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INS-Rep.-798
Nov. 1989

BRS Invariant Stochastic Quantization of Einstein Gravity

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Abstract

We study stochastic quantization of gravity in terms of a BRS invariant canonical operator formalism. By introducing artificially canonical momentum variables for the original field variables, a canonical formulation of stochastic quantization is proposed in the sense that the Fokker-Planck hamiltonian is the generator of the fictitious time translation. Then we show that there exists a nilpotent BRS symmetry in an enlarged phase space of the first-class constrained systems. The phase space is spanned by the the dynamical variables, their canonical conjugate momentum variables, Faddeev-Popov ghost and anti-ghost. We apply the general BRS invariant formulation to stochastic quantization of gravity which is described as a second-class constrained system in terms of a pair of Langevin equations coupled with white noises. It is shown that the stochastic action of gravity includes explicitly the DeWitt's type superspace metric which leads to a geometrical interpretation of quantum gravity analogus to nonlinear σ-models.

1. Introduction

Stochastic Quantization (in short, SQ) was first introduced by Parisi and Wu[1] as an interesting alternative quantization method.[2] There are two equivalent descriptions of SQ; One is based on a Langevin equation and the other is on a Fokker-Planck equation. A Langevin equation is appropriate for the numerical calculation, while it is not so suitable for the investigation of the symmetry properties of the system (such as renormalizability, unitarity, spontaneous symmetry breaking, etc.). On the other hand, Fokker-Planck (F-P) equation is constructed from the invariance principle under the symmetries of the system, thus it properly reflects the symmetry properties of the system. In this sense, the description in terms of the F-P equation is preferable for the investigation of the symmetry properties in SQ.

There are some other formulations of SQ which are motivated to clarify the symmetry properties in SQ.[3][4][5] Especially, a stochastic effective action is an useful bridge (we call it stochastic action in short) which connects those descriptions. In terms of the stochastic action, SQ has been recognized as a (D+1)-dimensional field theory and it provides us a canonical formulation of SQ, which is directly connected with the F-P equation. Furthermore, it is easy to apply the field theoretical tools developed in the ordinary quantization method to SQ, in particular, the renormalizability in SQ has been discussed in this context.[6-8]

In gauge theories, one of the most important symmetries for the consistent quantization of the system is the BRS symmetry[9][10] in the Faddeev-Popov effective action.[11] Recently the BRS symmetry of the stochastic action has been introduced for Yang-Mills field and the Ward-Takahashi identity in SQ is discussed.[12][13] In a previous paper[13] (referred to (I)), we gave a general formulation of the BRS invariant stochastic action for the first-class constrained systems and investigated the BRS structure for massless and massive Yang-Mills fields in SQ. For the gravitational field, the Langevin approach and the F-P approach have been discussed and recognized that (i) the structure of the Langevin equation takes an analogous form of a stochastic process on a Riemannian manifold,[14] (ii) In a formal sense, there exists a Zwanziger type gauge fixing which provides the Faddeev-Popov effective action in the infinite fictitious time limit.[17] It is also clarified how we obtain the Langevin equation of quantum gravity by starting from a Langevin system with a second-class constraint.[18]

The purpose of this paper is to investigate, in the context of SQ, the BRS symmetry in gravity and construct its BRS invariant formulation in terms of stochastic action by applying the general method in (I). Since the a BRS invariant stochastic action is obtained from a BRS invariant F-P equation or equivalently a F-P hamiltonian, we first investigate the BRS symmetry in the F-P hamiltonian for the first-class constrained systems which are defined in curved configuration space. Then we apply the method to the gravity case.

The paper is organized as follows. In section 2, we briefly review the equivalence between the three different descriptions of SQ. In section 3, we introduce two important concepts for SQ of the first-class constrained systems. One is a metric tensor G_{AB} and the other is a Killing-vector in a manifold of the configuration space. These concepts enables us to give a simple geometrical description of local gauge transformations and their geometrical interpretations. A variation of a configuration of the field variable is identified with the Killing-vector. We construct a F-P *equation which is invariant under the general coordinate transformation in* the configuration space. We also give an operator formalism for the invariant F-P equation and derive its path-integral representation which defines a stochastic action. In section 4, we give a general formulation of the BRS invariant F-P equation. The section 5 and 6 are devoted to the application of the general method to gravity. We consider a description for SQ of gravity by a pair of Langevin equations coupled with white noises. This procedure uniquely determine the metric tensor G_{AB} in the case of gravity. The BRS symmetry for the gravitational field is clarified in section 6. The section 7 is devoted to discussions. In the appendix A, we give the concrete description of the operator formalism in curved configuration spaces. Appendices B and C are devoted to some details on the path-integral representation. Especially we prove that our formulation is essentially equivalent to the approach by Namiki and Yamanaka[19]. We also show that the Parisi-Sourlas[20] supersymmetry in SQ

is realized in a operator ordering of the canonical formulation.

2. Fokker-Planck Hamiltonian and a Canonical Formulation in Stochastic Quantization

We first briefly review the equivalence between a Langevin equation, a F-P equation and a stochastic action.

We start from a Langevin equation for a quantum mechanical system,

$$\dot{q}^A = -\frac{\partial S_{cl}}{\partial q^A} + \eta^A, \qquad \left\langle \eta^A(t)\eta^B(t')\right\rangle_\eta = 2\delta^{AB}\delta(t-t'). \tag{2.1}$$

S_{cl} is the classical Euclidean action of the system. The dot ($\cdot$) denotes the derivative with respect to the fictitious time, t. The capital Roman subscript denotes the whole set of indices (including the space-time coordinates in the case of a quantum field theoretical system). The expectation value of an observable $O(q)$ is given by

$$\langle O(q)\rangle \equiv \lim_{t\to\infty}\langle O(q_\eta)\rangle_\eta, \tag{2.2}$$

where q^A_η is the solution of the Langevin equation (2.1).

A F-P equation

$$\dot{P}(q,t) = \frac{\partial}{\partial q^A}\left(\frac{\partial}{\partial q^A} + \frac{\partial S_{cl}}{\partial q^A}\right)P(q,t). \tag{2.3}$$

is equivalent to (2.1) under the definition of the distribution functional

$$\langle O(q_\eta)\rangle_\eta \equiv \int \mathcal{D}q O(q) P(q,t). \tag{2.4}$$

It is first shown by Parisi and Wu that the F-P equation (2.4) is rewritten as a "Schrödinger equation ", with an hermit hamiltonian, for the "wave functional "

$\Psi(q,t) \equiv e^{\frac{1}{2}S_{cl}}P(q,t),$

$$\dot{\Psi}(q,t) = -\mathcal{H}_{F-P}\Psi(q,t),$$
$$\mathcal{H}_{F-P} \equiv -(\frac{\partial}{\partial q^A})^2 + \frac{1}{4}(\frac{\partial S}{\partial q^A})^2 - \frac{1}{2}\frac{\partial^2 S}{\partial q^A \partial q^A}. \tag{2.5}$$

An advantage of this form is that the F-P hamiltonian $\mathcal{H}_{F-P}$ is formally positive semi-definite; necessary condition for the existence of the equilibrium limit. In fact, the Schrödinger equation (2.5) gives a simple proof for the equilibrium limit of the distribution functional, namely, it has an equilibrium solution $P_{eq} = e^{-S_{cl}}$. It also provides a basis for a canonical formulation of SQ in which $\mathcal{H}_{F-P}$ is the generator of the translation in the imaginary fictitious time. In order to realize the F-P hamiltonian in a phase space, we artificially introduce a canonical momentum p_A conjugate to the dynamical variable q^A and assume the commutation relation

$$[\mathrm{p}_A, \mathrm{q}^B] = -i\delta^B_A. \tag{2.6}$$

Thus we obtain an operator form of the F-P hamiltonian provided that we identify $\mathrm{p}_A \approx -i\frac{\partial}{\partial q^A}$.

Now we construct a stochastic action which connects the two pictures of SQ. Let us start with the generating functional of the white noise in (2.1)

$$Z[j] \equiv \int \mathcal{D}\eta \exp\{-\frac{1}{4}\int dt\eta^A\eta^A + \int dt j_A\eta^A\}. \tag{2.7}$$

By inserting the unity,

$$1 = \int \mathcal{D}q\delta(\dot{q}^A + \frac{\partial S_{cl}}{\partial q^A} - \eta^A)\det(\delta^{BC}\partial_t + \frac{\partial^2 S_{cl}}{\partial q^B \partial q^C}), \tag{2.8}$$

into the generating functional (2.7) and integrating out the white noise variable η^A, we obtain

$$Z[0] = \int \mathcal{D}q\mathcal{D}\pi\mathcal{D}\psi\mathcal{D}\bar{\psi}e^{\int K_{P-S}dt},$$
$$K_{P-S} \equiv -\pi_A\pi_A + i\pi_A(\dot{q}^A + \frac{\partial S_{cl}}{\partial q^A}) + \bar{\psi}_A(\dot{\psi}^A + \frac{\partial^2 S_{cl}}{\partial q^A \partial q^B}\psi^B). \tag{2.9}$$

The variable π_A is introduced to represent the δ-functional in (2.8). The Grassmanian variables ψ^A and $\bar{\psi}_A$ are also introduced to represent the determinant factor.

The generating functional (2.9) is also expressed in the following form

$$Z[0] = \int \mathcal{D}q\mathcal{D}p\exp\{\int dt(ip_A\dot{q}^A - H_{F-P}) - \frac{1}{2}S(+\infty) + \frac{1}{2}S(-\infty)\},$$
$$H_{F-P} \equiv p_A p_A + \frac{1}{4}(\frac{\partial S}{\partial q^A})^2 - \frac{1}{2}\frac{\partial^2 S}{\partial q^A \partial q^A}. \tag{2.10}$$

Here we have changed the integration variable π_A as follows

$$\pi_A = p_A + \frac{i}{2}\frac{\partial S_{cl}}{\partial q^A}, \tag{2.11}$$

and performed the integration of ψ and $\bar{\psi}$. This is a manifestation of the equivalence between the stochastic action and the F-P hamiltonian (2.5).

Here we add a comment on a supersymmetry in SQ. It is well-known that the stochastic action K_{P-S} in (2.9) possesses a supersymmetry, the Parisi-Sourlas (P-S) supersymmetry,[19]

$$\delta q^A = i\epsilon\psi^A,$$
$$\delta\psi^A = 0,$$
$$\delta\bar{\psi}_A = \epsilon\pi_A,$$
$$\delta\pi_A = 0. \tag{2.12}$$

This, first discussed in connection with the mechanism of the dimensional reduction, comes from the fact that the Langevin equation is coupled with the white noise. In this sense, the P-S symmetry is essential to justify that (2.9) describes the same system as that described by the Langevin equation (2.1). However, there is an alternative operator formalism in SQ in which the P-S symmetry is not referred.[5] In the approach, the contribution of the determinant factor in (2.8), the last term $\frac{1}{2}\frac{\partial^2 S}{\partial q^A \partial q^A}$ in (2.10), comes from the careful evaluation of the derivative with respect to the fictitious time based on the Ito's stochastic calculus[20]. In the Ito's stochastic calculus, we have

$$\left\langle \frac{d}{dt}S(q_\eta)\right\rangle_\eta = \left\langle \frac{\partial S}{\partial q^A}\dot{q}^A_\eta\right\rangle_\eta + \left\langle \frac{\partial^2 S}{\partial q^A \partial q^A}\right\rangle_\eta, \tag{2.13}$$

provided that the variable q^A_η is a function of the white noise. In this approach, we have just obtained the term $\frac{1}{2}\frac{\partial^2 S}{\partial q^A \partial q^A}$ in (2.10) from the surface term without referring to the P-S symmetry .[21] *In this sense, the P-S supersymmetry automatically* gives the same result as the Ito's stochastic calculus by regarding the variable q^A_0 independent of the white noise η^A. It is also useful to provide a basis to prove that SQ is in fact equivalent to the ordinary quantization method.[22][23]

In this paper, we mainly study the BRS invariant structure in the F-P hamiltonian (2.10) and the stochastic action

$$K \equiv ip_A \dot{q}^A - H_{F-P}. \tag{2.14}$$

Since this is obtained by integrating out the Grassmanian variables ψ^A and $\bar{\psi}_A$, P-S symmetry is implicit in this form.

3. First-Class Constrained Systems and the Invariant Fokker-Planck Equation

In this section, we discuss a method to construct a gauge invariant F-P equation. Here we introduce two important concepts; one is a metric tensor G_{AB} in the configuration space $\{q^A\}$, so called superspace metric and the other is a Killing-vector in the manifold $\{q^A, G_{AB}\}$. The Killing-vector is identified to the variation of the variable q^A under the local gauge transformation.

Let us consider the case that the classical action S_{cl} is invariant under the infinitesimal local gauge transformation

$$\delta q^A \equiv E^A_a \alpha^a. \tag{3.1}$$

Here α^a is a transformation parameter which is assumed to be independent of the fictitious time. E^A_a is a function of q^B. Note that the little Roman subscript also includes the space-time coordinates in the case of a quantum field theoretical system. The local gauge transformation of p_A should be determined such that the commutation relation (2.6) is preserved under the gauge transformation. Before

determining the transformation property of the canonical momentum variable, we consider the local gauge transformation as a special case of the general coordinate transformation in superspace

$$\delta_{g.c} q^A \equiv f^A(q). \tag{3.2}$$

Then we clarify the geometrical meaning of the local gauge transformation.

Following to (1), we introduce a metric tensor G_{AB} in the configuration space $\{q^A\}$. By using the superspace metric, it is easy to give a F-P equation to be invariant under the general coordinate transformation (3.2). We obtain an invariant F-P equation

$$\begin{aligned}\dot{P}(q,t) &= \nabla^A(\nabla_A + \frac{\partial S_{cl}}{\partial q^A})P(q,t), \\ &= \frac{1}{\sqrt{G}}\frac{\partial}{\partial q^A}\{\sqrt{G}G^{AB}(\frac{\partial}{\partial q^B} + \frac{\partial S_{cl}}{\partial q^B})\}P(q,t),\end{aligned} \tag{3.3}$$

where $G \equiv \det G_{AB}$, G^{AB} is the inverse of the metric tensor G_{AB}. We note that the distribution functional $P(q,t)$ in (3.3) is a scalar functional normalized by

$$1 = \int \mathcal{D}q\sqrt{G}P(q,t). \tag{3.4}$$

We also obtain an invariant F-P hamiltonian

$$\mathcal{H}_{F-P} \equiv -\nabla^A\nabla_A + \frac{1}{4}G^{AB}\frac{\partial S}{\partial q^A}\frac{\partial S}{\partial q^B} - \frac{1}{2}\nabla^A\frac{\partial S}{\partial q^A}, \tag{3.5}$$

which defines the "Schrödinger equation " in (2.5).

Now let us clarify the geometrical meaning of the local gauge transformation (3.1). The F-P equation (3.3) is transformed covariantly as a scalar under the general coordinate transformation provided that the superspace metric is transformed by

$$\delta_{g.c.} G^{AB} = \nabla^A f^B + \nabla^B f^A. \tag{3.6}$$

From the same reason as in the case of nonlinear σ-models, if and only if the transformation function f^A is a Killing-vector in superspace (the configuration space $\{q^A, G_{AB}\}$), the F-P equation is <u>invariant</u> under the general coordinate transformation (3.2) and (3.6).

From the correspondence, we identify the local gauge transformation (3.1) as the Killing-vector in the manifold $\{q^A, G_{AB}\}$. By solving the Killing-vector equation with respect to the metric G^{AB}

$$\nabla^A E^B_a + \nabla^B E^A_a = 0, \tag{3.7}$$

we obtain the solution G^{AB} (and G_{AB}) as a function of q^A for the given functional E^A_a. Then we arrive at a manifestly local gauge invariant F-P equation (3.3).

In what follows, we clarify the condition of the first-class constraints. The generator of the local gauge transformation (3.1) is given by

$$Q(\alpha)\Psi(q,t) \equiv -iE^A_a\alpha^a\frac{\partial}{\partial q^A}\Psi(q,t), \tag{3.8}$$

for a scalar wave function. By the help of the Killing-vector equation (3.7), the generator $Q(\alpha)$ is hermitian with respect to the inner product

$$\langle\Psi_1|Q(\alpha)|\Psi_2\rangle \equiv \int \mathcal{D}q\sqrt{G}\Psi_1(q)^*Q(\alpha)\Psi_2(q). \tag{3.9}$$

The condition of the first-class constraints is the closure of the algebra. We assume here

$$[Q_a, Q_b] = -iu^c_{ab}Q_c. \tag{3.10}$$

where $Q(\alpha) \equiv Q_a\alpha^a$. We also assume that the "structure constant " u^c_{ab} is independent of q^A and p_B. As we have shown in (I), these assumptions are satisfied for the case of massless and massive Yang-Mills fields. As we shall show later, in the case of the gravitational field, the condition (3.10) is also satisfied. The closed algebra (3.10) is equivalent to the condition

$$-\frac{\partial E^A_a}{\partial q^B}E^B_b + \frac{\partial E^A_b}{\partial q^B}E^B_a = u^c_{ab}E^A_c, \tag{3.11}$$

on the function E^A_a in (3.1). We note that the killing-vector equation (3.7) is used to derive the equation (3.11).

In the following, we construct an operator formalism for what we have explained above. Let us write the operator expression of the F-P equation (3.5) in a manifestly covariant form.

$$\mathbf{H}_{F\text{-}P} \equiv G^{-\frac{1}{2}}\mathbf{p}_A G^{\frac{1}{2}}G^{AB}\mathbf{p}_B + \frac{1}{4}G^{AB}\frac{\partial S}{\partial q^A}\frac{\partial S}{\partial q^B}(\mathbf{q}) - \frac{1}{2}\nabla^A\frac{\partial S}{\partial q^A}(\mathbf{q}). \tag{3.12}$$

Here the canonical momentum operator satisfies the commutation relation

$$[\mathbf{p}_A, \mathbf{q}^B] = -i\delta^B_A. \tag{3.13}$$

It is identified

$$\mathbf{p}_A \sim -i\frac{\partial}{\partial q^A}, \tag{3.14}$$

on the basis of a scalar wave functional. This relation implies that the operator is <u>not</u> hermitian with respect to the inner product (3.9). However we note that the eigen-values of p_A are real (see Appendix A).

The "Schrödinger equation " is given by

$$|\dot{\Psi}\rangle = -\mathbf{H}_{F\text{-}P}|\Psi\rangle. \tag{3.15}$$

We also obtain the operator expression for the generator of the local gauge transformation (3.1)

$$\mathbf{Q}(\alpha) \equiv \alpha^a E^A_a(\mathbf{q})\mathbf{p}_A. \tag{3.16}$$

By using the Killing-vector equation, the operator $\mathbf{Q}$ is shown to be hermitian. From the condition (3.12), the algebra (3.10) holds in the operator level with the definition; $\mathbf{Q} \equiv \mathbf{Q}_a\alpha^a$. The transformation property of $\mathbf{q}^A$ and the canonical momentum $\mathbf{p}_A$ are given by

$$\begin{aligned}\delta\mathbf{q}^A &\equiv [i\mathbf{Q}(\alpha), \mathbf{q}^A], \\ &= E^A_a(\mathbf{q})\alpha^a, \\ \delta\mathbf{p}_A &= -\frac{\partial E^B_a}{\partial q^A}(\mathbf{q})\mathbf{p}_B\alpha^a.\end{aligned} \tag{3.17}$$

Although the hermiticity of these operators are not discussed in (I), it depends on the definition of the inner product. In the appendices A and B, we also explain the inner product on which we have studied the BRS symmetry in (1).

In the following section, we show that how the BRS symmetry is realized in the canonical formulation.

4. BRS Symmetry in Stochastic Quantization of the First-Class Constrained Systems

In this section, we study the BRS symmetry in F-P hamiltonian in terms of the operator formalism developed in the previous section. We also derive the path-integral representation of a vacuum transition amplitude which gives a BRS invariant stochastic action for the first-class constrained systems.

The BRS transformation in SQ is constructed from the local gauge transformation by replacing

$$\alpha^a \rightarrow i\epsilon C^a, \tag{4.1}$$

where ϵ is a Grassmanian constant. C^a is the Faddeev- Popov ghost field. The nilpotent BRS transformation is given by[13]

$$\begin{aligned}
\delta_{BRS}\, \mathrm{q}^A &= i\epsilon E^A_a(\mathrm{q})\mathrm{C}^a, \\
\delta_{BRS}\, \mathrm{p}_A &= -i\epsilon \frac{\partial E^B_b}{\partial q^A} \mathrm{p}_B \mathrm{C}^b, \\
\delta_{BRS}\, \mathrm{C}^a &= -\frac{i\epsilon}{2}[\mathrm{C} \times \mathrm{C}]^a, \\
\delta_{BRS}\, \bar{\mathrm{C}}_a &= \epsilon(E^A_a \mathrm{p}_A - iu^c_{ab}\mathrm{C}^b\bar{\mathrm{C}}_c - \frac{i}{2}u^b_{ab}).
\end{aligned} \tag{4.2}$$

We note that the BRS transformation of the metric G^{AB} is induced by the BRS transformation of q^A since the metric G^{AB} is not a fundamental variable but the solution of the Killing-vector equation (3.7) as a function of q^A. It is easy to show the nilpotency of the BRS transformation (4.2) by using the algebra of constraints (3.10) and the Jacobi identity provided that the F-P ghost and anti-ghost satisfy the anti-commutation relation

$$\{i\bar{\mathrm{C}}_a, \mathrm{C}^b\} = -i\delta^b_a. \tag{4.3}$$

Here we comment on the derivation of the BRS transformation (4.2). The transformations of q^A and p_A are obtained by the replacement (4.1) in (4.14). The transformation property of the F-P ghost C^a is determined from the nilpotency condition of the transformations of q^A and p_A. The transformation for $\bar{\mathrm{C}}_a$ is uniquely determined as follows. In the present construction of the nilpotent BRS transformation, we start from the phase space (q^A, p_B). By regarding the F-P ghost and anti-ghost as the canonical conjugate pair, the phase space is enlarged to $(q^A, p_B, C^a, \bar{C}_b)$. This means that a charge which generates the BRS transformation of $\mathrm{q}^A, \mathrm{p}_B$ and C^a on the basis of the commutation relations (4.9) and (4.3) exists and it automatically gives the transformation of $\bar{\mathrm{C}}_a$. On the basis of this construction, the nilpotency of (4.2) is nontrivial, however, it is easily confirmed by an explicit calculation. The BRS transformation (4.2) is also derived from a consistent truncation of an extended BRS transformation in a (D+1)-dimensional formulation of SQ in which the multiplier field of the constraints Q_a in (3.8) and the Nakanishi-Lautorup field of a gauge fixing are introduced.[19] These auxiliary fields define a (D+1)-dimensional gauge multiplet with original dynamical variable, however, consistently truncated in a special class of gauge fixings leaving the nilpotent BRS transformation (4.2)* . Note that (4.2) is independent of the choice of gauge fixing in (D+1)-dimensional BRS invariant formulation. The similar structure of the BRS transformation is also discussed in a slightly different context by *Batalin-Fradkin-Vilkovisky.*[21]

The BRS charge which generates (4.2) is given by

$$\begin{aligned}
\mathrm{Q}_{BRS} &\equiv i(\delta'_{BRS}\, \mathrm{q}^A)\mathrm{p}_A - (\delta'_{BRS}\, \mathrm{C}^a)\bar{\mathrm{C}}_a, \\
&= -E^A_a \mathrm{C}^a \mathrm{p}_A + \frac{i}{2}[\mathrm{C} \times \mathrm{C}]^a \bar{\mathrm{C}}_a + \frac{i}{2}u^b_{ab}\mathrm{C}^a,
\end{aligned} \tag{4.4}$$

* There is a minor error in (1). The constant term, $-\frac{i}{2}u^b_{ab}$, in the BRS transformation of the anti-ghost $\bar{\mathrm{C}}_a$ is necessary for the hermiticity.

where we define $\delta_{BRS} \equiv \epsilon\delta'_{BRS}$. The BRS charge satisfies the nilpotency condition

$$\{\mathbf{Q}_{BRS}, \mathbf{Q}_{BRS}\} = 0. \tag{4.5}$$

We note that the BRS charge is hermit. Although the BRS transformation (4.2) is expressed in the basis of the non-hermit operator $\mathbf{p}_A$, it is easy to see that the hermit BRS transformation is realized in the basis of the hermit momentum operator $\hat{\mathbf{p}}_A$ by replacing $\mathbf{p}_A$ with $G^{\frac{1}{4}}\hat{\mathbf{p}}_A G^{-\frac{1}{4}}$ in (4.4) (see also the appendix A). In this basis the transformation takes more complicated form, because $\hat{\mathbf{p}}_A$ is not a vector under the general coordinate transformation, while the variation of $\hat{\mathbf{p}}_A$ under the BRS transformation is manifestly hermit.

The BRS invariant Fokker-Planck hamiltonian is given by

$$\mathbf{H}_{BRS} = \mathbf{H}_{F-P} + \{\mathbf{Q}_{BRS}, \bar{\mathbf{C}}_a \chi^a\}, \tag{4.6}$$

where χ^a is a gauge fixing function which is a function of only q^A. $\mathbf{H}_{BRS}$ is hermit provided that we choose an hermitian function χ^a. It defines the BRS invariant "Schrödinger equation "

$$|\dot{\Psi}\rangle = -\mathbf{H}_{BRS}|\Psi\rangle, \tag{4.7}$$

which is the basis of the present BRS invariant formulation

(4.6) is also invariant under the scale transformation of the ghost fields.

$$\begin{aligned} \delta_{gh} C^a &= \rho C^a, \\ \delta_{gh} \bar{C}_a &= -\rho \bar{C}_a. \end{aligned} \tag{4.8}$$

The invariance implies that the ghost number charge

$$\mathbf{Q}_{gh} \equiv i\frac{1}{2}(\mathbf{C}^a\bar{\mathbf{C}}_a - \bar{\mathbf{C}}_a\mathbf{C}^a), \tag{4.9}$$

conserves. The BRS charge and the ghost number charge satisfy the well-known

algebra

$$\begin{aligned} [\mathbf{Q}_{gh}, \mathbf{Q}_{gh}] &= 0, \\ [\mathbf{Q}_{gh}, \mathbf{Q}_{BRS}] &= -i\mathbf{Q}_{BRS}. \end{aligned} \tag{4.10}$$

From the algebra, one may require the Kugo-Ojima's subsidiary condition[25] ; $\mathbf{Q}_{BRS}|\Psi_{phys}\rangle = 0$ to specify the physical subsector in the whole Hilbert space. However we note that it may not be enough to define an unitary S-matrix in the context of SQ because we consider the asymptotic states with respect to the fictitious time. Thus, in addition to the subsidiary condition, it is necessary to specify a boundary condition on the true time coordinate in space-time. In this sense, the problem of the unitarity is remained yet to be solved. The BRS algebra provides the possible basis of the BRS cohomology.

Here, for the application to gravity, we derive the path-integral representation of (4.7). As we will show in the following section, in the case of the gravitational field, a nontrivial superspace metric G_{AB} exists and it satisfies the following special condition

$$\begin{aligned} G = \det G_{AB} &= \text{constant}, \\ \frac{1}{\sqrt{G}}\partial_B(\sqrt{G}G^{AB}) &= \partial_B G^{AB} = 0. \end{aligned} \tag{4.11}$$

The first condition implies that it is possible to reduce the invariant measure $\Pi_x\sqrt{G}dq^A$ to $\Pi_x dq'^A$, namely one can choose G = constant, by an appropriate choice of the variable q^A. This is well-known, especially for the gravitational field. The second condition is interpreted as a coordinate condition in superspace. In a Riemannian manifold, the coordinates which satisfy the condition $\frac{1}{\sqrt{G}}\partial_B(\sqrt{G}G^{AB}) = 0$ is called harmonic coordinate. In the following section, we explicitly show that $g_{\mu\nu}$ is a "harmonic coordinate " in the superspace of the 4-dimensional gravity.

If the conditions (4.11) hold, the path-integral representation of (4.7) takes a simple form because there is no problem in the operator orderings. By the help of the condition (4.11), we have

$$\begin{aligned} Z[0]_{BRS} &= \left\langle \Psi_f \right| e^{-H_{BRS}(t_f-t_i)} \left| \Psi_i \right\rangle, \\ &= \int \mathcal{D}q\mathcal{D}p\mathcal{D}C\mathcal{D}\bar{C}\Psi_f^*\Psi_i \exp\{ \int dt\,(i\dot{q}^A p_A - \dot{C}^a\bar{C}_a - H_{BRS})\}, \\ H_{BRS} &= G^{AB}p_A p_B + \frac{1}{4}G^{AB}\frac{\partial S_{cl}}{\partial q^A}\frac{\partial S_{cl}}{\partial q^A} - \frac{1}{2}G^{AB}\frac{\partial^2 S_{cl}}{\partial q^A \partial q^B} \\ &+ (E_a^A p_A - iu_{ab}^c C^b \bar{C}_c - \frac{i}{2}u_{ab}^b)\chi^a - i\bar{C}^a\frac{\partial \chi^a}{\partial q^A}E_b^A C^b. \end{aligned} \tag{4.12}$$

Note that the expression is correct only if the condition (4.11) holds. The path-integral representation (4.12) can be rewritten in the form in which the P-S supersymmetry is manifestly preserved.

Before closing this section, we comment on the gauge fixing function χ^a. Although χ^a is an arbitrary function of q^A, for a special class of the function χ^a, it can be shown that the equilibrium distribution functional of the F-P equation formally takes the form of the Faddeev-Popov effective action. In fact, the equilibrium distribution is given by the well-known functional,

$$\mathcal{P}_{Faddeev-Popov} \equiv \int e^{-S_{cl}-S_g}, \tag{4.13}$$

where $\int$ denotes $\int \mathcal{D}B\mathcal{D}c\mathcal{D}\bar{c}$. S_g denotes the gauge fixing term and the Faddeev-Popov determinant term;

$$\begin{aligned} S_g &= \hat{\delta}_{BRS}F, \\ \hat{\delta}_{BRS} &\equiv -iB_a\frac{\partial}{\partial \bar{c}_a} - \frac{1}{2}[c\times c]^a\frac{\partial}{\partial c^a} + E_a^A c^a\frac{\partial}{\partial q^A}, \\ F &= \bar{c}_a(\phi^a + \frac{\alpha}{2}B^a), \end{aligned} \tag{4.14}$$

where ϕ^a, a function of only q^A, denotes the gauge fixing function. The operator $\hat{\delta}_{BRS}$ is the generator of the BRS transformation in the ordinary quantization method which satisfies the nilpotency condition. In order to obtain the equilibrium distribution functional, it is sufficient to choose

$$\chi^a = -\int c^a\nabla^A(\partial_A F e^{-S_{tot}})\mathcal{P}^{-1}_{Faddeev-Popov}, \tag{4.15}$$

where $S_{tot} = S_{cl} + S_g$. The expression is a generalization of Yang-Mills[16][21] and gravity cases[17] discussed in terms of the Zwanziger type gauge in Langevin systems.

5. Stochastic Quantization of Gravity with the Langevin Equation

The purpose of this section is to illustrate how we apply the geometrical interpretation of the first-class constrained systems to the gravity case, especially, how we specify the superspace metric G_{AB}.[20] In order to introduce the superspace metric on a manifold of the configuration space $\{q^A\}$, we first consider a system in which two field variables $g_{\mu\nu}(x)$ and $\tilde{g}^{\mu\nu}(x)$ are independent each other and they satisfy the constraint

$$\tilde{g}^{\mu\lambda}g_{\lambda\nu} - \sqrt{g}\delta^\mu_\nu = 0, \tag{5.1}$$

where $g \equiv \det g_{\mu\nu}$. By solving the constraint, we have

$$\tilde{g}^{\mu\nu}(x) = \sqrt{g}g^{\mu\nu}(x). \tag{5.2}$$

By using these field variables, let us start with a pair of the Langevin equations

$$\begin{aligned} \dot{g}_{\mu\nu} &= -\gamma_1\frac{\Delta S_{cl}}{\Delta \tilde{g}^{\mu\nu}} + \eta_{\mu\nu}, \\ \dot{\tilde{g}}^{\mu\nu} &= -\gamma_2\frac{\Delta S_{cl}}{\Delta g_{\mu\nu}} + \tilde{\eta}^{\mu\nu}, \end{aligned} \tag{5.3}$$

where

$$\begin{aligned} S_{cl} &= S_E + \int d^4x\ \phi^\nu_\mu(\tilde{g}^{\mu\lambda}g_{\lambda\nu} - \sqrt{g}\delta^\mu_\nu), \\ S_E &\equiv \int d^4x\ \sqrt{g}R. \end{aligned} \tag{5.4}$$

The auxiliary field ϕ^μ_ν is introduced as a Lagrange multiplier field of the constraint (5.1). The derivative (or variation) Δ means that the variables $g_{\mu\nu}$ and $\tilde{g}^{\mu\nu}$ are regarded to be independent variables, that is

$$\frac{\Delta g_{\rho\sigma}(x')}{\Delta g_{\mu\nu}(x)} = \frac{\Delta\tilde{g}^{\rho\sigma}(x')}{\Delta\tilde{g}^{\mu\nu}(x)} = \frac{1}{2}(\delta^\mu_\rho\delta^\nu_\sigma + \delta^\mu_\sigma\delta^\nu_\rho)\delta^4(x;x'), \tag{5.5}$$

where the δ-function is a 4-dimensional bi-scalar density. We note that the functional derivative $\frac{\Delta}{\Delta\tilde{g}^{\mu\nu}}$ is a tensor while $\frac{\Delta}{\Delta g_{\mu\nu}}$ is a tensor density under the general coordinate transformation. (5.3) is invariant under the fictitious time independent general coordinate transformation in which the transformation parameter is independent of the fictitious time t, provided that the noise fields $\eta_{\mu\nu}$ and $\tilde{\eta}_{\mu\nu}$ are tensor and tensor density, respectively.

The correlations of the white noise fields are given by

$$\begin{aligned} \langle\tilde{\eta}^{\mu\nu}(x,t)\eta_{\rho\sigma}(x',t')\rangle &= \frac{2}{\gamma_3}(\delta^\mu_\rho\delta^\nu_\sigma + \delta^\mu_\sigma\delta^\nu_\rho)\delta^4(x;x')\delta(t-t'), \\ \langle\tilde{\eta}^{\mu\nu}(x,t)\tilde{\eta}^{\rho\sigma}(x',t')\rangle &= \langle\eta_{\mu\nu}(x,t)\eta_{\rho\sigma}(x',t')\rangle = 0, \end{aligned} \tag{5.6}$$

γ_1, γ_2 in (5.3) and γ_3 in (5.6) are constants which are determined from the requirement that (5.3) should reproduce a F-P equation of gravity.

Now we show that the pair of the Langevin equations (5.3) is equivalent to a Langevin equation for gravity coupled with a non-white noise. We first show that the multiplier field ϕ^μ_ν of the constraint is eliminated in the Langevin equations by using the consistency condition of the constraint. This implies that the constraint (5.1) is a second-class one. In fact, by the consistency condition

$$\frac{\partial}{\partial t}\left(\tilde{g}^{\mu\lambda}g_{\lambda\nu} - \sqrt{g}\delta^\mu_\nu\right) = 0, \tag{5.7}$$

and the constraint (5.1), we obtain

$$\begin{aligned} \dot{g}_{\mu\nu} = &-\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\left(\frac{\Delta S_E}{\Delta\tilde{g}^{\mu\nu}}\Big|_{\tilde{g}^{\mu\nu}=\sqrt{g}g^{\mu\nu}} - G_{\mu\nu\rho\sigma}\frac{\Delta S_E}{\Delta g_{\rho\sigma}}\Big|_{\tilde{g}^{\mu\nu}=\sqrt{g}g^{\mu\nu}}\right) \\ &+ \frac{1}{\gamma_1+\gamma_2}\left(\gamma_2\eta_{\mu\nu} - \gamma_1 G_{\mu\nu\rho\sigma}\tilde{\eta}^{\rho\sigma}\right), \end{aligned} \tag{5.8}$$

where

$$G_{\mu\nu\rho\sigma} = \frac{1}{2\sqrt{g}}(g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho} - g_{\mu\nu}g_{\rho\sigma}). \tag{5.9}$$

Here we note that the relation

$$\frac{\delta S_E}{\delta\tilde{g}^{\mu\nu}} = \frac{\Delta S_E}{\Delta\tilde{g}^{\mu\nu}}\Big|_{\tilde{g}^{\mu\nu}=\sqrt{g}g^{\mu\nu}} - G_{\mu\nu\rho\sigma}\frac{\Delta S_E}{\Delta g_{\rho\sigma}}\Big|_{\tilde{g}^{\mu\nu}=\sqrt{g}g^{\mu\nu}}, \tag{5.10}$$

holds under the constraint (5.1) due to the identity

$$\delta g_{\mu\nu} = -G_{\mu\nu\rho\sigma}\delta\tilde{g}^{\rho\sigma}. \tag{5.11}$$

Thus if we choose the constants as $-\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2} = \alpha$, we obtain

$$\begin{aligned} \dot{g}_{\mu\nu} &= \alpha\frac{\delta S_E}{\delta\tilde{g}^{\mu\nu}} + \xi_{\mu\nu}, \\ \xi_{\mu\nu} &= \frac{1}{\gamma_1+\gamma_2}\left(\gamma_2\eta_{\mu\nu} - \gamma_1 G_{\mu\nu\rho\sigma}\tilde{\eta}^{\rho\sigma}\right). \end{aligned} \tag{5.12}$$

The correlation of the noise variable $\xi_{\mu\nu}$ is evaluated in the sense of Ito's stochastic calculus as follows

$$\begin{aligned} \langle\xi_{\mu\nu}(x,t)\xi_{\rho\sigma}(x',t')\rangle &= -\frac{\gamma_1\gamma_2}{(\gamma_1+\gamma_2)^2}\Big\{\left\langle G_{\mu\nu\alpha\beta}\right\rangle\left\langle\eta_{\rho\sigma}\tilde{\eta}^{\alpha\beta}\right\rangle \\ &\qquad + \left\langle G_{\rho\sigma\alpha\beta}\right\rangle\left\langle\eta_{\mu\nu}\tilde{\eta}^{\alpha\beta}\right\rangle\Big\} \\ &= \frac{2}{\beta}\langle G_{\mu\nu\rho\sigma}\rangle\delta^4(x;x')\delta(t-t'), \end{aligned} \tag{5.13}$$

where γ_3 in (5.6) is chosen to be $\gamma_3 = \frac{2\alpha\beta}{\gamma_1+\gamma_2}$.

The Langevin equation (5.12) and the noise correlation (5.13) implies that the superspace metric G^{AB}, in the case of 4-dimensional gravity, is given by

$$\{G^{AB}\} = \{G_{\mu\nu\rho\sigma}\delta^4(x;x')\}. \tag{5.14}$$

The Langevin equations in (5.3) are not independent after eliminating the Lagrange multiplier field ϕ^μ_ν. We also have

$$\dot{g}^{\mu\nu} = \alpha \frac{\delta S_E}{\delta g_{\mu\nu}} + \tilde{\xi}^{\mu\nu},$$
$$\left\langle \tilde{\xi}_{\mu\nu}(x,t)\tilde{\xi}_{\rho\sigma}(x',t')\right\rangle = \frac{2}{\beta}\langle G^{\mu\nu\rho\sigma}\rangle \delta^4(x;x')\delta(t-t'), \tag{5.15}$$

where the inverse $G^{\mu\nu\rho\sigma}$ is defined by

$$G^{\mu\nu\rho\sigma} = \frac{1}{2}\sqrt{g}(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - g^{\mu\nu}g^{\rho\sigma}). \tag{5.16}$$

This implies that G_{AB} is given by

$$\{G_{AB}\} = \{G^{\mu\nu\rho\sigma}\delta^4(x;x')\}. \tag{5.17}$$

We here note that the superspace metric satisfies

$$\det\{G^{\mu\nu\rho\sigma}\delta^4(x;x')\} = \text{constant},$$
$$\frac{\delta}{\delta g_{\rho\sigma}}\{G^{\mu\nu\rho\sigma}\delta^4(x;x')\} = 0. \tag{5.18}$$

From the Langevin equation (5.12) and (5.15), we find that the choice of the metric tensor is correct and it also realized in the Langevin equations coupled with a white noise.

We have illustrated how to choose the metric tensor G_{AB} and G^{AB} in the configuration space $\{q^A\} = \{g_{\mu\nu}(x)\}$ in the case of the 4-dimensional gravity. At the end of this section, we comment on the D-dimensional gravity case. For the gravitational field in D-dimensional space-time, the suitable choice of the variables which give a BRS invariant path-integral measure are found to be[19]

$$\mathrm{g}_{\mu\nu} \equiv g^k g_{\mu\nu}; \qquad k = \frac{D-4}{4D},$$
$$\tilde{\mathrm{g}}^{\mu\nu} \equiv g^l g^{\mu\nu}; \qquad l = \frac{D+4}{4D}. \tag{5.19}$$

In the case of 4-dimension, (5.19) is consistent with the choice of the independent variables $g_{\mu\nu}$ and $\tilde{g}^{\mu\nu}$ (which is identified with $\tilde{g}^{\mu\nu} = \sqrt{g}g^{\mu\nu}$ by the constraint

(5.1)). In the case of the D-dimension, we choose (5.19) as the independent variables. By the same procedure explained in this section, we have the following results in the case of the D-dimensional gravity.[19]

$$\dot{\mathrm{g}}_{\mu\nu} = \alpha \frac{\delta S_E}{\delta \tilde{\mathrm{g}}^{\mu\nu}} + \xi_{\mu\nu},$$
$$\langle \xi_{\mu\nu}(x,t)\xi_{\rho\sigma}(x',t')\rangle = \frac{2}{\beta}\left\langle G^{(D)}_{\mu\nu\rho\sigma}\right\rangle \delta^D(x;x')\delta(t-t'), \tag{5.20}$$

where

$$G^{(D)}_{\mu\nu\rho\sigma} = \frac{1}{2}\mathrm{g}^{-\frac{2}{D}}(\mathrm{g}_{\mu\rho}\mathrm{g}_{\nu\sigma} + \mathrm{g}_{\mu\sigma}\mathrm{g}_{\nu\rho} - \frac{4}{D}\mathrm{g}_{\mu\nu}\mathrm{g}_{\rho\sigma}). \tag{5.21}$$

This implies that the metric tensor G^{AB} is given by

$$\{G^{AB}\} = \{G^{(D)}_{\mu\nu\rho\sigma}\delta^D(x;x')\}, \tag{5.22}$$

in the case of the D-dimensional gravity. It is also easy to confirm that the metric G_{AB} is

$$\{G_{AB}\} = \{G^{\mu\nu\rho\sigma}_{(D)}\delta^D(x;x')\},$$
$$G^{\mu\nu\rho\sigma}_{(D)} = \frac{1}{2}\mathrm{g}^{-\frac{2}{D}}(\tilde{\mathrm{g}}^{\mu\rho}\tilde{\mathrm{g}}^{\nu\sigma} + \tilde{\mathrm{g}}^{\mu\sigma}\tilde{\mathrm{g}}^{\nu\rho} - \frac{4}{D}\tilde{\mathrm{g}}^{\mu\nu}\tilde{\mathrm{g}}^{\rho\sigma}). \tag{5.23}$$

We note that only the metric tensor (5.22) and (5.23) satisfies the condition (5.18) in D-dimensional space-time.

Our approach is an analogue of the nonlinear σ-model case[20] and it successfully leads to the unique superspace metric if we choose the appropriate field variable for the gravitational field. A similar approach to introduce the independent field variables with a constraint has been discussed by remaining the multiplier field of the constraint to be governed by an additional Langevin equation.[11]

6. BRS Invariant Fokker-Planck Hamiltonian and the Stochastic Action for Gravity

In this section, we investigate SQ of gravity based on the BRS invariant stochastic quantization developed in the previous sections. We mainly investigate the BRS structure by applying the general method to construct the nilpotent BRS transformation and the Fokker-Planck hamiltonian which is invariant under the general coordinate transformation of gravity in the enlarged phase space of SQ.

We start with the general coordinate transformation for the gravitational field $g_{\mu\nu}$.

$$\begin{aligned}\delta_{g.c.}g_{\mu\nu} &\equiv \nabla_\mu\xi_\nu + \nabla_\nu\xi_\mu \\ &= \partial_\mu\xi^\rho g_{\rho\nu} + \partial_\nu\xi^\rho g_{\mu\rho} + \xi^\rho\partial_\rho g_{\mu\nu}.\end{aligned} \tag{6.1}$$

Here we consider a configuration space of the metric tensor $g_{\mu\nu}$; $\{q^A\} \equiv \{g_{\mu\nu}(x)\}$. The transformation function E_a^A in (3.1) is defined by

$$\delta_{g.c.}g_{\mu\nu} \equiv \int d^4x' E_{\mu\nu\,\rho}(x;x')\xi^\rho(x'). \tag{6.2}$$

It is not necessary to specify the transformation function $E_{\mu\nu\,\rho}(x;x')$ for the following discussion.

From the transformation rule (6.1), we derive the transformation of the canonical momentum variable and the algebra of the general coordinate transformation. The transformation property of the canonical momentum variable, let $p^{\mu\nu}(x)$ be the eigenvalue of canonical momentum operators, is given by

$$\begin{aligned}\delta_{g.c.}p^{\mu\nu}(x) &\equiv -\int d^4y d^4y' \frac{\delta}{\delta g_{\mu\nu}(x)} E_{\mu'\nu'\,\rho}(y;y')\xi^\rho(y')p^{\mu'\nu'}, \\ &= -\partial_\rho\xi^\mu p^{\rho\nu} - \partial_\rho\xi^\nu p^{\rho\mu} + \partial_\rho(\xi^\rho p^{\mu\nu}).\end{aligned} \tag{6.3}$$

We note that this is the transformation for a rank 2 tensor of weight ½ such as $\sqrt{g}g^{\mu\nu}$. The generator of the transformations (6.2) and (6.3) is given by

$$Q_{g.c.}(\xi) = \int d^4x \{\partial_\mu\xi^\rho \mathrm{g}_{\rho\nu} + \partial_\nu\xi^\rho \mathrm{g}_{\mu\rho} + \xi^\rho\partial_\rho \mathrm{g}_{\mu\nu}\}\mathrm{p}^{\mu\nu}(x). \tag{6.4}$$

This satisfies

$$\begin{aligned}[Q_{g.c.}(\xi_1), Q_{g.c.}(\xi_2)] &= -iQ_{g.c.}(v), \\ v^\mu &\equiv [\xi_1 \times \xi_2]^\mu, \\ &= \xi_2^\rho\partial_\rho\xi_1^\mu - \xi_1^\rho\partial_\rho\xi_2^\mu,\end{aligned} \tag{6.5}$$

provided that the commutation relation is given by

$$[\mathrm{p}^{\mu\nu}(x), \mathrm{g}_{\rho\sigma}(x')] = -i\frac{1}{2}(\delta_\rho^\mu\delta_\sigma^\nu + \delta_\sigma^\mu\delta_\rho^\nu)\delta^4(x;x'). \tag{6.6}$$

$[\xi_1 \times \xi_2]$ is just the Lee derivative of ξ_1 with respect to ξ_2.

From the general formula (4.2) of the BRS transformation in phase space, We finally obtain the BRS transformation for the gravitational field in SQ.

$$\begin{aligned}\delta_{BRS}\mathrm{g}_{\mu\nu} &= i\epsilon\{\partial_\mu \mathrm{c}^\rho \mathrm{g}_{\rho\nu} + \partial_\nu \mathrm{c}^\rho \mathrm{g}_{\mu\rho} + \mathrm{c}^\rho\partial_\rho \mathrm{g}_{\mu\nu}\}, \\ \delta_{BRS}\mathrm{p}^{\mu\nu} &= i\epsilon\{-\partial_\rho \mathrm{c}^\mu \mathrm{p}^{\rho\nu} - \partial_\rho \mathrm{c}^\nu \mathrm{p}^{\rho\mu} + \partial_\rho(\mathrm{c}^\rho \mathrm{p}^{\mu\nu})\}, \\ \delta_{BRS}\mathrm{c}^\mu &= i\epsilon \mathrm{c}^\rho\partial_\rho \mathrm{c}^\mu, \\ \delta_{BRS}\dot{\mathrm{c}}_\mu &= \epsilon\{-2\partial_\rho(\mathrm{g}_{\mu\sigma}\mathrm{p}^{\rho\sigma}) + \partial_\mu \mathrm{g}_{\rho\sigma}\mathrm{p}^{\rho\sigma} + i(\partial_\rho(\mathrm{c}^\rho\mathrm{c}^\mu) + \partial_\mu \mathrm{c}^\rho\dot{\mathrm{c}}_\rho)\}.\end{aligned} \tag{6.7}$$

The nilpotency of the BRS transformation is explicitly checked. We here assume the commutation relation (6.6) and the anti-commutation relation,

$$\{\mathrm{c}^\mu(x), \dot{\mathrm{c}}_\nu(x')\} = -\delta_\nu^\mu\delta^4(x;x'). \tag{6.8}$$

It is also easy to obtain the following BRS charge by the formula (4.4)

$$\begin{aligned}Q_{BRS} = &-\int d^4x(\partial_\mu \mathrm{c}^\rho \mathrm{g}_{\rho\nu} + \partial_\nu \mathrm{c}^\rho \mathrm{g}_{\mu\rho} + \mathrm{c}^\rho\partial_\rho \mathrm{g}_{\mu\nu})\mathrm{p}^{\mu\nu} \\ &+ i\int d^4x(\partial_\mu \mathrm{c}^\rho)\mathrm{c}^\mu \dot{\mathrm{c}}_\rho.\end{aligned} \tag{6.9}$$

The application of general method, with the superspace metric (5.17), is now straightforward. We here only show the results. The BRS invariant Fokker-Planck hamiltonian is given by

$$|\dot{\Psi}\rangle = \mathbf{H}_{BRS}|\Psi\rangle,$$

$$\mathbf{H}_{BRS} = -\int d^4x G_{\mu\nu\rho\sigma}\{p^{\mu\nu}p^{\rho\sigma} + \frac{1}{4}\frac{\delta S_E}{\delta g_{\mu\nu}(x)}\frac{\delta S_E}{\delta g_{\rho\sigma}(x)} - \frac{1}{2}\frac{\delta^2 S_E}{\delta g_{\mu\nu}(x)\delta g_{\rho\sigma}(x)}\}$$
$$+ \int d^4x [\{-2\partial_\rho(g_{\mu\sigma}p^{\rho\sigma}) + \partial_\mu g_{\rho\sigma}p^{\rho\sigma} + i(\partial_\rho(c^\rho\bar{c}^\mu) + \partial_\mu c^\rho\bar{c}_\rho)\}\lambda^\mu$$
$$- i\bar{c}_\rho\frac{\delta\lambda^\rho}{\delta g_{\mu\nu}(x)}(\partial_\mu c^\rho g_{\rho\nu} + \partial_\nu c^\rho g_{\mu\rho} + c^\rho\partial_\rho g_{\mu\nu})]. \qquad (6.10)$$

Here is an arbitrary gauge fixing function of $g_{\mu\nu}$. From the Fokker-Planck hamiltonian and the formula (4.12), we have the BRS invariant stochastic action for gravitational field in 4-dimensions.

$$K_{BRS} = \int d^4x (i\dot{g}_{\mu\nu}p^{\mu\nu} - \dot{c}^\mu\bar{c}_\mu)$$
$$+ \int d^4x G_{\mu\nu\rho\sigma}\{p^{\mu\nu}p^{\rho\sigma} - \frac{1}{4}\frac{\delta S_E}{\delta g_{\mu\nu}(x)}\frac{\delta S_E}{\delta g_{\rho\sigma}(x)} + \frac{1}{2}\frac{\delta^2 S_E}{\delta g_{\mu\nu}(x)\delta g_{\rho\sigma}(x)}\}$$
$$- \int d^4x [\{-2\partial_\rho(g_{\mu\sigma}p^{\rho\sigma}) + \partial_\mu g_{\rho\sigma}p^{\rho\sigma} + i(\partial_\rho(c^\rho\bar{c}^\mu) + \partial_\mu c^\rho\bar{c}_\rho)\}\lambda^\mu$$
$$- i\bar{c}_\rho\frac{\delta\lambda^\rho}{\delta g_{\mu\nu}(x)}(\partial_\mu c^\rho g_{\rho\nu} + \partial_\nu c^\rho g_{\mu\rho} + c^\rho\partial_\rho g_{\mu\nu})]. \qquad (6.11)$$

The BRS invariant stochastic action is the basis for both perturbative and non-perturbative analysis of quantum gravity in SQ. A peculiar feature in (6.11) is the appearance of the DeWitt's superspace metric. In the ordinary quantization method, it is introduced to define an invariant path-integral measure, while it does not appear in the covariant form in the canonical quantization. On the other hand, since (6.11) is explicitly includes the superspace metric, it leads to a geometric interpretation of quantum gravity with an analogue of nonlinear σ-model explained in section 3. The physical consequence of the geometrical picture is under investigation. In a perturbative sense, the renormalizability of 4-dimensional quantum gravity is not improved even if we consider the stochastic action (6.11). In fact, a simple power counting analysis shows that (6.11) is not renormalizable in loop expansions with the Ward-Takahashi (W-T) identity derived from the BRS symmetry. We note that, in addition to the BRS W-T identity there may also exist another W-T identity of the P-S supersymmetry which is important to construct the invariant F-P hamiltonian (3.12). This point will be published elsewhere.

We finally add a modified form of the stochastic action (6.11). The term $\frac{\delta^2 S_E}{\delta g_{\mu\nu}(x)\delta g_{\rho\sigma}(x)}$ in K_{BRS} is a singular term including $\delta^4(0)$ and its derivative. If we use an appropriate regularization method, such as the dimensional one, the term vanishes and we obtain a form which is similar to the stochastic action discussed in Ref.13 for Yang-Mills field. For the gravity case,

$$K_{BRS} = \int d^4x (i\dot{g}_{\mu\nu}\pi^{\mu\nu} - \dot{c}^\mu\bar{c}_\mu)$$
$$+ \int d^4x G_{\mu\nu\rho\sigma}\{\pi^{\mu\nu}\pi^{\rho\sigma} - i\pi^{\mu\nu}(x)\frac{\delta S_E}{\delta g_{\rho\sigma}(x)}\}$$
$$- \int d^4x [\{-2\partial_\rho(g_{\mu\sigma}\pi^{\rho\sigma}) + \partial_\mu g_{\rho\sigma}\pi^{\rho\sigma} + i(\partial_\rho(c^\rho\bar{c}^\mu) + \partial_\mu c^\rho\bar{c}_\rho)\}\lambda^\mu$$
$$- i\bar{c}_\rho\frac{\delta\lambda^\rho}{\delta g_{\mu\nu}(x)}(\partial_\mu c^\rho g_{\rho\nu} + \partial_\nu c^\rho g_{\mu\rho} + c^\rho\partial_\rho g_{\mu\nu})]. \qquad (6.12)$$

Here we have used the redefinition of the integration variable

$$\pi^{\mu\nu} = p^{\mu\nu} + \frac{i}{2}\frac{\delta S_E}{\delta g_{\mu\nu}}. \qquad (6.13)$$

We have discussed only the 4-dimensional space-time case. The extension of the result in the case of D-dimensional space-time is straightforward by using the fundamental variable defined in (5.19).

7. Discussions

In this paper, we developed the BRS invariant operator formalism of SQ for the first-class constrained systems. We obtained the BRS invariant F-P hamiltonian which is realized on a canonical formulation in an artificially enlarged phase space. We also derived the path-integral representation of the "vacuum transition amplitude " which gives the BRS invariant stochastic action. In the approach, SQ is recognized as a (D+1)-dimensional canonical hamilton formalism. The BRS symmetry is realized in the enlarged phase space ($q^A, p_B, C^a, \bar{C}_b$).

We applied the general formulation to the case of gravity and showed that there exists a non-trivial metric tensor in the configuration space $\{g_{\mu\nu}(x)\}$ of the gravitational field. Our starting point is a pair of the Langevin equations coupled with white noises. We introduced two independent variables of the gravitational fields, $g_{\mu\nu}$ and $\tilde{g}^{\mu\nu}$ (which is identified to $\sqrt{g}g^{\mu\nu}$ in 4-dimensional space-time by a second-class constraint). The choice of the independent variables is determined such that these variables give a BRS invariant path-integral measure. In the configuration space $\{g_{\mu\nu}, \tilde{g}^{\rho\sigma}\}$, the constraint defines a surface on which $\{g_{\mu\nu}\}$ is a natural coordinate and an unique superspace metric $G^{\mu\nu\rho\sigma}(x)\delta^4(x;x')$ is induced. The description is an analogue of the non-linear σ-model case. Since the Langevin equations are coupled with white noises, it may provide a possible basis for the numerical calculation of quantum gravity. It is also important to note that the variation of the gravitational field under the space-time general coordinate transformation is a Killing vector in the configuration space. This implies that in this respect the gravity is a nonlinear σ-model; the target space is the superspace in which the space time general coordinate transformation specifies the direction of the Killing vector. The geometric interpretation leads to the following analogy between the nonlinear σ-model and quantum gravity. In nonlinear σ-models, in general, although it is not renormalizable in 4-dimensional space-time, the renormalization is recognized as a deformation of the geometry, namely, a modification of the metric tensor in the target space. The present formulation suggests that the renormalization in SQ of gravity may be interpreted as a deformation of geometry

in superspace, namely, a change of the superspace metric $G_{\mu\nu:\rho\sigma}(x)\delta^4(x;x')$. This issue is under investigation in the BRS invariant formulation.

We obtained the BRS invariant formulation for SQ successfully, however, there remains many open questions. First of all, it should be noted that the BRS symmetry in SQ is essentially different from that in the ordinary quantization method except for the role in the renormalizability problem. The reason is as follows. In the ordinary quantization method, the BRS symmetry specifies the physical subsector in the whole Hilbert space and it provides a basis of the gauge independence of physics. On the other hand, as we explained in (I) and this paper, the BRS symmetry in SQ comes from a consistent truncation of a (D+1)-dimensional gauge symmetry. This may suggest that a priori it is not necessary to require the gauge independence of physics at a finite fictitious time. The fact that the special choice of the multiplier field (4.15) leads to the usual Faddeev-Popov effective action also supports this view point. Thus it should be clarified that how the BRS invariance leads to the unitary S-matrix in SQ, or equivalently how we use the BRS symmetry in constructing the S-matrix. In this context, it is also important to clarify the BRS cohomology by *requiring the Kugo-Ojima's subsidary condition in the present* BRS invariant formulation. We hope that the BRS symmetry in SQ developed in (I) and this paper is useful to solve these problems.

ACKNOWLEDGEMENTS

The author would like to thank Professor M. Ninomiya for the valuable comments and careful reading of the manuscript. He is also grateful to Professor N. Nakanishi, Dr. H. Nakazato, Dr. I. Ojima and Dr. J. Sakamoto for the helpful discussions. This work is supported in part by the Grant-in-Aid, Ministry of Education, Science and Culture No. 01790184. He is indebted to the Japan Society for the Promotion of Science for the financial aid.

APPENDIX A

In this section, we provide a basis of the operator formalism in SQ which is necessary to obtain its path-integral representation.

First we specify the definition of the abstract Hilbert space and operators. For the bra $(q|$ and the ket $|q\rangle$, we define

$$\begin{aligned} \mathbf{q}^A|q\rangle &= q^A|q\rangle, \\ 1 &= \int \mathcal{D}q|q\rangle\langle q|, \\ \langle q|q'\rangle &= \delta(q;q'), \end{aligned} \tag{A.1}$$

where the δ-function is a by-scalar density

$$\int \mathcal{D}q\delta(q;q') = \int \mathcal{D}q'\delta(q;q') = 1. \tag{A.2}$$

Since the bra $\langle q|$ and the ket $|q\rangle$ carry the weight $G^{\frac{1}{4}}$, we define a scalar wave function $\Psi(q)$ by

$$\langle q|\Psi\rangle \equiv G^{\frac{1}{4}}(q)\Psi(q). \tag{A.3}$$

The hermit conjugation of an operator $\dot{O}$ is defined by

$$\left\langle \Psi_1 \dot{\mathrm{O}} \Psi_2 \right\rangle^\dagger = \left\langle \Psi_2 \dot{\mathrm{O}}^\dagger \Psi_1 \right\rangle. \tag{A.4}$$

where the inner product is defined by (3.9). On this conjugate operator, an hermit canonical momentum operator $\dot{\mathbf{p}}_A$ is defined by

$$\left\langle q|\dot{\mathbf{p}}_A|q'\right\rangle \equiv -i\frac{\partial}{\partial q^A}\delta(q;q'), \tag{A.5}$$

which implies the differential operator

$$\dot{\mathbf{p}}_A \sim -iG^{-\frac{1}{4}}\frac{\partial}{\partial q^A}G^{\frac{1}{4}}, \tag{A.6}$$

on a scalar wave functional. Here the momentum operator is not a vector under the general coordinate transformation (3.3). As a vector (but a non-hermit) operator, we define

$$\mathbf{p}_A \equiv G^{\frac{1}{4}}\dot{\mathbf{p}}_A G^{-\frac{1}{4}}, \tag{A.7}$$

which is identified

$$\mathbf{p}_A \sim -i\frac{\partial}{\partial q^A}, \tag{A.8}$$

on a scalar wave functional. The both momentum operators satisfy the same commutation relation,

$$[\dot{\mathbf{p}}_A, \mathbf{q}^B] = [\mathbf{p}_A, \mathbf{q}^B] = -i\delta^B_A. \tag{A.9}$$

Although, $\mathbf{p}_A$ is not hermit, its transformation property is a vectorial one (see (3.17)). On the other hand, the transformation property of the hermitian operator $\dot{\mathbf{p}}$; $\delta\dot{\mathbf{p}}_A \equiv [i\mathbf{Q}(\alpha), \dot{\mathbf{p}}_A]$, takes more complicated form rather than (3.17).

To derive the path-integral representation (5.12), we define the complete set in terms of the momentum $|p\rangle$. From the definition (A.5) of the hermitian operator $\dot{\mathbf{p}}_A$, we have

$$\langle q|p\rangle = N_0 e^{ip_A q^A}, \tag{A.10}$$

for $|p\rangle$; $\dot{\mathbf{p}}_A|p\rangle = p_A|p\rangle$. Thus, we obtain

$$\begin{aligned} \langle p|p'\rangle &= \int \mathcal{D}q \langle p|q\rangle \langle q|p'\rangle, \\ &= \delta(p-p'), \end{aligned} \tag{A.11}$$

with an appropriate choice of the normalization N_0. This implies the following completeness condition

$$1 = \int \mathcal{D}p|p\rangle\langle p|. \tag{A.12}$$

In the last of this appendix, we note that the non-hermit operator (A.7) also has the same eigen-value p_A

$$\mathbf{p}_A G^{\frac{1}{4}}|p\rangle = p_A G^{\frac{1}{4}}|p\rangle. \tag{A.13}$$

APPENDIX B

In this appendix, we derive the path-integral representation (4.23) by stating from the "Schrödinger equation " (4.11). Let us consider a generating functional

$$Z[0] \equiv \langle \Psi_f | e^{-H_{F-P}(t_f - t_i)} | \Psi_i \rangle. \tag{B.1}$$

By inserting the complete sets (A.1) and (A.12), we obtain the path-integral representation. We note that

$$\begin{aligned}
&\left\langle p | \mathrm{H}_{F-P} | q \right\rangle, \\
&= \left\langle p \middle| G^{-\frac{1}{4}} \hat{\mathrm{p}}_A G^{\frac{1}{2}} G^{AB} \hat{\mathrm{p}}_B G^{-\frac{1}{4}} + \frac{1}{4} G^{AB} \frac{\partial S}{\partial q^A} \frac{\partial S}{\partial q^B}(\mathrm{q}) - \frac{1}{2} \nabla^A \frac{\partial S}{\partial q^A}(\mathrm{q}) \middle| q \right\rangle, \\
&= (p|q) \{ G^{AB} p_A p_B + i p_A \frac{\partial G^{AB}}{\partial q^B} - \frac{3}{16} G^{AB} \frac{1}{G^2} \frac{\partial G}{\partial q^A} \frac{\partial G}{\partial q^B} \\
&\quad + \frac{1}{4G} G^{AB} \frac{\partial^2 G}{\partial q^A \partial q^B} + \frac{1}{4G} \frac{\partial G}{\partial q^A} \frac{\partial G_{AB}}{\partial q^B} + \frac{1}{4} G^{AB} \frac{\partial S}{\partial q^A} \frac{\partial S}{\partial q^B}(q) \\
&\quad - \frac{1}{2} \nabla^A \frac{\partial S}{\partial q^A}(q) \}.
\end{aligned} \tag{B.2}$$

This gives the path-integral representation,

$$\begin{aligned}
Z[0] &= \int \mathcal{D}q \mathcal{D}p \left\langle \Psi_f | q_f \right\rangle \langle q_i | \Psi_i \rangle \exp\{ \int dt K_{F-P} \}, \\
K_{F-P} &= i \dot{q}^A p_A - G^{AB} p_A p_B - i p_A \frac{\partial G^{AB}}{\partial q^B} + \frac{3}{16} G^{AB} \frac{1}{G^2} \frac{\partial G}{\partial q^A} \frac{\partial G}{\partial q^B} \\
&\quad - \frac{1}{4G} G^{AB} \frac{\partial^2 G}{\partial q^A \partial q^B} - \frac{1}{4G} \frac{\partial G}{\partial q^A} \frac{\partial G_{AB}}{\partial q^B} - \frac{1}{4} G^{AB} \frac{\partial S}{\partial q^A} \frac{\partial S}{\partial q^B}(q) \\
&\quad + \frac{1}{2} \nabla^A \frac{\partial S}{\partial q^A}(q).
\end{aligned} \tag{B.3}$$

The expression is slightly complicated form due to the contribution which come from the operator ordering, however, it is rewritten in a simple form. To do this,

we introduce a momentum operator

$$\begin{aligned}
\Pi_A &= \mathrm{p}_A + \frac{i}{2} \frac{\partial S_{cl}}{\partial q^A}(\mathrm{q}), \\
&= e^{\frac{1}{2}S} \mathrm{p}_A e^{-\frac{1}{2}S}.
\end{aligned} \tag{B.4}$$

This has the same eigen-value as that of $\hat{\mathrm{p}}_A$, namely

$$\Pi_A G^{\frac{1}{4}} e^{\frac{1}{2}S(q)} |p) = p_A G^{\frac{1}{4}} e^{\frac{1}{2}S(q)} |p). \tag{B.5}$$

Then F-P equation takes the form

$$\mathrm{H}_{F-P} = G^{-\frac{1}{2}} \Pi_A G^{\frac{1}{2}} G^{AB} (\Pi_B + i \frac{\partial S}{\partial q^B}(\mathrm{q})). \tag{B.6}$$

Here, to avoid the complication of the operator ordering, we define a new inner product

$$(\phi_1 | \hat{O} | \phi_2) = \left\langle \phi_1 | G^{\frac{1}{4}} e^{-\frac{1}{2}S(q)} \hat{O} G^{-\frac{1}{4}} e^{\frac{1}{2}S(q)} | \phi_2 \right\rangle, \tag{B.7}$$

which gives the complete sets

$$\begin{aligned}
1 &= \int \mathcal{D}q |q)(q|, \\
&= \int \mathcal{D}p |p)(p|.
\end{aligned} \tag{B.8}$$

The path-integral representation of (3.1) is obtained by inserting the complete sets. We note

$$\begin{aligned}
&(p | \mathrm{H}_{F-P} | q), \\
&= (p|q) \{ G^{AB} p_A p_B - i p_A G^{AB} \frac{\partial S}{\partial q^B} + i p_A G^{-\frac{1}{2}} \frac{\partial}{\partial q^B} (G^{\frac{1}{2}} G^{AB}) \}.
\end{aligned} \tag{B.9}$$

Thus we obtain

$$\begin{aligned}
Z[0] &= \int \mathcal{D}q \mathcal{D}p \left\langle \Psi_f | q_f \right) (q_i | \Psi_i \rangle \exp\{ \int dt K_{eff} \}, \\
K_{eff} &= -G^{AB} \mathrm{p}_A p_B + i \mathrm{p}_A \{ \dot{q}^A + G^{AB} \frac{\partial S}{\partial q^B} - G^{-\frac{1}{2}} \frac{\partial}{\partial q^B} (G^{\frac{1}{2}} G^{AB}) \}.
\end{aligned} \tag{B.10}$$

In this expression and (B.3), the wave functions are related each other by

$$\langle q_i|\Psi_i\rangle = G^{\frac{1}{4}}(-\infty)e^{-\frac{1}{2}S(-\infty)}\langle q_i|\Psi_i\rangle,$$
$$\langle\Psi_f|q_f\rangle = G^{-\frac{1}{4}}(+\infty)e^{\frac{1}{2}S(+\infty)}\langle\Psi_f|q_f\rangle. \quad (B.11)$$

The stochastic action K_{eff} in (B.10) is the basis in (1). In (1), we implicitly assume the condition $\frac{\partial}{\partial q^B}(G^{\frac{1}{2}}G^{AB}) = 0$ which implies that the last term in K_{eff} vanishes.

APPENDIX C

In this appendix, we prove that the path-integral representation (B.3)(and (4.12)) is exactly equivalent to a stochastic action which is obtained from a slightly different operator formalism by Namiki and Yamanaka.[9]

Let us start with the Langevin equation[13]

$$\dot{q}^A = -G^{AB}\frac{\partial S_{cl}}{\partial q^B} + \frac{1}{\sqrt{G}}\frac{\partial}{\partial q^B}(\sqrt{G}G^{AB}) + h_I^A\eta^I,$$
$$\left\langle\eta^I(t)\eta^J(t')\right\rangle_\eta = 2\delta^{IJ}\delta(t-t'), \quad (C.1)$$

where

$$G^{AB} \equiv h_I^A h_I^B. \quad (C.2)$$

This equation is equivalent to the F-P equation (3.3) in the sense of the Ito's stochastic calculus as follows. In Ito's calculus, the fictitious time derivative of an expectation value of an observable $O(q)$ is given by

$$\frac{d}{dt}\langle O(q_\eta)\rangle_\eta$$
$$\equiv \int \mathcal{D}q\sqrt{G}O(q)\dot{P}(q,t).$$
$$= \left\langle\frac{\partial O}{\partial q^A}\dot{q}_\eta^A\right\rangle_\eta + \frac{1}{2}\left\langle\frac{\partial^2 O}{\partial q^A\partial q^B}\dot{q}_\eta^A\dot{q}_\eta^B\right\rangle_\eta,$$
$$= \left\langle-\frac{\partial O}{\partial q^A}\{G^{AB}\frac{\partial S_{cl}}{\partial q^B} - \frac{1}{\sqrt{G}}\frac{\partial}{\partial q^B}(\sqrt{G}G^{AB})\}\right\rangle_\eta + \left\langle\frac{\partial^2 O}{\partial q^A\partial q^B}\right\rangle_\eta\left\langle G^{AB}\right\rangle_\eta. \quad (C.3)$$

By expressing the expectation value in the r.h.s. of (c.3) with the distribution function, we find that (c.1) and (c.2) reproduces the F-P equation (3.3).

Although the Langevin equation (c.1) is not manifestly covariant, at a first sight, under the general coordinate transformation (3.2), we note that (c.1) is covariant because the term $\dot{q}^A$ is not a vector in the sense of the Ito's stochastic calculus. The term $\frac{1}{\sqrt{G}}\frac{\partial}{\partial q^B}(\sqrt{G}G^{AB})$ cancel the contribution which cones from the non-vectorial transformation property of $\dot{q}^A$. For this reason, we must be careful for the transformation property of $\dot{q}^A$ in the sense of the Ito' stochastic calculus if we use the invariance principle in constructing the Langevin equation.

In N-Y approach, a stochastic action is defined by (2.9) without the determinant factor of the unity (2.8). This implies the following stochastic action for the Langevin equation (c.1) and (c.2).

$$Z[0] = \int \mathcal{D}q\mathcal{D}\pi e^{\int K_{N-Y}dt},$$
$$K_{N-Y} \equiv -G^{AB}\pi_A\pi_B$$
$$+ i\pi_A\{\dot{q}^A + G^{AB}\frac{\partial S_{cl}}{\partial q^B} - \frac{1}{\sqrt{G}}\frac{\partial}{\partial q^B}(\sqrt{G}G^{AB})\}. \quad (C.4)$$

In this approach, note that the fictitious time derivative $\dot{q}^A$ still should be taken in the sense of Ito's calculus. The stochastic action is rewritten as

$$K_{N-Y} = ip_A\dot{q}^A + G^{AB}p_Ap_B - ip_B\frac{\partial G^{AB}}{\partial q^A} + \frac{3}{16}G^{AB}\frac{1}{G^2}\frac{\partial G}{\partial q^A}\frac{\partial G}{\partial q^B}$$
$$-\frac{1}{4G}\frac{\partial^2 G}{\partial q^A\partial q^B} - \frac{1}{4G}\frac{\partial G}{\partial q^A}\frac{\partial G_{AB}}{\partial q^B} - \frac{1}{4}G^{AB}\frac{\partial S}{\partial q^A}\frac{\partial S}{\partial q^B}(q) + \frac{1}{2}\nabla^A\frac{\partial S}{\partial q^A}(q)$$
$$-\frac{1}{2}\frac{d}{dt}(S - \frac{1}{2}lnG). \quad (C.5)$$

Here to derive the expression, we have introduced a new variable p_A by

$$\pi_A = p_A + \frac{i}{2}(\frac{\partial S_{cl}}{\partial q^A} - \frac{1}{2}\frac{1}{G}\frac{\partial G}{\partial q^A}), \quad (C.6)$$

and also used the relation in Ito's calculus

$$\frac{d}{dt}(S - \frac{1}{2}lnG)$$
$$= (\frac{\partial S_{cl}}{\partial q^A} - \frac{1}{2}\frac{1}{G}\frac{\partial G}{\partial q^A})\dot{q}^A + G^{AB}\frac{\partial^2}{\partial q^B}(\frac{\partial S_{cl}}{\partial q^B} - \frac{1}{2}\frac{1}{G}\frac{\partial G}{\partial q^B}). \quad (C.7)$$

The stochastic action (c.5) is exactly equivalent to (B.3) except the surface term contribution

$$G^{\frac{1}{4}}(+\infty)e^{-\frac{1}{2}S(+\infty)}G^{-\frac{1}{4}}(-\infty)e^{\frac{1}{2}S(-\infty)}. \qquad (C.8)$$

This exactly corresponds to the difference of the wave functions (B.11).

In this appendix, we showed that the N-Y approach is equivalent to the canonical formulation developed in this paper. Furthermore, we observed that the contribution from the P-S supersymmetry in SQ is properly included in the operator ordering in the canonical formulation.

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