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TOTAL CROSS SECTION OF HIGHLY EXCITED STRINGS *

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ABSTRACT

The unpolarized total cross section for the joining of two highly excited strings is calculated. The calculation is performed by taking the average over all states in the given excitation levels of the initial strings. We find that the total cross section grows with the energy and momentum of the initial states.

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In this paper, we calculate the unpolarized total cross section for the joining of two highly excited closed bosonic strings. The scattering proccess is illustrated in Fig. 1. By 'unpolarized cross section', we mean the cross section obtained by taking the average over all states in the given levels of the intial strings.

Recently, the behavior of high-energy strings[1] has attracted some attention. For example in refs. [1] the scattering of highly excited strings is studied and in refs. [2,3,4] the splitting and joining of strings is considered. On a related theme, there have been investigations on the behavior of a gas of fundamental strings, mainly in relation to a possible phase transition at the Hagedorn temperature. Work has been done by the use of the canonical ensemble[5] as well as the microcanonical one[6]. All these investigations assume an "ideal gas" of strings, thus neglecting interactions. We need, therefore, a method to study the properties of the system which respects string dynamics. Our final goal is to investigate a dynamical evolution of a system made of strings considering their interaction[7], in other words the string version of the nucleation problem[2]. To attack these problems, we have to know the feature of the string interaction, which are, after all, very simple. In the case of the closed string theory, they are described by three-string vertex, namely the decay and the combination. Characteristic features of the string theory are found in the scattering of the energetic strings[1]. As for the decay of the strings, several results have been obtained and they seem to match the physical intuition[2,4]. The combination of long strings is evaluated by Polchinski[3]. In the following, we will calculate the unpolarized total combinatiton rate of two higly excited (very massive) strings. Even though our calculation is performed in the fundamental string theory, our result will be applicable in the case of the cosmic string.

The quantity which we want to calculate is the unpolarized total cross section for the combination of two strings characterized by (n_1, k_1) and (n_2, k_2), where n is the level of the excitations in the oscillator modes and k is the momentum. This quantity is given formally as follows,

[1] Here, by high energy we mean both large kinetic energy and/or rest mass.

[2] Nucleation theory studies the formation of droplets in a supersaturated gas. The analogy is between the droplets and the excited strings.

$$
\begin{aligned}
\sigma_{tot}(n_1,k_1;n_2,k_2) &= \frac{1}{\rho_{n_1,n_1}}\sum_{\epsilon_1}\frac{1}{\rho_{n_2,n_2}}\sum_{\epsilon_2}\sum_{f}\frac{1}{J_i}(2\pi)^D\delta^D(k_1+k_2-p_f) \\
&\quad\times|\langle n_1,k_1,\epsilon_1;n_2,k_2,\epsilon_2|f,p_f\rangle|^2 \qquad (1)\\
&= \frac{2}{2E_1 2E_2 v_{rel}}\mathrm{Im}\Big\{4\pi\frac{1}{\rho_{n_1,n_1}}\sum_{\epsilon_1}\frac{1}{\rho_{n_2,n_2}}\sum_{\epsilon_2} \\
&\quad\times\langle n_1,k_1,\epsilon_1|V(k_2)_{n_2,\epsilon_2}V^{\dagger}(k_2)_{n_2,\epsilon_2}|n_1,k_1,\epsilon_1\rangle|\Big\} \ .
\end{aligned}
$$

The quantity ρ_{n_i,n_i} $(i=1,2)$ is the number of the states in the excitation level n_i or the degeneracy of the level, which has the asymptotic form $\sim c\cdot(\sqrt{n})^{-d-1}e^{\beta_H\sqrt{n}}$, where d is the number of the spatial dimensions, β_H is the so-called Hagedorn temperature and c is a constant. The first two summation, $\sum_{\epsilon_i}$, is the one over all states in the level n_i. Another summation $\sum_f$ is performed over all final states with momentum p_f. J_i is the flux of the initial states and v_{rel} is the relative velocity of the initial strings. To go from the first line to the second line, we have used the optical theorem and $\mathrm{Im}\{\ldots\}$ indicates the imaginary part.

The summation over all the level n_i states in eq.(1) is not straightforward. There is more than one way to carry out this summation. The first one is the direct calculation. This means to calculate the imaginary part of the four-point tree amplitude directly, taking care of the symmetry of the polarization tensors. The second one is to derive the four-point amplitude from the factorization of a two-loop vacuum amplitude by the states of the levels n_1 and n_2. In this method, the summation over the level n_i states is automatic. The third method is to extract a rule of summing over the level n_i states from the factorization of the one-loop amplitude. Using the rule derived from the one-loop factorization, we can calculate the unpolarized cross section in eq.(1). In the following, we will describe this last method, although we have also performed the direct calculation as a check.

As a first step, we have to derive a rule on the summation over the states at the level n from the factorization of the one-loop amplitude. Let us consider one-loop two-point amplitude,

$$
A_{2-pt} = (4\pi^2)^2\frac{1}{(2\pi)^D}\int d^2\tau\int d^2\nu\Big(\frac{2}{\mathrm{Im}\tau}\Big)^{D/2}Z(\tau)\langle V(k)_{n,\epsilon}V^{\dagger}(k)_{n,\epsilon}\rangle_{\tau,\nu}, \qquad (2)
$$

where τ and ν are the moduli parameter and the separation of the vertices, respectively. The dimension of the space-time, D, is 26 in the present case.

The quantity $Z(\tau)$ is a partiton function, $Z(\tau)=\sum_{n,m}\rho_{n,m}e^{2\pi i\tau(n-1)-2\pi i\bar{\tau}(m-1)}$, where $\rho_{n,m}$ is the number of the states with the excitation of the right and left oscillation modes n and m. We will use the generic form of the vertex operator,

$$
\begin{aligned}
&V(k)_{n,\epsilon} = \\
&= \Big(\frac{\kappa}{4\pi}\Big)\,\zeta_{(\epsilon,k)}^{\mu_1^{(1)}\mu_2^{(1)}\ldots\mu_1^{(2)}\ldots;\bar{\mu}_1^{(1)}\ldots\bar{\mu}_1^{(2)}\ldots}\,\partial_{\xi^{(1)}_{\mu_1^{(1)}}}\partial_{\xi^{(1)}_{\mu_2^{(1)}}}\ldots\partial_{\xi^{(2)}_{\mu_1^{(2)}}}\ldots\partial_{\bar{\xi}^{(1)}_{\bar{\mu}_1^{(1)}}}\ldots\partial_{\bar{\xi}^{(2)}_{\bar{\mu}_1^{(2)}}}\ldots \\
&\quad\times\exp\Big(-ik\cdot X+\sum_i\xi^{(i)}\cdot\partial^i X+\sum_i\bar{\xi}^{(i)}\cdot\bar{\partial}^i X\Big)\Big|_{\xi=\bar{\xi}=0} \ ,
\end{aligned}
$$

with

$$
\begin{aligned}
&\mu_j^{(i)};j\sim 1-n^{(i)} \ , \ \bar{\mu}_j^{(i)};j=1\sim\bar{n}^{(i)}, \\
&n=\sum_i in^{(i)}=\sum_i i\bar{n}^{(i)} \ , \ -k^2=8(n-1),
\end{aligned}
$$

where $\zeta_{(\epsilon,k)}$ is a polarization tensor and κ is a coupling constant. The number of symmetries of the polarization tensors, which we have to consider, grows rapidly with the increase of the level, and it is in fact this proliferation which renders difficult a brute force calculation using the known form of the vertex. Instead we will extract a rule for the summation over states in a given level from the factrization of the one-loop amplitude eq.(2). The factorization is obtained by extracting the parts of the one-loop amplitude which contribute in the limit $\tau_2=\mathrm{Im}\tau\to\infty$. That is:

$$
\begin{aligned}
A_{2-pt} &= (\pi\kappa)^2\int_{\tau_2\to\infty}d^2\tau\,\zeta_{(\epsilon,k)}^{\mu\ldots;\bar{\mu}\ldots}\zeta_{(\epsilon,k)}^{\dagger\mu'\ldots;\bar{\mu}'\ldots}\,\partial_{\xi}\ldots\partial_{\bar{\xi}}\ldots\partial_{\xi'}\ldots\partial_{\bar{\xi}'}\ldots \\
&\quad\times\Big\{\sum_n\rho_{n,n}\int_E\frac{d^Dp}{(2\pi)^D}e^{-\frac{\pi}{2}\tau_2(p^2+8(n-1))}\int_c\frac{1}{4\pi^2}d^2z|z|^{k\cdot p/2}|1-z|^{-k^2/2} \\
&\quad\times\exp\Big[\frac{\pi}{2}p\cdot(z^{-1}\xi'^{(1)}\bar{z}^{-1}\bar{\xi}'^{(1)}+\xi^{(1)}+\bar{\xi}^{(1)}) \\
&\quad+\sum_{i<j}(\xi^{(i)}\cdot\xi'^{(j)}z^{-j}(\partial^i X\partial^j X')_s+\bar{\xi}^{(i)}\cdot\bar{\xi}'^{(j)}\bar{z}^{-j}(\bar{\partial}^i X\bar{\partial}^j X')_s)\Big] \\
&\quad+O(\rho_{n,n-1})\Big\}\Big|_{\xi=\xi'=\bar{\xi}=\bar{\xi}'=0} \ , \qquad (3)
\end{aligned}
$$

where $\langle\ldots\rangle_s$ is the propagator of the string obtained from one-loop propagator by subtracting the 0-mode contributions and taking the limit $\tau_2\to\infty$. The integral $\int_E$ means Euclidean integration and $\int_c$ indicates the integration on the complex plane. As we can see from eq.(3), the modulus τ_2 plays a role of the Feynman parameter and $e^{-(\pi/2)\tau_2(p^2+8(n-1))}$ is the Feymann propagator of the level n state, after Wick rotation. In eq.(3), we have shown only the leading order term, which is proportional to $\rho_{n,n}$. The next order contribution to the level n factorization is proportional to $\rho_{n,n-1}$ and it is considered to be small compared with the leading term. As we will explain later, the next order terms do not contribute to the imaginary part in eq.(1). From the above expression of the amplitude, we can find a rule for the summing over the level n states. Using the unitarity relation[8]:

$$A_{1-loop}(1,2,\ldots,N) = \frac{1}{2}\sum_{n}\sum_{\epsilon_n}\int \frac{d^D k}{(2\pi)^D}\frac{-i}{k^2+M_n^2}A_{tree}(n,1,2,\ldots,N,n), \qquad (4)$$

the rule is given by:

$$\begin{aligned} &\left(\frac{4\pi}{\kappa}\right)^2 \int d^4\mu(z)\sum_{\epsilon_n} \langle V(p)_{n,\epsilon}(z_1)V(k)_{n',\epsilon'}(z_2)V^\dagger(k)_{n',\epsilon'}(z_3)V^\dagger(p)_{n,\epsilon}(z_4)\rangle \\ = \;& 4\rho_{n,n}\int d^4\mu(z)\; e^{-(p^2-8)\langle X(z_1)X(z_4)\rangle} \\ &\times\langle e^{ip\cdot X(z_1)}V(k)_{n',\epsilon'}(z_2)V^\dagger(k)_{n',\epsilon'}(z_3)e^{-ip\cdot X(z_4)}\rangle + O(\rho_{n,n-1}), \end{aligned} \qquad (5)$$

where $\langle X(z_a)\partial^i X(z_b)\rangle \doteq 0$ for $a=1,4$ $b=2,3$ $i\geq 2$, otherwise $\langle X(z)X(w)\rangle = -(1/2)\ln|z-w|$, $d^4\mu(z)$ is a integration measure of the amplitude and $\sum_{\epsilon_n}$ indicates summation over all states at level n and $\langle V\ldots V\rangle$ is the tree-level correlation.

Once we have found the rule for summing over the level n states, the evaluation of the cross section in eq.(1) is straightforward. The imaginary part of the quantity in the bracket in eq.(1) is given, in the limit $s=-(k_1+k_2)^2 \gg (\alpha')^{-1}$, by:

$$\begin{aligned} &\frac{1}{\rho_{n_1,n_1}}\sum_{\epsilon_1}\frac{1}{\rho_{n_2,n_2}}\sum_{\epsilon_2}\lim_{k_3\to -k_2}\lim_{k_4\to -k_1} 4\pi\left(\frac{4\pi}{\kappa}\right)^2 \mathrm{Im}\int d^4\mu(z) \\ &\times\langle V(k_1)_{n_1,\epsilon_1}(z_1)V(k_2)_{n_2,\epsilon_2}(z_2)V^\dagger(k_3)_{n_2,\epsilon_2}(z_3)V^\dagger(k_4)_{n_1,\epsilon_1}(z_4)\rangle \\ = \;& 4\pi\left(\frac{\kappa}{4\pi}\right)^2 4^2\mathrm{Im}\int d^2z|1-z|^{k_3\cdot k_4/2-4n_2}|z|^{k_1\cdot k_2/2}|_{s\to\infty,\, t\to 0} \\ = \;& 4\pi\left(\frac{\kappa}{4\pi}\right)^2\pi^2\left(n_1+n_2-2-\frac{s}{8}\right)^2 \quad . \end{aligned} \qquad (6)$$

As is shown in ref.[3], the imaginary part of the four point amplitude appears at the t-channel pole in the limit $t\to 0$. We can show that the next leading order terms in eq.(5) do not have t-channel poles. Therefore, only the leading term in eq.(5) contributes to the imaginary part and the result in eq.(6) turns out to be exact. Finally we obtain the unpolarized total cross section for the combination of two energetic closed bosonic strings in the following form:

$$\sigma_{tot}(n_1,k_1,n_2,k_2) = \frac{2}{2E_1 2E_2\, v_{rel}}(4\pi\kappa^2)\left(\frac{1}{4}k_1\cdot k_2\right)^2 \quad . \qquad (7)$$

For practical purposes, let us consider the interaction rate of two strings of level and momentum (n_1,k_1), (n_2,k_2) contained in a box of volume V_d. The interaction rate of these two strings is given by ,

$$R = \frac{(2\alpha')^{d/2}}{V_d}\left(v_{rel}\sigma_{tot}(n_1,k_1;n_2,k_2)\right) \quad . \qquad (8)$$

Integrating over the angular distribution, one then finds the behaviour of the average rate of the highly excited strings:

$$R_{ave} \sim E_1E_2 \quad . \qquad (9)$$

It is interesting to compare the behaviour of eq. (7) with energy with the corresponding one in the case of particles, for which

$$\sigma_{tot} = \frac{(2\pi)^4}{2E_1 2E_2\, v_{rel}}\sum_{f,\alpha}\prod_{n=1}^{N_f}\frac{d^3p_n}{(2\pi)^3 2E_n}|T_{fi}|^2\delta^4(p_i-p_f) \quad , \qquad (10)$$

where $\sum_{f,\alpha}$ means sum over all final states and their spins, N_f is the number of such states and T_{fi} is the relevant T-matrix. Neglecting the thresholds, at which the cross section increases due to the opening of new channels, we see that σ_{tot} for particles decreases when E_1 and E_2 grow, while σ_{tot} for strings increases as E_1 and E_2 grow. So, even using highly excited strings, increasing the energy does not probe smaller distances[1].

As we have mentioned in the beginning, one of the main motivations for this work has been as an ingredient for a realistic model of the dynamical evolution of the string distribution with respect to the energy, momentum, excitation level and so on. We are currently investigating this dynamical evolution of the string distribution and its rich structure both physically and mathematically. A system of interacting strings appear to have the potentiality of the most interesting dynamical systems, with the appearance of bifurcation, non-trivial orbits and possibly even chaotic behaviour. Some of our results on these subjects will appear soon.

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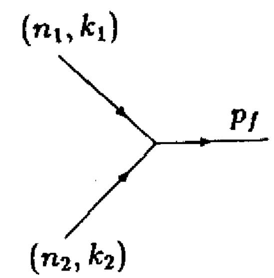

Figure 1

Two strings (n_1, k_1) and (n_2, k_2) combine to form a state with momentum p_f.

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