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TIME EVOLUTION OF THE MASS EXCHANGE IN GRAZING HEAVY-ION COLLISIONS

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ABSTRACT

On the basis of a macroscopical approach to the description of two interpenetrating quantal objects, the equations of two-fluid hydrodynamics for the cohesion stage of deeply inelastic heavy-ion collisions are formulated. The elasticity of the ions is analized in peripheral mass exchange reactions at intermediate energies. The system of closed equations of Newtonian mechanics, which simultaneously describes the motion of the ions along classical trajectories as well as the mass time evolution during the interaction period are derived and solved. The role of mass exchange in the friction force is discussed.

С.И. Бастуков, Ф. Деак, А. Киш, З. Шереш: Динамика массообмена в периферических столкновениях тяжелых ядер. KFKI-1989-54/A

АННОТАЦИЯ

На основе макроскопического описания динамики взаимопроникающих квантовых объектов выводятся уравнения двужидкостной гидродинамики для изучения стадии коалесценции сталкивающихся ионов в реакциях, сопровождающихся обменом нуклонами. В двужидкостной картине исследована эластичность ядер, проявляющаяся в периферических столкновениях. Основным результатом является формулировка самосогласованной классической динамики массообмена на стадии сцепления и относительного движения ядер по классическим траекториям.

Bastrukov S.I., Deák F., Kiss A., Seres Z.: A tömegcsere időbeli változása perifériális nehéz ion ütközésekben. KFKI 1989-54/A

KIVONAT

A dolgozatban két, egymásbamerülő kvantumobjektum leírásának makroszkopikus megközelítését ismertetjük a két-folyadék hidrodinamikai egyenletek alapján a mélyen rugalmatlan nehézion ütközésekre. Az ionok rugalmus tulajdonságát a perifériális tömegkicserélődési reakciók alapján kezeljük közepes ütközési energiáknál. Egy klasszikus mechanikai egyenletrendszerrel szimultán leírjuk az ionok klasszikus trajektoriáit, valamint a tömeg időbeli változását a kölcsönhatási periódus alatt. A tömeg kicserélődést a szokásos súrlódási erők alapján tárgyaljuk.

1. Introduction

At the present stage of heavy-ion collisions theory it is accepted that the common features of the deeply inelastic processes may be well understood on the basis of Newtonian dynamics, since the experimental cross sections at small angles are reminiscent of those for the Rutherford scattering. The inelastic component, which is responsible for transfer of energy and angular momentum from relative to intrinsic motion, during the cohesion period, is described by including friction forces. This approach is applied to the investigation of nucleus-nucleus reactions, when the mass of each ion remains unchanged before and after the collision [1–3]), as well as for processes, which are accompained by a flow of nuclear matter from one nucleus to the other [4–6]). In the first kind of collisions the nuclei are assumed to be rigid undeformable spheroids with constant masses. The friction coefficient is regarded to be constant and differs from zero only during the nuclei cohesion. Then all parameters of the system are fixed and the relevent variables are the variables of the relative motion. Therefore an equation of classical dynamics turns out to be sufficient to cover the whole process from approaching of the ions each other before touching and flying toward infinity after the collision.

In the framework of dynamical description of the mass exchange reactions one has to be restricted either to the amalgamation stage only, since the reduced mass of the nuclei ceases to be the same in the incoming and the outgoing channels, or introduce an additional condition at the moment of rupture to describe the the whole process of the collision [7]). The latter becomes clear if one consider the general form of the Newtonian equation, which governs the relative motion of ions along classical trajectories

$$\mu\frac{dV_i}{dt} = F_i - k_i V_i. \tag{1.1}$$

Here $\mu = \frac{M_1 M_2}{M_1+M_2}$ and V_i are the reduced mass and the ith component of relative velocity, respectively. The F_i stands for the ith component of the conservative forces acting between the nuclei and k_i are the friction coefficients which for different components of relative motion sometimes are choosen to be different. It is clear now that the reduced mass of the ions in the incoming channel is not equal to that in the outgoing channel due to the nucleon transfer, even when the total mass of the nuclei $M = M_1^{in} + M_2^{in} = M_1^{out} + M_2^{out}$ remains the same before and after the interaction. The problem is how to formulate a self-consistent *dynamical* description for the change of the ion's mass due to the particle transfer during the cohesion period and for the motion along classical trajectories before and after the contact. The present paper addresses these quastions.

It should be noted that the basic *kinematic* equations for the particle exchange processes can be exactly established also in the framework of classical mechanics, as it have been first shown rather long ago in ref.[8]), without special consideration of the wall or window dissipation mechanisms[9,10]).

In Sect.2 we analyze the degree of nuclear elasticity at intermediate energy of deeply inelastic processes on the basis of equations of the two-fluid hydrodynamics which are deduced there. In Sect.3 we obtain the closed system of the equations of classical dynamics for both processes, i.e. for the motion of the ions along Newtonian trajectories and for the mass exchange between them during the interaction period. The solutions of the mass transfer equations are considered there as well. Sect.4 contains the short discussion of the obtained results.

2. Two-fluid hydrodynamics of the cohesion stage

We shall start the analysis of the reaction with the stage when colliding nuclei are in close cohesion. For this purpose it seems to be useful the concept of the heteronuclear system or "heteronucleus" as a dynamically nonequilibrium state of the composite system. To be specific, we adopt that before the collision both the ions are characterized by different mean velocity, mass and energy densities as well as kinetic coefficients. From the hydrodynamical standpoint the amalgamation of the two nuclei may be provisionally regarded as a mixing process of two heterogeneous finite systems. The treatment of the general theory of such systems may be found, for example, in ref.[11]) and its applications to the heavy-ion collisions physics is given in [12]) and references therein. As in [12]) we shall assume that the heteronucleus has well defined single-connected shape. The total spatial volume, but not the shape of its surface, is regarded to be fixed during the whole period of the coalescence. The time of existing the heteronucleus has to be identified with the relaxation time the compound-nuclear to be reached. This is the main distinction between the compound-nucleus and the heteronucleus. The period of coalescence in deeply inelastic collisions at intermediate energies from 20 to 100 MeV/nucleon is shorter or of order of the relaxation time [13]). It is shorter also than a life-time of the thermodynamically equilibrated composite system [6,14]). Therefore one may assume that the heteronuclear state characterizes the whole period of close touching as well as in precompound reactions.

It should, however, be most decidedly emphasised that regarding the stage of cohesion of the ions in the two-liquid picture is no more than a convenient description, because of the mentioned distinctions between colliding nuclei, first of all between their densities, has the dynamical character, but not absolute one. Upon reaching the thermal equilibrium the heteronuclear phase descend into the compound one. The latter, as is known, has been well understood on the ground of one-fluid or homogeneous hydrodynamics - liquid drop model.

As it was shown in ref.[15]) the hydrodynamical description of two interpenetrating quantum objects may be given on the ground of two coupled Schrödinger-like equations

$$i\hbar\frac{\partial\Psi_1(\mathbf{r},t)}{\partial t} = -\frac{\hbar^2}{2m}\Delta\Psi_1(\mathbf{r},t) + U_{11}(\mathbf{r},t)\Psi_1(\mathbf{r},t) + W_{12}(\mathbf{r},t)\Psi_2(\mathbf{r},t), \qquad (2.1)$$

$$i\hbar\frac{\partial\Psi_2(\mathbf{r},t)}{\partial t} = -\frac{\hbar^2}{2m}\Delta\Psi_2(\mathbf{r},t) + U_{22}(\mathbf{r},t)\Psi_2(\mathbf{r},t) + W_{21}(\mathbf{r},t)\Psi_1(\mathbf{r},t), \qquad (2.2)$$

for macroscopic wave functions in Madelung representation

$$\Psi_\alpha(\mathbf{r},t) = \sqrt{\rho_\alpha(\mathbf{r},t)}e^{iS_\alpha(\mathbf{r},t)}. \qquad (2.3)$$

Here

$$\rho_\alpha(\mathbf{r},t) = |\Psi_\alpha(\mathbf{r},t)|^2, \qquad (2.4)$$

and

$$\mathbf{u}_\alpha(\mathbf{r},t) = \frac{\hbar}{m}\nabla S_\alpha(\mathbf{r},t), \qquad (2.5)$$

may be recognized as bulk density and mean velocity fields, respectively, i.e. as a fluid-dynamical variables of the ingredients of a two-liquid mixture. By m the reduced mass of particles setting in transport process is denoted and hereafter $\alpha = 1,2$. This way of interpreting the wave function of the system as a whole has been first given in the semi-microscopical Ginzgurg-Landau theory of superconductivity [16]). However, it does not exclude the orthodox quantum mechanical treatment of these equations. In particular, in ref.[17]) the system of eqs.(2.1), (2.2) has been proposed for the purely quantal description of the two-particle transfer reactions. The method of obtaining the one-fluid nuclear hydrodynamics by means of the Madelung transform is given in ref.[18]).

In eqs.(2.1) and (2.2) potentials U_{11} and U_{22} describe the collective degrees of freedom for each of the ions before and after the contact. They define the properties of the ions in the ground state and those induced by nearing of them. First of all they describe the static and dynamical *deformations*, which are important in the description of reactions nearby the Coulomb barrier. The potentials W_{12} and W_{21} stand for the amplitudes of the penetration of one nucleus into another and are responsible for the mass exchange. Both U and W are considered here to be real functions of the position and time, however, if one would regard them as complex functions the principal results of the present consideration valid as well. Besides, it is suggestive that the potentials W_{12} and W_{21} are different from zero only during the interaction time.

Substituting (2.3) and (2.5) into eqs.(2.1), (2.2) and separating real and imaginary parts of the result, we obtain the equations of two-fluid hydrodynamics

$$\frac{\partial \rho_1(\mathbf{r},t)}{\partial t} + \nabla(\rho_1(\mathbf{r},t)\mathbf{u}_1(\mathbf{r},t)) = -\eta_{12}(\mathbf{r},t)\sqrt{\rho_1(\mathbf{r},t)\rho_2(\mathbf{r},t)}, \tag{2.6}$$

$$\frac{\partial \rho_2(\mathbf{r},t)}{\partial t} + \nabla(\rho_2(\mathbf{r},t)\mathbf{u}_2(\mathbf{r},t)) = \eta_{21}(\mathbf{r},t)\sqrt{\rho_1(\mathbf{r},t)\rho_2(\mathbf{r},t)}, \tag{2.7}$$

$$\rho_1(\mathbf{r},t)\frac{d\mathbf{u}_1(\mathbf{r},t)}{dt} = \mathbf{f}_{21} + \frac{\eta_{12}(\mathbf{r},t)}{2}\sqrt{\rho_1(\mathbf{r},t)\rho_2(\mathbf{r},t)}(\mathbf{u}_1(\mathbf{r},t) - \mathbf{u}_2(\mathbf{r},t)), \tag{2.8}$$

$$\rho_2(\mathbf{r},t)\frac{d\mathbf{u}_2(\mathbf{r},t)}{dt} = \mathbf{f}_{12} + \frac{\eta_{21}(\mathbf{r},t)}{2}\sqrt{\rho_1(\mathbf{r},t)\rho_2(\mathbf{r},t)}(\mathbf{u}_1(\mathbf{r},t) - \mathbf{u}_2(\mathbf{r},t)), \tag{2.9}$$

where

$$\eta_{12}(\mathbf{r},t) = \frac{2W_{12}(\mathbf{r},t)}{\hbar}\sin(S_1(\mathbf{r},t) - S_2(\mathbf{r},t)), \tag{2.10}$$

$$\eta_{21}(\mathbf{r},t) = \frac{2W_{21}(\mathbf{r},t)}{\hbar}\sin(S_1(\mathbf{r},t) - S_2(\mathbf{r},t)), \tag{2.11}$$

and $\frac{d}{dt} = \frac{\partial}{\partial t} + (\mathbf{u}\nabla)$.

In the framework of the fluid-dynamical description, the quantities η_{12} and η_{21} have to be regarded as mass-diffusion coefficients, which determine the rate of dissolving of one nucleus into the other. These parameters should be adjusted from experiment. We shall assume henceforth that the η-parameters are constants. The terms $\mathbf{f}_{12}$ and $\mathbf{f}_{21}$ represent the densities of forces in the heteronuclear mixture and contain all the possible interactions, which are independent of the mean velocities $\mathbf{u}_\alpha$.

It is worthwhile to stress that the procedure of deriving the fluid-dynamical equations from the quantum mechanical two-level problem reflects the fact that these two descriptions do not contradict each other.

The main issue of this section is to test to what extent the two-fluid picture is suitable to describe the *grazing* collisions at intermediate energies from 20 to 100 MeV/nucleon. As is well established the period of the contact of ions in such kind of reactions is much shorter compared to those for headon one. In the latter case the system may approach rather closer to compound-nuclear formation. Therefore the coalescence of the nuclei in peripheral collisions should not lead to a considerable redistributions of the original fields of mean velocity and of bulk density. Bearing in mind this assumption, we shall consider the simplest case of spatially homogeneous mean velocity distributions and put

$$\mathbf{u}_\alpha = \mathbf{V}_\alpha(t) = \frac{d\mathbf{R}_\alpha(t)}{dt}, \tag{2.12}$$

where $\mathbf{R}_\alpha$ are the center-of-gravity positions of each ion in moment t, in the lab frame.

Let us consider the properties of elasticity of nuclei on the basis of the obtained equations in approximation of the spatially homogeneous density destributions when $\rho_\alpha = \rho_\alpha(t)$. There are two cases depending on value of the mass-diffusion parameter η, namely, either

$$\eta_{12} = \eta_{21} = \eta, \tag{2.13}$$

or

$$\eta_{12} \neq \eta_{21}. \tag{2.14}$$

It is easy to see that the condition (2.13) together with eqs.(2.6) and (2.7) gives

$$\rho_0 = \rho_1(t) + \rho_2(t) = constant, \tag{2.15}$$

that, in turn, just mean the total mass conservation in the course of collision, since we have assumed that the total volume of the heteronuclear stage is constant.

On account of (2.13), the time evolution of the density, for example first of the ions, is given by

$$\frac{d\rho_1(t)}{dt} = -\eta\sqrt{\rho_1(t)(\rho_0 - \rho_1(t))}. \tag{2.16}$$

Since eq.(2.16) is linear, the stability of the density oscillations can be investigated with respect to the harmonic perturbations of the form

$$\rho_\alpha(t) = \rho_\alpha^0(t) + \rho_\alpha' e^{-i\omega t}, \tag{2.17}$$

where $\rho_\alpha^0(t)$ are the densities of a local quasi-equilibrium state. In terms of the small density deviations $\delta\rho_1(t) = \rho_1' e^{-i\omega t}$, eq.(2.16) is replaced as

$$\frac{d\delta\rho_1(t)}{dt} = -\frac{\eta}{2\sqrt{\rho_1{}^0\rho_2{}^0}}(\rho_2{}^0 - \rho_1{}^0)\delta\rho_1(t), \tag{2.18}$$

Finally we find

$$\omega = i\frac{\eta}{2\sqrt{\rho_1{}^0\rho_2{}^0}}(\rho_2{}^0 - \rho_1{}^0), \tag{2.19}$$

i.e. the frequency turns out to be purely imaginary. The latter just means that the hydrodynamical or the sound pulses, in either the target and the projectile fragments will be damped out. Besides, under equalization of the densities of both the ingredients, the system as a whole loses the stability with respect to the sound vibrations. Thus, we found that if the total density is approximatelly constant during the interaction time, the nuclei behave themselves as two incompressible liquid drops or as inelastic rigid-like bodies. That is precisely what is observed in peripheral collisions at intermediate energies.

Let us now analyze the sound pulses in the case of different mass-diffusion coefficients (2.14). The simple combination of eqs.(2.6) and (2.7) leads to

$$\frac{d^2\sqrt{\rho_\alpha}(t)}{dt^2} = -\frac{\eta_{12}\eta_{21}}{4}\sqrt{\rho_\alpha}, \tag{2.20}$$

from where it follows that during the collision without total mass conservation (since the sum of right-hand sides of eqs.(2.6) and (2.7) does not equal to zero), in either ion arise the undamped sound vibrations with the same frequency

$$\omega = \frac{\sqrt{\eta_{12}\eta_{21}}}{2}, \tag{2.21}$$

and we may conclude that in the processes, when the total mass is not conserved, the nuclei to a great extent manifest themselves as a compressible liquid drops.

3. Time evolution of the mass in classical dynamics

The advantage of treating the composite system as a mixture of two liquids is that it allows to avoid the selection of the overlap region in order to obtain the mechanical variables and parameters through the hydrodynamical ones after integration over the full (constant) volume of the heteronucleus.

It is convenient to present the densities in the form

$$\rho_\alpha = M_\alpha(t)\phi_\alpha(\mathbf{r}), \tag{3.1}$$

where the dynamical specific volumes ϕ_α are normalized so that

$$\int_V (M_1(t)\phi_1(\mathbf{r}) + M_2(t)\phi_2(\mathbf{r}))dv = M_0, \tag{3.2}$$

First, we consider the case of the conservation of the total mass of either fragments in the ingoing and outcoming channels of the mass exchange reaction. The assumption about the homogeneous densities is then fulfilled. So we have

$$\int_V div\rho_\alpha \mathbf{u}_\alpha dv = \oint_\Sigma \rho_\alpha \mathbf{u}_\alpha ds = 0. \tag{3.3}$$

The integrals on r.h.s. of eqs.(2.6) and (2.7) can be presented in the form

$$\int_V \eta\sqrt{\rho_1(\mathbf{r},t)\rho_2(\mathbf{r},t)}dv = \lambda\sqrt{M_1(t)M_2(t)}, \tag{3.4}$$

since $\eta = \eta_{12} = \eta_{21}$. The constant λ denotes the drift mass parameter

$$\lambda = \int \eta\sqrt{\phi_1(\mathbf{r})\phi_2(\mathbf{r})}dv \ . \tag{3.5}$$

As a result, the total system of the dynamical equations in the lab frame can be represented as follows

$$\frac{dM_1(t)}{dt} = -\lambda\sqrt{M_1(t)M_2(t)}, \tag{3.6}$$

$$\frac{dM_2(t)}{dt} = \lambda\sqrt{M_1(t)M_2(t)}, \tag{3.7}$$

$$M_1(t)\frac{d\mathbf{V}_1(t)}{dt} = \mathbf{F}_{21} + \frac{\lambda}{2}\sqrt{M_1(t)M_2(t)}(\mathbf{V}_1(t) - \mathbf{V}_2(t)), \tag{3.8}$$

$$M_2(t)\frac{d\mathbf{V}_2(t)}{dt} = \mathbf{F}_{12} + \frac{\lambda}{2}\sqrt{M_1(t)M_2(t)}(\mathbf{V}_1(t) - \mathbf{V}_2(t)), \tag{3.9}$$

where eqs.(3.6), (3.7) determine the time evolution of mass of the ions during the cohesion, since the drift mass coefficient λ is not equal to zero during this period only. The rotational degrees of freedom are suggested to be frozen, since we are interested here in the processes when the creation of a binary system is scarcely probable. In eqs.(3.8) and (3.8) the forces $\mathbf{F}_{12}$ and $\mathbf{F}_{21}$ represent sums of non-dissipative, potential forces acting between nuclei, therefore $\mathbf{F} = \mathbf{F}_{12} = -\mathbf{F}_{21}$.

In order to obtain the standard form of eqs.(3.8), (3.9) in term of the relative velocity one should multiply eq.(3.8) by $M_2(t)$, and eq.(3.9) by $M_1(t)$. Next, from the first equation one subtracts the second one. Finally we have

$$\mu(t)\frac{d\mathbf{V}(t)}{dt} = \mathbf{F} - k(t)\mathbf{V}(t), \tag{3.10}$$

where $\mu(t) = \frac{M_1(t)M_2(t)}{M_1(t)+M_2(t)}$ is the reduced mass. Here $\mathbf{V}(t) = \mathbf{V}_1(t) - \mathbf{V}_2(t) = \frac{d\mathbf{R}(t)}{dt}$ and $\mathbf{R}(t) = \mathbf{R}_1(t) - \mathbf{R}_2(t)$ are the relative velocity and the relative distance between the centers-of-gravity of the colliding ions, respectively.

The friction coefficient k is

$$k(t) = \frac{\lambda}{2}\sqrt{M_1(t)M_2(t)}\frac{M_1(t) - M_2(t)}{M_1(t) + M_2(t)}, \tag{3.11}$$

Thus we have found that the friction coefficient k in the mass exchange peripheral reactions turns out to be function of time unlike the grazing elastic-like peripheral collicions when the mass each of the ions remains unchanged before and after the contact.

The solutions of eqs.(3.6), (3.7) require the knowledge of the mass of the nuclei in the incoming and the outgoing channels of reaction. Since the total mass of the nuclei

$M_0 = M_1{}^{in} + M_2{}^{in} = M_1{}^{out} + M_2{}^{out}$ before and after the collision is the same, eq.(3.7) may be replaced as follows

$$\frac{dM_1(t)}{dt} = -\lambda\sqrt{M_1(t)(M_0 - M_1(t))}, \tag{3.12}$$

Now it is clearly seen that the variables t and M_1 in eq.(3.12) are separable, so one can write

$$\int_{M_1{}^{in}}^{M_1{}^{out}} \frac{dM_1}{\sqrt{M_1(t)(M_0 - M_1(t))}} = -\lambda \int_0^{\tau_{int}} dt \tag{3.13}$$

where the integral on the l.h.s. is to be taken from $M = M_1^{in}$, i.e. mass of first fragment in the incoming channel to $M = M_1^{out}$ in the outgoing one. The time integration on the r.h.s. of eq.(3.13) covers the period of the direct contact of the ions, i.e. from *0* to the interaction time τ_{int}. Finally we get the following law for the time evolution for the mass of the first ion

$$M_1(t) = M_0 \frac{\left(\sqrt{\frac{M_1{}^{in}}{M_1{}^{out}}} - \tan(\frac{\lambda}{2}t)\right)^2}{1 + \left(\sqrt{\frac{M_1{}^{in}}{M_1{}^{out}}} - \tan(\frac{\lambda}{2}t)\right)^2}. \tag{3.14}$$

For the second one it just is $M_2(t) = M_0 - M_1(t)$. The arbitrariness of the drift mass parameter λ can be eliminated in all the obtained relations if τ_{int} is known. Indeed, when $t = \tau_{int}$ the final mass of the first fragment is $= M_1^{out}$, then from (3.14) we have for λ

$$\lambda = \frac{2}{\tau_{int}} \arctan\left(\sqrt{\frac{M_1{}^{in}}{M_2{}^{in}}} - \sqrt{\frac{M_1{}^{out}}{M_2{}^{out}}}\right). \tag{3.15}$$

For example, in case of pure Coulomb grazing collision [3]) the interaction period is $\tau_{int} = 1.58 \cdot 10^{-22} s$. We have found that λ must be of the order of $10^{21} - 10^{22} s^{-1}$. This value is in reasonable agreement with earlier estimations [19]).

Let us consider now the simple solution of the dynamical equation (3.10), which can be realized in fast processes, when one may neglect the forces of potential interactions **F**, i.e. when the relative motion of the ions during the interaction period is caused by the friction force only and when the relative velocity does not change away from its original direction. Then for the absolute value of **V** we have

$$\mu(t)\frac{dV(t)}{dt} = -\frac{\lambda}{2}\left(\sqrt{\frac{M_1(t)}{M_2(t)}} - \sqrt{\frac{M_2(t)}{M_1(t)}}\right)V(t). \tag{3.16}$$

Eq.(3.12) may be considered as the transformation from variable t to M_1, since we can express the latter as follows

$$dt = -\frac{dM_1}{\lambda\sqrt{M_1(t)(M_0 - M_1(t))}} = \frac{dM_2}{\lambda\sqrt{M_2(t)(M_0 - M_2(t))}}. \tag{3.17}$$

In terms of V and M_a the eq.(3.16) has the form

$$\frac{dV}{V} = -\frac{1}{2}\left(\frac{dM_1}{M_1} + \frac{dM_2}{M_2}\right). \tag{3.18}$$

The integrating the left-hand side over V from V^{in} to V^{out} and the right-hand side, in the same way as for eq.(3.13), yield

$$V^{out} = V^{in}\left(\frac{M_1{}^{in}M_2{}^{in}}{M_1{}^{out}M_2{}^{out}}\right)^{1/2}. \tag{3.19}$$

It is clearly seen that the obtained connection between relative velocities in the incoming and the outgoing channels does not depend explicitly on the drift mass parameter λ or on the friction coefficient k. In fact, this result first has been given in ref.[8]) on the basis of purely kinematic analysis of the mass transfer reaction. However the direct comparison of formulas from [8]) and (3.19) is troubled by the fact that in [8]) this link has been expressed in terms of number of particles entering in the transport process, whereas our result is given through masses of the initial and the final products of the reaction.

Let us turn to the case when the total mass is not conserved. As it was shown in previous section the ions may be pictured as a homogeneous compressible liquid drops. In this approximation the equations of classical dynamics are

$$\frac{dM_1(t)}{dt} = -\lambda_{12}\sqrt{M_1(t)M_2(t)}, \tag{3.20}$$

$$\frac{dM_2(t)}{dt} = \lambda_{21}\sqrt{M_1(t)M_2(t)}, \tag{3.21}$$

$$M_1(t)\frac{d\mathbf{V}_1(t)}{dt} = \mathbf{F}_{21} + \frac{\lambda_{12}}{2}\sqrt{M_1(t)M_2(t)}(\mathbf{V}_1(t) - \mathbf{V}_2(t)), \tag{3.22}$$

$$M_2(t)\frac{d\mathbf{V}_2(t)}{dt} = \mathbf{F}_{12} + \frac{\lambda_{21}}{2}\sqrt{M_1(t)M_2(t)}(\mathbf{V}_1(t) - \mathbf{V}_2(t)), \tag{3.23}$$

In terms of the reduced mass and the relative velocity, eqs.(3.22), (3.23) can be rewritten in the same form as eq.(3.10), however the friction coefficient is

$$k(t) = \frac{\mu(t)}{2}\left(\lambda_{21}\sqrt{\frac{M_1(t)}{M_2(t)}} - \lambda_{12}\sqrt{\frac{M_2(t)}{M_1(t)}}\right), \tag{3.24}$$

Eqs.(3.20) and (3.21) may be represented also as follows

$$\frac{d\sqrt{M_1}}{dt} = -\frac{\lambda_{12}}{2}\sqrt{M_2}, \tag{3.25}$$

$$\frac{d\sqrt{M_2}}{dt} = \frac{\lambda_{21}}{2}\sqrt{M_1}, \tag{3.26}$$

and both can be reduced to

$$\frac{d^2\sqrt{M_\alpha}}{dt^2} = -\omega^2\sqrt{M_\alpha}, \tag{3.27}$$

where

$$\omega = \frac{\sqrt{\lambda_{12}\lambda_{21}}}{2}. \tag{3.28}$$

By taking into account that $M_\alpha(t=0) = M_\alpha{}^{in}$ and $M_\alpha(t=\tau_{int}) = M_\alpha{}^{out}$, we have found that during the cohesion the mass of the ions will evolve as

$$M_\alpha = M_\alpha^{in}\cos^2\omega t - \frac{\sqrt{M_\alpha^{in}M_\alpha^{out}}}{\sin\omega\tau_{int}}\left[1-\sqrt{\frac{M_\alpha^{in}}{M_\alpha^{out}}}\cos\omega\tau_{int}\right]\sin 2\omega t +$$

$$+\frac{M_\alpha^{out}}{\sin^2\omega\tau_{int}}\left[1-\sqrt{\frac{M_\alpha^{in}}{M_\alpha^{out}}}\cos\omega\tau_{int}\right]^2\sin^2\omega t. \tag{3.29}$$

It is easy to check that the connection between V^{out} and V^{in} in this case is given by the expression (3.19) as well, under the conditions mentioned above.

The equation of time evolution of the energy may be obtained from eq.(3.10), after its scalar multiplication by $\mathbf{V}$, i.e.

$$\frac{dE(t)}{dt} = \mathbf{F}\cdot\mathbf{V} - g(t)E(t). \tag{3.30}$$

Here $E(t) = \mu(t)\frac{V^2(t)}{2}$ is the energy of the relative motion and the function $g(t)$ is determined as follows

$$g(t) = \frac{d\ln\mu(t)}{dt} - \left(\lambda_{21}\sqrt{\frac{M_1(t)}{M_2(t)}} - \lambda_{12}\sqrt{\frac{M_2(t)}{M_1(t)}}\right). \tag{3.31}$$

This equation may be used for finding of Q-value of the reaction.

In figs.1 and 2 we have ploted the time dependences of the mass and the friction coefficient for the reactions $^{14}N\ (^{167}Ho, X)Y$, investigated at the K500 cyclotron of MSU [20–22]), where as final products Y one has detected ^{6-8}Li, $^{7,9,10}Be$, $^{8,10-13}B$ and $^{10-14}C$. The calculations have been carried out for the processes with total mass conservation. The mass drift parameter λ have been adjusted from eq.(3.15) and we have found that $\lambda = 10^{21}s^{-1}$. This is in reasonable agreement with estimations of the interaction time collected in ref.[3]). The more general case require the knowledge of three parameters λ_{12}, λ_{21} and τ_{int}. However we may note that if $\lambda_{21} = 3\cdot10^{21}s^{-1}$ and $\lambda_{12} = 0.3\cdot10^{21}s^{-1}$ and the interaction time $\tau_{int} = 0.2 \cdot 10^{-21} s$ are choosen, the time dependence of the mass and the friction coefficient, calculated with aid of (3.29) and (3.24), respectively, have almost the same forms as in figs.1 and 2. It is seen that the smooth change of the projectile mass, i.e. ^{14}N, causes the smooth decrease of the friction coefficient during the interaction time. We stress again that the knowledge of the λ, strongly infuence the period of mass transfer, however the problem of the consistent its calculations is beyond the scope of the presented theory.

4. Conclusions

The principal results of this study are as follows.

i) In the framework of a macroscopical description of two interpenetrating quantal objects we have set up the two-fluid hydrodynamics for investigating the coalescence stage of heavy-ion collisions at intermediate energies. On the basis of the obtained equations we have shown that in grazing processes the colliding nuclei may be modelized to be incompressible liquid drops or a quasi-rigid inelastic species, when the total mass of the fragments in the incoming and the outgoing channels of the reaction is conserved. In peripheral as well as in headon collisions, when the total mass is not preserved, both the target and the projectile to many extant display themselves as two homogeneous compressible liquid drops. These qualitative conclusions are well supported by all the known theoretical and experimental investigations of the grazing dissipative processes.

ii) From the equations of two-liquid hydrodynamics we have derived the self-consistent equations of classical dynamics for the two nuclei the reduced mass of which, due to the mass exchange, before and after reaction is not to be the same. The obtained equations of Newtonian mechanics obey the requirement of invariancy when changing from a right-hand system to a left-hand system in accordance with the statement formulated in ref.[1]). Therefore we may conclude that these equations govern the whole process of the collision. The procedure of obtaining the Newtonian equations permits to write down them in the same form as they have been first introduced heuristically for nucleus-nucleus collisions without particle exchange [1,2]).

iii) We have found also that in the mass exchange reactions the friction force has to be proportional to the first order of the relative velocity of colliding nuclei. The peculiar feature of the mass exchange grazing processes is the fact that the friction coefficient turns out to be a function of time and essentially depends upon the time evolution of mass of the ions. The obtained results are to be justified when the target and the projectile masses do not change. In the latter processes the friction coefficient is a constant. It worthwhile to note that the presented approach allows to study the change of the observable characteristics such as cross section, angular distribution, energy and angular momentum loss, Q-value and so in dependence on number of particles taking part in the transport from one ion to the other. The final results may be represented in terms of the mass and the relative velocity of the ions in the incoming and the outgoing channels of the mass exchange processes.

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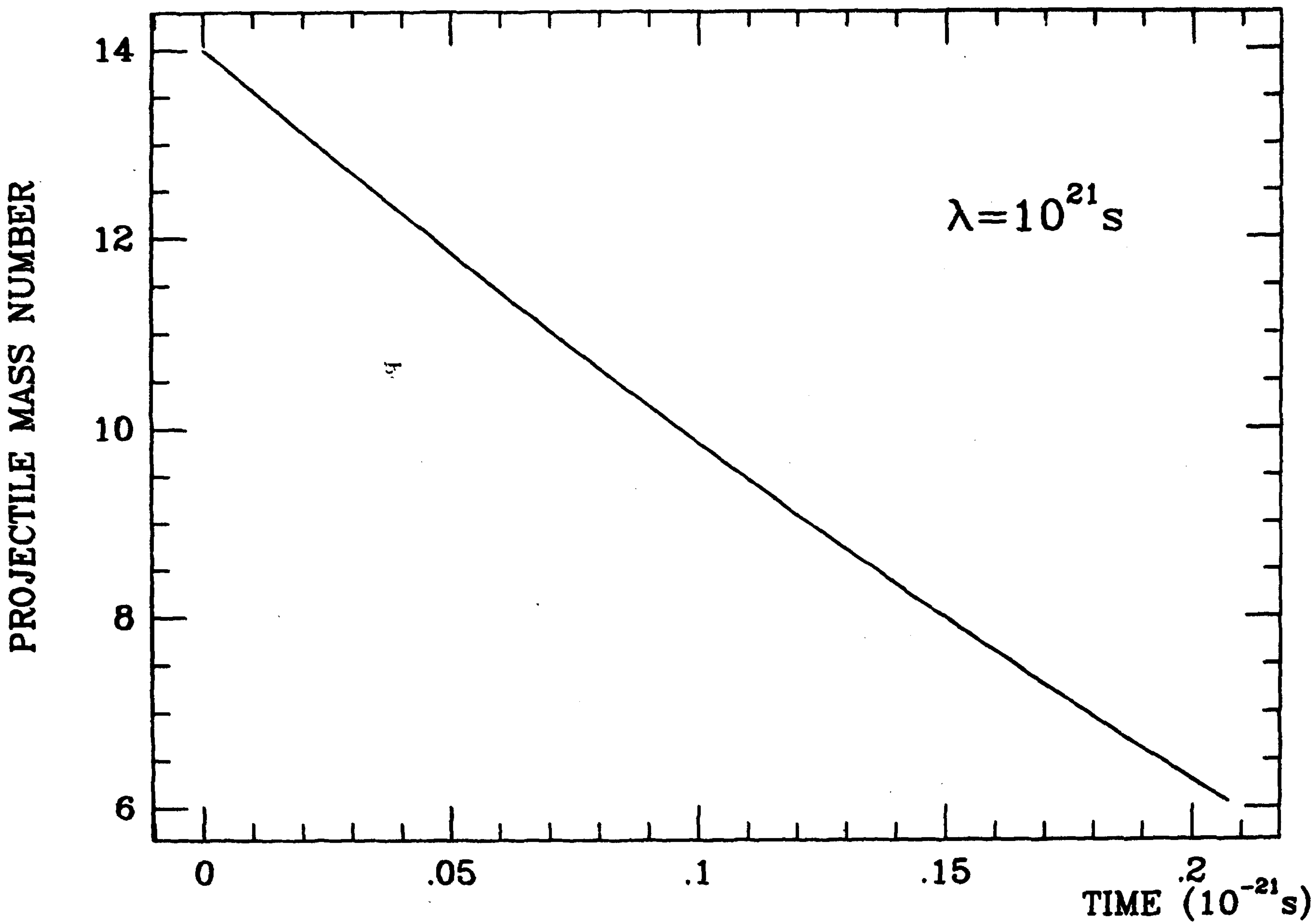

Fig.1. Time evolution of the mass of ^{14}N on collision with ^{165}Ho

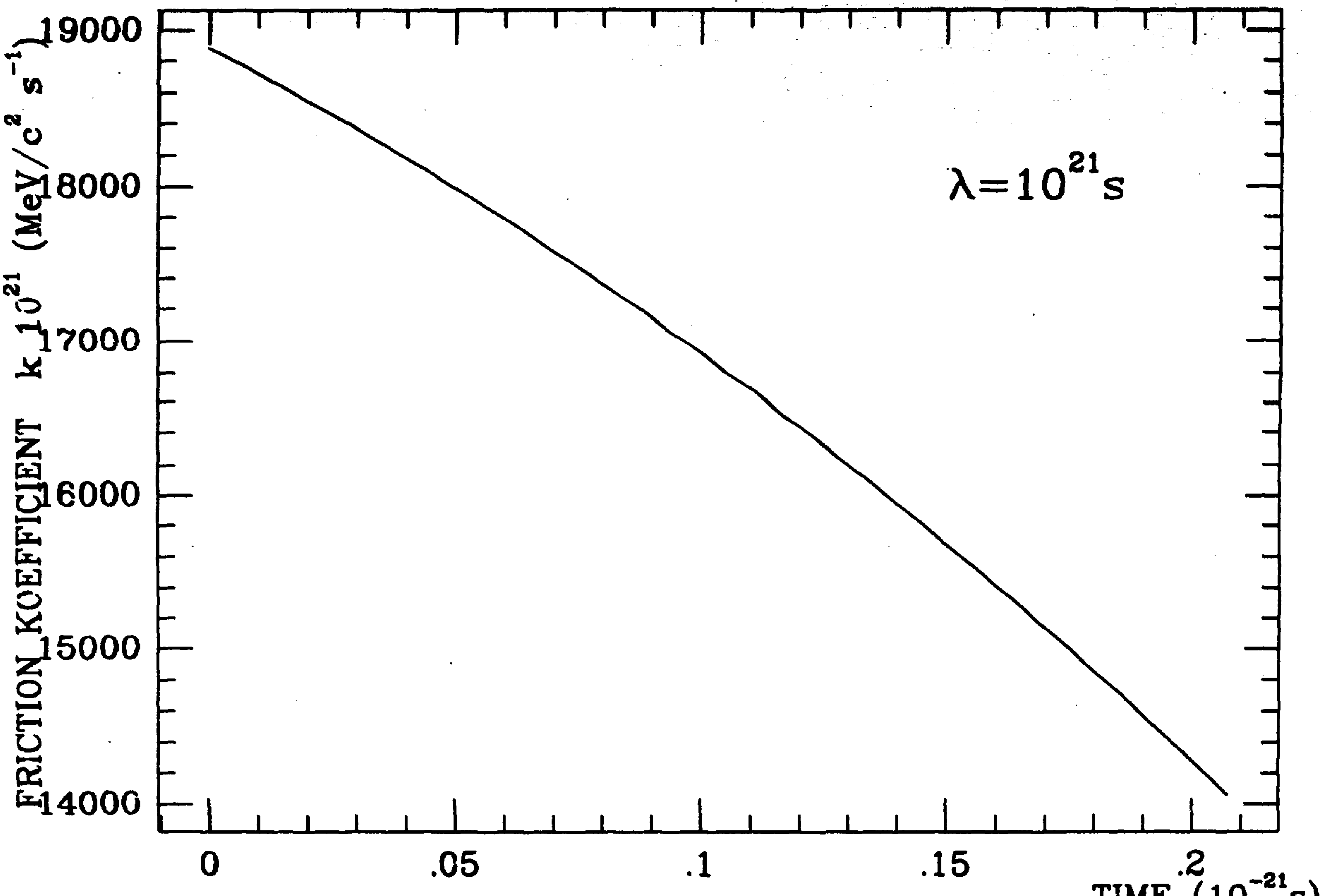

Fig.2. Time dependence of the friction coefficient in reaction $^{14}N+^{165}Ho$

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