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SUPERCONDUCTIVITY IN MIXED BOSON–FERMION SYSTEMS *

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ABSTRACT

The superconductivity of mixed boson–fermion systems is studied using a simple boson–fermion transformation model. The critical temperature of the superconducting transition is calculated over a wide range of the narrow boson band position relative to the Fermi level. The BCS scenario and boson condensation picture are recovered in two limiting cases of high and low positions of boson band, respectively, with modifications due to boson–fermion interaction.

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1. INTRODUCTION

The interest in the superconductivity in the strongly correlated electron systems has been revived by the discovery of the high temperature superconductivity in oxide superconductors[1]. The extensive theoretical investigation of strongly correlated electron systems pioneered by Anderson[2] has revealed the following scenario usually occuring in the one-band models: The strong interaction converts low lying excitations into charged bosons which then form a two-particle condensate. For instance, in the generalized Hubbard model with N-flavours and strong on-site repulsion doping leads to the appearance of scalar Bose excitations (holons). These holons in the same plane repel each other, while holons in adjacent planes attract each other to form a two-particle Bose condensate[3,4]. A similar situation occurs in dimer models of the spin liquid state. In these models holes on different sublattices carry opposite "quasi-charges" [5,6] and interact with each other following the Coulomb law, which results in Bose condensation of "quasi-neutral" molecules consisting of two holes [7]. Thus in all these models a Bose-condensate of doubly charged (w.r.t. a real electrical field) Bose particles is formed.

In more complicated two-band models one should expect that apart from these Bose excitations, a broad band of fermion excitations is also present. According to the conservation law, a single fermion cannot decay into a boson, but two fermions can be transformed into a bosonic molecule. This qualitative picture is consistent with the experimental observation that the usual broad fermion band coexists in cuprates with narrow-band excitations of unclear origin. For instance, it is tempting to ascribe the strong absorption of infrared light in doped cuprates at around 0.3 eV to the interaction with these Bose particles [8].

All the above reasoning justifies a theoretical study of phenomenological models in which a broad band of fermion excitations (which can be identified to oxygen p-orbitals in real materials) coexists with a narrow band of Bose particles with double charges (which can be formed from the subsystem of copper electrons by strong interaction). In the simplest case the interation between Bose and Fermi subsystems can be described as a transformation of two fermions into a boson and vice versa. The behaviour of the system is governed by the strength of the interation and the position of the narrow boson band with respect to the Fermi level. If the interaction is weak and the boson band is above the Fermi level, there will be no bosons in the ground state, but the virtual process of creating bosons from fermion pairs will give rise to a weak attraction between fermions and a BCS-type superconductivity. When the Bose band energy is decreased below the Fermi level, fermions will flow into it forming bosons. In this case a Bose system with fixed number of bosons (governed by the relative position of the boson band to the Fermi level) is formed and superconductivity in the entire system occurs due to the superfluidity of the Bose subsystem. This Bose subsystem differs considerably from the ususal diluted Bose systems because the interaction of bosons with fermions changes the spectrum of the former and leads to their decay.

2. THE MODEL HAMILTONIAN

The strong anisotropy of the high temperature superconductors allows us to consider the two-dimensional model as the zeroth-order approximation. We will limit ourselves to this approximation in the present paper. The model Hamiltonian can be written as

$$H = \int d^2\mathbf{r}\, b^\dagger(\mathbf{r}) \left[-\frac{1}{2m}\frac{\partial^2}{\partial \mathbf{r}^2} + (E - 2\mu) \right] b(\mathbf{r}) + \sum_{\mathbf{k},\sigma} \psi^\dagger_{\mathbf{k}\sigma}\psi_{\mathbf{k}\sigma} (\epsilon(\mathbf{k}) - \mu)$$
$$+ \int d^2\mathbf{r}\, V \left[b^\dagger(\mathbf{r})\psi_\sigma(\mathbf{r})\psi_{-\sigma}(\mathbf{r}) + h.c. \right]$$
$$+ \frac{1}{2}\int d^2\mathbf{r}\, d^2\mathbf{r}_1\, U(\mathbf{r}-\mathbf{r}_1) b^\dagger(\mathbf{r}) b^\dagger(\mathbf{r}_1) b(\mathbf{r}_1) b(\mathbf{r}) \qquad (1)$$

The Hamiltonian of this type was considered earlier [9,10]. We assume that the short-range repulsion $U(\mathbf{r}-\mathbf{r}_1)$ in the last term of (1) leads to a strong repulsion of bosons on the same site. In this case we can replace it by $U\delta(\mathbf{r}-\mathbf{r}_1)$. The energy parameter E is the separation between the bottoms of the boson and the fermion bands. We suppose also that the boson band is only slightly filled so that in the momentum dependence of the energy only the quadratic term $\mathbf{k}^2$ is kept. The third term of (1) describes the interband processes, i.e., the transformation of bosons into fermions and vice versa. The character of the superconducting transition depends on the amplitude of such process V. We suppose it is small compared with the Fermi energy ϵ_F so that an integration over the fermion states can be performed to yield the effective action in the imaginary time given as

$$S = \int d\mathbf{r} \int_0^{1/T} d\tau \left\{ b^\dagger(\mathbf{r},\tau) \left[\frac{\partial}{\partial\tau} - \frac{1}{2m}\frac{\partial^2}{\partial \mathbf{r}^2} + E - 2\mu \right] b(\mathbf{r},\tau) \right.$$
$$\left. + \int d\mathbf{r}_1 \int_0^{1/T} d\tau_1 b^\dagger(\mathbf{r},\tau) b(\mathbf{r}_1,\tau_1) \Pi(\mathbf{r}-\mathbf{r}_1,\tau-\tau_1) + \frac{1}{2} b^\dagger(\mathbf{r},\tau) b^\dagger(\mathbf{r},\tau) b(\mathbf{r},\tau) b(\mathbf{r},\tau) \right\}, \qquad (2)$$

where the polarization operator Π is defined as

$$\Pi(\mathbf{r}-\mathbf{r}_1,\tau-\tau_1) = 4\langle\psi^\dagger_\uparrow(\mathbf{r}_1\tau_1)\psi_\uparrow(\mathbf{r}\tau)\rangle^2 V^2. \qquad (3)$$

Calculating the expectation value (3) as for the idea gas, for $T > T_c$, the superconducting transition temperature, we obtain

$$\Pi(\mathbf{k},\omega) = 4\nu V^2 \pi T \sum_\omega \int \frac{d\Omega_p}{2\pi} \frac{1}{|\omega_1| + i\,\mathbf{pk}/2m}\, \theta(|\omega_1| - |\omega|/2), \qquad (4)$$

where ν is the density of states at the Fermi level, $\omega_1 = 2\pi T(n_1 + 1/2)$, $\omega = 2\pi T n_2$ with n_1, n_2 as integers. For $\omega = 0$ and small values of k, the polarization operator becomes

$$\Pi(k,0) = 4\nu V^2 \left[\ln\left(\frac{\gamma\epsilon_F}{\pi T}\right) - k^2\xi^2 \right], \qquad (5)$$

where

$$\xi^2 = \frac{7\zeta(3)}{16\pi^2 T^2 m^2}\frac{1}{k^2}\int \frac{d\Omega_p}{2\pi} (\mathbf{pk})^2 - \left(\frac{v_F}{T}\right)^2$$

and $\zeta(3)$ Riemann zeta function, $\ln\gamma = 0.57...$ the Euler constant, v_F the Fermi velocity.

The total density of particles $N = N_F + 2N_B$ does not depend on the temperature and is determined by the chemical composition of the compound. Denoting the Fermi energy of system with $N_B = 0$ as $\epsilon_F{}^0$ we find from the particle number conservation that

$$N_B = \nu(\varepsilon_F^0 - \varepsilon_F) = -\nu\,\delta\varepsilon_F \tag{6}$$

On the other hand, the boson density can be found from the boson Green's function as

$$N_B = \langle b^\dagger b \rangle = T\sum_\omega \int \frac{d^2k}{(2\pi)^2} G_B(\omega,k)\, e^{i\omega\delta} \tag{7}$$

with δ approaching zero from the positive side. The boson Green's function G_B itself can be determined from the effective action (2). Formulas (6) and (7) combined yield the equation to determine the chemical potential μ, while the critical temperature T_c for the superconducting transition is the minimal temperature when the equation $\langle b \rangle = 0$ has a solution.

3. TRANSITION TEMPERATURE IN LIMITING CASES

The renormalized chemical potential for bosons μ^* is given by

$$\mu^* = 2\mu - E + \Pi(0,0) \tag{8}$$

For the case of low boson density, μ^* is small, so that the chemical potential for fermions μ will be close to $(E - \Pi(0,0))/2$ and as follows from (6)

$$N_B = \nu(\varepsilon_F^0 - \frac{E - \Pi(0,0)}{2}) \tag{9}$$

For small enough V, the superconducting transition temperature T_c concides with the Bose condensation temperature for a two-dimensional non-ideal gas and is given by [11]

$$T_c = 4\pi N_B / 2m \ln\ln(\frac{1}{N_B a^2}), \tag{10}$$

where a^2 is the area of the unit cell.

Now consider the influence of the boson-fermion interaction upon the transition temperature T_c. We first discuss the case of high transition temperature so that the main contribution to the integral (7) for the boson density comes from the region of small k. In this case equations (6) and (7) can be simplified to yield

$$\nu(\varepsilon_F^0 - \frac{E}{2}) = T\int^{\sqrt{4\pi mT}} \frac{k\,dk}{2\pi} \frac{1}{\frac{k^2}{2m} + \Pi(k,0) + E - 2\mu + \Sigma(k)}, \tag{11}$$

where the self-energy correction $\Sigma(k)$ is due to the hard core boson repulsion. This correction term will lead to the convergence of the integral (11) at small $k \le k_c$. For large $k > k_c$, this correction is small and can be neglected. This charasteritic value k_c can be found from the estimate

$$k_c^2/2m \sim \Gamma, \tag{12}$$

where Γ is the renormalized boson-boson scattering amplitude which plays the role of the effective interaction between bosons at large distance [11]. In two-dimensional systems the scattering amplitude is logarithmic and does not depend on the details of the interaction, namely

$$\Gamma = \frac{m}{\ln(1/na^2)} \tag{13}$$

Inserting this expression into the estimation (12) for k_c we get the cut-off for the logarithmically divergent integral (11). Evaluating this integral with logarithmic accuracy we find

$$4\nu(\varepsilon_F^0 - \frac{E}{2}) = T\int_{(\ln 1/na^2)^{-1}}^{\sqrt{4\pi mT}} \frac{k\,dk}{2\pi} \frac{1}{k^2/2m + \Pi(k,0) - \Pi(0,0)} \tag{14}$$

If the condition

$$\frac{k_c^2}{2m} \gg \Pi(0,0) - \Pi(k_c,0) \tag{15}$$

is satisfied, the expression for the transition temperature (10) is recovered from (12) and (14). In the opposite limit the effective mass of bosons m^* for small momenta $k \ll \xi^{-1}$ is determined by the interaction of bosons with fermion system and is equal to

$$m^* = \frac{1}{8\nu V^2 \xi^2} \tag{16}$$

Equation (11) is transformed in this case into

$$4\nu\left(\varepsilon_F^0 - \frac{E - \Pi(0,0)}{2}\right) = \int_{k_c}^{1/a} \frac{d^2k}{(2\pi)^2} \frac{1}{\exp\{[\frac{k^2}{2m} + \Pi(0,0) - \Pi(k,0)]/T\} - 1}$$

$$= \frac{T m^*}{\pi} \ln\left(\frac{1}{k_c \xi}\right) + \frac{T m}{2\pi} \exp\left(-\frac{\Pi(0,0)}{T}\right)\left| 1 - \exp\left(-\frac{1}{2mTa^2}\right)\right| \qquad (17)$$

Depending on the values of parameters m/m^* and V, either the first or the second term on the right hand side of (17) may appear to be essential.

So far we have considered the limiting case when the boson density N_B is high enough. Upon the increase of the energy E the boson density decreases, and the right hand side of (17) becomes small for $E > 2\varepsilon_F^0$, so that the equation for determining the temperature of Bose condensation turns into the BCS formula, i.e.,

$$\varepsilon_F^0 - \frac{E}{2} + 2\nu V^2 \ln \frac{\gamma \varepsilon_F^0}{\pi T} = 0 \qquad (18)$$

In the limiting case $m \to \infty$ and $m^* \to 0$, (17) reduces to the expression for the transition temperature obtained by the self-consistent field method [10] as given by

$$T_c = \Pi(0,0) / \ln(1/4\pi a^2 N_B). \qquad (19)$$

4. CONCLUDING REMARKS

We have carried out the theoretical study of a two-band model consisting of a narrow bosonic band and a broad fermionic band. Two limiting cases are possible depending on the relative position of the bosonic band w.r.t. the fermion band: If the bosonic level is high compared with the Fermi level, the superconductive transition is mainly driven by the fermion attraction and can be described by the BCS theory. In the opposite case the superconducting transition is due to Bose condensation. These bosons, however, have a unusual spectrum, renormalized by the interaction with fermions. We have obtained an expression for the transition temperature valid in the entire range of the boson band position.

Physical phenomena which could be observed in such systems to differentiate them from the usual superconductors are related to the presence of an additional mode with a small gap. This mode can be described as a relative oscillation of the fermion and boson condsensates. Since this oscillation is neutral as a whole, its gap is not strongly influenced by the long range Coulomb interaction as in the case of the gapless mode in the ordinary superconductors. The concrete physical effects due to this plausible mode remain to be studied.

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