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Intermittency in Multihadron Production: An Analysis using Stochastic Theories

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Abstract

Multiplicity data of the NA22, KLM, and UA1 collaborations are analysed by means of probability distributions derived in the framework of pure birth stochastic equations. The intermittent behaviour of the KLM and UA1 data is well reproduced by the theory. A comparison with the negative binomial distribution is also made.

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1 Introduction

The hypothesis that multiparticle production at high energies shows intermittent behaviour, as recently put forward by Bialas and Peschanski [1], has met considerable interest. In order to extract the physical information contained in the multiplicity fluctuations in a given rapidity interval, they considered the factorial momente F_i, relying on well-known concepts of fluid dynamics [2,3]:

$$\ln\langle F_i\rangle \to f_i \cdot \ln(1/\delta y) \tag{1}$$

for $\delta y \to 0$, with

$$\langle F_i\rangle = \langle n(n-1)\ldots(n-i+1)\rangle/\langle n\rangle^i. \tag{2}$$

Here δy (or $\delta\eta$) is the rapidity (pseudorapidity) interval considered and f_i is the slope parameter. This idea has been applied to data in high energy collision [4,5] as well as cosmic ray experiments [6]: Very recently, the NA22 [7], KLM [8], and UA1 [9] collaborations and the Buffalo group [10] published experimental data showing a clear signal of intermittency.

On the theoretical side it has been shown in [11,12] that certain physical mechanisms like branching and cascade-type evolution, cold quark-gluon plasma, Schwinger tunneling mechanism, etc. will give intermittent behaviour. Carruthers and Minh Duong-Van proposed [13] that the concept of self-similarity should be adapted for the description of multiparticle production in hadronic reactions. Self-similarity is the basic property of all physical mechanisms which lead to intermittency, and naturally explains the observed multiplicity fluctuations. An alternative description of the dependence of the multiplicity moments on δy by quantum statistical correlations has been proposed in [14].

The purpose of our paper is to analyse the data for the normalized factorial moments using the probability distribution of [15], as following from the pure birth (PB) stochastic differential equation with specified initial conditions. The motivations are: Firstly, as has been shown in [16], the multiplicity distributions in different pseudorapidity intervals are well explained by this model. Secondly, the stochastic diagram related to the PB process has the property of self-similarity (see Fig. 1). In addition, we shall also compare the data with the negative binomial (NB) distribution, because both apprcaches show similar properties, and add some comments on the Furry distribution.

2 Probability Distribution from PB Stochastic Equation

The basis of our considerations is the PB stochastic equation [15–17]:

$$\partial P(n;t)/\partial t = \lambda(n-1)P(n-1;t) - \lambda n P(n;t) \tag{3}$$

with the initial condition at $t = 0$

$$P(n;t=0) = \langle m\rangle^n e^{-\langle m\rangle}/n!. \tag{4}$$

Here t is the evolution variable and λ is the parameter characterizing the PB process. As demonstrated in [15], the solution of eq. (3) with initial condition (4) gives a good description of the multiplicity distributions at high energies. The generating function of the $P(n;t)$, defined as

$$Q(z;t) = \sum_{n=0}^{\infty} P(n;t) z^n \tag{5a}$$

is given by

$$Q(z;t) = \exp\{\frac{\langle n\rangle(z-1)}{1-\langle n\rangle(z-1)/\xi(t)}\} \tag{5b}$$

with

$$\xi(t) = (1/\langle n\rangle - 1/\langle m\rangle)^{-1} = \langle n\rangle/p, \qquad \langle n\rangle = \langle m\rangle(p+1) \tag{6a}$$

$$p = e^{\lambda t} - 1. \tag{6b}$$

In the following we shall not explicitly write the t dependence. From eq. (5b) follows

$$P(n=0) = Q(z=0) = \exp\{-\frac{\langle n\rangle}{1+\langle n\rangle/\xi}\} \tag{7a}$$

$$P(n) = \frac{1}{n!}\frac{\partial^n Q}{\partial z^n}\bigg|_{z=0} = \frac{\xi}{n}\frac{(\langle n\rangle/\xi)^n}{(1+\langle n\rangle/\xi)^{n+1}}\exp\{-\langle n\rangle/(1+\langle n\rangle/\xi)\}L_{n-1}^{(1)}\{-\xi/(1+\langle n\rangle/\xi)\} \tag{7b}$$

$$(n = 1, 2, \ldots)$$

where $L_n^{(1)}(x)$ is the associated Laguerre polynomial. The i-th normalized factorial moment is then written as

$$\langle F_i\rangle = \frac{1}{\langle n\rangle^i}\frac{\partial^i Q(z)}{\partial z^i}\bigg|_{z=1} = \Gamma(i)(1/\xi)^{i-1}L_{i-1}^{(1)}(-\xi). \tag{8}$$

Especially for $i = 2$, we obtain

$$\langle F_2\rangle = \langle n(n-1)\rangle/\langle n\rangle^2 = 1 + 2/\xi. \tag{9}$$

Knowing the data for $\langle F_2\rangle$ or $\ln\langle F_2\rangle$, one can calculate the higher moments $\langle F_i\rangle$ with the help of eq. (8).

One can also obtain an explicit expression for the slope parameter f_i by expanding eq. (8) and using the following approximation $[O(1/\xi^3)]$

$$\ln\langle F_i\rangle \simeq \ln\{1 + \frac{i(i-1)}{\xi} + \frac{i(i-1)^2(i-2)}{2!\xi^2} + \frac{i(i-1)^2(i-2)^2(i-3)}{3!\xi^3}\}. \tag{10}$$

As $\xi^{-1} = \alpha + \beta\ln(1/\delta y)$ is satisfied by the data, we have the following expression for f_i in eq. (1), if $\beta \ll 1$, $\beta < \alpha$:

$$f_i = B/A, \tag{11}$$

where

$$\begin{aligned} A &= 1 + i(i-1)\alpha + i(i-1)^2(i-2)\alpha^2/2! + i(i-1)^2(i-2)^2(i-3)\alpha^3/3!, \\ B &= \{i(i-1) + i(i-1)^2(i-2)\alpha + i(i-1)^2(i-2)^2(i-3)\alpha^2/2!\}\beta. \end{aligned}$$

From this follows that the solution (7) of eq. (3) has the property of intermittency. Eq. (11) shows that the slope depends on both the intercept (α) and slope (β) of ξ^{-1}.

3 Normalized Factorial Moments in NB Distribution

As the NB distribution gives a good description of the multiplicity data [18], we shall use it also in the analysis of the δy dependence of the moments. Here the normalized factorial moments are

$$\begin{aligned} \langle F_2\rangle &= 1+1/k, \\ \langle F_3\rangle &= (1+1/k)(1+2/k) = \langle F_2\rangle\cdot(1+2/k), \\ \langle F_i\rangle &= \langle F_{i-1}\rangle\cdot(1+\frac{i-1}{k}), \end{aligned} \quad (12)$$

where the parameter k can be interpreted as the number of cells. We shall determine it by using $\ln\langle F_2\rangle$ as input.

To obtain an explicit expression for f_i, we expand eq. (12), as in eq. (10), using the following approximation $[O(1/k^3)]$

$$\ln\langle F_i\rangle \simeq \ln\{1+\frac{i(i-1)}{2k}+\frac{i(i-1)(i-2)}{2k^2}\cdot\frac{(3i-1)}{12}+\frac{i^2(i-1)^2(i-2)(i-3)}{48k^3}\}. \quad (13)$$

As $k^{-1}=\alpha'+\beta'\ln(1/\delta y)$ holds, we have the following ratio for f_i in eq. (1) in the case of $\beta'\ll 1$ and $\beta'\ll\alpha'$

$$f_i = B'/A', \quad (14)$$

where

$$\begin{aligned} A' &= 1+i(i-1)\alpha'/2+i(i-1)(i-2)(3i-1)\alpha'^2/24+i^2(i-1)^2(i-2)(i-3)\alpha'^3/48, \\ B' &= \{i(i-1)/2+i(i-1)(i-2)(3i-1)\alpha'/12+i^2(i-1)^2(i-2)(i-3)\alpha'^2/16\}\beta'. \end{aligned}$$

Eq. (14) shows that the NB distribution also has the property of intermittency.

4 Data Analysis

We now analyse the data for the multiplicity moments of the NA22, KLM, and UA1 collaborations by using the PB distribution, eq. (8), and the NB distribution, eq. (12). Using the $\langle F_2\rangle$ data as input we calculate the higher moments $\langle F_i\rangle$, $i\geq 3$.

In Figs. 2a and b we show the results of our analysis of the KLM data on p-Em (200 GeV) and ^{16}O-Em(Ag,Br) (200 GeV/n) collisions [8]. The data for ^{16}O-Em(Ag,Br) are fairly well explained by eqs. (8) aand (16). On the other hand, in the case of the p-Em data only the slopes of the $\ln\langle F_i\rangle$ are well reproduced, whereas their absolute normalizations slightly disagree.

The UA1 data for $\ln\langle F_i\rangle$, $i=2-5$, at $\sqrt{s}=630$ GeV [9] have very good statistics. They are shown in Fig. 3, together with our predictions. For the PB distribution, eq. (8), we have very good agreement. The slight discrepancy with the NB distribution, eq. (12), may be attributed to the small values of the k parameter ($k=1.7-2$ for

$0 \leq \ln(1/\delta\eta) \lesssim 3$). The slopes of $\ln\langle F_i\rangle$ agree. Since the absolute normalization of $\ln\langle F_i\rangle$ is experimentally less well determined, however, no firm conclusion can be drawn, whether the PB distribution or the NB distribution better reproduces the data.

In order to extract the numerical values for the slope parameters, we fit our results for $\ln\langle F_i\rangle$, $i = 3, 4, 5$, by a stright line. The f_i obtained in this way are given in Table 1 and compared with those from the KLM and UA1 experiments. We also give in Table 1 the numerical values following from eqs. (11) and (14). As one can see, these explicit expressions for the f_i are good approximations.

We also compare in Table 1 the NA22 π^+p and K^+p data at 250 GeV for $\ln\langle F_i\rangle$, $i = 3, 4, 5$ [7] with the predictions of eqs. (8), (11), (12) and (14). In this case neither the PB nor the NB distribution can reproduce the slopes of $\ln\langle F_i\rangle$ in a satisfactory manner. In particular there is a large disagreement between the data for the slopes of $\ln\langle F_4\rangle$ and $\ln\langle F_5\rangle$ and the theoretical predictions for $\ln(1/\delta y) > 1$. But there the data have large errors.

We have also analysed the data of the Buffalo group [10] and examined the behaviour of $\ln\langle F_i\rangle$ as well as $\ln\langle C_i\rangle$, $\langle C_i\rangle = \langle n^i\rangle/\langle n\rangle^i$, vs. $\ln(1/\delta\eta)$, as suggested in [19]. We find that $\ln\langle C_i\rangle$ show intermittency more clearly.

5 Furry Distribution

This distribution is the solution of eq. (3) with the initial condition at $t = 0$

$$P(n; t = 0) = \delta_{n,k} \tag{15}$$

instead of eq. (4) [15]. In this case the number of independent sources is fixed, there is no fluctuation (i.e. superposition of many contributions) at $t = 0$. The normalized factorial moments are given by

$$\langle F_i\rangle = k\sum_{j=0}^{i}\binom{i}{j}\frac{(k+i-j-1)!}{(k-j)!}p^{i-j}/\langle n\rangle^i, \tag{16}$$

where $\langle n\rangle = k(p+1)$, with p given by eq. (6a). As in the previous calculations, we use $\langle F_2\rangle$ as input. We can determine the ratio $(p-1)/\langle n\rangle = \rho$ from

$$\langle F_2\rangle = 1 + (p-1)/\langle n\rangle = 1 + \rho, \tag{17a}$$

$$\rho = \langle F_2\rangle - 1.$$

In terms of ρ, the $\langle F_i\rangle$ can be expressed as follows

$$\langle F_3\rangle = 1 + 3\rho + 2\rho^2 + O(2\rho/\langle n\rangle + 2/\langle n\rangle^2), \tag{17b}$$

$$\langle F_4\rangle = 1 + 6\rho + 11\rho^2 + 6\rho^3 + O\{(8\rho + 12\rho^2)/\langle n\rangle + (8 + 12\rho)/\langle n\rangle^2\}, \tag{17c}$$

$$\langle F_5\rangle = 1 + 10\rho + 35\rho^2 + 50\rho^3 + 24\rho^4 + O\{(20\rho + 80\rho^2 + 72\rho^3)/\langle n\rangle + (20 + 80\rho + 96\rho^2)/\langle n\rangle^2 + +48\rho/\langle n\rangle^3 + 24/\langle n\rangle^4\}. \tag{17d}$$

It is worth mentioning that eqs. (17) without the terms of $O(1/\langle n\rangle \sim 1/\langle n\rangle^4)$ are equivalent to the $\langle F_i\rangle$ of the NB, eq. (12). The slope parametrs could be determined by means of eqs. (17), provided a relation between $\langle n\rangle$ and $\ln(1/\delta y)$ is found.[1]

6 Concluding Remarks

The normalized factorial moment in the PB stochastic process, eq. (8), and the NB distribution, eq. (12), have been used to analyse the data of the multiplicity moments of the NA22, KLM, and UA1 collaborations. The PB distribution can well explain the data by KLM and UA1. The NB distribution also gives good results provided the k parameter is not too small. The capability of the PB, as well as the NB distribution to describe the intermittent behaviour seen in the experimental data is attributed to the property of self-similarity, inherent in both approaches.

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[1]The slope parameters f_i depend on the multiplicity. For example, in the subset of UA1 data with $n_{ch} < 15$ the f_i are similar to those of NA22 ($n_{ch} \lesssim 7-8$). Intermittency may be stronger in events with a small number of sources, whereas the effect will be averaged out if a large number of sources is involved [9]. In our language a source is equivalent to an "excited hadron" at $t = 0$.

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Figure Captions and Table Caption

Fig. 1 Stochastic diagram of eq. (3). Self-similarity between the larger box and the internal box is indicated. Evolution variables t and t_f are introduced [16].

Fig. 2 Analysis of the data of the KLM collaboration: $\ln\langle F_i\rangle$ vs. $\ln(1/\delta\eta)$, $\ln\langle F_i\rangle$ from eqs. (8) and (12).
a) p + Em collision at 200 GeV.
b) O + Em(Ag,Br) collision at 200 GeV/n.

Fig. 3 Analysis of the data of the UA1 collaboration at $\sqrt{s}$ = 630 GeV: $\log_{10}\langle F_i\rangle$ vs. $\log_2(1/\delta\eta)$, $\ln\langle F_i\rangle$ from eqs. (8) and (12).

Table 1 Comparison of the data for the slope parameters f_i with our results from the PB and the NB distributions. Predictions for the f_i are given as following from eqs. (11) and (14). Theoretical values according to eqs. (8) and (12) are obtained by a straight-line fit to our results for $\ln\langle F_i\rangle$.

KLM p + Em at 200 GeV

f_i	Exp	PB		NB	
		Eq. (11)	Eq. (8)	Eq. (14)	Eq. (12)
$i=2$	0.027 ± 0.002	input	input	input	input
3	0.063 ± 0.011	0.0663	0.0652	0.0700	0.0692
4	0.129 ± 0.030	0.113	0.110	0.124	0.121
5	0.202 ± 0.060	0.163	0.161	0.178	0.180

KLM O + (Ag,Br) interaction at 200 GeV/nucleon

f_i	Exp	PB		NB	
		Eq. (11)	Eq. (8)	Eq. (14)	Eq. (12)
$i=2$	0.020 ± 0.001	input	input	input	input
3	0.052 ± 0.004	0.0522	0.0512	0.0545	0.0536
4	0.099 ± 0.008	0.0929	0.0901	0.0997	0.0972
5	0.158 ± 0.014	0.139	0.135	0.151	0.148
6	0.234 ± 0.022	0.186	0.183	0.200	0.205

UA1 $\bar{p}p$ at $\sqrt{s} = 630$ GeV

f_i	Exp	PB		NB	
		Eq. (11)	Eq. (8)	Eq. (14)	Eq. (12)
$i=2$	0.011 ± 0.001	input	input	input	input
3	0.025 ± 0.003	0.0256	0.0255	0.0273	0.0272
4	0.050 ± 0.005	0.0424	0.0421	0.0468	0.0465
5	0.077 ± 0.011	0.0587	0.0602	0.0636	0.0677

NA22 ($n_{ch} < 7-8$) at $\sqrt{s} = 22$ GeV

f_i	Exp	PB		NB	
		Eq. (11)	Eq. (8)	Eq. (14)	Eq. (12)
$i=2$	0.0127 ± 0.0008	input	input	input	input
3	0.0499 ± 0.0022	0.0315	0.0312	0.0332	0.0330
4	0.148 ± 0.007	0.0542	0.0534	0.0590	0.0583
5	0.328 ± 0.019	0.0785	0.0783	0.0855	0.0872

Table 1

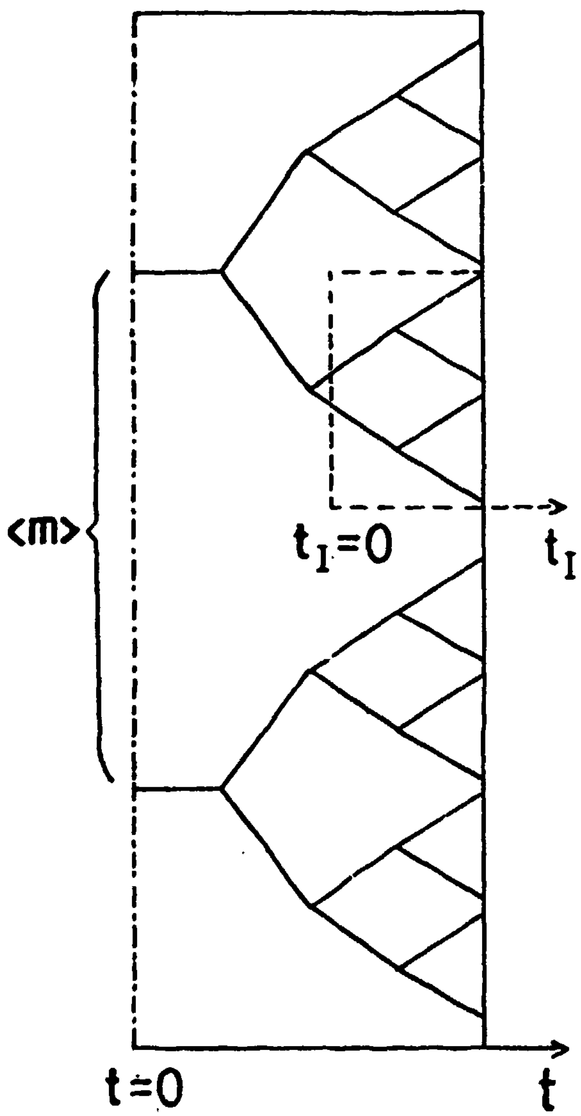
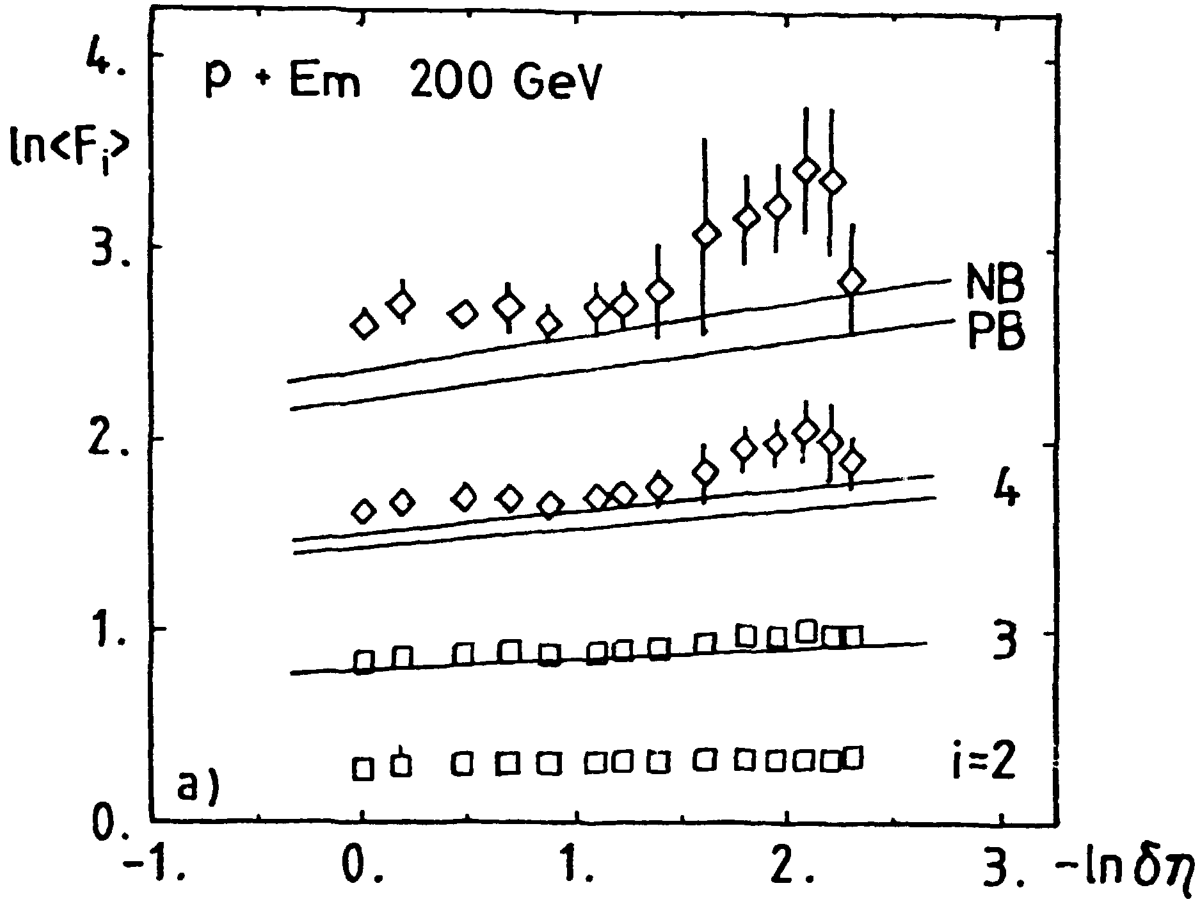


Fig.1



p + Em 200 GeV
ln<Fi>
NB
PB
4
3
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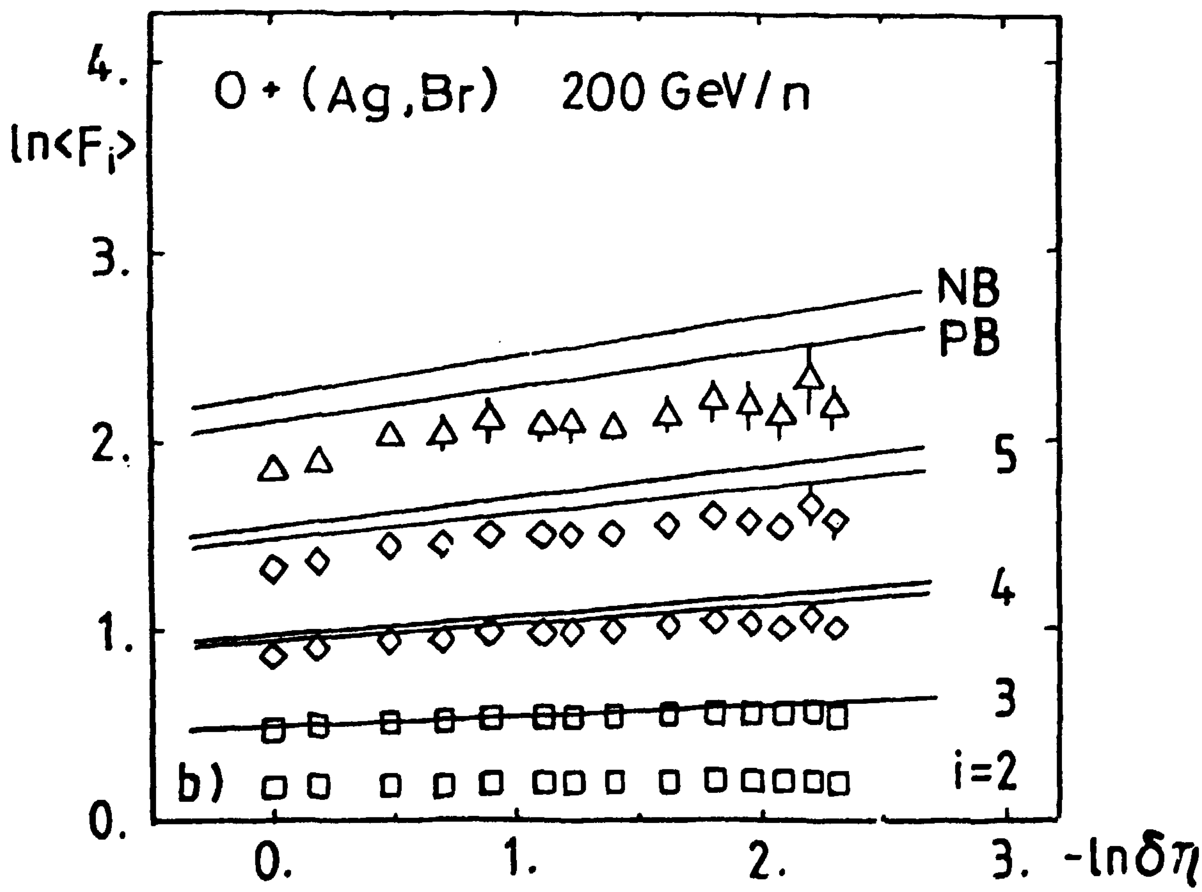
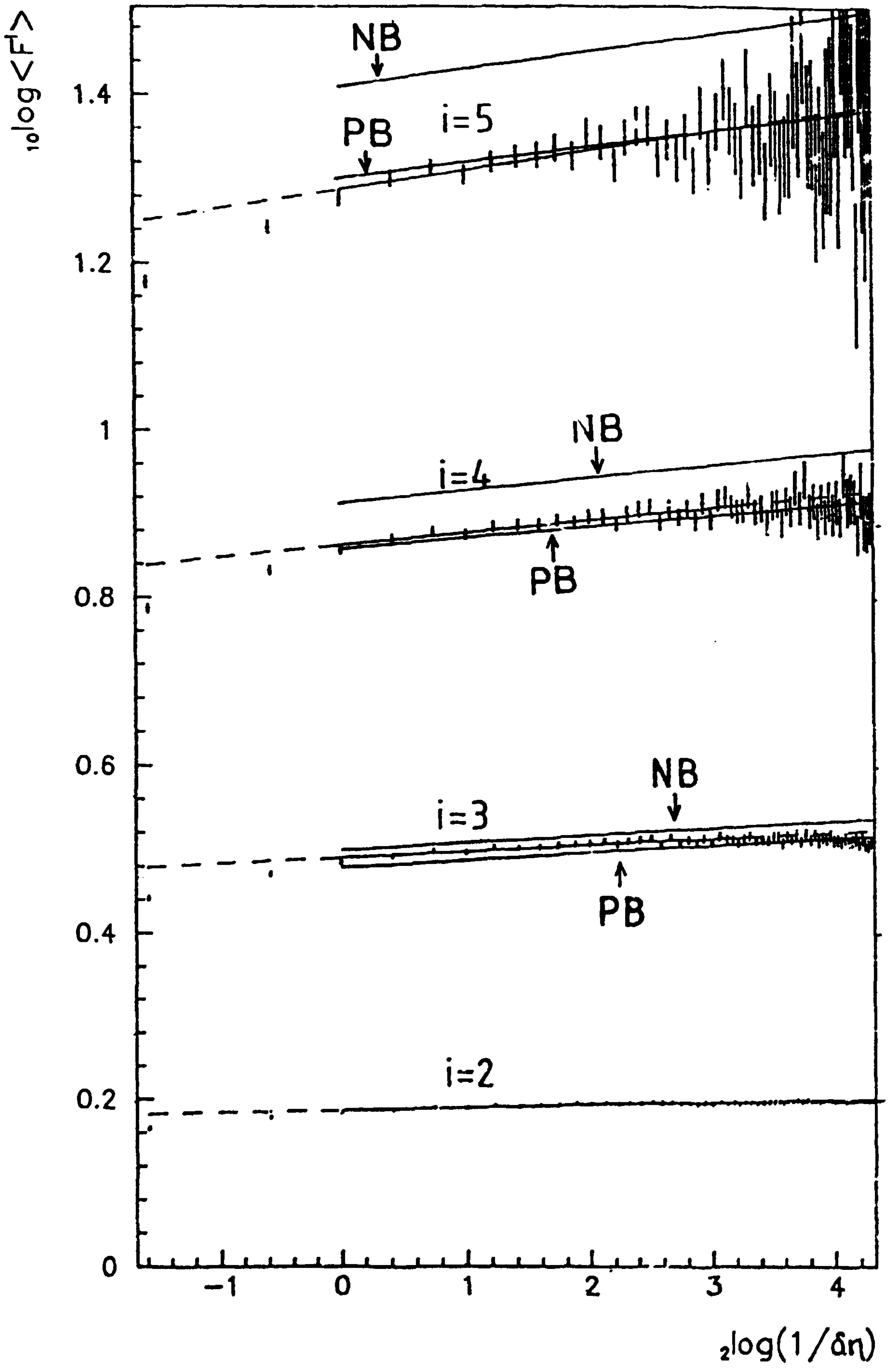
3.
2.
1.
0.
-1.
0.
1.
2.
3. -ln δη



O + (Ag,Br) 200 GeV/n
ln<Fi>
NB
PB
5
4
3
i=2
b)
4.
3.
2.
1.
0.
0.
1.
2.
3. -ln δη

Fig.2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1.2
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0.8
0.6
0.4
0.2
0
−1
0
1
2
3
4
₂log(1/δη)
NB
PB
i=5
NB
i=4
PB
NB
i=3
PB
i=2

Fig.3