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DYNAMICS OF NUCLEAR FUEL ASSEMBLIES IN VERTICAL FLOW CHANNELS

DYNMOD CODE:
A USER'S MANUAL AND PROGRAM DESCRIPTION

V.A. MASON

Chalk River Nuclear Laboratories
Chalk River, Ontario
January 1988
ATOMIC ENERGY OF CANADA LIMITED

DYNAMICS OF NUCLEAR FUEL ASSEMBLIES IN VERTICAL FLOW CHANNELS

DYNMOD CODE:
A USER'S MANUAL AND PROGRAM DESCRIPTION

V.A. Mason*


Research Company
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1988 January

AECL-9682
DYNMOD est un programme de calcul conçu pour prédire le comportement dynamique des assemblages de combustible nucléaire dans un écoulement axial.

On décrit dans le présent rapport les calculs exécutés avec DYNMOD et les données d'entrée que demande le programme. En outre, on y présente des exemples d'emploi de DYNMOD et une brève évaluation de la précision du modèle dynamique.

Ce rapport a pour but de servir de manuel de référence pour les utilisateurs de DYNMOD.

* Travaux effectués lors du stage auprès des Springfields Nuclear Power Development Laboratories, UKAEA (AEARU), Springfields, Preston, Angleterre.
DYNAMICS OF NUCLEAR FUEL ASSEMBLIES IN VERTICAL FLOW CHANNELS

DYNMOD CODE:

A USER'S MANUAL AND PROGRAM DESCRIPTION

V.A. Mason*

Abstract

DYNMOD is a computer program designed to predict the dynamic behaviour of nuclear fuel assemblies in axial flow.

The calculations performed by DYNMOD and the input data required by the program are described in this report. Examples of DYNMOD usage and a brief assessment of the accuracy of the dynamic model are also presented.

It is intended that the report will be used as a reference manual by users of DYNMOD.


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# TABLE OF CONTENTS

## INTRODUCTION

### SECTION 1: DYNMOD DESCRIPTION

1. **DYNMOD MAIN PROGRAM**
   - 1.1 The Dynamic Model
   - 1.2 Input and Output

2. **SUBROUTINE PINNED**
   - 2.1 Description
   - 2.2 Input and Output

3. **SUBROUTINE FORCES**
   - 3.1 Description
   - 3.2 Input and Output

4. **SUBROUTINE RECMOB**
   - 4.1 Description
   - 4.2 Input and Output

5. **SUBROUTINE RANDOM**
   - 5.1 Description
   - 5.2 Input and Output

6. **SUBROUTINE CALFOR**
   - 6.1 Description
   - 6.2 Input and Output

## SECTION 2: DYNMOD INPUT PARAMETERS

1. **INPUT DATA FORMAT**

2. **DYNMOD MAIN PROGRAM (DATA SETS 1, 2 and 3)**
   - 2.1 Title Card
   - 2.2 DATA SET 1
   - 2.3 DATA SET 2
   - 2.4 DATA SET 3
   - 2.5 SUBROUTINE PINNED: DATA SET 4
   - 2.6 DYNMOD Card Load Order

3. **SUBROUTINE FORCES**
   - 3.1 DATA SET 5
   - 3.2 FORCES Card Load Order
4. SUBROUTINE RECMOB
   4.1 DATA SET 6
   4.2 RECMOB Card Load Order

5. SUBROUTINE RANDOM : DATA SETS 7 And 8
   5.1 DATA SET 7
   5.2 DATA SET 8
      5.2.1 DFORCE (NxM) (DATA SET 8, OPTION 1)
      5.2.2 PSDFOR (NxN) (DATA SET 8, OPTION 2)
      5.2.3 UNCORF (NxM) (DATA SET 8, OPTION 3)
   5.3 RANDOM Card Load Order

6. SUBROUTINE CALFOR : DATA SETS 9, 10 and 11
   6.1 DATA SET 9
   6.2 DATA SET 10 (Optional)
   6.3 DATA SET 11
      6.3.1 DISMAT (NxM) (DATA SET 11, OPTION 1)
      6.3.2 DISPOW (NxN) (DATA SET 11, OPTION 2)
   6.4 CALFOR Card Load Order

7. MULTICASE RUNS USING DYNMOD

SECTION 3 : DYNMOD EXAMPLES
1. NATURAL FREQUENCIES AND MODESHAPES : DYNMOD MAIN PROGRAM
2. ADDITIONAL CONSTRAINTS : SUBROUTINE PINNED
3. TRANSIENT AND STEADY HARMONIC RESPONSE : SUBROUTINE FORCES
4. RECEPTANCE AND MOBILITY : SUBROUTINE RECMOB
5. RANDOM FORCED RESPONSE : SUBROUTINE RANDOM
   5.1 Discrete Force Spectra Input
   5.2 Cross Power Spectral Density of Force Input
   5.3 Auto P.S.D. of Force Input
6. FORCE CALCULATIONS : SUBROUTINE CALFOR
   6.1 Discrete Displacement Spectra Input
   6.2 Cross Power Spectral Density of Displacement Input
SECTION 4: DYNMOD ACCURACY

1. NATURAL FREQUENCIES

2. MODESHAPES

3. VISCOS DAMPING

4. STRUCTURAL DAMPING

5. STEADY FORCED RESPONSE

6. TRANSFER FUNCTIONS

7. RANDOM FORCES

8. FORCE CALCULATIONS

REFERENCES

APPENDIX I: DYNMOD FLOW CHARTS

APPENDIX II: DYNMOD INPUT PARAMETERS AND VARIABLES

APPENDIX III: LAMPS DEBUGGING PACKAGE AND ERROR CODES

APPENDIX IV: LAMPS FUNCTIONS AND SUBROUTINES USED IN DYNMOD

APPENDIX V: DYNMOD INPUT CHART
INTRODUCTION

DYNMOD is a computer program designed to predict the dynamic characteristics and stability of nuclear fuel assemblies in axial flow.

The computer model was constructed originally to analyse the dynamic behaviour of CANDU-BLW reactor fuel strings. However, the program can also be used to model the vertical booster fuel rods of the CANDU-PHW reactors and the fuel assemblies of the British AGR (during on-load refuelling), the British SGHWR and the Italian Cirene reactor. Using a slightly modified program, the fuel assembly dynamics of various other nuclear reactor systems, such as the fast breeder reactor, can be examined. In addition, the modelling techniques incorporated in DYNMOD can be used in the non-nuclear field to predict the vibration characteristics of pipework, heat exchanger components, and self-propelled cylindrical bodies moving in air or water.

It is intended that DYNMOD be used as a design tool to improve the dynamics of nuclear fuel assemblies. A fuel designer can use the analytical model to provide a qualitative prediction of the dynamic characteristics of a proposed fuel assembly. If the results are satisfactory, a prototype assembly can be built and modelled accurately. It will then be possible to investigate numerically the effects of design changes, breakages, channel contacts, irradiation deformations, etc., on the vibration characteristics and dynamic stability of the fuel. The program has been structured such that any transfer of data between the computer model and Fast Fourier Transform spectrum analysis equipment (e.g., Hewlett Packard 3582) will be straightforward.

The mathematical formulation of the dynamic model, using matrix operator calculus, is described in AECL report number AECL-5976, 'Dynamics of nuclear fuel assemblies in vertical flow channels: computer modelling and associated studies'.

The main program and associated sub-programs in DYNMOD are written exclusively in the LAMPS matrix processor language. LAMPS runs under the CDC 6600 SCOPE operating system at the Chalk River Nuclear Laboratories and is described in AECL report number AECL-5977, 'LAMPS: a FORTRAN based matrix processor'.

This user's manual is divided into four sections. The first contains a brief discussion of the nature of the calculations performed by DYNMOD. A description of the structural and hydrodynamic modelling parameters which are used to idealize the fuel assembly, flow channel and fluid is presented in the second section. A detailed description of the input and output data format is included in this section. This is followed by a series of examples which demonstrate the use of the DYNMOD main program and the package of sub-program options. The final section describes some of the methods used to assess the accuracy of the dynamic model during its development.
A comprehensive appendix has been included in the manual. Appendix I contains simplified flow charts of the main program and sub-programs of DYNMOD. The meanings and characteristics of the program variables are tabulated in Appendix II. In order to assist the user to locate input data errors, the LAMPS debugging package error codes and a list of the relevant LAMPS functions and subroutines used in DYNMOD have been included in Appendices III and IV, respectively (ref. 2). Finally, a DYNMOD input chart is provided in Appendix V for the checking of data card load order. A listing of DYNMOD can be found in reference 1.

Examples of core space requirements and program run-times, for a variety of DYNMOD calculations, are presented in Section 3.
SECTION 1

DYNMOD DESCRIPTION
SECTION I
DYNMOD DESCRIPTION

DYNMOD consists of a main program, one dedicated subroutine (PINNED) and four optional subroutine packages (FORCES, RECMOB, RANDOM and CALFOR).

The main program of DYNMOD, with subroutine PINNED, will calculate the natural frequencies and corresponding mode shapes of constrained reactor fuel assemblies in axial flow. Sub-programs FORCES and RANDOM calculate the response of the fuel assembly to a variety of forcing functions. The sub-program RECMOB is designed to calculate the receptance and mobility spectra of the fuel string in fluid. CALFOR will predict the nature of the lateral forces, fluid or otherwise, acting on a fuel assembly from a knowledge of its response and transfer function characteristics.

If any of the subroutine options are superfluous to a problem, they can be removed from the program. DYNMOD will then occupy less space in the computer core. (e.g. If FORCES, RANDOM and CALFOR are not required for a particular calculation, then these subroutines, plus statements DYNMOD 433, 442 and 446 (reference 1), can be removed from DYNMOD before loading.)

The calculations performed by DYNMOD are discussed in more detail below. A full analytical description of the model is presented in ref. 1.

1. DYNMOD MAIN PROGRAM

1.1 The Dynamic Model

It is assumed that the reactor fuel assembly can be represented by an articulated structure consisting of a string of interconnected fuel bundles. The bundle string stands above or hangs below a single support. Each bundle is described in terms of its mass, geometric dimensions, inherent structural damping and hydrodynamic mass. (N.B. The 'bundle' idealization, though originally proposed for the CANDU-BLWR dynamic model, works equally well on fuel assemblies consisting of long integral components, e.g. the British SGHWR fuel stringer.) The structural stiffnesses of the fuel string are incorporated in the computer model as interbundle bending stiffnesses, bundle end plate stiffnesses and bundle shear stiffnesses. A matrix equation of motion of the idealized fuel assembly is generated by the substitution of energy and generalized force terms into the non-conservative Lagrangian equation.

The energy terms introduced into Lagrange's equation are the gravitational and strain potential energies of the fuel assembly, the kinetic energy of the structure and the lateral kinetic energy of the fluid entrained by the fuel string. Structural damping is incorporated in the
model as an array of generalized forces. The generalized hydrodynamic forces are derived from the pressure drop forces, the normal and longitudinal friction forces, and the lateral drag forces along the fuel assembly. A generalized base drag force at the end of the fuel assembly and a non-conservative inviscid force, attributed to the loss in lateral momentum flux at the free end, are also included in the model.

The matrix equation of motion is formulated in terms of two sets of generalized coordinates, one representing the angular deflections of the bundles from the vertical, and the other the angular deflections of the bundle end faces from the horizontal. Solution of the equations of motion gives the natural frequencies and mode shapes of the fuel assembly in axial flow.

In general, because of the nature of the coefficient matrices in the equations of motion, each element of an eigenvector will be characterized by its relative phase in addition to its relative amplitude. Therefore, 2N equations will be required to determine all elements in each mode of oscillation of a N degree of freedom system. For a fuel assembly idealized into N bundles and having N degrees of freedom there will be 2N natural frequencies (obtained from the eigenvalues) and 2N normalized mode shapes (obtained from the eigenvectors).

1.2 Input and Output

The main program requires three sets of data (DATA SETS 1, 2, and 3 in "DYNMOD FLOW CHART", Appendix I).† The first consists of plotting and fault finding (debugging) parameters (7 variables). Structural parameters, which uniquely describe the idealized fuel assembly, are contained in the second data set (14 variables). The third data set consists of fluid and fluid-structure interaction parameters (7 variables). (All input parameters are defined in Section 2.)

These data allow DYNMOD to calculate and output (print and/or plot) the natural frequencies and mode shapes of a fuel assembly, supported at one end, in axial flow. This information describes the free vibration characteristics and dynamic stability of the fuel string.

Examination of a DYNMOD natural frequency will indicate whether the mode is damped-stable, undamped, fluid elastically unstable, or buckled (ref. 1, chapter 3).

Like the natural frequencies, the mode shape elements can be complex numbers. In most parallel flow situations, for a parameter mode, the vibration phase at the ends of adjacent bundles will be different. Hence, the "Data Set" consists of a string of matrices input by a single call of the LAMPS subroutine READn (...). However, in the case of DATA SETS 8 (Option 2), 10 and 11 (Option 2), subroutine READn (...) is called once during each cycle of the relevant frequency loop. For example, with reference to the flow chart of subroutine CALFOR in Appendix I, READn (...) will be called m times at the position marked DATA SET 10 for spectra containing m frequency points.
the motion described by a complex eigenvector (non stationary mode) might resemble a wave travelling along the fuel assembly.

The natural frequencies and mode shapes usually appear in 'modal pairs'. These describe the nature of the waves travelling in both directions along the fuel assembly. In the absence of hydrodynamic effects and structural damping, both waves described by a modal pair are the same (i.e., the two travelling waves become a standing wave).

After the natural frequencies and mode shapes have been output, the computer core space used by DYNMOD is optimized. This process involves erasing unwanted matrices. However, some matrices, for example the eigenvalues and eigenvectors of the idealized fuel assembly, are saved. These are then used in the subprogram options.

2. SUBROUTINE PINNED

2.1 Description

In certain designs of nuclear reactor, the fuel assemblies are constrained at more than one position. For example the British SGHWR fuel stringer is designed to be constrained at both ends. Similarly, the fuel assemblies of many pressure tube reactor systems are held in firm or intermittent contact with their flow channels by the high velocity coolant flow.

By using subroutine PINNED in DYNMOD it is possible to model fuel strings that are constrained at the end of the last bundle and/or pinned at the end of any intermediate bundle.

2.2 Input and Output

Subroutine PINNED requires one set of data (6 Variables: DATA SET 4 in the Subroutine PINNED flow chart). The input data contains a description of the constraint locations and the stiffness and structural damping parameters associated with any additional end constraints. Subroutine PINNED evaluates a coordinate reduction matrix which is used to define a new set of bundle angle coordinates. In effect this reduces the number of degrees of freedom of the system. The number of degrees of freedom lost is equal to the number of additional constraints acting on the bundle string.

There is no printed or plotted output from PINNED. The coordinate reduction matrix and the stiffness and damping parameters are transferred to the main program in DYNMOD. Here, the coordinate reduction matrix is used to operate on the coefficient matrices of the equations of motion. The end stiffness parameters are used in the strain potential energy matrix and the structural damping factors are incorporated in the total potential energy matrix via the vector of generalized structural damping forces.
3. SUBROUTINE FORCES

3.1 Description

Subroutine forces consist of two main parts. The first part will calculate the transient response of a fuel assembly in axial flow to any number of steady, phase related harmonic forces of the same frequency. These calculations assume that the fuel assembly is at its equilibrium position at the start of the forcing. (It is possible to modify the matrix operations in this subroutine to evaluate the transient response of the structure to other types of applied force, e.g., force ramps, impulses, etc. This feature may be useful for the solution of fuel-pressure tube interaction problems.) All transient response output is in time domain. The second part of FORCES calculates the steady state response of a fuel assembly in axial flow to any number of steady, phase related harmonic forces of the same frequency. Steady state response output is in frequency domain; it is the limiting response of a fuel string to gradually applied harmonic forces. (N.B., The sudden application of harmonic forces can sometimes cause a fuel assembly to experience transient excursions beyond its steady state vibration limits. Such transients are quickly damped in engineering structures.)

3.2 Input and Output

FORCES requires one data set (5 variables: DATA SET 5 in Subroutine FORCES flow chart). These data indicate whether a transient or steady harmonic response calculation is required. It is possible to specify both. The frequency, amplitudes, relative phases, and locations of the lateral harmonic forces are defined at input. This information is sufficient for DYNMOD to calculate the transient and steady harmonic responses. For transient response calculations, a time step and observation time limit must also be provided in DATA SET 5.

Transient response output consists of a single $N \times (n_T+1)$ matrix, printed in LAMPS format, where $N$ is the number of bundles in the idealized fuel assembly and $n_T$ is the number of calculation time increments. The rows of this matrix describe the displacement with time of the bundle ends from their equilibrium position. The columns show the shape of the fuel assembly at a particular instant in time.

Steady response output for a particular forcing frequency consists of a single column vector of complex numbers. Each complex number describes the amplitude and relative phase of the steady vibration at the end of a fuel bundle. For $N$ bundles there will be $N$ complex numbers in the response vector.

Output from FORCES consists of absolute values of displacement, e.g., metres, inches, etc. The output is not normalized.
4. SUBROUTINE RECMOB

4.1 Description

RECMOB is designed to calculate the receptance (or compliance, displacement per unit force) and mobility (velocity per unit force) between points on the fuel assembly over a specified range of frequencies. These transfer function spectra can be evaluated with or without axial flow. (Receptance and mobility are constant functions of the properties of an unchanging dynamic system.) The subroutine can be used for the accurate determination of dynamic modeling parameters by fitting the calculated transfer functions to experimental data. Equally a knowledge of the transfer functions will enable the engineer to predict the response of a fuel assembly to applied forces in the relevant spectral range.

4.2 Input and Output

Subroutine RECMOB requires one data set (4 variables: DATA SET 6 in subroutine FORCES flow chart). At input RECMOB is provided with frequency range information and the point along the fuel string at which the pseudo unit lateral force acts (electrodynamic shaker position). The spectral information consists of the minimum and maximum frequencies in the range of interest, and a frequency increment value which determines the degree of resolution of the calculated transfer functions.

At output, RECMOB prints column vectors of magnitude and phase for both receptance and mobility together with the frequency at which the transfer functions were calculated. The vectors of real numbers therefore describe the auto and cross transfer functions with respect to the force application point (shaker position). (i.e., For an N bundle string, there will be one value of auto-transfer function and N-1 values of cross-transfer function in the column vector.) Each element in a vector corresponds to a particular bundle end position.

Both receptance and mobility are printed at output. This information is useful when it is necessary to distinguish between modes that could be critically damped, low frequency damped, low frequency unstable or buckled in axial flow. An examination of the relative phase between displacement and velocity — obtained from the receptance and mobility calculations — often resolves the nature of the vibration. (e.g., For a damped-stable vibration, the velocity and displacement are in quadrature; whereas for a buckled mode, the velocity is in phase with the displacement.)

5. SUBROUTINE RANDOM

5.1 Description

RANDOM is designed to calculate the response of a fuel assembly to complicated forcing functions (e.g., fluid turbulence forces). Because
it is often very difficult to predict how the forces acting on a fuel string will vary with time, it is usual to describe the forces in a statistical fashion. The forces acting on a fuel assembly in parallel flow will rarely take the form of discrete spectra. Very often they are of a random nature. The method usually adopted for the statistical representation of these forces is to average their power spectral density matrix over many time records.

DYNMOD, with subroutine RANDOM, will evaluate the fuel string response to forces when they can be described as

(i) discrete force spectra,
(ii) averaged power spectral densities,

or, for the case of uncorrelated forces,

(iii) averaged auto-power spectral densities.

All response calculations in RANDOM require the complete receptance matrix of the fuel assembly. This (N x N) matrix, for a N bundle string, completely describes the receptance characteristics of the idealized structure. It is a function of frequency, and is calculated in RANDOM for each spectral line of interest.

If a fluid forced response calculation is attempted, all relevant hydrodynamic parameters must be input into the DYNMOD main program. Omission of these data will cause RANDOM to evaluate a simple structural receptance matrix instead of the required "fluid-structure interaction" receptance matrix.

At output RANDOM provides

(i) the auto power spectral density of response,
(ii) the mean square and the root mean square values of the response at each bundle end.

A simple modification to RANDOM will cause the cross power spectral density of response to be output.

Alternatively, for a discrete force spectrum input, the output consists of

(i) the complex discrete response spectra,
(ii) the auto power (intensity) spectra,

and (iii) the mean square and root mean square values of the response for each bundle end.

5.2 Input and Output

Subroutine RANDOM REQUIRES TWO DATA SETS. The first, DATA SET 7, inputs 4 variables; the second, DATA SET 8, inputs 1 variable (Subroutine RANDOM Flow Chart). DATA SET 7 describes the frequency range information and indicates which of the three types of response calculation is required.
The frequency data consist of the number of spectral lines of interest, the value of the lowest frequency point and the spacing of the frequency points.

DATA SET 8 contains a description of the applied forces. These might be discrete force spectra, cross power spectral densities, or auto power spectral densities, depending upon which of the three calculation options has been requested in DATA SET 7. Any number of forces of any frequency, amplitude and relative phase can be accommodated by subroutine RANDOM.

The primary output for discrete force spectra inputs consists of a discrete spectrum of response for each bundle end. This is printed in row vector form, one for each bundle end, and contains amplitude and phase information. Manipulation of these data provides similar row vectors which describe the intensity (auto power) spectra of the response at each bundle end. After printing the intensity spectra, RANDOM calculates and prints two column vectors. These correspond to the mean square and root mean square values of the response along the fuel assembly.

Similarly, the output for averaged power spectral density calculations consists of row vectors of auto power spectral density at each bundle end. These curves are then integrated by RANDOM to give the column vectors of mean square and root mean square displacement.

6. SUBROUTINE CALFOR

6.1 Description

Subroutine CALFOR calculates the lateral forces acting on a fuel assembly. If comprehensive response data are available, and the receptance matrix of the assembly in axial flow is known (by experiment or calculation), then the characteristics of the resolved forces can be deduced. Once the points of action and magnitudes of the forces are known, their importance can be assessed. Further DYNMOD experiments, in which dynamic modelling parameters of the fuel assembly are altered, will indicate the modifications required to reduce the intensity of the forces and/or minimize their effect.

CALFOR will evaluate the lateral forces resolved at each fuel bundle end when the response data can be expressed in discrete spectra form or auto and cross power spectral density form.

At output, for discrete response spectra data, the subroutine prints

(i) the cross power (intensity) matrix of the resolved forces for each spectral line,

(ii) the discrete force spectrum for each bundle end,

(iii) the auto power (intensity) spectrum of force for each bundle end, and
(iv) the mean square values and root mean square values of the forces resolved at the ends of the fuel bundles.

If the response data are input in cross power spectral density form, CALFOR provides all the above information (with the intensity spectrum changed to power spectral density) except for the discrete force spectra.

6.2 Input and Output

For general usage, subroutine CALFOR requires two data sets. If experimentally determined receptance matrices are to be used in the calculations, these must be input via a third data set.

On progressing through CALFOR, DATA SET 9 marks the first data input point (5 variables: Subroutine CALFOR Flow Chart). This data set contains frequency range information, similar to that used in subroutine RANDOM. It also indicates the nature of the response data to be input and whether or not receptance matrices are to be calculated. When experimental transfer functions are to be used in CALFOR, the optional DATA SET 10 is accessed in the frequency loop. This inputs a single (NxN) receptance matrix (for a fuel string of N bundles) for each of m spectral lines.

DATA SET 11 is input last. This contains the measured response data from which the forces can be deduced. If discrete response spectra are to be input in DATA SET 11, they are input as a single (Nxm) matrix of complex numbers; the rows of which describe the response spectra at each bundle end. However, in the case of cross power spectral density of response data, a single (NxN) complex matrix is input for each of the m cycles of the frequency loop.

When the input data consist of discrete response spectra, a column vector of resolved forces is calculated for each spectral line. If required, each of these vectors can be cross multiplied to give the (NxN) force intensity matrix. These square matrices can be printed for each of the m spectral lines.

All of the column vectors of force are horizontally joined in spectral order to form a (NxM) resolved force matrix. The rows of this matrix describe the discrete spectrum of resolved forces, in amplitude and phase, acting at each bundle end. This information is printed and comprises the primary output of CALFOR.

The scalar multiplication of the resolved force matrix with its complex conjugate gives the intensity matrix of the resolved forces. Each row of this matrix is printed to give the force intensity spectrum for each bundle end.

Further calculations allow CALFOR to print the column vectors of mean square and root mean square resolved forces along the fuel assembly.

If DATA SET 11 contains cross power spectral density of response data, CALFOR calculates (and optionally prints) the cross p.s.d. matrix of the resolved forces at each frequency point. Column vectors are formed from the diagonals of these matrices. The vectors are horizontally
joined in spectral order to form a \((N \times m)\) auto p.s.d. matrix of the resolved forces; CALFOR then prints the rows of the matrix to give the auto p.s.d. of resolved force at each bundle end. Row by row integration of the auto p.s.d. force matrix gives a column vector of mean square forces along the fuel assembly. The mean square, and root mean square column vectors of resolved force are printed by CALFOR before core optimization.
SECTION 2

DYNMOD INPUT PARAMETERS
SECTION 2
DYNMOD INPUT PARAMETERS

This section describes the input data required by DYNMOD. Where possible, diagrams and examples have been included to help clarify the meanings of the program variables. A summary of the section is presented in APPENDIX V, "DYNMOD INPUT CHART".

S.I. units are used for data input/output examples (plotting parameter matrices XL and YL, however, use inches). All matrix descriptions assume that the fuel assembly has been idealized as a N bundle string.

1. INPUT DATA FORMAT

All input to DYNMOD is via the LAMPS subroutine READn(......), ref. 2. This allows complex number matrices to be input in free format.

The following example demonstrates the flexibility of Subroutine READn (......).

Consider the LAMPS statement

CALL READ3(2HEX, EX, 3HWHY, WHY, 4HZEDD, ZEDD).

Here EX, WHY and ZEDD are integer names (called matrix pointers) which are used in the program to represent matrices. If EX and ZEDD are column vectors and WHY is a rectangular matrix, then the data set (or packet) shown in figure 1 might be used for input.

The data are read column by column, the shape of the matrix being defined by the integers in parentheses after each variable name; e.g., EX is a 12 element column vector, WHY is a rectangular matrix of 4 rows and 3 columns.

Operator * is used to avoid repetition of input data, i.e., 3*7 is equivalent to 7...7...7.

Complex numbers are punched in the form (real part)±(imaginary part), with no spaces within the number; hence 1.6-2.1 is input as (1.6-12.1). To avoid ambiguity, imaginary numbers may be input as either (0.0)±(imaginary number) or (imaginary number)±, where ± is seen by the computer as √-1; i.e., 0.0±1E6 is equivalent to i×10^6 and 10.7± is equivalent to i10.7. Other free format options are discussed in ref. 2.

Each data set is terminated by the dollar sign $.

The matrices stored by a LAMPS program after reading the sample card of figure 1 will be
After reading the data cards, SUBROUTINE READn(...) prints the matrices. This information is used to check for data input errors.

A typical example of a comprehensive DYNMOD input data pack is shown in figure 2. The optional "DATA SET 10" has been omitted (see sub-section 5.2).

The data in figure 2 cause all of the DYNMOD accessory subroutines to be used. Very often only limited information is required. The example of figure 3 shows the data cards required to perform a calculation in the subprogram CALFOR.

2. DYNMOD MAIN PROGRAM (DATA SETS 1, 2 and 3)

2.1 Title Card

The plotter and lineprinter output can be titled by making the first card of the input data deck a 'comment' card. A sample comment card is shown in figure 4.

If * is punched in column 1, a few lines are skipped before printing the title.

However, if / is punched in the first column of the comment card the title is printed at the head of the first page of lineprinter output (i.e., the page containing the natural frequencies).

2.2 DATA SET 1

Data for the seven plotting and fault finding parameters are provided by DATA SET 1. This set of data is input by SUBROUTINE READ 7(...) in DYNMOD statements 173 to 179. The seven scalar matrices served by DATA SET 1 are named DBUG, PLOT, BWR, SDR, XL, YL and NPERFR; these matrices are defined below. The order of the matrix is given in parentheses after the matrix name.

\[
\begin{pmatrix}
5.0 & 12.0 & 12.0 & 10.6 \\
5.0 & 12.0 & 12.0 & 620 \\
2.1-17.2 & 12.0 & -1.9 & 620 \\
2\times10^3 & 12.0 & 8.7+12.0 & 620 \\
2\times10^3 & 0.0 & \\
2\times10^3 & l+i & \\
2\times10^3 & l+i & \\
2\times10^3 & l+i & \\
2\times10^3 & l+i & \\
17.3+i1.6\times10^3 & 0.0 & \\
\end{pmatrix}
\]

and \[ZEDD = \begin{pmatrix}
0.0 \\
l+i \\
l+i \\
l+i \\
l+i \\
l+i \\
0.0 \\
0.0
\end{pmatrix}\]
**DBG (lxl)**

Matrix DBG may contain either 0 or 1.

- **DEBUG = (0)** No debugging information is required
- **DEBUG = (1)** Extra information will be printed by DYNMOD for debugging (fault finding). This information consists of
  1. Matrix \( V_R \) and the determinant of matrix \( V_{22} \) (the first to be inverted in DYNMOD); section 2.8, ref. 1.
  2. The coefficient matrices of the equations of motion and the determinant of matrix \( A_j \); sections 2.8 and 2.9, ref. 1.
  3. The eigenvalues and eigenvectors of the dynamic system.

**PLOT (lxl)**

Matrix PLOT may contain either 0 or 1.

- **PLOT = (0)** No graph plots are supplied in the output.
- **PLOT = (1)** Graph plots are supplied, in addition to the printed information, in the output. The plots are
  1. An Argand diagram of the complex eigenvalues
  2. The modeshapes of the fuel assembly. (In general, the modes are non-stationary, i.e., the eigenvectors contain complex elements. Therefore PLOT causes two sets of modeshapes to be plotted. The first is a graphical representation of the modes projected on the real plane. The second is an 'excursion envelope' representation of the modeshapes obtained by plotting the moduli of the complex eigenvector elements; the sign of the modulus of a complex element is set equal to that of the real part.)

Examples of the graph plot output from DYNMOD are shown in figures 5, 6 and 7.

**BWR (lxl)**

Matrix BWR contains bundle width ratio information of the graph plot option. A single number representing the ratio (bundle length/bundle width) is input into BWR.

**SDR (lxl)**

Matrix SDR contains the successive deflection ratio used by the graph plot option. This ratio is used to artificially reduce the peak deflections of successive mode shapes plotted by DYNMOD. The Jth modeshape normalization will be multiplied by the factor \((SDR)_J^{-1}\).
A single number representing the ratio is input into SDR.

**XL (1x1)**

Matrix XL contains a single number which represents the length of the plot frame, parallel to the X axis, in inches.

**YL (1x1)**

Matrix YL contains a single number which represents the length of the plot frame, parallel to the Y axis, in inches.

**NPERFR (1x1)**

Matrix NPERFR contains a single integer which corresponds to the number of modeshapes to be plotted per plot-frame.

A sample DATA SET 1 card is shown in figure 8.

### 2.3 DATA SET 2

The information used to define the idealized fuel assembly structure is provided by DATA SET 2. SUBROUTINE READ 14(....) in DYNMOD statements 184 to 197 inputs data for the fourteen matrices Y, G, L, M, NEL, DIAM, KCST, KEND, KPAR, GCST, GEND, GPAR, DCTOPX, DETOPX; these matrices are defined below. Figures 9 and 10 demonstrate the construction of DATA SET 2.

**Y (1x1)**

Matrix Y indicates the method used to support the fuel assembly.

- Y = (1) Fuel assembly hangs below its support (e.g., SGHWR)
- Y = (-1) Fuel assembly stands above its support (e.g. CANDU-BLW Reactor)
- Y = (0) Effect of gravity on the fuel assembly is ignored.

**G (1x1)**

Matrix G contains the acceleration due to gravity.

- G = (9.806) metres/s².

**L (Nxl)**

L is a column vector of bundle lengths (metres).

**M (Nx1)**

M is a column vector of bundle masses (kg).
NEL (Nx1)

NEL is a column vector describing the number of fuel elements in each fuel bundle along the string.

DIAM (Nx1)

Matrix DIAM defines the diameters (metres) of the fuel elements in each bundle along the fuel string.

KCST (Nx1)

KCST is a column vector of interbundle bending stiffnesses (originally the central support tube stiffnesses of a CANDU-BLW Reactor fuel string), (Newton metres/radian).

This parameter is most often used to represent the stiffness of long integral components which are idealized as a string of bundles; see section 13.5 ref. 1.

KEND (Nx1)

Column vector KEND describes the stiffnesses associated with the rocking and bending of adjacent bundle endplates, (Newton metres/radian).

KPAR (Nx1)

KPAR is a column vector of bundle shear (parallelogramming) stiffnesses, (Newton metres/radian).

GCST (Nx1)

Matrix GCST is a column vector of structural damping factors associated with the interbundle bending stiffnesses.

GEND (Nx1)

The elements of GEND are the structural damping factors associated with the bundle end plate stiffnesses.

GPAR (Nx1)

GPAR is a column vector of structural damping factors associated with the shear stiffness of the fuel bundles.

DCTOPX (1x1)

Matrix DCTOPX contains an additional structural damping factor associated with the 'interbundle' bending stiffness at the fuel assembly support.
DETOPX (1x1)

Matrix DETOPX contains an extra structural damping factor associated with the bending and rocking stiffness between the first bundle end plate and the support.

Sample DATA SET 2 cards corresponding to the data of figures 9 and 10 are shown in figure 11. (N.B. In figure 11, because matrices Y and G are not dimensioned, the input subroutine assumes that they each contain a single number. Matrices M, NEL, DIAM, etc., are automatically dimensioned as four element column vectors by matrix L (4).)

2.4 DATA SET 3

DATA SET 3 supplies information for the seven fluid parameter matrices, RHO, U, CDCFCB, DIAMH, VM, DEQ and FEF. Subroutine READ 7(...) in DYNMOD statements 201 to 207 inputs the matrix data. The fluid and fluid-structure interaction matrices are defined below.

**RHO (Nxl)**

RHO is a column vector of the mean fluid densities adjacent to each fuel bundle, (kg/metre$^3$).

**U (Nxl)**

U is a column vector of the flow velocities adjacent to each bundle along the fuel string, (metres/s).

**CDCFCB (Nx3)**

Matrix CDCFCB contains the following column vectors:

(i) $C_D$ (Nxl), the bundle drag coefficients in still fluid (M/s),
(ii) $C_F$ (Nxl), the coefficients of friction of the bundles, and
(iii) $C_B$ (Nxl), the base drag coefficient. (This single number is input as a column vector for convenience.)

$C_D$, $C_F$ and $C_B$ are horizontally joined to form the rectangular matrix CDCFCB.

**DIAMH (Nxl)**

DIAMH is a column vector of the hydraulic diameters in the region of the fuel bundles (4 x Flow area/Wetted perimeter; metres).

**VM (Nxl)**

VM is a column vector of the hydrodynamic masses of the bundles along the fuel assembly (kg).
DEQ (Nx1)

DEQ describes the equivalent diameter of the free end of the fuel assembly (metres). This single number is input as a vector for convenience.

FEF (Nx1)

Matrix FEF contains the free end factor (end shape factor) of the fuel assembly. The shape factor is associated with the non-conservative lateral force at the end of the fuel string and can assume a value between 0 and 1.

A more detailed description of the matrices in DATA SET 3 is available in ref. 1.

Sample DATA SET 3 cards are shown in figure 12.

2.5 SUBROUTINE PINNED: DATA SET 4

The number of additional fuel assembly constraints is described by DATA SET 4. SUBROUTINE READ6(.....) in PINNED statements 53 to 58 inputs the six matrices BPIN, JPIN, KCBOT, KEBOT, DCBOTX, DEBOTX; these are described below.

BPIN (1x1)

Matrix BPIN, the end bundle constraint indicator, may contain either 0 or 1.

BPIN = (0) Not end-pinned. This setting indicates that the fuel string is not constrained at the end.

BPIN = (1) End-pinned. A pinned joint at the end of the fuel assembly is modelled by this setting. The joint is free to move vertically.

JPIN (1x1)

Matrix JPIN, the intermediate bundle constraint indicator, may contain any integer between 0 and N-1 inclusive.

JPIN = (0) Not mid-pinned. This setting indicates that no intermediate bundles along the fuel assembly are constrained.

JPIN = (J) Mid-pinned. A pinned joint at the end of bundle J is modelled by this setting. The joint is free to move in the x direction.

KCBOT (1x1)

Matrix KCBOT contains a single number describing the 'interbundle' bending stiffness at the fuel assembly end constraint (Newton metres/radian).
KEBOT (lxl)

Matrix KEBOT contains a single number describing the bending and rocking stiffness of the endplate of bundle N at the fuel string end constraint (Newton metres/radian).

DCBOTX (lxl)

The element of matrix DCBOTX is an additional structural damping factor associated with the 'interbundle' bending stiffness at the fuel assembly end constraint, KCBOT. The function of DCBOTX is identical to that of matrix DCTOPX in DATA SET 2.

DEBOTX (lxl)

The element of matrix DEBOTZ is an additional structural damping factor associated with the endplate bending and rocking stiffness KEBOT at the end constraint of the fuel assembly. The function of DEBOTX is identical to that of matrix DETOPX in DATA SET 2.

A sample DATA SET 4 card is shown in fig. 13. The data in this card are not relevant to the fuel assembly depicted in figs. 9 and 10.

2.6 DYNMOD Card Load Order

The order of loading for the DYNMOD main program data sets is as follows:

1. TITLE CARD : /REACTOR FUEL DYNAMICS (for example)
2. DATA SET 1 : DBUG, PLOT, BWR, SDR, XL, YL, NPERFR $
3. DATA SET 2 : Y, G, L, M, NEL, DIAM, KCST, KEND, KPAR, GCST, GEND, GPAR, DCTOPX, DETOPX $
4. DATA SET 3 : RHO, U, CDCFCB, DIAMH, VM, DEQ, FEF $
5. DATA SET 4 : BPIN, JPIN, KCBOT, KEBOT, DCBOTX, DEBOTX $
6. DATA SET 5 : DEFAULT CARD (WHICH = 0) $
7. DATA SET 6 : DEFAULT CARD (SHAKER = 0) $
8. DATA SET 7 : DEFAULT CARD (DATA = 0) $
9. DATA SET 9 : DEFAULT CARD (TYPE = 0) $

(Note: DATA SETS 1 to 4 must precede all data sets peculiar to the DYNMOD accessory subroutines, i.e., FORCES, RECMOB, RANDOM and CALFOR.)
3. SUBROUTINE FORCES

3.1 DATA SET 5

SUBROUTINE READ5(......) in FORCES statements 62 to 66 inputs the five matrices WHICH, TMAX, SAMPLE, OMEGA and FORCE. These matrices are used for steady harmonic and transient response calculations and describe the characteristics of the applied harmonic forces.

WHICH (lxl)

Matrix WHICH indicates the type of forced response calculation required and may contain a 0, 1, 2, or 3.

WHICH = (0). No response. The response to harmonic forces is not required.

WHICH = (1). Transient response. This setting indicates that the transient response to the applied harmonic forces is required.

WHICH = (2). Steady state response. The steady harmonic response to the applied forces is calculated.

WHICH = (3). Steady and transient response. Steady harmonic and transient response calculations are performed.

TMAX (lxl)

Matrix TMAX describes the transient response observation time (seconds) after the application of the harmonic forces.

SAMPLE (lxl)

SAMPLE contains the time interval between calculations of the fuel bundle positions (seconds). Plots of the bundle positions against time provide the transient response of the fuel assembly; the transient response is a time domain function. The ratio TMAX/SAMPLE gives the number of points on the transient response curves after application of the forces.

OMEGA (lxl)

Matrix OMEGA contains the frequency of the applied forces (Hz).

FORCE (Nx1)

FORCE is a column vector of the harmonic forces applied at the bundle ends (Newtons). The elements of FORCE are complex numbers which describe the amplitude and relative phase of the forces applied to the fuel assembly. e.g. If the element of OMEGA is \( f \) and the \( J^{th} \) element of FORCE is \( (a+ib) \), then the harmonic force applied to the bottom of bundle \( J \) is given by \( (a^2+b^2)\frac{1}{2} \exp i(2\pi ft + \tan^{-1}(b/a)) \).
A typical DATA SET 5 card for a four bundle string might be that shown in figure 14.

3.2 FORCES Card Load Order

The order of loading for the FORCES data set is as follows:

1. DATA SETS : 1, 2, 3, and 4 (see sub-section 2.6) $ $ $ $
2. DATA SET 5 : WHICH, TMAX, SAMPLE, OMEGA, FORCE $
3. DATA SET 6 : DEFAULT CARD (SHAKER = 0) $
4. DATA SET 7 : DEFAULT CARD (DATA = 0) $
5. DATA SET 9 : DEFAULT CARD (TYPE = 0)$.

4. SUBROUTINE RECMOB

4.1 DATA SET 6

The four matrices SHAKER, FMIN, FMAX, and BANWID are defined by DATA SET 6. They are input by SUBROUTINE READ 4(...) in RECMOB statements 56 to 59 and are used for the calculation of the frequency domain functions receptance and mobility.

SHAKER (1x1)

Matrix SHAKER contains the number of the fuel bundle at the end of which the electrodynamic shaker or pseudo unit lateral force acts.

For SHAKER = (0), transfer functions are not calculated.

FMIN (1x1)

FMIN defines the lower frequency limit of the transfer functions (Hz).

FMAX (1x1)

FMAX defines the upper frequency limit of the transfer functions (Hz).

BANWID (1x1)

Matrix BANWID contains the frequency interval between transfer function calculations (Hz).

A typical DATA SET 6 example is shown in figure 15.
4.2 RECMOB Card Load Order

The order of loading for the RECMOB data set is as follows:

1. DATA SETS 1, 2, 3, 4 and 5 (see sub-sections 2.6 and 3.2) $ $ $ $ 
2. DATA SET 6 : SHAKER, FMIN, FMAX, BANWID $ 
3. DATA SET 7 : DEFAULT CARD (DATA = 0) $ 
4. DATA SET 9 : DEFAULT CARD (TYPE = 0) $ 
5. SUBROUTINE RANDOM : DATA SETS 7 and 8 

5.1 DATA SET 7 

SUBROUTINE READ 4(....) in RANDOM statements 75 to 78 inputs the four matrices DATA, POINTS, FMIN and BANWID. These matrices define the nature of the forced response problem.

DATA (lx1)

The element of matrix DATA can assume a value of 0, 1, 2 or 3.

DATA = (0) No Response. This setting indicates that complicated forced response calculations are not required.

DATA = (1) Discrete spectra. The response to forces described in discrete spectral form is to be evaluated.

DATA = (2) Averaged power spectral density format. This setting indicates that the response to forces expressed in averaged power spectral density form is required.

DATA = (3) Uncorrelated random. The response to uncorrelated forces expressed in averaged auto power spectral density format is to be calculated.

POINTS (lx1)

Matrix POINTS defines the number of discrete spectral lines (or the number of channels for continuous spectra) in the force data.

FMIN = (lx1)

FMIN defines the lower frequency limit of the force spectra (Hz).

BANWID (lx1)

The frequency interval between adjacent spectral lines or channels is defined by matrix BANWID (Hz).
Typical DATA SET 7 cards are shown in figures 16 and 17.

5.2 DATA SET 8

The data contained in DATA SET 8 completely describes the characteristics of the applied forces. Depending upon the integer stored in the matrix DATA (in DATA SET 7), the force information will be input by either RANDOM statement 116, 183 or 205, i.e., CALL READ 1(6H DFORCE, DFORCE), CALL READ 1(6H PSDFOR, PSDFOR) or CALL READ 1(6H UNCORF, UNCORF) respectively. These three force matrix options are described below.

5.2.1 DFORCE (NxM) (DATA SET 8, OPTION 1)

If the element of matrix DATA is set equal to 1, then the (NxM) matrix DFORCE must be supplied in DATA SET 8. DFORCE (Newtons) is a single rectangular matrix containing discrete force spectra information. The columns of DFORCE contain N complex numbers which describe the amplitudes and relative phases of the forces acting on the fuel assembly, for each of M spectral lines. (The integer M is equal to the element of matrix POINTS in DATA SET 7.) The column vectors of force are joined horizontally, in spectral order, to form the matrix DFORCE.

With reference to the data in figure 18, a typical DATA SET 8 (Option 1) card for a 4 bundle string and 5 discrete frequency points is shown in figure 19.

5.2.2 PSDFOR (NxN) (DATA SET 8, OPTION 2)

If the element of matrix DATA is set equal to 2, then the (NxN) matrix PSDFOR must be supplied in DATA SET 8. PSDFOR is a square matrix of the averaged cross power spectral densities of the forces acting on the fuel assembly at a particular frequency (Newtons²/Hz). In any problem, there must be M DATA SET 8 cards where M is the number of frequency points of interest (i.e., The element of matrix POINTS in DATA SET 7 is equal to M).

With reference to the data in figure 20, a typical DATA SET 8 (Option 2) card pack, for a two bundle string and eight frequency points, is shown in figure 21.

5.2.3 UNCORF (NxM) (DATA SET 8, OPTION 3)

If the element of matrix DATA is set equal to 3, then the (NxM) matrix UNCORF must be supplied in DATA SET 8. UNCORF (Newtons²/Hz) is a single rectangular matrix describing the spectral nature of the uncorrelated forces acting on the fuel assembly. The rows of UNCORF are the averaged auto-power spectral densities of force acting at each bundle end. The columns of UNCORF contain N real numbers which are the auto p.s.d.s of
the forces acting on the fuel assembly for each of the M spectral lines. (Integer M is equal to the element of matrix POINTS in DATA SET 7.)

Figure 22 shows the construction of matrix UNCORF for a 36 bundle string and 12 frequency points; a corresponding DATA SET 8 (Option 3) card is shown in fig. 23.

5.3 RANDOM Card Load Order

The order of loading for the RANDOM data sets for M spectral lines is as follows:

(a) Discrete response calculations.
1. DATA SETS 1, 2, 3, 4, 5 and 6 (see sub-sections 2.6, 3.2 and 4.2) $ $$ $$ $$ $$
2. DATA SET 7 : DATA, POINTS, FMIN, BANWID $
3. DATA SET 8 : DFORCE $
4. DATA SET 9 : DEFAULT CARD (TYPE = 0) $

(b) Response to correlated forces
1. DATA SETS 1, 2, 3, 4, 5 and 6 (see sub-sections 2.6, 3.2 and 4.2) $ $$ $$ $$ $$
2. DATA SET 7 : DATA, POINTS, FMIN, BANWID $
3. DATA SET 8 : PSDFOR 1 (PSDFOR (NxN) at spectral line number 1) $
4. " " : PSDFOR 2 $
5. " " : PSDFOR 3 $
: :
M+2. " : PSDFOR M $
M+3. DATA SET 9 : DEFAULT CARD (TYPE = 0) $

(c) Response to uncorrelated forces.
1. DATA SETS 1, 2, 3, 4, 5 and 6 (see sub-sections 2.6, 3.2 and 4.2) $ $$ $$ $$ $$
2. DATA SET 7 : DATA, POINTS, FMIN, BANWID $
3. DATA SET 8 : UNCORF $
4. DATA SET 9 : DEFAULT CARD (TYPE = 0) $
6. **SUBROUTINE CALFOR : DATA SETS 9, 10 and 11**

6.1 **DATA SET 9**

The five matrices **TYPE**, **EXPER**, **POINTS**, **FMIN** and **BANWID** are input by SUBROUTINE READ 5(...) in CALFOR statements 81 to 85. These matrices identify the nature of the resolved force calculations to be performed.

**TYPE** = (0). No force calculations.
This setting indicates that resolved force calculations are not required.

**TYPE** = (1). Discrete spectra.
Calculate the resolved forces from discrete response spectra.

**TYPE** = (2). Discrete spectra.
As for **TYPE** = (1) but supply at output the cross power (intensity) spectrum of the resolved forces at each frequency.

**TYPE** = (3). Cross power spectra.
Calculate the power spectral density of the resolved forces from averaged auto and cross power spectral density data.

**TYPE** = (4). Cross power spectra.
As for **TYPE** = (3) but supply at output the cross power spectral densities of the forces at each frequency.

**EXPER** (1x1)

Matrix **EXPER** specifies the nature of the transfer function data to be used in the calculations. It can contain the value 0 or 1.

**EXPER** = (0).
Calculate the transfer function matrix. This setting indicates that the square receptance matrix of the fuel assembly, with or without axial flow, is calculated by CALFOR for each frequency of interest.

**EXPER** = (1)
Experimental transfer function matrix. When matrix **EXPER** is set to this value the optional DATA SET 10 is used to input experimental receptance matrix data. In this case the receptance calculations are not performed.
POINTS (1x1)

Matrix POINTS defines the number of discrete spectral lines (or the number of channels for continuous spectra) in the measured response data.

FMIN (1x1)

Matrix FMIN defines the lower frequency limit of the response spectra to be input (Hz).

BANWID (1x1)

The element of matrix BANWID is the frequency interval between adjacent spectral lines or channels in the input response data (Hz).

A typical DATA SET 9 card is shown in figure 24.

6.2 DATA SET 10 (Optional)

DATA SET 10 is optional and is only required when EXPER (DATA SET 9) is set equal to 1.

TRAMAT (NxN)

A single matrix TRAMAT is input by each call of SUBROUTINE READ 1(...) in the statement CALFOR 101. TRAMAT, in this case, is an experimentally measured receptance matrix (metres/Newton).

The receptance matrix for M spectral lines is a three dimensional (NxNxM) array of complex numbers. At each frequency point TRAMAT consists of a square (NxN) matrix. Therefore, statement CALFOR 101 must be activated M times during a resolved force calculation involving M frequency points. Section 6.4 describes the data loading procedure for the two different types of force calculation.

An example of a DATA SET 10 package describing the receptance of a two degree of freedom structure over five frequency points is shown in figure 25 (the data are arbitrary).

6.3 DATA SET 11

The matrices input by DATA SET 11 describe the nature of the response data to be used in the resolved force calculations. Response data can be input by either CALFOR statement 125 or 213 (i.e., CALL READ 1(6H DISMAT, DISMAT) or CALL READ 1 (6H DISPOW, DISPOW) respectively) depending upon the value of the element of matrix TYPE.
6.3.1 DISMAT (NxM) (DATA SET 11, OPTION 1)

DISMAT (metres) is a single rectangular matrix containing discrete response spectra data. If TYPE = (1) or (2), then the matrix DISMAT must be supplied in DATA SET 11. DISMAT is a complex displacement matrix consisting of N displacement values, one for each bundle end, for each of M spectral lines. (The element of matrix POINTS in DATA SET 9 is set equal to M.) The columns of DISMAT contain displacement amplitude and relative phase information at a particular spectral line. Rows of DISMAT describe the discrete displacement spectra observed at each bundle end.

An example of a typical DATA SET 11 (Option 1) card for a three bundle assembly and four spectral lines is shown in figure 26. The construction of this data card is similar to that of DATA SET 8 (Option 1) with displacement (real and imaginary parts) substituted for force; see figs. 18 and 19.

6.3.2 DISPOW (NxN) (DATA SET 11, OPTION 2)

DISPOW (metres\(^2/\text{Hz}\)) is a cross power spectral density matrix of displacement for each of M frequency points.

If TYPE = (3) or (4), then the matrix DISPOW must be supplied in DATA SET 11.

At each frequency point DISPOW is a square (NxN) matrix. A set of M DATA SET 11 cards must be supplied in order to define the complete (NxNxM) array of complex numbers describing the cross power spectral density of displacement for the fuel assembly. The construction of the DISPOW matrices is similar to that of PSDFOR in DATA SET 8, OPTION 2; see figures 20 and 21. There the complex numbers describing the cross p.s.d. of displacement are substituted for the cross p.s.d. of force elements.

A typical DATA SET 11 (Option 2) card pack for a two bundle string and five frequency points is shown in fig. 27.

6.4 CALFOR Card Load Order

The order of loading for the CALFOR data sets, for M spectral lines, is as follows:

(a) Discrete force spectra calculations with calculated receptance matrix.

1. DATA SETS 1, 2, 3, 4, 5, 6, 7 and 8 (see sub-sections 2.6, 3.2, 4.2 and 6.3) $ $$ $$ $$
2. DATA SET 9 : TYPE, EXPER, POINTS, FMIN, BANWID $
3. DATA SET 11 : DISMAT $
(b) Discrete force spectra calculations with experimental receptance matrix.
   1. DATA SETS 1, 2, 3, 4, 5, 6, 7 and 8 (see sub-sections 2.6, 3.2, 4.2 and 6.3) $$$$$$$$
   2. DATA SET 9 : TYPE, EXPER, POINTS, FMIN, BANWID $
   3. DATA SET 10 : TRAMAT 1 (TRAMAT (NxN) at spectral line number 1) $.
   4. DATA SET 11 : DISMAT $
   5. DATA SET 10 : TRAMAT 2 $
   6. DATA SET 10 : TRAMAT 3 $

   : : :
   M+3.DATASET 10 : TRAMAT M $

(c) Cross p.s.d. force calculations with calculated receptance matrix.
   1. DATA SETS 1, 2, 3, 4, 5, 6, 7 and 8 (see sub-sections 2.6, 3.2, 4.2 and 6.3) $$$$$$$$
   2. DATA SET 9 : TYPE, EXPER, POINTS, FMIN, BANWID $
   3. DATA SET 11 : DISPOW 1 (DISPOW (NxN) at spectral line number 1) $
   4. DATA SET 11 : DISPOW 2 $
   5. DATA SET 11 : DISPOW 3 $

   : : :
   M+2.DATASET 11 : DISPOW M $

(d) Cross p.s.d. force calculations with experimental receptance matrix.
   1. DATA SETS 1, 2, 3, 4, 5, 6, 7 and 8 (see sub-sections 2.6, 3.2, 4.2 and 6.3) $$$$$$$$
   2. DATA SET 9 : TYPE, EXPER, POINTS, FMIN, BANWID $
   3. DATA SET 10 : TRAMAT 1 $
   4. DATA SET 11 : DISPOW 1 $
   5. DATA SET 10 : TRAMAT 2 $
   6. DATA SET 11 : DISPOW 2 $

   : : :
   2M+1.DATASET 10 : TRAMAT M $
   2M+2.DATASET 11 : DISPOW M $
7. MULTICASE RUNS USING DYNMOD

DYNMOD can be used in multicase mode; however, great care must be exercised in ordering the data cards. Depending upon the information required, a single DYNMOD case can use as many as 11 data sets. Each data set provides information for the LAMPS READn (....) subroutine. In each multicase run a set of data cards must be provided every time READn (....) is called. If a single DYNMOD run requires M data sets, then 5 loops through the program in multicase mode will require 5M sets of data cards.

Once all the matrices have been defined in the first case (or loop) of a DYNMOD run, the data sets of subsequent cases need only contain the matrix alterations or dummy data. (i.e. For dummy data simply input one matrix of the same value as in the previous case and terminate with a dollar sign.) A separate data card must be used for each data set.
SECTION 3

DYNMOD EXAMPLES
Examples of the use of DYNMOD are presented in this section. In each case the input data required for a particular calculation and the output provided by the computer program are included. Some examples discuss the response and stability characteristics of CANDU-BLW and UKAEA-SCHW reactor fuel assemblies. The results presented in these cases are purely hypothetical and are not relevant to current or proposed fuel designs.

Other examples of DYNMOD usage and the methods used to interpret the program output are discussed in AECL Report 5976 (ref.1).

1. NATURAL FREQUENCIES AND MODESHAPES: DYNMOD MAIN PROGRAM

The free vibration and dynamic stability characteristics of a SCHW reactor fuel assembly in axial flow are examined in this example. A pinned constraint at the end of the fuel stringer and structural damping effects have been included in the dynamic model. The single phase coolant flow rate is 27.72 kg/s. A sketch of the idealized fuel assembly is shown in figure 28. The four sets of data cards which describe the fuel and channel system are not required. The data cards WHICH = 0, SHAKER = 0, DATA = 0, and TYPE = 0 indicate that calculations by the accessory sub-programs are not required. All input information is printed by the LAMPS input subroutine and can be seen in figure 30.

DYNMOD calculates and prints the 56 natural frequencies and 56 modeshapes of the fuel assembly (i.e., 2 x 28, the number of degrees of freedom of the system; 29 bundles, end-pinned); these are presented in figures 31 and 32. Because of the volume of output, only the first four pairs of modeshapes are shown.

The program execution time was 10.2 seconds and the highest storage position used in the complex array SPACE was 31261; ref.2.

Figures 6 and 7 are typical examples of the graph plotter output from DYNMOD. The plot in figure 6 depicts the real part of the first 15 modeshapes of an end-pinned SCHWR stringer in axial flow. Figure 7 is a plot of the moduli of the first 15 complex modeshapes of a SCHWR stringer in axial flow which has pinned constraints at both ends of the fuel bundle; i.e., because of the axial flow, the elements of the eigenvectors of the structure are complex numbers (the modeshapes are non-stationary.)
2. ADDITIONAL CONSTRAINTS: SUBROUTINE PINNED

The cards shown in figure 33 cause DYNMOD to calculate the natural frequencies and modeshapes of a uniform clamped-clamped beam in the absence of structural damping, fluid and gravity. The beam, idealized into 12 bundles, is 11 metres long, has a mass of 1100 kg and a flexural rigidity of $5 \times 10^4$ Newton.metres$^2$. (Methods used to idealize integral structural members, like beams, rods, pipes, etc., are discussed in section 13.5 of ref.1.) The DYNMOD output for the clamped beam data is presented in figure 34.

Two alternative DATA SET 4 cards are shown at the bottom of fig. 33. These describe a clamped-intermediate pinned beam and a pinned-pinned -pinned beam respectively. The first three modeshapes and natural frequencies of the constrained beams are shown in fig. 35.

For the case of the clamped-clamped beam, the program execution time was 1.2 seconds and the highest storage location reached in array SPACE was 5941.

3. TRANSIENT AND STEADY HARMONIC RESPONSE: SUBROUTINE FORCES

The transient and steady response of a uniform cantilever to a single harmonic force applied to the tip are calculated in this example. Eight sets of data are required; these are shown in figure 36. The cantilever idealization is identical to the uniform beam in example 2 above. Structural damping, hydrodynamic and gravitational effects have again been ignored.

After the natural frequencies and modeshapes have been output, a rectangular matrix is printed, column by column, under the heading 'Transient Harmonic Response'; see figure 37. The Jth row of this matrix represents the time domain response of the cantilever observed at the end of bundle J. Figure 38 shows the transient response plot for the cantilever tip. The number of points plotted in the time domain correspond to the number of columns in the transient response matrix.

All elements of the matrix are complex numbers. This is because exponential notation is used to describe the applied forces in DYNMOD. The harmonic force applied at the cantilever tip in this example is a 'rotating vector' given by $(1.0 + 10.0) \exp 1.2\pi 0.1t = \exp 10.2\pi t = \cos 0.2\pi t + i \sin 0.2\pi t$. Therefore, the real parts of the transient response matrix elements give the response to the force $F_C = \cos 0.2\pi t$ Newtons, while the imaginary parts give the response to the force $F_S = \sin 0.2\pi t$ Newtons. The time domain responses to forces $F_C$ and $F_S$ are given by curves C and S respectively in figure 38.

DYNMOD will calculate the transient response of a structure to any number of lateral forces of any amplitude and relative phase. The response
output is projected on the real and imaginary planes as in the simple example above.

The steady harmonic response to the applied force in the example is printed in fig. 39. This consists of a single column vector of complex numbers representing the amplitude and relative phase of the response at each bundle end. Element J of the vector describes the steady harmonic response at the end of bundle J. (If this complex number is given by \(a + ib\) then the steady vibration amplitude is \((a^2 + b^2)^{\frac{1}{2}}\) and the phase angle, relative to the applied force, \(\tan^{-1}(\frac{b}{a})\).) A plot of the steady response is shown in figure 39; the relative phase angles are very small and have been ignored.

The calculations required 2.1 seconds for execution and used a maximum of 6776 locations in the complex array SPACE.

4. RECEPTANCE AND MOBILITY: SUBROUTINE RECMOB

The receptance and mobility of a uniform pinned-pinned beam excited at the centre can be calculated by DYNMOD using the data cards shown in figure 40. Again, the beam is identical to that used in examples 2 and 3 above, however, in this case a structural damping factor of 0.1 is assumed. After the natural frequencies and modeshapes have been printed, DYNMOD outputs the transfer functions in the form shown in figure 41, i.e., for each spectral line, column vectors of magnitudes and relative phase angles of receptance and mobility, relative to the shaker or exciter position, are printed. The \(J^{th}\) element in a column corresponds to the transfer function magnitude or phase angle at the end of bundle \(J\). Two driven-point receptance spectra are plotted in figure 42. These correspond to shaker positions of 2.5 m and 5.5 m (mid-point) from a beam support.

The transfer function calculations for 80 spectral lines required 8.7 seconds execution time and used a maximum of 5941 locations in the complex array SPACE.

5. RANDOM FORCED RESPONSE: SUBROUTINE RANDOM

Three data input options are available in RANDOM for the description of the applied forces. Examples of each option are given below.

5.1 Discrete Force Spectra Input

This example calculates the response of a pinned-pinned beam to a distributed force. The beam is similar to that described in example 4 above. The distributed forces acting on the beam are 12, 24, 36, 48 and 60 Newtons at the five spectral lines, 1, 2, 3, 4, and 5 Hz respectively, e.g., the 36 Newton force is resolved into twelve in-phase forces...
of 3 Newtons at 3 Hz at each bundle end, the 48 N. force is resolved
into twelve in-phase forces of 4 Newton at 4 Hz, etc. Figure 43 shows
the input data needed for the calculation. DATA SET 8 inputs the force
spectra as a (12 x 5) matrix DFORCE. After printing the natural fre-
cuencies and modeshapes, the response spectrum at the end of each bundle
is output. This is followed by the mean power spectrum of response at
the end of each bundle and the mean square and root mean square values
of response along the beam. The response output is shown in figure 44.

The program execution time was 2.4 seconds and a maximum of 8586 storage
locations were used in array SPACE.

5.2 Cross-Power Spectral Density of Force Input

This option is used to input correlated force data. Using the data cards
shown in figure 45 DYNMOD will calculate the response of an inverted
CANDU-BLW reactor fuel assembly to the forces described in figure 46. The
force data are input in cross power spectral density form. This entails
supplying the (12 x 12) P.S.D. matrix PSDFOR with information for each
of the ten spectral lines of interest. Matrix PSDFOR, which is printed
on input by DYNMOD, is shown in figure 47 for the first two spectral lines.
After printing the complex natural frequencies and modeshapes, the auto-
power spectrum of displacement at each bundle end is printed. The auto-
power spectra output is shown in figure 48 together with the mean and
root mean square values of the response. The results are plotted in figure
49.

These calculations required 11.0 seconds execution time and the maximum
storage location reached in array SPACE was 14206.

5.3 Auto P.S.D. of Force Input

When the force data are uncorrelated-random, this input option is the
most convenient to use. The data cards used to model the spectra of
three uncorrelated forces applied to the ends of bundles 2, 3, and 4 of
the pinned-pinned beam in example 4 (without structural damping) can be
seen in figure 50.

Here the matrix UNCORF represents the auto p.s.d. of the uncorrelated
forces. UNCORF is shown printed in fig. 51. Each force spectrum con-
tains 15 frequency points and resembles a triangular profile with a
peak at zero frequency.

Figures 52 and 53 show the autopower p.s.d. of displacement at the end
of each bundle, and the mean and root mean square values of the response
along the beam. The values of auto p.s.d. of displacement are printed
in rows in spectral order. This example required 10.9 seconds execution
time and the calculations occupied a maximum of 16666 storage locations
in array SPACE.
6. FORCE CALCULATIONS: SUBROUTINE CALFOR

The data input and output for the resolved force calculations is very similar to that required by subroutine RANDOM. Two basic options are available for the input of displacement data; these are discussed below.

6.1 Discrete Displacement Spectra Input

For this simple example, it is assumed that displacement measurements have been made along the length of a pinned-pinned beam at a frequency of 1 Hz. The beam is similar to that idealized in example 4. All displacement measurements coincide with the bundle ends in the dynamic model. Figure 54 shows the data cards required to deduce the nature of the resolved forces responsible for the beam deflections. The in-phase displacements are input into the column vector DISMAT.

After printing the natural frequencies and modeshapes of the beam, DYNMOD outputs the cross power (intensity) spectrum of force for each spectral line (one in this case). This matrix is shown in figure 55 and its output is optional. Because the twelve bundle beam has only eleven degrees of freedom, the cross power matrix is reduced to the order (11 x 11) as explained in ref. 1.

The discrete force spectrum and auto power (intensity) spectrum of force are then printed. These data are followed by the mean square and root mean square values of the forces resolved at the bundle ends; see fig. 55.

The calculation required 1.6 seconds execution time and the highest storage location reached in array SPACE was 5939.

6.2 Cross Power Spectra Density of Displacement Input

It is possible to calculate the nature of the resolved forces acting on a structure from a knowledge of the cross p.s.d. of its response to the forces. In the example described here it is assumed that an averaged cross p.s.d. matrix of displacements has been measured along a pinned-centre pinned-pinned uniform beam. The cross p.s.d. displacement matrix was measured for two frequencies. Each displacement matrix DISPOW is of order (12 x 12) corresponding to the 12 bundle idealization of the beam. All elements of DISPOW, at both frequencies, are unity. The data cards required by DYNMOD to calculate the resolved forces are shown in fig. 56.

At output, if requested, a reduced cross p.s.d. matrix of the resolved forces is provided for each spectral line; these are shown in figure 57. Because the idealized bundle string has 10 degrees of freedom the reduced cross p.s.d. force matrices are of order (10 x 10).

Next, the auto p.s.d. of the applied forces are printed for each bundle end. Finally this is followed by the mean square and root mean square
values of the forces resolved along the beam; see figure 58. (Lateral forces coinciding with constraint positions are assumed zero.)

1.7 seconds execution time was required for these calculations and a maximum of 5927 storage locations in array SPACE were used.
SECTION 4

DYNMOD ACCURACY
SECTION 4

DYNMOD ACCURACY

This section briefly describes the methods used to check the accuracy of the dynamic modelling program, DYNMOD. All the test examples outlined below involve the comparison of DYNMOD output with analytical calculations of the vibration characteristics of uniform beams in the absence of axial flow. For cases involving more complicated structures, with or without axial flow, experimental results must be obtained for comparison with the DYNMOD output data. Some examples of computed versus experimental results are presented in sub-section 6.

1. NATURAL FREQUENCIES

The first three natural frequencies of a uniform cantilever, calculated analytically and by DYNMOD are shown in the table below.

<table>
<thead>
<tr>
<th>NATURAL FREQUENCY</th>
<th>MODE 1</th>
<th>MODE 2</th>
<th>MODE 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical</td>
<td>0.103</td>
<td>0.648</td>
<td>1.815</td>
</tr>
<tr>
<td>Computed</td>
<td>0.103</td>
<td>0.651</td>
<td>1.831</td>
</tr>
<tr>
<td>% Discrepancy</td>
<td>0.01</td>
<td>0.49</td>
<td>0.88</td>
</tr>
</tbody>
</table>

The cantilever was 11 metres long with a mass of 1100 kg and a flexural rigidity of $5 \times 10^4$ Nm². It was assumed to be in a gravity and flow free environment. A sketch of the 12 bundle cantilever idealization is shown in figure 59. (The integral beam was modelled according to the recommendations in chapter 13 of ref. 1.)

Figure 60 is a plot of normalized first mode natural frequency against the number of bundles in the cantilever idealization. The curve clearly shows the improvement in accuracy obtained by increasing the number of degrees of freedom of its structure. The bundle idealizations used in this numerical experiment are shown in figure 61; the boost in accuracy obtained by using 'half bundles' at the beam ends is clearly seen in figure 60.

2. MODESHAPES

Figure 62 shows the first three modeshapes of a simply supported uniform beam calculated by DYNMOD. A twelve bundle idealization is used.
for the beam, with 'half-bundles' at the ends. The normalized mode-
shapes of the beam are given by $\sin \frac{n\pi x}{L}$, where $n$ is the mode number, and 
x is the distance along the beam of length $l$. The table in figure 63 compares the mode-
shapes predicted by DYNMOD with the analytical values. Such a high degree of accuracy is achieved because the bundle masses are distributed and not lumped.

3. VISCOUS DAMPING

In DYNMOD, viscous damping has been incorporated as a drag force per unit length given by the expression

$$\frac{1}{2} \rho \cdot n \cdot D \cdot C_D \cdot \frac{\partial v}{\partial t},$$

where $\rho$ is the fluid density,

$n$ is the number of fuel elements in the relevant bundle,

$D$ is the diameter of the fuel elements,

and $C_D$ is the viscous drag coefficient at very low relative velocities (bundle to fluid), $\frac{\partial v}{\partial t}$.

The driven-point receptance of the mid-point of a simply supported viscous damped beam was calculated analytically for the first natural frequency. This result was then compared with the value computed by DYNMOD to test the accuracy of the viscous damping formulation in the dynamic model. The calculations are described below.

At resonance, force and displacement are in quadrature. The work done by an applied force at the beam centre equals the energy dissipated by the damping mechanism.

With reference to figure 64, the instantaneous rate of doing work at time $t$ is $FV \sin^2\omega t$, where $F$ is the applied force magnitude, $V$ is the velocity magnitude at the point of application of the force and $\omega/2\pi$ is the resonant frequency.

The work done per cycle by the applied force in overcoming damping is

$$\int_0^T FV \sin^2 \omega t \, dt = \frac{FVT}{2},$$

where $T = \frac{2\pi}{\omega}$.

An approximation for the energy dissipated by damping for a 12 bundle beam idealization can be calculated as follows.
At time $T$ the rate of dissipating energy by damping at bundle $i$ is 
$\frac{F_{Di} V_{Bi} \Sin^2 wT}{2}$, where $F_{Di}$ is the peak viscous drag force acting on bundle $i$ and $V_{Bi}$ is the magnitude of the bundle velocity; see figure 65. Therefore, as before, the energy dissipated by bundle $i$ per cycle is 

$$\frac{F_{Di} V_{Bi} T}{2}$$ \hspace{1cm} (4.3)

The energy dissipated by damping for all the bundles in the beam is

$$\sum_{i=1}^{12} \frac{F_{Di} V_{Bi} T}{2}$$ \hspace{1cm} (4.4)

The viscous drag force per unit length is given by equation 4(1). If the fluid surrounding the beam of diameter 0.047 m has a density of 1000 kg /m$^3$, then for a drag coefficient of $C_D = 1.0$ m/s, the drag force acting on bundle $i$ is $23.55 V_{Bi} \ell_i$ Newtons, where $\ell_i$ is the bundle length in metres.

The energy dissipated by damping at bundle $i$, from equation 4(3) is

$$\frac{(23.55 V_{Bi} \ell_i) (V_{Bi} T)}{2} = 11.8 \frac{V_{Bi}^2 \ell_i}{f}$$ \hspace{1cm} (4.5)

(N.m), where $f$ in this example is the first natural frequency of the beam.

If the beam has a mass of 1100 kg, a length of 11 metres and a flexural rigidity of $5 \times 10^4$ Nm$^2$, then $f = 0.29$ Hz and equation 4(5) becomes

$$40.6 \frac{V_{Bi}^2 \ell_i}{f}$$ \hspace{1cm} (4.6)

Because

$$V_{Bi} = (D_{N_i} S)w,$$

where $D_{N_i}$ is the normalized amplitude of the motion at the mid point of bundle $i$ and $S$ is a normalizing factor (which converts $D_{N_i}$ into absolute deflection), the work done by the damping forces acting on bundle $i$ is

$$40.6 w^2 D_{N_i}^2 \ell_i S^2$$ \hspace{1cm} (N.m) \hspace{1cm} (4.7)

Hence the work done by the damping forces on the beam per cycle at the first natural frequency is

$$40.6 w^2 S^2 (\sum_{i=1}^{12} D_{N_i}^2 \ell_i)$$ \hspace{1cm} (N.m) \hspace{1cm} (4.8)

At resonance expressions 4(2) and 4(8) can be equated

$$40.6 w^2 S^2 (\sum_{i=1}^{12} D_{N_i}^2 \ell_i) = \frac{FVT}{2} = \frac{W. D_{NPeak} S}{0.58}$$ \hspace{1cm} (4.9)
where \( F = 1 \) Newton and \( D_{N_{\text{Peak}}} \) is the normalized deflection at the mid point of the beam.

4(9) reduces to the following form

\[
S = \frac{D_{N_{\text{Peak}}}}{42.91 \left( \sum_{i=1}^{12} D_{N_i}^2 \ell_i \right)} \text{ (metres)} \tag{4(10)}
\]

As the modeshapes of a simply supported beam are sinusoidal

\[
\sum_{i=1}^{12} D_{N_i}^2 \ell_i \text{ is easily evaluated.}
\]

Finally, the auto receptance \( R_0 \) at the mid point of the beam is given by

\[
S \cdot D_{N_{\text{Peak}}}
\]

Hence, from 4(10)

\[
R_0 = \frac{D_{N_{\text{Peak}}}}{42.91 \left( \sum_{i=1}^{12} D_{N_i}^2 \ell_i \right)^2} \text{ (mm/N)} \tag{4(11)}
\]

Evaluation of 4(11) gives \( R_0 = 4.321 \text{ mm/Newton} \). The DYNMOD result is 4.295 mm/Newton. The discrepancy in the two results is 0.6%.

Figure 66 shows the effect of viscous drag coefficient on the auto receptance of the mid point of a simply supported beam in the region of the first natural frequency.

4. STRUCTURAL DAMPING

The numerical experiments used to test the structural damping facility in DYNMOD were similar to the viscous damping tests. The deflection at the mid point of the simply supported uniform beam of section 4.3 is given by the expression (Vibration and Shock in Damped Mechanical Systems - J. Snowdon)

\[
y_o = F_o \left( \lambda a \right) \left( \text{Sinh}(\lambda a)\text{Cos}(\lambda a) - \text{Cosh}(\lambda a)\text{Sin}(\lambda a) \right) e^{i\omega t} \tag{4(12)}
\]

\[
\frac{2w^2 M_B}{\text{Cosh}(\lambda a) \text{ Cos}(\lambda a)}
\]

where \( F_o \) is the force applied at the beam centre and \( w \) is its angular frequency, \( a \) is half the beam length, and \( M_B \) is the beam mass.
A is given by

\[ A = \left(\frac{4\pi w^2}{E\nu g}\right)^{\frac{1}{2}} = \left(\frac{\nu 100}{K}\right)^{\frac{1}{2}} \]

where \( K \) is the inter-bundle bending stiffness for the 12 bundle beam idealization. The driven point receptance at the mid point of the beam was calculated with and without structural damping for a frequency of 0.28 Hz (the first natural frequency is 0.29 Hz).

(i) No structural damping.

Evaluation of expression 4(12) gives a receptance value of 7.86 mm/N @ 0.0°Arc; DYNMOD provides the value 7.97 mm/N @ 0.0°Arc. The discrepancy between the results is 1.4%. This includes rounding errors incurred in the evaluation of 4(12).

(ii) With structural damping.

A structural damping factor of \( g = 0.25 \) was assumed for each bundle. The damping is incorporated as an imaginary interbundle stiffness, i.e., \( \lambda \) in 4(12) is now given by

\[ \lambda = \left(\frac{\nu 100}{K^*}\right)^{\frac{1}{2}} \]

where \( K^* = K(1+ig) \).

For a uniform beam this approach is compatible with both of the methods described in ref.(1) for the introduction of structural damping into the dynamic model.

Evaluation of 4(12) gives

\[ \frac{F_0(\lambda a)}{2\nu^2 M_B} = (2.244-i 0.138) \times 10^{-4} \]

and

\[ \left(\frac{\sinh(\lambda a) - \cos(\lambda a) - \cosh(\lambda a) \sin(\lambda a)}{\cosh(\lambda a) \cos(\lambda a)}\right) = (-3.097 + i 18.824). \]

Hence \( y_o = (-5.73 + i 120.23) \times 10^{-4} \) m/N

\[ = 2.103 \text{ mm/N @ 74.2°Arc}. \]

DYNMOD computations give a value of 2.140 mm/N @ 74.2°Arc.

The receptance magnitude discrepancy between the two results is 1.8%.

Figure 67 shows the auto receptance at the mid point of a simply supported uniform beam in the region of the first natural frequency for a variety of structural damping factors.
5. STEADY FORCED RESPONSE

A steady load of 1 Newton was applied to the tip of the cantilever described in sub-section 1. The deflection of the cantilever beam was calculated analytically and by DYNMOD. The results are compared in the table below.

<table>
<thead>
<tr>
<th>Distance from clamped end of cantilever (m)</th>
<th>0.5</th>
<th>1.5</th>
<th>2.5</th>
<th>3.5</th>
<th>4.5</th>
<th>5.5</th>
<th>6.5</th>
<th>7.5</th>
<th>8.5</th>
<th>9.5</th>
<th>10.5</th>
<th>11.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical</td>
<td>0.03</td>
<td>0.24</td>
<td>0.64</td>
<td>1.20</td>
<td>1.92</td>
<td>2.77</td>
<td>3.73</td>
<td>4.78</td>
<td>5.90</td>
<td>7.07</td>
<td>8.27</td>
<td>8.87</td>
</tr>
<tr>
<td>DYNMOD</td>
<td>0.21</td>
<td>0.62</td>
<td>1.19</td>
<td>1.92</td>
<td>2.76</td>
<td>3.72</td>
<td>4.78</td>
<td>5.90</td>
<td>7.07</td>
<td>8.27</td>
<td>8.88</td>
<td></td>
</tr>
</tbody>
</table>

Analytical values were obtained using the expression, \( y = \frac{P}{6EI} x^2 (3L-x) \), where \( P \) is the load applied at the cantilever tip, \( y \) is the deflection a distance \( x \) from the clamp and \( EI \) is the flexural rigidity of the cantilever of length \( L \).

The steady response of the cantilever tip to harmonic loads, in the region of the first natural frequency, can be seen in figure 39.

6. TRANSFER FUNCTIONS

Figures 42, 66 and 67 are auto-receptance plots for a uniform beam. Comparison of these completed curves with analytical results serves as a useful check on the accuracy of DYNMOD. In general, more detailed tests of the structural model will involve the fitting of computed curves to experimental data. The mobility curves of figures 68 and 69 show computed values fitted to experimental data (ref.1). Figure 68 shows the driven point mobility at the mid-point of a structure consisting of two CANDU-BLW Reactor fuel bundles attached to a clamped central support rod. The computed and measured driven point mobilities of a full length CANDU-BLW Reactor LS-3 fuel string are compared in figure 69. Each of these examples demonstrates the basic accuracy of the dynamic model. A comparison of the computed and measured transfer functions of fuel assemblies in axial flow is not yet available.

7. RANDOM FORCES

a) As a preliminary test of subroutine RANDOM, a variety of trivial cases were run to ensure that the results from subroutines RANDOM and FORCES
were similar. For example, a distributed harmonic load of 1N/m at 1Hz was applied to the idealized simply supported beam described in section 4.2. The response of the beam was calculated by FORCES and RANDOM and the deflection results are printed in figure 70. It can be seen that the two outputs are in excellent agreement.

b) Further simple tests of subroutine RANDOM were undertaken to ensure that the three input options of DATA SET 8 give similar response results, e.g., the data cards

\[
\begin{align*}
\text{DFORCE} & \quad (12,1) \quad 5*0 \quad 1.414 \quad 6*0 \quad $ \quad \text{(Newtons)}, \\
\text{PSDFOR} & \quad (12,12) \quad 65*0 \quad 1 \quad 78*0 \quad $ \quad \text{(Newtons}^2/\text{Hz}), \\
\text{and} \quad \text{UNCORF} & \quad (12,1) \quad 5*0 \quad 1 \quad 6*0 \quad $ \quad \text{(Newtons}^2/\text{Hz}),
\end{align*}
\]

for discrete force spectra, correlated PSD and uncorrelated PSD of force respectively, give the same output data. (In the cases of PSDFOR and UNCORF, BANWID = 1.0.)

c) The accuracy of the response to complicated forcing functions was obtained by comparing the output of RANDOM with that from the AECL program PIPEAU (ref.3; PIPEAU was developed for the analysis of heat exchanger tube vibrations). Two examples are described below.

(i) Using the simply supported beam idealization of earlier sections, the mean square deflection of the mid point of the beam to the forcing function described in figure 71 was calculated by PIPEAU and DYNMOD.

The results were as follows

\[
\begin{array}{|c|c|}
\hline
\text{DFORCE} & 1.37 \times 10^{-5} \quad \text{(m}^2) \\
\hline
\text{PIPEAU} & 1.31 \times 10^{-5} \quad \text{(m}^2) \\
\hline
\text{DISCREPANCY} & 4.5\% \\
\hline
\end{array}
\]

In the PIPEAU calculation, only the contributions from the first three modes were considered. Had the responses of the other normal modes been considered, the percentage discrepancy would be less.

(ii) The RMS response of the beam centre to the power spectral density of force functions shown in figure 72 are as follows

\[
\begin{array}{|c|c|}
\hline
\text{Force function} & \text{RMS mid point displacement} \\
\hline
\text{A} & \text{DYNMOD} \quad 6.20 \text{ mm} \\
\text{} & \text{PIPEAU} \quad 6.05 \text{ mm} \\
\text{} & \text{DISCREPANCY} \quad 2.5\% \\
\hline
\end{array}
\]
Force function | RMS mid point displacement
---|---
B | 11.99 mm
DYNMOD | PIPEAU
11.66 mm | DISCREPANCY
2.8% |
C | 8.97 mm
DYNMOD | PIPEAU
8.64 mm | DISCREPANCY
3.8% |
D | 16.21 mm
DYNMOD | PIPEAU
15.72 mm | DISCREPANCY
3.0%

(The forces A, B, and C are assumed to be uncorrelated)

In all these examples, the percentage discrepancies in the results would be less if the responses of all the normal modes had been considered in the PIPEAU calculations.

8. FORCE CALCULATIONS

An assessment of the accuracy of the resolved force results, calculated by subroutine CALFOR, was obtained by a 'reverse comparison' with subroutine RANDOM.

The response of a structure to an arbitrary forcing function is calculated by RANDOM. The displacement values are then supplied as input for CALFOR and resolved force data are evaluated. Finally, these forces are compared with the arbitrary force values used originally by RANDOM.

Consider the following example. Application of a distributed harmonic load of 11.0 N/m @ 1.0 Hz to the uniform beam described in section 4.7 gives the harmonic response described in figure 70. If this displacement data, correct to 6 significant figures, are now used as input for CALFOR, the resolved forces printed in figure 55 are obtained. These force results are in good agreement with the original distributed load of 1.0 N/m.
It is interesting to note that inaccuracies in defining the displacement data required by CALFOR can result in a rapid divergence in the accuracy of the calculated resolved forces. In the case of the above example, the resolved force values obtained when the displacement data are specified to 3 significant figures (instead of 6) are shown in figure 73. The discrepancy between the individual resolved force results of figures 55 and 73 is clearly seen.

REFERENCES


FIGURE 1: Typical input data cards for programs written in the LAMPS matrix processor language. (Note: the lowest card in the figure is the first in the data set.)
**FIGURE 2:** A typical comprehensive DYNMOD input data card pack. Data set 10 is optional and is not shown here. The lowest card in the figure is the first in the card deck.
FIGURE 3: Input data cards for natural frequencies, modeshapes and resolved force calculations. Subroutines Pinned, Forces, Remob and Random have been avoided by using the Data set 4, 5, 6 and 7 cards shown here.
FIGURE 4: DYNMOD Title card.
This card is the first in the input data pack.
FIGURE 5: ARGAND Diagram of complex eigenvalues.
(All vibration modes are unstable or buckled.)
FIGURE 6: First 15 modeshapes of an end-pinned SGHWR fuel assembly in axial flow. The real parts of the complex modeshapes are plotted.
FIGURE 7: First 15 modeshapes of a SGHWR fuel assembly end-pinned and pinned at the top of the fuel bundle. The moduli of the complex modeshapes are plotted.
FIGURE 8: Data set 1 card. (DYNMOD Main Program.)
**Idealized Fuel String**

1.3 m

**Gravitational Acceleration**

7 kgs

0.5 m

30 kgs

30 kgs

0.25 m

5 kgs

**Bundle Cross-Sections**

0.06 m

**Matrix Construction**

\[ Y = (1) \text{ (i.e. top supported)} \]

\[ G = (9.806) \text{ m/s}^2 \]

\[ L = \begin{bmatrix} 1.30 \\ 0.50 \\ 0.50 \\ 0.25 \end{bmatrix} \text{ m} \]

\[ M = \begin{bmatrix} 7.0 \\ 30.0 \\ 30.0 \\ 5.0 \end{bmatrix} \text{ kg} \]

\[ NEL = \begin{bmatrix} 1 \\ 18 \\ 18 \\ 1 \end{bmatrix} \]

\[ DIAM = \begin{bmatrix} 0.06 \\ 0.02 \\ 0.02 \\ 0.11 \end{bmatrix} \text{ m} \]

**Figure 9 Data Set 2,**

Structural Parameters for a Four Bundle Assembly,

(Physical Shape and Dimensions)
FIGURE 10  DATA SET 2 (CONTINUED).
STRUCTURAL PARAMETERS FOR A FOUR BUNDLE ASSEMBLY.
(STIFFNESS AND STRUCTURAL DAMPING).

MATRICE CONSTRUCTION

\[
\begin{align*}
\text{K}_{\text{CST}} &= \begin{bmatrix}
\text{K}_{\text{CST}1} \\
\text{K}_{\text{CST}2} \\
\text{K}_{\text{CST}3} \\
\text{K}_{\text{CST}4}
\end{bmatrix} = \begin{bmatrix}
10^5 \\
7.5 \times 10^3 \\
7.5 \times 10^3 \\
7.5 \times 10^3
\end{bmatrix} \text{ Nm/rad} \\
\text{K}_{\text{END}} &= \begin{bmatrix}
\text{K}_{\text{END}1} \\
\text{K}_{\text{END}2} \\
\text{K}_{\text{END}3} \\
\text{K}_{\text{END}4}
\end{bmatrix} = \begin{bmatrix}
10^5 \\
10^5 \\
10^5 \\
10^5
\end{bmatrix} \text{ Nm/rad} \\
\text{K}_{\text{PAR}} &= \begin{bmatrix}
\text{K}_{\text{PAR}1} \\
\text{K}_{\text{PAR}2} \\
\text{K}_{\text{PAR}3} \\
\text{K}_{\text{PAR}4}
\end{bmatrix} = \begin{bmatrix}
10^5 \\
6.5 \times 10^3 \\
6.5 \times 10^3 \\
10^5
\end{bmatrix} \text{ Nm/rad} \\
\text{G}_{\text{CST}} &= \begin{bmatrix}
\text{G}_{\text{CST}1} \\
\text{G}_{\text{CST}2} \\
\text{G}_{\text{CST}3} \\
\text{G}_{\text{CST}4}
\end{bmatrix} = \begin{bmatrix}
0.005 \\
0.005 \\
0.005 \\
0.005
\end{bmatrix} \\
\text{G}_{\text{END}} &= \begin{bmatrix}
\text{G}_{\text{END}1} \\
\text{G}_{\text{END}2} \\
\text{G}_{\text{END}3} \\
\text{G}_{\text{END}4}
\end{bmatrix} = \begin{bmatrix}
0.008 \\
0.06 \\
0.06 \\
0.06
\end{bmatrix} \\
\text{G}_{\text{PAR}} &= \begin{bmatrix}
\text{G}_{\text{PAR}1} \\
\text{G}_{\text{PAR}2} \\
\text{G}_{\text{PAR}3} \\
\text{G}_{\text{PAR}4}
\end{bmatrix} = \begin{bmatrix}
0.003 \\
0.05 \\
0.05 \\
0.007
\end{bmatrix} \\
\text{DCTOPX} &= (0.001) \\
\text{DETOPX} &= (0.002)
\end{align*}
\]
FIGURE 11: Data set 2 cards. (DYNMOD Main Program.)
The lowest card in the figure is the first in the data set.
FIGURE 12: Data set 3 cards. (DYNMOD main program.)
The cards read upwards.
FIGURE 13: Data set 4 cards. (Subroutine PINNED.)
FIGURE 14: Data set 5 cards. (Subroutine FORCES.)
FIGURE 15 CONSTRUCTION OF DATA SET 6. WITH REFERENCE TO A TYPICAL DYNMOD RECEPTANCE PLOT. (SUBROUTINE RECMOB).
FIGURE 16  CONSTRUCTION OF DATA SET 7.
WITH REFERENCE TO FORCES EXPRESSED AS DISCRETE SPECTRA.
(FOR A N BUNDLE STRING THERE WILL USUALLY BE N DISCRETE
FORCE SPECTRA CURVES).
(SUBROUTINE RANDOM).
AVERAGED CROSS P.S.D. FORCE BETWEEN TWO BUNDLE ENDS \((N^2/Hz)\) (ARBITRARY SCALE)

- MAGNITUDE
- RELATIVE PHASE

X MAGNITUDE
O RELATIVE PHASE

**Figure 17** CONSTRUCTION OF DATA SET 7 WITH REFERENCE TO FORCES EXPRESSED IN AVERAGED POWER SPECTRAL DENSITY FORMAT. (FOR A N BUNDLE STRING THERE WILL USUALLY BE N² P.S.D. CURVES) (SUBROUTINE RANDOM)
\[ f = P \exp(i\omega t) \]

\[ \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} \exp(i\omega t) \]

\[ D\text{FORCE} = \begin{bmatrix} (5+i2) & (3+i4) & (3-i) & (3+i5) & (1-i2) \\ (1-i3) & (5-i2) & (4+i5) & (3-i2) & (1+i) \\ (1+i) & (3+i5) & (3-i3) & (5+i5) & (1-i2) \\ (3+i4) & (2-i4) & (3-i2) & (1-i4) & (4+i2) \end{bmatrix} \]

**FIGURE 18** DATA SET 8 (OPTION 1).

DISCRETE SPECTRA OF FORCES ACTING ON A FOUR BUNDLE STRING.

(SUBROUTINE RANDOM)
FIGURE 19: Data set 8 (Option 1) Cards. (Subroutine RANDOM)
The cards read upwards.
FIGURE 20  DATA SET 8 (OPTION 2).
AVERAGED POWER SPECTRAL DENSITIES OF THE FORCES ACTING ON A TWO BUNDLE STRING
Power spectral densities of Figure 20 converted into the eight p.s.d. matrices PSDFOR (One for each frequency).

FIGURE 21: Data set 8 (Option 2) Card Pack. (Subroutine RANDOM.)
The cards read upwards.
FIGURE 22 DATA SET 8 (OPTION 3)
AVERAGED POWER SPECTRAL DENSITIES OF THE UNCORRELATED FORCES ACTING ON A THREE BUNDLE STRING
FIGURE 23: Data set 8 (Option 3) Cards. (Subroutine RANDOM)
The cards read upwards.
FIGURE 25: Data set 10 (Optional) cards. (Subroutine CALFOR)
The cards read upwards.
FIGURE 26: Data set 11 (Option 1) cards. (Subroutine CALFOR)
The cards read upwards.
FIGURE 27: Data set 11 (Option 2) Cards. (Subroutine CALFOR)
The cards read upwards.
FIGURE 28 SGHWR FUEL STRINGER: IDEALIZED FOR USE IN THE DYNAMIC MODELLING PROGRAM DYNMOD
FIGURE 29: Data cards required to calculate the natural frequencies and
modeshapes of a SGHWR fuel stringer.
FIGURE 30 (Parts 1, 2, 3, 4, 5): INPUT DATA FOR SGWR FUEL VIBRATION CHARACTERISTICS.
<table>
<thead>
<tr>
<th>Enr.,</th>
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<tbody>
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<td>.53,</td>
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</tbody>
</table>

END OF DATA SET 3.

FIGURE 50 (cont)
(PART 4)
APIN = ( 1.000000, 0.0 )
JPIN = ( 0.0, 0.0 )
KCRAT = ( 0.0, 0.0 )
KFROT = ( 0.0, 0.0 )
SHAKER = ( 0.0, 0.0 )

END OF DATA SET 6
(DEFAULT SETTING)

NCROT = ( 0.0, 0.0 )
DFROT = ( 0.0, 0.0 )
DATA = ( 0.0, 0.0 )

END OF DATA SET 4.

END OF DATA SETS 7 AND 2.
(DEFAULT SETTING)

WHICH = ( 0.0, 0.0 )
TYPE = ( 0.0, 0.0 )

END OF DATA SET 5.
(DEFAULT SETTING)

END OF DATA SETS 9, 10 AND 11.
(DEFAULT SETTING)

Figure 30 (cont)
(Part 5)
NOTE: The negative imaginary part of the eigenvalues does not indicate instability in this case. (The negative sign occurs because imaginary stiffnesses have been used to model structural damping.)

ALL THESE MODES ARE STABLE

<table>
<thead>
<tr>
<th>MODE</th>
<th>NATURAL FREQUENCY</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIRST</td>
<td>5.03Hz</td>
</tr>
<tr>
<td>SECOND</td>
<td>10.26Hz</td>
</tr>
<tr>
<td>THIRD</td>
<td>15.78Hz</td>
</tr>
</tbody>
</table>

FIGURE 31: The natural frequencies of a SGHWR fuel string in axial flow corresponding to the input data of Figure 30.
First four modeshape pairs of a SGHR fuel stringer in axial flow corresponding to the data in Figure 30.
FIGURE 33: Data cards required for the calculation of the natural frequencies and
modeshapes of a clamped-clamped uniform beam.
FIGURE 34: Natural frequencies and first four mode shape pairs of a clamped-clamped beam.
UNIFORM CLAMPED CLAMPED BEAM

UNIFORM CLAMPED-MID-PINNED BEAM

UNIFORM PINNED-PINNED-PINNED BEAM

FIGURE 35 FIRST THREE NATURAL FREQUENCIES AND MODESHAPES OF A UNIFORM BEAM CORRESPONDING TO THE INPUT DATA OF FIGURE 33
FIGURE 36: Data cards required for the calculation of the transient and steady harmonic response of a forced cantilever. The cards read upwards.
FIGURE 37: Transient response of a uniform cantilever to a harmonic force applied at the tip. The first fifteen time-steps are shown.
CASE 1
FORCING FREQUENCY NEAR FIRST RESONANCE

\( F = 1 \text{N}, \ f = 0.1 \text{Hz} \)
(FIRST NATURAL FREQUENCY = 0.1034 Hz)

RESPONSE TO
SINE FUNCTION

RESPONSE TO
COSINE FUNCTION

FIGURE 38  TRANSIENT RESPONSE OF A UNIFORM CANTILEVER TO HARMONIC FORCES APPLIED AT THE TIP
UNIFORM BEAM EXPERIMENT

STEADY HARMONIC RESPONSE = 

\( (.1909108E-04, .3165394E-22) \)
\( (.3640675E-02, .2812198E-20) \)
\( (.1037652E-01, .2274724E-20) \)
\( (.1976076E-01, -.2244101E-20) \)
\( (.3133609E-01, -.7325730E-20) \)
\( (.4466108E-01, -.1070314E-19) \)
\( (.5931928E-01, -.1211925E-19) \)
\( (.7492966E-01, -.1267527E-19) \)
\( (.9115818E-01, -.1427107E-19) \)
\( (.1077300, -.1834569E-19) \)
\( (.1244424, -.2322644E-19) \)
\( (.1328100, -.2575912E-19) \)

**Figure 39**  STEADY HARMONIC RESPONSE OF A UNIFORM CANTILEVER TO A FORCE OF 1 NEWTON @ 0.1 Hz APPLIED AT THE TIP. THE CANTILEVER IS DESCRIBED BY THE DATA IN FIGURE 36
FIGURE 40: Data cards required for the calculation of the driven-point receptance and mobility at the midpoint of a simply supported uniform beam. The cards read upwards.
FIGURE 41: Receptances and mobilities along a simply supported beam for two forcing frequencies. The beam is described by the data in Figure 40.
FIGURE 42  TWO DRIVEN POINT RECEPANCES OF A UNIFORM SIMPLY SUPPORTED BEAM.  
THE BEAM IS DESCRIBED BY THE INPUT DATA OF FIGURE 40.
FIGURE 43: Data cards required for the calculation of the spectral response of a simply supported uniform beam to distributed forces expressed as discrete spectra. The two cards shown above are substituted for the "DATA 0 $" card in Figure 40.
FIGURE 44: Uniform simply supported beam response output corresponding to the input data of Figure 43.
FIGURE 64 (Cont'd)
FIGURE 45: Data cards required to calculate the response of an inverted CANDU-7U reactor fuel assembly to the forces described in Figure 46. The cards read upwards.
FIGURE 46  FLUID FORCES ACTING ON AN INVERTED CANDU-BLW REACTOR FUEL ASSEMBLY
FIGURE 47: Cross power spectral density of force matrix PSDFOR, constructed from the data in Figure 46, for the first two spectral lines.
FIGURE 48: Response of a CANDU-BLW reactor fuel assembly to the forces described in Figure 46.
FIGURE 49  RESPONSE OF CANDU-BLW REACTOR FUEL ASSEMBLY TO THE FORCES IN FIG. 46

(NATURAL FREQUENCIES OF ASSEMBLY)
FIGURE 50: Data cards required to calculate the response of a simply supported beam to uncorrelated forces expressed in auto power spectral density format. The cards shown above are substituted for the "DATA 0 $" card in Figure 40. The cards read upwards.
<table>
<thead>
<tr>
<th>F1</th>
<th>F2</th>
<th>F3</th>
<th>F4</th>
<th>F5</th>
<th>F6</th>
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**FIGURE 51:** Uncorrelated force input data expressed in averaged power spectral density format. The forces are applied at three of the twelve bundle ends of the uniform pinned-pinned beam of Section 3.4.
FIGURE 52:
The response of the uniform pinned-pinned beam to the forces described in Figure 51.
FIGURE 53: The mean square and RMS displacement of the uniform pinned-pinned beam of Section 3.4 to the uncorrelated forces described in Figure 51.
FIGURE 54: Data cards required for the calculation of the resolved forces acting on a simply supported uniform beam. The displacement data is in discrete spectrum form. Only one frequency point is considered in the example. The cards shown above are substituted for the "TYPE 0 $" card in Figure 40. The data cards read upwards.
FIGURE 55: Resolved forces (Newtons) calculated by subroutine CALFOR.
FIGURE 56: Data cards required for the calculation of the resolved forces acting on a pinned-centre pinned-pinned uniform beam. The displacement data is supplied in averaged cross power spectral density format. Two frequency points are considered. The first card shown above is substituted for the data set 4 card in Figure 40. The remaining cards are substituted for the "TYPE 0 $" card in Figure 40. The data cards read upwards.
FIGURE 57: Cross power spectral density of the resolved forces calculated from displacement information.
FIGURE 58: Auto power spectral densities of the forces resolved at the bundle ends of a uniform beam. The mean square and RMS values of the forces are also shown. The forces were calculated from the displacement information described in Figure 59.
FIGURE 59  ACCURATE IDEALIZATION OF A UNIFORM CANTILEVER

FIGURE 60  THE EFFECT OF BUNDLE NUMBER ON COMPUTED FREQUENCY ACCURACY

FIGURE 61  'EQUAL LENGTH' BUNDLE IDEALIZATION OF A CANTILEVER
FIGURE 62  FIRST THREE MODE SHAPES OF A SIMPLY SUPPORTED UNIFORM BEAM

1. NORMALIZED DISPLACEMENT

2. NORMALIZED DISPLACEMENT

3. NORMALIZED DISPLACEMENT
FIGURE 63: First three modeshapes of a simply supported uniform beam; A comparison of DYNMOD (D) and analytical (A) results. The values are identical to the fifth place of decimals.
APPLIED FORCE AND VELOCITY (ARBITRARY UNITS) AT THE MID-POINT OF A SIMPLY SUPPORTED UNIFORM BEAM

Figure 64 Forcing

DRAG FORCE AND MEAN BUNDLE VELOCITY (ARBITRARY UNITS) CORRESPONDING TO BUNDLE i

Figure 65 Damping
FIGURE 66 AUTO-RECEPTANCE OF THE MID-POINT OF A SIMPLY SUPPORTED UNIFORM BEAM FOR VARIOUS DRAG COEFFICIENTS (CD)

**TEST CONDITIONS**

END PINNED UNIFORM BEAM IDEALIZED AS A 12 BUNDLE STRING (WITH HALF BUNDLES AT ENDS).

BUNDLE LENGTH = 1 m, BUNDLE MASS = 100 kg
KCST = 5 x 10⁴ N·m, KPAR = 5 x 10⁶ N·m,
KEND = 0.0 N·m, NO STRUCTURAL DAMPING.
DRAG COEFFICIENT CD = 0.0 TO 2.0 m/s

FREQUENCY SHIFTS DOWNWARDS AS DAMPING INCREASES
Figure 67: Auto-receptance of the mid-point of a simply supported uniform beam for various structural damping factors (GCST).
FIGURE 68  DRIVEN POINT MOBILITY OF A TWO BUNDLE ASSEMBLY

MEASURED

DYNMOD

KCST = 2600 N·m
KPAR = 8700 "
KEND = 1800 "
GCST = 0.00 "
GPAR = 0.09 "
GEND = 1.38 "
FIGURE 69 COMPARISON OF THE MEASURED AND PREDICTED MOBILITIES OF AN INVERTED CANDU-BLW REACTOR FUEL ASSEMBLY
FIGURE 70: Response of a simply supported beam to a distributed force of 11.0 N. @ 1.0 Hz. On the left hand side are the results from subroutine RANDOM, on the right hand side are the results from subroutine FORCES.

(NOTE: 12 x 1.0 N loads are shown. One of these acts at a constraint therefore the distributed force is 11.0 N.)
IDEALIZED SIMPLY SUPPORTED UNIFORM BAR IN FLUID

DIMENSIONS (HYPOTHETICAL)
LENGTH = 11 metres
MASS = 1100 kgs
VISCOUS DAMPING RATIO = 0.236
(O.D. = 1 inch, E = 20x10^6 lbs/sq. inch)

APPLIED RANDOM
FORCE, F
(N^2/Hz)
(AVERAGED POWER
SPECTRAL DENSITY
FORMAT)

FIGURE 71 DESCRIPTION OF A RANDOM FORCING FUNCTION APPLIED
AT THE MID-POINT OF A UNIFORM, SIMPLY SUPPORTED
BAR IMMERSED IN FLUID.
SIMPLY SUPPORTED UNIFORM BAR

FORCE FUNCTION A

FORCE FUNCTION B

FORCE FUNCTION C

FORCE FUNCTION D

(F_A, F_B, AND F_C ARE ASSUMED UNCORRELATED)

FIGURE 72 DESCRIBES SOME FORCING FUNCTIONS ACTING ON THE SIMPLY SUPPORTED UNIFORM BAR OF FIGURE 71
FIGURE 73: Resolved forces (NEWTONS) calculated by subroutine CALFOR.
APPENDIX I

DYNMOD FLOW CHARTS
1. DYNMOD MAIN PROGRAM FLOW CHART

START

DECLARE LAMPS FUNCTIONS

RESERVE CORE SPACE FOR LAMPS

DECLARE MATRIX POINTERS, NON-LAMPS FUNCTIONS AND VARIABLES

DEFINE STATEMENT FUNCTIONS

DEFINE MATRIX CONSTANTS

SET DEFAULT VALUES

INPUT (DATA SET 1) DEBUGGING AND PLOTTING PARAMETERS

D1

INPUT (DATA SET 2) STRUCTURAL PARAMETERS

D2

INPUT (DATA SET 3) FLUID PARAMETERS

CALCULATE THE COORDINATE TRANSFORM MATRIX CMAT

IS THE FUEL ASSEMBLY CONSTRAINED?

P1

NO

CALL SUBROUTINE PINNED

CALCULATE THE COORDINATE REDUCTION MATRIX R

CALCULATE THE CONSTRAINT STIFFNESS AND STRUCTURAL DAMPING

D2

PROCEED TO EVALUATE THE STRUCTURAL LAGRANGIAN TERMS

CALCULATE THE STRUCTURAL KINETIC ENERGY COEFFICIENT MATRIX

CALCULATE GRAVITATIONAL AND BENDING STRAIN POTENTIAL ENERGY COEFFICIENT MATRICES

ANY ADDITIONAL CONSTRAINT STIFFNESS AND DAMPING?

YES

D3

ADD CONSTRAINT STIFFNESS AND DAMPING TERMS TO STRAIN ENERGY MATRICES

NO

D4

END
FORMULATE THE TOTAL LAGRANGIAN POTENTIAL ENERGY TERM INCLUDING STRUCTURAL DAMPING FORCES

PROCEED TO EVALUATE THE HYDRODYNAMIC LAGRANGIAN TERMS

CALCULATE THE FLUID KINETIC ENERGY COEFFICIENT MATRICES

CALCULATE THE GENERALIZED HYDRODYNAMIC MOMENTS

ASSEMBLE THE MATRIX EQUATION OF MOTION

IS THE FUEL ASSEMBLY CONSTRAINED?

NO

YES

USE MATRIX R TO REDUCE THE ORDER OF THE MATRIX EQUATION OF MOTION

REARRANGE THE MATRIX EQUATION OF MOTION INTO A FIRST ORDER DIFFERENTIAL EQUATION

SOLVE THE MATRIX EQUATION OF MOTION TO GIVE THE EIGENVALUES AND EIGENVECTORS OF THE FUEL ASSEMBLY IN AXIAL FLOW

USE THE COORDINATE REDUCTION MATRIX TO GENERATE THE ORIGINAL BUNDLE ANGLES

OUTPUT NATURAL FREQUENCIES AND MODESHAPES

IS THE OUTPUT TO BE PLOTTED?

CALL PLOTTING SUBROUTINES

YES

OPTIMIZE CORE AND SAVE MATRICES NEEDED FOR FORCE CALCULATIONS

SUBROUTINE PACKAGES FOR FORCED RESPONSE, TRANSFER FUNCTIONS, AND RESOLVED FORCES

CALCULATE TRANSIENT AND STEADY RESPONSE

CALL SUBROUTINE FORCES

OUTPUT
CALCULATE TRANSFER FUNCTIONS RECEPTANCE AND MOBILITY

CALL SUBROUTINE RECMOB

OUTPUT

CALCULATE THE RESPONSE TO RANDOM FORCES

CALL SUBROUTINE RANDOM

OUTPUT

CALCULATE RESOLVED FORCES FROM RESPONSE DATA

CALL SUBROUTINE CALFOR

OUTPUT

OPTIMIZE CORE

ANY MORE DATA?

YES

STOP

NO
2. SUBROUTINE PINNED FLOW CHART

CALLED FROM DYNMOD

START

DECLARE LAMPS FUNCTIONS

ACCESS LAMPS CORE SPACE IN DYNMOD

DECLARE MATRIX POINTERS AND VARIABLES

SET DEFAULT VALUES

INPUT (DATA SET 4) LOGICAL OPERATORS CONSTRAINT STIFFNESSES AND DAMPING VALUES

IS THE FUEL ASSEMBLY CONSTRAINED?

NO

RETURN TO DYNMOD

YES

PROCEED TO CALCULATE THE COORDINATE REDUCTION MATRIX R

IS THE FUEL ASSEMBLY CONSTRAINED AT THE FREE END ONLY?

NO

CALCULATE THE END CONSTRAINT COORDINATE REDUCTION MATRIX

YES

CALCULATE THE INTERMEDIATE BUNDLE CONSTRAINT COORDINATE REDUCTION MATRIX A1

PA

PB
Is the fuel assembly constrained at the free end and an intermediate bundle?

Yes

Modify the coordinate reduction matrix $R_j$ to accommodate the end constraint

Calculate the matrix describing the extra structural damping end terms

Stop

Return the coordinate reduction matrix, constraint stiffness, and extra structural damping terms to DYNMOD
3. SUBROUTINE FORCES FLOW CHART

CALLED FROM DYNMOD

START

DECLARE LAMPS FUNCTIONS

ACCESS LAMPS CORE SPACE IN DYNMOD

DECLARE MATRIX POINTERS AND VARIABLES

SET DEFAULT VALUES

INPUT (DATA SET 5) LOGICAL OPERATORS AND APPLIED FORCE DATA

ARE FORCED RESPONSE CALCULATIONS REQUIRED?

IF THE FUEL ASSEMBLY IS CONSTRAINED THE MATRICES ARE REDUCED ACCORDINGLY

CALCULATE THE GENERALIZED APPLIED FORCES

IS THE STEADY STATE RESPONSE REQUIRED ALONE?

NO

PROCEED TO CALCULATE THE TRANSIENT RESPONSE IN TIME DOMAIN

OUTPUT THE FUEL
ASSEMBLY DISPLACEMENT AFTER EACH TIME INCREMENT

OPTIMIZE CORE

YES

CALCULATE THE
ANGULAR DEFLECTION
AND ANGULAR VELOCITY
MATRICES AT THE
SPECIFIED TIME \( t \)

F_1

F_2

F_4

F_A
132

IS THE STEADY STATE RESPONSE TO THE APPLIED FORCES REQUIRED ALSO?

PROCEED TO CALCULATE THE STEADY STATE HARMONIC RESPONSE

CALCULATE THE STEADY AMPLITUDE AND RELATIVE PHASE OF THE RESPONSE

OUTPUT THE AMPLITUDE AND PHASE INFORMATION AT THE FREQUENCY (SPECTRAL LINE) OF INTEREST

OPTIMIZE CORE

RETURN TO DYNMOD
4. SUBROUTINE RECMOB FLOW CHART

Called from DYMMOD

Start

Declare Lamps Functions

Access Lamps Core Space in DYMMOD

Declare Matrix Pointers and Variables

Set Default Values

Input (Data Set 6) Logical Operators and Frequency Range Information

Are Transfer Functions Required?

Yes → If the fuel assembly is constrained, the matrices are reduced accordingly

No → Does the shaker (pseudo lateral force) act at a constraint?

Yes → Construct a pseudo lateral force vector. This gives a unit force at the shaker position

No → Obtain the generalized form of the force vector

Set the variable frequency \( f \) to \( f = F_{MIN} \)

Frequency Loop

\( R_1 \)

\( R_2 \)

\( R_3 \)

\( R_4 \)
CALCULATE THE STEADY RESPONSE TO THE UNIT APPLIED FORCE AT THE FREQUENCY \( f \)

EVALUATE THE AUTO AND CROSS MOBILITIES AND RECEPANCES AT THE FREQUENCY \( f \)

CONVERT COMPLEX TRANSFER FUNCTIONS TO POLAR FORM (i.e., AMPLITUDE AND RELATIVE PHASE)

OUTPUT COLUMN VECTORS OF MOBILITIES AND RECEPANCES ALONG THE FUEL ASSEMBLY FOR A SINGLE SHAKER POSITION

IS \( 1 > f_{\text{max}} \) ?

ADD FREQUENCY INCREMENT (\( \Delta f_{\text{MIN}} \)) TO \( f \)

OUTPUT, PRINT THE MESSAGE "MOBILITIES AND RECEPANCES ARE ZERO"

OPTIMIZE CORE

STOP

RETURN TO DYNMOD
5. SUBROUTINE RANDOM FLOW CHART

CALLED FROM DYNMOD

START

DECLARE LAMP* FUNCTIONS

ACCESS LAMPS CORE SPACE IN DYNMOD

DECLARE MATRIX POINTERS AND VARIABLES

SET DEFAULT VALUES

INPUT DATA SET 7
LOGICAL OPERATORS AND FREQUENCY RANGE INFORMATION

IF THE FUEL ASSEMBLY IS CONSTRAINED THE MATRICES ARE REDUCED ACCORDINGLY

SET VARIABLE FREQUENCY f TO FMIN

FREQUENCY LOOP

EVALUATE THE COMPLETE (N X N) RECEPTANCE MATRIX

IS A RANDOM FORCED RESPONSE REQUIRED?

NO

YES

R5

R6

R10

R6

R6
IS THE FORCE DATA SUPPLIED IN CORRELATED OR UNCORRELATED POWER SPECTRAL DENSITY FORM?

NO

INPUT (DATA SET 8, OPTION 1) DISCRETE FORCE SPECTRA

CALCULATE THE STEADY AMPLITUDE AND RELATIVE PHASE OF THE RESPONSE AT FREQUENCY f

IS f > FMAX?

NO

ADD FREQUENCY INCREMENT (BANWIN) TO f

YES

CONSTRUCT THE RESPONSE SPECTRUM MATRIX

RC

ARE THE FORCES UNCORRELATED?

YES

RB

NO

R7

R6
OUTPUT THE DISCRETE SPECTRUM OF RESPONSE FOR EACH BUNDLE END

CALCULATE THE INTENSITY SPECTRUM OF RESPONSE

OUTPUT THE INTENSITY SPECTRUM FOR EACH BUNDLE END

CALCULATE THE MEAN SQUARE DISPLACEMENTS AT EACH BUNDLE END

OUTPUT THE COLUMN VECTOR OF MEAN SQUARE DISPLACEMENTS

EVALUATE THE R.M.S. VALUES OF RESPONSE

OUTPUT THE ROOT MEAN SQUARE VECTOR OF RESPONSE

OPTIMIZE CORE
INPUT (DATA SET 6, OPTION 2) CROSS POWER SPECTRAL DENSITY OF FORCE FOR A GIVEN FREQUENCY $f$

CALCULATE THE AUTO POWER SPECTRAL DENSITY OF THE RESPONSE AT FREQUENCY $f$

IS $f > F_{\text{MAX}}$ ?

NO

ADD FREQUENCY INCREMENT (BANWID) TO $f$

YES

$R_7$

$R_9$

$R_9$
INPUT (DATA SET 8, OPTION 3)
UNCORRELATED FORCE SPECTRA (AUTO POWER SPECTRAL DENSITY)

CALCULATE THE AUTO POWER SPECTRAL DENSITY OF RESPONSE AT FREQUENCY f

IS f > FMAX ?

YES

ADD FREQUENCY INCREMENT (BANWID) TO f

NO

OUTPUT THE AUTO P.S.D. OF RESPONSE FOR EACH BUNDLE END

CALCULATE THE MEAN SQUARE VALUE OF THE RESPONSE

OUTPUT THE VECTOR OF MEAN SQUARE VALUES

CALCULATE THE R.M.S. VALUE OF THE RESPONSE

OUTPUT THE R.M.S. VALUES OF RESPONSE

OPTIMIZE CORE

STOP

RETURN TO DYNAMO
6. SUBROUTINE CALFOR FLOW CHART

- CALLED FROM DYNMOD
- START
- DECLARE LAMPS FUNCTIONS
- ACCESS LAMPS CORE SPACE IN DYNMOD
- DECLARE MATRIX POINTERS AND VARIABLES
- SET DEFAULT VALUES
- INPUT (DATA SET 9) LOGICAL OPERATORS AND FREQUENCY RANGE INFORMATION
- ARE FORCE CALCULATIONS REQUIRED?
- IF THE FUEL ASSEMBLY IS CONSTRAINED THE MATRICES ARE REDUCED ACCORDINGLY
- SET VARIABLE FREQUENCY $f$ TO $F_{\text{MIN}}$
- FREQUENCY LOOP

$c_1$

$c_2$

$c_3$

$c_4$
IS THE RECEPTANCE MATRIX TO BE CALCULATED?

NO

EXPERIMENTALLY DETERMINED TRANSFER FUNCTION MATRICES ARE AVAILABLE

YES

INPUT (DATA SET 10, OPTIONAL) RECEPTANCE MATRIX AT THE FREQUENCY \( f \)

CALCULATE THE RECEPTANCE MATRIX FOR FREQUENCY \( f \)

IS THE RESPONSE DATA SUPPLIED IN CROSS POWER SPECTRAL DENSITY FORM?

YES

NO

C_A

C_B

C_3
INPUT (DATA SET 11, OPTION 1)

DISCRETE RESPONSE SPECTRUM

CALCULATE THE RESOLVED FORCE VECTOR AT FREQUENCY $f$

IS THE CROSS POWER MATRIX OF FORCE REQUIRED?

YES

CALCULATE THE CROSS-POWER MATRIX (INTENSITY MATRIX) OF FORCE AT THE FREQUENCY $f$

NO

OUTPUT CROSS POWER (INTENSITY) MATRIX OF FORCE AT THE FREQUENCY $f$

HORIZONTALLY JOIN THE FORCE VECTORS IN SPECTRAL ORDER TO FORM A TWO DIMENSIONAL RESOLVED FORCE MATRIX

$C_B$

$C_C$
OUTPUT THE DISCRETE FORCE SPECTRUM FOR EACH BUNDLE END

CALCULATE THE AUTO-POWER (INTENSITY) SPECTRA OF FORCE

OUTPUT THE AUTO-POWER (INTENSITY) SPECTRUM OF FORCE FOR EACH BUNDLE END

CALCULATE THE MEAN SQUARES OF THE RESOLVED FORCES

OUTPUT THE MEAN SQUARE VALUES OF THE FORCES FOR EACH BUNDLE END

EVALUATE THE ROOT MEAN SQUARE VALUES OF THE RESOLVED FORCES

OUTPUT THE R.M.S. VALUES OF RESOLVED FORCES FOR EACH BUNDLE END
INPUT (DATASET 11, OPTION 2)
CROSS-POWER SPECTRAL DENSITY OF RESPONSE AT FREQUENCY $f$

CALCULATE THE CROSS POWER SPECTRAL DENSITY OF THE RESOLVED FORCES AT FREQUENCY $f$

IS THE CROSS P.S.D. OF FORCE REQUIRED AS OUTPUT?

YES
OUTPUT THE CROSS P.S.D. OF RESOLVED FORCES AT THE FREQUENCY $f$

NO

EXTRACT THE DIAGONAL OF THE CROSS P.S.D. MATRIX TO GIVE THE AUTO P.S.D. COLUMN VECTOR OF FORCE AT FREQUENCY $f$

HORIZONTALLY JOIN THE FORCE VECTORS IN SPECTRAL ORDER TO FORM A TWO DIMENSIONAL AUTO P.S.D. OF RESOLVED FORCE MATRIX

C3
YES

ADD FREQUENCY INCREMENT (FMAX) TO F

OUTPUT THE AUTO POWER SPECTRUM OF RESOLVED FORCE FOR EACH BUNDLE END

CALCULATE THE MEAN SQUARE VALUES OF THE RESOLVED FORCES

OUTPUT THE MEAN SQUARE VALUES OF THE FORCES ACTING AT EACH BUNDLE END

CALCULATE THE R.M.S. VALUES OF THE RESOLVED FORCES

OUTPUT THE R.M.S. VALUES OF THE RESOLVED FORCES ACTING AT EACH BUNDLE END

OPTIMIZE CORE

STOP

RETURN TO DYNMOD

YES

IS I > FMAX?
### APPENDIX II: DYNMOD INPUT PARAMETERS AND VARIABLES

<table>
<thead>
<tr>
<th>MATRIX VARIABLE NAME</th>
<th>ELEMENT DIMENSIONS</th>
<th>UNITS</th>
<th>MATRIX ORDER *</th>
<th>DATA SET NUMBER</th>
<th>INPUT OPTION</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>BANWID</td>
<td>T⁻¹</td>
<td>Hz</td>
<td>(1x1)</td>
<td>6,7,9</td>
<td>-</td>
<td>Frequency interval</td>
</tr>
<tr>
<td>BPIN</td>
<td>-</td>
<td>-</td>
<td>(1x1)</td>
<td>4</td>
<td>-</td>
<td>End bundle constraint indicator</td>
</tr>
<tr>
<td>BWR</td>
<td>-</td>
<td>-</td>
<td>(1x1)</td>
<td>1</td>
<td>-</td>
<td>Plotter bundle length/width ratio</td>
</tr>
<tr>
<td>CDCFCB</td>
<td>L⁻¹ T⁻¹,</td>
<td>m/s</td>
<td>(N×3)</td>
<td>3</td>
<td>-</td>
<td>Horizontally joined coefficient vectors of viscous drag (CD), friction (CF) and base drag (CB)</td>
</tr>
<tr>
<td>DATA</td>
<td>-</td>
<td>-</td>
<td>(1x1)</td>
<td>7</td>
<td>-</td>
<td>Force input descriptor</td>
</tr>
<tr>
<td>DBUG</td>
<td>-</td>
<td>-</td>
<td>(1x1)</td>
<td>1</td>
<td>-</td>
<td>Program debugging indicator</td>
</tr>
<tr>
<td>DCBOTE</td>
<td>-</td>
<td>-</td>
<td>(1x1)</td>
<td>4</td>
<td>-</td>
<td>Extra interbundle bending stiffness structural damping factor at bottom constraint</td>
</tr>
<tr>
<td>DCTOPX</td>
<td>-</td>
<td>-</td>
<td>(1x1)</td>
<td>2</td>
<td>-</td>
<td>Extra interbundle bending stiffness structural damping factor at bottom constraint</td>
</tr>
<tr>
<td>DEBOTE</td>
<td>-</td>
<td>-</td>
<td>(1x1)</td>
<td>4</td>
<td>-</td>
<td>Extra endplate flexure stiffness structural damping factor at bottom constraint</td>
</tr>
<tr>
<td>DEQ</td>
<td>L</td>
<td>m</td>
<td>(N×1)</td>
<td>3</td>
<td>-</td>
<td>Equivalent diameter of the free end</td>
</tr>
<tr>
<td>DETOPX</td>
<td>-</td>
<td>-</td>
<td>(1x1)</td>
<td>2</td>
<td>-</td>
<td>Extra endplate flexure stiffness structural damping factor at top constraint</td>
</tr>
<tr>
<td>DFORCE</td>
<td>MLT⁻²</td>
<td>N</td>
<td>(N×M)</td>
<td>8</td>
<td>1</td>
<td>Discrete spectra of applied forces</td>
</tr>
<tr>
<td>DIAM</td>
<td>L</td>
<td>m</td>
<td>(N×1)</td>
<td>2</td>
<td>-</td>
<td>Fuel element diameters</td>
</tr>
<tr>
<td>DIAMH</td>
<td>L</td>
<td>m</td>
<td>(N×1)</td>
<td>3</td>
<td>-</td>
<td>Bundle/pressure tube hydraulic diameters</td>
</tr>
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</table>

(Cont'd)
<table>
<thead>
<tr>
<th>MATRIX VARIABLE NAME</th>
<th>ELEMENT DIMENSIONS</th>
<th>UNITS</th>
<th>MATRIX ORDER *</th>
<th>DATA SET NUMBER</th>
<th>INPUT OPTION</th>
<th>MATRIX DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>DISMAT</td>
<td>L</td>
<td>m</td>
<td>(MxM)</td>
<td>11</td>
<td>1</td>
<td>Discrete spectra of displacement response</td>
</tr>
<tr>
<td>DISPOW</td>
<td>$L^2T^2$</td>
<td>m$^2$/Hz</td>
<td>(NxN)</td>
<td>11</td>
<td>2</td>
<td>Cross power spectral density of displacement response</td>
</tr>
<tr>
<td>EXPER</td>
<td>-</td>
<td>-</td>
<td>(1x1)</td>
<td>9</td>
<td>-</td>
<td>Transfer function data descriptor</td>
</tr>
<tr>
<td>FEF</td>
<td>-</td>
<td>-</td>
<td>(Nx1)</td>
<td>3</td>
<td>-</td>
<td>Free end shape factor</td>
</tr>
<tr>
<td>FMAX</td>
<td>$T^{-1}$</td>
<td>Hz</td>
<td>(1x1)</td>
<td>6</td>
<td>-</td>
<td>Upper frequency limit of transfer function</td>
</tr>
<tr>
<td>FMIN</td>
<td>$T^{-1}$</td>
<td>Hz</td>
<td>(1x1)</td>
<td>6,7,9</td>
<td>-</td>
<td>Lower frequency limit of spectra</td>
</tr>
<tr>
<td>FORCE</td>
<td>$MLT^{-2}$</td>
<td>N</td>
<td>(Nx1)</td>
<td>5</td>
<td>-</td>
<td>Applied harmonic forces</td>
</tr>
<tr>
<td>G</td>
<td>$LT^{-2}$</td>
<td>m/s$^2$</td>
<td>(1x1)</td>
<td>2</td>
<td>-</td>
<td>Gravitational acceleration</td>
</tr>
<tr>
<td>GCST</td>
<td>-</td>
<td>-</td>
<td>(Nx1)</td>
<td>2</td>
<td>-</td>
<td>Structural damping factors associated with interbundle bending stiffnesses</td>
</tr>
<tr>
<td>GEND</td>
<td>-</td>
<td>-</td>
<td>(Nx1)</td>
<td>2</td>
<td>-</td>
<td>Structural damping factors associated with bundle endplate flexure stiffness</td>
</tr>
<tr>
<td>GPAR</td>
<td>-</td>
<td>-</td>
<td>(Nx1)</td>
<td>2</td>
<td>-</td>
<td>Structural damping factors associated with bundle shear</td>
</tr>
<tr>
<td>JPIN</td>
<td>-</td>
<td>-</td>
<td>(1x1)</td>
<td>4</td>
<td>-</td>
<td>Intermediate bundle constraint indicator</td>
</tr>
<tr>
<td>KCBOT</td>
<td>$ML^2T^{-2}$</td>
<td>N.m/rad</td>
<td>(1x1)</td>
<td>4</td>
<td>-</td>
<td>Interbundle bending stiffness at bottom constraint</td>
</tr>
<tr>
<td>KCST</td>
<td>$ML^2T^{-2}$</td>
<td>N.m/rad</td>
<td>(Nx1)</td>
<td>2</td>
<td>-</td>
<td>Interbundle bending stiffness</td>
</tr>
<tr>
<td>KEBOT</td>
<td>$ML^2T^{-2}$</td>
<td>N.m/rad</td>
<td>(1x1)</td>
<td>4</td>
<td>-</td>
<td>Endplate flexure stiffness at bottom constraint</td>
</tr>
<tr>
<td>KEND</td>
<td>$ML^2T^{-2}$</td>
<td>N.m/rad</td>
<td>(Nx1)</td>
<td>2</td>
<td>-</td>
<td>Endplate flexure stiffnesses</td>
</tr>
<tr>
<td>KPAR</td>
<td>$ML^2T^{-2}$</td>
<td>N.m/rad</td>
<td>(Nx1)</td>
<td>2</td>
<td>-</td>
<td>Bundle shear stiffnesses</td>
</tr>
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(Cont'd)
<table>
<thead>
<tr>
<th>MATRIX VARIABLE NAME</th>
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<th>UNITS</th>
<th>MATRIX ORDER</th>
<th>DATA SET NUMBER</th>
<th>INPUT OPTION</th>
<th>MATRIX DESCRIPTION</th>
</tr>
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<tbody>
<tr>
<td>L</td>
<td>L</td>
<td>m</td>
<td>(Nx1)</td>
<td>2</td>
<td>-</td>
<td>Bundle lengths</td>
</tr>
<tr>
<td>M</td>
<td>M</td>
<td>kg</td>
<td>(Nx1)</td>
<td>2</td>
<td>-</td>
<td>Bundle masses</td>
</tr>
<tr>
<td>NEL</td>
<td>-</td>
<td>-</td>
<td>(Nx1)</td>
<td>2</td>
<td>-</td>
<td>Numbers of fuel elements per bundle</td>
</tr>
<tr>
<td>NPERFR</td>
<td>-</td>
<td>-</td>
<td>(1x1)</td>
<td>1</td>
<td>-</td>
<td>Number of modes plotted per frame</td>
</tr>
<tr>
<td>OMEGA</td>
<td>T&lt;sup&gt;-1&lt;/sup&gt;</td>
<td>Hz</td>
<td>(1x1)</td>
<td>5</td>
<td>-</td>
<td>Frequency of the applied harmonic forces</td>
</tr>
<tr>
<td>PLOT</td>
<td>-</td>
<td>-</td>
<td>(1x1)</td>
<td>1</td>
<td>-</td>
<td>Graph plot indicator</td>
</tr>
<tr>
<td>POINTS</td>
<td>-</td>
<td>-</td>
<td>(1x1)</td>
<td>7,9</td>
<td>-</td>
<td>Number of frequency points of interest in a calculation</td>
</tr>
<tr>
<td>PSDFOR</td>
<td>M&lt;sup&gt;2&lt;/sup&gt;L&lt;sup&gt;-2&lt;/sup&gt;T&lt;sup&gt;-3&lt;/sup&gt;</td>
<td>N&lt;sup&gt;2&lt;/sup&gt;/Hz</td>
<td>(NxN)</td>
<td>8</td>
<td>2</td>
<td>Averaged cross power spectral density of the applied forces</td>
</tr>
<tr>
<td>RHO</td>
<td>ML&lt;sup&gt;-3&lt;/sup&gt;</td>
<td>kg/m&lt;sup&gt;3&lt;/sup&gt;</td>
<td>(Nx1)</td>
<td>3</td>
<td>-</td>
<td>Fluid densities</td>
</tr>
<tr>
<td>SAMPLE</td>
<td>T</td>
<td>s</td>
<td>(1x1)</td>
<td>5</td>
<td>-</td>
<td>Interval between time-step calculations</td>
</tr>
<tr>
<td>SDR</td>
<td>-</td>
<td>-</td>
<td>(1x1)</td>
<td>1</td>
<td>-</td>
<td>Mode shape plot successive deflection ratio</td>
</tr>
<tr>
<td>SHAKER</td>
<td>-</td>
<td>-</td>
<td>(1x1)</td>
<td>6</td>
<td>-</td>
<td>Number of bundle to which the electrodynamic shaker is attached</td>
</tr>
<tr>
<td>TMAX</td>
<td>T</td>
<td>s</td>
<td>(1x1)</td>
<td>5</td>
<td>-</td>
<td>Transient response observation time</td>
</tr>
<tr>
<td>TRAMAT</td>
<td>M&lt;sup&gt;-1&lt;/sup&gt;L&lt;sup&gt;-2&lt;/sup&gt;T</td>
<td>m/N</td>
<td>(NxN)</td>
<td>10</td>
<td>optional</td>
<td>Cross receptance matrix</td>
</tr>
<tr>
<td>TYPE</td>
<td>-</td>
<td>-</td>
<td>(1x1)</td>
<td>9</td>
<td>-</td>
<td>Response input data descriptor for resolved force calculations</td>
</tr>
<tr>
<td>U</td>
<td>L T&lt;sup&gt;-1&lt;/sup&gt;</td>
<td>m/s</td>
<td>(Nx1)</td>
<td>3</td>
<td>-</td>
<td>Flow velocities</td>
</tr>
<tr>
<td>UNCORF</td>
<td>M&lt;sup&gt;2&lt;/sup&gt;L&lt;sup&gt;2&lt;/sup&gt;T&lt;sup&gt;-3&lt;/sup&gt;</td>
<td>N&lt;sup&gt;2&lt;/sup&gt;/Hz</td>
<td>(NxM)</td>
<td>8</td>
<td>3</td>
<td>Uncorrelated applied forces</td>
</tr>
<tr>
<td>VM</td>
<td>M</td>
<td>kg</td>
<td>(Nx1)</td>
<td>3</td>
<td>-</td>
<td>Hydrodynamic (virtual or added) masses along the fuel string</td>
</tr>
</tbody>
</table>

(cont'd)
<table>
<thead>
<tr>
<th>VARIABLE NAME</th>
<th>ELEMENT ORDER</th>
<th>UNITS</th>
<th>MATRIX ORDER</th>
<th>DATA SET NUMBER</th>
<th>INPUT OPTION</th>
<th>MATRIX DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>WHICH</td>
<td>-</td>
<td>-</td>
<td>(1x1)</td>
<td>5</td>
<td>-</td>
<td>Harmonic forced response calculation indicator</td>
</tr>
<tr>
<td>XL</td>
<td>L</td>
<td>in.</td>
<td>(1x1)</td>
<td>1</td>
<td>-</td>
<td>Plot frame length in X direction</td>
</tr>
<tr>
<td>Y</td>
<td>-</td>
<td>-</td>
<td>(1x1)</td>
<td>2</td>
<td>-</td>
<td>Gravitational acceleration direction indicator</td>
</tr>
<tr>
<td>YL</td>
<td>L</td>
<td>in.</td>
<td>(1x1)</td>
<td>1</td>
<td>-</td>
<td>Plot frame length in Y direction</td>
</tr>
</tbody>
</table>

* N is the number of bundles in the fuel string.

* M is the number of spectral lines in frequency domain.
APPENDIX III: LAMPS DEBUGGING PACKAGE AND ERROR CODES

NOTE:  i) arg\textsubscript{i} refers to the value of the i'th argument when the routine was involved.

ii) * these codes will occur only at the third code in a message from ALLOC.

<table>
<thead>
<tr>
<th>errcode</th>
<th>routine</th>
<th>form of message</th>
<th>explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>TRANS</td>
<td>*</td>
<td>directory overflow</td>
</tr>
<tr>
<td></td>
<td>ADJOINT</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>ALLOC</td>
<td>3.0 a r</td>
<td>directory overflow</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.1 a r</td>
<td>SPACE overflow</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>where a is the number of calls of available</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>space (negative for case 3.1),</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>r is the identifying code of the routine</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>which is called ALLOC (e.g. 2 refers to</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>TRANS).</td>
</tr>
<tr>
<td>8</td>
<td>COPY</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>DIAG</td>
<td>12.0 arg\textsubscript{1} 12</td>
<td>arg not square</td>
</tr>
<tr>
<td>21</td>
<td>IDENT</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>INVERT</td>
<td>24.0 arg\textsubscript{1} 0</td>
<td>arg not square</td>
</tr>
<tr>
<td>25</td>
<td>JOINH</td>
<td>25.0 arg\textsubscript{1} arg\textsubscript{2}</td>
<td>arguments not conformable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(different # rows)</td>
</tr>
<tr>
<td>26</td>
<td>JOINV</td>
<td>26.0 arg\textsubscript{1} arg\textsubscript{2}</td>
<td>arguments not conformable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(different # cols)</td>
</tr>
<tr>
<td>27</td>
<td>MODAL</td>
<td>27.0 arg\textsubscript{1} 0</td>
<td>arg\textsubscript{1} not square</td>
</tr>
<tr>
<td></td>
<td></td>
<td>27.1 req'd avail space space</td>
<td>not enough storage</td>
</tr>
<tr>
<td></td>
<td></td>
<td>27.2 AR IERR</td>
<td>error IERR in COMLR2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(see EISPACK documentation)</td>
</tr>
<tr>
<td>28</td>
<td>MULT</td>
<td>28.0 arg\textsubscript{1} arg\textsubscript{2}</td>
<td>not conformable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(#cols arg\textsubscript{1} ≠ # rows arg\textsubscript{2})</td>
</tr>
<tr>
<td>29</td>
<td>CONST</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ONE</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SCALAR</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SCALARC</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>OP</td>
<td>30.0 arg\textsubscript{1} arg\textsubscript{3}</td>
<td>different # cols</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30.1 arg\textsubscript{1} arg\textsubscript{3}</td>
<td>different # rows</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30.2 arg\textsubscript{1} arg\textsubscript{3}</td>
<td>zero divide</td>
</tr>
<tr>
<td>errcode</td>
<td>routine</td>
<td>form of message</td>
<td>explanation</td>
</tr>
<tr>
<td>---------</td>
<td>-----------</td>
<td>-----------------</td>
<td>--------------------------------------------------</td>
</tr>
<tr>
<td>35</td>
<td>SUBMAT</td>
<td>35.0 arg₁ implied # rows</td>
<td>implied # rows (cols)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>35.1 arg₁ implied # cols</td>
<td>exceeds the dimension of the matrix arg₁</td>
</tr>
<tr>
<td>39</td>
<td>UPPER</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>43</td>
<td>NORMLIZ</td>
<td>43.0 arg₁ 0</td>
<td>invalid pointer arg₁</td>
</tr>
<tr>
<td>44</td>
<td>GRAF</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>DELRC</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>47</td>
<td>D</td>
<td>47.0 arg₁ 0</td>
<td>arg not a vector</td>
</tr>
<tr>
<td>50</td>
<td>READx</td>
<td>50.0 cardcol 0</td>
<td>invalid # cols</td>
</tr>
<tr>
<td></td>
<td>INPUT</td>
<td>50.1 cardcol 0</td>
<td>invalid # rows</td>
</tr>
<tr>
<td>63</td>
<td>GRAF2</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>91</td>
<td>IOTA</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>92</td>
<td>SOLVEX</td>
<td>92.1 arg₁ arg₂</td>
<td>inconsistent equations</td>
</tr>
<tr>
<td></td>
<td>SOLVEH</td>
<td></td>
<td></td>
</tr>
<tr>
<td>93</td>
<td>RESHAPE</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>94</td>
<td>STROW</td>
<td>94.0 arg₁ arg₃</td>
<td>not conformable</td>
</tr>
<tr>
<td></td>
<td>STCOL</td>
<td>94.1 arg₁ arg₃</td>
<td>not conformable</td>
</tr>
<tr>
<td>95</td>
<td>TRIINV</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>96</td>
<td>CREATE</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>97</td>
<td>GRAFM</td>
<td>97.0 arg₁ arg₂</td>
<td>not conformable</td>
</tr>
</tbody>
</table>

**NOTE:**

the explanation 'not conformable' means that the shapes of matrix arguments do not conform e.g., for matrix multiplication the number of columns of arg₁ must equal the number of rows of arg₂.
### APPENDIX IV: FUNCTIONS USED IN DYNMOD

<table>
<thead>
<tr>
<th>FUNCTION</th>
<th>DESCRIPTION</th>
</tr>
</thead>
</table>
| ADD      | Elemental or ordinary addition  
(K = ADD(M,N) means $K = M + N$) |
| ADD3     | Ordinary addition of three matrices  
(K = ADD3(L,M,N) means $K = L + M + N$) |
| ARGAND   | An output function  
(K = ARGAND(M) means plot an argand diagram of the complex elements of matrix M) |
| ASSIGN   | Protect a matrix  
(K = ASSIGN(K) means protect matrix K) |
| ASSIGNC  | Protect a matrix and then erase all unprotected matrices in the computer core. |
| CONJ     | Evaluate the complex conjugate  
(K = CONJ(M) means $K = M^*$) |
| COPY     | Copy a matrix  
(K = COPY(M) means that $K$ is a matrix whose elements are identical to $M$ but $K$ and $M$ occupy different portions of core) |
| CREATE   | Generate a LAMPS matrix from a complex array  
(K = CREATE(C,JP,JQ,JPDIM) means create a (JPxJQ) matrix $K$ which is a copy of the complex matrix C where C is declared as a complex array and JPDIM is the dimensioned number of rows of C) |
| D        | Convert a row or column vector into a diagonal matrix  
(K = D(L) means that $K$ is a diagonal matrix whose $i$th element on the diagonal is the $i$th element of the vector matrix L or L) |
| DELCOL   | Delete a column of a matrix  
(K = DELCOL(M,J) means that $K$ equals $M$ with column $J$ deleted) |
<table>
<thead>
<tr>
<th>FUNCTION</th>
<th>DESCRIPTION</th>
</tr>
</thead>
</table>
| DELETE   | Returns a matrix to the unprotected state  
          \[(K=\text{DELETE}(K) \text{ means that } K \text{ is returned to the unprotected state)}\] |
| DELRC    | Delete a row and a column of a matrix  
          \[(K=\text{DELRC}(M,I,J) \text{ means } K \text{ equals } M \text{ with row } I \text{ and column } J \text{ deleted})\] |
| DELROW   | Delete a row of a matrix  
          \[(K=\text{DELROW}(M,I) \text{ means } K \text{ equals } M \text{ with row } I \text{ deleted})\] |
| DET      | Calculate the determinant of a matrix  
          \[(C=\text{DET}(M) \text{ means calculate the complex determinant } C \text{ of } M)\] |
| DIAG     | Convert a diagonal matrix into a column vector  
          \[(K=\text{DIAG}(M) \text{ means that } K \text{ is a column vector consisting of the diagonal elements of } M)\] |
| DN       | Elemental division  
          \[(K=\text{DIV}(M,N) \text{ means } K=M/N)\] |
| ELEMENT  | Return a matrix element  
          \[(C=\text{ELEMENT}(M,I,J) \text{ means that the complex number } C \text{ is equal to element } (I,J) \text{ of } M)\] |
| GETCOL   | Returns the number of columns of a matrix  
          \[(M\text{COL}=\text{GETCOL}(M) \text{ means that the integer } M\text{COL} \text{ is equal to the number of columns of matrix } M)\] |
| GETROW   | Returns the number of rows of a matrix  
          \[(M\text{ROW}=\text{GETROW}(M) \text{ means that the integer } M\text{ROW} \text{ is equal to the number of rows of matrix } M)\] |
| IDENT    | Generate an identity matrix  
          \[(K=\text{IDENT}(IP) \text{ means that matrix } K \text{ has ones on the diagonal and zeros elsewhere; } K \text{ is of order } (IP\times IP))\] |
| INVERT   | Calculate the inverse of a matrix  
          \[(K=\text{INVERT}(M) \text{ means } K=M^{-1})\] |
| JOINH    | Join two matrices horizontally  
          \[(K=\text{JOINH}(M,N) \text{ means } K=(M,N))\] |
<table>
<thead>
<tr>
<th>FUNCTION</th>
<th>DESCRIPTION</th>
</tr>
</thead>
</table>
| JOINV    | Join two matrices vertically  
            \( (K=\text{JOINV}(M,N) \text{ means } \mathbf{K} = \begin{pmatrix} \mathbf{M} \\ \mathbf{N} \end{pmatrix}) \) |
| LOWER    | Generate a matrix with ones on and below the main diagonal and zeros elsewhere  
            \( (K=\text{LOWER}(IP) \text{ means form the } (IPxIP) \text{ matrix } \mathbf{K} \text{ as defined above}) \) |
| MODAL    | Solves the eigenvalue problem \( (\mathbf{K} - \lambda \mathbf{I}) \mathbf{X} = 0 \) where \( \mathbf{K} \) is a \( (nxn) \) square matrix.  
            \( (K=\text{MODAL}(M,\text{IOUT}) \text{ means that } \overline{\text{IOUT}} \text{ is returned as a column vector of the eigenvalues } \lambda \text{ in ascending order of modules. } \mathbf{K} \text{ is the normalized modal matrix of } \mathbf{M} \text{ such that the } j^{th} \text{ column of } \mathbf{K} \text{ is the normalized eigenvector } \overline{\mathbf{x}}_j \text{ corresponding to the } j^{th} \text{ eigenvalue}) \) |
| MULT     | Matrix multiplication  
            \( (K=\text{MULT}(M,N) \text{ means } \mathbf{K} = \mathbf{M} \cdot \mathbf{N}) \) |
| MULT3    | Matrix multiplication  
            \( (K=\text{MULT3}(L,M,N) \text{ means } \mathbf{K} = \mathbf{L} \cdot \mathbf{M} \cdot \mathbf{N}) \) |
| NEG      | Elemental negation  
            \( (K=\text{NEG}(M) \text{ means } \mathbf{K} = -\mathbf{M}) \) |
| NORMLIZ  | Normalizes the columns of a matrix  
            \( (K=\text{NORMLIZ}(M) \text{ means that the elements of each column of } \mathbf{M} \text{ are divided by the magnitude of the column; this leaves the normalized matrix } \mathbf{K}) \) |
| ONE      | Generate a matrix of ones  
            \( (K=\text{ONE}(IP,IQ) \text{ means } \mathbf{K} \text{ is an } (IPxIQ) \text{ matrix of ones}) \) |
| ONEBLO   | Generates a matrix with ones below the diagonal  
            \( (K=\text{ONEBLO}(IP) \text{ means that matrix } \mathbf{K} \text{ has ones below the diagonal but zeros on and above it}) \) |
| ONEBOV   | Generates a matrix with ones above the diagonal  
            \( (K=\text{ONEBOV}(IP) \text{ means that matrix } \mathbf{K} \text{ has ones above the diagonal but zeros on and below it}) \) |
| OUTPUT   | An output function  
            \( (K=\text{OUTPUT}(K) \text{ means print } \mathbf{K} \text{ on the lineprinter}) \) |
<table>
<thead>
<tr>
<th>FUNCTION</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>OUTPUTC</td>
<td>An output function ( (K=OUTPUTC(TITLE,J,K) \text{ means print } K \text{ together with } J \text{ one-word titles, one above each of the first } J \text{ columns (} 1&lt;J&lt;4)) )</td>
</tr>
<tr>
<td>OUTPUTL</td>
<td>An output function ( (K=OUTPUTL(TITLE,J,K) \text{ means print } K \text{ together with a title consisting of } J \text{ words } (1&lt;J&lt;8)) )</td>
</tr>
<tr>
<td>OUTPUTT</td>
<td>An output function ( (K=OUTPUTT(TITLE,K) \text{ means print } K \text{ together with a title of up to } 10 \text{ characters}) )</td>
</tr>
<tr>
<td>PAGE</td>
<td>An output function ( (K=OUTPUT(PAGE(M)) \text{ means skip to a new page on the lineprinter and print } M) )</td>
</tr>
<tr>
<td>POWER</td>
<td>Elemental exponentiation ( (K=POWER(M,C) \text{ means } K \text{ is the matrix } M \text{ raised to the power } C, \ K=\text{ } M^C) )</td>
</tr>
<tr>
<td>POWERN</td>
<td>Elemental exponentiation ( (K=POWERN(M,N) \text{ means } K \text{ is the matrix } M \text{ raised to the integer power } N \text{ i.e. each element of } M \text{ is raised to the power } N) )</td>
</tr>
<tr>
<td>SCALAR</td>
<td>Matrix generation ( (K=SCALAR(R,\text{AI}) \text{ means that } K \text{ is a (1x1) matrix whose real part is } R \text{ and whose imaginary part is } \text{AI}) )</td>
</tr>
<tr>
<td>SCALARC</td>
<td>Matrix generation ( (K=SCALARC(C) \text{ means that } K \text{ is a (1x1) matrix containing the complex number } C) )</td>
</tr>
<tr>
<td>SDIAGHI</td>
<td>Generates a matrix with ones on the superdiagonal ( (K=SDIAGHI(IP) \text{ means that } K \text{ has ones on the superdiagonal and zeros elsewhere; } K \text{ is of the order } (IPxIP)) )</td>
</tr>
<tr>
<td>SDIAGLO</td>
<td>Generates a matrix with ones on the subdiagonal ( (K=SDIAGLO(IP) \text{ means that } K \text{ has ones on the subdiagonal and zeros elsewhere; } K \text{ is of order } (IPxIP)) )</td>
</tr>
<tr>
<td>FUNCTION</td>
<td>DESCRIPTION</td>
</tr>
<tr>
<td>----------</td>
<td>-------------</td>
</tr>
</tbody>
</table>
| SIGNMOD  | Forms the moduli of the elements of a matrix  
(K=SIGNMOD(M) means that the element $K_{ij}$ of $K$ is equal to the modulus of element $M_{ij}$ of $M$. $K_{ij}$ has the sign of the real part of $M_{ij}$) |
| SMULT    | Elemental multiplication  
(K=SMULT(M,N) means $K = M \times N$) |
| SMULT3   | Elemental multiplication  
(K=SMULT3(L,M,N) means $K = L \times M \times N$) |
| SPECIAL  | Generates a matrix of zeros with unity added to one of the elements  
(K=SPECIAL(M,N,J,K) means that $K$ is a (MxN) matrix with unity in position (J,K) and zeros elsewhere) |
| SUB      | Elemental or ordinary subtraction  
(K=SUB(M,N) means $K = M - N$) |
| SUBCOL   | Generates a column vector from the column of a matrix  
(K=SUBCOL(M,J) means that $K$ is a column vector matrix equal to column J of matrix $M$) |
| SUBMAT   | Generates a submatrix from a matrix  
(K=SUBMAT(M,I,J,JP,JQ) means that $K$ is a (JPxJQ) submatrix of $M$. The element (1,1) of $K$ corresponds to element (I,J) of $M$) |
| SUBROW   | Generates a row vector from the row of a matrix.  
(K=SUBROW(M,I) means that $K$ is a row vector matrix equal to row I of matrix $M$) |
| TRANS    | Form the transpose of a matrix  
(K=TRANS(M) means $K = M^T$) |
| TRIINV   | Calculates the inverse of a tridiagonal matrix  
(K=TRIINV(M,FUZZ,LSW,NZ,IOUT) means that $K$ is the inverse of the tridiagonal matrix $M$. If $M$ is singular then LSW is set equal to TRUE, IOUT gives the indices of the zero rows and NZ is the number of rows in IOUT. All numbers $\leq$ FUZZ are regarded as zero) |
| ZERO     | Generates a matrix of zeros  
(K=ZERO(IP,IQ) means that $K$ is an (IPxIQ) matrix of zeros) |
### LAMPS SUBROUTINES USED IN DYNMOD

<table>
<thead>
<tr>
<th>SUBROUTINE</th>
<th>DESCRIPTION</th>
</tr>
</thead>
</table>
| CLEANUP | Erases all unprotected matrices from core  
(Call CLEANUP) |
| DELETEX | Return matrices to the unprotected state  
(Call DELETEX(M,N) means that all matrices created between $\mathbf{M}$ and $\mathbf{N}$ are returned to the unprotected state) |
| GRAFM | Parallelogram plotting routine  
(Call GRAFM(M,N,L,K1,K2,K3,K4,K5); for a full description see AECL Report 6086.) |
| INITIAL | A subroutine used for matrix storage management; see AECL Report 6086.  
(Call INITIAL(HISPACE,HIDIR)) |
| READn | Input subroutine  
(Call READn(6HTITLE1,M1,...,6HTITLEn,Mn) means that the n matrices $\mathbf{M}_1,\ldots,\mathbf{M}_n$ are input in free format. The matrix titles appear on the data cards with the matrix data). |
| SHOWMEM | Indicates the amount of core space required by the program  
(Call SHOWMEM prints out the highest location reached in the LAMPS matrix storage array SPACE since the start of the job) |
| STORE | Changes a matrix by inserting a complex number  
(Call STORE(M,I,J,C) inserts a complex number C in the element location (I,J) of matrix $\mathbf{M}$) |
APPENDIX V: DYNMOD INPUT CHART

A comprehensive run of DYNMOD would involve the preparation of 11 sets of data. The output data would include

(i) the free vibration and dynamic stability characteristics of the fuel assembly,

(ii) the transient and steady response of the assembly to harmonic forces,

(iii) the receptances and mobilities of the fuel string,

(iv) the response of the structure to complicated forcing functions (expressed in statistical form), and

(v) the calculation of the resolved forces acting on the fuel assembly from a knowledge of its response characteristics.

The DYNMOD Input Chart indicates the order of appearance of the DATA SETS in the input data card pack.

All matrices input by DYNMOD are shown in the Chart. These are grouped into the relevant DATA SETS.

The logical operator matrices in each DATA SET are identified and their effect on the program execution is explained.
START

DATA SET 1 | DBUG | PLOT | BWR, SDR, XL, YL, NPERFR

NO EXTRA FAULT - FINDING INFORMATION IS REQUIRED | DBUG=(0) | PLOT=(0) | NO GRAPH PLOTS ARE SUPPLIED IN THE OUTPUT

EXTRA FAULT - FINDING INFORMATION IS REQUIRED | DBUG=(1) | PLOT=(1) | GRAPH PLOTS ARE SUPPLIED IN THE OUTPUT

DATA SET 2

G, L, M, NEL, DIAM, KCST, KEND, KPAR, GCST, GEND, GPAR, DCTOPX, DETOPX

FUEL STANDS ABOVE ITS SUPPORT | Y=(-1)

GRAVITY EFFECTS ARE IGNORED | Y=(0)

FUEL HANGS BELOW ITS SUPPORT | Y=(1)

DATA SET 3 | RHO, U, COCFCB, DIAMH, VM, DEQ, FEF

SUBROUTINE PINNED

DATA SET 4 | BPIN | JPIN | KCBOT, KEBOT, DCBOTX, DEBOTX

FUEL ASSEMBLY END FREE | BPIN=(0) | JPIN=(0) | INTERMEDIATE FUEL BUNDLES ARE FREE

FUEL ASSEMBLY END CONSTRAINED | BPIN=(1) | JPIN=(J) | THE END OF BUNDLE J IS CONSTRAINED

CALCULATE THE NATURAL FREQUENCIES AND MODESHAPES OF THE NUCLEAR FUEL ASSEMBLY IN AXIAL FLOW

DYNMOD ACCESSORY SUBROUTINES
DYNMOD MAIN PROGRAM

ACCESSORY SUBROUTINES

SUBROUTINE FORCES

DATA SET 5 WHICH

WHICH = (1) TRANSIENT RESPONSE CALCULATIONS

WHICH = (2) STEADY HARMONIC RESPONSE CALCULATIONS

WHICH = (3) STEADY AND TRANSIENT RESPONSE CALCULATIONS

WHICH = (0) TRANSFER DIRECT TO SUBROUTINE RECMOB AND DATA SET 6

FORCED RESPONSE CALCULATIONS

SUBROUTINE RECMOB

DATA SET 6 SHAKER

SHAKER = (1) ELECTRODYNAMIC SHAKER INSTALLED AT END OF BUNDLE 1

SHAKER = (0) TRANSFER DIRECT TO SUBROUTINE RANDOM AND DATA SET 7

RECEPTANCE AND MOBILITY CALCULATIONS

TO RANDOM
FROM RECMOB

SUBROUTINE RANDOM

DATA SET 7 DATA POINTS, FMIN BANWID

DATA = (1) DISCRETE RESPONSE SPECTRA CALCULATED

DATA = (2) CALCULATE RESPONSE TO CORRELATED FORCES EXPRESSED IN AVERAGED P.S.D. FORM

DATA = (3) RESPONSE TO RANDOM UNCORRELATED FORCES CALCULATED

DATA = (0) TRANSFER DIRECT TO SUBROUTINE CALFOR AND DATA SET 9

DATA SET 8, OPTION 3 UNCORF

DATA SET 8, OPTION 2 PSDFOR

DATA SET 8, OPTION 1 QFORCE

TO CALFOR
FROM RANDOM

SUBROUTINE CALFOR

DATA SET 9

TYPE

EXPER

POINTS, FMIN, BANWID

EXPER = (1)

INPUT EXPERIMENTAL
RECEPTANCE DATA
VIA DATA SET 10

EXPER = (0)

RECEPTANCE MATRICES
ARE CALCULATED
BY CALFOR

DATA SET 10

TRANAT

TYPE = (1) OR (2)

CALCULATE THE
RESOLVED FORCES
FROM DISCRETE
RESPONSE SPECTRA

TYPE = (2)

PRINT
CROSS INTENSITY
SPECTRUM OF FORCES AS
ADDITIONAL OUTPUT

TYPE = (3) OR (4)

CALCULATE THE P.S.D.
OF THE RESOLVED FORCES
FROM THE AVERAGED
P.S.D. OF RESPONSE

TYPE = (4)

PRINT
CROSS P.S.D.'S OF THE
RESOLVED FORCES AS
ADDITIONAL OUTPUT

DATA SET 11, OPTION 2

DISPOW

DATA SET 11, OPTION 1

DISNAT

END
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