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ABSTRACT

The superconformal algebras of meromorphic vector fields with multipoles, the central extension and the relevant abelian differential of the third kind on super Riemann sphere were constructed. The background of our theory is concerned with the interaction of closed superstrings.

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In recent papers [1,2,], the algebra of meromorphic vector fields with multipoles on Riemann surfaces have been constructed and the properties of the relevant abelian differentials of the third kind have been investigated. The algebras with central extension may play important roles in string theory and conformal fields theory [3]. In the paper [4], we have discussed the superconformal algebra with three poles without central term. Now, in this note, we will concetrate upon the corresponding super version of the algebra of meromorphic vector field with multipoles and the central term on Riemann sphere. First, we will construct the superconformal algebra of meromorphic vector fields with multipoles on super Riemann sphere. Secondly, we will construct an abelian differential of the third kind and use it to introduce the concept of Euclidean time and to give a picture of interaction of superstring.

As the super version of Riemann Roch theorem has not been well understood, we will use $\lambda = -1$, and $\lambda = -\frac{1}{2}$ differentials, which given in the reference [1], to construct the superconformal algebra of the meromorphic vector fields with multipoles on super Riemann sphere. According to Giddings and Nelson[5], super Riemann surface is a 1|1 complex supermanifold with a superconformal structure. And the class of superconformal structures is the class of supercomplex structures in addition to the requirement that the transition functions be analytic and satisfy extra condition

$$D_{\theta_\alpha} Z_\beta = \theta_\beta D_{\theta_\alpha} \theta_\beta \tag{1}$$

where $Z_\alpha = f_{\alpha\beta}(Z_\beta, \theta_\beta), \theta_\alpha = \phi_{\alpha\beta}(Z_\beta, \theta_\beta), D_\theta = \frac{\partial}{\partial\theta} + \theta\frac{\partial}{\partial z}$.

In our case, we chose N-different points $P_i(i = 1, 2, ..., N)$ with local coordinate $w_i(P_i) = 0, \theta_i(P_i) = 0$ on super Riemann sphere, here

$$\begin{aligned} z = w_1 = w_2 + z_2 = \dots = w_{N-1} + z_{N-1} = -\frac{1}{w_N} \\ \frac{1}{z}\theta = \frac{1}{w_1}\theta_1 = \frac{1}{w_2 + z_2}\theta_2 = \frac{1}{w_{N-1} + z_{N-1}}\theta_{N-1} = \theta_N \end{aligned} \tag{2}$$

this gluing conditions obviously satisfy (1). By use of $\lambda = -1$ and $\lambda = -\frac{1}{2}$ differentials in addition to the requirement (2), we can introduce a basis H^i_n and $G^i_r(i = 1, 2, ..., N - 1)$ of superfields with multipoles on super Riemann sphere, which in neighberhoods of the points P_i will have the following form:

$$\begin{aligned} H_n^1 &= w_1^{1-n}\frac{\partial}{\partial w_1} + \frac{1}{2}(1-n)w_1^n\theta_1\frac{\partial}{\partial\theta_1} \\ &= (z-z_1)^{1-n}\frac{\partial}{\partial z} + \frac{1}{2}(1-n)(z-z_1)^n\theta_1\frac{\partial}{\partial\theta_1} \\ &\quad here \quad z_1 = 0, \qquad n \in Z \end{aligned} \tag{3}$$

$$\begin{aligned} H_n^i &= w_i^{1-n}\frac{\partial}{\partial w_i} + \frac{1}{2}(1-n)w_i^n\theta_i\frac{\partial}{\partial\theta_i} \\ &= (z-z_i)^{1-n}\frac{\partial}{\partial z} + \frac{1}{2}(1-n)(z-z_i)^n\theta_i\frac{\partial}{\partial\theta_i} \\ &\quad here \quad n \geq 2, \qquad (i = 2,3,...,N-1) \end{aligned} \tag{4}$$

$$\begin{aligned} G_r^1 &= w_1^{1-r}(\frac{\partial}{\partial\theta_1} - \theta_1\frac{\partial}{\partial w_1}) \\ &= (z-z_1)^{1-r}(\frac{\partial}{\partial\theta_1} - \theta_1\frac{\partial}{\partial z}) \\ &\quad here \quad r \in Z \end{aligned} \tag{5}$$

$$\begin{aligned} G_r^i &= w_i^{1-r}(\frac{\partial}{\partial\theta_i} - \theta_i\frac{\partial}{\partial w_i}) \\ &= (z-z_i)^{1-r}(\frac{\partial}{\partial\theta_i} - \theta_i\frac{\partial}{\partial z}) \\ &\quad here \quad n \geq 2, \qquad (i = 2,3,...,N-1) \end{aligned} \tag{6}$$

With respect to the basis H_n^i, G_r^i, we can get the following algebra relations:

$$\begin{aligned} [H_n^i, H_m^i] &= (n-m)H_{n+m}^i \\ [H_n^i, G_r^i] &= (\frac{1}{2}n + \frac{1}{2} - r)G_{n+r}^i \\ \{G_r^i, G_s^i\} &= -2H_{r+s-1}^i \\ [H_n^i, H_m^j] &= \sum_{k=2}^{n+1}(2n-k)X_{nm}^{ij}(k)H_k^i - \sum_{k=2}^{m+1}(2m-k)X_{mn}^{ji}(k)H_k^j \\ [H_n^i, G_r^j] &= \sum_{k=2}^{n+1}(\frac{3}{2}n + \frac{1}{2} - k)X_{nr}^{ij}(k)G_k^i - \sum_{k=2}^{r+1}(\frac{3}{2}r + \frac{1}{2} - \frac{k}{2})X_{rn}^{ji}(k)G_k^j \\ \{G_r^i, G_s^j\} &= \sum_{k=2}^{r+1}(-2)Y_{rs}^{ij}(k)H_{k-1}^i + \sum_{k=2}^{s+1}(-2)Y_{sr}^{ji}(k)H_{k-1}^j \end{aligned} \tag{7}$$

where $i \neq j$, $\quad i,j = 1,2,...,N-1$,

$$X_{nm}^{ij}(k) = \begin{cases} C_{n+1-k}^{1-m}(z_i - z_j)^{k-m-n}, & n \geq 2 \\ (1-\delta_{2-k,0})C_{k-m-n}^{1-m}(z_i - z_j)^{k-m-n}, & i = 1 \quad and \quad n \leq 1 \end{cases} \tag{8}$$

$$Y_{rs}^{ij}(k) = \begin{cases} (1-\delta_{2-k,0})C_{r+1-k}^{1-s}(z_i - z_j)^{k-r-s}, & r \geq 2 \\ C_{k-r-s}^{1-s}(z_i - z_j)^{k-r-s}, & i = 1 \quad and \quad r \leq 1 \end{cases} \tag{9}$$

$$C_m^n = \frac{n!}{n!(n-m)!} \tag{10}$$

Let $\hat{H}_n^i$, $\hat{G}_r^i$ be the central extension form of H_n^i, G_r^i respectively, then the relations of the algebra with central term is following;

$$\begin{aligned} [\hat{H}_n^i, \hat{H}_m^i] &= (n-m)\hat{H}_{n+m}^i + \frac{1}{12}c\delta_{i,1}\delta_{m+n,0}(n^3 - n) \\ [\hat{H}_n^i, \hat{G}_r^i] &= (\frac{1}{2}n + \frac{1}{2} - r)\hat{G}_{n+r}^i \\ \{\hat{G}_r^i, \hat{G}_s^i\} &= -2\hat{H}_{r+s-1}^i + \frac{1}{3}c\delta_{i,1}\delta_{r+s,0}(r^2 - \frac{1}{4}) \\ [\hat{H}_n^i, \hat{H}_m^j] &= \sum_{k=2}^{n+1}(2n-k)X_{nm}^{ij}(k)\hat{H}_k^i - \sum_{k=2}^{m+1}(2m-k)X_{mn}^{ji}(k)\hat{H}_k^j \\ &\quad + \frac{1}{24}c\delta_{i,1}C_{1+n}^{1-m}(z_i - z_j)^{-n-m}(n^3 - n) \\ [\hat{H}_n^i, \hat{G}_r^j] &= \sum_{k=2}^{n+1}(\frac{3}{2}n + \frac{1}{2} - k)X_{nr}^{ij}(k)\hat{G}_k^i - \sum_{k=2}^{r+1}(\frac{3}{2}r + \frac{1}{2} - \frac{k}{2})X_{rn}^{ji}(k)\hat{G}_k^j \\ \{\hat{G}_r^i, \hat{G}_s^j\} &= \sum_{k=2}^{r+1}(-2)Y_{rs}^{ij}(k)\hat{H}_{k-1}^i + \sum_{k=2}^{s+1}(-2)Y_{sr}^{ji}(k)\hat{H}_{k-1}^j \\ &\quad + \frac{1}{6}c\delta_{i,1}\delta_{r+s,0}C_{1+r}^{1-s}(z_i - z_j)^{-r-s}(r^2 - \frac{1}{4}) \end{aligned} \tag{11}$$

In the paper [5], a $(\frac{p}{2}, 0)$-type holomorphic superdifferential is defined as;

$$d\theta^p\phi_{(z,\theta)} + d\theta^{p-1}\eta D_\theta\phi_{(z,\theta)} \tag{12}$$

where $\eta = dz + \theta d\theta$

From that we define an abelian differential of the third kind with multipoles on super Riemann sphere as

$$\omega = \prod_{i=1}^{N}C_i(z-z_i)^{-1}dz + \frac{\partial f_{(z)}}{\partial z}\theta dz + f_{(z)}d\theta \tag{13}$$

where C_i is a constant, $f_{(z)}$ is a holomorphic function.

Using the abelian differential, according to the method of Krichever and Novikov [6], We can define the "Euclidean time" on super Riemann sphere as

$$\tau = \mathrm{Re}\int_{p_0}^{p}\omega \tag{14}$$

where P_0 is an arbitary point, obviously, the function is univalent.

Let us give this time an explanation, if $\psi_{(\tau,\sigma)}$ is a field defined on super Riemann sphere, then $\psi_{(t,\theta,\sigma)}$ (where $\tau = t + \theta$) satisfies the canonical equation

$$\begin{aligned}\frac{\partial\psi_{(t,\theta,\sigma)}}{\partial t} &= [H, \psi_{(t,\theta,\sigma)}] \\ \frac{\partial\psi_{(t,\theta,\sigma)}}{\partial\theta} &= [Q, \psi_{(t,\theta,\sigma)}]\end{aligned} \tag{15}$$

where H is Hamiltonian, Q is supersymmetry charge.

We define a one-parameter family C_τ of contours as follows

$$C_\tau = \{P \in S^2, \tau_{(p)} = \tau\} \tag{16}$$

By a suitable choose of C_i, we can get for $t \to -\infty$ the contour C_τ being a small circle surrounding $Z = Z_1, Z_2, ..., Z_l$, $t \to +\infty$ the contours C_τ are small circles surrounding $Z = Z_{l+1}, ..., Z_N$. This implies an interacting picture of closed superstrings. Imagine a small circle surrounding P_i be a closed superstring, then the following evolution of the circle from $t \to -\infty$ to $t \to +\infty$ is equivalent to L closed superstring splitting into $N - L$ closed superstrings.

Above, we have extended the results [1] to one kind of super case, Ramond sector. For Neveu-Schwarz sector of multipoles, a similar result will be obtained if one use such a way to expand the half integer binomial of NS sector. It is clear that the background of our theory concerns with the interaction of closed superstrings, so it is very interesting to study the relations between the symmetry involving in our algebra and the symmetry involving G. Felder [7].The relation between $\lambda - differential$ with multipoles and N-point correlation function is under investigation.

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After completion of this work, we receive the papers of R. Dick [8] and M. Schlichenmaier [9], they also got some results in the conformal case.

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