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NUCLEON AND ISOBAR PROPERTIES IN A RELATIVISTIC HARTREE-FOCK CALCULATION WITH VECTOR RICHARDSON POTENTIAL AND VARIOUS RADIAL FORMS FOR SCALAR MASS TERMS

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ABSTRACT. Mean field models of the nucleon and the delta are established with the two-quark vector Richardson potential along with various prescriptions for a running quark mass. This is taken to be a one-particle operator in the Dirac-Hartree-Fock formalism. An effective density dependent one body potential $U(\rho)$ for quarks at a given density ρ inside the nucleon is derived. It shows an interesting structure. Asymptotic freedom and confinement properties are built-in at high and low densities in $U(\rho)$ and the model dependence is restricted to the intermediate densities.

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1 Introduction

In a remarkable paper Witten [1] argued that it may be meaningful to talk about the relativistic mean field for quarks in a baryon. The justification follows from large N theories. As an example, the one electron Coloumb problem is exact in atomic physics [2] if one employs the idea that the number of dimensions, N, is large. It was pointed out by t'Hooft [3] that in QCD, treating the number of colours, N, as a large parameter, one gets a good explanation of the qualitative facts
(1) the existence of an infinite number of sharp resonances, with an interaction which is weak and (2) the Zweig rule on the suppression of $s\bar{s}$, $c\bar{c}$, $b\bar{b}$, and other massive quark pair production.

Witten extended these ideas to baryons and suggested that a mean field theory may be appropriate since in the large N limit, a baryon consists of a large number of valence quarks. Furthermore the large N prescription suggests that the interaction between the quarks through gluons should be scaled down by 1/N as the number of colours is taken to be large. This implies a "statistical" model for the baryon with a large number of valence particles with small interaction between them. The problem reduces to solving that of a relativistic potential from a complex field theoretical one. The potential is an effective gluon exchange interaction which behaves like the fourth component of a Lorentz vector. Witten [1] showed that in 1+1 dimension relativistic QCD may be exactly solved in the mean field approximation. In 3+1 dimension the actual potential form has to be borrowed from phenomenology. The Richardson potential [4] is suitable for this purpose. This potential was shown to lead to a reasonable meson spectrum even for the pion [5]. In the baryon sector for three strange quarks with mass m =150 MeV it leads to a reasonable description of the Ω^- as shown by Dey, Dey and Le Tourneux [6].

When one looks at the nucleon and the isobar with u,d quarks of negligible masses (m = 10 MeV) there is a problem [6]. The particles are not confined even though the Richardson potential contains a linearly rising confining part. This is due to the fact that the effective single particle confining potential obtained self consistently from Richardson potential is a vector one. The same problem was encountered by Crater and Van Alstine [5] who suggested a somewhat arbitrary prescription of taking a half-vector half-scalar form for the linear part. This choice also leads to a cancellation of spin orbit effects at long range, thus preventing partial multiplet inversions for the lighter mesons. The same procedure was adopted by Dey, Dey and Le Tourneux [6] and reasonable results were obtained for the isobar after subtraction of the centre of mass kinetic energy (T_{cm}). The half-scalar and half-vector form of the linear part of this two-body Richardson potential implies an effective mass m(r) and an effective vector potential U(r), respectively, both growing with distance. For example, m(r) reaches a value of 200 MeV around r ≈ 1 fm and continues to grow. It was suggested by Shuryak [7] from an analysis of QCD sumrule and also from the instanton size that the chiral symmetry breaking scale is about 0.3 fm rather than 1 fm. To elucidate, let us sketch the arguments given by Shuryak [7].

A general analysis of chiral symmetry breaking can be made starting from the small distances and using the operator product expansion (OPE) methods. Politzer [8] was the first to consider this and he calculated corrections to the quark propagator. Quark mass was obtained as a function of q^2 and quark condensate. The result however is gauge dependent. For light quarks it is most convenient to work in x-representation. The gauge condition used is the so called fixed point gauge

$$X_{\mu} A_{\mu} = 0 \qquad (1)$$

which was independently suggested by Fock, Schwinger and many others [9].

In this gauge (translational invariance is violated) the free quark propagator and its correction due to condensate is given by [7]

$$S(x) = -\hat{x}/(2\pi^2 x^4) - i\langle\bar{\psi}\psi\rangle \qquad \ldots(2)$$

the second term contributes to the effective mass as can be seen by comparing this with the fermion propagator expansion upto the first power of mass

$$S(x) = -\hat{x}/(2\pi^2 x^4) - im/(4\pi^2 x^2) \qquad (3)$$

This gives a co-ordinate dependent "effective mass" as follows [7]

$$m_{eff}(x) = -4\pi^2 x^2 \langle\bar{\psi}\psi\rangle \qquad (4)$$

With phenomenological value of the condensate, $\langle\bar{\psi}\psi\rangle \sim (-255\ MeV)^3$, we realise that $m_{eff}(x) \sim 350$ MeV at $x \sim 0.14$ fm

It may seem unreasonable to expect u and d quark effective masses to pick up their dynamical values at such short distances (~ 15 fm) without distrupting precocious scaling in deep inelastic scattering In actual fact as pointed out by Shuryak [7] the scale of chiral symmetry breaking turns out to be more like 0.35 fm as can be seen from the eqn (2)

$$x \sim x_{chiral} = (-2\pi^2\langle\bar{\psi}\psi\rangle)^{1/3} \qquad (5)$$

This corresponds to a momentum of about 4 GeV/c which is not too unreasonable, considering the idealised treatment from which the result has been derived

A lower value of r_0 (even 0.35 fm) means that we can work with a constant average quark mass as well Furthermore, the constituent quark models are well justified if the quarks acquire a mass in such a short distance Our numerical results show better agreement with experiments when the chiral symmetry breaking scale is longer than 0.35 fm and the rise is slow

Inspired by the arguments given above we saturate the quark mass at a distance r_0, less than the confinement scale, where the mass attains its constituent value. We have used this r_0 as a parameter and studied its variation on the ground state properties of the nucleon. We are separating the scalar part and the vector part. This was also done in references 5 and 6 but there both the scalar and the vector part were confining. In this work we would like to test numerically the argument that " Instantons do not lead to confinement, only to finite quark mass renormalisation [7]" Assuming the confinement is due to gluons we have a confining vector potential. Chiral symmetry breaking yields the finite mass term. For the vector part we have retained the phenomenological qq potential due to Richardson [4], and we derived the effective potential self consistently. The idea is to describe a baryon consisting of 3 quarks with running mass moving in an effective mean field obtained from a 2-body qq interaction self consistently.

Since we have considered the u, d quarks ($m_u=m_d$)only the baryon mass after correcting for the center of mass motion should be the average of N and Δ. The splitting has been calculated perturbatively by putting the colour magnetic interaction. This way of computing N-Δ mass difference is not free from overcounting because we already have a vector potential and one gluon exchange interaction is a vector by nature. In this model it is more realistic to compare the energy with Δ mass. Shuryak and Rosner [10] have shown that lowering of N will occur because of instanton contribution as this is attractive in the di-quark S=0 channel. And this may lower the nucleon by an amount $3\alpha/2$ with α about 200 Mev, i.e. without the spin-dependent instanton interaction the nucleon should be degenerate with Δ. The important point made by Shuryak and Rosner is that non-perturbative configurations such as instantons can play a crucial role in the description of spin-spin forces, particularly for light hadrons. The pattern of these effects can be very hard to disentangle from those more familiar ones due to chromomagnetic interactions arising from one-gluon exchange. In view of their comments we cannot attach too much importance to our calculation of N-Δ mass difference.

In the Friedberg and Lee non-topological soliton bag model [11] quarks are described as independent particles confined by the scalar soliton potential. Running mass of a quark starts from a value near zero and reaches to a maximum given by $g\sigma_v$. The picture is similar in our case. In addition, we have a confining vector part obtained self-consistently from a 2-body interaction. This helps the confinement further. One could easily notice this by changing the 2- body interaction parameter A. For example if A is decreased the r.m.s. radius of the nucleon increases. When the nucleon is embedded in nuclear matter the radius increases as the nuclear density increases [12,13]. Before solving the nuclear matter binding from a quark picture even crudely as done by Guichon [14] and by Frederico et al [15], we have studied the behaviour of the nucleon mass and its ground state properties with change of vector as well as scalar potential independently. We can draw certain conclusions which are important for further calculations. For example, the single particle mean field can easily be obtained as a function of quark density. Extension of this work is in progress [16].

2 Results and discussion

In our calculations the vector potential remains the same as in reference [6] but the scalar potential has been replaced by different phenomenological forms as well as the form given by eqn (4), saturated to the constituent quark mass at a certain distance r_0. Since we are considering u and d quarks only, the the scalar part saturates to 350-500 MeV. The forms we use are

(1) given by Shuryak as in eqn (4)

(2) $m(r) = V - V\,[\,1\,/\,(\,1 + \exp\,(r-c)/a\,)\,]$ ...(6)

where $V = 260$ MeV, $c = 0.1$ fm., $a = 0.02$ fm. .

(3) $m(r)$ = constant throughout

(4) $m(r) = M(1 - \exp(-r^2/R^2))$ ...(7)

The N - Δ mass difference is calculated from the one gluon exchange colour magnetic interaction energy only [17]. The energy-shift is given by

$$\Delta E_N = 6\,\alpha_c \times \sum \langle \lambda_i^a \lambda_j^a \rangle \langle \sigma_i \sigma_j \rangle \int_0^a \frac{\mu(m_i, r)\, \mu(m_j, r)}{4}\, dr \quad ..(8)$$

where $\mu(m_i, x) = -2/3 \int_0^x r^3\, G_i(r)\, F_i(r)\, dr$ with G and F the large and small components of the ground state wave-functions

$$\phi_{1s1/2m}(r) = \left[\frac{1}{4\pi}\right]^{1/2} \begin{bmatrix} i\, G(r)\, \chi_m \\ \vec{\sigma} \cdot \hat{r}\, F(r)\, \chi_m \end{bmatrix} \quad (9)$$

Results for $m(r)$ a la Shuryak (eqn 4) are tabulated for 2 sets of parameters (Table 1.) The r_o for the two sets are 0.15 and 0.35 fm respectively. The saturated quark mass m_s for both the sets is 260 MeV. The average value of the quark mass is shown by $\langle m(r) \rangle$. We get identical results if we repeat our calculations with a constant quark mass m_c given by this average value for the set (a). This is of course not surprising. As shown in Table 1 the r_o, the saturated distance in set (a) is very small. For all practical purposes we could take the quark mass to be a constant. This is not the case when r_o is taken to be slightly larger. Since there is no difference between the two m_s values, it is the different r_o which changes the wave function. The average mass is lower by 20 MeV from the saturated mass m_s in the latter case. The energy decreases by 100 MeV. For further increase in r_o the energy will be lower. But we prefer to stick to somewhat large energies to keep room for pionic interactions. After all, chiral symmetry is broken in our model and there must be a resulting Goldstone pion In any case the energies we get are not too unreseanable We can see that r_0 cannot be lower than about 0 35 fm

The mass form given by eqn (6) is designed to obtain results similar to that given by the set(a) of eqn (4) This is done to smoothen the kink in the first The running mass given by eqn (7) was used by Brevik [18] In Table 2 we show the corresponding numbers Like Brevik we also found that the numerical results do not depend on the individual value of M and R for sufficiently large R but only on the ratio A = M/R^2 We find that the average mass varies, but not too much, between 120 to 140 MeV Constant mass m_c is required to be larger to give the same radius as expected The energy is lowered more than 200 MeV The decrease increases with A decrease also the $m_c-\langle m(r)\rangle$ In other words, effective quark mass decreases with A Comparing our results with the ref 17 we observe that the additon of a vector potential has helped to decrease the radius For all A, ϵ_A is rather small Reasonable results are obtained for A=250 MeV, with a large $m_c \simeq 0$ 9

Last but not the least interesting is the graph $U(\rho)$ versus ρ given in fig 1 The probability of finding a quark within a nucleon between r and $r+dr$ is given by

$$\rho(r) = G(r)^2 + F(r)^2 \qquad (10)$$

At small distances the density $\rho(r)$ increases but the effective single particle vector interaction $U(r)$ decreases This is in accordance with asymptotic freedom On the other hand as r increses U increases due to confinement effect and $\rho(r)$ decreases rapidly Only in the intermediate range of ρ less than 3 fm^{-3} (corresponding to r greater than 0.6 fm) one finds the U to be dependent on the chosen prescription for the mass $m(r)$

3 Summary and conclusions

In this paper relativistic Hartree-Fock calculations for quarks have been done using the two body Richardson potential the confining vector part along with different chiral symmetry breaking mass term in the scalar part. By decreasing the vector potential we see that the quark mass effectively decreases and the radius increases. This could mean a higher nuclear density [14]. It is interesting to note that the density dependent single particle interaction $U(\rho)$ falls off sharply with quark density $\rho(r)$ for a scalar potential which breaks chiral symmetry at a shorter distance. In our opinion this $U(\rho)$ can be utilized in quark physics calculation much in the same spirit as Skyrme force is used in nuclear physics.

To summerize a co-ordinate dependent mass $m(r)$ is found to confine current quarks with a selfconsistent vector potential and give results which are acceptable. To obtain a Lorentz covariant theory, one should solve for a scalar field σ coupled to quarks with a Lagrangian consisting of $\mathcal{L}_{\sigma}$ and $\mathcal{L}_{-11}$. One can then connect this $\sigma(r)$ with the scalar potential of relativistic HF models with an aim to apply to nuclear matter. This will be described elsewhere [16].

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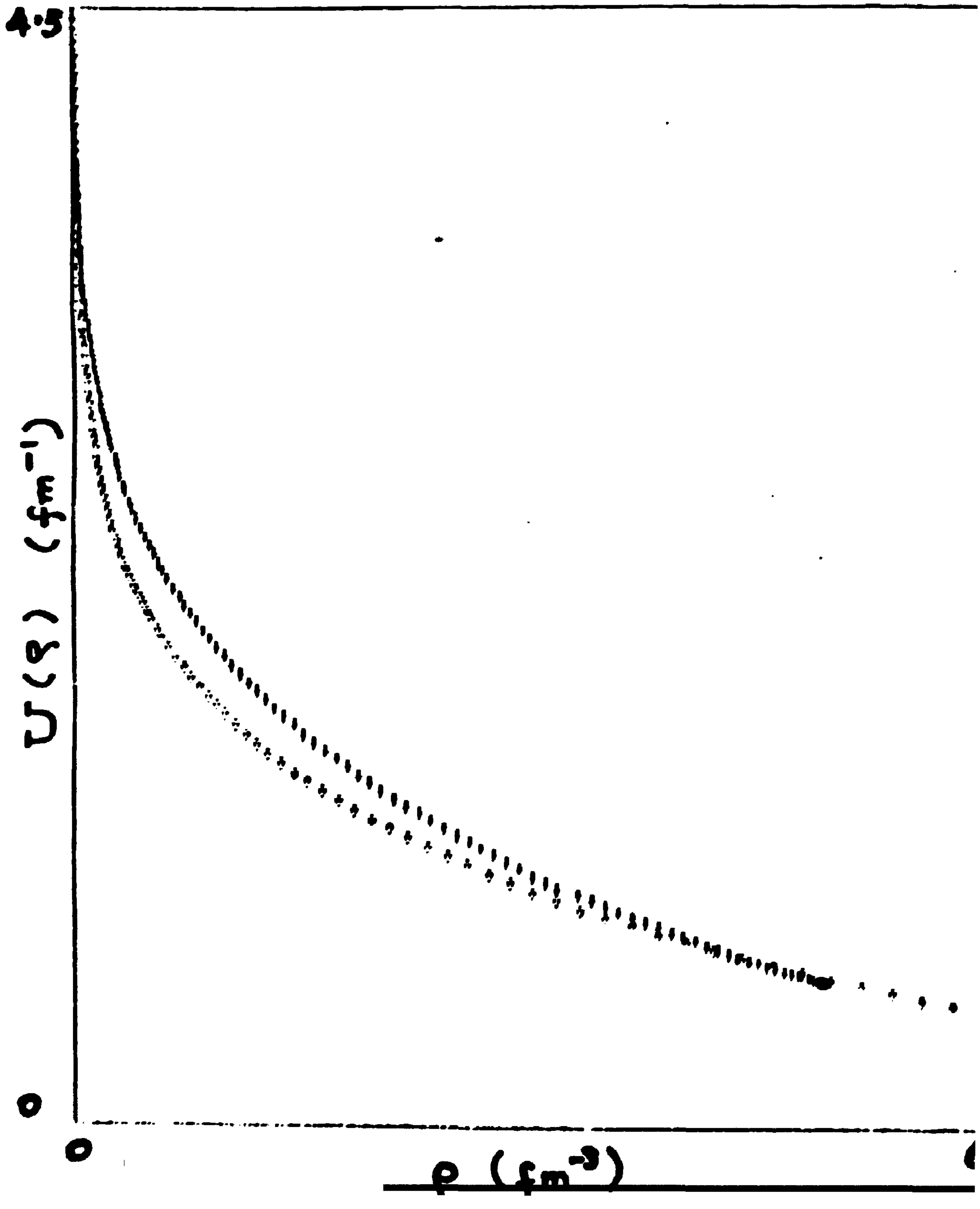
Figure caption

Fig 1 The self consistent single particle potential U as a function of the quark density ρ The lower curve gives the results for Shuryak's prescription for the mass M(r) (Table 1a) whereas the upper curve gives the result for the M(r) of Brevik [18] Λ is 300 MeV



4·5
0
U(ρ) (fm⁻¹)
0
ρ (fm⁻³)

Table 1 Mass form is given by eqn (4). m_c is the equivalent quark mass required to have the same radius $\Delta m = m_c - \langle m(r) \rangle$

(1a) $r_0 = 15\text{fm}$, $\langle \bar{\psi}\psi \rangle_0 = (-225\ \text{MeV})^3$, $\langle m(r) \rangle = 258$ MeV, $\Delta m \simeq 0$,
$m_s = 260$ MeV

A (MeV)	$\sqrt{\langle r^2 \rangle}$ (fm)	B_A	μ (n m)	$E_{HF} - T_{cm}$ (MeV)	ΔE_H (MeV)
300	91	1.02	2.19	1564	$286\alpha_c$
250	1.02	1.15	2.45	1382	$227\alpha_c$
225	1.10	1.22	2.56	1294	$185\alpha_c$

(1b) $r_0 = .35\text{fm}$, $\langle \bar{\psi}\psi \rangle_0 = (-128\ \text{MeV})^3$, $\langle m(r) \rangle \simeq 240\text{-}248$ MeV,
$\Delta m \simeq 45\text{-}48$ MeV $m_s = 260$ MeV

A (MeV)	$\sqrt{\langle r^2 \rangle}$ (fm)	B_A	μ (n m)	$E_{HF} - T_{cm}$ (MeV)	ΔE_H (MeV)
300	88	0.974	2.13	1492	$324\alpha_c$
250	97	1.11	2.39	1325	$261\alpha_c$
225	1.05	1.18	2.51	1246	$222\alpha_c$

Table 2 Mass form is given by eqn (8) $A = M/K^2 = 648\ \text{fm}^{-3}$ m_c is the equivalent mass to get the same radius

A (MeV)	$\langle m(r) \rangle$ (MeV)	m_c (MeV)	$\sqrt{\langle r^2 \rangle}$ (fm)	B_A	μ (n m)	$E_{HF} - T_{cm}$ (MeV)	ΔE_H (MeV)
300	120	218	976	782	2.65	1345	$347\alpha_c$
250	131	261	1.02	895	2.79	1112	$306\alpha_c$
225	137	298	1.04	948	2.83	1010	$353\alpha_c$

FOOTNOTES

F1 The proof that $\lambda > 1/4$ is a necessary condition for the existence of a non-trivial solution of our integral equation may be obtained following the same steps of section III of ref. (6).