EVALUATION OF HIGHER ORDER CORRECTIONS TO JET PRODUCTION

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ABSTRACT

A complete \( O(\alpha_s^3) \) calculation of all parton parton scattering subprocesses contributing to inclusive production of a hadron or a jet within a small opening angle \( \delta \) at large transverse momenta is presented. Numerical results for jet production within \( \delta = .2 \) at present \( pp \) colliders are given.

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Introduction
Hadron jets from hadronic colliders are an important tool to test in a quantitative way the QCD improved parton model. Moreover present and future data will provide eventual evidence for new physics (SUSY, compositeness, strong Higgs sector...) only if QCD radiative corrections are under reasonable control. Although an important ambiguity is due to the experimental uncertainty in the measurement of the jet energy in the calorimeter, theoretical uncertainties, like the scales of parton densities and of the strong coupling constant, can be reduced by performing a complete $O(\alpha_s^3)$ calculation of the partonic subprocess contributing to inclusive production of a hadron, i.e.:

$$H_1(K_1) + H_2(K_2) \rightarrow H_3(K_3) + X \quad (1)$$

or jet production

$$H_1(K_1) + H_2(K_2) \rightarrow \text{jet}(P) + X \quad (2)$$

The jet is defined\(^{1}\) as an arbitrary set of hadrons within a small cone of opening angle $\delta$ and with a fixed total momentum.

At the Born level, i.e. $O(\alpha_s^2)$, the calculation involves a two body scattering and does not depend on the cone size. In the early 80's preliminary evaluations\(^{1,2}\) at next to leading order involved only scattering of non identical quarks. Based on the calculation of full order $O(\alpha_s^3)$ matrix elements\(^3\) for all $2\rightarrow 2$ and $2\rightarrow 3$ parton subprocesses in $n = 4-2\epsilon$ dimensions our group\(^4\) has computed the next to leading order corrections for all scattering processes. The extension to general jet kinematical configurations ($\delta$ not small) is in progress whereas results restricted to gluons concerning inclusive jet cross-section using a different jet definition in terms of $R = \sqrt{\Delta \eta^2 + \Delta \phi^2}$ are available\(^5\).

Calculational Method

The method followed involves first algebraic manipulations using MACSYMA and/or REDUCE. Starting from the expressions\(^3\) of the matrix elements squared in $n$ dimensions for the $2\rightarrow 3$ parton subprocesses one performs the phase space integrations of the real subprocesses, then cancels the $1/e^2$ divergences by adding the virtual corrections. The leftover $1/e$ terms, corresponding to mass singularities, are absorbed into the structure and fragmentation functions beyond the leading order for one hadron inclusive production. In the case of jet production the collinear singularities associated to the radiation emitted by the outgoing partons are automatically cancelled by adding the contributions of one and two partons produced within the cone. The collinear singularity associated with the incoming parton line $p'_1$ is proportional to:

$$\frac{\alpha_s}{2\pi} \int_0^1 \frac{dx}{x} \sigma_{p'_1 p p_1 \rightarrow p_1}$$

where
\[ H_{\text{pip'}}(x) = \frac{P_{\text{pip'}}(x)}{x} \left\{ -\frac{1}{e} \ln 4\pi - \gamma_E + \ln \frac{M^2}{\mu^2} \right\} + f_{\text{pip'}}^*(x) \]

where \( P_{\text{pip'}} \) is the Altarelli Parisi kernel\(^6\). \( M \) is the factorization scale at which structure functions are evolved and \( \mu \) is the renormalization scale at which the running coupling constant is taken. The finite \( O(\alpha_s) \) corrections \( f_{\text{pip'}} \) for structure functions and \( d_{\text{pip'}} \) for fragmentation functions) are only available for quarks\(^7\). Since we have freedom for gluons we can either put \( f_{\text{pip'}} = d_{\text{pip'}} = 0 \) (except for \( p_i = p'_i = \text{quark} \)) or incorporate the relevant kinematical factors found for quarks\(^7\) i.e. take \( P_{\text{pip'}}(x) \ln \left\{ \frac{1-x}{x} \right\} \) for \( f_{\text{pip'}} \) and \( P_{\text{pip'}}(x) \ln \left\{ (1-x)x^2 \right\} \) for \( d_{\text{pip'}} \). Of course the evolution of distributions should be done using the same prescription.

Finally the cross-section can be expressed in terms of convolution of partonic cross-sections free of singularities with evolved structure functions. The detailed expressions can be found in ref. 4).

**Numerical Results**

We will discuss some phenomenological applications for proton antiproton collisions at \( \sqrt{s} = 630 \text{ GeV} \) and 1.8 TeV for \( \theta_{\text{cm}} = 90^\circ \) using the Duke Owens set of structure functions for \( \Lambda = 200 \text{ MeV} \). As it is well known gluon effects play an important role at these energies. The two questions we will address are the absolute size of the \( O(\alpha_s^3) \) terms and the resultant theoretical uncertainty of the \( O(\alpha_s^3) + O(\alpha_s^4) \) cross-sections depending upon the renormalization and factorization mass scales.

It should be stressed that in contrast to Drell Yan processes where the Born level is a QED or electroweak one (i.e. independent of \( \alpha_s \)) for jet production, since the Born cross-section is an \( O(\alpha_s^2) \) process, it may be misleading to express the size of next to leading corrections in terms of \( K \) factor.

The dependence on the jet size is logarithmic. Therefore the cross-section diverges to negative infinity as \( \delta \to 0 \). When \( \delta \) becomes smaller the corrections become larger and negative, meaning that higher order perturbative corrections cannot be neglected.

We give in fig.1 (resp. fig.2) the scaled jet cross-section \( p_T \frac{d\sigma}{dp_T} \) as a function of \( \eta = \frac{2p_T}{\sqrt{s}} \) \( (p_T \) being the transverse momentum of the jet) choosing \( \mu = M = p_T/2 \) (resp. \( \mu = M = 2p_T \)) for \( \delta = .2 \) at \( \sqrt{s} = 630 \text{ GeV} \). A comparison of the two figures shows that when higher order corrections are included the cross-section is quite stable. This was not the case for the Born cross-section since for a scale \( p_T/2 \) the results were higher by a factor 2-3 compared to the scale \( 2p_T \). We have checked that these results are not affected if we take the Diemoz et al. set of structure functions\(^9\). Similar conclusions hold at Tevatron energy. The authors of ref. 5) have got the same result for the pure gluonic case. For one hadron inclusive cross-section the sensitivity to \( \mu \) and \( M \) is also reduced in a similar way when \( O(\alpha_s^3) \) are included (see fig.3). The "preferred scale" at the Born level seems to be \( \mu = M = 2p_T \) for which the \( K \) factor is of the order of one.
Fig. 1: The scaled jet cross-section $p_T^4 E \frac{d\sigma}{dp_T^2}$ as a function of $\eta = \frac{2p_T}{\sqrt{s}}$ at $\sqrt{s} = 630$ GeV for $\mu = M = \frac{p_T}{2}$. Dashed curve: Born cross-section. Full curve: $O(\alpha_s^2 + \alpha_s^3)$ prediction for all subprocesses. Small dashed curve: $O(\alpha_s^2 + \alpha_s^3)$ prediction for all quark quark subprocesses. Dot-dashed curve: $O(\alpha_s^2 + \alpha_s^3)$ prediction for quark gluon subprocess. Long dashed curve: $O(\alpha_s^2 + \alpha_s^3)$ prediction for gluon gluon subprocess.

Fig. 2: Same caption as for fig. 1 for $\mu = M = 2p_T$. 
Fig. 3: The inclusive cross-section $P_T^A \frac{d\sigma}{d^3p}$ as a function of $\eta = \frac{2p_T}{\sqrt{s}}$ at $\sqrt{s} = 630$ GeV for $q\bar{q} \rightarrow q\bar{q}+X(a)$, $qg \rightarrow g+X(b)$ and $gg \rightarrow g+X(c)$. Dashed curves: Born predictions for $\mu = M = \frac{3p_T}{4}$ (upper curve) and $\mu = M = 2p_T$ (lower curve). Full curves: $O(\alpha_s^2 + \alpha_s^3)$ for $\mu = M = \frac{3p_T}{4}$ (upper curve) and $\mu = M = 2p_T$ (lower curve).
The contribution due to gluon gluon and quark gluon subprocesses is dominant at small $\eta$ as expected but the latest one is never negligible up to $\eta = .5$. The choice for finite next to leading terms in gluon structure functions affects mainly gluon gluon subprocesses at large $p_T$. Nevertheless since the gluon gluon contribution is smaller by a few orders of magnitude the scaled jet cross-section is poorly sensitive to this theoretical uncertainty.

Conclusion

We have presented the results of a full calculation of $O(\alpha_s^3)$ radiative corrections to high $p_T$ parton parton scattering processes. The theoretical uncertainty in the prediction of the jet cross-section has been sizeably reduced, showing an excellent stability of $O(\alpha_s^2) + O(\alpha_s^3)$ cross-section upon change of the renormalization and factorization mass scales in the range $p_T/2 \leq \mu, M \leq 2p_T$. The next to leading corrections are reasonably small.

References