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PARITY VIOLATION AND SUPERCONDUCTIVITY IN DOPED MOTT INSULATORS *

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ABSTRACT

We study parity violating states of strongly correlated two-dimensional electronic systems. On the basis of mean field theory for the SU(2N)-symmetric generalization of the system involved we give the arguments supporting the existence of these states at a filling number different from one-half. We derive an effective Lagrangian describing the long wavelength dynamics of magnetic excitations and their interaction with charged spinless holes. We establish that the ground state of a doped system is superconducting and discuss the phenomenological manifestations of the parity violation.

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I. Introduction

The discovery of high temperature superconductivity (HTSC) has stimulated development of a number of classical problems of the condensed matter theory. Apparently, the problem of the nature of the ground state of strongly correlated electronic systems seems to be the most intriguing. We shall call a system "strongly correlated" if there is such a strong interaction that may be considered as a number of constraints imposed on the dynamical variables. As a consequence, the total set of variables is excessive and there exist gauge transformations keeping the constraints and the Hamiltonian of a system invariant. The gauge theory description of the strongly correlated systems has first been proposed by Anderson and Baskaran [1] and by Wiegmann [2] in the context of the HTSC and Mott insulator theory.

It could be conjectured that asymptotical limit of infinitely strong correlations properties of the ground state and of the low-lying excitations are completely determined by the filling factor ν , which is, by definition, a ratio of the number of particles to the number of quantum states the system. For $\nu = 1/2$ (one particle per site) a sufficiently general example of a strongly correlated system is provided by the so-called Hubbard antiferromagnet. It is described the Hamiltonian for spin operators placed on the sites of a regular lattice

$$\mathcal{H}_m = \sum_{i,j} J_{ij} \vec{S}_i \vec{S}_j + \mathcal{H}_m' \qquad (1.1)$$

where $\mathcal{H}_m'$ includes the fourth and higher order contributions in spin operators; these contributions coincide with the operator structures arising at $\nu = 1/2$ in the t/U-expansion of the standard Hubbard model

$$H = -\sum_{ij} t_{ij} c^{+}_{i\sigma} c_{j\sigma} + U \sum_{i} c^{+}_{i\sigma} c_{i\sigma} c^{+}_{i,-\sigma} c_{i,-\sigma} \tag{1.2}$$

where $c^{+}_{i\sigma}$ is a creation operator for a spin σ electron at a site i and t_{ij} is a hopping matrix. We shall regard the coupling constants J_{ij} and the coefficients in $\mathcal{H}_m'$ as variable parameters. For fillings different from one-half charge transfer processes should be taken into account. Apart from the substitution

$$\vec{S}_i = \frac{1}{2} c^{+}_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta} \tag{1.3}$$

to the Hamiltonian governing the dynamics of spin degrees of freedom we also include a hopping term

$$H_\lambda = -\sum_{ij} t_{ij} (1 - c^{+}_{i,-\sigma} c_{i,-\sigma}) c^{+}_{i\sigma} c_{j\sigma} (1 - c^{+}_{j,-\sigma} c_{j,-\sigma}) \tag{1.4}$$

into the total Hamiltonian of a strongly correlated system.

In the next section, we consider possible ground states of the Hamiltonian (1.2), which could retain their original properties in the vicinity of ν = 1/2 too. Special attention will be paid to the parity violating (flux) ground states [3-6].

In section 3, we construct mean field theory for a SU(2N)-symmetric generalization of the model presented by (1.1), (1.4). In section 4, we derive an effective long wave length action describing magnetic excitations and their interaction with holes (for ν < 1/2). In section 5, we study the pairing of holes in parity violating states with a short range antiferromagnetic order. At last, in section 6, we discuss physical manifestations of parity violation.

2. Symmetry properties of spin disordered ground states

We shall consider topologically rigid ground states

remaining their typical properties at small variations of parameters of a system and define distinct classes of universality in the space of all possible Hamiltonians. First, we shall describe some types of nontrivial magnetic ordering which arise as the mean field solutions, and which are the likely candidates for the role of the ground state [7].

It could be shown for the minimal Heisenberg model (J_{ij}=J > 0 for i,j nearest neughbour only, $\mathcal{H}_m'$ = 0) that at zero temperature the Neel long range order (LRO) takes place if a site spin S is large enough (S > 1 for a square and S > 3/2 for a triangular lattice [8]). For the problem of magnetic CuO lauers in HTSC materials the case of a S = 1/2 spin square lattice antiferromagnet is the most interesting. Although there are no exact data numerous computer simulations point to the existence of the Neel LRO, which is cherecterized by the local magnetization $|\langle \vec{S}_i \rangle| \simeq 0.3$ [9]. This brings us to the conclusion that in the case of the minimal Heisenberg model quantum fluctuations are not strong enough to destroy the LRO. Quantum fluctuations become stronger when the coordinate space dimension, or the spin S magnetude, or the number of nearest neighbour sites being not nearest neighbours for each other decreases. For a given S=1/2 spin square lattice case then LRO could be destroyed if the frustrating interactions of nonnearest neighbours is introduced. Adding an antiferromagnetic coupling of next nearest neighbours to the minimal Heisenberg exchange interaction we obtain a model [6,10]

$$H = J \sum_{n.n.} \vec{S}_i \vec{S}_j + J' \sum_{n.n.n.} \vec{S}_i \vec{S}_j \tag{2.1}$$

At some value of J$'$ the Neel LRO breaks down (classically,

$J'_c = 1/2J$). In a classical system a helicoidal order arises at $J'>J'_c$. Quantum fluctuations produce a disordered state ($\langle\vec{S}_i\rangle=0$ for any site i) in the range of the coupling constant J'; the width of this range being O (exp(-S*const)) [10]. Introducing higher order terms into the Hamiltonian (2.1) we could single out the ground state favour one disordered ground out of a number of similar competative disordered states.

Classification of disordered spin states is conventionally done in terms of possible types of spin rotation symmetry proprerties of the two-point correlation function $\langle S_i^a S_j^b \rangle$ allow for a special "spin nematic" magnetic ordering [11]. Whereas the three-point correlation function $\langle S_i^a S_j^b S_k^c \rangle$ is needed to characterize "scalar" and "tensor" magnetic ordering types [12].

It has been believed for a long time that the two-dimensional quantum magnetism could be described in the same fashion as the three-dimensional magnetism. In the conventional approach the possibility of the wave functions to have a nontrivial quantum U(1) holonomy has not been considered. This specifically two-dimensional symmetry property could be found through averages of substantially nonlocal functionals of spin operators. We shall below give some typical examples of these functionals.

At the same time the existence of the quantum holonomy reveals itself in the transformation rules of the ground state under the action of the translations $T_{\vec{a}}$ on arbitrary lattice vectors $\{\vec{a}\}$ supplemented by the time reversal R. The simplest disordered state is invariant with respect to any generalized translation $\mathcal{T}_{\vec{a}} = T_{\vec{a}}R$. It has been shown in [7] that the properties of such a state enable one to treat it as a quantum paramagnetic state [2]. At the same time it could be considered as the original homogeneous RVB state conjectured by Anderson [13].

Another example is a state invariant only with respect to some subgroup of all $\mathcal{T}_{\vec{a}}$ translations. This is the dimerized or "the molecular crystal" state, built from solid (nonresonating) singlet valence bonds [6].

The nontrivial holonomy is peculiar for the states forming projective irreducible representations of the "finite Heisenberg group" [14], the generators $\mathcal{T}_{\vec{a}}$ being "magnetic translations":

$$\mathcal{T}_{\vec{a}}\mathcal{T}_{\vec{b}} = e^{i\Phi[\vec{a},\vec{b}]}\mathcal{T}_{\vec{b}}\mathcal{T}_{\vec{a}} \qquad (2.2)$$

where $\vec{a}$ and $\vec{b}$ are the vectors of some two-dimensional lattice. The phase Φ plays the role of a fictitious "magnetic" field flux (the flux of frustrations) through an elementary plaquette of a lattice. At a time reversal and a coordinate two-dimensional space parity inversion ((x,y) --> (-x,y)) the flux Φ changes its sign. It means that the considered "flux" state isn't invariant under these discrete transformations. As has been shown in [7] the flux state is sort of a quantum spin liquid [2]. To reveal the holonomy of the flux state it is convenient to introduce an average value of the operator of a cyclic spin permutation around some closed contour C on a lattice. This operator could be represented as a product of consecutive interchanges of nearest neighbours and for the s=1/2 spin case it has the form $\prod_{\langle ij\rangle\in C}\left(\frac{1+4\vec{S}_i\vec{S}_j}{2}\right)$. For the quantum paramagnetic states as well as for the dimerized states the average value of the "Wilson loop"

$$W(C) = \left\langle \prod_{(i,j)\in C} \frac{1+4\vec{S}_i\vec{S}_j}{2} \right\rangle \tag{2.3}$$

obeys an "area law"

$$W(C) = \exp\left(-S(C)/\xi^2\right) \tag{2.4}$$

Here S(C) is an area bounded by C and ξ is some correlation length (in lattice units).

In contrast, for the flux state the Wilson loop exhibits a "perimeter law"

$$W(C) \sim \exp\left(-L(C)/\xi + i\Phi S(C)\right) \tag{2.5}$$

where L(C) is the length of C and the flux Φ is given by the composition rule (2.2). The phase $\Phi S(C)$ could be interpreted as a solid angle subtended by the classical spin vectors belonging to the contour C. It is a Berry phase of the spin wave functional emerging at permutation of spins around a closed contour C and exhibition a nontrivial U(1) holonomy.

For fillings ν different from one-half one could conjecture that the ground state of the Hamiltonian $H_m + H_h$ (see (1.1) and (1.4)) could also be parity breaking. Then (2.3) obeys the same perimeter law (2.5) in this state where instead of $\vec{S}_i$ one should use the substitution (1.3). In seems likely that for an appropriate choice of $\mathcal{H}_m$ the parity violating ground state of an undoped system does not break down at small doping. At the same time one could speculate that doping does breaks down the Neel LRO and produces the parity violating flux phase [2,6]. We do not intend to study this

question here, but simply note, that a contribution analogous to (2.5) appears also in a contour average.

$$B(C) = tr\left\langle \prod_{i\in C}\left(\frac{1+2\vec{\sigma}\vec{S}_i}{2}\right)\right\rangle \tag{2.6}$$

where the trace is taken the indices of the $\vec{\sigma}$ -matrices. The spin operator functional (2.6) is a Berry phase accompanying a hole rotation around C. A virtual hole rotation around a plaquette is a local source of the frustration flux.

In the next section we shall bring some arguments in favour of the existence of a parity ground state for a doped case.

3. Mean field theory for doped Mott insulators

Strictly, the conclusions based on any mean field theory of a real SU(2)-symmetric system are only qualitatively reliable. To have a reliable calculational scheme, we shall introduce an additional parameter, which is a large 2N rank of the symmetry group SU(2N). In other words we will consider a model, generalizing (1.1), (1.4) for 2N-coloured fermions c_{in} (n=1,...,2N) [7,15,16]. At a one-half filling we have a model of a generalized SU(2N) antiferromagnet. A generator T_{nm} of the SU(2N) group in a completely antisymmetric N degree representation is placed on each lattice site. For filling ν < 1/2 we allow for the states with no more than N fermions per site. That is, we have on each site one of the representations [N], [N-1],..., [0], where [M] denotes a representation given by the Young scheme consisting of a column of the length M. To generalize the S > 1/2 spin case for the SU(2)-symmetry group one could attribute an additional 2S-valued "flavour" index. Then we would obtain a representation [M, 2S] given by the rectangular Young scheme having M rows of the length 2S. Hamiltonian of the generalized model is

$$H = -\sum_{i,j} t_{ij} \Pi c^+_{in} c_{jn} \Pi + \sum_{i,j} J_{ij} T_i^{ab} T_j^{ba} + \dots \tag{3.1}$$

where the projector Π restricts the values of the occupation numbers of sites to less than (N+1) and the dots stand for higher order terms in T_{nm}. It is convenient to use a fermionic representation for SU(2N) generators

$$T_i^{nm} = c^+_{in} c_{im} \tag{3.2}$$

For the restrictions imposed by the projector Π to hold we introduce a neutral fermion Ψ_{in} and a SU(2N) singlet "slave" boson b_i

$$c_{in} \Pi = \Psi_{in} b_i^+ \tag{3.3}$$

which are subject to the local constraint

$$\sum_{n=1}^{2N} \Psi^+_{in} \Psi_{in} + b_i^+ b_i = N \tag{3.4}$$

Note that b_i is not a standard boson, but rather a linear combination of N bosons required to represent the root vectors of the SU(2N) group E_α [17]. The hopping term in (3.1) could be expanded in the Hubbard generators X^{ab}, where a, b = 1,...,, ,..$\sum_{M=0}^{N}$ dim [M]. The set of X^{ab} generators could be considered as a Z_2-graded superalgebra and odd generators could be represented by products of 2N-coloured fermions and root vectors E_α. In this turn, root vectors admit a bosonic representation of the generalized Holstein-Primakoff type. It is remarkable that for the hopping term in (3.1) one should have only a special diagonal combination $\sum_c X_i^{ac} X_j^{cb}$, which could be expressed by the single boson field b_i. The overall description of a generalized "slave"-boson representation will be given elsewhere [18].

In terms of the Ψ_n and b fields the Hamiltonian (3.1)

takes a form

$$\mathcal{H} = -\sum_{ij} t_{ij} \Psi^+_{in} \Psi_{jn} b_j^+ b_i - \sum_{ij} J_{ij} \Psi^+_{in} \Psi_{jn} \Psi^+_{jm} \Psi_{im} + \dots \tag{3.5}$$

In what follows we shall consider a two-dimensional bipartite lattice. For one-half filling quanum fluctuations destroy the two-sublattice staggered (Neel) LRO at N >> 1. It is evident from the fact that the energy of the Neel state is E $\sim$ -N, whereas for any singlet valence bound state E$\sim$ -N^2 [6]. Valence bound states could be characterized by a bound variable

$$\Delta_{ij} = \Psi^+_{in} \Psi_{jn} \tag{3.6}$$

In contrast to the Neel order parameter $(-)^i \Psi^+_{in} \Psi_{im}$, fluctuations around some mean field configuration of Δ_{ij} are suppressed at large N.

For arbitrary filling it is consistent to use the field Δ_{ij} to perform the Hubbard-Stratonovich transformation and to linearize the fermionic Hamiltonian (3.5). After this procedure we obtain a Lagrangian [19]

$$\begin{aligned}\mathcal{L} = &-\sum_i \Psi^+_{in} \partial_0 \Psi_{in} - \sum_i b_i^+ \partial_0 b_i - \sum_{ij} \Big(t_{ij} b_j^+ b_i \Delta_{ij} + \\ &+ J_{ij} \Psi^+_{in} \Psi_{jn} \Delta_{ij} + J_{ij} |\Delta_{ij}|^2 + \frac{t_{ij}^2}{J_{ij}} b_i^+ b_i b_j^+ b_j \Big) - \mathcal{H}'_m(\Psi^*) + \\ &+ \sum_i i\, a_0(i) \left(\Psi^+_{in} \Psi_{in} + b_i^+ b_i - N \right) + \mu \sum_i b_i^+ b_i \end{aligned} \tag{3.7}$$

where a_0 (i) is a Lagrange multiplier responsible for the condition (3.4) and μ is a chemical potential of holes.

The Lagrangian (3.7) is invariant under local U(1)-transformations:

$$\Psi_{in} \to e^{i\varphi(i)} \Psi_{in}, \quad b_i \to e^{i\varphi(i)} b_i, \tag{3.8}$$
$$\Delta_{ij} \to e^{i(\varphi(i)-\varphi(j))} \Delta_{ij}, \quad a_0(i) \to a_0(i) + i\, \partial_0 \varphi(i)$$

In contrast with the local U(1) symmetry found in [1] the invariance under the transformations (3.8) holds at any filling. Note that instead of (3.7) a modified supersymmetric mean field theory apart from Δ_{ij} involving bilinears $\psi^+_{in} b_j$, $b_i^+ \psi_{jn}$ and $b_i^+ b_j$ could be considered. It will be made elsewhere [18]. The ground state Δ_{ij} configuration could be defined by an effective potential $V(\Delta)$, determined as

$$e^{-\int d\tau\, U(\Delta)} = \int \mathcal{D}\psi_n \mathcal{D}\psi_n^+ \mathcal{D}b\mathcal{D}b^+ e^{\int \mathcal{L} d\tau}$$

Performing integration over fermions, we obtain a contribution

$$U_F(\Delta) = \sum_{i,j} J_{ij} |\Delta_{ij}|^2 + 2N \sum_\lambda \varepsilon_\lambda \theta(\varepsilon_F - \varepsilon_\lambda) + \mathcal{H}_m'(\Delta) \qquad (3.9)$$

In (3.9) are eigenvalues of the operator

$$H_{ij} = i\alpha_0(i)\delta_{ij} + \Delta_{ij} J_{ij} \qquad (3.10)$$

and ε_F denotes the highest occupied energy level at a given filling ν.

Due to the symmetry (3.8) any Δ_{ij} configuration should be characterized in a gauge invariant way. The overall description is given by the values of $|\Delta_{ij}|$ for all bonds and a set of phase circulations (fluxes)

$$\Phi_c = \mathrm{Im}\ \ln \prod_{\langle ij \rangle \in C} \Delta_{ij} \qquad (3.11)$$

of the products of Δ_{ij} taken over closed contours C.

We shall describe any distribution of fluxes by means of a configuration $\bar{\Delta}_{ij}$ which is specified up to an arbitrary gauge transformation. For a periodical distribution of fluxes the Δ_{ij} configuration could also be chosen as periodical.

We shall call the periodicity region as a "magnetic" elementary cell.

The mean field solution also contains the mean value of the Lagrange multiplier field $\alpha_0(i) = (-1)^i m$. The quantity m determines a gap in the spectrum of triplet magnetic excitations ("spin waves"). It has been assumed in [7] that by analogy between a long wave length limit of the theory (3.7) and 2N-flavour QED_{2+1} the mass

$$m \sim J \exp(-N \cdot \mathrm{const}) \qquad (3.12)$$

is dynamically generated. This is a chiral symmetry breaking phenomenon in multiflavour 2+1-dimensional QED [20].

At one-half filling different configurations of $\bar{\Delta}_{ij}$ could be identified with the SU(2N) generalized preferable ground states described in the previous section. The distributions with zero total flux correspond to quantum paramagnetic states. For the simplest case of a homogeneous RVB state $\bar{\Delta}_{ij}$ is real at all bonds and the mean field spectrum is $\varepsilon(\vec{k}) = \Delta \sum_{\vec{\mu}} \exp(i\vec{\mu}\vec{k})$, where $\vec{\mu}$ are the primitive lattice vectors and $\vec{k}$ takes values in a whole Brillouin zone ($-\pi \le k_x, k_y \le \pi$). In dimerized states for any given site i $\bar{\Delta}_{ij}$ is non-zero on only one of the bonds going out of this site. There is no gapless modes in this state: $\varepsilon(\vec{k}) = \Delta$.

In a homogeneous flux state the flux Φ is related to each plaquette P. Its value is the same as the one of the phase in the composition rule for "magnetic translation" operators (2.2). The mean field spectrum ε_λ is given by a solution of the problem of a charged particle moving on a lattice in an external magnetic field Φ. This ancient quantum mechanical problem is often called the Hofstadter problem [21].

The energy spectrum is regular at rational values of the flux: $\Phi = 2\pi\, p/q$, where integers p and q have no common factors. The spectrum consists of q distinct bands, each band being q-fold degenerate. The wave vector $\vec{k}$ takes values in a reduced Brillouin magnetic zone whose volke is $(2\pi)^2/q$. The mass (3.12) only determines the boundaries of the spectrum which for even q consists of q/2 positive and q/2 negative energy bands $\mathcal{E}_\lambda = \pm\sqrt{m^2 + \xi_\alpha^2(\vec{k})}$, where $\xi_\alpha(\vec{k})$, $\alpha = 1,\dots,q/2$ is the spectrum of a positively defind operator $D_{\alpha\alpha'} = \sum_\beta \Delta_{\alpha\beta}\Delta_{\beta\alpha'}$. The level structure has a certain ultrametricity property: quantum numbers of the levels vary drastically at any small shift of a filling number ν, but the total energy as a function of ν changes slightly.

For arbitraty fillings the flux state has lower values of the effective potential $\mathcal{U}$ (3.9) than the homogeneous RVB state at least at $\mathcal{H}_m' = 0$. This follows from the remarkable fact stated in [22] that the total energy $E(\nu, \Phi)$ of the system of spinless noninteracting fermions with density occupying levels of the Hofstadter problem has the minimum at a flux

$$\Phi = 2\pi\nu z \tag{3.13}$$

Here z is a structural factor which is equal to 1,2 and 1/2 for a square, hexagonal and triangular lattice respectively and is a number of lattice sites belonging to a plaquatte.

There is a large class of higher order spin interactions which stabilize the flux state. One example which is obvious at $\nu = 1/2$ has been suggested in [6]. Its SU(2N)-generalized is a sixth order term which corresponds to the coupling of sites shown in fig.1 :

$$H_m' = -J'\, tr(T_1 T_2 T_3)\, tr(T_4 T_5 T_6) + \dots \tag{3.14}$$

with $J' > 0$ and the dots denote the contribution of all other triangles of the given plaquettes. We should note that although for one-half filling the flux Φ equals π, the ground state is quite different from the so-called "s+id"-phase. The latter could be described by a distribution of Δ_{ij} with $\prod_{ij\in P} \Delta_{ij} = -|\Delta|^4$ [15,23], but in this state the parity is actually not violated. The flux through any triangle formed by three sites of a plaquette vanishes at the "s+id"-phase, but equals $\pi/2$ at the considered flux state.

The total effective potential also includes the contribution $\mathcal{U}_B(\Delta)$ resulting from the integration over the bosonic field b_i. The density of bosons is equal to $\rho = N(1-2\nu)$ $(0 < \rho < 1)$. At $\rho \ll 1$ $\mathcal{U}_B$ has the following expansion

$$\mathcal{U}_B(\Delta) = \min \mathcal{E}_\lambda' \rho + O(\rho^2) \tag{3.15}$$

where $\min \mathcal{E}_\lambda'$ is the lowest eigenvalue of the Hofstadter problem for the Hamiltonian operator $\mathcal{H}_{ij}' = t_{ij}\Delta_{ij} + i\alpha_0(i)\delta_{ij}$ with Φ given by (3.13) and the interaction effects lead to the term $O(\rho^2)$. We shall concentrate on the case $|t_{ij}| \lesssim J_{ij}$. The motion of holes does not frustrate a spin system significantly and second nonlocal interaction of holes of the strength t_{ij}^2 / J_{ij} is weak. Then we arrive at the conclusion that for ρ small enough ($\rho < \rho_c(t/J)$) the ground state distribution of fluxes is determined by the minima of $\mathcal{U}_F(\Delta)$. By tuning the parameters of the Hamiltonian H'm it is possible to render one of the states mentioned above favourable. For a pure "t-J model" ($H_m' = 0$) the demerized state is, likely, favourable for large N [6,16]. In what follows we shall suppose that the Hamiltonian H_m' is closen in such a way that in the vicinity of the one-half filling the ground state is a parity breaking flux state with Φ given by (3.13). Similar conjecture has been in a series of other papers [17,24].

4. Effective action for doped Mott insulators

In this section we shall derive an effective action which describes the long wave length dynamics of the Mott–Hubbard-like system. To do this we should perform integration over the short wavelength degrees of freedom. This renormalization procedure will be carried out in the so-called "coherent states representation" [25]. Here we shall confine ourselves to the physical case of the SU(2) symmetry group and a biparticle lattice. The case of an arbitrary rank symmetry group will be presented elsewhere [18].

We introduce, following [2], the basis of states of a site i if a double occupation is forbidden: $|+_i\rangle$, $|-_i\rangle$ - for a fermion with up (down) spin and $|0_i\rangle$ - for a hole. Hubbard generators $X^{ab} = |a\rangle\langle b|$, where a, b = +, -, 0 form a Z_2-graded ortho–simplectic superalgebra osp (1/2)

$$\{X^{ab}, X^{cd}\}_{\pm} = \delta^{bc} X^{ad} \pm \delta^{ad} X^{cb} \tag{4.1}$$

where the bracket becomes an anticommutator if both X's are Z_2-odd ($X^{0,\pm}$, $X^{\pm 0}$), but is a commutator in any other case.

Any state of a system is represented by an action of a direct product of group elements ascribed to different sites on a vacuum state $|\Psi_0\rangle$:

$$|\Psi_G\rangle = \prod_i G_i |\Psi_0\rangle \tag{4.2}$$

We choose the vacuum state $|\Psi_0\rangle$ to be the Neel state without holes

$$|\Psi_0\rangle = \prod_{a\in A} |+_a\rangle \otimes \prod_{b\in B} |-_b\rangle \tag{4.3}$$

where A and B are two sublattices.

In this representation the Lagrangian is

$$\mathcal{L} = -\langle G|\frac{\partial}{\partial t}|G\rangle - \langle G|H|G\rangle \tag{4.4}$$

It can be rewritten as

$$\mathcal{L} = -\,\mathrm{str}\left(PG^{-1}\frac{\partial}{\partial t}G\right) - \mathrm{str}\left(HGPG^{-1}\right) \tag{4.5}$$

where $P = |\Psi_0\rangle\langle\Psi_0|$.

The osp (1/2) invariant renormalization procedure is performed by the substitution G ---> $\tilde{G}g$, where the supermatrix g contains the "fast" degrees of freedom and $\tilde{G}$ contains the "slow" ones. This division is correct if the correlation radius R_c in system is large. For large N we see from (3.12) that $R_c \sim 1/m \gg 1$. Form N=1 the procedure is formally legitimate in the vicinity of the zero-temperature phase transition into a disordered state.

Acting on the vacuum state by the group element G we obtain an orbit manifold $Q = GPG^{-1}$. It is a homogeneous superspace and the coordinates introduced on this space are physical fields. By construction, the matrix Q satisfies the conditions $Q^2 = Q$ and strQ=1. Then it can be parametrized by two SU(2)-doublets: z_α (boson) and χ_α (fermion) and its expansion in the generators X^{ab} is

$$Q = z_\alpha \bar{z}_\beta (1 - \chi^+_\gamma \chi_\gamma) X^{\alpha\beta} + \chi^+_\alpha X^{0\alpha} + X^{\alpha 0}\chi_\alpha - \chi^+_\gamma \chi_\gamma X^{00} \tag{4.6}$$

where

$$\bar{Z}_\alpha Z_\alpha = 1 \tag{4.7}$$

and

$$\epsilon_{\alpha\beta} Z_\alpha \chi_\beta = 0 \tag{4.8}$$

The bosonic field represents an average value of the spin operator at a given site:

$$\langle G | \vec{S}_i | G \rangle = \frac{1}{2} \vec{m}_i = \frac{1}{2} \bar{Z}_\alpha \vec{\sigma}_{\alpha\beta} Z_\beta \tag{4.9}$$

and a fermionic field describes holes

The constraint (4.8) permits an interpretation that a hole carries charge but not a spin, or that its spin is uniquely determined by spins surrounding a hole.

In terms of Z_α and χ_α the Lagrangian of our model has the form

$$\mathcal{L} = -\sum_i \chi_\alpha^+ \frac{\partial}{\partial\tau} \chi_\alpha - \sum_i \bar{Z}_\alpha \frac{\partial}{\partial\tau} Z_\alpha + \sum_{ij} t_{ij} \chi_{i\alpha}^+ \chi_{j\alpha} - \dots \tag{4.10}$$

$$- \sum_{ij} J_{ij} |\bar{Z}_{i\alpha} Z_{j\alpha}|^2 (1 - \chi_{i\gamma}^+ \chi_{i\gamma})(1 - \chi_{j\delta}^+ \chi_{j\delta}) + \dots$$

After the substitution $G \dashrightarrow \tilde{G}g$ we obtain an osp(1/2)-covariant expression

$$\mathcal{L} = -\sum_i \mathrm{str}\left(q_i \tilde{G}_i^{-1} \frac{\partial}{\partial\tau} \tilde{G}_i\right) - \sum_{ij} \mathrm{str}\left(\tilde{G}_i q_i \tilde{G}_i^{-1} \Lambda_{ij} \tilde{G}_j q_j \tilde{G}_j^{-1} \Lambda_{ji}\right) + \dots \tag{4.11}$$

where $q_i = g_i P_i g_i^{-1}$ and

$$\Lambda_{ij} \otimes \Lambda_{ji} = J_{ij} \; \hat{\mathbb{1}}_B \otimes \hat{\mathbb{1}}_B + \frac{1}{2} t_{ij} \left(\hat{\mathbb{1}}_F \otimes \hat{\mathbb{1}}_B + \hat{\mathbb{1}}_B \otimes \hat{\mathbb{1}}_F\right),$$

$$\mathbb{1}_B = \begin{pmatrix} 1 & \\ & 1 \\ & & 0 \end{pmatrix}, \qquad \mathbb{1}_F = \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix}$$

in this basis of states $\{ |a\rangle \}$ $(a = 0, \pm)$ introduced above. We then extract from (4.11) a part L_{gr}, which depends on gradients of the slow variables in a continuous limit. It can be

written in terms of a vector superfield

$$\mathcal{A}_\mu = -i\left\{(Z^+ d_\mu Z)_{\alpha\beta} X^{\alpha\beta} + X^{0\alpha} Z^+_{\alpha\beta} d_\mu \chi_\beta + h.c. + \chi_\gamma^+ d_\mu \chi_\gamma X^{00}\right\} \tag{4.12}$$

The quaternionic matrix $Z_{\alpha\beta} = \begin{pmatrix} z_1 & z_2 \\ -\bar{z}_2 & \bar{z}_1 \end{pmatrix}$ in (4.12) is an element of the SU(2) group in the fundamental representation.

The effective Lagrangian is given by the expansion

$$\mathcal{L}_{eff}(\mathcal{A}_\mu) = \langle \mathcal{L}_{gr} \rangle + \frac{1}{2} \left\langle \mathcal{L}_{gr} \int_0^\beta \mathcal{L}_{gr}(\tau)\, d\tau \right\rangle + \dots \tag{4.13}$$

where the brackets denote a functional averaging over the fast variables $q_i(\tau)$:

$$\langle q_i^{ab}(\tau) \dots q_j^{cd}(0) \rangle = \langle\langle X_i^{ab}(\tau) \dots X_j^{cd}(0) \rangle\rangle \tag{4.14}$$

the double brackets standing for an exact irreducible correlation function. To obtain an expansion up to the fourth order in gradients, one should keep the following terms in (4.13)

$$\mathcal{L}_{eff} = -\sum_i \chi_{i\alpha}^+ \frac{\partial}{\partial\tau} \chi_{i\alpha} - S \sum_i (-)^i \tilde{\bar{Z}}_{i\alpha} \frac{\partial}{\partial\tau} \tilde{Z}_{i\alpha} - \sum_{ij} \rho_{ij} A^+_{ij} A^-_{ij} +$$

$$+ \int_0^\beta d\tau \sum_{ij,kl} \Pi_{ij,kl}(\tau) A_{ij}(\tau) A_{kl} + \sum_{ij} t_{ij} \chi_{i\alpha}^+ \chi_{j\alpha} + \int_0^\beta d\tau \sum_{ijk} R_{ijk}(\tau) A_{ij}(\tau) \left(\bar{Z}_{k\alpha} \frac{\partial}{\partial\tau} Z_{k\alpha}\right) \tag{4.15}$$

where $Z_{i\alpha}$ and $\chi_{i\alpha}$ parametrize $\tilde{Q}_i = \tilde{G}_i \left(\frac{P_+ - P_-}{2}\right) \tilde{G}_i^{-1}$. The composite fields

$$A^+_{ij} = -i \tilde{\bar{Z}}_{i\alpha} \tilde{\bar{Z}}_{j\beta} \epsilon^{\alpha\beta}, \quad A^-_{ij} = (A^+_{ij})^*$$

$$e^{iA_{ij}} = \tilde{\bar{Z}}_{i\alpha} \tilde{Z}_{j\alpha} \tag{4.16}$$

form an SU(2) triplet. The coefficients of the expansion (4.15) are determined by the short-range processes and are given by the formulae

$$\rho_{ij} = J_{ij} \; \mathrm{str} \langle\langle (\mathbb{1}_B X_i(0) \mathbb{1}_B X_j(0)) \rangle\rangle, \tag{4.17}$$

$$\Pi_{ij,k\ell}(\tau)=\frac{1}{2}J_{ij}J_{k\ell}\,Str\langle\langle \mathbb{I}_B X_i(\tau)\mathbb{I}_B X_j(\tau)\mathbb{I}_B X_\ell(0)\mathbb{I}_B X_k(0)\rangle\rangle \tag{4.18}$$

$$R_{ij,k}(\tau)= J_{ij}\,Str\langle\langle \mathbb{I}_B X_i(\tau)\mathbb{I}_B X_j(\tau)\mathbb{I}_B X_k(0)\rangle\rangle + \tag{4.19}$$

$$+t_{ij}\,Str\langle\langle \mathbb{I}_B X_i(\tau)\mathbb{I}_F X_j(\tau)\mathbb{I}_F X_k(0)\rangle\rangle$$

$$\tilde{t}_{ij}= \frac{1}{2}\sum_{\ell,k} t_{i\ell}t_{kj}\int_0^\beta d\tau\, Str\langle\langle \mathbb{I}_B X_i(\tau)\mathbb{I}_F X_\ell(\tau)\mathbb{I}_B X_k(0)\mathbb{I}_F X_j(0)\rangle\rangle \tag{4.20}$$

The value of the coefficient S is determined by the intermediate asymtotics of the correlation function

$$Str\langle\langle \mathbb{I}_B X_i(0)\mathbb{I}_B X_j(0)\rangle\rangle \to (-1)^{|i-j|}S^2$$

if $1 \ll i-j \ll R_c$.

To obtain the continuous limit of Expression (4.15) we should take into account the existence of a short-range Neel order extending up to the distances of the order of R_c.

In terms of $\tilde{Z}_{i\alpha}$, parametrizing $\tilde{a}_i$, the constraint (4.8) has different form for A and B sublattices

$$\epsilon_{\alpha\beta}\tilde{Z}_{i\alpha}\tilde{\chi}_{i\beta}=0, \quad i\in A$$
$$\bar{\tilde{Z}}_{i\alpha}\tilde{\chi}_{i\alpha}=0, \quad i\in B \tag{4.21}$$

Then we should introduce two fermion species which correspond to the A and B sublattices, respectively

$$\tilde{\chi}_{i\alpha}=\begin{cases}\tilde{Z}_{i\alpha}u_i, & i\in A\\ \epsilon_{\alpha\beta}\bar{\tilde{Z}}_{i\beta}v_i, & i\in B\end{cases} \tag{4.22}$$

We should also divide the fluctuations of slow variables $Z_{i\alpha}$ into antiferromagnetic $\vec{n}=\frac{1}{2}(\vec{m}(a)-\vec{m}(b))$ and ferromagnetic $\vec{\ell}=\frac{1}{2}(\vec{m}(a)+\vec{m}(b))$ vectors. Taking the continuous limit we obtain from (4.15)

$$\mathcal{L}_{eff}=\int d^2x\,\Big\{-\chi^+\Big(\frac{\partial}{\partial\tau}+i\tau^3A_0\Big)\chi - S\Big([\vec{n},\frac{\partial}{\partial\tau}\vec{n}]\,\vec{\ell}\Big)-$$
$$-\frac{1}{2}\rho_{\mu\nu}\big(\nabla_\mu\vec{n}\nabla_\nu\vec{n}+\vec{\ell}^{\,2}\delta_{\mu\nu}\big)-\varkappa_{0\mu}F_{0\mu}^2-\varkappa_{\mu\nu}F_{\mu\nu}^2+ \tag{4.23}$$
$$+\frac{i\vartheta_1}{8\pi^2}(A_\mu\dot{A}_\nu)\epsilon^{\mu\nu}+\frac{i\vartheta_2}{4\pi^2}\epsilon^{\mu\nu}A_0\partial_\mu A_\nu+\frac{1}{M_{\mu\nu}}(\nabla_\mu-iA_\mu\tau^3)\chi^+(\nabla_\nu+iA_\nu\tau^3)\chi\Big\}$$

Here $\mu,\nu = x, y$ are the space components of the gradients. The fields $A_0=-i\bar{Z}_\alpha\frac{\partial}{\partial\tau}Z_\alpha$ and $A_\mu=-i\bar{Z}_\alpha\partial_\mu Z_\alpha$ are components of a U(1) gauge vector field with strength components $F_{0\mu}=\partial_\tau A_\mu-\partial_\mu A_0$; $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$. Its meaning will be clarified below.

We have combined two spinless fermionic fields u(a) and v(b) into an "isospinor" $(\chi_1(x),\chi_2(x))$. In the following we shall assume that matrices $\vec{\tau}$ act in this basis. The coefficients appeared in (4.23) can be deduced from the formulae (4.17)-(4.20) and have the following form

$$\rho_{\mu\nu}=\frac{1}{2}\sum_{i,j}(i-j)_\mu(i-j)_\nu\rho_{ij}, \tag{4.24}$$

$$\frac{1}{M_{\mu\nu}}=\frac{1}{4}\sum_{ij}\tilde{t}_{ij}(i-j)_\mu(i-j)_\nu, \tag{4.25}$$

$$\varkappa_{\mu\nu}=\frac{1}{8}\sum_{i,j,k,\ell}\int_0^\beta d\tau\,\Pi_{ij,k\ell}(\tau)\times\big[(i-j)^\mu(k-\ell)^\mu(k-i)^\nu(\ell-j)^\nu- \tag{4.26}$$
$$-(i-j)^\mu(k-\ell)^\nu(k-i)^\nu(\ell-j)^\mu\big],$$

$$\varkappa_{0\mu}=-\frac{1}{4}\sum_{i,j,k,\ell}\int_0^\beta \Pi_{ij,k\ell}(\tau)(i-j)^\mu(k-\ell)^\mu\tau^2d\tau \tag{4.27}$$

$$\vartheta_1=i\pi^2\sum_{i,j,k,\ell}\int_0^\beta d\tau\,\Pi_{ij,k\ell}(\tau)\,\tau\,(i-j)^\mu(k-\ell)^\nu\epsilon_{\mu\nu} \tag{4.28}$$

$$\vartheta_2=i\pi^2\sum_{i,j,k,\ell}\int_0^\beta d\tau\,R_{ij,k}(\tau)\,(i-j)^\mu(i-k)^\nu\epsilon_{\mu\nu} \tag{4.29}$$

It could be shown that due to the SU(2) rotational invariance of the correlation functions there is a Ward identity relating three- and four-point correlators. It results to the equality

$$\vartheta_1 = \vartheta_2 \qquad (4.30)$$

Below we shall use a common notation ϑ for both coefficients. The imaginary part of the Lagrangian (4.23) is than a famous Chern- Simons term [26]. For the case of a one-half filling it has been derived by an analogous approach in [7]. Assuming that fluctuations of $\vec{l}$ are small we integrate them out and obtain a final form of the long wave-langth effective Lagrangian of the nonlinear σ-model coupled with fermions

$$\mathcal{L}_{eff} = \mathcal{L}_\sigma + \mathcal{L}_F$$

$$\mathcal{L}_\sigma = -\frac{1}{2}\left\{ \left(\frac{\partial}{\partial\tau}\vec{n}\right)^2 \frac{1}{c^2} + \rho_{\mu\nu}\partial_\mu \vec{n}\,\partial_\nu \vec{n} \right\} - \varkappa_{0\mu} F_{0\mu}^2 - \varkappa_{\mu\nu} F_{\mu\nu}^2 + \frac{i\vartheta}{8\pi^2}\left(A_\mu \partial_0 A_\nu + 2A_0 \partial_\mu A_\nu\right)\epsilon^{\mu\nu} \qquad (4.31)$$

$$\mathcal{L}_F = \chi^+(-\partial_0 + iA_0\tau^3)\chi + \frac{1}{M_{\mu\nu}}\chi^+(\partial_\mu + iA_\mu\tau^3)(\partial_\nu + iA_\nu\tau^3)\chi \qquad (4.32)$$

where $c^2 = \rho_{\mu\mu}/S^2$.

These formulae are the most general. The effective Lagrangian describes fluctuations around any definite ground state, whose properties are coded in the coefficients (4.24)-(4.29). In the following we shall assume that L_{eff} is symmetric with respect to the SO(2) rotations of the space coordinates. Then we will suppose that

$$\varkappa_{0x} = \varkappa_{0y} \sim Ja^2; \quad \rho_{xx} = \rho_{yy}, \quad \rho_{xy} = 0; \quad c = {}^2\rho/S^2 \sim Ja,$$

$$M_{xx} = M_{yy}, \quad M_{xy} = 0.$$

At arbitrary filling L_σ is not relativistically invariant ($\varkappa_{0x} = \varkappa_{0y} \neq \varkappa_{xy}$), but at one-half filling this symmetry is restored [7]. The effective Lagrangian (4.31), (4.32) is also gauge invariant under the action of the U(1) tranformations,

$$A_\mu \to A_\mu + \partial_\mu \varphi, \quad A_0 \to A_0 + \partial_\tau \varphi$$

$$\chi \to e^{i\varphi\tau_3}\chi$$

leading $\vec{n}$ uncharged.

At the Neel ordered state the $\vec{n}$-field is correlated in such a way that $\langle \vec{n}(x)\,\vec{n}(0)\rangle \sim$ Const at $x \to \infty$ and there are spin waves at all distances. In this case the higher order terms in (4.31) are irrelevant with respect to the first two terms. In the absence of the LRO spin wave excitations (triplet combinations of "z-quanta") have a mass $m \sim 1/R^c$ ($\langle \vec{n}(x)\,\vec{n}(0)\rangle \sim e^{-x/R_c}$) and at distances larger than R_c the $\vec{n}$-field is not well defined. Formally one could consider that the constraint (4.7) is weakened, because the corresponding Lagrange multiplier acquires a vacuum expectation value O(m) and z_α becomes a constrained complex spinor. As a result, the gauge field A_μ initially defined as a singlet combination of "Z-quanta" becomes independent at large distances.

The field z_α can be integrated out, which will produce terms $F_{0\mu}^2$ and $F_{\mu\nu}^2$. Then the long wave length dynamics of the singlet magnetic excitations is governed by the action for an abelian gauge field (A_0, A_μ) with the Chern-Simons term (C.-S.):

$$\mathcal{L}_g = -\varkappa_{\mu\nu} F_{\mu\nu}^2 - \varkappa_{0\mu} F_{0\mu}^2 + i\vartheta\,\mathcal{L}_{CS} \qquad (4.33)$$

In addition, we shall make comments on Expression (4.32), describing an interaction of holes with magnetic excitations. It is of a critical importance that a hole imbedded into an antiferromagnet, has no definite spin even for the case of a short range Neel order. Due to the presence of sublattices in the vicinity of any given site the spin of a hole is not a good quantum number. This phenomenon takes place even in a weak coupling limit. Namely, for a hole moving on the spin density wave background only one spin component along the direction of an induced magnetization is conserved [27].

However a hole is described by a two-component wave function in the continuous limit. The components describe the hole emerging either on the locally introduced A or B sublattices of a bipartite lattice. Defined in this manner the wave function is not a spinor with respect to the group of spin rotations. It explains why the Lagrangian (4.32) is not invariant under an arbitrary transformation $\chi \to U\chi$, where $U \in SU(2)$.

In general, an interaction of holes with magnetic degrees of freedom could include couplings with a whole SU(2)-triplet $g A_\mu \tau^3 + g'(A_\mu^+ \tau^- + h.c.)$, where $A_\mu^{\pm}$ are the continuous versions of the lattice composite fields $A_{ij}^{\pm}$ introduced in (4.16). The vertices of this type appear in the continuous limit of a two- band Emery model [28]. The bare (unrenormalized) vertices of this model have the form

$$g\psi^+\tau^a\ell^a\psi + g'\psi^+\tau^a\nabla_i\psi\,[\vec{n},\nabla_i\vec{n}]^a$$

It can be shown that the conclusions made for the case of L given by (4.32) retain their validity in a more general case at $g' \neq g$.

Now we pass over to the evaluation of the coefficients which

enter Expressions (4.31), (4.32). In the SU(2N)-symmetric case the coefficients (4.17)-(4.20) could be found when the correlators $\langle\langle \chi_i(\tau)\ldots\chi_j(0)\rangle\rangle$ are calculated by means of a 1/N-expansion around the mean field approximation.

First of all we shall find a value of ϑ which is the parameter determining such quantum numbers of quasiparticles as spin and statistics. Using formula (4.29) we obtain an expression for this quantity

$$\vartheta = i\pi^2\int_0^\beta d\tau\sum_{i,j,k}\langle\langle \vec{S}_i(\tau)\Big\{\big[\vec{S}_j(0),\vec{S}_k(0)\big]J_{jk} + \overbrace{\big(\chi_j^+\vec{\sigma}\chi_k - \chi_k^+\vec{\sigma}\chi_j\big)}^{t_{jk}}\Big\}\rangle\rangle \qquad (4.34)$$

The sum in the figure brackets is the total spin current density. A state, which is characterized by vanishing of this current has been termed a spiral phase in [29]. There is a long range helicoidal order in this state, a helix pitch Q equals $1/\sqrt{\rho}$. In the flux states considered above spins have their own "twist" even in the absence of holes and (4.34) differs from zero. By calculating ϑ in the mean field approximation one could obtain [7] that ϑ is proportional to the Hall conductivity $\sigma_{xy}(\nu)$ of the Hofstadter problem (3.10)

$$\vartheta = 2\pi\,\sigma_{xy}(\nu) \qquad (4.35)$$

We stress that for the parity conserving ground state ϑ vanishes. The quantity $\sigma_{xy}(\nu)$ has a topological meaning [30]: it is the first Chern class of a vector bundle with the base being the space of eigenfunctions of the Hofstadter problem. Being a Berry phase ϑ has no 1/N-corrections, so the mean field calculation has exact results even for N=1. The Hall conductivity is given in the case of a square lattice and a rational flux $\Phi = 2\pi\, P/q$ by the Diophantine equation [31]

$$\Phi/_{2\pi}\, \sigma_{xy}(\nu) - S = \nu \quad (4.36)$$

where S is an iteger and $|\sigma_{xy}(\nu)| \leq q/2$. This condition provides unambiguous solutions of (4.35) except for the case of even q and $\nu = 1/2$. The mean field value of the flux (3.13) leads to the solution $\sigma_{xy}(\nu) = 1$. Consequently, in flux states $\vartheta = 2\pi$ independently of a filling factor ν. For the SU(2N)-symmetric case, we would obtain $\vartheta = 2\pi N$. This result has previously been obtained for the case of a one-half filling in [7]. Then we conclude that the neutral magnetic excitations ("spinons") have a spin equal to $S = \pi/2\vartheta = 1/4$ [7]. These are the "semions" predicted by Laughlin et al. [32] for the case $\nu = 1/2$. We shall see in section 5 that the charged quasiparticle also has a spin S = 1/4 and it could be considered as a bound state of a charged fermion and a neutral "spinon". When two semions of a collection of such quasiparticles are interchanged the wave function acquires a phase factor $e^{i\pi/2}$.

The "electric" and "magnetic" susceptibilities $\varkappa_{xx} = \varkappa_{oy}$ and $\varkappa_{xy}$ differ by a value $O(\rho)$. For small doping we shall neglect this difference and use $\varkappa \sim J^{-1}$.

The effective bandwidth of a hole M^{-1} given by Equation (4.25) could be evaluated as

$$M^{-1} = \min(J, t^2/J) \quad (4.37)$$

In the limit $t \gg J$ the result (4.27) follows from a selfconsistent solution of (4.25) [33]. Its physical origin is that in the presence of a short range Neel order a coherent movement of a hole becomes possible only due to the quantum fluctuations of the surrounding spins overflopping with a

frequency O(J). In the opposite limit $t \ll J$, the second order of perturbation theory for the hopping term of (1.2) is sufficient to obtain (4.37).

We should stress that at $t \gtrsim J$ the above result holds for a "naked" hole state with the lowest possible total spin component $|\sum_i S_i^z| = 1/2$. At $t \gg J$ this state has larger energy than the polaronic state with $|\sum_i S_i^z| = O((t/J)^{1/2})$ [34]. If $t \gg J$ one should start from the reference vacuum state $|\Psi_o\rangle$ which insludes ferromagnetic bubbles around each of the holes and one obtains $M^{-1} \sim J(J/t)^{1/4}$.

Note, however that numerical simulations show, that even at $t \lesssim 150J$ a "naked" hole state probably has a lower energy [35].

In the next section we shall consider a system of holes and study their tendency to pairing produced by thespin interaction.

5. Pairing in disordered spin states

In general the presence of a short range Neel order results in a short distance attraction of holes placed on different sublattices. This attraction takes place since holes interrupt smaller number of antiferromagnetic bonds when the holes are close to each other, than when they are remote. In the continuum limit it leads to the conclusion that holes with different eigenvalues of τ^3 attact. Then in (4.32) the components of an isospin χ serve as opposite charges coupled through an abelian gauge field A_μ.

In the homogeneous RVB state fluctuations of the magnetic degrees of freedom are described by Eq. (4.33) with $\vartheta = 0$. A natural generalization of (4.33) for large values of $F_{\mu\nu}$, $F_{o\mu}$ is the action of the compact QED_{2+1}

$$\mathcal{L} = \varkappa \left(F_{0\mu}^{2} - \cos F_{12} \right)^{2} \qquad (5.1)$$

It is known [36] that in this theory the confinement phenomenon takes place. An "area" law for Wilson loops (2.4) is manifestation of the confinement. External particles having opposite charges are bound by a linear potential. If there is a finite density of holes the attraction potential is partially screened. Its Fourier tranform is given by the formula

$$V_{12}(k) = -\operatorname{Re} D_{00}(k)\ \tau_1^3 \tau_2^3$$

where $D_{\mu\nu}(k) = \langle A_\mu(k) A_\nu(-k) \rangle$ and $\tau^3_{1,2}$ are the charges of interacting particles. Here and below $k_\mu = (\omega, c\vec{k})$, $\mu = (0, x, y)$. In the region $\omega \ll k v_F$, where $v_F \sim \sqrt{\rho}/M$ is a Fermi velocity of holes, and $k \ll 2k_F$, the correlator $D_{\mu\nu}(k)$ is renormalized in the RPA approximation and has a form [37]

$$D_{\mu\nu}(k) = -\frac{(\delta_{\mu\nu} - k_\mu k_\nu / k^2)}{\omega^2 + c^2 \vec{k}^2 + \omega_{pe}^2 \frac{\omega}{k v_F} \left(1 - \frac{k^2}{4k_F^2}\right)^{-1/2}} \qquad (5.2)$$

In (5.2) $\omega^2_{pe} = \Delta_s \varepsilon_F$ is the plasma frequency $\varepsilon_F \sim \rho/M$ is a Fermi energy of holes and $\Delta_s = c/\xi$ is a correlation length of spin singlet A_μ fluctuations (ξ is the same as in (2.4), (2.5)). If $k v_F < \omega < (ck, \omega_{pe})$ then we have [37]

$$D_{\mu\nu}(k) = -\frac{(\delta_{\mu\nu} - k_\mu k_\nu / k^2)}{\omega^2 + \omega_{pe}^2 + c^2 \vec{k}^2} \qquad (5.3)$$

and there arises a screened attraction acting on distances $O(\xi)$. The corresponding potential leads to a pairing of holes

in a state with zero angular momentum ("s -wave").

The mechanism of pairing in the parity violating flux states which we are dealing with seems to be less trivial. In general the correlator $D_{\mu\nu}(k)$ has an additional antisymmetric term

$$D_{\mu\nu}(k) = V(k^2)\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) + i\mu \frac{k^\lambda \epsilon^{\mu\nu\lambda}}{k^2}\, U(k^2) \qquad (5.4)$$

where $\mu = \frac{1}{\varkappa} \frac{\nu^2}{2\pi}$ is the topological mass for the A_μ field. The second term in (5.4) describes an instantaneous Bohm- Aharonov interaction and contains an unphisical pole at $\omega = |\vec{k}| = 0$ [38]. In the relativistic invariant case

$$V(k^2) = U(k^2) = -\frac{\varkappa}{\omega^2 + \mu^2 + c^2 \vec{k}^2} \qquad (5.5)$$

The pair interaction potential derived from (5.4), (5.5) has the form

$$\begin{aligned}
&\Gamma_{\alpha\beta,\gamma\delta}(\vec{p}_+, -\vec{p}_-; \vec{q}_+, -\vec{q}_-) = \\
&= (\tau^3 \otimes \tau^3)_{\alpha\gamma,\beta\delta} \left\{ V(\bar{p} - \bar{q}) + U(\bar{p} - \bar{q}) \frac{p_\mu q_\nu \epsilon^{\mu\nu}}{(\bar{p} - \bar{q})^2} \right\} + \\
&+ (1 \otimes 1)_{\alpha\gamma,\beta\delta}\, W(\bar{p} - \bar{q}) - (\bar{q} \to -\bar{q},\ \gamma \leftrightarrow \delta),
\end{aligned} \qquad (5.6)$$

where $\bar{q}_\pm = \bar{q} \pm \frac{\bar{Q}}{2}$, $\bar{p}_\pm = \bar{p} \pm \frac{\bar{Q}}{2}$ and $\bar{Q}$ is the center of mass momentum. In the relativistic invariant case the functions $U(r)$, $V(r)$ and $W(r)$ are given by the formulae

$$\begin{aligned}
V(r) &= c^2 \frac{1}{2\pi\varkappa} K_0(\mu r) \\
U(r) &= \frac{2\pi}{\nu^2} \frac{1}{2M} \left(1 - \mu r K_1(\mu r)\right), \\
W(r) &= \left(\frac{2\pi}{\nu^2}\right)^2 \frac{1}{4Mr^2} \left(1 - \mu r K_1(\mu r)\right)^2
\end{aligned} \qquad (5.7)$$

To prove that the potential (5.6) leads to pairing we write down the Bethe-Salpeter equation for the two-particle wave function with $\vec{Q} = 0$:

$$\Psi_{\alpha\beta}(\vec{p}|E) = \sum_{\vec{q}} \Gamma_{\alpha\beta,\gamma\delta}(\vec{p},-\vec{p};\vec{q},-\vec{q}) \langle \chi_\gamma(\vec{p}) \chi_\delta(-\vec{p}) \rangle \tag{5.8}$$

assuming that the pair energy E is close to the bound state energy value E_b . In the "ladder" approximation (see fig.2) the BS equation is

$$\Psi_{\alpha\beta}(\vec{p}|E) = \sum_{\omega,\vec{q}} \Gamma_{\alpha\beta,\mu\nu}(p,-p;q,-q)\, G_{\mu\gamma}(q)\, G_{\nu\delta}(-q)\, \Psi_{\gamma\delta}(\vec{q}|E) + O(\Psi^3) \tag{5.9}$$

For the values of E close to E_b the higher order terms in (5.9) could be neglected. More accurately one should renormalize the bare potential Γ by summing up all the two-particle irreducible diagrams which could not be splitted by cutting two fermionic lines. Performing this procedure it is convenient to introduce the so-called T-matrix instead of the vertex Γ . The T-matrix is a physical scattering amplitude and is finite even for singular potentials which do not have a well defined Fourier tranform. This is necessary, for example, for the potentials having a hard repulsive core. The T-matrix is expressed via Γ by the equation

$$T = \Gamma + \Gamma * G_0 * T \tag{5.10}$$

where $G_0(\vec{k}|E) = (E - 2\varepsilon_k + i\varepsilon)^{-1}$ and the asterisk in (5.10) denotes a convolution operation. In Ref. [39] for the problem of a two-dimensional pairing a universal asymptotical form of

the T-matrix has been used

$$T_0(E) = \frac{4\pi}{m} \ln^{-1}(E/E_b) \tag{5.11}$$

It was assumed in (5.11) that in a two-particle (vacuum) problem there is a a bound state with the energy E . Formula (5.11) has used in [39] and there has been claimed that the pairing instability in the presence of a finite density of particles occurs if and only if the two-particle bound state does exist.

This result is not correct because the universal form of the T-matrix (5.11) is not achieved when (5.9) is solved. The crossed diagrams which are beyond the ladder approximation give only small corrections due to the infrared finiteness of the theory in the presence of the Chern-Simons term.

Difiding Equation (5.9) onto the symmetric Ψ_S and antisymmetric Ψ_A isospin components one could see that an attraction exists in both channels for particles with opposite τ^3 isospin components ($\tau_1^3 = -\tau_2^3$). For these states Equation (5.9) could be rewritten as

$$\Psi_{A,S}(\vec{p}|E) = \sum_{\vec{q}} \frac{(1-2n_{\vec{q}})\Psi_{A,S}(\vec{q}|E)}{2(\varepsilon_q - \varepsilon_F) - E + i0} \Big\{ \Big[V(\vec{p}-\vec{q}) + U(\vec{p}-\vec{q}) \frac{p_\mu q_\nu \epsilon_{\mu\nu}}{(p-q)^2} - W(\vec{p}-\vec{q}) \Big] \pm (\vec{q} \leftrightarrow -\vec{q}) \Big\} \tag{5.12}$$

where $n_{\vec{q}} = (1 + \exp(\varepsilon_q - \varepsilon_F)/T)^{-1}$ is the Fermi distribution function. Let us explain the consistency of our approach. We start from the theory (4.32), (4.33) of "charged" fermions coupled with the abelian gauge field, and, hence, we introduce the bare Fermi surface. The notion of a fractional statistics occurs if we attempt to calculate the amplitude of rotation of one particle around another. For a large interparticle separation this amplitude is simply $e^{2\pi^2 i\, k/\theta}$, where k is a linking number of the contours representing the classical space-time trajectories of the particles [40]. This "statistical transformation" is an effect of an exchange by long wavelength "photons", whose dynamics is governed by the Chern-Simons term only. By solving Equation (5.12) we not only take this statistical effect into account but also study the influence of the "nontopological" term $F_{\mu\nu}^2$. We see that using single-valued fermionic wave functions (instead of the "pseudo-wave functions" popular in literature on the subject ·) we should treat the statistical Bohm-Aharonov interaction as a shift of a pair angular momentum in the radial part of the fermionic wave function.

In (5.12) the potentials $W(\vec{p}-\vec{q})$ and $U(\bar{p}-\bar{q})\, p_\mu q_\nu \epsilon_{\mu\nu}/(\bar{p}-\bar{q})^2$ decrease as $1/r^2$ in the coordinate representation. Then it is natural to regard them as "renormalization" of the pair angular momentum. To see this let us introduce the function $\chi_{A,S}(q|E) = \frac{1-2n_q}{2(\varepsilon_q-\varepsilon_p)-E+i\varepsilon}\Psi_{A,S}(q|E)$, then tranform (5.12) into the coordinate representation and pick out the harmonics with the angular momentum ℓ. The harmonics satisfies the equation

$$\left(-\frac{1}{M}\frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\right)+\frac{\ell^2}{Mr^2}-2\varepsilon_F\right)\chi^\ell_{A,S}(r)=\left(E+V(r)+U(r)\frac{\ell}{r^2}-W(r)\right)\chi^\ell_{A,S} \qquad (5.13)$$

with l even for χ_A and l odd for χ_S.

It follows from (5.7) that at distances larger than μ^{-1} the angular momentum l is shifted by a value of a double "fractional spin" $2S = \mathcal{I}/\theta$. Transferring the terms decreasing

as $1/r^2$ from the right hand side to the left we obtain

$$\left[-\frac{1}{M}\frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\right)+\frac{(\ell+2S)^2}{Mr^2}\right]\chi^\ell_{A,S}(r)=\left(E-V(r)\right)\chi^\ell_{A,S}(r) \qquad (5.14a)$$

Transforming (5.14) back into the momentum representation we obtain

$$\tilde{\Psi}_\ell(p)=\frac{1}{2\pi}\int_0^\infty\frac{q\,dq\,(1-2n_q)}{2(\varepsilon_q-\varepsilon_F)-E+i0}\,\tilde{\Psi}_\ell(q)\,V_{\ell+2S}(q,p) \qquad (5.14b)$$

where $\tilde{\Psi}_\ell(p)$ is an angular harmonics of the Fourier transform of the multivalued function $\tilde{\Psi}(\vec{r}) = \sum_\ell \Psi_\ell(r)\, e^{i\varphi(\ell+2S)}$

and

$$V_{\ell+2S}(\bar{q},\bar{p})=\int_0^{2\pi}\frac{d\varphi}{2\pi}V(\bar{p}-\bar{q})\,e^{i\varphi(\ell+2S)}=\frac{c^2}{\varkappa\mu^2}\left(\frac{pq}{\mu^2+p^2+q^2}\right)^{|\ell+2S|} \qquad (5.15)$$

As we have noted above one could use single-valued functions $\Psi_\ell(r)$, but assuming that $\ell = n+2S$, where n is an integer Then the solution of (5.14) would have nontrivial monodromy properties in a complex plane of a radial variable r. For the kernal of Equation (5.14) one would obtain (5.15) with ℓ substituted by an integer n. Equation (5.14) has a solution E = $= -\Delta_\ell$, where

$$\Delta_\ell = \varepsilon_F \exp\left[-4\pi\left(\frac{\mu^2\varkappa}{c^2 M}\right)\left(\frac{\mu^2}{2M\varepsilon_F}\right)^{|\ell+2S|}\right] \qquad (5.16)$$

determines the value of the gap generated by the pairing with an angular momentum l. Although the coupling is not weak the exponent could be made arbitrarily small for low enough hole density. At finite temperatures the pairing with momentum l occurs at $T \sim \Delta_{\ell}$. It follows from (5.16) that pairs with angular momenta l and $-(1+4S)$ simultaneously appear at decreasing temperature. Then we get that at $\vartheta = 2\pi$ the partial waves with l=0 and l=$-\,\mathrm{Sign}\,\vartheta$ are degenerate.

For real systems an intersite Coulomb repulsion between the holes should be taken into account. A screened Coulomb potential $U_{sc}(r)$ could be approximated by a hard-core part of the total potential $\tilde{V}(r) = V(r) + U_{sc}(r)$. The presence of the hard-core part of $\tilde{V}(r)$ lifts up the degeneracy of the s- and p-waves. The p-wave state becomes favourable because the wave function with $|l| = 1$ vanishes at r=0 and this state is less influenced by $U_{sc}(r)$, than the s-wave state. Since all hole pairs have angular momenta of the same sing ($l = -\,\mathrm{Sgn}\,\vartheta$) the superconducting condensate is odd under parity and time reversal .

Now we shall consider a possibility of the existence of a bound state of an isolated hole pair. If it exists it would be consistent to suppose that at hole densities $\rho < \rho_c$ all holes are bound into pairs at zero temperature. The critical density ρ_c is determined by the mean bound state size L_{ℓ}, $\rho_c \sim L_{\ell}^{-2}$. The system behaves like a bose-gas with a weak repulsion. Then at the temperature T_B which is lower than the binding energy E a phase transition of the Kosterlitz-Thouless type occurs and the superconducting state of the preexisting bound pairs is formed.

In the original model (1.1), (1.4) where the intersite Coulomb interaction is absent the bound state of isolated pair really exists. We would like to study the dependence of E_{ℓ}, L_{ℓ} on the "topological mass" μ, which is the critical scale parameter of the problem. Then we give the results for the

SU(2N)-generalized version (3.5) of our model which correspond to $\vartheta = 2N$ and $\mu/M \sim N\,\mathcal{J}/M$ (lattice constant is unity).

The binding energy and the mean size of the bound state can be evaluated as

$$E_{\ell} \sim \frac{1}{M} L_{\ell}^{-2} \sim \frac{\mu^2}{M} \exp\left(-\frac{4\pi}{M\int_0 V(r)\, d^2 r}\right) \sim \frac{\mathcal{J}^2 N^2}{\mu} \exp\left(-\mathrm{const}\,\frac{\mathcal{J}N^2}{M}\right) \quad (5.17)$$

On the other hand, a weakly interacting two-dimensional bose-gas has a phase transition at the temperature [41]

$$T_B = \frac{4\pi \rho_B}{NM} \left[\ln\left(\ln \frac{\mu^2}{\rho_B}\right)\right]^{-1} \quad (5.18)$$

In (5.18) ρ_B is a temperature-dependent density of bound pairs. This formula is obtained under the assumption that the logarithmic factor is large. The equilibrium bound pairs concentration could be obtained from the equation [42]

$$\rho_B = \frac{\partial}{\partial\lambda} \sum_q \int \frac{d\omega}{\pi} \frac{1}{e^{(\varepsilon(q)-2\lambda)/T} - 1}\, \delta(\vec{q}, \omega) \quad (5.19)$$

where

$$\delta(\vec{q},\omega) = -\mathrm{Im}\, \ln\left[\chi(\vec{q},\omega) - \mathrm{Re}\,\chi(\vec{q}, E_{\ell})\right] \quad (5.20)$$

is the phase shift of the scattering of holes and

$$\chi(\vec{q},\omega) = \sum_{\vec{k}} \frac{1 - n_{\vec{q}+\vec{k}} - n_{\vec{q}-\vec{k}}}{\omega - \varepsilon_{\vec{q}+\vec{k}} - \varepsilon_{\vec{q}-\vec{k}} + 2\lambda + i\varepsilon} \quad (5.21)$$

is dynamic susceptibility.

The analysis of the Equations (5.19)-(5.21) shows that at $T \ll E_\ell$ and $\rho \ll \rho_c$ all holes are coupled into pairs ($\rho_B = \frac{1}{2}\rho$) and the transition temperature is given by (5.18). At $\rho \sim \rho_c \sim L_\ell^{-2}$ the boson density ρ_B strongly decreases, providing the crossover of $T_c(\rho)$ between the law (5.18) and

$$T_c \sim \Delta_0 = \Delta_{synt} \sim \rho/M \exp\left(-\frac{N^3}{\sqrt{\rho}} J^2/M\right) \tag{5.22}$$

In the realistic case N=1 one would obtain $\rho_c \sim \exp(-\frac{J}{M}\text{const})$ and for $t \sim J$ the bose-condensation mejchanism of the bound pairs would be correct even for large doping. However, taken into account, the Coulomb repulsion could prevent pairing and the bose-condensation picture would not be realized. The fact if the Coulomb repulsion would retrain the system from a Coupler-like instability at a temperature of (5.22) depends on the details of screening and so deserves further investigation.

Solving Equation (5.9) for pairs with nonzero center-of-mass momentum $\vec{Q}$ and introducing an external electromagnetic field A the substitution $\vec{Q} \longrightarrow \vec{Q} - 2e\vec{A}_{ext}$, we get the Ginsburg-Landau functional

$$F[\Psi, \vec{A}_{ext}] = c\,|(\vec{\nabla} - 2ie\vec{A}_{ext})\Psi|^2 + a|\Psi|^2 + b|\Psi|^4 + \frac{1}{8\pi}(\text{rot}\,\vec{A}_{ext})^2 + c'(\text{rot}\,\vec{A}_{ext})_z|\Psi|^2 \tag{5.23}$$

Here $\Psi(\vec{x})$ is a superconductiong order parameter. We assume that a pairing with $l = \text{sgn}\,\vartheta$ takes place. Then in addition to the standard terms (5.23) contains also the last parity breaking term. The coefficient of this term is

$$c' \sim \mu_B \,\text{sgn}\,\vartheta \min\left(\frac{t}{J}, \frac{J}{t}\right) \tag{5.24}$$

where μ_B is the Bohr magneton. This term is produced by the orbital currents of rotating pairs.

It follows from (5.23) that orbital supercurrents create a magnetic field. As a consequence, the two-dimensional layer has a magnetic moment which is equal to

$$M \sim \rho \mu_B \min\left(\frac{t}{J}, \frac{J}{t}\right) \tag{5.25}$$

per site.

We should note that our explicit calculation of F does not produce another possible parity breaking contribution $\sim$ $\sim [\vec{J} \times \vec{E}]_z$, where $\vec{E}$ is an electric field and $\vec{J}$ is a charge current of condensate pair s. The reason is that in a translation invariant system with the same ratio of charge to mass for all particles, a spatially constant electric field influences the center of mass coordinate. Since the interval interactions do not affect the center of mass motion, the response to a uniform electric field should be the same as for the free particles and certainty it should be parity even [43].

The parity violating pairing could have numerous microscopic manifestations. One of them is a rotation of a polarization vector of an electromagnetic wave reflected from the lauer [43,44].

The dependence of the lower critical field H_{c_1} on the direction of an external magnetic field is another observable effect. It follows from the fact that Abrikosov vortices with left-side and right-side chiralities have different energies

$$E_{L(R)} = c\,\Phi^2 \pm c'\,\Phi \tag{5.26}$$

where Φ is a flux carried by a vortex

6.DISCUSSION

In this section we shall briefly discuss the other possible mechanism of superconductivity, where the parity violation plays a crucial role. It is a scenario of the anyon superconductivity considered in a number of papers [43,45-47]. Anyons are particles obeying fractional statistics: if two anyons are interchanged, their wave function acquires a phase factor $e^{i\varphi}$ with arbitrary ϕ. This system could be described by the Lagrangian

$$\mathcal{L} = -\frac{1}{2m}|(\vec{\nabla}-i\vec{A})\psi|^2 + \bar{\psi}\left(-\frac{\partial}{\partial\tau} - iA_0\right)\psi + + \frac{i\vartheta}{8\pi^2} A_\mu \partial_\nu A_\lambda \epsilon^{\mu\nu\lambda} \tag{6.1}$$

written in terms of a one-component fermion ψ and an abelian gauge field A_μ. An equation of motion followed from (6.1)

$$\frac{\vartheta}{4\pi^2}\, \mathrm{rot}\,\vec{A} = \bar{\psi}\psi \tag{6.2}$$

is a local variant of the condition (3.13).

Due to the locality of the condition (6.2) there is a gapless collective mode in the system [43,46]. It is essentially a consequence of a compressibility of the anyon liquid [45]. If the anyons are electrically charged this mode becomes a longtitudinal component of a photon and leads to the appearence of the massless pole in a correlation function of electromagnetic currents $\langle J_\mu(q) J_\nu(-q)\rangle \sim \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}$ [46,47]. It means that a photon becomes massive and the Meissner effect takes place.

Although the anionic picture is very attractive, we should note that it could not be realized in disordered flux states with a short range antiferromagnetic order which we have studied in this paper. As we have shown in section 4 in the presence of the short range Neel order two species of fermions arise in a continuons theory. In the mean field approximation each of the components $\tau_3 = \pm 1$ creates its own fictitious field with a strength $B(x) = \pi\rho(x)\,\mathrm{sgn}\,\tau_3$ but these fields cancel each other. Then the RPA approximation used in [43,46] is not applicable and the superconductivity occurs in accordance with the mechanism proposed in section 5.

At temperatures $T_c < T < T_m$, where $T_m \sim J$ is the temperature when the parity violating flux state appears there are normal charged excitations in the system. The carriers are singly charged spin s=1/4 "semions". Then in an external electric field they would produce an anomalous fractional quantum Hall effect at zero magnetic field [7,44]. The quantization in a fictitious (frustration) "magnetic" field with flux (3.13) leads to Hall conductivity with an even denominator

$$\sigma_{xy} = \frac{e^2}{h}\,\frac{1}{2}\ (\text{integer})$$

To find the conditions which allow for the anyonic scenario we shall propose the phase diagram of the t-J model extended by additional spin interactions H_m'. In the region where $t/J \lesssim \rho^{-2}$ the system is described by an abelian gauge field coupled to two-component fermions. By increasing t/J we destroy the short range Neel order. We conjecture that at intermediate values of t/J a new phase of a strongly frustrated antiferromagnet, which is described by a nonabelian SU(2) gauge field, could take place.

At $t/J > \rho^{-2}$ we arrive at a fully frustrated spin system.

It is widely believed that Nagaoka limit $t/J \to \infty$ corresponds to fully polarized feromagnetic state for a finite number of holes. At increasing doping a phase transition into a paramagnetic state at $t/J \to \infty$ has been predicted. Recently, Wiegmann has conjectured that in the limit $t/J \to \infty$ some kind of a flux state occurs [17]. This flux state can be built on the basis of a ferromagnetic Nagaoka state if a skyrmion of a small size around each hole emerges. Due to the ferromagnetic background the system is described by a one-component fermion field in a continuum limit. To find a preferable spin configuration in the Nagaoka limit one should minimize the lattice Hamiltonian

$$H = \sum_{ij} t_{ij} \chi_i^+ \chi_j \sqrt{\frac{1+\vec{n}_i \cdot \vec{n}_j}{2}}\, e^{iA_{ij}} \qquad (6.3)$$

where A_{ij} is a solid angle subtended by unit vectors $\vec{n}_i$, $\vec{n}_j$ and a fixed axis $\vec{Z}_0$. According to the conjecture of [17] a gain in energy due to the nonzero flux of A_{ij} [22] is larger than a loss resulting from narrowing of a band ($t_{ij} \to t_{ij}\sqrt{(1+\vec{n}_i \cdot \vec{n}_j)/2}$).

Then we suppose that at $t/J > \rho^{-2}$ the holes could behave like a one-component anyonic system and in this region the anionic superconductivity could take plase. For large enough doping we expect that the considered system ceases to be strongly correlated and is in a metallic paramagnetic state. An expected phase diagram is shown in Fig.3 .

In conclusion, we have studied the parity violating flux states of a doped Mott insulator. Assuming an existence of a short range Neel order we have derived an effective long wavelength action which contains the Chern-Simons term.

The Chern-Simons term provides fractional spin and statistics for the unbound charged excitations, which appear at temperatures higher than T . At smaller temperatures the holes tend to pair and in the ground state there is a parity odd superconducting condensate. We also discuss the conditions when alternative anionic superconductivity arises.

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Figure Captions

Fig. 1 Coupling of triangles belonging to different plaquettes which stabilizes the $\phi = \pi$ flux state.

Fig. 2 Bethe-Salpeter equation for the two-particle bound state.

Fig. 3 Phase diagram of the generalized t-$\mathcal{J}$ model:
1 - Paramagnetic metallic state for large doping;
2 - U(1) gauge theory with two species of fermions;
3 - U(1) gauge theory with one fermion (anyon liquid);.
4 - Suppsed SU(2)-invariant theory.

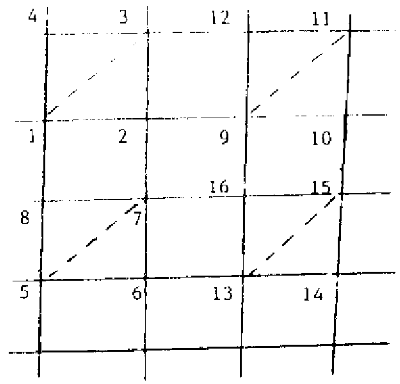

Fig.1

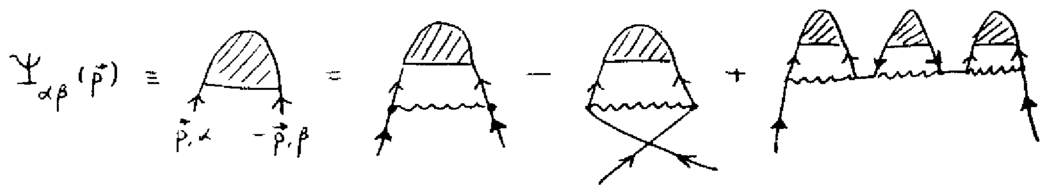

Fig.2

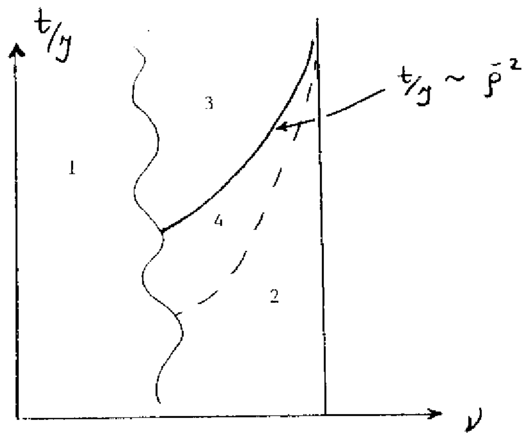

Fig.3

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