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# BRS Symmetry in Stochastic Quantization of the Gravitational Field*

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* Talk given at the Conference on Probabilistic Methods in Quantum Field Theory and Quantum Gravity, Cargese, France - August 1989.

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# BRS Symmetry in Stochastic Quantization of the Gravitational Field*

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**Abstract**

We study stochastic quantization of gravity in terms of a BRS invariant canonical operator formalism. By introducing artificially canonical momentum variables for the original field variables, a canonical formulation of stochastic quantization is proposed in a sense that the Fokker-Planck hamiltonian is the generator of the fictitious time translation. Then we show that there exists a nilpotent BRS symmetry in an enlarged phase space for gravity ( in general, for the first-class constrained systems ). The stochastic action of gravity includes explicitly an unique DeWitt's type superspace metric which leads to a geometrical interpretation of quantum gravity analogous to nonlinear $\sigma$-models.

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## 1. Introduction

Stochastic Quantization in short, SQ was first introduced by Parisi and Wu[1] as an interesting alternative quantization method.[2] It is formulated in terms of a Langevin equation and the Fokker-Planck ( F-P ) equation. There are, in addition, some other formulations of SQ[3][4][5] which are motivated to clarify the symmetry properties such as renormalizability, unitarity, spontaneous symmetry breaking and so on. Especially, a stochastic action is an useful tools not only to prove the equivalence between SQ and the ordinary quantization but also to apply the field theoretical methods, for example, the renormalizability in SQ has been discussed in this context.[6][7][8][9]

In gauge theories, the BRS symmetry[10][11] in the Faddeev-Popov effective action[12] is one of the most important symmetries for the consistent quantization ( especially, for renormalizability and unitarity ). Recently, the BRS symmetry has been introduced in the stochastic action for Yang-Mills field and the Ward-Takahashi identity in SQ is discussed.[13][14] However, there remains many problems, except for the renormalizability, such as the unitarity ( how to define the *S-matrix* in SQ ).

In this short note, we describe the BRS symmetry in SQ of gravity which is realized in an artificially enlarged phase space for a canonical formulation of SQ. We explain the geometric interpretation of quantum gravity. It is clarified by formulating SQ of gravity as a second-class constrained system. The BRS invariant F-P hamiltonian and the canonical operator formalism provides a possible basis of the BRS cohomology, we expect it also important for the unitarity problem in the context of SQ.

To illustrate a general structure of the Langevin equation and F-P equation for gravity, let us first consider a stochastic process of a quantum mechanical system on a Riemannian manifold ( or equivalently, a constrained surface ) $\{q^A, G_{AB}\}$. We assume that the drift force is given by the gradient of a scalar function $S$ on the manifold. The Langevin equation is given by[15]

$$\dot{q}^A = -G^{AB}\partial_B S + \frac{1}{\sqrt{G}}\partial_B(G^{AB}\sqrt{G}) + \xi^A. \tag{1.1}$$

The *noise variables* $\xi^A$ *is defined by*

$$\begin{aligned} \xi^A &\equiv \mathrm{h}^A_I \eta^I, \\ \left\langle \eta^I(t)\eta^J(t')\right\rangle &= 2\delta^{IJ}\delta(t-t'), \end{aligned} \tag{1.2}$$

where $\mathrm{h}^A_I$ satisfies $G^{AB} = \mathrm{h}^A_I \mathrm{h}^B_I$. The Langevin equation is transformed covariantly under the general coordinate transformation. The non-covariant term in (1.1) is necessary for the covariance because, in Ito's stochastic calculus[16] , $\dot{q}^A$ is not a vector quantity. In fact, the F-P hamiltonian equivalent to (1.1) is written in the manifestly covariant form

$$\dot{P}(q,t) = \frac{1}{\sqrt{G}}\partial_A G^{AB}\sqrt{G}(\partial_B + \partial_B S)P(q,t), \tag{1.3}$$

for the probability distribution functional $P(q,t)$ which is a scalar density. (1.1) and (1.3) are also the basis of the *second-class constrained systems* ( *nonlinear* $\sigma$-models ). For SQ of gravity, the analogous structure of the Langevin equation for gravity was first discussed in Ref.17. In a formal sense, the Faddeev-Popov effective action of gravity in the infinite fictitious time limit has been discussed in a slightly different context.[18] We here show that the Langevin equation of gravity really possesses the structure of (1.1) by formulating a pair of the Langevin equations with a second-class constraint.[19] Our approach is an analogue of the nonlinear $\sigma$-model in which a constraint defines a ( constrained ) surface and there appears an induced metric on the surface. The BRS structure of the F-P hamiltonian is also studied in this context.[20]



## 2. SQ of Gravity as a Second-Class Constrained System

We first introduce two independent field variables, $\hat{g}_{\mu\nu}(x)$ and $\hat{h}^{\mu\nu}(x)$, Then we show that a second-class constraint defines a constrained surface, namely, the configuration space of the gravitational field, and an unique superspace metric is induced from the consistency condition of the constraint. In this procedure, the choice of the field variable is important. We specify it by the requirement that the field variable defines the BRS invariant path-integral measure ( it is also invariant under the general coordinate transformation ). For D-dimensional gravity, the field variables which define the BRS invariant path-integral measure are given by[21]

$$\begin{aligned} \hat{g}_{\mu\nu} &\equiv g^k g_{\mu\nu}; \qquad k = \frac{D-4}{4D}, \\ \hat{g}^{\mu\nu} &\equiv g^l g^{\mu\nu}; \qquad l = \frac{D+4}{4D}. \end{aligned} \tag{2.1}$$

*Then we require the constraint as follows*

$$\hat{h}^{\mu\lambda}\hat{g}_{\lambda\nu} - \hat{g}^{\frac{2}{D}}\delta^\mu_\nu = 0, \tag{2.2}$$

where $\sqrt{g} = \hat{g}^{\frac{2}{D}}$. This leads

$$\hat{h}^{\mu\nu}(x) = \hat{g}^{\mu\nu}(x), \tag{2.3}$$

where $\hat{g}^{\mu\nu}$ is defined in (2.1).

For these independent variables, $\hat{g}_{\mu\nu}(x)$ and $\hat{h}^{\mu\nu}(x)$, we assume a pair of the Langevin equations with the constraint (2.2)

$$\begin{aligned} \dot{\hat{g}}_{\mu\nu} &= -\gamma_1 \frac{\Delta S_{cl}}{\Delta \hat{h}^{\mu\nu}} + \hat{\eta}_{\mu\nu}, \\ \dot{\hat{h}}^{\mu\nu} &= -\gamma_2 \frac{\Delta S_{cl}}{\Delta \hat{g}_{\mu\nu}} + \hat{\eta}^{\mu\nu}, \end{aligned} \tag{2.4}$$

where $\gamma_1$ and $\gamma_2$ are some constants. The classical action $S_{cl}$ consists of the Einstein action and the second-class constraint

$$S_{cl} = S_E + \int d^D x\ \phi^\nu_\mu(\tilde{h}^{\mu\lambda}\tilde{g}_{\lambda\nu} - \tilde{g}^{\frac{1}{D}}\delta^\mu_\nu). \tag{2.5}$$

Here the auxiliary field $\phi^\mu_\nu$ is introduced as a Lagrange multiplier field of the constraint (2.2). In the derivative ( or variation ) $\Delta$, the variables $\tilde{g}_{\mu\nu}$ and $\tilde{h}^{\mu\nu}$ are regarded to be independent each other and

$$\frac{\Delta\tilde{g}_{\rho\sigma}(x')}{\Delta\tilde{g}_{\mu\nu}(x)} = \frac{\Delta\tilde{h}^{\mu\nu}(x')}{\Delta\tilde{h}^{\rho\sigma}(x)} = \frac{1}{2}(\delta^\mu_\rho\delta^\nu_\sigma + \delta^\mu_\sigma\delta^\nu_\rho)\delta^D(x;x'). \tag{2.6}$$

The pair of the Langevin equations (2.4) is invariant under the fictitious time independent general coordinate transformation in which the transformation parameter is <u>independent</u> of the fictitious time $t$, provided that the noise fields $\tilde{\eta}_{\mu\nu}$ and $\tilde{\eta}^{\mu\nu}$ are transformed by the same transformation rules as those of $\tilde{g}_{\mu\nu}$ and $\tilde{g}^{\mu\nu}$, respectively.

The correlations of the white noise variables are given by

$$\begin{aligned}\langle\tilde{\eta}^{\mu\nu}(x,t)\tilde{\eta}_{\rho\sigma}(x',t')\rangle &= \frac{2}{\gamma_3}(\delta^\mu_\rho\delta^\nu_\sigma + \delta^\mu_\sigma\delta^\nu_\rho)\delta^D(x;x')\delta(t-t'),\\ \langle\tilde{\eta}^{\mu\nu}(x,t)\tilde{\eta}^{\rho\sigma}(x',t')\rangle &= \langle\tilde{\eta}_{\mu\nu}(x,t)\tilde{\eta}_{\rho\sigma}(x',t')\rangle = 0.\end{aligned} \tag{2.7}$$

$\gamma_1$, $\gamma_2$ in (2.4) and $\gamma_3$ in (2.7) are suitably determined. Note that the transformation property of the white noises are consistent with the correlation (2.7).

Now it is shown that the pair of the Langevin equations (2.4) is reduced to an equation by showing that the multiplier field $\phi^\mu_\nu$ of the constraint is eliminated by the consistency condition of the constraint. This implies that the constraint (2.2) is a second-class one. In fact, by the consistency condition

$$\frac{\partial}{\partial t}\left(\tilde{h}^{\mu\lambda}\tilde{g}_{\lambda\nu} - \tilde{g}^{\frac{1}{D}}\delta^\mu_\nu\right) = 0, \tag{2.8}$$

and the constraint (2.2), we obtain[19]

$$\begin{aligned}\dot{\tilde{g}}_{\mu\nu} &= \alpha\frac{\delta}{\delta\tilde{g}^{\mu\nu}}S_E + \tilde{\xi}_{\mu\nu},\\ \tilde{\xi}_{\mu\nu} &= \frac{1}{\gamma_1+\gamma_2}\left(\gamma_2\tilde{\eta}_{\mu\nu} - \gamma_1\hat{G}_{\mu\nu\rho\sigma}\tilde{\eta}^{\rho\sigma}\right),\end{aligned} \tag{2.9}$$



where $\hat{G}_{\mu\nu\rho\sigma}$ is given by

$$\hat{G}_{\mu\nu\rho\sigma} \equiv \frac{1}{2}\tilde{g}^{-\frac{2}{D}}(\tilde{g}_{\mu\rho}\tilde{g}_{\nu\sigma} + \tilde{g}_{\mu\sigma}\tilde{g}_{\nu\rho} - \frac{4}{D}\tilde{g}_{\mu\nu}\tilde{g}_{\rho\sigma}). \tag{2.10}$$

We here chose the constants $-\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2} = \alpha$. This implies that the Langevin equation of gravity really possesses the structure of (1.1). We also note that the corresponding term to the non-covariant term in (1.1) vanishes in this case because the tensor satisfies a "coordinate condition "

$$\frac{\delta}{\delta\tilde{g}_{\rho\sigma}}\{\hat{G}_{\mu\nu\rho\sigma}\delta^D(x;x')\} = 0. \tag{2.11}$$

It also satisfies the condition; $\det\{\hat{G}_{\mu\nu\rho\sigma}\delta^D(x;x')\} = \text{constant}$, due to the appropriate choice of the field variable (2.1). The correlation of the noise variable $\tilde{\xi}_{\mu\nu}$ is evaluated in the sense of Ito's stochastic calculus as follows

$$\begin{aligned}\left\langle\tilde{\xi}_{\mu\nu}(x,t)\tilde{\xi}_{\rho\sigma}(x',t')\right\rangle &= -\frac{\gamma_1\gamma_2}{(\gamma_1+\gamma_2)^2}\Big\{\left\langle\hat{G}_{\mu\nu\alpha\beta}\right\rangle\left\langle\tilde{\eta}_{\rho\sigma}\tilde{\eta}^{\alpha\beta}\right\rangle\\ &\qquad + \left\langle\hat{G}_{\rho\sigma\alpha\beta}\right\rangle\left\langle\tilde{\eta}_{\mu\nu}\tilde{\eta}^{\alpha\beta}\right\rangle\Big\}\\ &= \frac{2}{\beta}\left\langle\hat{G}_{\mu\nu\rho\sigma}\right\rangle\delta^D(x;x')\delta(t-t'),\end{aligned} \tag{2.12}$$

where $\gamma_3$ in (2.7) is chosen to be $\gamma_3 = \frac{2\alpha\beta}{\gamma_1+\gamma_2}$.

The equations in (2.4) are not independent each other after eliminating the Lagrange multiplier field $\phi^\mu_\nu$, we also have

$$\begin{aligned}\dot{\tilde{g}}^{\mu\nu} &= \alpha\frac{\delta}{\delta\tilde{g}_{\mu\nu}}S_E + \tilde{\xi}^{\mu\nu},\\ \left\langle\tilde{\xi}^{\mu\nu}(x,t)\tilde{\xi}^{\rho\sigma}(x',t')\right\rangle &= \frac{2}{\beta}\left\langle\hat{G}^{\mu\nu\rho\sigma}\right\rangle\delta^D(x;x')\delta(t-t'),\end{aligned} \tag{2.13}$$

where

$$\hat{G}^{\mu\nu\rho\sigma} \equiv \frac{1}{2}\tilde{g}^{-\frac{2}{D}}(\tilde{g}^{\mu\rho}\tilde{g}^{\nu\sigma} + \tilde{g}^{\mu\sigma}\tilde{g}^{\nu\rho} - \frac{4}{D}\tilde{g}^{\mu\nu}\tilde{g}^{\rho\sigma}). \tag{2.14}$$

From (2.9) and (2.13), we find that the choice of the superspace metric is properly realized in terms of the pair of the Langevin equations coupled with white noises, namely, the metric tensor $G^{AB}$ is given by

$$\{G^{AB}\} = \{\tilde{G}_{\mu\nu\rho\sigma}\delta^D(x;x')\},$$
$$\{G_{AB}\} = \{\tilde{G}^{\mu\nu\rho\sigma}\delta^D(x;x')\}. \tag{2.15}$$

The procedure we have illustrated above is equivalent to that of non-linear $\sigma$-model, for example $O(N)$ non-linear $\sigma$-model. We first consider an enlarged configuration space which is spanned by $\{\tilde{g}_{\mu\nu}\}$ and $\{\tilde{h}^{\mu\nu}\}$. Then we require a second-class constraint. it defines the induced metric $\tilde{G}_{\mu\nu\rho\sigma}\delta^D(x;x')$ on the configuration space of the gravitational field which is parametrized by the coordinate $\{\tilde{g}_{\mu\nu}\}$.

## 3. BRS Symmetry and the Invariant Fokker-Planck Hamiltonian for Gravity

Here, we mainly investigate the BRS structure in SQ of gravity by constructing the nilpotent BRS transformation and the BRS invariant F-P hamiltonian in an enlarged phase space of SQ.[18]

We first define an enlarged phase space to realize a canonical formulation in which the F-P hamiltonian is the generator of the fictitious time translation. By introducing the canonical momentum variable to the gravitational field, we assume the commutation relation

$$[\mathrm{p}^{\mu\nu}(x), \mathrm{g}_{\rho\sigma}(x')] = -i\frac{1}{2}(\delta^\mu_\rho\delta^\nu_\sigma + \delta^\mu_\sigma\delta^\nu_\rho)\delta^4(x;x'). \tag{3.1}$$

In the following, we consider the 4-dimensional gravity case and the extension to the D-dimensional case is straightforward. By the usual procedure, we obtain the F-P hamiltonian which gives the stochastic "Schrödinger equation " equivalent to the Langevin equation (2.9)

$$\mathrm{H}_{F-P} = -\int d^4x G_{\mu\nu\rho\sigma}\{\mathrm{p}^{\mu\nu}\mathrm{p}^{\rho\sigma} + \frac{1}{4}\frac{\delta S_E}{\delta g_{\mu\nu}(x)}\frac{\delta S_E}{\delta g_{\rho\sigma}(x)} - \frac{1}{2}\frac{\delta^2 S_E}{\delta g_{\mu\nu}(x)\delta g_{\rho\sigma}(x)}\}. \tag{3.2}$$

We note that this is an hermitian operator. The last term comes from the integration of the Grassmanian variables to realize the Parisi-Sourlas supersymmetry in SQ.[11] It includes a singular term with $\delta^4(0)$ and its derivative. Thus if we use an appropriate regularization method, such as the dimensional one, it may vanishes.



Since the F-P hamiltonian is invariant under the space-time general coordinate transformation provided the transformation parameter is <u>independent</u> of the fictitious time, there exists a conserved charge, namely the generator of the general coordinate transformation. It is given by

$$\mathrm{Q}_{g.c.}(\xi) = \int d^4x\{\partial_\mu\xi^\rho \mathrm{g}_{\rho\nu} + \partial_\nu\xi^\rho \mathrm{g}_{\mu\rho} + \xi^\rho\partial_\rho \mathrm{g}_{\mu\nu}\}\mathrm{p}^{\mu\nu}(x), \tag{3.3}$$

where $\xi_\mu$ is the transformation parameter. We note that the transformation of the canonical momentum generated by (3.3) is the same as that of a tensor <u>density</u> with upper indices. It preserves the commutation relation (3.1) unchanged. The charge commutes with the F-P hamiltonian and satisfies the well-known algebra in gravity.

$$[\mathrm{Q}_{g.c.}(\xi_1), \mathrm{Q}_{g.c.}(\xi_2)] = -i\mathrm{Q}_{g.c}(v), \tag{3.4}$$

where

$$v^\mu \equiv [\xi_1 \times \xi_2]^\mu,$$
$$= \xi_2^\rho\partial_\rho\xi_1^\mu - \xi_1^\rho\partial_\rho\xi_2^\mu. \tag{3.5}$$

$[\xi_1 \times \xi_2]$ is just the Lee derivative of $\xi_1$ with respect to $\xi_2$. The algebra is sufficient for the existence of the BRS symmetry in the phase space.

The BRS transformation in SQ is constructed from the general coordinate transformation by replacing

$$\xi_\mu \to i\epsilon c_\mu, \tag{3.6}$$

where $\epsilon$ is a Grassmanian constant. $c_\mu$ is the Faddeev-Popov ghost field. We here introduce its canonical conjugate momentum, F-P anti-ghost field, to realize the BRS symmetry in the enlarged phase space by assuming that they satisfy the anti-commutation relation

$$\{c^\mu(x), \bar{c}_\nu(x')\} = -\delta^\mu_\nu \delta^4(x;x'). \tag{3.7}$$

Here we comment on the derivation of the BRS transformation. The transformations of $g_{\mu\nu}$ and $p^{\mu\nu}$ are obtained by the replacement (3.6). The transformation property of the F-P ghost $c^\mu$ is determined from the nilpotency condition of the transformations of $g_{\mu\nu}$ and $p^{\mu\nu}$. The transformation for $\bar{c}_\mu$ is uniquely determined as follows. In the present construction of the nilpotent BRS transformation, we start from the phase space $(g_{\mu\nu}, p^{\mu\nu})$. By regarding the F-P ghost and anti-ghost as the canonical conjugate pair, the phase space is enlarged to $(g_{\mu\nu}, p^{\mu\nu}, c^\mu, \bar{c}_\nu)$. This means that there exists a charge which generates the BRS transformation of $g_{\mu\nu}, p^{\mu\nu}$ and $c^\mu$ on the basis of the commutation relations (3.1) and (3.7). Thus it automatically gives the transformation of $\bar{c}_\mu$. we obtain[20]

$$\begin{aligned}
\delta_{BRS} g_{\mu\nu} &= i\epsilon\{\partial_\mu c^\rho g_{\rho\nu} + \partial_\nu c^\rho g_{\mu\rho} + c^\rho \partial_\rho g_{\mu\nu}\},\\
\delta_{BRS} p^{\mu\nu} &= i\epsilon\{-\partial_\rho c^\mu p^{\rho\nu} - \partial_\rho c^\nu p^{\rho\mu} + \partial_\rho(c^\rho p^{\mu\nu})\},\\
\delta_{BRS} c^\mu &= i\epsilon c^\rho \partial_\rho c^\mu,\\
\delta_{BRS} \bar{c}_\mu &= \epsilon\{-2\partial_\rho(g_{\mu\sigma}p^{\rho\sigma}) + \partial_\mu g_{\rho\sigma}p^{\rho\sigma} + i(\partial_\rho(c^\rho \bar{c}^\mu) + \partial_\mu c^\rho \bar{c}_\rho)\}.
\end{aligned} \tag{3.8}$$

On the basis of this construction, the nilpotency of the BRS transformation is *nontrivial, however, it is easily confirmed by an explicit calculation.* The BRS transformation is also derived from a consistent truncation of an extended BRS transformation in a (D+1)-dimensional formulation of SQ in which the multiplier field of the constraints $Q_{g.c.}$ in (3.3) and the Nakanishi-Lautorup field of a gauge fixing are introduced.[22] These auxiliary fields define a (D+1)-dimensional gauge multiplet with original dynamical variable, however, consistently truncated in a special class of gauge fixings leaving the nilpotent BRS transformation (3.8). Note that (3.6) is independent of the choice of gauge fixing in (D+1)-dimensional BRS invariant formulation. We also note that, for the first-class constrained systems, in general there exsists the structure of the BRS symmetry described here.[23] The similar structure of the BRS transformation is also discussed in a slightly different context by Batalin-Fradkin-Vilkovisky.[24]



The BRS charge which generates (3.8) is given by

$$\begin{aligned}
Q_{BRS} = &-\int d^4x(\partial_\mu c^\rho g_{\rho\nu} + \partial_\nu c^\rho g_{\mu\rho} + c^\rho \partial_\rho g_{\mu\nu})p^{\mu\nu}\\
&+ i\int d^4x(\partial_\mu c^\rho)c^\mu \bar{c}_\rho.
\end{aligned} \tag{3.9}$$

where we define $\delta_{BRS} \equiv \epsilon\delta'_{BRS}$. The BRS charge satisfies the nilpotency condition

$$\{Q_{BRS}, Q_{BRS}\} = 0. \tag{3.10}$$

We note this charge is hermitian. The nilpotency provides a possible basis of the BRS cohomology.

The BRS invariant Fokker-Planck hamiltonian is given by

$$H_{BRS} = H_{F\text{-}P} + \{Q_{BRS}, \bar{c}_\mu \chi^\mu\}, \tag{3.11}$$

where $\chi^\mu$ is a gauge fixing function which is a function of only $q^A$. $H_{BRS}$ is hermitian provided that we choose an hermitian functional $\chi^\mu$. It defines the BRS invariant "Schrödinger equation "

$$|\dot{\Psi}\rangle = -H_{BRS}|\Psi\rangle, \tag{3.12}$$

which is the basis of the present BRS invariant formulation.

(3.11) is also invariant under the scale transformation of the ghost fields.

$$\begin{aligned}
\delta_{gh} c^\mu &= \rho c^\mu,\\
\delta_{gh} \bar{c}_\mu &= -\rho \bar{c}_\mu.
\end{aligned} \tag{3.13}$$

The transformation is generated by the ghost number charge

$$Q_{gh} \equiv i\frac{1}{2}(c^\mu \bar{c}_\mu - \bar{c}_\mu c^\mu). \tag{3.14}$$

The BRS charge and the ghost number charge satisfy the well-known algebra

$$\begin{aligned} [Q_{gh}, Q_{gh}] &= 0, \\ [Q_{gh}, Q_{BRS}] &= -iQ_{BRS}. \end{aligned} \tag{3.15}$$

From the algebra, one may require the Kugo-Ojima's subsidiary condition[25],

$$Q_{BRS}|\Psi_{phys}\rangle = 0, \tag{3.16}$$

to specify the physical subsector in the whole Hilbert space. However we note that it may not be enough to define an unitary S-matrix in the context of SQ because we consider the asymptotic states with respect to the fictitious time. Thus, in addition to the subsidiary condition, it is necessary to specify a boundary condition on the true time coordinate in space-time. In this sense, the problem of the unitarity is remained yet to be solved.

We also obtain the path-integral representation of the "vacuum transition amplitude " which gives the BRS invariant stochastic action ( see Ref.20).

## 4. Discussions

In this note, we described the BRS invariant operator formalism for SQ of gravity. We obtained the BRS invariant F-P hamiltonian which is realized in an artificially enlarged phase space as the generator of the fictitious time translation. In the approach, SQ is recognized as a (D+1)-dimensional canonical hamilton formalism. The BRS symmetry is realized in the enlarged phase space ( $g_{\mu\nu}, p^{\mu\nu}, c^{\mu}, \bar{c}_{\nu}$ ) for 4-dimensional gravity.

We also showed that there exists a non-trivial metric tensor in the configuration space of the gravitational field. Our starting point is a pair of the Langevin equations coupled with white noises. We introduced two independent variables of the gravitational fields, $\hat{g}_{\mu\nu}$ and $\hat{g}^{\mu\nu}$. The choice of the independent variables is determined such that these variables give a BRS invariant path-integral measure.



In the configuration space $\{\tilde{g}_{\mu\nu}, \tilde{g}^{\rho\sigma}\}$, the constraint (2.2) define a surface on which $\{\tilde{g}_{\mu\nu}\}$ is a natural coordinate and the unique superspace metric $\tilde{G}^{\mu\nu\rho\sigma}(x)\delta^{D}(x;x')$ (2.14) is induced. The description is an analogue of the non-linear $\sigma$-model case. It is also important to note that the variation of the gravitational field under the space-time general coordinate transformation is a Killing vector in the configuration space. This implies that in this respect gravity is a nonlinear $\sigma$-model; the target space is the superspace in which the space-time general coordinate transformation specifies the direction of the Killing vector.[26] The geometric interpretation leads to the following analogy between the nonlinear $\sigma$-model and quantum gravity. In *nonlinear $\sigma$-models, in general, although it is not renormalizable in 4-dimensional* space-time, the renormalization is recognized as a deformation of the geometry, namely, a modification of the metric tensor in the target space. The present formulation suggests that the renormalization in SQ of gravity may be interpreted as a deformation of geometry in superspace, namely, a change of the superspace metric $\tilde{G}_{\mu\nu\rho\sigma}(x)\delta^{D}(x;x')$.

There remains many open questions to be solved. Especially, it should be clarified that how the BRS invariance leads to the unitary S-matrix in SQ, or equivalently how we use the BRS symmetry in constructing the S-matrix. In this context, it is also important to clarify the BRS cohomology by requiring the Kugo-Ojima's subsidiary condition in the present BRS invariant formulation. We hope that the BRS symmetry in SQ described here is useful to solve the problem.

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