in the review articles of Ref. [23].

EDMN from strong CP violation: QCD is defined by the Lagrangian

$$\mathcal{L}_0 = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + i\bar{q}_L\gamma^\mu D_\mu q_L + i\bar{q}_R\gamma^\mu D_\mu q_R \tag{32}$$

where $G^a_{\mu\nu}$ is the gluonic field strength tensor, $q_{L,R}$ are the vectors of left- and right-handed quark fields and D_μ is the covariant derivative. In addition, there is a mass term

$$-\mathcal{L}_M = \bar{q}_L M q_R + h.c. \tag{33}$$

with a mass matrix M coming from spontaneous symmetry breaking in the electroweak sector. However, this is not yet the whole story since because of the non-trivial topological structure of the QCD gauge group $SU(3)$ one gets a whole class of gauge-invariant vacua [24]

$$|\theta\rangle = \sum_{n\in\mathbf{Z}} e^{-in\theta}|n\rangle \tag{34}$$

labelled by a parameter $\theta \in \mathbf{R}$. $|n\rangle$ ($n \in \mathbf{Z}$) are nontrivial vacuum states of the gauge field characterized by a winding number n. Equivalently, one can take into account the vacuum structure (34) by adding a term

$$\mathcal{L}_\theta = \theta \frac{g_s^2}{32\pi^2} G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

with

$$\tilde{G}^a_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}G^{a\rho\sigma} \tag{35}$$

to the Lagrangian (g_s is the $SU(3)$ gauge coupling constant). Because of the anomaly the generator of chiral transformations $\hat{Q}_5$ which is time-independent in the limit of zero quark masses is not gauge-invariant and changes the vacuum:

$$e^{i\alpha\hat{Q}_5} q_{\substack{L\\R}} e^{-i\alpha\hat{Q}_5} = e^{\pm i\alpha} q_{\substack{L\\R}}, \qquad e^{i\alpha\hat{Q}_5}|\theta\rangle = |\theta + 2N_f\alpha\rangle \tag{36}$$

where N_f is the number of flavours. This has an effect when the quark mass term (33) is diagonalized. By $SU(N_f)$ transformations on $q_{L,R}$ one can achieve

$$M \rightarrow e^{i\rho/N_f}\hat{M} \tag{37}$$

with

$$\rho = \arg\det M$$

where $\hat{M}$ is diagonal and positive. Because of (36) the remaining phase ρ cannot be removed without changing the vacuum angle θ at the same time. Choosing $\alpha = \rho/2N_f$ in (36) we get

$$|\theta\rangle \rightarrow |\theta + \rho\rangle \equiv |\bar{\theta}\rangle$$

and

$$-\mathcal{L}_M \rightarrow \bar{q}_L \hat{M} q_R + h.c. \tag{38}$$

For simplicity we have not changed notation when making a basis change. Equ. (38) shows that θ and the phase of the quark mass determinant cannot be separated. Therefore the physical phase is given by $\bar{\theta} = \theta + \rho$. Now one can write down the complete QCD Lagrangian

$$\mathcal{L}_{\mathrm{QCD}} = \mathcal{L}_0 + \mathcal{L}_{\hat{M}} + \mathcal{L}_{\bar{\theta}} \tag{39}$$

where $\mathcal{L}_{\bar{\theta}}$ violates P, CP and T. In the SM one is not allowed to set $\bar{\theta} = 0$ because there is hard CP violation and in order to ensure renormalizability one has to put *all* terms into the Lagrangian compatible with gauge symmetry.

Equ. (39) is not suited for making a perturbative expansion for small $\bar{\theta}$. To get the CP violating perturbation one can use Dashen's theorem [25] and chiral symmetry breaking given by

$$\langle 0|\bar{q}_{La}q_{Rb}|0\rangle = -\Delta\delta_{ab}. \tag{40}$$

Confining ourselves to $N_f = 3$ there is a freedom of $SU(3)_{L,R}$ flavour rotations $X_{L,R}$ in the mass term

$$-\tilde{\mathcal{L}}_M = e^{i\bar{\theta}/3}\bar{q}_L X_L^\dagger \hat{M} X_R q_R + h.c. \tag{41}$$

Here the $\bar{\theta}$ parameter has been shifted into the mass term. According to [25] the correct perturbation $\delta\mathcal{L}_{\rm CP}$ is found by minimizing $\langle 0|\tilde{\mathcal{L}}_M|0\rangle$ with respect to $X_{L,R}$ and inserting the solution into (41). In the approximation of small $\bar{\theta}$ one obtains [26]

$$\delta\mathcal{L}_{\rm CP} =\simeq -i\bar{\theta}\left(\sum_{q=u,d,s}\frac{1}{m_q}\right)^{-1}(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s). \tag{42}$$

Equ. (42) gives a contribution to the EDMN but has no effect in $K^0\bar{K}^0$ since it does not change flavour. Many different methods can be used to calculate d_n from (42). E.g. in Ref. [26] intermediate negative-parity states are inserted in first order perturbation theory in $\bar{\theta}$ whereas in Ref. [27] chiral perturbation theory is employed. Both methods lead to similar results, $d_n(\bar{\theta}) \sim 3\cdot 10^{-16}\bar{\theta}$ e. cm in Ref. [26] and $5\cdot 10^{-16}\bar{\theta}$ e.cm in Ref. [27], respectively.

Finally we can conclude our discussion of the EDMN in the following way. For the parameter $\bar{\theta}$ of strong CP violation there is an upper bound from the experimental limit on the EDMN of order

$$|\bar{\theta}| < 10^{-10} - 10^{-9}. \tag{43}$$

This means that with strong CP violation the EDMN can have any value in the SM within the experimentally allowed range. If, on the other hand, we knew that $\bar{\theta}$ vanishes for some reason (e.g. there could be an axion in the larger theory where the SM is embedded [24]) then the EDMN would be very small with a generous upper bound of

$$|d_n| < 10^{-30}\ \text{e.cm} \quad (\bar{\vartheta} = 0). \tag{44}$$

3.2 Left-Right Symmetric Model

In left-right symmetric models the SM gauge group is enlarged to $SU(2)_L\times SU(2)_R\times U(1)$ to restore parity symmetry [28] at high energies. In such models there are six vector bosons $\vec{W}_L$, $\vec{W}_R$ which couple to left- and right-handed fermion doublets, respectively. By spontaneous breakdown of the gauge group left and right vector bosons mix[1]. Thus instead of $W^\pm_{L,R}$ we have the physical states $W^\pm_{1,2}$. From the $K_L K_S$ mass difference one

[1]To avoid some problems with neutrino masses it might be desirable to decouple parity breaking from $SU(2)_R$ breaking and to have parity breaking at a much higher scale [29].

can show that [30] the mass of the nearly right-handed vector boson W_2 must be in the TeV range, $M_2 \gtrsim 2.5$ TeV. The left-right mixing angle ξ is given by

$$\xi \simeq 2\frac{|vw|}{|v|^2+|w|^2}\left(\frac{M_1}{M_2}\right)^2 \sim 2\frac{m_b}{m_t}\left(\frac{M_1}{M_2}\right)^2 \lesssim 1.5\cdot 10^{-4} \tag{45}$$

where v, w are the vacuum expectation values of the $(1/2,1/2,0)$ scalar multiplet (we confine ourselves to a minimal version with only one such multiplet).

The charged current interaction is given by

$$\begin{aligned} -\mathcal{L}_{cc} &= \frac{g}{\sqrt{2}}\{\cos\xi\bar{u}_L\gamma^\mu V_L d_L - e^{-i\lambda}\sin\xi\bar{u}_R\gamma^\mu V_R d_R\}W^+_{1\mu} + \\ &+ \frac{g}{\sqrt{2}}\{e^{i\lambda}\sin\xi\bar{u}_L\gamma^\mu V_L d_L + \cos\xi\bar{u}_R\gamma^\mu V_R d_R\}W^+_{2\mu} \end{aligned}$$

with

$$e^{i\lambda} = -vw^*/|vw|. \tag{46}$$

Because of left and right vector bosons there are two mixing matrices V_L and V_R where the first one is the usual KM matrix. Since in (46) there are left- and right-handed couplings there is in general an EDMN already at the one-loop level and we can use the formulas of Sect. 2 to calculate the EDM of quarks. W_2 can be neglected in this context because of its heavy mass. Calculation shows that the dominant contributions to d_n are given by [14] the EDM of the d quark through W_1 and charm quark exchange

$$d_q(W_1,c) \simeq \frac{eg^2\sin 2\xi}{64\pi^2 M_1^2}\cdot\frac{10}{3}\cdot m_c\ \mathrm{Im}(e^{i\lambda}V_{Lcd}V^*_{Rcd}) \tag{47}$$

and by the exchange contribution

$$d_n^{ex}(W_1) = \frac{eg^2\sin 2\xi}{48\pi^{3/2}M_1^2}\sqrt{2M_q\omega}\ \mathrm{Im}(e^{i\lambda}V_{Lud}V^*_{Rud}) \tag{48}$$

which has been calculated in the oscillator model. In this model a fit of baryon masses gives $\sqrt{M_q\omega} \simeq 0.3$ GeV. To get a bound on (47) and (48) we assume

$$|V_{Lij}| = |V_{Rij}| \tag{49}$$

which is supported by manifest CP invariance. Then we can estimate [14]

$$|d_d(W_1,c)| \lesssim 0.5\cdot 10^{-25}\ \text{e.cm}\left(\frac{4\ \text{TeV}}{M_2}\right)^2\left(\frac{100\ \text{GeV}}{m_t}\right) \tag{50}$$

and

$$|d_n^{ex}(W_1)| \lesssim 2\cdot 10^{-25}\ \text{e.cm}\left(\frac{4\ \text{TeV}}{M_2}\right)^2\left(\frac{100\ \text{GeV}}{m_t}\right). \tag{51}$$

We would like to finish the discussion of left-right symmetric models with a remark on the relation (49). In the literature sometimes $V_{Lij} = V_{Rij}$ is used for estimates which is called "manifest left-right symmetry" in contrast to (49) which is dubbed "pseudomanifest left-right symmetry". To our knowledge $V_L = V_R$ does not come from a symmetry and is thus a mere assumption for convenience which is, however, not very useful for CP estimates (e.g. it would give CP conservation in $\mathcal{L}_{cc}$ for two generations). In contrast to this (49) can be derived from left-right symmetry which gives hermitian Yukawa coupling matrices in the quark sector and usual CP invariance supplying real Yukawa couplings. Taken both conditions together we get real, symmetric coupling matrices. This case we call manifest CP invariant. Therefore the quark mass matrices $M_{n,p}$ are symmetric but in general complex because of the vacuum expectation values. For such matrices the left and right diagonalization matrices can be chosen to obey the relation [31]

$$U_L^{n,p} = U_R^{n,p*} \tag{52}$$

in

$$U_L^{n\dagger} M_n U_R^n = \hat{M}_d, \qquad U_L^{p\dagger} M_p U_R^p = \hat{M}_u \tag{53}$$

where $\hat{M}_{d,u}$ are diagonal and positive. Consequently, one gets

$$V_L = U_L^{p\dagger} U_L^n = (U_R^{p\dagger} U_R^n)^* = V_R^* \tag{54}$$

in the phase convention (52). Therefore, in an arbitrary convention Equ. (49) is valid which is therefore a natural relation deriving from P and CP invariance before spontaneous symmetry breaking.

3.3 Minimal Supersymmetric Extension of the SM

The minimal supersymmetric (SUSY) version of the SM based on $N = 1$ supergravity [32] contains soft supersymmetry breaking terms which mix left and right squark fields, the partners of left- and right-handed quarks. Because of this the gaugino-fermion-sfermion interaction, the SUSY counterpart of the usual gauge interaction of fermions, leads to an EDM already at the one-loop level. Let us consider here the gluino-down quark-down squark interaction. The mass matrix of down squarks $\tilde{d} = (\tilde{d}_L, \tilde{d}_R)^T$ looks like

$$M_{\tilde{d}}^2 = \begin{pmatrix} \mu_L^2 1 + \hat{M}_d^2 & A^* m_{3/2} \hat{M}_d \\ A m_{3/2} M_d & \mu_R^2 1 + \hat{M}_d^2 \end{pmatrix} \tag{55}$$

in a basis where the down quark mass matrix is diagonal. The mass parameters $\mu_{L,R}$ are of the same order of magnitude as the gravitino mass $m_{3/2}$. A is a parameter of the soft supersymmetry breaking part induced by the super Higgs mechanism. It is complex in general. Another complex parameter is introduced by the gluino mass $m_{\tilde{g}}$ appearing in the Majorana mass term

$$\frac{1}{2} \tilde{g}_L^{aT} C^{-1} \tilde{g}_L^a m_{\tilde{g}} + h.c. \tag{56}$$

Defining phases φ_A and $\varphi_{\tilde{g}}$ by

$$A = |A| e^{-i\varphi_A}, \qquad m_{\tilde{g}} = |m_{\tilde{g}}| e^{-i\varphi_{\tilde{g}}} \tag{57}$$

one can easily show that there is only one phase

$$\varphi_s = \varphi_A - \varphi_{\tilde{g}} \quad (58)$$

which is physical. φ_s appears in the $\tilde{g}d\tilde{d}$ interaction with the general form

$$\mathcal{L}_{\tilde{g}d\tilde{d}} = i\sqrt{2}g_s\tilde{d}_i^\dagger\bar{\tilde{g}}^a\frac{\lambda^a}{2}(\Gamma_L P_L + \Gamma_R P_R)_{ij}d_j + h.c. \quad (59)$$

where λ^a ($a = 1,\ldots,8$) are the Gell-Mann matrices. Using Form. (21) to estimate the EDM of the d quark we get

$$d_d \sim \frac{eg_s^2}{16\pi^2 M_{\tilde{d}}^2}|m_{\tilde{g}}|\vartheta_{LR}\sin\varphi_s \quad (60)$$

where

$$\vartheta_{LR} \sim |A|m_d/M_{\tilde{d}} \quad (61)$$

is the mixing angle between left and right down squark. With $|A| \sim 1$ and $M_{\tilde{d}} \sim |m_{\tilde{g}}| \sim m_{3/2} \sim 100$ GeV one gets an upper bound of order [33,34]

$$|\varphi_s| \lesssim 10^{-3}. \quad (62)$$

Thus the new phase φ_s provided by the SUSY extension of the SM is even restricted by the EDMN.

Taking into account one-loop corrections one can show that also the KM phase appears in the squark mass matrix. It is remarkable, however, that it does not contribute to d_d [34]. It is evident from the above estimate that even with squark and gaugino masses in the TeV range the EDMN can be as large as the experimental limit. With such high SUSY masses the EDMN is probably the only CP violating detectable effect of supersymmetry which remains at low energies.

3.4 Weinberg Model

This model [35] combines the principles of spontaneous CP violation and flavour conservation in the neutral Higgs sector and the SM is enlarged only in the scalar sector. To realize this idea three Higgs doublets ϕ_i ($i = 1,2,3$) are necessary. Neutral flavour conservation is achieved by a horizontal symmetry leading to the Yukawa couplings

$$-\mathcal{L}_Y = \bar{Q}_L\Gamma\phi_1 n_R + \bar{Q}_L\Delta\tilde{\phi}_2 p_R + h.c. \quad (63)$$

$\tilde{\phi}_2$ is defined as $\tilde{\phi}_2 \equiv i\tau_2\phi_2^*$, Q_L denotes the $SU(2)$ quark doublet fields and n_R, p_R are the quark singlets in the weak basis. ϕ_3 couples only in the Higgs sector. One can easily show that in this model the KM matrix V is real [36] and therefore ε and ε' are generated by flavour changing charged scalar H_i^+ ($i = 1,2$) interactions which are given by [37]

$$\mathcal{L}_H = 2^{3/4}G_F^{1/2}\bar{u}\{V\hat{M}_d X_i H_i^+ P_R + \hat{M}_u V \quad H_i^+ P_L\}d + h.c. \quad (64)$$

The constants X_i, Y_i ($i = 1,2$) parametrize mixing in the charged Higgs sector. It has been shown that in order to reproduce ε and ε' one has to invoke either long distance contributions (η, η') to ε [38] or the vacuum expectation values of the scalar doublets have to obey the hierarchy $|v_1| \ll |v_2| < |v_3|$ [39].

The CP violating parameter Im $(X_1^* Y_1) = -$ Im$(X_2^* Y_2)$ of the charged Higgs sector is constrained by ε and ε'/ε. Taking this into account one can derive a lower bound [40]

$$|d_n| \gtrsim 10^{-25} \text{ e.cm} \tag{65}$$

dangerously close to the upper limit trom experiment in the scheme containing long distance contributions to ε. For the hierarchy of vacuum expectation values there is a similar result. In the Weinberg model $d_n \simeq 4d_d/3$ because the u quark contribution is suppressed by quark mass ratios and charged scalars do not contribute to d_n^{ex} in the oscillator model [14].

It is true that for ε and ε' only charged scalars play a rôle because of neutral flavour conservation but for the EDMN there are also contributions from the neutral scalars. In principle, looking at the couplings, these are suppressed by $(m_d/m_c \sin\Theta_c)^2 \sim 10^{-3}$ compared to those of the charged Higgs scalars [41]. However, it was recently emphasized by Weinberg [42] that there is one CP violating purely gluonic operator

$$\mathcal{O} = \frac{1}{6} C f_{abc} G^a{}_{\mu\rho} G^b{}_\nu{}^\rho G^c{}_{\lambda\sigma} \varepsilon^{\mu\nu\lambda\sigma} \tag{66}$$

of dimension 6 whose coefficient involves neither small quark masses nor small mixing angles. The dominant contributions to C come from graphs of the type of Fig. 7. Approximating the propagator of the neutral scalar ϕ_2^0 (Equ. (63)) by

$$\frac{1}{v_2^2} \int d^4x e^{-iq\cdot x} \langle 0|T(\phi_2^0(x)\phi_2^0(0))|0\rangle \simeq i \frac{\sqrt{2}\, G_F z}{q^2 - m_{H^0}^2 + i\varepsilon} \tag{67}$$

the EDMN through neutral Higgs exchange is estimated by [42]

$$d_n(H^0) \sim \frac{e\mathcal{M}C}{4\pi} \sim 10^{-19} \text{ Im } z\, h\left(\frac{m_t}{m_{H^0}}\right) \text{ e.cm.} \tag{68}$$

Here $\mathcal{M} \sim 1$ GeV is the chiral symmetry breaking scale, h is a function contained in C coming from the two-loop graphs involving the t quark and ϕ_2^0 (Fig. 7) and z is a dimensionless parameter characterizing CP violation in scalar mixing. With the lower bound $h > 0.015$ for $m_{H^0} < 10 m_t$ and Equ. (68) one gets an upper bound

$$|\text{Im } z| < 10^{-4}. \tag{69}$$

It is not clear to us whether this bound has an impact on the parameters in the charged Higgs sector and thus on the discussion of CP violation of charged scalar interactions or if Im z can be varied independently.

In the literature [43] other contributions to $d_n(H^0)$ have been considered which according to Ref. [42] correspond to the operator $G^a_{\mu\nu} G^{a\mu\nu} G^b_{\rho\sigma} \tilde{G}^{b\rho\sigma}$. Since this one is of dimension 8 its contribution to the EDMN should be suppressed compared to $\mathcal{O}$ Equ. (66).

3.5 Lee Model

This model contains two Higgs doublets instead of one in the SM and it has therefore the minimal Higgs sector for spontaneous CP violation [44]. There is no flavour conservation in the neutral Higgs couplings to quarks. Therefore either the masses of the neutral scalars must be very heavy or their couplings must be fine-tuned. In general there is CP violation in the KM matrix and in the neutral and charged Higgs sectors. With fine-tuning ($m_{H^0} \sim W$ boson mass) one gets an EDMN of order [45]

$$d_n(H^+) \sim 10^{-26} \text{ e.cm.} \tag{70}$$

3.6 SM with Two Scalar Doublets and Neutral Flavour Conservation

In this model there must be hard CP violation through complex Yukawa couplings as in the SM. It is easy to show that [47]

$$d_q(\text{1-loop}) = 0. \tag{71}$$

There is also no exchange contribution to the EDMN at the tree level. Consequently the EDMN must be very small similar to the situation in the SM.

4 Conclusions

At last we would like to summarize the most significant points of our discussion.

i) For the EDM of an elementary or composite particle to be non-zero it is necessary to have P *and* T/CP violation[2].

ii) Models where a non-zero one-loop EDM is possible and with scales of order of the W mass have an EDMN naturally close to the experimental bound or larger.

iii) Among the models discussed in this paper the experimental limit on the EDMN leads to constraints on CP violating phases in SUSY models and the neutral Higgs sector of the Weinberg model.

iv) In the Weinberg model CP violation in $K^0\bar{K}^0$ and the EDMN through charged scalars involve both the same CP violating parameter. Using experimental information on ε and ε'/ε not much freedom is left for the EDMN and one gets the lower bound $|d_n(H^+)| \gtrsim 10^{-25}$ e.cm which is just compatible with experiment.

v) With a non-zero experimental result for the EDMN one cannot distinguish between different models. Particularly, strong CP violation could always be responsible for it (this would not be the case for a leptonic EDM).

[2]Note that the "EDM of molecules" is a totally different notion which has nothing to do with P or T violation. For a clarification of this point see Ref. [6].

Considering the last point of the summary it is rather difficult to tell what one could learn from measuring a non-zero EDMN. But without doubt a non-zero EDMN would show us that CP violation is a general phenomenon – as it is the case in our theoretical models – not confined to the $K^0\bar{K}^0$ system. In this way theory and experiment would come one step closer to each other.

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References

[1] J.H. Christensohn, J.W. Cronin, V.L. Fitch and R. Turlay, Phys. Rev. Lett. *13* (1964) 138.

[2] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. *49* (1973) 652.

[3] S.L. Glashow, Nucl. Phys. *22* (1961) 579;
S. Weinberg, Phys. Rev. Lett. *19* (1967) 1264;
A. Salam, Proc. 8th Nobel Symposium, Aspenäsgården, 1968, ed. N. Svartholm (Almqvist and Wiksell, Stockholm, 1968).

[4] E.M. Purcell and N.F. Ramsey, Phys. Rev. *78* (1950) 807.

[5] L. Landau, Nucl. Phys. *3* (1957) 127.

[6] H. Rupertsberger, preprint UWThPh-1989-68.

[7] A. Messiah,"Quantum Mechanics", vol. II, p. 581, North-Holland Publ. Comp., Amsterdam, 1966.

[8] M.A. Bouchiat and C. Bouchiat, J. Physique *35* (1974) 899.

[9] A. Messiah, "Quantum Mechanics", vol. II, p. 669-672, North-Holland Publ. Comp., Amsterdam, 1966.

[10] N.M. Monyonko and J.H. Reid, Prog. Theor. Phys. *73* (1985) 734.

[11] I.S. Altarev etal., JETP Lett. *44* (1986) 460.

[12] K.F. Smith etal., Phys. Lett. *B234* (1990) 191.

[13] Review of Particle Properties, Phys. Lett. *B204* (1988).

[14] G. Ecker, W. Grimus and H. Neufeld, Nucl. Phys. *B229* (1983) 421.

[15] J. Ellis, M.K. Gaillard and D.V. Nanopoulos, Nucl. Phys. *B109* (1976) 213.

[16] E.P. Shabalin, Sov. J. Nucl. Phys. *28* (1978) 75.

[17] E.P. Shabalin, Sov. J. Nucl. Phys. *31* (1980) 864.

[18] I.B. Khriplovich, Phys. Lett. *B173* (1986) 193.

[19] D.V. Nanopoulos, A. Yildiz and P. Cox, Phys. Lett. *87B* (1979) 53;
B.F. Morel, Nucl. Phys. *B157* (1979) 23;
E.P. Shabalin, Sov. J. Nucl. Phys. *32* (1980) 228;
N.G. Deshpande, G. Eilam and W.L. Spencer, Phys. Lett. *108B* (1982) 42;
M.B. Gavela et al., Phys. Lett. *109B* (1982) 83.

[20] M.B. Gavela et al., Phys. Lett. *109B* (1982) 215.

[21] J.O. Eeg and I. Picek, Phys. Lett. *130B* (1983) 308; Nucl. Phys. *B244* (1984) 77.

[22] M.B. Gavela et al., Z. Phys. *C23* (1984) 251.

[23] E.P. Shabalin, Sov. Phys. Usp. *26* (1983) 297;
J.E. Eeg, in Proc. of the Fifth Moriond Workshop, La Plagne, ed. J. Tran Thanh Van, Editions Frontière, Gif-sur-Yvette (1985), p. 287;
X.-G. He, B.H.J. McKellar and S. Pakvasa, in Proc. of AIP Conf. (1990), p. 964;
M.B. Gavela et al., "CP Violation and the Neutron Electric Dipole Moment", contribution to the Blois Conf. (May 1989) to celebrate the 25th anniversary of the discovery of CP violation, preprint LPTHE Orsay 89/29.

[24] For a recent review see R.D. Peccei, "The Strong CP Problem" in the review volume on CP violation, ed. C. Jarlskog, World Scientific Publishing Co., Singapore (1988).

[25] R. Dashen, Phys. Rev. *D3* (1971) 1879.

[26] V. Baluni, Phys. Rev. *D19* (1979) 2227.

[27] R. Crewther, P. Di Vecchia and E. Witten, Phys. Lett. *88B* (1979) 123; *91B* (1980) 487 (E).

[28] J.C. Pati and A. Salam, Phys. Rev. *D10* (1975) 275;
R.N. Mohapatra and J.C. Pati, Phys. Rev. *D11* (1975) 566 and 2558;
R.N. Mohapatra and G. Senjanović, Phys. Rev. Lett. *40* (1980) 912; Phys. Rev. *D23* (1981) 165.

[29] D. Chang, R.N. Mohapatra and M.K. Parida, Phys. Rev. Lett. *52* (1984) 1072; Phys. Rev. *D30* (1984) 1052.

[30] G. Beall, M. Bander and A. Soni, Phys. Rev. Lett. *48* (1982) 848;
G. Ecker and W. Grimus, Nucl. Phys. *B258* (1985) 328.

[31] I. Schur, Am. J. Math. *67* (1945) 472.

[32] For a review see H.P. Nilles, Phys. Rep. *110* (1984) 1.

[33] W. Buchmüller and D. Wyler, Phys. Lett. *121B* (1983) 321;
J. Polchinski and M.B. Wise, Phys. Lett. *125B* (1983) 393.

[34] J.-M. Gérard, W. Grimus, A. Masiero, D.V. Nanopoulos and Amitava Raychaudhuri, Nucl. Phys. *B253* (1985) 93.

[35] S. Weinberg, Phys. Rev. Lett. *37* (1976) 657.

[36] G.C. Branco, Phys. Rev. Lett. *44* (1980) 504;
H. Neufeld, W. Grimus and G. Ecker, Int. J. Mod. Phys. *A3* (1988) 603.

[37] C.H. Albright, J. Smith and S.-H.H. Tye, Phys. Rev. *D21* (1980) 711.

[38] Y. Dupont and T.N. Pham, Phys. Rev. *D28* (1983) 2169;
J.F. Donoghue and B.R. Holstein, Phys. Rev. *D32* (1985) 1152.

[39] G.C. Branco, A.J. Buras and J.-M. Gérard, Phys. Lett. *155B* (1985) 192; Nucl. Phys. *B259* (1985) 306.

[40] I.I. Bigi and A.I. Sanda, Phys. Rev. Lett. *58* (1987) 1604.

[41] N.G. Deshpande and E. Ma, Phys. Rev. *D16* (1977) 1583.

[42] S. Weinberg, Phys. Rev. Lett. *63* (1989) 2333.

[43] A.A. Anselm et al., Phys. Lett. *152B* (1985) 116;
T.P. Cheng and L.-F. Li, Phys. Lett. *B234* (1990) 165;
G. Valencia, preprint BNL-43377 (1989).

[44] T.D. Lee, Phys. Rev. *D8* (1973) 1226.

[45] J. Liu and L. Wolfenstein, Nucl. Phys. *B289* (1987) 1.

[46] A.F. Abbott, P. Sikivie and M.B. Wise, Phys. Rev. *D21* (1980) 1393.

[47] Ph.A.S. de Sousa Gerbert, Nucl. Phys. *B272* (1986) 581.

Figure Captions

Figs. 1a,b: Generic diagrams giving a one-loop EDM of a fermion f (B = exchanged boson, F = exchanged fermion).

Fig. 2: A typical two-loop contribution to the EDM of the d quark in the SM (W = charged vector boson). These contributions add up to zero.

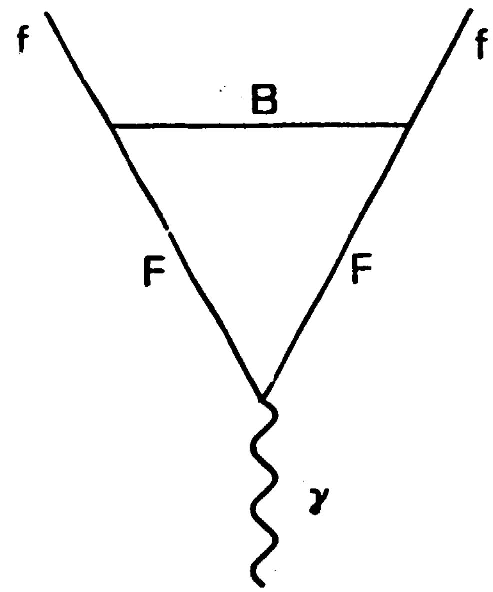
Fig. 3: A typical diagram contributing to the EDM of quarks at the lowest order where the sum of all these contributions does not vanish (G = gluon).

Fig. 4: The simplest type of an exchange diagram for the EDMN.

Fig. 5: EDMN exchange diagram involving a penguin graph.

Fig. 6: EDMN exchange diagram involving a photopenguin.

Fig. 7: Graph contributing to the coefficient C of the operator Equ. (66) in the Weinberg model.



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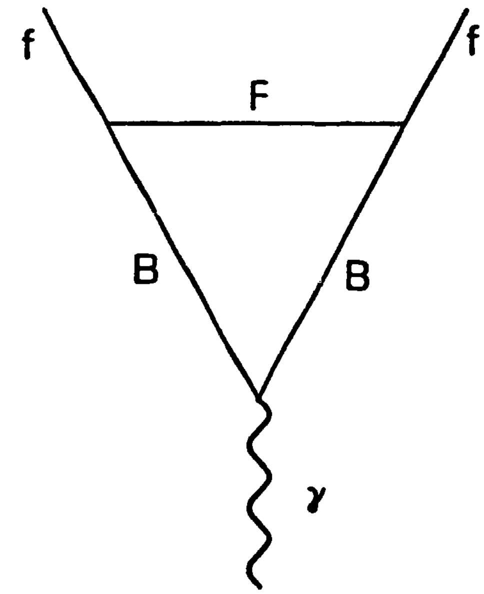


Fig.1b

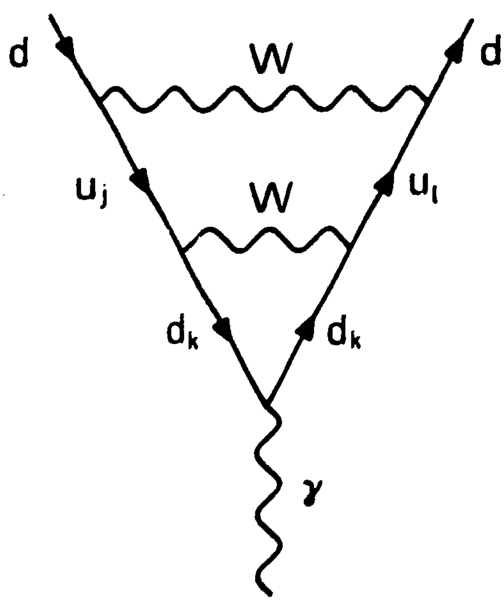
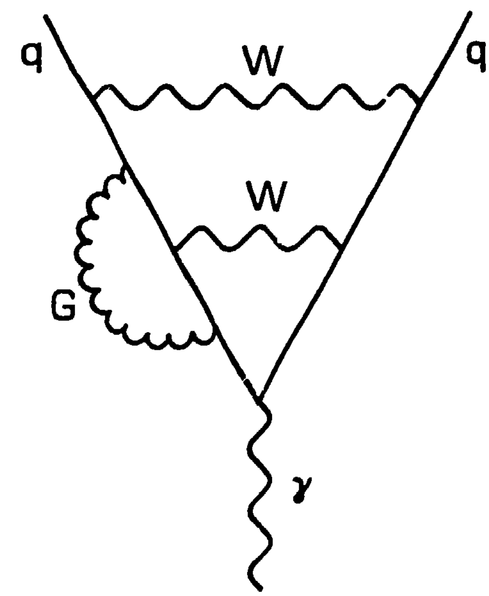


Fig.2



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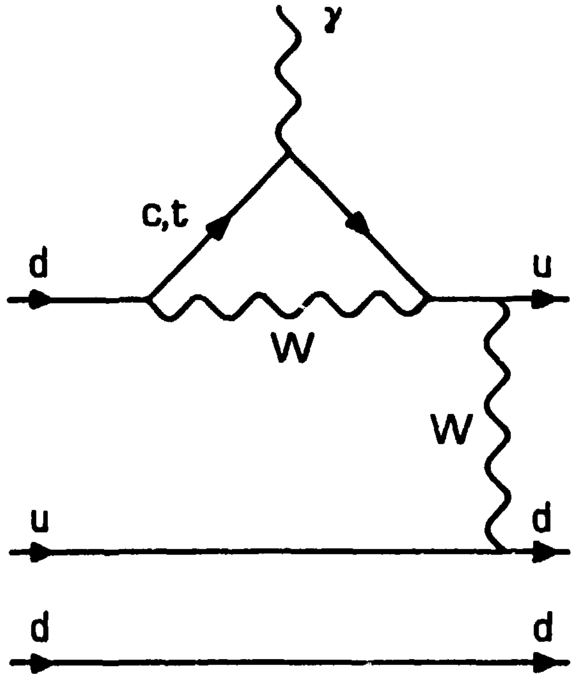

Fig.4

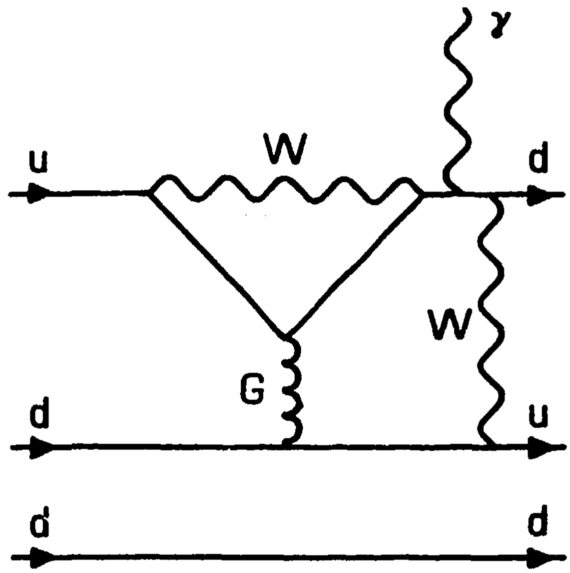

Fig.5

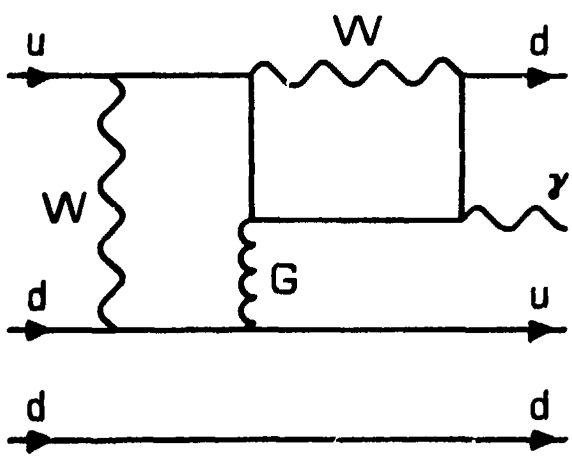

Fig.6

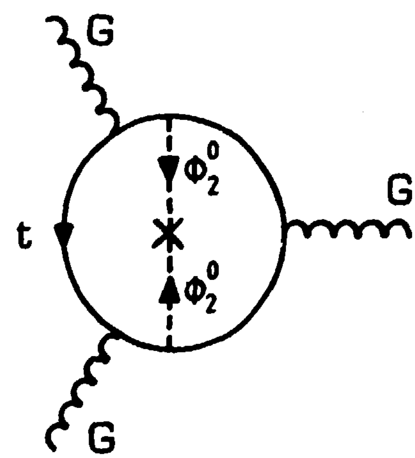

Fig.7